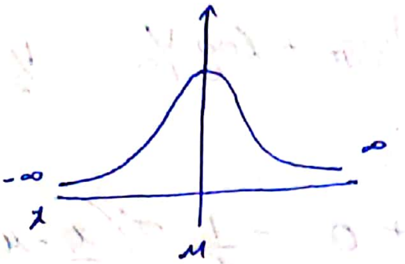
Univariate Gaussian distribution :-

$$P(x/\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$p(x)$

i) $\int_{-\infty}^{\infty} P(x) = 1$

ii) $P(x) \geq 0$



Multivariate :-

$x = [x_1 \; x_2 \; x_3]$ ↓ column matrix

Σ - covariance matrix tells ~~an~~ relationship b/w dimensions.

d - dimensions.

$$P(x/\mu, \Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|} e^{-\frac{1}{2}[x-\mu]^T \cdot \Sigma^{-1} \cdot [x-\mu]}$$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}$

· Mahalanobis distance $= [x-\mu]^T \cdot \Sigma^{-1} \cdot [x-\mu]$

If all dimension are ~~ide~~, independent Then Σ = unit matrix.

i.i.d. inividual identical distribution.

Let $x_1, x_2, \ldots x_N$ be i.i.d

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2\left[\frac{x-\mu}{\sigma}\right]^2}$$

Joint sourcing func of the samples (likeiwood function) $= \prod_{i=1}^{N} P(x_i)$

$$\log \text{Likiwood} = \Sigma \log P(x_i)$$

$$= \Sigma \log \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2} \right]$$

$$= \Sigma \left[\log \frac{1}{\sqrt{2\pi}} + \log \frac{1}{\sigma} - \frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2 \right]$$

$$\frac{d}{d\mu} \log \text{Likiwood} = \Sigma \left(0 + 0 - \cancel{\frac{1}{2}} \cancel{2} \left(\frac{x_i-\mu}{\sigma}\right) \frac{\times 1}{\sigma} \right) = 0$$

$$= \Sigma (x_i - \mu) = 0$$

$$\Sigma x_i = n\mu$$

$$\mu = \frac{1}{n} \Sigma x_i$$

$$\frac{d}{d\sigma} \log(\text{Likiwood}) = \Sigma \left(0 + \frac{1}{\frac{1}{\sigma}} \cdot \frac{-1}{\sigma^2} + \cancel{\frac{1}{2}} \cancel{2} \frac{(x_i-\mu)}{\sigma} \cdot \frac{x_i-\mu}{\sigma^2} \right) = 0$$

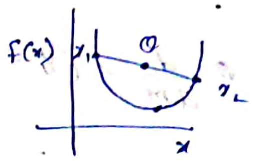
$$\frac{\sigma N}{\sigma} = \frac{\Sigma (x_i - \mu)^2}{\sigma^{3}{}^2}$$

$$\sigma^2 = \frac{1}{N} \Sigma (x_i - \mu)^2$$

convex optimisation.

min $f_0(x)$ $\quad x \in \mathbb{R}^n$ (n dimension vector)

~~such that~~ s.t $f_1(x) \le 0$



$y = ax^2 + bx + c$

when function is a convex if any two points of $f(x)$ the line joing on the two points is witin the curve then the function is called convex function.

$\theta x_1 + (1-\theta)x_2 \quad 0 \le \theta \le 1$

↓

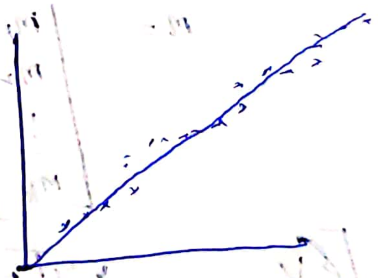
Be any point in the line segment b/w x_1 & x_2.

~~$f(x_1) \le f(\theta x_1 + (1-\theta)x_2) \le f(x_2)$~~

If $f(\theta x_1 + (1-\theta)x_2) \le f(x_1)\theta + (1-\theta)f(x_2)$ Then the function is convex. for every x_1, x_2.

1) Least squares problem.

2) Linear programming



$y = mx + c$

The line at which the distance sum of square of distance is minimum.

$$(y_1 - (mx_1+c))^2 + (y_2 - (mx_2+c))^2 + \cdots + (y_n - (mx_n+c))^2$$

$$\Sigma \left(y_i - (m(x_i + c))\right)^2$$

$$\frac{d}{dm} \Sigma \left(y_i - m(x_i + c)\right)^2$$

$$= \Sigma\, 2\left(y_i - m(x_i + c)\right)\left(x_i + \cancel{c}\right) = 0$$

$$\frac{d}{dc} \Sigma \left(y_i - m(x_i + c)\right)^2$$

$$= \Sigma\, 2\left(y_i - m(x_i + c)\right)\left(-1\right) = 0$$

$$[Y - (mX + c)]^T [Y - (mX + c)] \qquad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \quad X = \begin{bmatrix} x \\ x_1 \\ \vdots \\ x_N \end{bmatrix}$$

=

Data matrix $X = \begin{bmatrix} 1 & & \\ 1 & & \\ \vdots & & \\ 1 & & \end{bmatrix}_{N \times K}$ let K be no of attributes.

$$m = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_K \end{bmatrix}_{K \times 1}$$

$$\theta = [(Y - Xm)]^T [Y - Xm]$$

let $Y - Xm = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix} = D$

$\Rightarrow \quad D^T D = \sum_{i=1}^{N} d_i^2$

2) Linear programming.

$\min \quad c^T x \qquad x \in R^k$

constrains. $\quad a_i^T x \leq 0 \qquad 1 \leq i \leq N$

$\Downarrow$

$Ax \leq 0$

Quadric optimisation

$\min \quad x^T A x$

A - positive definite matrix.

equality constrains.

$h_i(x) = 0$

inequality $\; g_i(x) \leq 0$

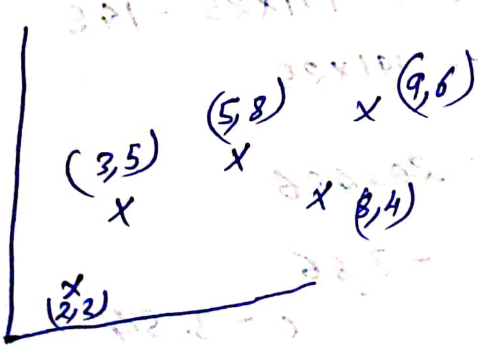
$\rightarrow$ Find if i) $x \log x$
ii) e^{ax}
iii) x^{α} $\quad \alpha \geq 1$, $\alpha \leq 0$ $\quad$ are convex.

~~$x \log x$~~

~~let x_1, x_2 θ~~

~~$x \log x$ $\quad \theta x_1 + (1-\theta) x_2$~~

$\rightarrow$



Find m, c.

$y = mx + c$

2,2
3,5
5,8
9,6
8,4

$(2-(2m+c))^2 + (5-(3m+c))^2 + (8-(5m+c))^2 + (6-(9m+c))^2 + (4-(8m+c))^2$

$= (4-4m-2c)(+2) + (10-6m-2c)(+3) + (16-10m-2c)(+5) + (12-18m-2c)(+9) + (8-16m-2c)(+8) = 0$

$290 - \cancel{382}\,366\,m - 54c = 0 \rightarrow (1)$

$50 - 54m - 10c = 0 \rightarrow (2)$

$54m = 50 - 10c$

$m = \frac{50-10c}{54}$

$290 - \cancel{382}\,191 \frac{(50-10c)}{\cancel{54}\,27} - 54c = 0$

$145 - 191\frac{(25-5c)}{27} - 27c = 0$

$145 - 191 \times 25 + 955c - 729c = 0$

$226c = 191 \times 25 - 145$

$c = 191 \times 25$

$c = \cancel{20.486}$

$m = \cancel{-2.86}$

$c = 3.54$

$m = 0.26$

3.54

i) $x \log x$

$\boxed{x_1 < x_2}$

$x_1 \log x_1$

i)

ii) e^{ax}

iii) x^{α} $\alpha_0 >$

α-ev

i) $x \log x$

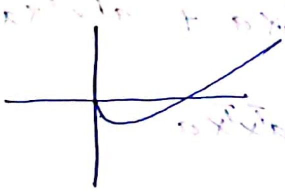
$\boxed{x_1 < x_2}$

$\theta x_1 + (1-\theta) x_2$

$\theta x_1 \log x_1 + (1-\theta) x_2 \log x_2$

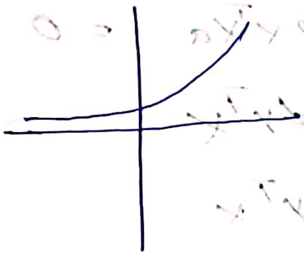
$x_1 \log x_1$ $\quad$ $x_2 \log x_2$

i)



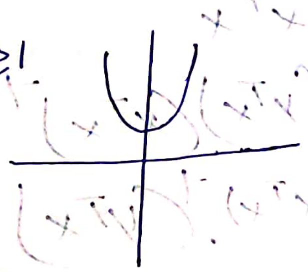
convex

ii) e^{ax}



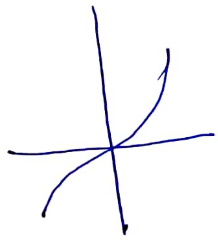
convex

iii) x^{α} $\quad$ $\alpha \geq 1$



α-even

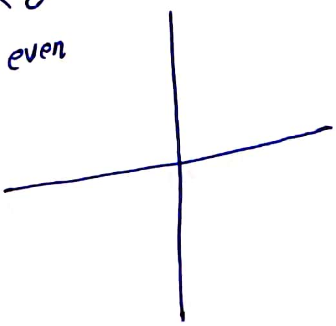
convex



α-odd $\quad$ not convex.

$\alpha < 0$

α - even



α - odd

$$[Y - Xa]^T [Y - Xa]$$

$$= y^T y - (a^T x^T y)^T - y^T x a + a^T x^T x a$$

$$= y^T y - 2 y^T X a + a^T x^T x a$$

$$\frac{\delta}{\delta a}\left[y^T y - 2 y^T X a + \underset{1\times k}{a^T} \underset{k\times k}{x^T x} \underset{k\times 1}{a} \right]$$

$$= 0 - 2 y^T X + 2 x^T X a = 0$$

$$\cancel{2} x^T X a = \cancel{2} y^T X$$

$$x^T X a = y^T X$$

$$a = \frac{y^T X}{x^T X}$$

$$a = (y^T X)(x^T x)^{-1}$$

$$(x^T x)^{-1} (y^T X)$$

(o - b)^T

(a^T b^T)

N×

k×1

1×k

1×1

~~Affine sets~~:

Convex
Affine

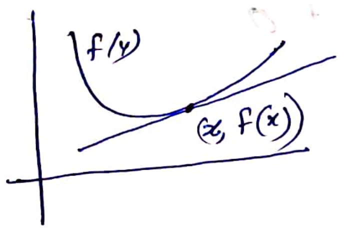
Con~~eave~~ cave → double derivative is −ve.

(or) convex → double derivate is +ve

Conditions:-

First order derivative:

$$f(y) \geq f(x) + \nabla f(x)^T (y-x)$$



→ The Tranget of a point x on f(x) is always below the curve. ⇒ for any point y on the f(y) is greater than the ~~value~~ interval of that y on the line.

Secound order derivative:-

$$ax_1^2 + bx_2^2 + cx_1x_2$$

$$x \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \begin{bmatrix} 2ax_1 + cx_2 \\ 2bx_2 + cx_1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2a & c \\ c & 2b \end{bmatrix}$$

single derivative

double derivative.

$\nabla^2 f(x) \geq 0$ → convex

$\nabla^2 f(x) > 0$ → strictly convex.

$$f(x) = \|Ax - b\|^2$$

$$= \|(Ax - b)^T (Ax - b)\|$$

$$f(x) = x^T A^T A x - x^T A^T b - b^T A x + b^T b$$

$$= x^T A^T A x - 2b^T A x + b^T b$$

$$\frac{\delta f(x)}{\delta x} = \cancel{2x^T x} \; 2A^T A x - 2b^T A + 0$$

$$= 2A^T (Ax - b)$$

$$\frac{\delta f(x)}{\delta x^2} = 2A^T (A)$$

$$= 2A^T A$$

for any matrix it is ≥ 0

then it's ~~a~~ moduls ≥ 0 (M)

$$x^T M x \geq 0.$$

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$\rightarrow f(x,y) = x^2/y$ convex $\forall\, y > 0$

$$\frac{\delta f(x,y)}{\delta x} = \frac{2x}{y} \qquad \frac{\delta f(x,y)}{\delta y} = \frac{-x^2}{y^2}$$

$$\frac{\delta f(x,y)}{\delta X} = \begin{bmatrix} \frac{2x}{y} \\ \frac{-x^2}{y^2} \end{bmatrix} \qquad \begin{bmatrix} 2/y & -\frac{2x}{y^2} \\ -\frac{2x}{y^2} & +\frac{2x^2}{y^3} \end{bmatrix}$$

$$\frac{4x^2}{y^4} - \frac{4x^2}{y^4} \geq 0$$

convex.

$$\frac{4x^2}{y^2}\left(\frac{1}{y^2} \geq 1\right)$$

$\rightarrow$ log sum exp

$$f(x) = \log\left[\sum_{k=1}^{n} e^{x_k}\right]$$

$$\nabla f(x) = \frac{1}{\sum_{k=1}^{n} e^{x_k}} \begin{bmatrix} e^{x_1} \\ \vdots \\ e^{x_n} \end{bmatrix}$$

$$-\frac{e^{x_1} e^{x_2}}{y^3}$$

$$\nabla^2 f(x) = \frac{\left(\sum_{k=1}^{n} e^{x_k}\right)^2 - (e^{x_1})^2}{}$$

$$\left[\begin{matrix} \frac{y^2 - (e^{x_1})^2}{y^3} & \frac{e^{x_1}(e^{x_2})^2}{y^3} \\ \frac{-(e^{x_1})e^{x_2}}{y^3} & \\ \vdots & \end{matrix}\right.$$

≥ 0

$n \times n$

$\rightarrow$ min $5x^2 + 10y^2 - 8xy = 0$

$2x - 5y = 6$

$5x^2 + 10y^2 - 8xy = 0$

$2x - 6 = 5y$

$25x^2 + \frac{2}{\cancel{5}}(2x-6)^2 - \frac{8}{\cancel{5}}x(2x-6) = 0$

$25x^2 + 8x^2 + 72 - \cancel{48x} - 16x^2 + \cancel{48x} = 0$

$17x^2 = -72$

$\Rightarrow$ $f(x) = \log\left(\sum_{k=1}^{n} e^{x_k}\right)$ log sum exp

prove that this function is convex $x \in R^n$

$x =$ vector $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$

$$\frac{\delta}{\delta x} f(x) = \begin{bmatrix} \frac{1}{\sum_{k=1}^{n} e^{x_k}} e^{x_1} \\ \vdots \\ \vdots \\ \frac{1}{\sum_{k=1}^{n} e^{x_k}} e^{x_n} \end{bmatrix}$$

$$\frac{\delta f(x)}{\delta x^2} = \begin{bmatrix} \frac{y^2 - e^{(x_1)^2}}{y^3} & \frac{-e^{x_1+x_2}}{y^3} & \cdots \\ \frac{-e^{x_1+x_2}}{y^3} & \frac{y^2 - e^{2x_2}}{y^3} & \cdots \\ \frac{-e^{x_1+x_3}}{y^3} & \vdots & \\ \vdots & \vdots & \end{bmatrix} \geq 0$$

$$\frac{y^2 - e^{2x_1}}{y^3}\left(\frac{y^2 - e^{2x_2}}{y^3}\right)$$

$$y^4 - y^2\left(e^{2(x_1)} + e^{2x_2}\right) + \cancel{2e^{2(x_1+x_2)}} - e$$

$$\left(\frac{y^2 - e^{2(x_1+x_2)}}{y^3}\right)^2 \geq 0$$

$$-e^{x_2+x_3} \quad -e^{x_1+x_2} \quad \left(\frac{y^2 - e^{2x_2}}{y^3}\right) - \frac{e^{x_1+x_3}}{y^6}$$

$$\frac{-e^{x_1+2x_2+x_3}}{y^3 6} \quad - y^2\left(e^{x_1+x_3} + e^{x_1+2x_2+x_3}\right)$$

$$\frac{y^2 e^{x_1+x_3} - 2e}{y^6} \quad e^{x_1+x_3}(\cdot \geq 0$$

$x A x^T \geq 0$

$x^T A^T x$

$x^T A x \geq 0$

$x A x^T + x^T A x \geq 0$

$\sum a_i v_i^2 - (\sum a_i v_i)^2 \geq 0$

$a_1 = \frac{e^{x_1}}{\sum e^{x_i}}$

$\frac{e^{x_1}}{e^{x_1} + e^{x_2} + e^{x_3}}$

$\sum a_i v_i^2 \geq (\sum a_i v_i)^2$

$\sum a_i^2 v_i^2 +$

$a_1 v_1^2 + a_2 v_2^2 + a_3 v_3^2$

$a_1^2 v_1^2 + a_2^2 v_2^2 + a_3^2 v_3^2$
$+ 2 a_1 a_2 v_1 v_2 + 2 a_2 a_3 v_2 v_3$
$2 a_1 a_3 v_1 v_3$

$\frac{a_1 + a_2 + a_3}{1}$

Linear Programming

$x \in R^n$. n dimension.

Find min $c^T x$ $\Rightarrow$ max $c^T x$

$Ax \geq b$, $x \geq 0$ $\qquad$ $Ax \leq b$ $x \geq 0$

$$A = \underbrace{\begin{bmatrix} a_{11} & a_{1j} & \cdots & a_{1n} \\ \vdots & & & \\ a_{i1} & a_{ij} & & a_{in} \\ a_{m1} & a_{mj} & - & a_{mn} \end{bmatrix}}_{m \times n} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{n \times 1} \geq \underset{m \times 1}{b}$$

$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \geq b_1$

$a_{i1} x_1 + \cdots a_{ij} + a_{in} x_n \geq b_i$

a

simplex Method :-

max $4x_1 + 6x_2$

such that (s.t) $-x_1 + x_2 \le 11$

$x_1 + x_2 \le 27$

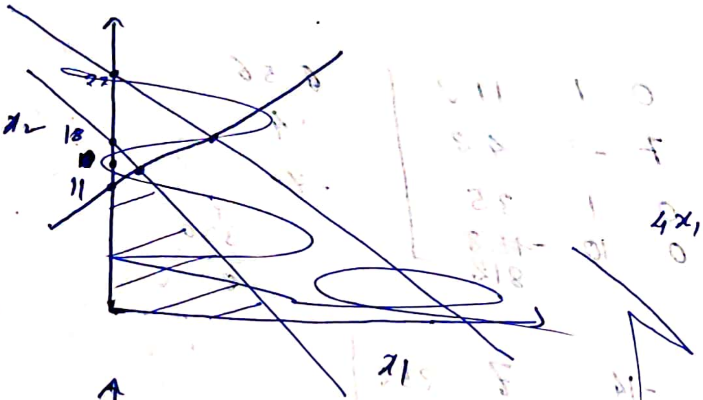
$2x_1 + 5x_2 \le 90$

$x_2 = 11 + x_1$

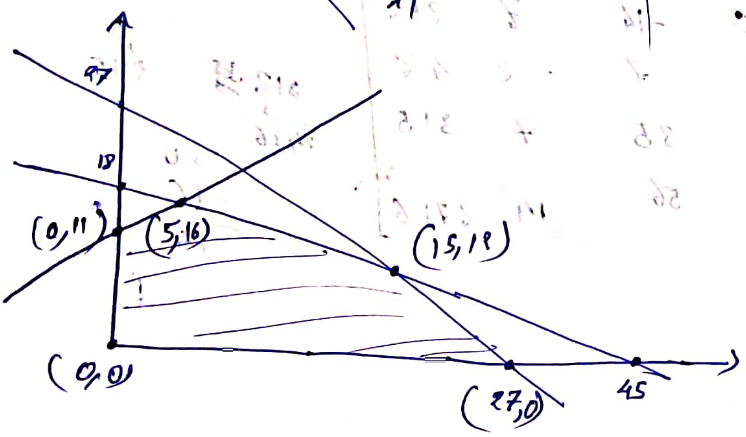
$x_2 \le 27 - x_1$

$5x_2 \le 90 - 2x_1$

$x_2 \le 18$



$4x_1 + 6x_2$



$-x_1 + x_2 + s_1 = 11$

$x_1 + x_2 + s_2 = 27$

$2x_1 + 5x_2 + s_3 = 90$

s_1, s_2, s_3 are slag Variabls.

max => $Z = 4x_1 + 6x_2 + 0s_1 + 0s_2 + 0s_3$

$Z - 4x_1 - 6x_2 - 0s_1 - 0s_2 - 0s_3 = 0$

$$A = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & \\ -1 & 1 & 1 & 0 & 0 & 11 \\ 1 & 1 & 0 & 1 & 0 & 27 \\ 2 & 5 & 0 & 0 & 1 & 90 \\ \hline -4 & -6 & 0 & 0 & 0 & 0 \end{array}$$

$11/1 \leftarrow$ least +ve.

$27/1$

$90/5 = 18$

↑ max -ve

$R_2 = R_2 - R_1$
$R_3 = R_3 - 5R_1$

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 11 \\ 2 & 0 & -1 & 1 & 0 & 16 \\ \boxed{7} & 0 & -5 & 0 & 1 & 35 \\ -10 & 0 & +6 & 0 & 0 & 466 \end{bmatrix}$$

-11
8
5

Z = maximise

$$\begin{bmatrix} 0 & 7 & 2 & 0 & 1 & 112 \\ 0 & 0 & 3 & 7 & -2 & 42 \\ 7 & 0 & -5 & 0 & 1 & 35 \\ 0 & 0 & \cancel{-95} \; -84 & 0 & 10 & \cancel{-112} \; 812 \end{bmatrix}$$

56
14
-7

350
462
812

$2x_1$
$4x_1$
$3x_1$

Z

→ (0,0)
→ Artificial
na slay
$s_1 s_2$

$$\begin{bmatrix} 0 & 21 & 0 & -14 & 7 & 252 \\ 0 & 0 & 3 & 7 & -2 & 42 \\ 21 & 0 & 0 & 35 & 7 & 315 \\ 0 & 0 & 0 & 56 & 14 & 2716 \end{bmatrix}$$

105
210

336
84

812
3
2436

280

30
-16
14

→ 2 phase

$-1a_1$
$-1a_2$
$0 \; s_1$
s_2

simplex Method :-

max $4x_1 + 6x_2$

such that (s.t) $-x_1 + x_2 \le 11$

$x_1 + x_2 \le 27$

$2x_1 + 5x_2 \le 90$

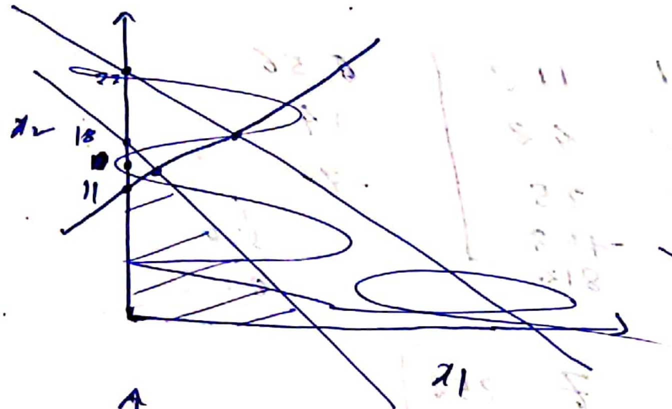
$x_2 = 11 + x_1$

$y = mx + c$

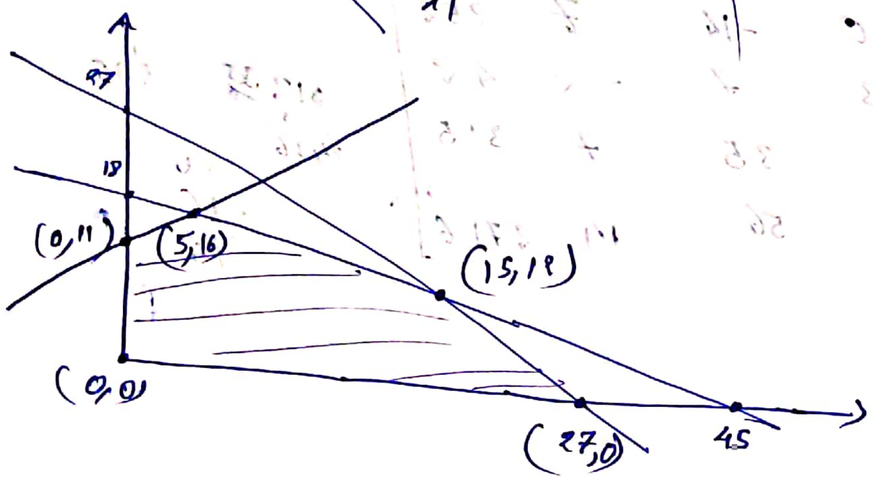
$x_2 \le 27 - x_1$

$5x_2 \le 90 - 2x_1$

$x_2 \le 18$



$4x_1 + 6x_2$



72
60
132

16
+6

20,
116

$-x_1 + x_2 + S_1 = 11$

$x_1 + x_2 + S_2 = 27$

$2x_1 + 5x_2 + S_3 = 90$

S_1, S_2, S_3 are slag Variables.

$\max \Rightarrow Z = 4x_1 + 6x_2 + 0S_1 + 0S_2 + 0S_3$

$\underline{Z} - 4x_1 - 6x_2 - 0S_1 - 0S_2 - 0S_3 = 0$

$$A = \begin{array}{cccccc} x_1 & x_2 & S_1 & S_2 & S_3 & \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} -1 & 1 & 1 & 0 & 0 & 11 \\ 1 & 1 & 0 & 1 & 0 & 27 \\ 2 & 5 & 0 & 0 & 1 & 90 \\ \hline -4 & -6 & 0 & 0 & 0 & \underline{0} \end{array}\right]$$

$11/1 \leftarrow$ least +ve.

$27/1$

$90/5 = 18$

↑ max -ve

$R_2 = R_2 - R_1$
$R_3 = R_3 - 5R_4$

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 11 \\ 2 & 0 & -1 & 1 & 0 & 16 \\ \boxed{7} & 0 & -5 & 0 & 1 & 35 \\ -10 & 0 & +6 & 0 & 0 & 466 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 7 & 2 & 0 & 1 & 112 \\ 0 & 0 & 3 & 7 & -2 & 42 \\ 7 & 0 & -5 & 0 & 1 & 35 \\ 0 & 0 & \cancel{-46}\ -8 & 0 & 10 & \cancel{-112}\ 812 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 21 & 0 & -14 & 7 & 252 \\ 0 & 0 & 3 & 7 & -2 & 42 \\ 21 & 0 & 0 & 35 & 7 & 315 \\ 0 & 0 & 0 & 56 & 14 & 2716 \end{bmatrix}$$

56
14
-7

112
70

350
462
812

105
210

336
84

812
3
2436

280

-16
14

$Z =$ maximise $-2x_1 - 3x_2$

$3x_1 + 2x_2 = 14$

$2x_1 - 4x_2 \geq 2$

$4x_1 + 3x_2 \leq 19$

$2x_1 - 4x_2 - s_1 = 2 \quad s_1 \geq 0 \quad - (1)$

$4x_1 + 3x_2 + s_2 = 19 \quad s_2 \geq 0 \quad - (2)$

$3x_1 + 2x_2 + 0s_1 + 0s_2 = 14$

$Z + 2x_1 + 3x_2 + 0s_1 + 0s_2 = 0$

→ (0,0) is no used beause it valids the consai's.

→ Artifial ~~stat~~ varibles is used we add when ther no slag varble.

~~s_1, s_2~~ $s_1 \to$ subrus varib a.

$s_2 \to$ ~~stat sat~~ slag varibls. +

$3x_1 + 2x_2 + a_1 = 14 \qquad 14$

$2x_1 - 4x_2 - s_1 + a_2 = 2 \qquad 2$

$4x_1 + 3x_2 + s_2 = 0$

→ 2 phase - I phase → miminize a_1, a_2.

$Z' = -a_1 - a_2$

$Z' + a_1 + a_2 = 0$

	0 x_1	0 x_2	0 s_1	0 s_2	-1 a_1	-1 a_2		
$-1\ a_1$	3 0	2	0	0	1	0	14	14/3
$-1\ a_2$	2	-4	-1	0	0	1	2	1
$0\ s_2$	4 0	3	0	1	0	0	19	19/4
$C_j - Z_j$	-5	-2	-1	0	0	0		

↑

		0	0	0	0	-1	-1	0
		x_1	x_2	s_1	s_2	a_1	a_2	6
-1	a_1	0	8	3/2	0	1	-3/2	11
0	x_1	1	-2	-1/2	0	0	1/2	1
0	s_2	0	11	2	1	0	-2	15 ←
		0	8	3/2	0	0	-3/2	+11

-11

-1	a_1	0	0	1/22	-8/11	1	-1/22	$11 - \frac{120}{11}$	1/11
0	x_1	1	0	3/22	2/11	0	1/2 + 4/11	$1 - \frac{30}{11}$	-19/
0	x_2	0	1	2/11	1/11	0	-2/11	15/11	
		0	0	1/22	-8/11	0	-1 + 1/22		

		-2	-3	0	0		
		x_1	x_2	s_1	s_2		
0	s_1	0	0	1	-8/11	1/11	
-2	x_1	1	0	0	2/11	-41/11	41/2
-3	x_2	0	1	0	1/11	15/11	15 ← leaf.
		0	0	0	7/11		
					↑		

		-2	-3	0	0	c
0	s_1	0	8	1	0	11
-2	x_1	1	-2	0	0	-7/11
0	s_2	0	1	0	1/11	15/11
		0	1	0	0	142/11
			↑			

$\frac{142}{11}$

Maximize $z = -5x_1 - 3x_2 + 7x_3$

$x_1 \le 4$

s.t $2x_1 + 4x_2 + 6x_3 = 7$

$3x_1 - 5x_2 + 3x_3 \le 5$

$-4x_1 - 9x_2 + 4x_3 \le -4$

$x_1 \ge -2 \qquad 0 \le x_2 \le 4$

x_3 - free.

$2x_1 + 4x_2 + 6x_3 = 7$

$x_3 = \frac{7 - 2x_1 - 4x_2}{6}$ $\quad 3x_1 - 5x_2 + 3x_3 + s_1 = 5$

$-4x_1 - 9x_2 + 4x_3 + s_2 = -4$

$z = -5x_1 - 3x_2 + \frac{7}{6}(7 - 2x_1 - 4x_2)$

$3x_1 - 5x_2 + \frac{1}{2}(7 - 2x_1 - 4x_2) \le 5 \rightarrow (1)$

$-4x_1 - 9x_2 + \frac{2}{3}(7 - 2x_1 - 4x_2) \le -4 \rightarrow (2).$

$x_2 \le 4 \rightarrow (3)$

$z = -5x_1 - 3x_2 + \frac{7}{6}(7 - 2x_1 - 4x_2)$

$x_1 + 2 \ge 0$

$z = -5x_1 - 3x_2 + \frac{49}{6} - \frac{7}{3}x_1 - \frac{14}{3}x_2 + 10 + \frac{14}{3}$

Z

$\cancel{3x_1 7}$

$3x_1 - 7x_1 - 5x_2 - 2x_2 + 7/2 \le 5$

$-4x_1 - 7x_2 + \frac{7}{2} + s_1 = 5$

$-4x_1 - 9x_2 + \frac{14}{3} - s_2 = 4$

$+4x_1 + \frac{4}{3}x_1 + 9x_2 + \frac{8}{3}x_2$

$2x_1 + 4x_2 + 6x_3' - 6x_3'' + a_1 = 11$

$3x_1 - 5x_2 + 3x_3' - 3x_3'' + x_4 = 11$

$4x_1 + 9x_2 - 4x_3' + 4x_3'' - x_5 + a_2 = 12$

$x_2 + x_6 = 4$

		0	0	0	0	0	0	0	-1	-1	
		x_1	x_2	x_3'	x_3''	x_4	x_5	x_6	a_1	a_2	C
-1	a_1	2	4	6	-6	0	0	0	1	0	11
0	x_4	3	-5	3	-3	1	0	0	0	0	11
-1	a_2	4	$\boxed{9}$	-4	4	0	-1	0	0	1	12
0	x_6	0	1	0	0	0	0	1	0	0	4
	-6	6	13	2	-2	0	-1	0	1	1	
			↑								

	0	0	0	0	0	0	0	-1	-1	
	x_1	x_2	x_3'	x_3''	x_4	x_5	x_6	a_1	a_2	C
-1 a_1	2/9	0	$\boxed{70/9}$	-70/9	0	4/9	0	1	-4/9	75/9
0 x_4	47/9	0	7/9	-7/9	1	-5/9	0	0	5/9	159/9
0 x_2	4/9	1	-4/9	$\boxed{4/9}$	0	-1/9	0	0	1/9	12/9
0 x_6	-4/9	0	4/9	-4/9	0	1/9	1	0	-1/9	24/9
-1	2/9	0	70/9	-70/9	0	4/9	0	0	-13/9	
			↑							

	x_1	x_2	x_3'	x_3''	x_4	x_5	x_6	a_1	a_2	C
x_3'	2/70	0	1	-1	0	4/70	0	1/70	-4/70	51/70
x_4	463/70	0	0	0	1	-54/70	0	-1/70	α	1549/70
	288/70	1	0	0	0	-54/70	0	α	α	1049/70
	-288/70	0	0	0	0	54/70	1	α	α	104/70

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