

Effects of the local information on the resource-epidemic dynamics on multiplex networks

Jun Wang¹, Ruijie Wang^{2,*}, Die Wu³

1. School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China

2. Aba Teachers University, Aba 623002, China

3. School of Computer Science, Sichuan Normal University, Chengdu 610101, China

junwang31@qq.com, ruijiewang001@163.com, dwu.cse@gmail.com

Corresponding Author: Ruijie Wang Email: ruijiewang001@163.com

Abstract—The effects of the local-information-driven awareness on the resource-epidemic coupled dynamics is studied in this paper. The double-layer multiplex networks are adopted to depict the coupled dynamics of the resource allocation and epidemic spreading, and a coupled model based on the modified SIS model is proposed. Through extensive Monte Carlo simulations, we find that on a network without inter-layer degree correlation, there is a cross point of the basic infection rate, when the value of the basic infection rate is lower than the cross point, the breakout of the epidemic can be postponed with the decrease of the level of awareness, while when it is larger than the point, the spread size of the epidemic can be reduced with the increase of the level of awareness. In addition, we also find that the inter-layer degree correlation can reduce the final spread size of the epidemic.

Keywords—multiplex networks; local information; awareness; resource-epidemic spreading

I. INTRODUCTION

With the great progress made in the theory of complex networks, nearly all of the natural phenomena can be described by the language of the complex networks, in which the individuals can be abstracted by nodes and relationships between individuals can be represented by edges [1]. Typical examples include the social network, technological network and transportation network [2]. Based on this, we can not only research the evolution law of the natural phenomena by studying the structure feature of the networks, but also model the spreading dynamics of information, disease etc. taking place on the networks. To model the diffusion of information or the spreading of disease on the networks, there are two basic mathematic models, namely the susceptible-infected-susceptible (SIS) and susceptible-infected-recovered(SIR) models [3]. In the SIS model, the nodes are divided into two subsets according to the node state, namely the susceptible and infected states. The virus transmits from the infected nodes to their susceptible neighbors with rate β at each time step, and at the same time the infected nodes recover to the susceptible state

with recovery rate μ . While, in the SIR model, there are three subsets of nodes in the network, namely the nodes may be in one of the three states, susceptible, infected and recovered. The only difference between the SIS and SIR model is that the infected nodes recovery to the recovered state at each time step.

In recent years, with the acquisition of big data and the great improvement made in computing power, the research on complex network has developed from single-layer to multi-layer and temporal networks [4]. Accordingly, the research of the spreading dynamics has also evolved from the single dynamics on single networks to coupled dynamics on multiplex networks and temporal networks [5]. For the coupled dynamics on multiplex networks, the typical represents are the coevolution of information and disease [6].

The ultimate goal of studying the epidemic spreading on complex networks is suppression the spread of the disease. Consequently, the immunization strategies based on the theory of complex networks have been studied extensively. Among them, the typical research representatives include the targeted immunization [8], acquaintance immunization [9] and the information-driven vaccination [10]. Recently, the research of the immunization strategies from the aspects of resource allocation have been a hotspot in the fields of complex networks [12]. Such as, Preciado et al. [13] studied the strategy of dynamical resource allocation on disease suppression. Based on the mechanism of a real-time feedback of the epidemic, they solved the problem of optimal resource allocation for the control of disease. In addition, Chen et al. [14] investigated the influence of the self-awareness on the coupled dynamics of resource allocation and disease spreading on complex networks, they found an optimal heterogeneity of self-awareness for epidemic control. In addition, they also studied the interplay between resource allocation and epidemic spreading from the aspects of individual resource support [15].

In this paper, we study the effects of the awareness on the spreading dynamics of the epidemic by considering that the awareness of self-protection of the individuals is stimulated by the local information of the

epidemic. Intuitively, during a pandemic, the information about the state of the epidemic obtained from neighbors will make a more impressive impression on a person. Consequently, the fraction of the infected neighbors is adopted to represent the local information about the epidemic in this work. We assume that each person will have an initial probability of resource allocation, and with the increase of the infection density of neighbors, the probability of resource allocation decreases. Furthermore, we assume that as long as an individual allocates resources to help others, its own risk of infection will increase, i.e., the effective rate of transmission will increase. Finally, to mimic the resource allocation process and epidemic spreading, the double-layer multiplex networks are adopted in this research, and a modified SIS model is used to model the process of the epidemic. By adopting extensive Monte Carlo simulations, we find that on a network without inter-layer degree correlation, there is a cross point of the basic infection rate, when the value of the basic infection rate is lower than the cross point, the breakout of the epidemic can be postponed with the decrease of the level of awareness. While when it is larger than the point, the spread size of the epidemic can be reduced with the increase of the level of awareness. In addition, we also find that the inter-layer degree correlation can reduce the final spread size of the epidemic.

II. MODEL DESCRIPTION

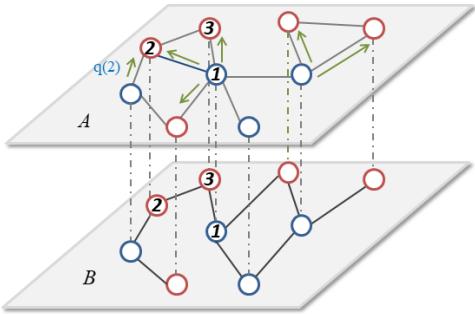


Fig. 1. (Color online) Schematic plot of a typical multiplex network that is composed of two layers. Layer A is the network of social relationships among individuals, and layer B is the physical contact among individuals. The blue and red circles indicates the healthy and infected nodes respectively. Resources are allocated from the healthy nodes to the infected nodes in layer A indicates by the arrows in the figure, q stands for the resource quantity. Virus transmits from the infected nodes to the susceptible ones in layer B . There is one-to-one correspondence between the interlayer nodes.

In this section, the double-layer multiplex networks are built. As shown in Fig. 1, layers A and B represent the social and contact subnetworks, respectively. Each node in two layer represents an individual in real world, and an edge in social (contact) layer depicts the friendship (contact relationship) between each pair of individuals. There is one-to-one correspondence between the nodes in social and contact subnetworks.

The total nodes in the network is indicated as N . In the construction process of the multiplex network, the uncorrelated configuration model [16] is used. To build a double-layer network without inter-layer degree correlation, we follow the following steps: first of all, the network size is set as $N=10000$, and two degree sequences are generated from two degree distributions $P_A(k)$ and $P_B(k)$, respectively. Note that, in this paper, we only focus on the networks with heterogeneous degree distribution, thus we have $P_A(k)=P_B(k)\sim k^{-\gamma}$. Next, in each subnetwork, two pair of nodes (the degree number of each node must larger than 1) are selected randomly, and an edge is assigned to this pair of nodes, meanwhile the degree number of the two nodes decreases by one. This process continues until the degree number of all are zero.

For the process of the resource allocation, each node in the social layer is assumed to generate one unit resource during one time step, and then it allocates the resource to its infected neighbors based on its awareness. As shown in Fig. 1, the green arrow indicates the allocation directions. The quantity of resource allocation is dependent on the fraction of infected neighbors τ_i , which is expressed as:

$$\tau_i = \frac{m_i}{|\Pi_i|}, \quad (1)$$

where m_i and Π_i represent the number of infected neighbors and the set of neighbors respectively. Initially, each healthy node has a uniform probability of resource allocation, which is denoted as q_0 . With the evolution of the epidemic spreading, the allocation willingness of each individual is affected by the information of the epidemic state. We consider the local information of the fraction of infected neighbors in this paper. The quantity of resource that a healthy node i allocates to its infected neighbors at time t can be written as:

$$q_i(t) = q_0(1-\alpha)^{\tau_i} * \Omega_i(t), \quad (2)$$

where α represents the awareness level of the individuals, and $\Omega_i(t)$ is the total quantity of resources of node i at time t . For the sake of simplicity and without loss of generality, the value of $\Omega_i(t)$ is set as $\Omega_i(t)=1$. In addition, we assume that the healthy nodes allocated resources evenly to all infected neighbors. Thus the resource quantity that a healthy node i can allocate to an infected neighbor j is:

$$\omega_{i \rightarrow j}(t) = q_i(t) \frac{1}{m}, \quad (3)$$

In the contact subnetwork, the spreading process of the epidemic is modeled by a modified SIS model. In this model, the effective infection rate between a pair of

susceptible and infected nodes is affected by the resource quantity that the susceptible node allocates out. The more resources a node allocates, the more likely it is to be infected. We define the basic infection rate of the epidemic as β , and the effective infection rate of a susceptible node as λ . Then, we have the following expression:

$$\lambda_i = q_i \beta, \quad (4)$$

For the recovery process of the epidemic, each infected node recovers to the susceptible state with a basic rate μ at each time step. Since resources can promote the recovery rate of the infected nodes, we consider that the actual recovery rate $\mu_i(t)$ of node i is a function of the resource quantity that it receives at time t . Consequently, $\mu_i(t)$ can be written as:

$$\mu_i(t) = 1 - (1 - \mu)^{\varepsilon \omega_i(t) + 1}, \quad (5)$$

where the parameter ε represents the resource utilization rate. In the stationary state, the fraction of infected nodes can be calculated as:

$$\rho = \frac{1}{N} \sum_{i=1}^N \rho_i. \quad (6)$$

III. RESULTS

We explore the effects of the local information on the coupled dynamics of the resource allocation and disease spreading on multiplex networks by performing extensive Monte Carlo simulations. Note that, to determine the threshold of the epidemic, we adopt the susceptibility measure [17] that is defined as:

$$\chi = N \frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle} \quad (7)$$

where $\langle \dots \rangle$ represents the average of all simulation realizations.

A. Results on networks without inter-layer degree correlation

First of all, we carry out the research on the networks without inter-layer degree correlation. The degree distributions of the two subnetworks are identical, namely $P_A(k) = P_B(k) \sim k^{-\gamma}$ is the power exponent that is set to be $\gamma = 2.4$ in this section. The other parameters of the network structure are as follow: the average degrees of the two subnetworks are $\langle k \rangle \equiv \langle k_A \rangle = \langle k_B \rangle = 10$, and the maximum and minimum degrees of the network are $k_{\max} = \sqrt{N}$ and $k_{\min} = \sqrt{N}$, respectively.

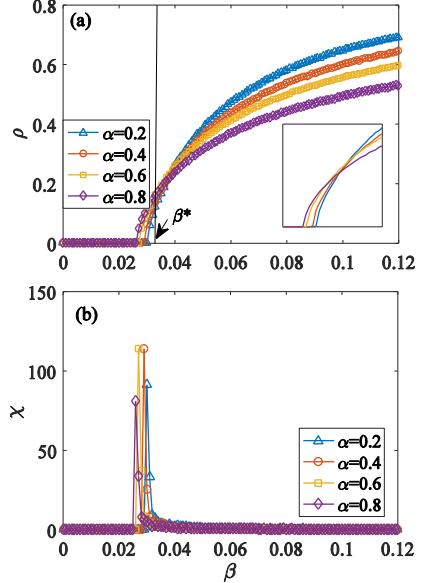


Fig. 2. (Color online) Influence of the local information driven awareness on the coupled dynamics on networks without inter-layer degree correlation. (a) The fraction of infected nodes ρ as a function of the infection rate β for $\alpha = 0.2, 0.4, 0.6$ and $\alpha = 0.8$ respectively, the arrow indicates the critical value β^* . Inset shows the enlarged information around the cross point. (b) The value of χ as a function of β . Each point in the figure is obtained by averaging over 500 independent simulations

Figure 2 exhibits the fraction of infected nodes at the steady state for four typical values of awareness. To perform the simulations, the other parameters are as follow: the basic recovery rate is set as $\mu = 0.1$, the resource utilization is $\varepsilon = 0.6$ and the initial willingness of resource allocation is $q_0 = 0.8$. To reduce fluctuations due to experimental errors, each data in the figure is obtained by averaging over 500 independent realizations. From figure 2 (a), we find that there is a critical value of the basic infection rate $\beta = 0.037$ on the parameter plane that the where all the curves intersect. Specifically, when $\beta < \beta^*$, the epidemic threshold decreases β_c with the increase of α . As shown in figure 2 (b), the peaks of the curves indicate the location of β_c . While, when $\beta > \beta^*$, the fraction of infected nodes ρ decreases with α . The results in this section suggest that when there is a relatively small value of infection rate, individuals should not be too cautious and afraid of the epidemic, and they should be encouraged to increase the mutual assistance of resources to control the large-scale spread of the disease to the greatest extent. While when there is a relatively large value of infection rate of the disease, the disease spreads rapidly through the network, and there are a high proportion of infected nodes in the network. Under this circumstance, individuals should be on high alert for disease, and they should keep the resources for self-protection.

B. Results on networks with inter-layer degree correlation

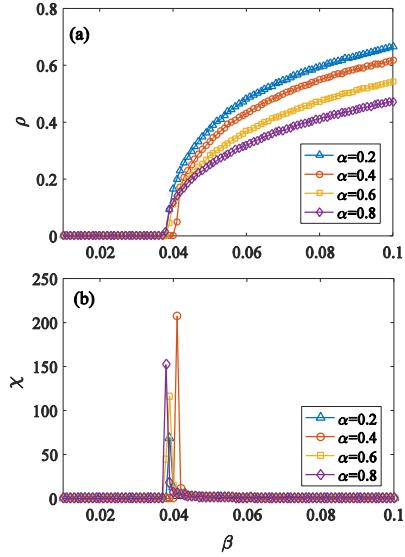


Fig. 3. (Color online) The impact of the inter-layer degree correlation on the coupled dynamics. (a) The fraction of infected nodes ρ as a function of β for four typical values of α , the inter-layer degree correlation is $\pi_d = 0.8$. (b) The relationship between the susceptibility measure χ and the basic infection rate β . Data is obtained by averaging the results of 200 independent simulations.

Next, we study the effects of the inter-layer degree correlation on the results. First of all, we construct the correlation networks by the following steps: first of all, two degree sequences are generated from the degree distribution defined in the previous section. And then, the two sequences are sorted in ascending order for a network with positive correlation, or one of the sequences is sorted in descending order and the other one is sorted in ascending order for a network with negative correlation. Next, one of the sequences is randomly selected and shuffled with probability p . At last, the nodes are connected as the previous section. Following the above steps, a double-layer network with a certain degree correlation π_d is obtained.

In figure 3 (a), we display the fraction of infected nodes at the steady state ρ as a function of basic infection β when the two layer correlated positively, i.e. $\pi_d = 0.8$. We find that the cross phenomenon almost disappears, and the epidemic threshold are nearly identical for all values of α , as shown in figure 2 (b). Moreover, we also find that when there is a relatively large infection rate β , the fraction of infected nodes ρ decreases with the increase of α , which is accordance with the results obtained on the networks without inter-layer degree correlation.

Next, we explore the effects of the negative degree correlation on the results. As shown in figure 4, the degree correlation is set to be $\pi_d = -0.8$. We find that

the phenomena are similar with the results obtained on networks without degree correlation. There is also a critical point β^* . When $\beta < \beta^*$, the epidemic threshold increases with the decrease of α , while when $\beta > \beta^*$, the spread size ρ decreases with the increase of α .

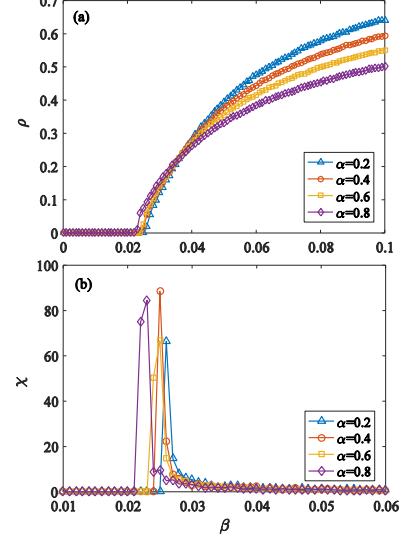


Fig. 4. (Color online) (Color online) The impact of the inter-layer degree correlation on the coupled dynamics. (a) The fraction of infected nodes ρ as a function of β for four typical value of α , the inter-layer degree correlation is $\pi_d = -0.8$. (b) The relationship between the susceptibility measure χ and the basic infection rate β . Data is obtained by averaging the results of 200 independent simulations.

At last, we study systematically the effects of the inter-layer degree correlation on the coupled dynamics by exploring the relationship between the values of ρ and π_d . As shown in figures 5 (a) and (b), the values of the basic infection rate are set to be $\beta = 0.03$ and $\beta = 0.06$ respectively. We find that under the two infection rates, the fraction of infected nodes ρ decreases monotonously with the increase of the value of π_d . In addition, there is a boundary for each subplot that is denoted by the dotted line, when the value of π_d exceeds the value indicated by the boundary, the fraction of infected nodes drops sharply. The result obtained in this section indicates that a more positive inter-layer degree correlation is more conducive to the containment of the epidemic.

IV. CONCLUSIONS

In summary, we have studied the effects of the information-driven awareness on the resource-epidemic dynamics on the multiplex networks. We have considered that during the pandemic, the awareness of the individuals is stimulated by the information about the epidemic, which alters the behavior of the

individuals. Specifically, the awareness of the individuals affects the mutual assistance of resources, a

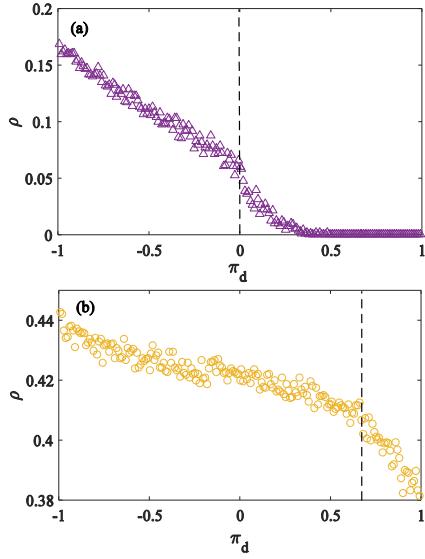


Fig. 5. (Color online) The fraction of infected nodes ρ as a function of inter-layer degree correlation π_d for $\beta=0.03$ (a) and $\beta=0.06$ (b) respectively. The vertical dotted lines in the two subfigures split the parameter pane into two parts.

lower level of awareness indicates a higher level of resource allocation, on the contrary, a lower level of resource allocation. In addition, we have focused on the local information that is represented by the fraction of infected neighbors. To depict the coupled dynamics, the double-layer multiplex networks that are composed of the social and contact layers are adopted in this paper. The spread of the epidemic has been modeled by a resource based SIS model, in which the probability of being infected of a susceptible node is affected by the resource quantity allocated by it. Based on Monte Carlo simulations, we have found that on the networks without inter-layer degree correlation, there is a cut-off point β^* at which the dynamics of the two sides are completely opposite. When $\beta < \beta^*$, with the decrease of the level of awareness, the epidemic threshold increases, which suggests that when the infection rate is low, the individuals should be encouraged to increase the mutual assistance of resources. While when $\beta > \beta^*$, the fraction of infected nodes decreases with the increase of awareness, which indicates that under this circumstance, individuals should be on high alert for disease, and they should keep the resources for self-protection. Moreover, we have also studied the inter-layer degree correlation on the results and found that, a more positive inter-layer degree correlation is more conducive to the containment of the epidemic. The conclusions obtained in this paper is of practical significance for the development of control strategies for the outbreak of epidemics.

ACKNOWLEDGMENT

This work was supported by the Special Cultivation Project of the University Level Scientific Research Project of Aba Teachers University (No. ASZ21-03), and the National Natural Science Foundation of China (No. 62002250).

REFERENCES

- [1] S. Boccaletti, V. Latora, Y. Moreno et al., "Complex networks: Structure and dynamics," *Phys. Rep.*, vol. 424, no. 4-5, pp. 175–308, 2006.
- [2] M. Newman, "Networks: an introduction," United States: Oxford University Press Inc., New York, 2010.
- [3] R. Pastor-Satorras, C. Castellano, P. Van Mieghem, and A. Vespignani, "Epidemic processes in complex networks," *Rev. Mod. Phys.*, vol. 87, no. 3, p. 925, 2015.
- [4] P. Holme, J. Saramaki, "Temporal networks," *Phys. Rep.*, vol. 519, no. 3, pp. 97–125, 2012.
- [5] L. Zhu, W. Liu, Z. Zhang, "Interplay between epidemic and information spreading on multiplex networks," *Math. Comput. Simul.*, vol. 188, pp. 268–279, 2021.
- [6] C. Granell, S. Gomez, and A. Arenas, "Dynamical interplay between awareness and epidemic spreading in multiplex networks," *Phys. Rev. Lett.*, vol. 111, no. 12, p. 128701, 2013.
- [7] H. Zhang, "The effects of global awareness on the spreading of epidemics in multiplex networks," *Physica A*, vol. 492, pp. 1495–1506, 2018.
- [8] P. Holme, B. J. Kim, C. N. Yoon, S. K. Han, "Attack vulnerability of complex networks," *Phys. Rev. E*, vol. 65, no. 5, p. 056109, 2002.
- [9] R. Cohen, S. Havlin, D. Ben-Avraham, "Efficient immunization strategies for computer networks and populations," *Phys. Rev. Lett.*, vol. 91, no. 5, p. 247901, 2003.
- [10] X. Wang, X. Zhu, X. Tao et al., "Anomalous role of information diffusion in epidemic spreading," *Phys. Rev. Res.*, vol. 3, no. 1, p. 013157, 2021.
- [11] R. Wang, "Information-driven resource allocation for epidemic controlling on multiplex networks," 2022 IEEE 6th Information Technology and Mechatronics Engineering Conference (ITOEC), vol. 6, pp. 1643–1647, 2022.
- [12] I. J. Rao, M. L. Brandeau, "Optimal allocation of limited vaccine to control an infectious disease: Simple analytical conditions," *Math. Biosci.*, vol. 337, p. 108621, 2021.
- [13] A. Y. Lokhov, D. Saad, "Optimal deployment of resources for maximizing impact in spreading processes," *Proc. Natl. Acad. Sci.*, vol. 114, no. 39, pp. E8138–E8146, 2017.
- [14] X. Chen, R. Wang, D. Yang, et al. "Effects of the Awareness-Driven Individual Resource Allocation on the Epidemic Dynamics," *Complexity*, vol. 2020, p. 8861493, 2020.
- [15] X. Zhu, Y. Liu, S. Wang et al., "Allocating resources for epidemic spreading on metapopulation networks," *Appl. Math. Comput.*, vol. 411, pp. 126531, 2021.
- [16] M. Catanzaro, M. Boguná, and R. Pastor-Satorras, "Generation of uncorrelated random scale-free networks," *Phys. Rev. E*, vol. 71, no. 2, p. 027103, 2005.
- [17] X. Chen, C. Yang, L. Zhong, and M. Tang, "Crossover phenomena of percolation transition in evolution networks with hybrid attachment," *Chaos*, vol. 26, no. 8, p. 083114, 2016.