Plane Sweep Stereo

Setting:

Cameras are not rectified

Cameras are calibrated (K_l, $[R;t]_l$, $[R;t]_r$, K_r are known)

Standard Stereo:

for each pixel
  for each disparity
    compute match cost

Plane Sweep Stereo:

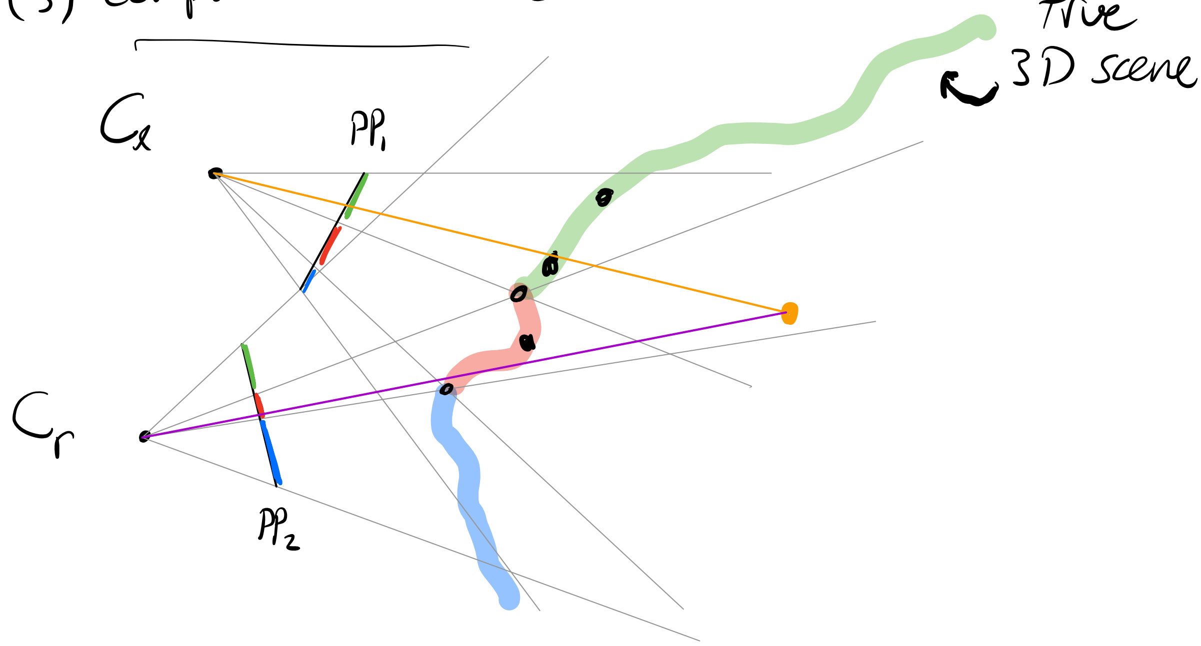
for each disparity
  for each pixel
    compute match cost

Intuition:

(1) "unproject" a pixel to a hypothesized depth d

(2) "reproject" that 3D point back into the other camera

(3) compute match score



Details:

Given: $K_L^{3\times3}$ $K_R^{3\times3}$ $[R|t]_L^{3\times4}$ $[R|t]_R^{3\times4}$

(1) "unproject":

- ✓ convert pixels to camera coords
- ✓ move to depth d
- ✓ put in world coords

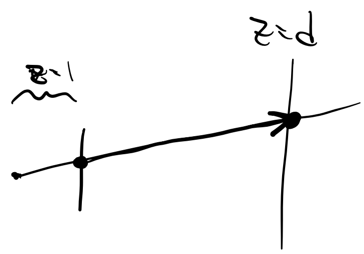
$$\left(K_L [R|t]_L\right)^{-1}$$

$$[R|t]^{-1} \cdot K_L^{-1}$$

$$\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \approx \underset{4\times4}{\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}^{-1}} \; d \; K_L^{-1} \begin{bmatrix} x_{img} \\ y_{img} \\ 1 \end{bmatrix}_L$$

$\uparrow$ add 1

$\uparrow$ $\begin{bmatrix} x_{cam} \\ y_{cam} \\ 1 \end{bmatrix}$



(2) "Reproject"

Given: $\overbrace{K_L^{3\times3} \; K_R^{3\times3}}^{\text{intrinsics}} \; \overbrace{[R|t]_L^{3\times4} \; [R|t]_R^{3\times4}}^{\text{Extrinsics}}$

- world to cam
- cam to pixel

$$\underset{3\times1}{\begin{bmatrix} x_{img} \\ y_{img} \\ 1 \end{bmatrix}_R} \approx \underset{3\times3}{K_R} \; \underset{3\times4}{[R|t]_R} \; \underset{4\times1}{\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}}$$

$\uparrow$ x^{cam}

(3) compute match score

Insight: "unproject-reproject" is a homography!

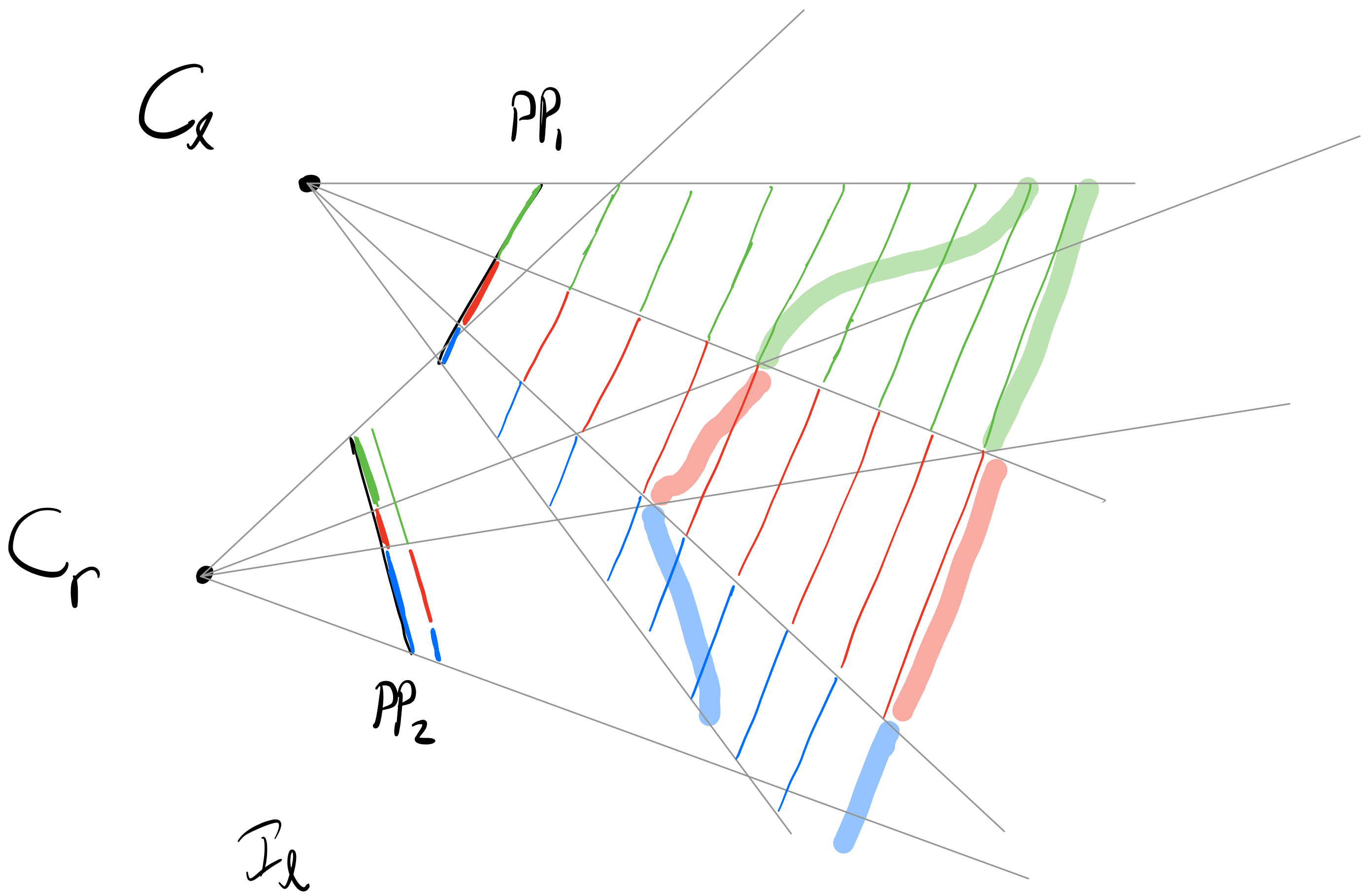
Strategy: (1) unproject corners of Left img

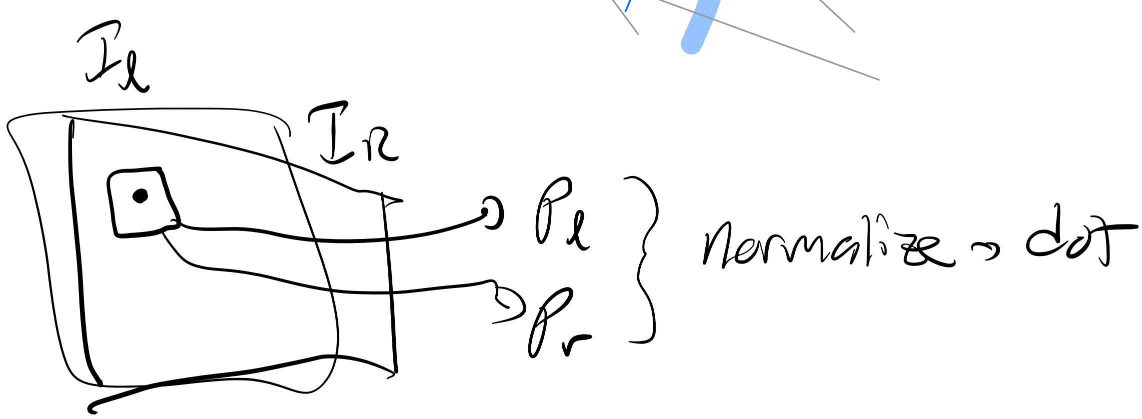
(2) reproject into R cam

(3) fit H

(4) Warp

(5) Compute NCC





$$[R\,;\,t] = \begin{pmatrix} R & \begin{matrix}0\\0\\0\end{matrix} \\ 0\,0\,0 & 1 \end{pmatrix} \begin{pmatrix} I_{3\times3} & t \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{pmatrix} R & R\cdot t \\ 0\,0\,0 & 1 \end{pmatrix}$$

(translate
then
rotate)

$$\mathrm{inv} \begin{pmatrix} R & R\cdot t \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{bmatrix} R^T & -t \\ 0\,0\,0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \vec{u} & \vec{v} & \vec{w} & \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{bmatrix} R & t \\ 0\,0\,0 & 1 \end{bmatrix}^{-1}$$

$$= \left(\begin{bmatrix} I & t \\ 0 & 1 \end{bmatrix} \begin{pmatrix} R & 0 \\ 0 & 1 \end{pmatrix} \right)^{-1}$$

$$= \begin{pmatrix} R^T & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I & -t \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} R^T & R^T t \\ 0 & 1 \end{bmatrix}$$

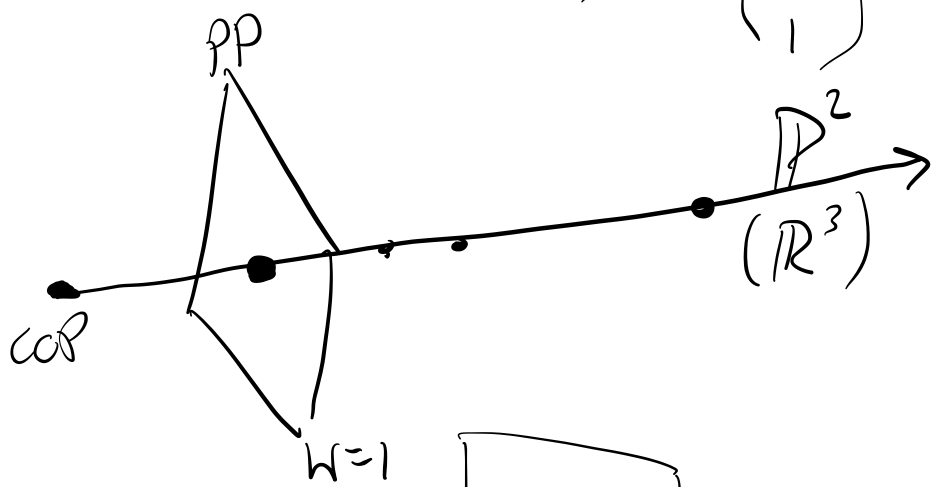
$$\begin{bmatrix} \hat{R} & \hat{t} \\ 0 & 1 \end{bmatrix}^{-1} \stackrel{\checkmark}{=} \begin{bmatrix} \hat{R}^T & -\hat{R}^T t \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} \vec{u} & \vec{v} & \vec{w} & \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

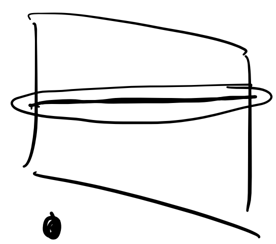
$$\begin{bmatrix} \hat{R}\hat{R}^T & -\hat{R}\hat{R}^T t + t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Projective Geometry

Homog. coords $\begin{bmatrix}x\\y\end{bmatrix} \to \begin{bmatrix}x\\y\\1\end{bmatrix} \to \begin{bmatrix}x'\\y'\\w'\end{bmatrix} \to \begin{bmatrix}x/w\\y/w\end{bmatrix}$

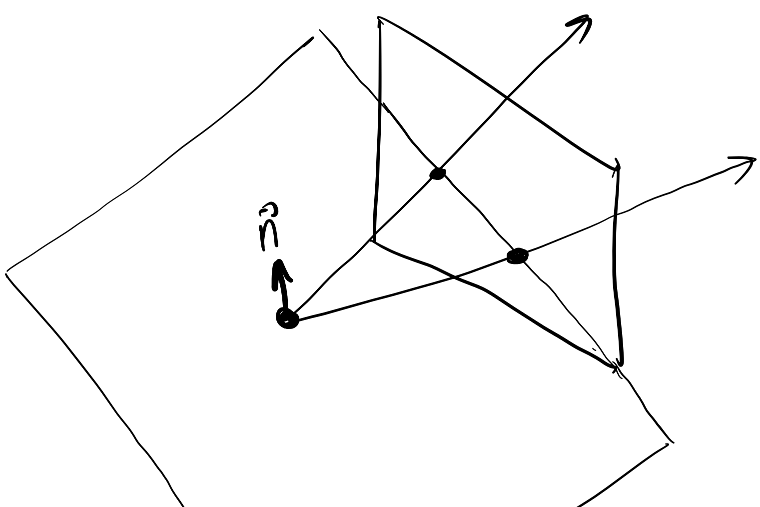
normalize





A point (0D) in $\mathbb{P}^2$ is (like) a ray (1D) in $\mathbb{R}^3$ — through the origin!

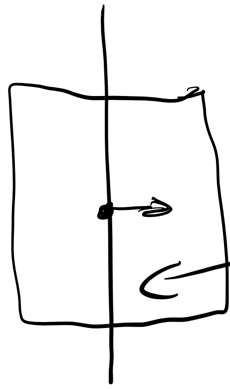
A line (1D) in $\mathbb{P}^2$ is (like) a plane (2D) through the origin in $\mathbb{R}^3$



$\vec{n} = \vec{l} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$ax + by + c = 0$

Example: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



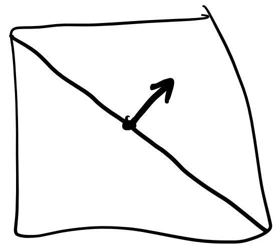
$1x + 0y + 0 = 0$

$x = 0$

HW #3:

$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad \underline{1}x + \underline{1}y + \underline{0} = \underline{0}$

$y = -x$



HW #4

$y = -2x + 400$

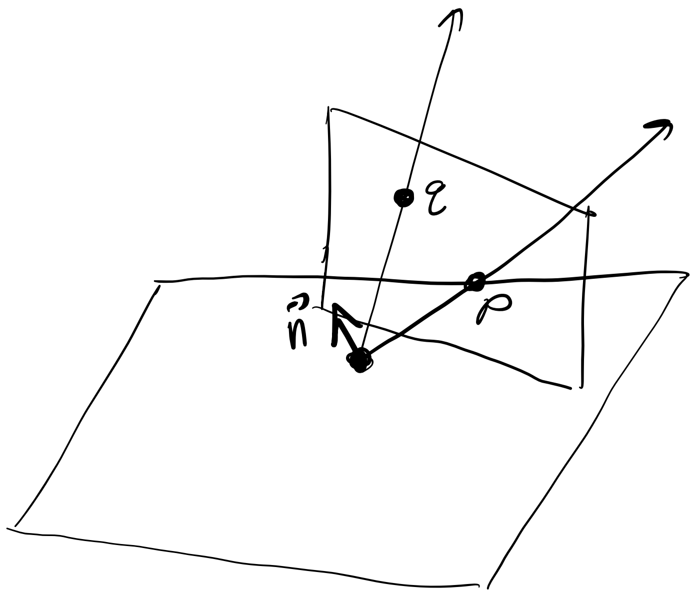
$2x + y - 400 = 0 \rightarrow \begin{bmatrix} 2 \\ 1 \\ -400 \end{bmatrix}$

Note: $\begin{bmatrix} 2 \\ 1 \\ -400 \end{bmatrix} \rightarrow \begin{bmatrix} 4 \\ 2 \\ -800 \end{bmatrix}$

Points on lines; Lines through points

$$p \cdot \ell = 0$$

Geometrically

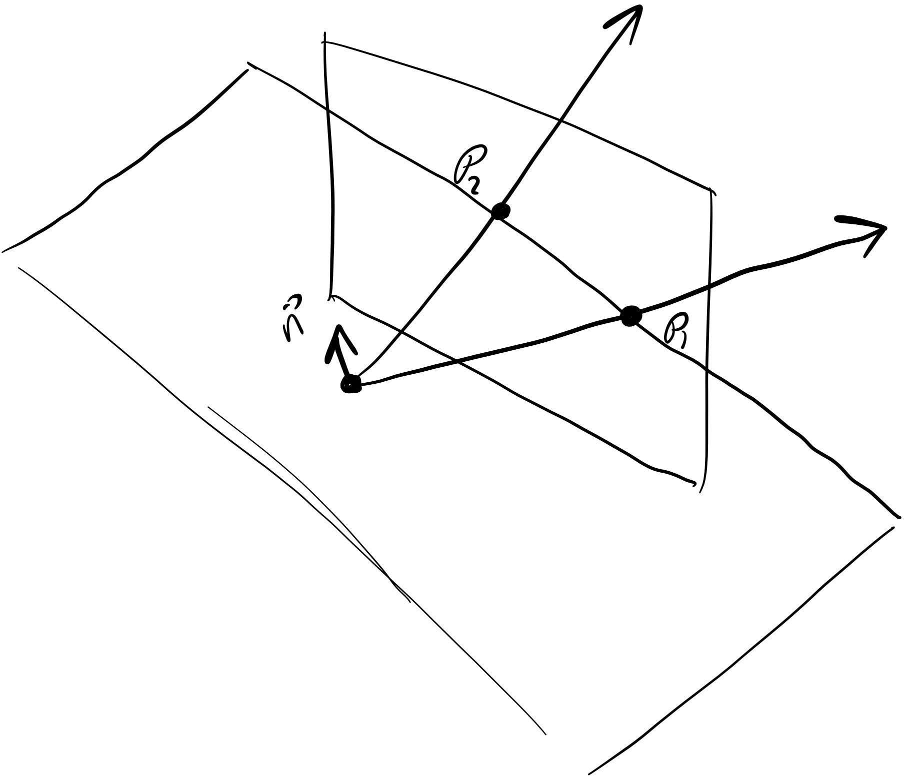


HW #5 $\quad ax + by + c = 0$

$$(p_x \quad p_y \quad p_w)$$

$$p \cdot \ell = \frac{p_x}{p_w} a + \frac{p_y}{p_w} b + c = 0$$

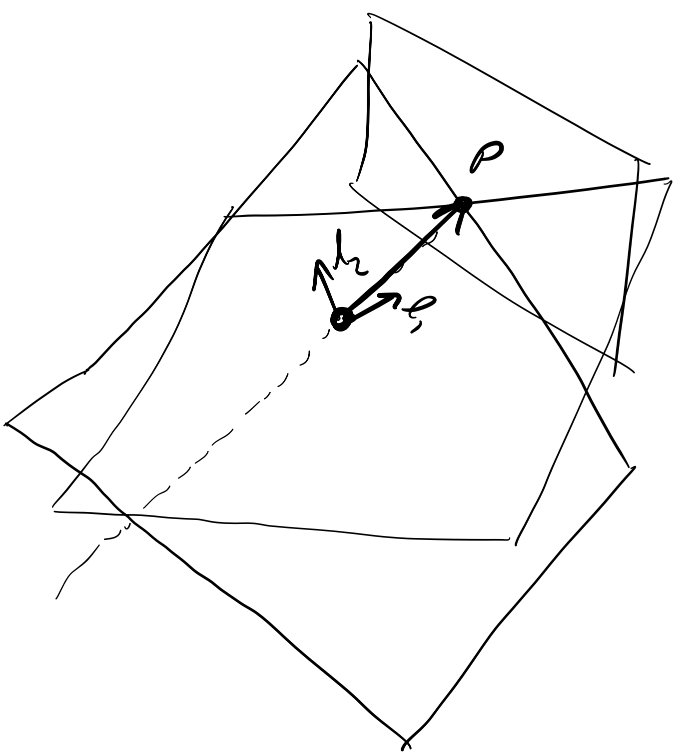
Point-Line Duality



The line through 2 pts is the plane spanned by their vectors.

cross product!?

$$l = P_1 \times P_2$$



$$P = l_1 \times l_2$$

(70, 70) (0, 40)

$$\begin{pmatrix} 70 \\ 70 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 40 \\ 1 \end{pmatrix}$$