# Ratings encoding

In the table below, each row represents a user's ratings of movies:  $\checkmark$  (check) indicates the person liked the movie,  $\checkmark$  (x) that they didn't, and  $\bullet$  (dot) that they didn't rate it one way or another (neutral rating or didn't watch). Can encode these ratings numerically with 1 for  $\checkmark$  (check), -1 for  $\checkmark$  (x), and 0 for  $\bullet$  (dot).

| Person           | Fyre | Frozen II    | Picard | Ratings written as a 3-tuple |
|------------------|------|--------------|--------|------------------------------|
| $\overline{P_1}$ | Х    | •            | ✓      |                              |
| $P_2$            | 1    | $\checkmark$ | X      |                              |
| $P_3$            | 1    | ✓            | ✓      |                              |
| $P_4$            | •    | X            | ✓      |                              |

## **Definitions**

| Term                       | Notation Example(s)  | We say in English   |  |  |
|----------------------------|--|---|--|--|
| sequence                   | $x_1, \ldots, x_n$   | A sequence $x_1$ to $x_n$   |  |  |
| summation                  | $x_1, \dots, x_n$ $\sum_{i=1}^n x_i \text{ or } \sum_{i=1}^n x_i$                  | The sum of the terms of the sequence $x_1$ to $x_n$   |  |  |
| all reals                  | $\mathbb{R}$   | The (set of all) real numbers (numbers on the number line)  |  |  |
| all integers               | $\mathbb{Z}$   | The (set of all) integers (whole numbers including negatives, zero, and positives)  |  |  |
| all positive integers      | $\mathbb{Z}^+$   | The (set of all) strictly positive integers   |  |  |
| all natural numbers        | N  | The (set of all) natural numbers. <b>Note</b> : we use the convention that 0 is a natural number.   |  |  |
| piecewise rule definition  | $f(x) = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$ | Define $f$ of $x$ to be $x$ when $x$ is nonnegative and to be $-x$ when $x$ is negative   |  |  |
| function application       | f(7) $f(z)$ $f(g(z))$  | f of 7 <b>or</b> $f$ applied to 7 <b>or</b> the image of 7 under $f$ $f$ of $z$ <b>or</b> $f$ applied to $z$ <b>or</b> the image of $z$ under $f$ $f$ of $g$ of $z$ <b>or</b> $f$ applied to the result of $g$ applied to $z$ |  |  |
| absolute value square root | $\begin{array}{c}  -3  \\ \sqrt{9} \end{array}$                                    | The absolute value of $-3$<br>The non-negative square root of 9   |  |  |

# Data types

| Term  | Examples:            |                         |
|---|----------------------|-------------------------|
|   | (add additional      | examples from class)    |
| set   | $7 \in \{43, 7, 9\}$ | $2 \notin \{43, 7, 9\}$ |
| unordered collection of elements                        |                      |                         |
| repetition doesn't matter                               |                      |                         |
| Equal sets agree on membership of all elements          |                      |                         |
| n-tuple   |                      |                         |
| ordered sequence of elements with $n$ "slots" $(n > 0)$ |                      |                         |
| repetition matters, fixed length                        |                      |                         |
| Equal n-tuples have corresponding components equal      |                      |                         |

#### string

ordered finite sequence of elements each from specified set repetition matters, arbitrary finite length  $Equal\ strings\ have\ same\ length\ and\ corresponding\ characters\ equal$ 

### $Special\ cases:$

When n = 2, the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted  $\lambda$ .

A set with no elements is called the **empty set** and is denoted  $\{\}$  or  $\emptyset$ .

### Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

$$\{x \in U \mid ...x...\}$$

or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x... \mid x \in U\}$$

We use the symbol  $\in$  as "is an element of" to indicate membership in a set.

**Example sets**: For each of the following, identify whether it's defined using the roster method or set builder notation and give an example element.

### Defining functions ratings

Recall our representation of Netflix users' ratings of movies as n-tuples, where n is the number of movies in the database. Each component of the n-tuple is -1 (didn't like the movie), 0 (neutral rating or didn't watch the movie), or 1 (liked the movie).

Consider the ratings  $P_1 = (-1, 0, 1), P_2 = (1, 1, -1), P_3 = (1, 1, 1), P_4 = (0, -1, 1)$ 

Which of  $P_1$ ,  $P_2$ ,  $P_3$  has movie preferences most similar to  $P_4$ ?

One approach to answer this question: use **functions** to define distance between user preferences.

For example, consider the function  $d_0$ : given by

$$d_0(((x_1, x_2, x_3), (y_1, y_2, y_3))) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$$

Extra example: A new movie is released, and  $P_1$  and  $P_2$  watch it before  $P_3$ , and give it ratings;  $P_1$  gives  $\checkmark$  and  $P_2$  gives  $\checkmark$ . Should this movie be recommended to  $P_3$ ? Why or why not?

Extra example: Define a new function that could be used to compare the 4-tuples of ratings encoding movie preferences now that there are four movies in the database.