What Is Accumulation of Semantics?

- Parametric functions allow for the decoupling of return value
  - But how can you parameterize the direction of evaluation?
  - This goes lower than type level constructs

```
eval_forward(syntax, state): (Int, state')
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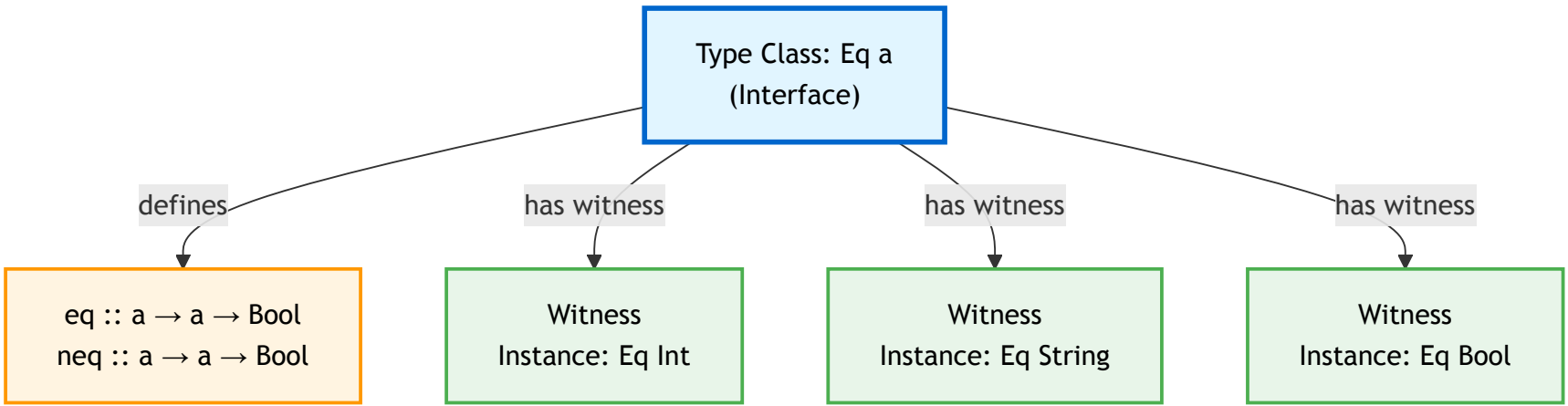
  - This goes lower than type level constructs

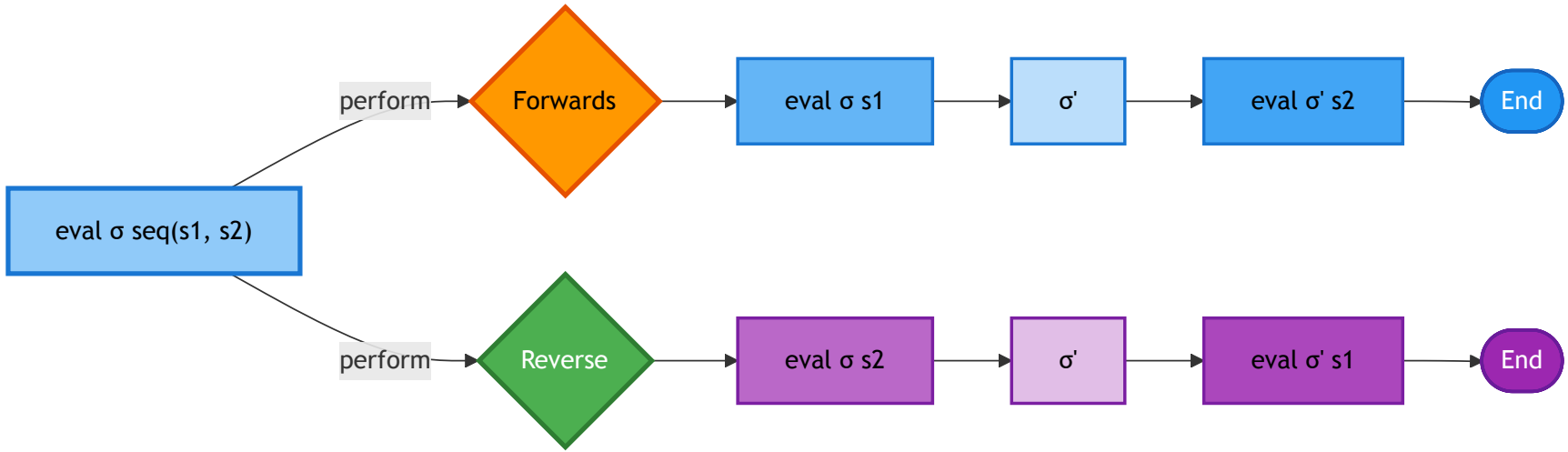
```
eval_bidirectional_generic[D](syntax, state): (D, state')
```

Interfaces and Witnesses

- Type Classes allow polymorphism via the definition of an interface
  - A witness is an implementation that obeys this interface

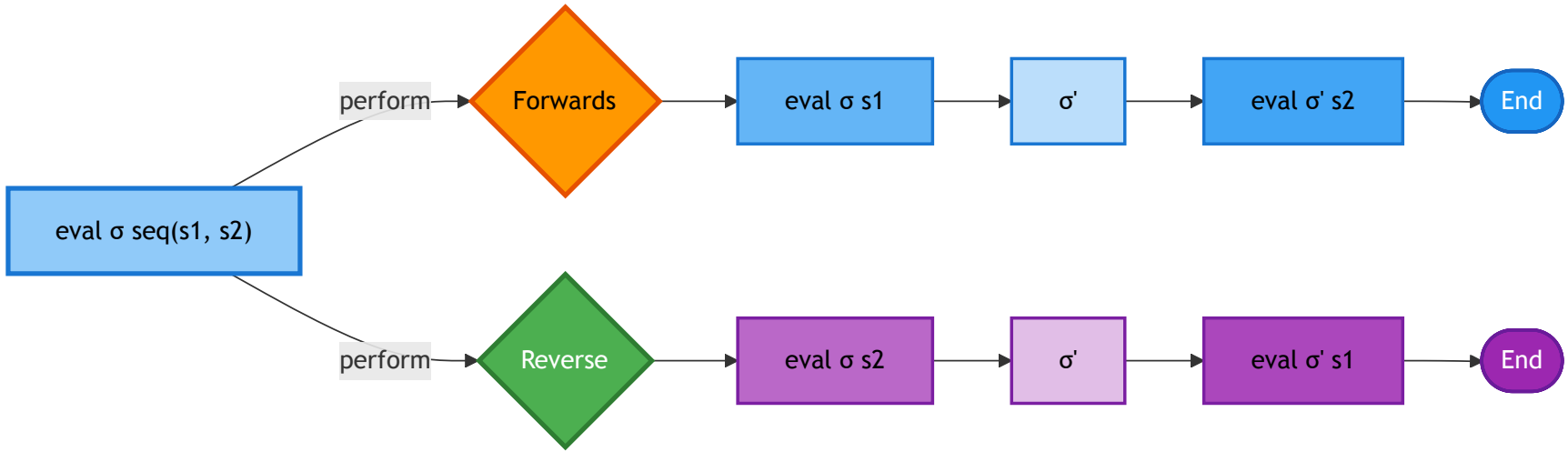


Resumptions for Bidirectionality



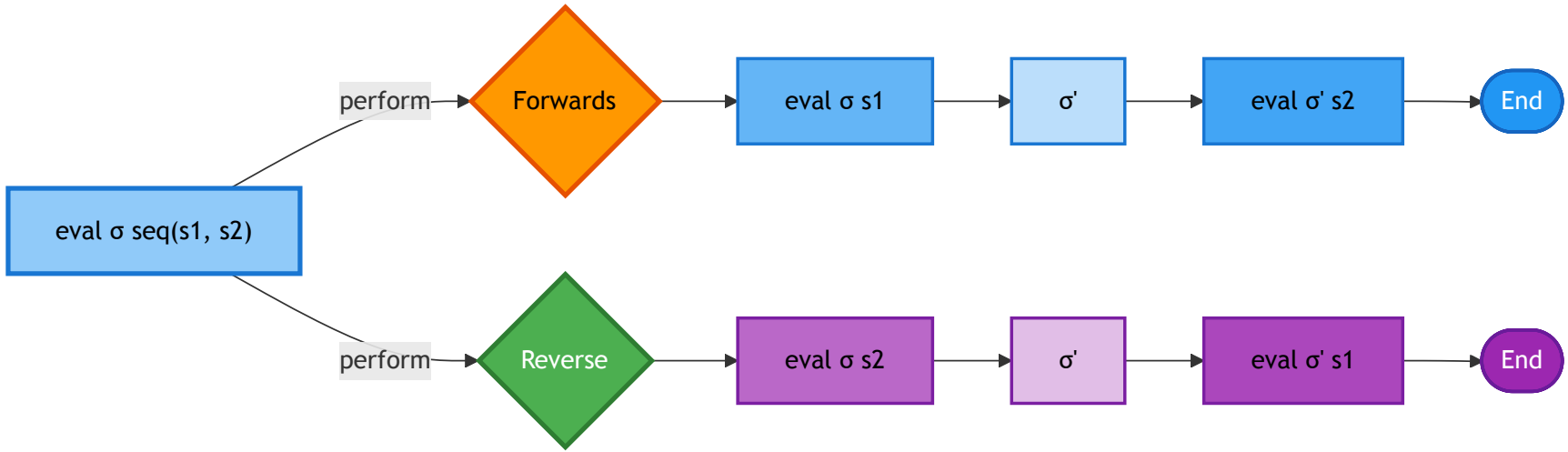

```
def handle_forwards(s1, s2, k) =
    k(k(eval(s1,σ)), s2)
```

Resumptions for Bidirectionality




```
def handle_forwards(s1, s2, k) =
    k(σ`, s2)
```

Resumptions for Bidirectionality



σ``

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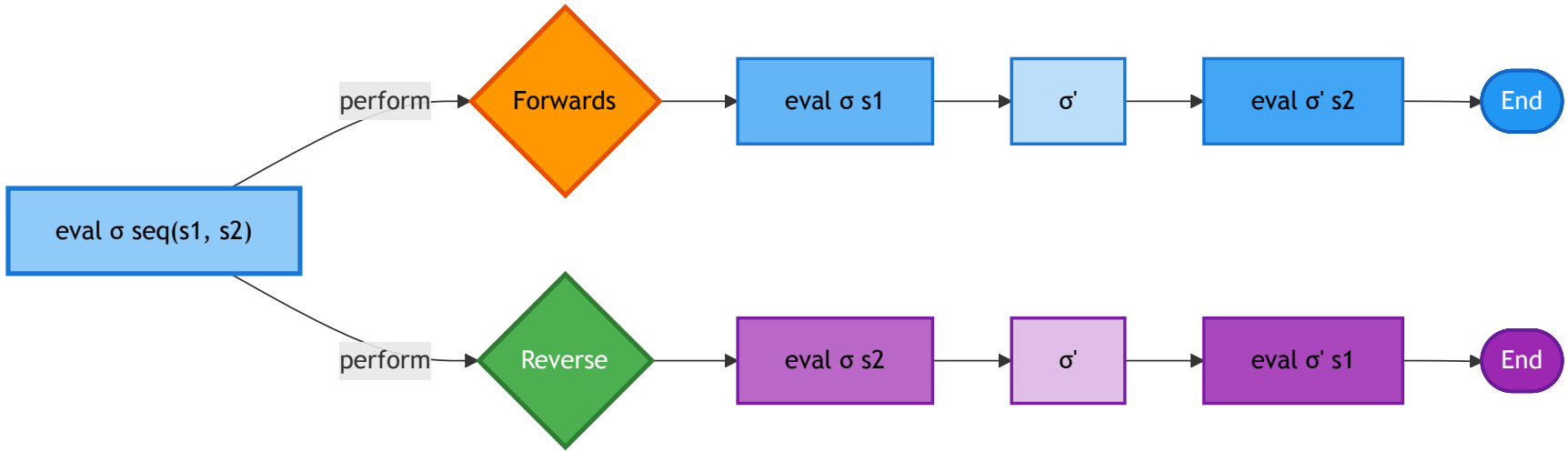
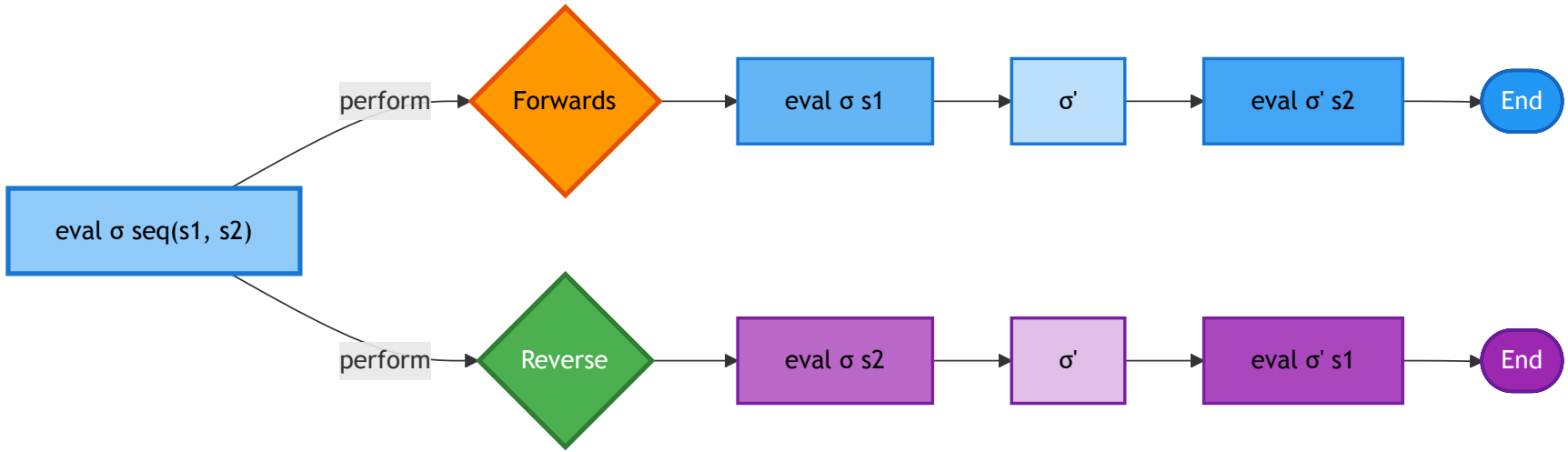



```
def handle_backwards(s1, s2, k) =
    k(k(eval(s2,σ)), s1)
```

Resumptions for Bidirectionality

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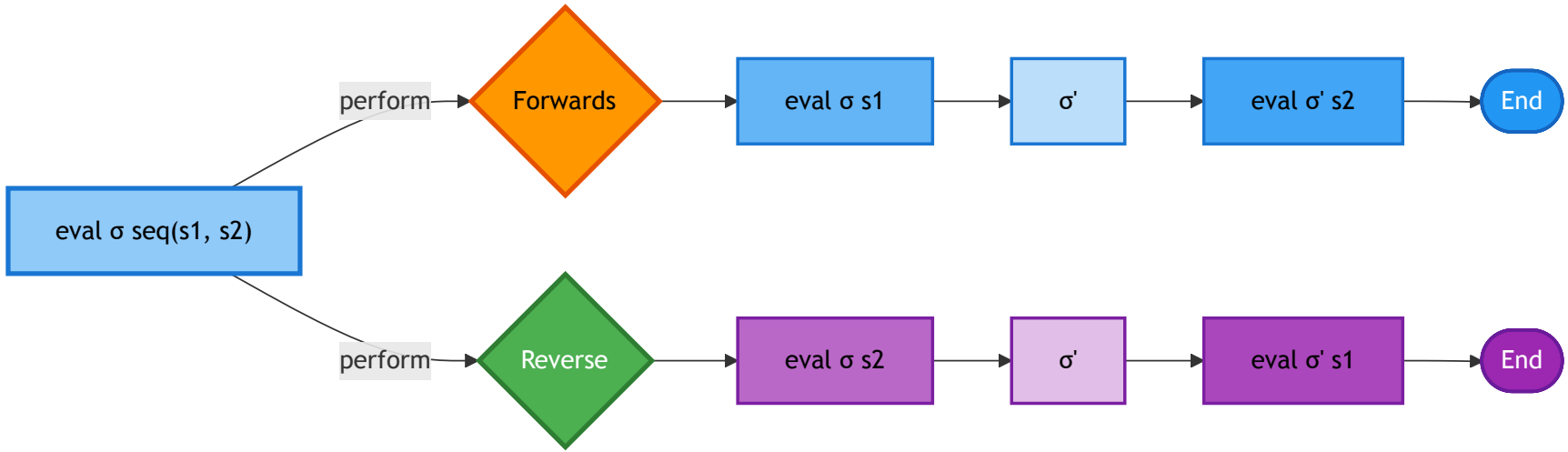


σ``

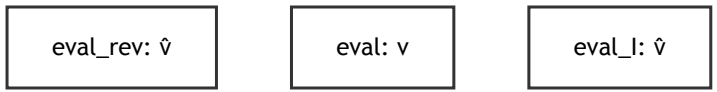
Example Language

```
Expressions e ::= e
                | cst(n)
                | e1 + e2
                | if(e1) e2 else e3
                | var(x)
                | seq(e1,e2)

Int         n ::= int
Ident       x ::= string
```

Monolithic

```
def eval(e: expr, env:...): ...
```



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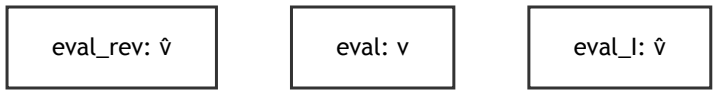
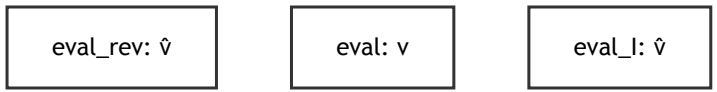


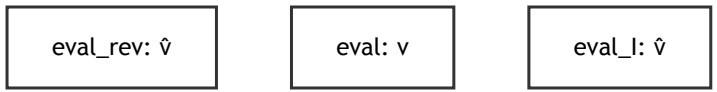

```
def eval(e: expr, env): int = match e
    ...
    | seq(e1, e2) => eval(e2, eval(e1, env))
```

Monolithic




```
def eval_I(e: expr, env): Interval = match e
    ...
    | seq(e1, e2) => eval(e2, eval(e1, env))
```

Monolithic

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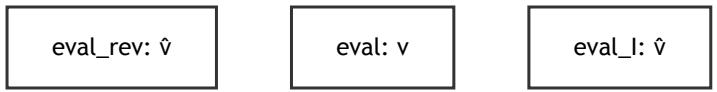


```
def eval_rev(e : expr, env_out): Set[str] = match e
    ...
    | seq(e1, e2) => eval(e1, eval(e2, env_out))
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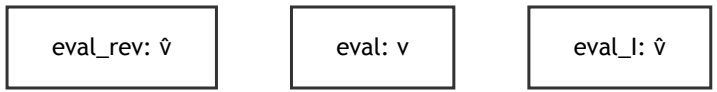
Monolithic

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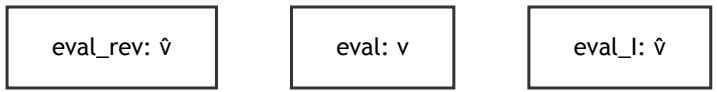


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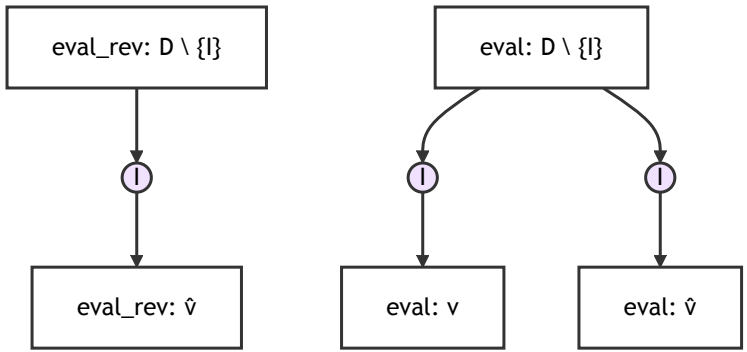


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eval(e: expr, env): D \ {I} = match e
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    | plus(e1, e2) => plusI(eval(e1,env), eval(e2,env))
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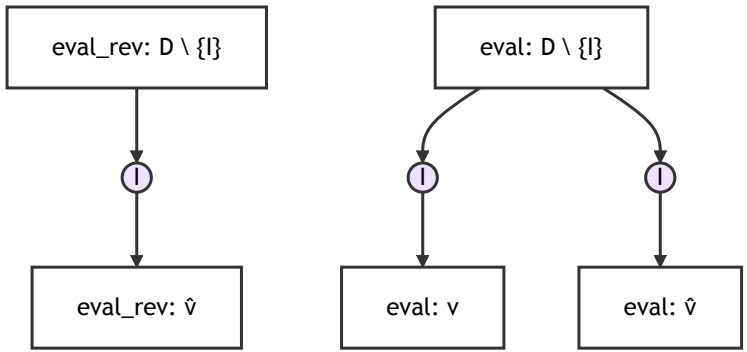
Domain Generic

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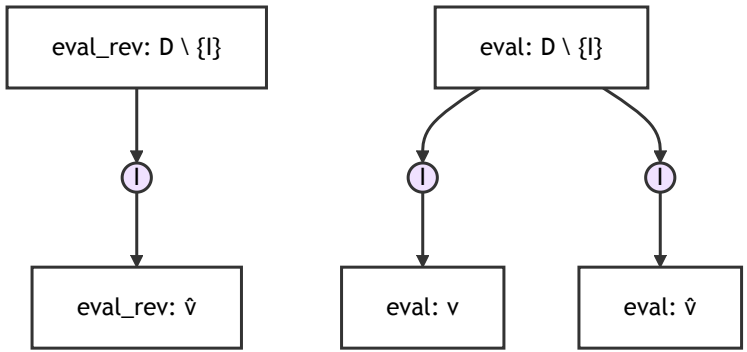


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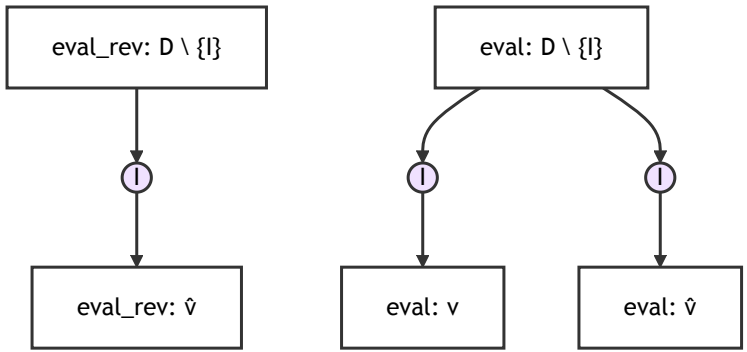


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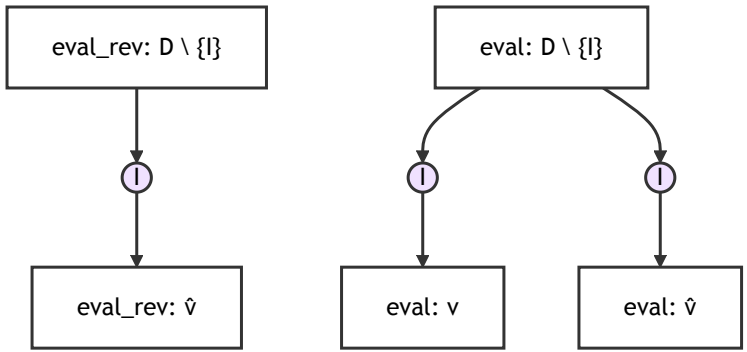


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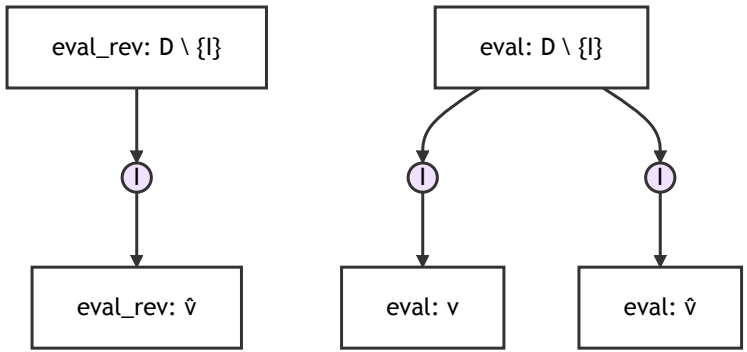


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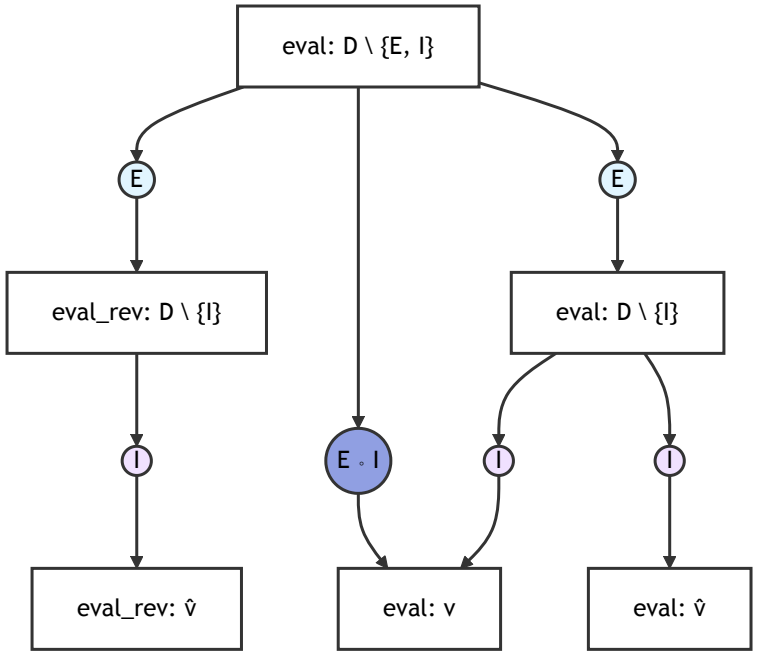
Complete Parametricity

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eval(e: expr, env): D \ {E,I} = match e
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    | var(x) => varE(env, x)
    | plus(e1, e2) => plusE(env, e1, e2)
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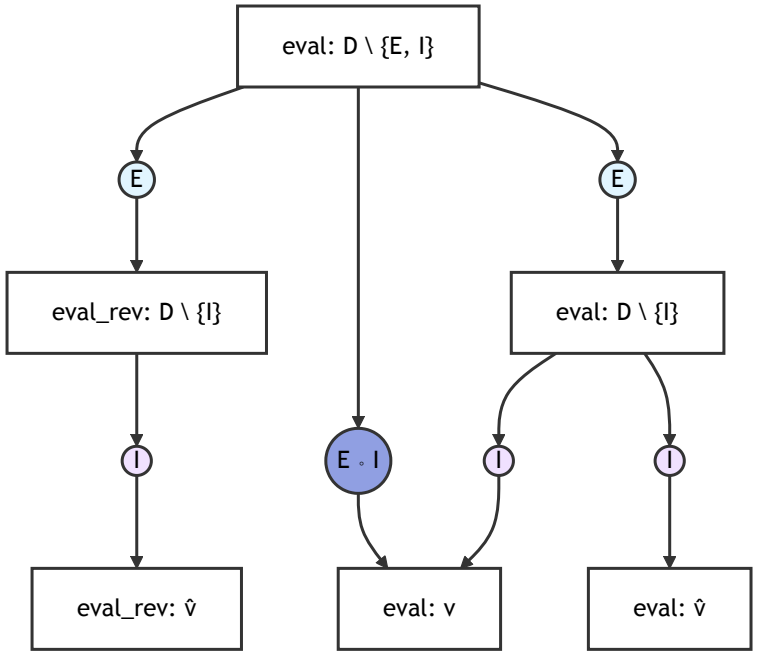


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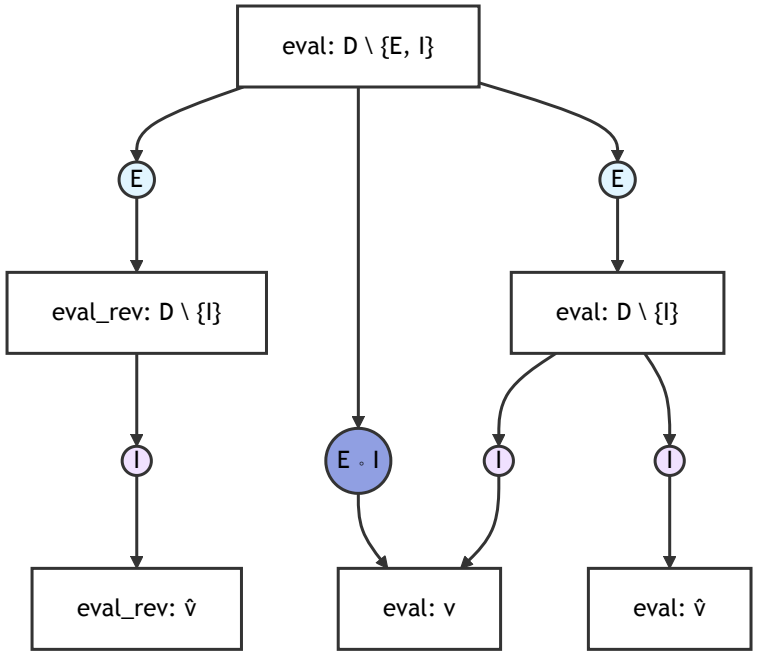


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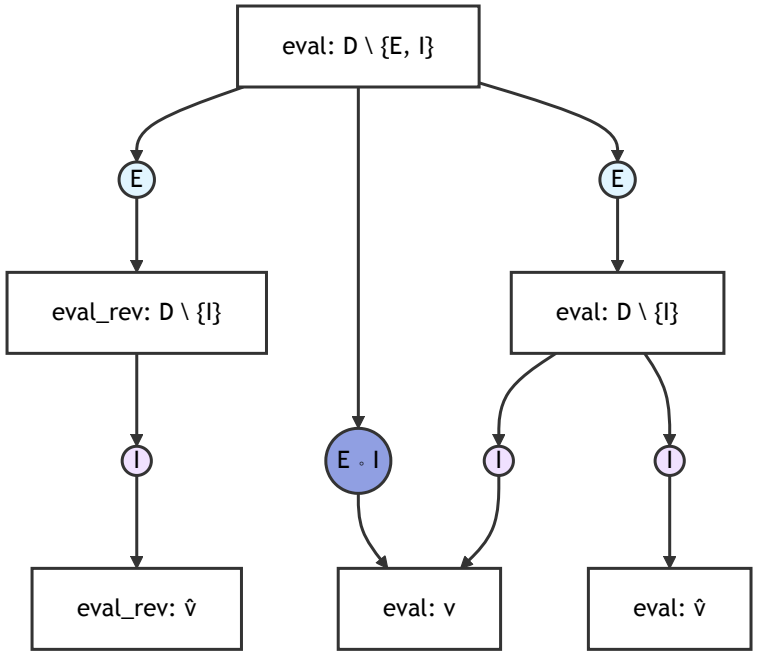


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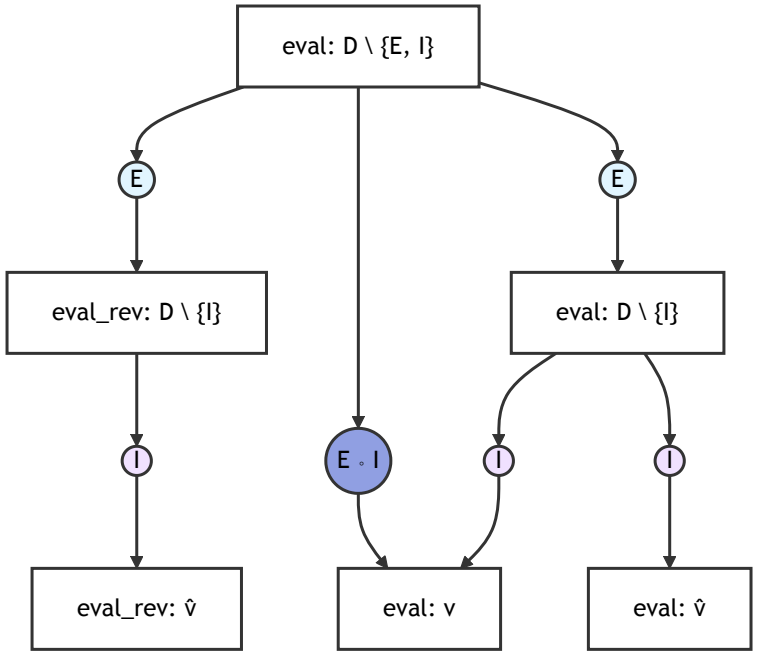


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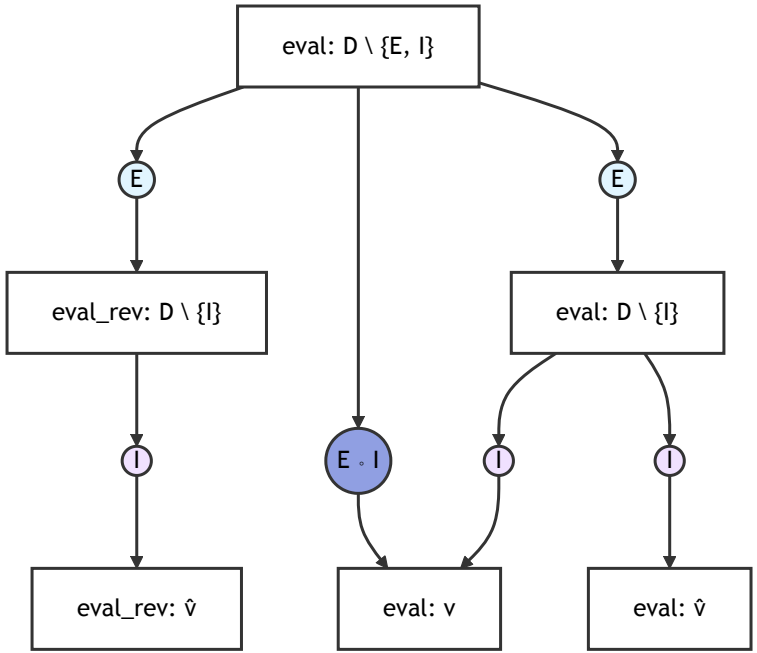


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    (g, st1) = k(st, e1) # e1 ⇓ st
    st2 = k(st1, e2)     # e2 ⇓ st2
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    ifI(g, st2 , st3) # pass resulting states to intro
```

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def ifI(g, st2, st3) = match g
    | True  -> st2 # only use "then" state
    | False -> st3 # only use "else" state
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def ifI(g, st2, st3) =
    joinL( # deterime how to return / combine states
        assumeL(g, st2),
        assumenotL(g, st3)
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Lowering Handlers

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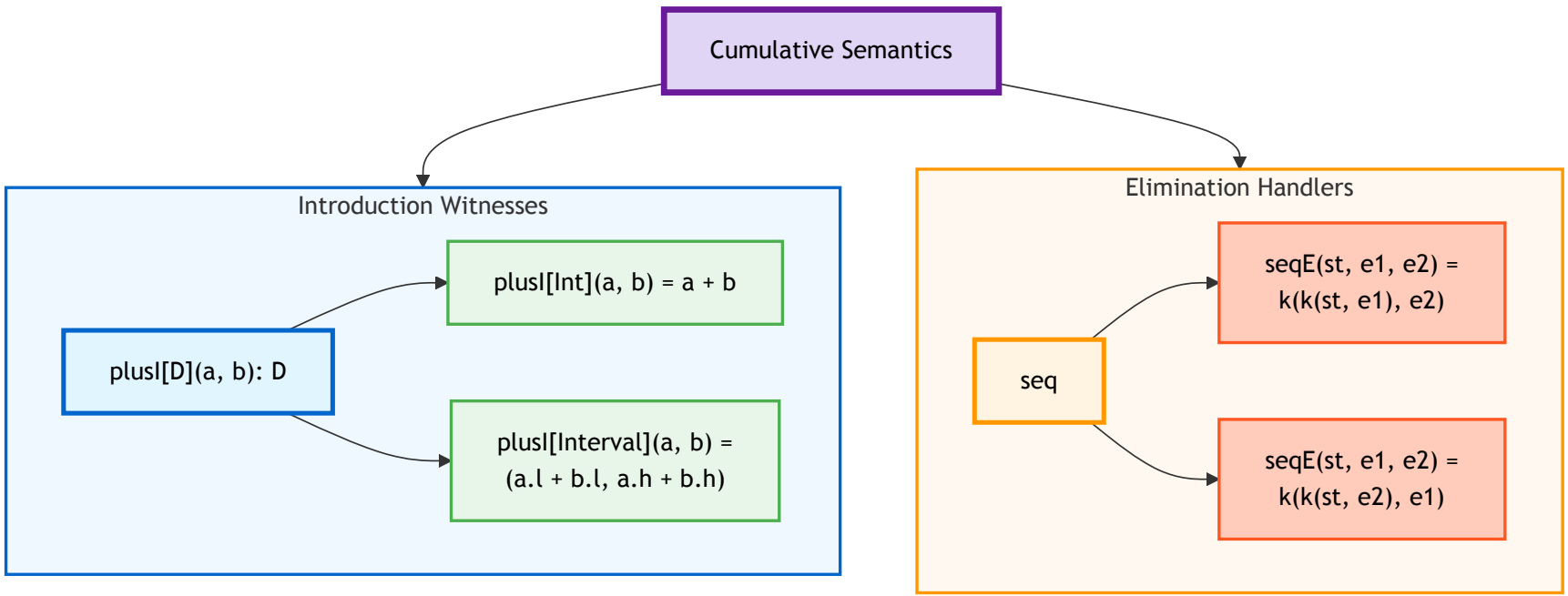
Cumulative Abstract Semantics

- *Elimination* handlers leverage continuations to eliminate the source syntax
- *Introduction* witnesses provide abstract-domain specific semantics
- *Lowering* witnesses provide flow insensitive, reusable abstract domain operators



Changes to the Recipe

Standard Recipe:

1. Syntax
2. Concrete Interpreter
3. Collect Semantics
4. Abstract Domain
5. Abstract Interpreter

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New Recipe:

1. Syntax
2. Generic Interfaces
3. Concrete (Elim & Intro)
4. Collecting (Elim)
5. Abstract Domain (Lowering)
6. Abstract Interpreter (Intro, *Elim*)

Performance

- running the following Python program in the **Concrete Domain** with our
  - Lean4 typeclass and CPS implementation *compiles to a C++ binary*
  - Effekt scoped effekt system implementation *compiles to LLVM*

```
x = 0
while x < 1000000
    x += 1
print(x)
```

```
# running python test code with performance tool 'hyperfine'
hyperfine 'python3 while_perf.py'
```

Performance

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```
x = 0
while x < 1000000
    x += 1
print(x)
```

```
Benchmark 1: python3 while_perf.py
  Time (mean ± σ):      53.3 ms ±   6.7 ms    [User: 48.5 ms, System: 3.3 ms]
  Range (min … max):    45.7 ms …  76.8 ms    61 runs
```

Performance

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```
x = 0
while x < 1000000
    x += 1
print(x)
```

```
# run lean4 binary with performance tool 'hyperfine'
hyperfine "./.lake/build/bin/cumulativesemantics --while_perf --iterations 1000000"
```

Performance

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```
x = 0
while x < 1000000
    x += 1
print(x)
```

```
Benchmark 1: ./.lake/build/bin/cumulativesemantics --while_perf --iterations 1000000
  Time (mean ± σ):     321.2 ms ±   6.3 ms    [User: 315.1 ms, System: 2.7 ms]
  Range (min … max):   313.6 ms … 333.6 ms    10 runs
```

Performance

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```
x = 0
while x < 1000000
    x += 1
print(x)
```

```
# run effekt LLVM binary with perfomance tool 'hyperfine'
hyperfine ./out/concrete
```

Performance

- running the following Python program in the **Concrete Domain** with our
  - Lean4 typeclass and CPS implementation *compiles to a C++ binary*
  - Effekt scoped effekt system implementation *compiles to LLVM*

```
x = 0
while x < 1000000
    x += 1
print(x)
```

```
Benchmark 1: ./out/concrete
  Time (mean ± σ):     624.9 ms ±  10.1 ms    [User: 622.6 ms, System: 1.9 ms]
  Range (min … max):   605.0 ms … 639.6 ms    10 runs
```

Questions?