Plane Sweep Stereo

$$\underset{3\times1}{X_c} \leftarrow \underset{3\times4}{[\overset{3\times3}{R} \mid \overset{3\times1}{t}]}\ \underset{4\times1}{X_w}$$

extrinsics

Setting:

intrinsics 3x3

Cameras are not rectified

Cameras are calibrated ($K_L, [R;t]_L, [R;t]_R, K_R$ are known)

Standard Stereo:

for each pixel
- for each disparity
  - compute match cost

Plane Sweep Stereo:

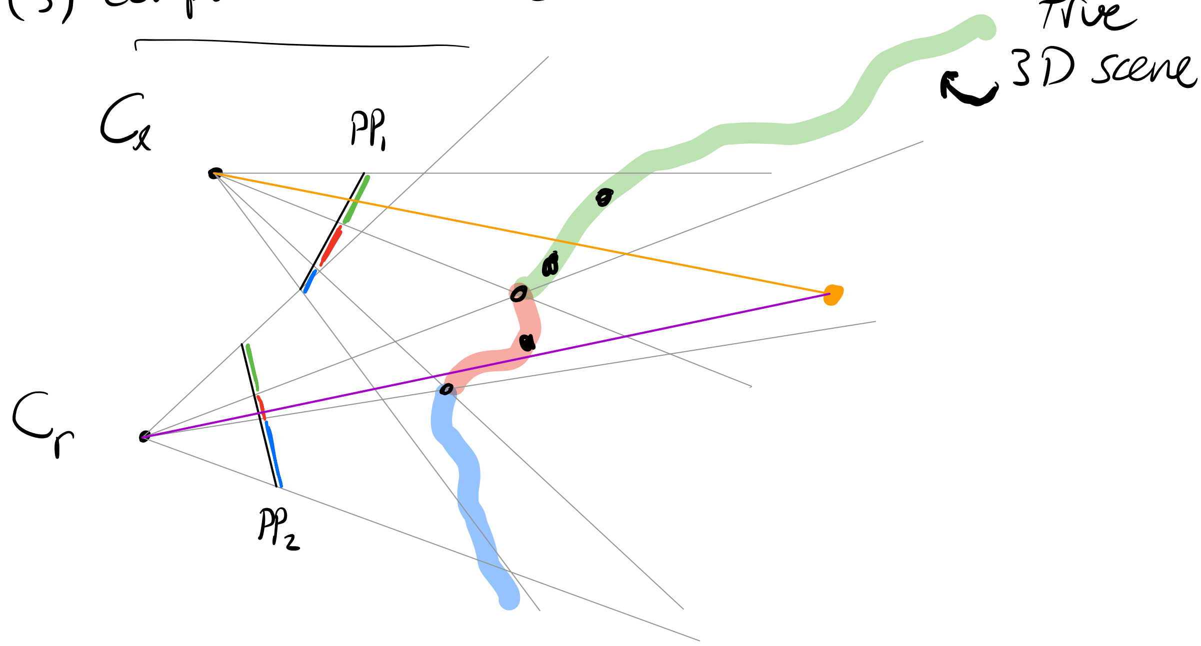
for each depth
- for each pixel
  - compute match cost

Intuition:

(1) "unproject" a pixel to a hypothesized depth d

(2) "reproject" that 3D point back into the other camera

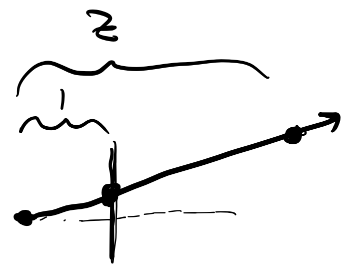
(3) compute match score



Details:

Given: $K_L^{3\times3}$ $K_R^{3\times3}$ $[R|t]_L^{3\times4}$ $[R|t]_R^{3\times4}$

(1) "unproject":



- convert pixels to camera coords
- move to depth d
- put in world coords

$$\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \approx \begin{bmatrix} R_L & t_L \\ \vec{0} & 1 \end{bmatrix}^{-1} \; d \; K_L^{-1} \begin{bmatrix} x_{img} \\ y_{img} \\ 1 \end{bmatrix}_L$$

$\begin{bmatrix} R^T & -R^T t \end{bmatrix}$

aug w/ 1

x^{cam}

3×1

(2) "Reproject"

Given: $K_L^{3\times3}$ $K_R^{3\times3}$ $[R|t]_L^{3\times4}$ $[R|t]_R^{3\times4}$

- world to cam
- cam to pixel

$$\begin{bmatrix} x_{img} \\ y_{img} \\ 1 \end{bmatrix}_R \approx K_R \; [R|t]_R \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

x^{cam}

3×1

(3) compute match score

Insight: "unproject-reproject" is a homography!

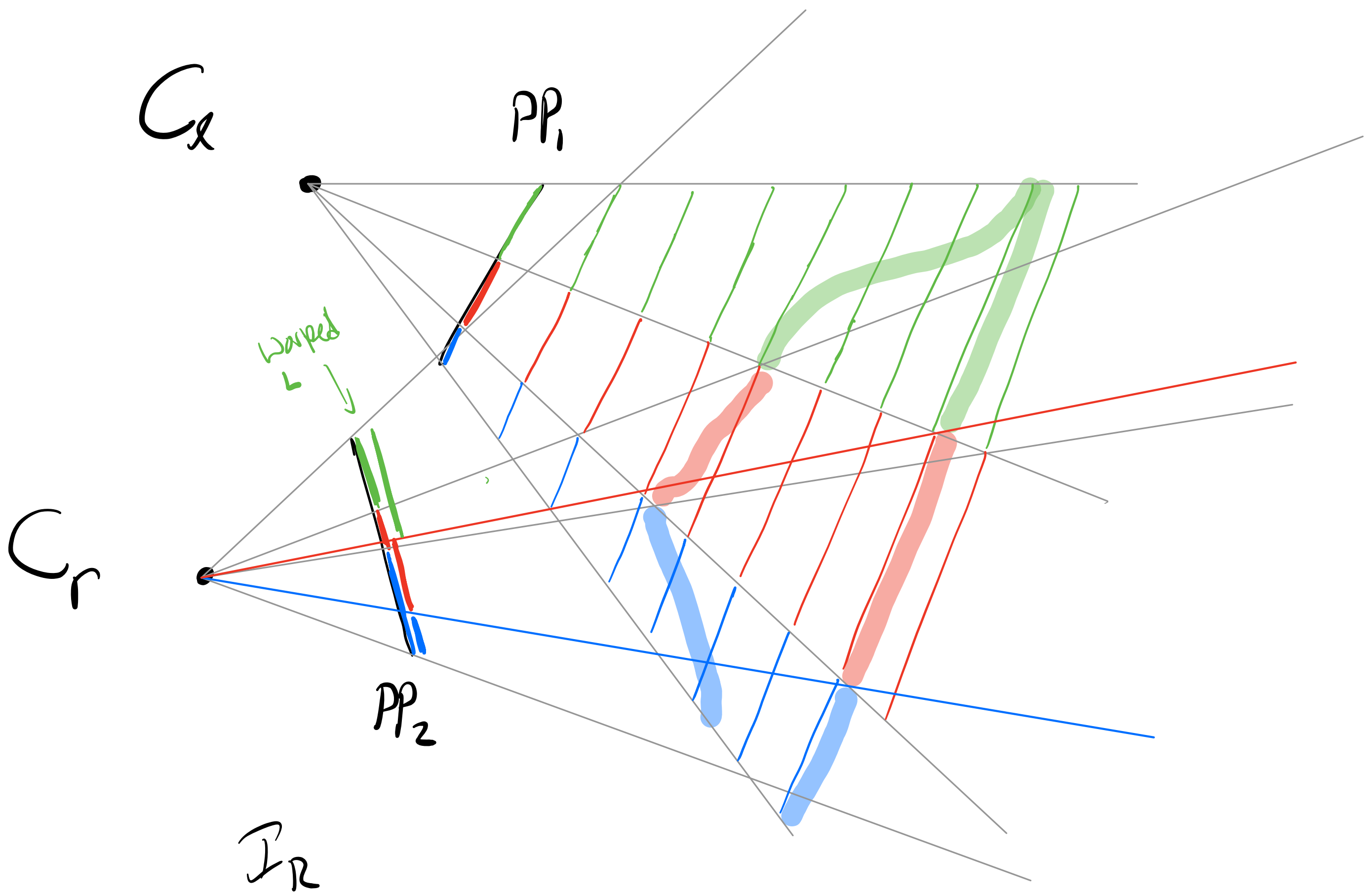
Strategy: (1) unproject 4 corners of L img

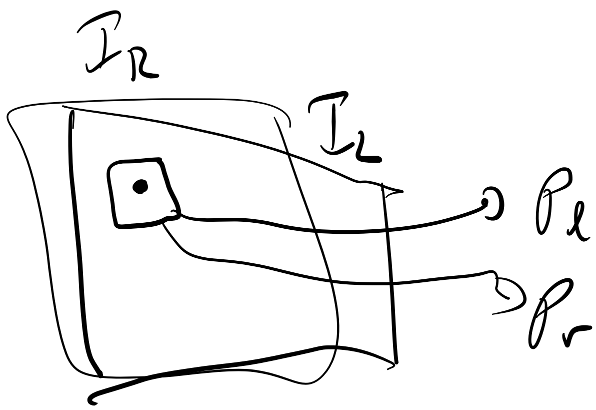
(2) reproject into R img

(3) fit H to correspondences

(4) Warp L to R using H

(5) Compute NCC





normalize → dot

$$[R\,;\,t] = \begin{pmatrix} R & \begin{matrix}0\\0\\0\end{matrix} \\ 0\,0\,0 & 1 \end{pmatrix} \begin{pmatrix} I_{3\times 3} & t \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{pmatrix} R & R\cdot t \\ 0\,0\,0 & 1 \end{pmatrix}$$

(translate $\underline{\text{then}}$ rotate)

$$\operatorname{inv} \begin{pmatrix} R & R\cdot t \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{bmatrix} R^T & -t \\ 0\,0\,0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \vec{u} & \vec{v} & \vec{w} & \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{bmatrix} R & t \\ 0\,0\,0 & 1 \end{bmatrix}^{-1}$$

$$= \left(\begin{bmatrix} I & t \\ 0 & 1 \end{bmatrix} \begin{pmatrix} R & 0 \\ 0 & 1 \end{pmatrix} \right)^{-1}$$

$$= \begin{pmatrix} R^T & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I & -t \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} R^T & R^T t \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{R} & \hat{t} \\ 0 & 1 \end{bmatrix}^{-1} \overset{\checkmark}{=} \begin{bmatrix} \hat{R}^T & -\hat{R}^T t \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} \vec{u} & \vec{v} & \vec{w} & \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

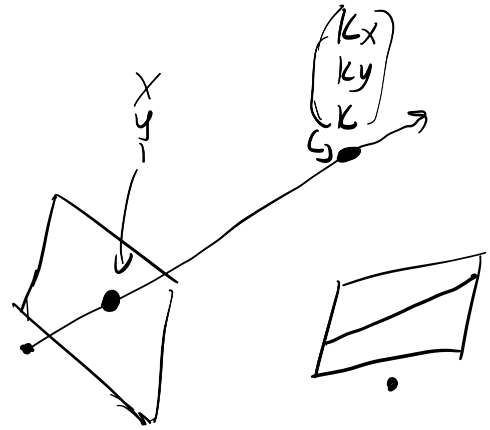
$$\begin{bmatrix} \hat{R}\hat{R}^T & -\hat{R}\hat{R}^T t + t \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Projective Geometry:

Homogeneous points $\begin{bmatrix}x\\y\end{bmatrix} \to \begin{bmatrix}x\\y\\1\end{bmatrix} \to \begin{bmatrix}x'\\y'\\w'\end{bmatrix} \xrightarrow[\text{normalize}]{} \begin{bmatrix}x'/w'\\y'/w'\end{bmatrix}$

$\mathbb{P}^2$

Homogeneous <u>Lines</u>

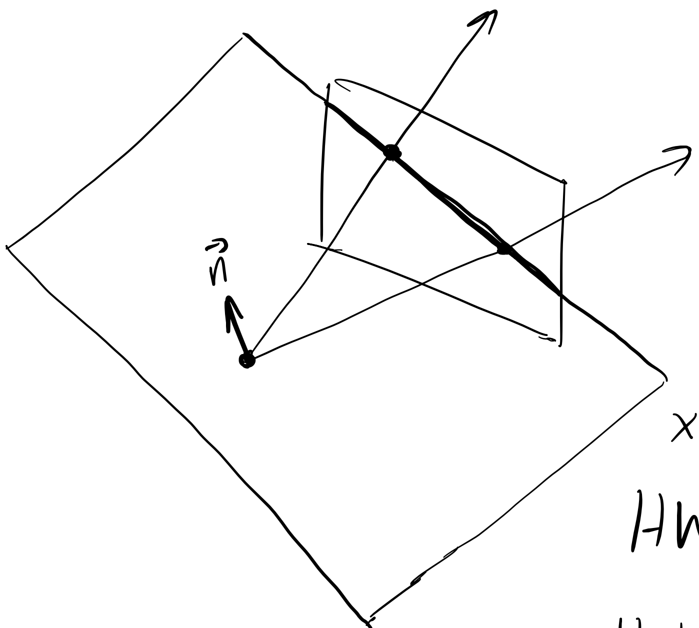


A <u>point</u> (0D) in $\mathbb{P}^2$ corresponds to a <u>ray</u> (1D) through the origin in $\mathbb{R}^3$

A <u>Line</u> (1D) in $\mathbb{P}^2$ corresponds to a <u>plane</u> (2D) through the origin in $\mathbb{R}^3$

$\vec{l} = \begin{bmatrix}a\\b\\c\end{bmatrix}$ projects to

$ax+by+c=0$



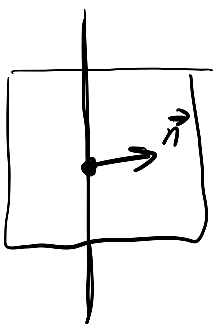
$\vec{l} = \vec{n} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$\begin{bmatrix}1\\1\\0\end{bmatrix}$

$x+y=0$

HW#3 $y=-x$

$\begin{bmatrix}1\\0\\0\end{bmatrix}$



$1x+0y+0=0$

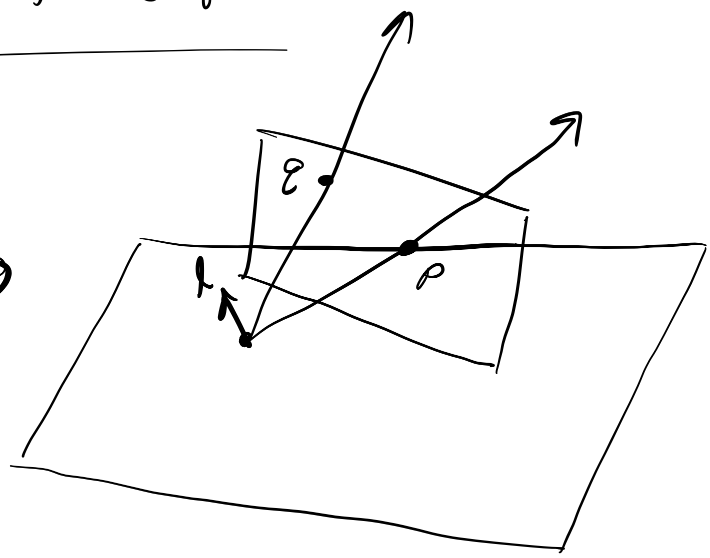
$x=0$

HW#4 $y=-2x+400$

$2x+y-\underline{400}=0$

$\begin{bmatrix}2\\1\\-400\end{bmatrix}$

Points on lines; Lines through points



If point p is on line l, $p \cdot l = 0$

Geometric Argument:

if l is orthogonal to p, p lies in the plane

HW#

Algebraic Argument:

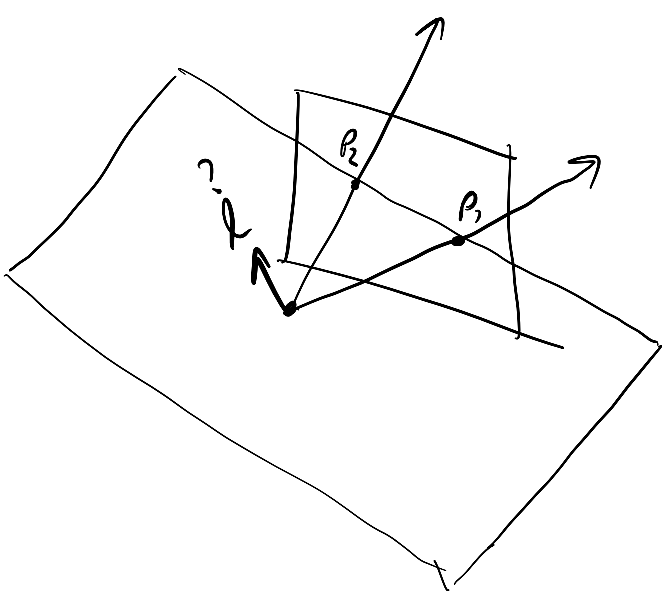
$$l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad ax + by + c = 0$$

$$p \cdot l \qquad p = \begin{bmatrix} x \\ y \\ w \end{bmatrix} \qquad \frac{x}{w} \quad \frac{y}{w}$$

$$a\frac{x}{w} + b\frac{y}{w} + c = 0$$

$$ax + by + cw = 0$$

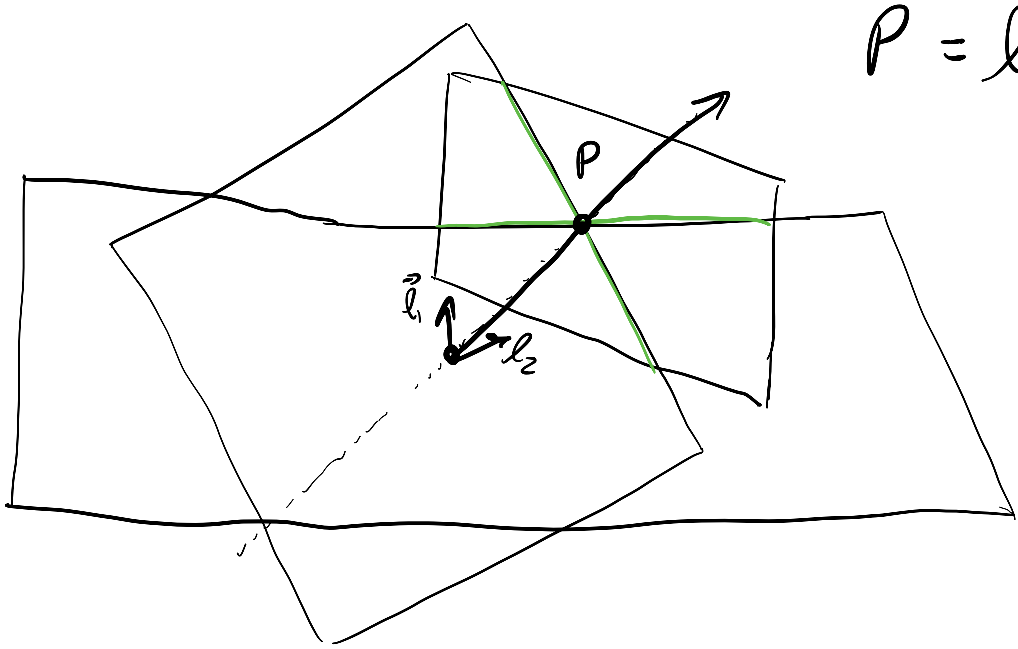
Point-Line Duality



$$l = p_1 \times p_2$$

HW #6 $\begin{bmatrix} 70 \\ 70 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 40 \\ 1 \end{bmatrix}$

$$P = l_1 \times l_2$$



$$\begin{bmatrix} 2 \\ 1 \\ -400 \end{bmatrix}$$