Proceedings of the *Open source GIS - GRASS users conference 2002* - Trento, Italy, 11-13 September 2002

The solar radiation model for Open source GIS: implementation and applications

Jaroslav Hofierka[*,**], Marcel Šúri[*,***]

[*] GeoModel s.r.o., M. Marečka 3, 841 07 Bratislava, Slovak Republic, http://www.geomodel.sk

[**] Department of Geography and Geoecology, Faculty of Humanities and Natural Science, University of Prešov, 17. novembra 1, 081 16 Prešov, Slovak Republic, tel. +421 905 494487, fax +421 2 65315915, e-mail hofierka@geomodel.sk

[***] European Commission Joint Research Centre, Institute for Environment and Sustainability, 210 20 Ispra (VA), Italy, tel. +39 0332 789977, fax +39 0332 786394, e-mail marcel.suri@jrc.it.

1 Introduction

Solar radiation, incident to the earth's surface, is a result of complex interactions of energy between the atmosphere and surface. At global scale, the latitudinal gradients of radiation are caused by the geometry of the earth and its rotation and revolution about the sun. At regional and local scales, terrain (relief) is the major factor modifying the distribution of radiation. Variability in elevation, surface inclination (slope) and orientation (aspect) and shadows cast by terrain feature creates strong local gradients. The spatial and temporal heterogeneity of incoming solar energy determines dynamics of many landscape processes, e.g. air and soil temperature and moisture, snow melting, photosynthesis and evapotranspiration, with direct impact to the human society. Accurate and spatially distributed solar radiation data are desired for various applications (environmental science, climatology, ecology, building design, remote sensing, photovoltaics, land management, etc.).

There are several hundreds of ground meteorological stations directly or indirectly measuring solar radiation throughout the Europe. To derive spatial databases from these measurements different **interpolation techniques** are used, such as spline functions, weighted average procedures or kriging [1, 2, 3]. In mountainous regions, the use of additional information gained from satellite images may improve the quality of solar radiation interpolation using co-kriging approach [4, 5]. Spatially continuous irradiance values can be also derived directly from **meteorological geostationary satellites** (e.g. METEOSAT). Processing of satellite data provides less accurate values (compared to ground measurements), but the advantage is a coverage over vast territories at temporal resolution of 0.5-12 hours [6, 7]. Other techniques of generating spatial databases are **solar radiation models** integrated within geographical information systems (GIS). They provide rapid, cost-efficient and accurate estimations of radiation over large territories, considering surface inclination, aspect and shadowing effects. Coupling radiation models with GIS and image processing systems improves their ability to process different environmental data and co-operate with other models. A significant progress has been made toward developing solar radiation models in the last two decades [8].

One of the first GIS-based solar radiation models was *SolarFlux* [8, 9] developed for ARC/INFO GIS. Similar initiative was made by implementation of solar radiation algorithms into commercially available GIS Genasys using AML script [10]. Another approach for computing all three components of radiation was realised in a standalone model *Solei* under MS Windows that was linked to GIS IDRISI via data format [11, 12]. All three mentioned models use rather simple empirical formulas. Some of their parameters are spatially-averaged (lumped) and therefore are not suitable for calculations over large areas. More advanced methods for ecological and biological applications are used in *Solar Analyst*, developed as an ArcView GIS extension module [13]. In the pre-processing phase, based on the DTM, the model generates an upward-looking hemispherical viewshed. The similar procedure for generating a sunmap for every raster cell makes calculation considerably faster. The model is suitable for detailed-scale studies. It is not flexible enough for calculation of atmospheric transmissivity and diffuse proportion as it allows to set parameters available only for the nearest weather stations or just typical values. This makes its use for larger areas rather limited. The *SRAD* model [14, 15] was designed to model a complex set of short-wave and long-wave interactions of solar energy with Earth surface and atmosphere. Although based on a simplified representation of the underlying physics, the main solar radiation factors are considered and the model is able to characterize the spatial variability of the landscape processes. However, it is designed for modelling of topo- and mesoscale processes and the calculation over large territories is also limited.

Coming out of progress made and discussed during the preparation of the European Solar Radiation Atlas [16] and information on solar radiation models [17], it was decided to develop a new GIS-based model, denoted as *r.sun*. It is based on previous work done by Hofierka [18] and prepared for an open source environment of GRASS GIS. Its features eliminate the limitations of the previously discussed models. The set of parameters provides a sufficient flexibility to be used in various applications. At the same time, the open source implementation creates a basis for its future improvements.

The aim of this paper is to present capabilities, structure and performance of the *r.sun* solar radiation model and to show examples of its application. All three components of potential solar radiation are computed (beam, diffuse and reflected) for clear-sky conditions. The overcast conditions can be simulated as well using a clouds attenuation factor. The model is applicable for large regions, spatial variation of solar radiation due to terrain and terrain-shadowing effects are also included. The examples present application of solar radiation model for planning the photovoltaic systems in Central and Eastern Europe and for solar radiation modelling in mountainous terrain of Slovakia.

2 Interactions of solar radiation with landscape

According to the widely accepted terminology [19] the two concepts of solar (i.e. short-wave) radiation are used in this paper. The term **irradiance** is used to consider the solar power (instantaneous energy) falling on unit area per unit time [$W.m^{-2}$]. The term **irradiation** is used to consider the amount of solar energy falling on unit area over a stated time interval [$Wh.m^{-2}$]. The same symbols are used for irradiance and irradiation, the two concepts can be differentiated by context or by the attached units.

The interaction of solar radiation with the earth's atmosphere and surface is determined by 3 groups of factors:

1 the earth's geometry, revolution and rotation (declination, latitude, solar hour angle);
2 terrain (elevation, surface inclination and orientation, shadows);
3 atmospheric attenuation (scattering, absorption) by:
  3.1. gases (air molecules, ozone, CO_2 and O_2),
  3.2. solid and liquid particles (aerosols, including non-condensed water),
  3.3. clouds (condensed water).

The first group of factors determines the available extraterrestrial radiation based on solar position above horizon and can be precisely calculated using astronomic formulas.

The radiation input to the earth's surface is then modified by its terrain topography, namely slope inclination and aspect, as well as shadowing effects of neighbouring terrain features. This group of factors can be also modelled at high level of accuracy. The elevation above sea level determines the attenuation of radiation by the thickness of the atmosphere.

Intensity of the extraterrestrial solar radiation traversing through the earth's atmosphere is attenuated by various atmospheric constituents, namely gases, liquid and solid particles and clouds. The path length through atmosphere is also critical. Because of its dynamic nature and complex interactions the atmospheric attenuation can be modelled only at a certain level of accuracy that decreases from top to bottom.

The attenuation by gas constituents (factors 3.1) describes clear and dry (Rayleigh) atmosphere and is given by its relative optical air mass and optical thickness (m and $\delta_R(m)$ respectively, see below). These parameters can be determined at a good level of precision.

The attenuation by solid and liquid particles (factors 3.2) is described by the Linke turbidity (T_{LK}). It indicates the optical density of hazy and humid atmosphere in relation to a clean and dry atmosphere. In other words T_{LK} is the number of clean dry air masses that would result in the same extinction then real hazy and humid air. Due to a dynamic nature of the turbidity factor, its calculation and subsequent averaging leads to a certain degree of generalisation. There are clear seasonal changes of the turbidity (lowest values in winter, highest in summer), the values of turbidity factor always differ from place to place in a similar degree of magnitude and these differences are also correlated with the terrain elevation. It increases with an intensity of industrialisation and urbanisation. The values of Linke turbidity for different landscapes or world regions can be found in literature [e.g. 16, 19, 30] or in http://www.soda-is.com/ [20]).

Clouds (factor 3.3) are the strongest attenuates. Theoretical analysis of the attenuation of solar radiation passing through clouds requires great deal of information regarding instantaneous thickness, position and number of layers of clouds, as well as their optical properties. Therefore a simple empirical techniques are used to estimate the attenuation of cloud cover.

The radiation, selectively attenuated by the atmosphere, which is not reflected or scattered and reaches the surface directly is **beam** (direct) radiation. The scattered radiation that reaches the ground is **diffuse** radiation. The small part of radiation that is reflected from the ground onto the inclined receiver is **reflected** radiation. These three components of radiation together create **global** radiation.

In many applications a study of solar radiation under clear (i.e. cloudless) skies is very important. Maximum insolation is obtained when skies are absolutely clean and dry and relatively less radiation is received when aerosols are also present. Omitting the clouds attenuation factor (factor 3.3) leads to clear-sky radiation values.

3 Clear-sky radiation

The *r.sun* model estimates global radiation under clear-sky conditions from the sum of its beam, diffuse and reflected components. While the calculation of the beam component is quite straightforward, the main difference between various models (namely for inclined surfaces) is in treatment of the diffuse component. This component depends on climate and regional terrain conditions and therefore in Europe is often the largest source of estimation error. As the theoretical background of the *r.sun* model is based on the work undertaken for development of European Solar Radiation Atlas (ESRA) [15, 20, 21], underlying equations for diffuse radiation implemented in *r.sun* reflect especially European climate conditions. Computing of diffuse component to other regions might be associated with higher estimation error. Taking into consideration the existing diffuse models and their limitation, the ESRA team selected the diffuse model for inclined surfaces by Muneer [23] as it has a sound theoretical basis and thus more potential for later improvement. The ground reflected radiation contributes to inclined surfaces by only several percents and is sometimes ignored. In the *r.sun* model we have adopted solutions suggested by the ESRA team for both diffuse and reflected radiation.

3.1 Beam radiation

Outside the atmosphere, at the mean solar distance, the beam irradiance, also known as the solar constant (I_0), is 1367 $W.m^{-2}$. The earth's orbit is lightly eccentric and the sun-earth distance varies slightly across the year. Therefore a correction factor ε, to allow for the varying solar distance, is applied in calculation of the **extraterrestrial irradiance** G_0 normal to the solar beam [$W.m^{-2}$]:

$$G_0 = I_0\, \varepsilon \tag{1}$$

where:

$$\varepsilon = 1 + 0.03344 \cos (j' - 0.048869) \tag{2}$$

the day angle j' is in radians:

$$j' = 2\, \pi\, j/365.25 \tag{3}$$

and j is the day number which varies from 1 on January 1st to 365 (366) on December 31st.

The **beam irradiance normal to the solar beam** B_{0c} [$W.m^{-2}$], attenuated by the cloudless atmosphere, is calculated as follows:

$$B_{0c} = G_0 \exp \{-0.8662\, T_{LK}\, m\, \delta_R(m)\} \tag{4}$$

The term -0.8662 T_{LK} is the air mass 2 Linke atmospheric turbidity factor [dimensionless] corrected by Kasten [24]. The parameter m in equation (4) is the relative optical air mass [-] calculated using the formula [25]:

$$m = (p/p_0)/(\sin h_0^{ref} + 0.50572\, (h_0^{ref} + 6.07995)^{-1.6364}) \tag{5}$$

where h_0^{ref} is the corrected solar altitude h_0 (an angle between the sun and horizon) in degrees by the atmospheric refraction component Δh_0^{ref}:

$$\Delta h_0^{ref}=0.061359\, (0.1594+1.123\, h_0 + 0.065656\, h_0^2)/(1 + 28.9344\, h_0 + 277.3971\, h_0^2)$$
$$h_0^{ref} = h_0 + \Delta h_0^{ref} \tag{6}$$

The p/p_0 component in (5) is correction for given elevation z [m]:

$$p/p_0 = \exp (-z/8434.5) \tag{7}$$

The parameter $\delta_R(m)$ in (4) is the Rayleigh optical thickness at air mass m and is calculated according to the improved formula by Kasten [24] as follows:

for m <= 20:

$$\delta_R(m) = 1/(6.6296 + 1.7513\, m - 0.1202\, m^2 + 0.0065\, m^3 - 0.00013\, m^4) \tag{8}$$

for m > 20

$$\delta_R(m) = 1/(10.4 + 0.718\, m) \tag{9}$$

The **beam irradiance on a horizontal surface** B_{hc} [$W.m^{-2}$] is then calculated as:

$$B_{hc} = B_{0c} \sin h_0 \tag{10}$$

where h_0 is the solar altitude angle given by equation (13).

The **beam irradiance on an inclined surface** B_{ic} [$W.m^{-2}$] is calculated as:

$$B_{ic} = B_{0c} \sin \delta_{exp} \tag{11}$$

or

$$B_{ic} = B_{hc} \sin \delta_{exp}/\sin h_0 \tag{12}$$

where δ_{exp} is the solar incidence angle measured between the sun and an inclined surface defined in equation (15).

The **position of the sun in respect to a horizontal surface** is given by the two co-ordinates – solar altitude h_0 (an angle between the sun path and a horizontal surface), and solar azimuth A_0 (a horizontal angle between the sun and meridian - measured from East), and is calculated as follows [26, 27]:

$$\sin h_0 = C_{31} \cos T + C_{33} \qquad (13)$$
$$\cos A_0 = (C_{11} \cos T + C_{13})/((C_{22} \sin T)^2 + (C_{11} \cos T + C_{13})^2)^{1/2}$$

where:

$$C_{11} = \sin \varphi \cos \delta \qquad (14)$$
$$C_{13} = -\cos \varphi \sin \delta$$
$$C_{22} = \cos \delta$$
$$C_{31} = \cos \varphi \cos \delta$$
$$C_{33} = \sin \varphi \sin \delta$$

In the *r.sun* model we have implemented the sun declination δ [rad] defined as [28]:

$$\delta = \arcsin (0.3978 \sin (j' - 1.4 + 0.0355 \sin (j' - 0.0489))) \qquad (15)$$

where the calculation of the day angle j' [radians] is explained in equation (3). The hour angle T [rad] is calculated from the local solar time t expressed in decimal hours on the 24 hour clock as:

$$T = 0.261799 (t - 12)$$

The **position of the sun in respect to an inclined surface** is defined by the angle δ_{exp} [26, 27]. If an inclined surface is defined by the inclination angle (slope) γ_N and the azimuth (aspect) A_N (an angle between the projection of the normal on the horizontal surface and East) then:

$$\sin \delta_{exp} = C'_{31} \cos (T - \lambda') + C'_{33} \qquad (16)$$

where:

$$C'_{31} = \cos \varphi' \cos \delta \qquad (17)$$
$$C'_{33} = \sin \varphi' \sin \delta$$

and:

$$\sin \varphi' = - \cos \varphi \sin \gamma_N \cos A_N + \sin \varphi \cos \gamma_N \qquad (18)$$
$$\mathrm{tg}\, \lambda' = - (\sin \gamma_N \sin A_N)/(\sin \varphi \sin \gamma_N \cos A_N + \cos \varphi \cos \gamma_N).$$

The hour angle of the time of sunrise/sunset over a horizontal surface $T_h^{r,s}$ can be calculated then as:

$$\cos T_h^{r,s} = -C_{33}/C_{31} \qquad (19)$$

The hour angle of the time of sunrise/sunset over an inclined surface $T_i^{r,s}$ can be calculated by analogy:

$$\cos (T_i^{r,s} - \lambda') = -C'_{33}/C'_{31}. \qquad (20)$$

3.2 Diffuse radiation

As the cloudless sky becomes more turbid, the diffuse irradiance increases, while the beam irradiance decreases. The estimate of the **diffuse component on a horizontal surface** D_{hc} [W.m^{-2}] is made as a product of the normal extraterrestrial irradiance G_0, a diffuse transmission function Tn dependent only on the Linke turbidity factor T_{LK}, and a diffuse solar altitude function F_d dependent only on the solar altitude h_0 [16]:

$$D_{hc} = G_0\, Tn(T_{LK})\, F_d(h_0) \qquad (21)$$

The estimate of the transmission function $Tn(T_{LK})$ gives a theoretical diffuse irradiance on a horizontal surface with the sun vertically overhead for the air mass 2 Linke turbidity factor. The following second order polynomial expression is used:

$$Tn(T_{LK}) = -0.015843 + 0.030543\ T_{LK} + 0.0003797\ T_{LK}^2 \qquad (22)$$

The solar altitude function is evaluated using the expression:

$$F_d(h_0) = A_1 + A_2 \sin h_0 + A_3 \sin^2 h_0 \qquad (23)$$

where the values of the coefficients A_1, A_2 and A_3 are only depended on the Linke turbidity factor T_{LK} defined in the following expressions:

$$\begin{array}{ll} A_1' = 0.26463 - 0.061581\ T_{LK} + 0.0031408\ T_{LK}^2 & \\ A_1 = 0.0022/Tn(T_{LK}) & \text{if } A_1'\ Tn(T_{LK}) < 0.0022 \\ A_1 = A_1' & \text{if } A_1'\ Tn(T_{LK}) >= 0.0022 \\ A_2 = 2.04020 + 0.018945\ T_{LK} - 0.011161\ T_{LK}^2 & \\ A_3 = -1.3025 + 0.039231\ T_{LK} + 0.0085079\ T_{LK}^2 & \end{array} \qquad (24)$$

The model for estimating the clear-sky **diffuse irradiance on an inclined surface** D_{ic} [W.m^{-2}] [23] distinguishes between sunlit, potentially sunlit and shadowed surfaces. The equations are as follows:

a) for sunlit surfaces and non-overcast sky (h_0 in radians):

if $h_0 >= 0.1$ (i.e. 5.7°)

$$D_{ic} = D_{hc} \{F(\gamma_N)(1 - K_b) + K_b \sin \delta_{exp}/\sin h_0\} \qquad (25)$$

if $h_0 < 0.1$ (i.e. 5.7°)

$$D_{ic} = D_{hc} \{F(\gamma_N)(1 - K_b) + K_b \sin \gamma_N \cos A_{LN}/(0.1 - 0.008\ h_0)\} \qquad (26)$$

where

$$\begin{array}{ll} A^*_{LN} = A_0 - A_N & \\ A_{LN} = A^*_{LN} & \text{if } -\pi <= A^*_{LN} <= \pi \\ A_{LN} = A^*_{LN} - 2\pi & \text{if } A^*_{LN} > \pi \\ A_{LN} = A^*_{LN} + 2\pi & \text{if } A^*_{LN} < -\pi \end{array} \qquad (27)$$

b) for surfaces in shadow ($\delta_{exp} < 0$ and $h_0 >= 0$):

$$D_{ic} = D_{hc}\ F(\gamma_N) \qquad (28)$$

where $F(\gamma_N)$ is a function accounting for the diffuse sky irradiance that may be calculated by the following equation (γ_N in radians):

$$F(\gamma_N) = r_i(\gamma_N) + (\sin \gamma_N - \gamma_N \cos \gamma_N - \pi \sin^2(\gamma_N/2))\ N \qquad (29)$$

where $r_i(\gamma_N)$ is a fraction of the sky dome viewed by an inclined surface [dimensionless]:

$$r_i(\gamma_N) = (1 + \cos \gamma_N)/2 \qquad (30)$$

and value of N for surfaces in shadow is 0.25227. For sunlit surfaces under clear sky the term N is calculated as:

$$N = 0.00263 - 0.712\ K_b - 0.6883\ K_b^2 \qquad (31)$$

The K_b is a measure of the amount of beam irradiance available (proportion between beam irradiance and extraterrestrial solar irradiance on a horizontal surface):

$$K_b = B_{hc}/G_{0h} \qquad (32)$$

where G_{0h} [W.m^{-2}] is calculated as:

$$G_{0h} = G_0 \sin h_0 \qquad (33)$$

3.3 Ground reflected radiation

The estimation of the clear-sky diffuse ground reflected irradiance on an inclined surface (R_i) relies on an isotropic assumption. The ground reflected clear-sky irradiance received on an inclined surface [W.m^{-2}] is proportional to the global horizontal irradiance G_{hc}, to the mean ground albedo ρ_g and a fraction of the ground viewed by an inclined surface $r_g(\gamma_N)$ [16]:

$$R_i = \rho_g\, G_{hc}\, r_g(\gamma_N) \tag{34}$$

where:

$$r_g(\gamma_N) = (1 - \cos \gamma_N)/2 \tag{35}$$

and global irradiance on a horizontal surface G_{hc} [W.m^{-2}] is given as a sum of its beam and diffuse component:

$$G_{hc} = B_{hc} + D_{hc} \tag{36}$$

In [16, page 141] typical albedo values for a variety of ground surfaces are listed. As default the values of 0.2 and 0.15 are mostly used.

4 Radiation under overcast conditions

Overcast radiation can be calculated from the clear-sky values by application of a factor parametrising the attenuation caused by cloudiness. The explicit calculation can be found in [29, 30]. However, the cloudiness observation by a meteorological service routine is usually prone to subjective errors and does not describe sufficiently the physical nature and dynamic spatial-temporal pattern of different types of cloud cover. Therefore a simpler parameter has to be used. For an assessment of global irradiance/irradiation on a horizontal surface for overcast conditions a **clear-sky index** K_c is used [31, 32]:

$$K_c = G_h/G_{hc} \tag{37}$$

The index represents a ratio between global radiation under overcast (G_h) and clear-sky (G_{hc}) conditions and its values are usually within interval 0.25 – 1.00. There are several methods of estimation the clear-sky index:

1. The clear-sky index can be calculated for ground stations where both values of measured (or from other climatologic data estimated) global radiation G_h and clear-sky radiation G_{hc} are known [31].

2. Simple regression-based formula for calculation the ratio of global radiation for a given cloud amount G_h to the clear-sky radiation G_{hc} can be found. For example, based on the analysis of 10-years hourly records in Hamburg, Kasten and Czeplak [31] have found this empirical equation:

$$K_c = 1 - 0.75\, (C/8)^{3,4} \tag{38}$$

where C is cloudiness expressed in okta.

3. The K_c can be derived from the cloud index calculated from short-wave surface irradiance measured by satellites. The Heliosat method [33, 34, 35] is basically driven by the strong complementarity between the planetary albedo recorded by the METEOSAT radiometer and the surface radiant flux. The planetary albedo increases with increasing atmospheric turbidity and cloud cover. The cloud index as a measure of cloud cover is then derived for each pixel:

$$n = (\rho - \rho_c)/(\rho_t - \rho_c) \tag{39}$$

where ρ is the albedo of the pixel in question and the cloud-free (ρ_c) and total cloudy case (ρ_t). Beyer et al. [34] have shown that a close relationship exists between the clear-sky index and the cloud index:

$$K_c = a\,n + b \tag{40}$$

where the coefficients a and b result from a linear regression with ground data.

In the first two cases to obtain a spatially continuous database an interpolation technique is needed. The third approach implicitly results in a raster map.

When a raster map of clear-sky index is available, the global irradiance/irradiation on a horizontal surface can be calculated as follows:

$$G_h = K_c\, G_{hc} \tag{41}$$

To compute the overcast solar radiation for inclined surfaces the diffuse D_h and beam B_h components of overcast global radiation as well as the clear-sky index have to be treated separately. The ratio of diffuse to the global radiation D_h/G_h for clear and overcast skies changes according to the cloudiness. In Europe the values of the ratio D_h/G_h are typically in interval 0.3 – 1.0 [31]. The underlying physical processes are quite complicated and computationally represented only by estimations. In literature there are many empirical equations for calculating D_h/G_h in literature, e.g. [16, 31, 36]. However for many meteorological stations the diffuse and beam components are already calculated either from direct measurements of diffuse irradiance or from cloudiness, sunshine or other meteorological data. Having these data available the diffuse and beam components of the clear-sky index for each station can be calculated as follows:

$$\begin{aligned} K^b{}_c &= B_{hs}/B_{hc} \\ K^d{}_c &= D_{hs}/D_{hc} \end{aligned} \tag{42}$$

where subscript *s* in B_{hs} and D_{hs} is meant to distinguish between data measured on meteorological stations and the modelled values B_h, and D_h. The clear-sky index defines for both components a fraction of the respective clear-sky radiation reduced by cloudiness. Having the raster maps of $K^b{}_c$ and $K^d{}_c$ available the overcast beam and diffuse components for horizontal plane can be computed as follows:

$$\begin{aligned} B_h &= B_{hc}\, K^b{}_c \\ D_h &= D_{hc}\, K^d{}_c \end{aligned} \tag{43}$$

Finally the computation of overcast radiation for inclined surfaces is analogous to the procedure described for clear-sky model in the section 3. The only difference is that instead of clear-sky values B_{hc}, D_{hc} and G_{hc} the overcast B_h, D_h and G_h are used. In other words – for computing overcast beam radiation B_i, in equation (12), the B_{hc} values are substituted by B_h. For computing the diffuse component D_i, in equations (25), (26), (28) the D_{hc} values are substituted by D_h and in equation (32) the B_{hc} values are substituted by B_h. For computing the ground reflected radiation, in equation (34), the G_{hc} values are replaced by G_h.

5 Implementation in GRASS GIS

The presented solar radiation model is a substantial improvement of the older version [18], which application was limited only to small areas and clear-sky beam radiation. The new model provides a solution for all three components of global solar radiation under clear-sky or overcast conditions. Large areas can be modelled accurately using spatially variable parameters, and shadowing effects of terrain can be modelled by new effective shadowing algorithm.

The *r.sun* works in two modes. In the **mode 1** - for the instant time - it calculates a solar incident angle [degrees] and solar irradiance values [$W.m^{-2}$]. In the **mode 2** the daily sum of solar irradiation [$Wh.m^{-2}.day^{-1}$] and duration of the beam irradiation are computed within a given day. By scripting the two modes can be used separately or in a combination to provide estimates for any desired time steps or intervals. The model accounts for a sky obstruction by local relief features using an optional shadowing parameter. Details of the command (synopsis, description, notes) can be found on r.sun manual page.

5.1 Input parameters

The model requires only a few mandatory input parameters – digital terrain model (elevation, slope, aspect – *elevin*, *slopein*, *aspin*), day number *day* (for mode 2), and additionally a local solar time *time* (for mode 1). However, several other parameters can be set to fit the specific user needs. These parameters have default values that are used unless they are overridden by user settings as a single value or a name of the raster. The table 1 presents a list of all input parameters.

Parameter name	Type of input	Description	Mode	Units	Interval of values
elevin	raster	elevation	1, 2	metres	0 – 8900
aspin	raster	aspect (panel azimuth)	1, 2	decimal degrees	0 – 360
slopein	raster	slope (panel inclination)	1, 2	decimal degrees	0 – 90
linkein	raster	Linke atmospheric turbidity	1, 2	dimensionless	0 - ≈7
lin	single value	Linke atmospheric turbidity	1, 2	dimensionless	0 - ≈7
albedo	raster	ground albedo	1, 2	dimensionless	0 – 1
alb	single value	ground albedo	1, 2	dimensionless	0 – 1
latin	raster	latitude	1, 2	decimal degrees	-90 – 90
lat	single value	latitude	1, 2	decimal degrees	-90 – 90
coefbh	raster	clear-sky index for beam component	1, 2	dimensionless	0 – 1
coefdh	raster	clear-sky index for diffuse component	1, 2	dimensionless	0 – 1
day	single value	day number	1, 2	dimensionless	0 – 366
declin	single value	solar declination	1, 2	radians	-0.40928 – 0.40928
time	single value	local (solar) time	1	decimal hours	0 – 24
step	single value	time step	2	decimal hours	0.01 – 1.0
dist	single value	sampling distance coefficient for shadowing	1, 2	dimensionless	0.1 – 2.0

Table 1: *r.sun* input parameters

Solar declination is computed internally using equation (15) and day number unless an explicit value of *declin* is used. In the case that user's data are localised in GRASS location with defined projection, *r.sun* uses internal GRASS function to get geographical latitude for every raster cell. Otherwise the user can set the latitude as a constant *lat* for the whole computed region or, as a raster *latin* representing spatially distributed values over larger region. Similarly, the Linke turbidity factor and ground albedo can be set as a spatially averaged (single) values *lin*, *alb* or spatially distributed parameters *linkein*, *albedo*. The *step* parameter defines time step used for all-day irradiation calculation from sunrise to sunset. The default value is 0.5 hour. The shadowing effect of terrain can be taken into account using the *–s* flag. The *dist* parameter defines the sampling density at which the visibility of a raster cell is computed in the direction of solar beam. The values above 1.0 are suitable for fast, but less accurate estimates, while values less 1.0 for slower and more precise calculations. It is recommended to use values in the range 0.5 – 1.5.

5.2 Model outputs

According to the setting of output parameters the model automatically recognises between modes 1 and 2. When calculating in **mode 1** the solar incident angle *incidout*, and solar irradiance raster maps *beam_rad*, *diff_rad* and *refl_rad* are computed. Calculation in **mode 2** gives the sums of solar irradiation within a specified day for selected components of global irradiation *beam_rad*, *diff_rad* and *refl_rad*. A raster map showing duration of beam irradiation *insol_time* can be computed as well.

Besides clear-sky irradiances/irradiations, the model can calculate overcast radiation on conditions that *coefbh* and *coefdh* input raster maps are defined, expressing the beam and diffuse components of clear-sky index (equations 42).

The incidence angle and irradiance/irradiation maps can be computed without considering the terrain shadowing by default or with shadowing effects by setting the flag *-s*. In mountainous areas this can lead to very different results especially at low sun altitudes. The value of a shadowed area is written to the output maps as zero. The table 2 presents a list of all output raster maps.

Besides output raster maps, the model stores basic solar radiation parameters used in the computation in *r.sun_out.txt* local text file. Currently it contains day number, solar constant, extraterrestrial irradiance, solar declination, interval of latitude, times of sunrise and sunset, time step, interval of used Linke turbidity and ground albedo.

Solar radiation modelling for periods longer or shorter than one day can be done using UNIX shell scripting within GRASS GIS environment. The example can be found in [37, page 326].

Parameter name	**Description**	**Mode**	**Units**
incidout	solar incidence angle	1	decimal degrees
beam_rad	beam irradiance	1	$W.m^{-2}$
diff_rad	diffuse irradiance	1	$W.m^{-2}$
refl_rad	ground reflected irradiance	1	$W.m^{-2}$
insol_time	duration of the beam irradiation	2	min.
beam_rad	beam irradiation	2	$Wh.m^{-2}.day^{-1}$
diff_rad	diffuse irradiation	2	$Wh.m^{-2}.day^{-1}$
refl_rad	ground reflected irradiation	2	$Wh.m^{-2}.day^{-1}$

Table 2: *r.sun* output raster maps

6 Applications

To demonstrate the model's capabilities we briefly present two applications. They cover territory of Central and Eastern Europe at different spatial resolutions. Spatial data are represented by large raster maps with parameters representing terrain, latitude, Linke atmospheric turbidity, radiation, and clear-sky index. The first application presents building solar radiation database that was prepared for an assessment of potential photovoltaic electricity production in 10 European countries. The second one demonstrates modelling of solar irradiance in Slovakia at detailed scale.

6.1 Mapping the potential for photovoltaic systems in Central and Eastern Europe

Solar energy is one of the possible environmentally sustainable resources for producing electricity using photovoltaic (PV) systems. However, there are still barriers to their widespread use due to initial capital costs that make them uncompetitive with the conventional systems. Therefore their design and location has to be precisely planned and calculated. Within a project "Environment and Solar Energy Resource" held in European Commission Joint Research Centre in Ispra a map-based inventory of the suitability of areas for installation of PV systems was realised [38]. The territory covers 10 European Union Candidate Countries (Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia). A part of these works was dedicated to development of a spatial solar database using the *r.sun* model and interpolation techniques *s.surf.rst* and *s.vol.rst* implemented in GRASS GISS. In the assessment the building-integrated horizontal panels and south-facing panels inclined at angles of 15, 25 and 40 degrees were considered.

For the assessment of PV systems performance raster maps of monthly means and annual mean of daily sums of global irradiation were computed in the following steps:

1. computing of clear-sky global irradiation on a horizontal surface;
2. calculating and spatial interpolation of clear-sky index and computing raster maps of overcast global irradiation on a horizontal surface;
3. computing the diffuse and beam components of overcast global irradiation and computing raster maps of global irradiation on inclined surfaces.

Step 1: Clear-sky global irradiation on a horizontal surface

All raster data were integrated in a GIS project in the Lambert equal area azimuthal map projection. A dimension of raster maps was 2145 x 1410 cells with raster resolution 1 km. The elevation data were derived from the USGS GTOPO30 digital terrain model. The latitude raster map was interpolated from densely distributed points using the regularised spline with tension *s.surf.rst*. The ground albedo was considered as a constant 0.15.

One of the critical data set, needed in modelling is the Linke turbidity. Due to its inherent spatial and temporal variability usually only time averages are estimated from data measured at meteorological stations and on satellites. For our purpose monthly means of the Linke turbidity factor T_{LK} for 317 sites over the area were excerpted from the SoDa web database [20]. The accuracy of the data from this source is reported to RMSE = 0.7 T_{LK} units. To eliminate effect of the elevation, before interpolation, the T_{LK} values were normalised to the elevation at sea level [39]:

$$T_{LKn} = T_{LK} + 0.00035\ z \tag{44}$$

where z is the elevation of the raster cell. The spatially distributed data layers of T_{LKn} were interpolated using *s.surf.rst* [40] to derive 12 raster files. Following the inverse to the equation (44) was performed to obtain T_{LK} values from T_{LKn} using the digital terrain model.

The daily sum of clear-sky global irradiation G_{hc} [Wh.m^{-2}.day^{-1}] available for a solar panel at horizontal position was computed in mode 2. The example of command syntax for calculation of January mean is presented below:

```
r.sun -s elevin=cc.ele aspin=asp0 slopein=slo0 linkein=cc.lin01
   latin=cc.fi beam_rad=01.0b diff_rad=01.0d declin=-0.36146 day=17
   step=0.25 dist=0.7
```

```
r.mapcalc 01.0g = '01.0b + 01.0d'
```

The time step of 0.25 hour was considered to be satisfactory for daily estimates. The day numbers and declination values for each month were set according to recommendations of ESRA team [16, p. 108]. To compromise between time of computation and accuracy, the distance for calculation of shadowing was set to 0.7.

Step 2: Global irradiation on a horizontal surface

The global irradiation for overcast conditions was calculated using the clear-sky index K_c and solar irradiation database developed from meteorological measurements for a set of European meteorological stations by the ESRA team [16]. For the study area the data from 182 meteorological station were available, comprising geographical position and monthly means of global (G_{hs}), beam (B_{hs}) and diffuse (D_{hs}) irradiation on a horizontal surface. The subscript *s* is used here to distinguish values from meteorological stations from those estimated by the model. The clear-sky irradiation values G_{hc}, B_{hc} and D_{hc}

computed in the step 1 were added to this database and for each meteorological station the clear-sky index was computed:

$$K_c = G_{hs}/G_{hc} \tag{45}$$

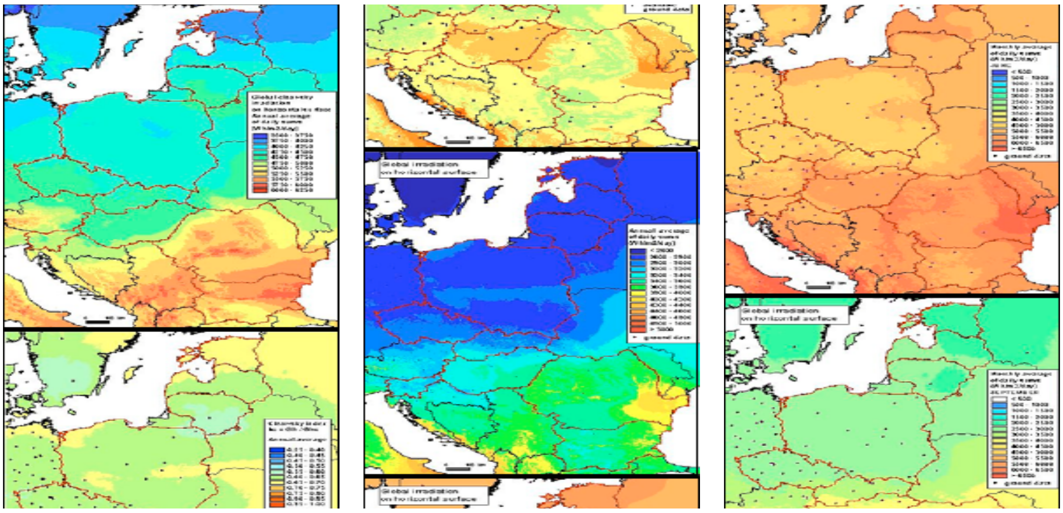

Figure 1: Annual mean of (a) daily sums of clear-sky global irradiation G_{hc}, (b) clear-sky index K_c and (c) daily sums of overcast irradiation G_h (colour tables for G_{hc} and G_h are different)

The clear-sky index is correlated with elevation, mostly in summer and winter, therefore it was decided to use a multivariate spatial interpolation of the point values to account for changes in vertical dimension. For this purpose the method *s.vol.rst*, implemented in GRASS GIS, was used [41]. The position of input points is defined using x,y,z values, where the z value represents elevation derived from the digital terrain model. The interpolation results are very sensitive to the correct selection of parameters of tension, smoothing and conversion factor for z-values (see s.vol.rst manual page). To obtain the most accurate results the crossvalidation procedure was applied separately for each of the 12 monthly point data sets of K_c [41]. Based on this methodology, the resulting raster maps show better interpolation results than using 2D-interpolation [42].

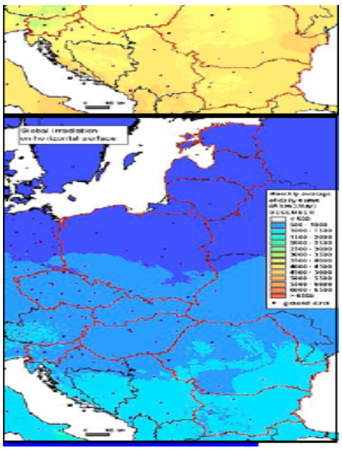

Figure 2: Monthly means of daily sums of global irradiation input to solar panels at horizontal position in June, September and December

The raster maps of overcast global irradiation on a horizontal surface were then calculated (figures 1 and 2):

$$G_h = K_c\, G_{hc} \tag{46}$$

Step 3: Global irradiation on inclined surfaces

To compute global irradiation on inclined planes G_i at real (overcast) atmospheric conditions the beam and diffuse components for conditions of mean monthly cloudiness have to be estimated. The reason is that the ratio of diffuse component to global radiation is different for clear-sky and overcast conditions.

From the data for meteorological stations the ratio D_{hs}/G_{hs} was calculated and raster maps were interpolated using *s.vol.rst* in the same way as it was done for the clear-sky index K_c (figure 3). The raster maps of diffuse and beam components of global irradiation for overcast conditions were computed:

$$D_h = G_h\, D_{hs}/G_{hs} \tag{47}$$
$$B_h = G_h - D_h$$

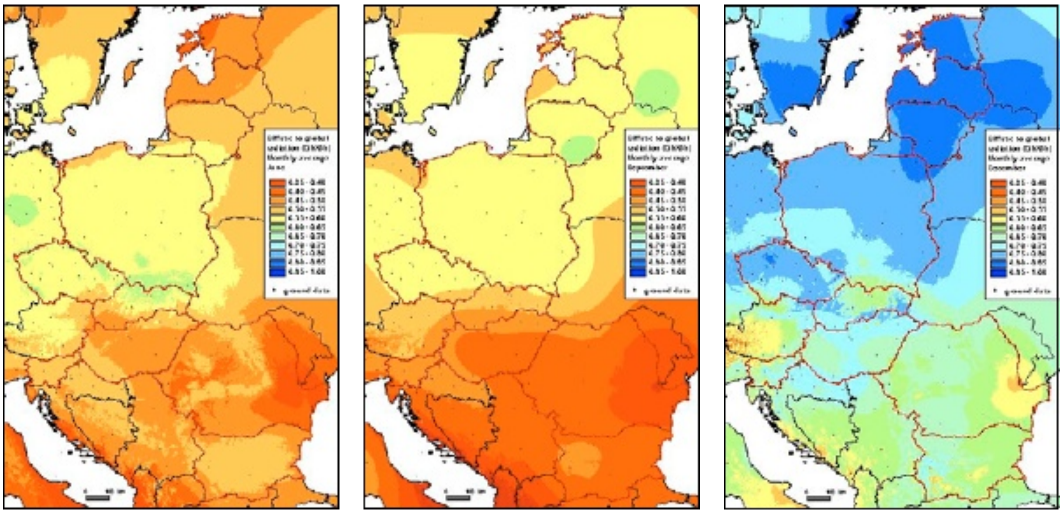

Figure 3: Ratio of monthly mean of diffuse to global irradiation on horizontal surface for June, September and December

Following the diffuse and beam components of the clear-sky index (implemented in *r.sun* as *coefbh* and *coefdh*) were calculated from the raster maps of clear-sky and overcast irradiations:

$$K^d_c = D_h/D_{hc} \tag{48}$$
$$K^b_c = B_h/B_{hc}$$

Having the raster maps of both components of the clear sky index, the monthly means of daily sums of global irradiation input [$Wh.m^{-2}.day^{-1}$] to southwards-oriented solar panels were computed for 3 inclination angles – 15, 25, and 40 degrees (figure 4). The example of command syntax for January and inclination angle of 15 degrees is as follows:

```
r.sun -s elevin=cc.ele aspin=asp270 slopein=slo15 linkein=cc.lin01
   alb=0.15 latin=cc.fi coefbh=kcb01 coefdh=kcd01 beam_rad=01.15b
   diff_rad=01.15d refl_rad=01.15r declin=-0.36146 day=17 step=0.25
   dist=0.7

r.mapcalc 01.15g = ‘01.15b + 01.15d + 01.15r’
```

The solar database for Central and Eastern Europe consists of raster maps representing 12 monthly means and 1 annual mean of daily sums of clear-sky global irradiation, Linke turbidity, clear-sky index and all components of overcast irradiation for horizontal surfaces, as well as those inclined at angles of 15, 25, and 40 degrees.

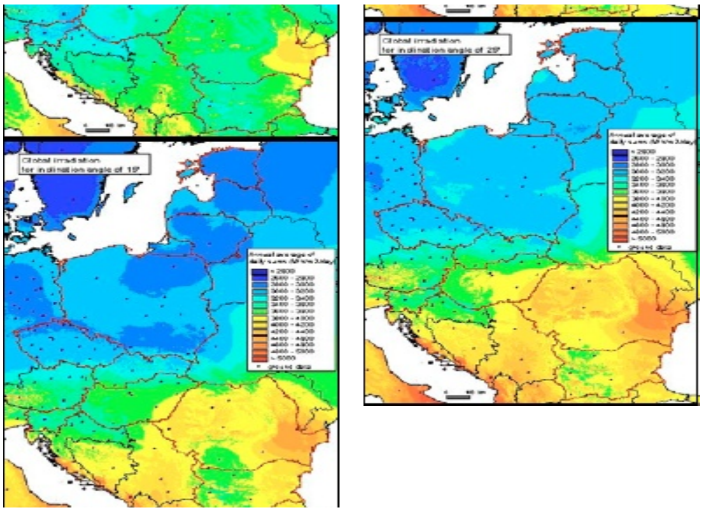

Figure 4: Annual means of daily sums of global irradiation input to solar panels at horizontal position and those inclined at angle of 15 and 25 degrees.

6.2 Solar irradiance distribution for ecological studies in Slovakia

In this example we have used digital terrain model of Slovakia with spatial resolution of 100 m. Size of raster maps is 4300 x 2150 cells. The raster maps of latitude and Linke atmospheric turbidity were calculated using the same methodology and data as explained in the first application. The clear-sky irradiance values for March 21st (spring equinox) were computed at 8.00 and 12.00 local solar time (figure 5 and 6). The effect of terrain shadowing is presented on figure 7. The results show considerable differences in spatial pattern due to terrain shadows in mountainous areas. In order to see details, the figures show only the subset of Slovakia, in its mountainous central part.

The command syntax for calculation of irradiance [$W.m^{-2}$] with shadowing is as follows:

```
r.sun -s elevin=sk.ele aspin=sk.asp slopein=sk.slo linkein=sk.lin03
   latin=sk.fi beam_rad=0321_12.bc diff_rad=0321_12.dc
   refl_rad=0321_12.rc incidout=sk0321-12.inc alb=0.15 declin=0 day=80
   time=12.00 dist=0.3

r.mapcalc 0321_12h.gc = '0321_12.bc + 0321_12.dc + 0321_12.rc'
```

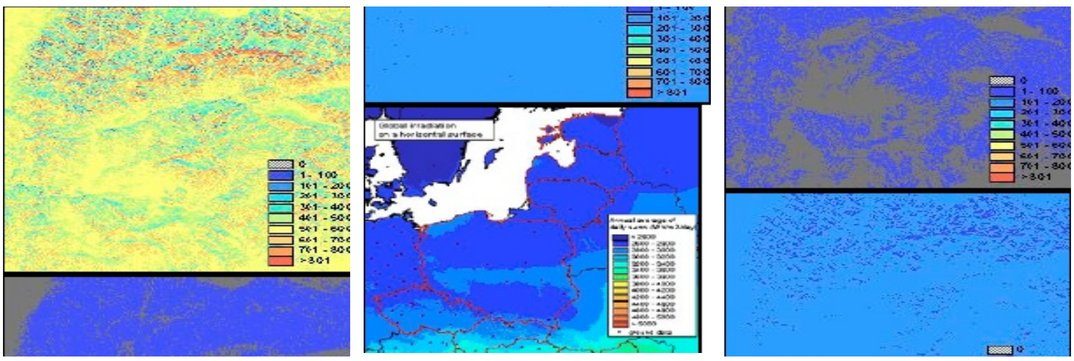

Figure 5: Three components of the clear-sky irradiance at 12.00 hour in March 21st
(a) beam, (b) diffuse and (c) reflected [$W.m^{-2}$]

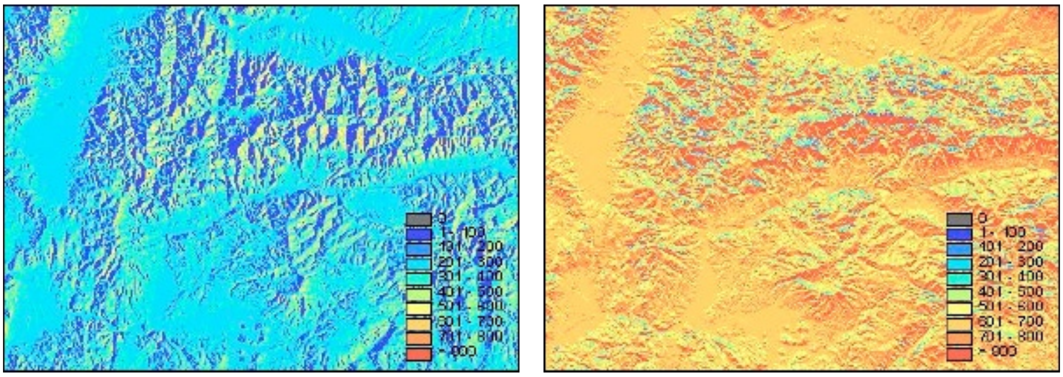

Figure 6: Clear-sky global irradiance at 8.00 and at 12.00 hour in March 21st [W.m^{-2}]

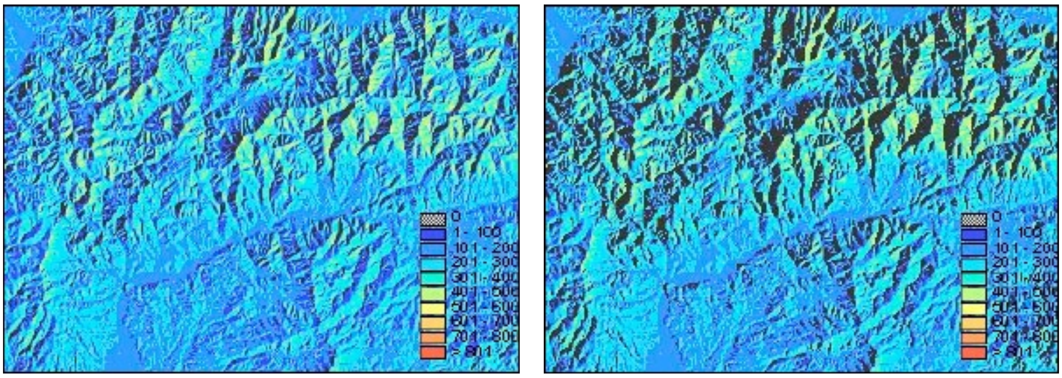

Figure 7: Clear-sky beam irradiance at 8.00 hour in March 21st
(a) shadowing not applied, (b) shadowing applied [W.m^{-2}]

7 Conclusions

The *r.sun* is a complex and flexible solar radiation model, fully integrated within open source environment of GRASS GIS. It calculates all three components of solar irradiance/irradiation (beam, diffuse and reflected) for clear-sky as well as overcast conditions. The implemented equations follow the latest European research in solar radiation modelling. Integration in GRASS GIS enables to use interpolation tools that are necessary for data preparation.

The model is especially appropriate for modelling of large areas with complex terrain because all spatially variable solar parameters can be defined as raster maps. The model can be used easily for long-term calculations at different map scales – from continental to detailed. Two operational modes enable the user account for temporal variability of solar radiation within a day or within a year (using shell scripts). These features offer wide variety of possible applications as documented on the two examples.

Open source code enables to make modifications and improvements in future, according to research development in solar radiation modelling or to fit better specific user needs.

8 Acknowledgements

This work has been carried out under the EC JRC Enlargement Action Programme as project number 52 entitled “Environment and the solar energy resource” in a co-operation with GeoModel s.r.o. The authors would like to thank to Tomáš Cebecauer, Ewan D. Dunlop and Arwyn R. Jones for the discussions and helpful comments.

9 References

[1] Hutchinson, M.F., Booth, T.H., McMahon, L.P., Nin, H.A., 1984, Estimating monthly mean values of daily total solar radiation for Australia. *Solar Energy*, 32: 277-290.

[2] Hulme, M., Conway, D., Jones, P.D., Jiang, T., Barrow, E.M., Turney, C., 1995, A 1961-1990 climatology for Europe for climate change modelling and impact applications. *International Journal of Climatology*, 15: 1333-1364.

[3] Zelenka, A., Czeplak, G., D'Agostino, V., Josefson, W., Maxwell, E., Perez, R., 1992, *Techniques for supplementing solar radiation network data*. Technical Report, International Energy Agency, # IEA-SHCP-9D-1, Swiss Meteorological Institute, Switzerland.

[4] D'Agostino, V., Zelenka, A., 1992, Supplementing solar radiation network data by co-kriging with satellite images. *International Journal of Climatology*, 12: 749-761.

[5] Beyer, H.G., Czeplak, G., Terzenbach, U., Wald, L., 1997, Assessment of the method used to construct clearness index maps for the new European solar radiation atlas (ESRA). *Solar Energy*, 61: 389-397.

[6] Noia, M., Ratto, C.F., Festa, R., 1993, Solar irradiance estimation from geostationary satellite data: I. Statistical models, *Solar Energy*, 51: 449-456.

[7] Noia, M., Ratto, C.F., Festa, R., 1993, Solar irradiance estimation from geostationary satellite data: II. Physical models. *Solar Energy*, 51: 457-465.

[8] Dubayah, R., Rich, P.M., 1995, Topographic solar radiation models for GIS. *International Journal of Geographical Information Systems*, 9: 405-419.

[9] Hetrick, W.A., Rich, P.M., Barnes, F.J., Weiss, S.B., 1993, GIS-based solar radiation flux models. *American Society for Photogrammetry and Remote Sensing Technical papers*. GIS, Photogrammetry and Modeling, 3: 132-143.

[10] Kumar, L., Skidmore, A.K., Knowles, E., 1997, Modelling topographic variation in solar radiation in a GIS environment. *International Journal of Geographical Information Science*, 11, 5: 475-497.

[11] Mészároš, I., 1998, Modelovanie príkonu slnečnej energie na horské povodie. *Acta hydrologica Slovaca*, 1: 68-75.

[12] Miklánek, P., 1993, The estimation of energy incom in grid points over the basin using simple digital elevation model. *Annales Geophysicae*, European Geophysical Society, Springer, Suppl. II, 11: 296.

[13] Fu, P., Rich, P.M., 2000, *The Solar Analyst 1.0 user manual*. Helios Environmental Modeling Institute, http://www.hemisoft.com.

[14] Wilson, J.P., Gallant, J.C., 2000, Secondary topographic attributes. In: Wilson, J.P., Gallant, J.C., eds., *Terrain analysis; principles and applications*, New York (Wiley): 87-132.

[15] McKenney, D.W, Mackey, B.G., Zavitz, B.L., 1999, Calibration and sensitivity analysis of a spatially-distributed solar radiation model. *International Journal of Geographical Information Science*, 13, 1: 49-65.

[16] Scharmer, K., Greif, J., eds., 2000, *The European solar radiation atlas*. Vol. 2: Database and exploitation software. Paris (Les Presses de l'École des Mines).

[17] Muneer, T., 1997, *Solar Radiation and Daylight Models for Energy Efficient Design of Buildings*, Oxford (Architectural Press).

[18] Hofierka, J., 1997, Direct solar radiation modelling within an open GIS environment. *Proceedings of the Joint European GI Conference 1997*, Vienna: 575-584.

[19] Page, J.K., ed., 1986, Prediction of solar radiation on inclined surfaces. *Solar Energy R&D in the European Community, Series F: Solar radiation data*, Vol. 3, Dordrecht (D. Reidel Publishing Company).

[20] Wald, L., 2000, SODA: a project for the integration and exploitation of networked solar radiation databases. *European Geophysical Society Meeting, XXV General Assembly*, Nice, France, 25-29 April 2000, http://www.soda-is.com/.

[21] Page, J.K., Albuisson, M., Wald, L., 2001, The European solar radiation atlas: a valuable digital tool. *Solar Energy*, 71: 81-83.

[22] Rigollier, Ch., Bauer, O., Wald, L., 2000, On the clear sky model of the ESRA – European Solar radiation Atlas – with respect to the Heliosat method. *Solar Energy*, 68: 33-48.

[23] Muneer, T., 1990, Solar radiation model for Europe. *Building services engineering research and technology*, 11: 153-163.

[24] Kasten, F., 1996, The Linke turbidity factor based on improved values of the integral Rayleigh optical thickness, *Solar Energy*, 56: 239-244.

[25] Kasten, F., Young, A.T., 1989, Revised optical air mass tables and approximation formula. *Applied Optics*, 28: 4735-4738.

[26] Krcho, J., 1990, *Morfometrická analýza a digitálne modely georeliéfu*. Bratislava (VEDA).

[27] Jenčo, M., 1992, Distribúcia priameho slnečného žiarenia na reoreliéfe a jej modelovanie pomocou komplexného digitálneho modelu reliéfu. *Geografický časopis*, 44: 342-355.

[28] Gruter, J.W., ed., 1984, *Radiation nomenclature*. In. 2nd Solar Energy Programme of the CEC, Project F, Solar Radiation Data, CEC, Brussels.

[29] Becker, S., 2001, Calculation of direct solar and diffuse radiation in Israel. *International Journal of Climatology*, 21: 1561-1576.

[30] Kitler, R., Mikler, J., 1986, Základy využívania slnečného žiarenia. Bratislava (Veda).

[31] Kasten, F., Czeplak, G., 1980, Solar and terrestrial radiation dependent on the amount and type of cloud. *Solar Energy*, 24: 177-189.

[32] Hammer, A., Heinemann, D., Westerhellweg, A. et al., 1998, Derivation of daylight and solar irradiance data from satellite observations. *Proceedings of the 9th Conference on satellite meteorology and oceanography*, Paris, May 1998: 747-750, http://www.satel-light.com/core.htm.

[33] Cano, D., Monget, J.M., Albuisson, M., Guillard, H., Regas, N., Wald, L., 1986, A method for the determination of the global solar radiation from meteorological satellite data. *Solar Energy*, 37: 31-39.

[34] Beyer, H.G., Costanzo, C., Heinemann, D., 1996, Modifications of the Heliosat procedure for irradiance estimates from satellite images. *Solar Energy*, 56: 207-212.

[35] Hammer, A., Heinemann, D., Westerhellweg, A. et al., 1998, Derivation of daylight and solar irradiance data from satellite observations. *Proceedings of the 9th Conference on satellite meteorology and oceanography*, Paris, May 1998: 747-750.

[36] Hrvoľ, J., 1991, Eine neue beziehung für die Berechnung der monatlichen durchschnittsummen der diffusen Strahlung. *Acta Meteorologica Univeristas Comenianae*, XIX: 3-14.

[37] Neteler, M., Mitasova, H., 2002, *Open Source GIS: A GRASS GIS Approach*, Boston (Kluwer Academic Publishers).

[38] Šúri, M., Dunlop, E.D., Jones, A.R., 2002, GIS-based inventory of the potential photovoltaic output in Central and Eastern Europe. *PV in Europe. From PV Technology to Energy Solutions*, Conference and Exhibition, 7-11 October 2002, Rome, submitted presentation.

[39] WMO, Meteorological aspects of utilization of solar radiation as an energy source, 1981, *World Meteorological Organisation Technical Note* No. 172, 557, Geneva, Switzerland.

[40] Mitasova, H., Mitas, L., 1993, Interpolation by Regularized Spline with Tension: I. Theory and Implementation. *Mathematical Geology*, 25: 641 – 655.

[41] Hofierka, J., Parajka, J., Mitasova, M., Mitas, L., 2002, Multivariate iterpolation of precipitation using regularized spline with tension. *Transactions in GIS*, 6: 135-150.

[42] Šúri, M., Hofierka, J., 2002, Solar radiation database for Central and Eastern Europe using solar radiation model and multivariate interpolation. *paper in preparation*.

Annex: Applied symbols

Position of the raster cell (solar surface)

φ	geographical latitude [rad]
z	elevation above sea level [m]
γ_N	slope angle [rad]
A_N	aspect (orientation, azimuth) - angle between the projection of the normal on the horizontal surface and East [rad]
φ'	relative geographical latitude of an inclined surface [rad]
λ'	relative geographical longitude [rad]

Parameters of the surface

ρ_g	mean ground albedo
ρ_c	on satellite measured planetary albedo under cloud-free (clear) sky
ρ_t	on satellite measured planetary albedo under total cloudy sky

Date-related parameters

j	day number 1-365 (366)
j'	Julian day number expressed as a day angle [rad]
T	hour angle of the local solar time [rad]
$T_h^{r,s}$	hour angle of time of sunrise and sunset over the local horizon [rad]
$T_i^{r,s}$	hour angle of time of sunrise and sunset over the inclined raster cell (surface) [rad]
δ	solar declination [rad]
ε	correction of the variation of the sun-earth distance from its mean value

Solar position

h_0	solar altitude – an angle between the sun and horizon [rad]
h_0^{ref}	solar altitude corrected by atmospheric refraction [rad]
A_0	solar azimuth – an angle between the sun and meridian, measured from East [rad]
A_{LN}	angle between the vertical surface containing the normal to the surface and vertical surface passing through the centre of the solar disc [rad]
δ_{exp}	solar incidence angle - an angle between the sun and the (inclined) surface [rad]

Solar radiation

I_0	solar constant (1367 $W.m^{-2}$)
G_0	extraterrestrial irradiance/irradiation on a surface perpendicular to the solar beam [$W.m^{-2}$] or [$Wh.m^{-2}$]
G_h	$G_h = B_h + D_h$ – global solar irradiance/irradiation on a horizontal surface [$W.m^{-2}$] or [$Wh.m^{-2}$]

G_i — $G_i = B_i + D_i + R_i$ – global solar irradiance/irradiation on an inclined surface [$W.m^{-2}$] or [$Wh.m^{-2}$]
B_h — beam irradiance/irradiation on a horizontal surface [$W.m^{-2}$] or [$Wh.m^{-2}$]
B_i — beam irradiance/irradiation on an inclined surface [$W.m^{-2}$] or [$Wh.m^{-2}$]
D_h — diffuse irradiance/irradiation on a horizontal surface [$W.m^{-2}$] or [$Wh.m^{-2}$]
D_i — diffuse irradiance/irradiation on an inclined surface [$W.m^{-2}$] or [$Wh.m^{-2}$]
R_i — ground reflected irradiance/irradiation on an inclined surface [$W.m^{-2}$] or [$Wh.m^{-2}$]

note — *subscript "c" means clear-sky values*
subscript "s" means values measured or calculated from meteorological data

K_c — clear-sky index representing a ratio between overcast global irradiance/irradiation (G_h) and clear-sky irradiance/irradiation (G_{hc}) [-]
K^b_c — beam component of the clear-sky index [-]
K^d_c — diffuse component of the clear-sky index [-]

Parameters of the atmosphere

p/p_0 — correction of station elevation [-]
T_{LK} — Linke turbidity factor [-]
T_L — corrected Linke turbidity factor ($T_L = 0.8662\ T_{LK}$), see [24]
T_{LKn} — Linke turbidity factor normalised to the sea level [-]
m — relative optical air mass [-]
$\delta_R(m)$ — Rayleigh optical thickness [-]

Parameters of the radiation transmission

$F_d(h_0)$ — diffuse solar elevation function
$Tn(T_{LK})$ — diffuse transmission function
$F(\gamma_N)$ — function accounting for the diffuse sky irradiance distribution
K_b — proportion between beam irradiance and extraterrestrial solar irradiance on a horizontal surface
$r_i(\gamma_N)$ — fraction of the sky dome viewed by an inclined surface [-]
$r_g(\gamma_N)$ — fraction of the ground viewed by an inclined surface [-]