Image & Video Processing

Image Enhancement
(Spatial Filtering)

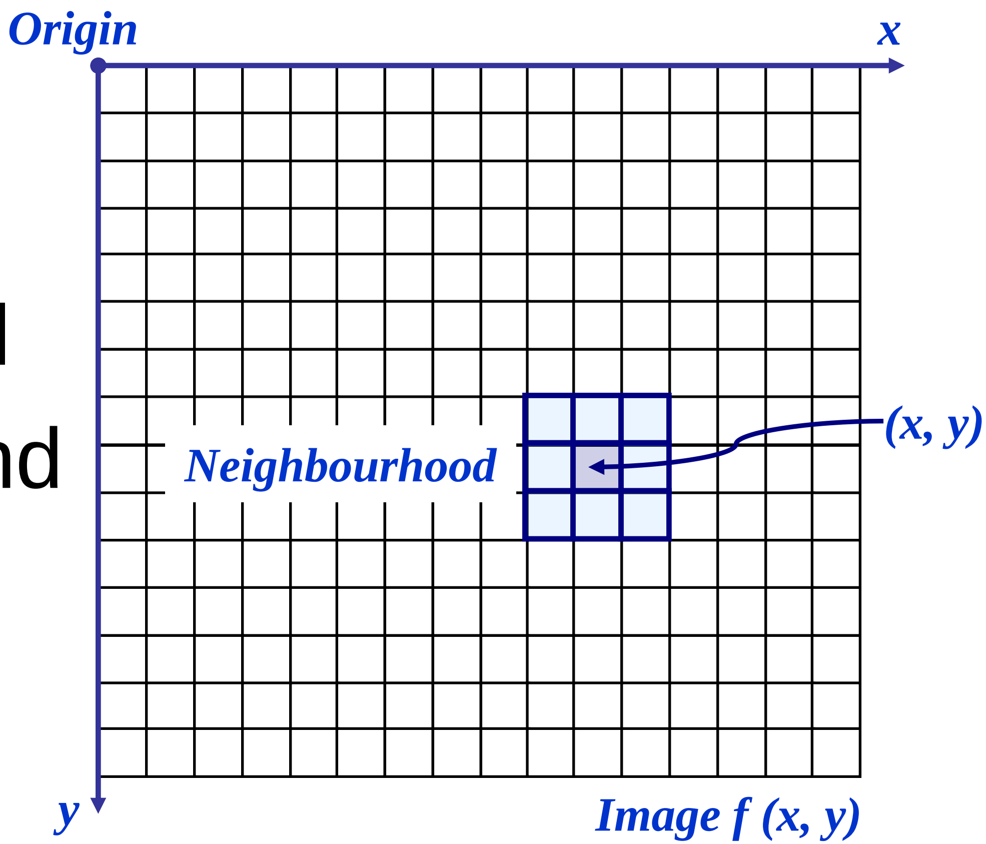
Contents

In this lecture we will look at spatial filtering techniques:

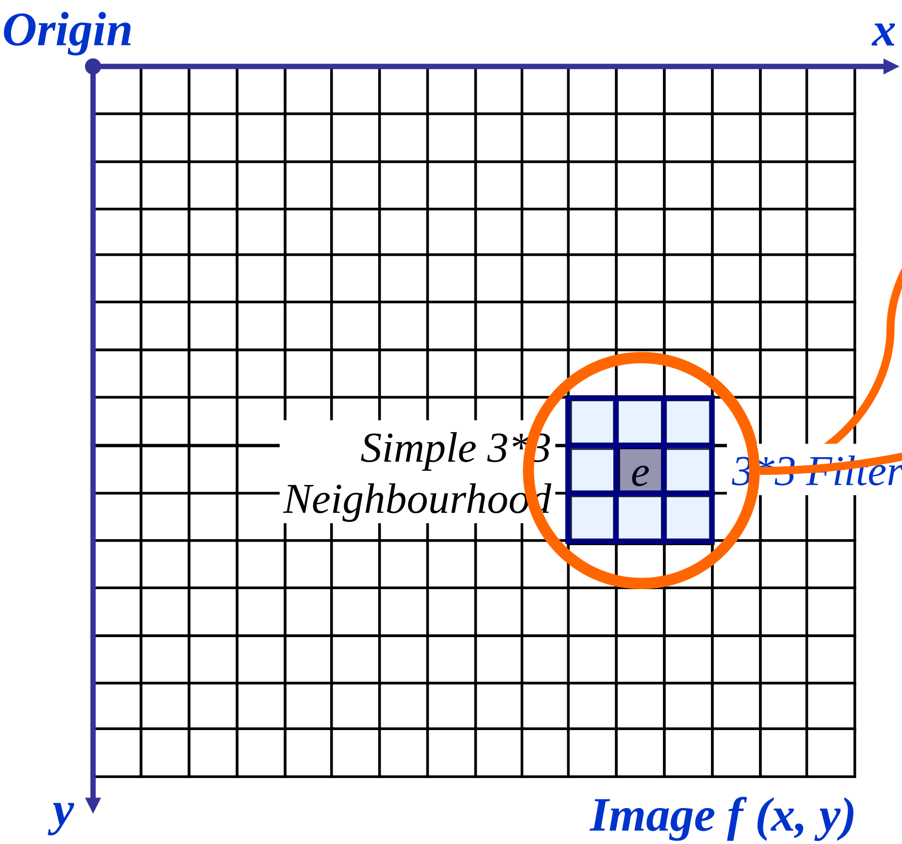
- Neighbourhood operations
- Spatial filtering : Correlation and convolution
- Smoothing operations
- Sharpening filters
  - 1^{st} derivative filters
  - 2^{nd} derivative filters
- Combining filtering techniques

Neighbourhood Operations

- Neighbourhood operations simply operate on a larger neighbourhood of pixels than point operations
- Neighbourhoods are mostly a rectangle around a central pixel
- (Although any size and shape are possible)



The Spatial Filtering Process



a	b	c
d	e	f
g	h	i

Original Image Pixels

*

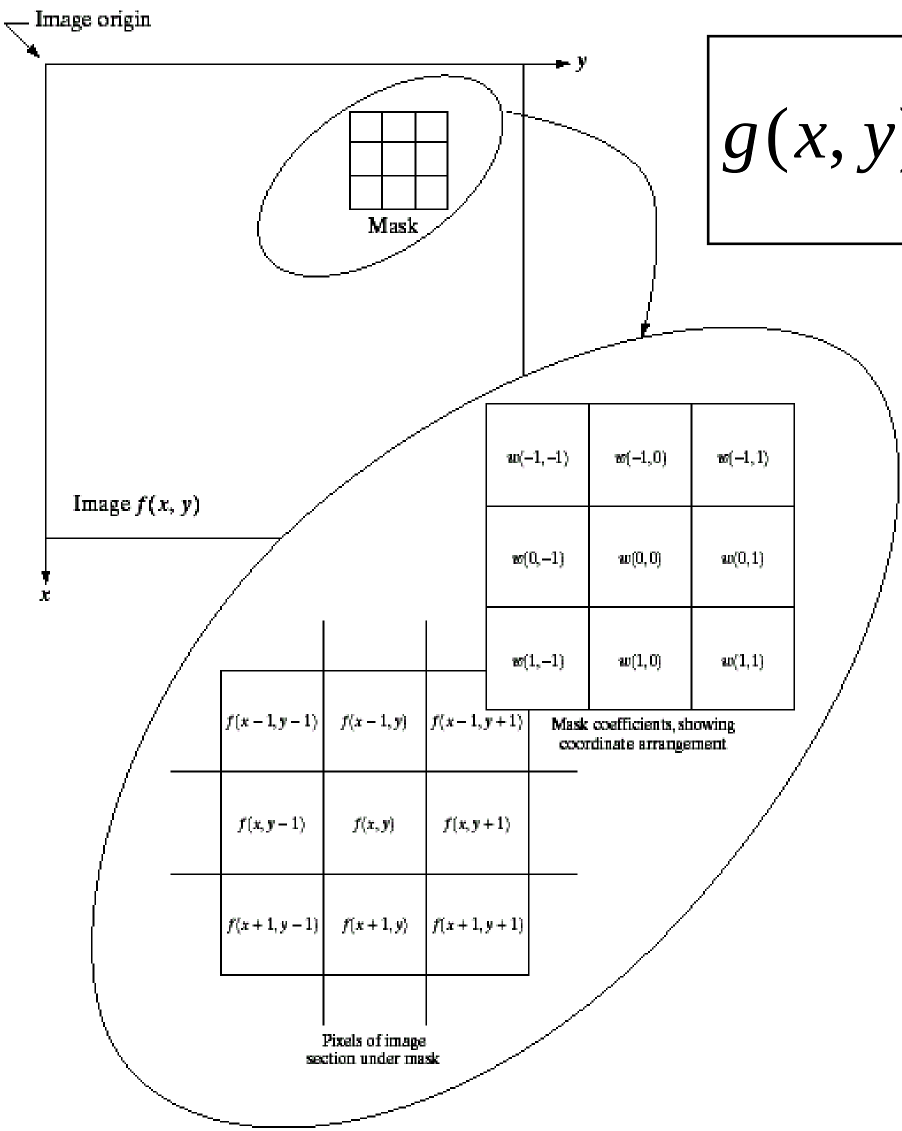
r	s	t
u	v	w
x	y	z

Filter

$$e_{processed} = v*e + r*a + s*b + t*c + u*d + w*f + x*g + y*h + z*i$$

The above is repeated for every pixel in the original image to generate the filtered image

Spatial Filtering: Equation Form



$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Filtering can be given in equation form as shown above

Notations are based on the image shown to the left

Correlation & Convolution

The filtering we have been talking so far is referred to as *correlation* with the filter itself

Convolution is a similar operation, with just one subtle difference

a	b	c
d	e	f
g	h	i

Original Image Pixels

$*$

r	s	t
u	v	w
x	y	z

Filter

$$\begin{aligned} e_{processed} = \; & v*e + \\ & z*a + y*b + x*c + \\ & w*d + u*f + \\ & t*g + s*h + r*i \end{aligned}$$

For symmetric filters it makes no difference

Smoothing Spatial Filters

One of the simplest spatial filtering operations we can perform is a smoothing operation

- Simply average all of the pixels in a neighbourhood around a central value
- Especially useful in removing noise from images
- Also useful for highlighting gross detail

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

Simple averaging filter

Weighted Smoothing Filters

More effective smoothing filters can be generated by allowing different pixels in the neighbourhood different weights in the averaging function

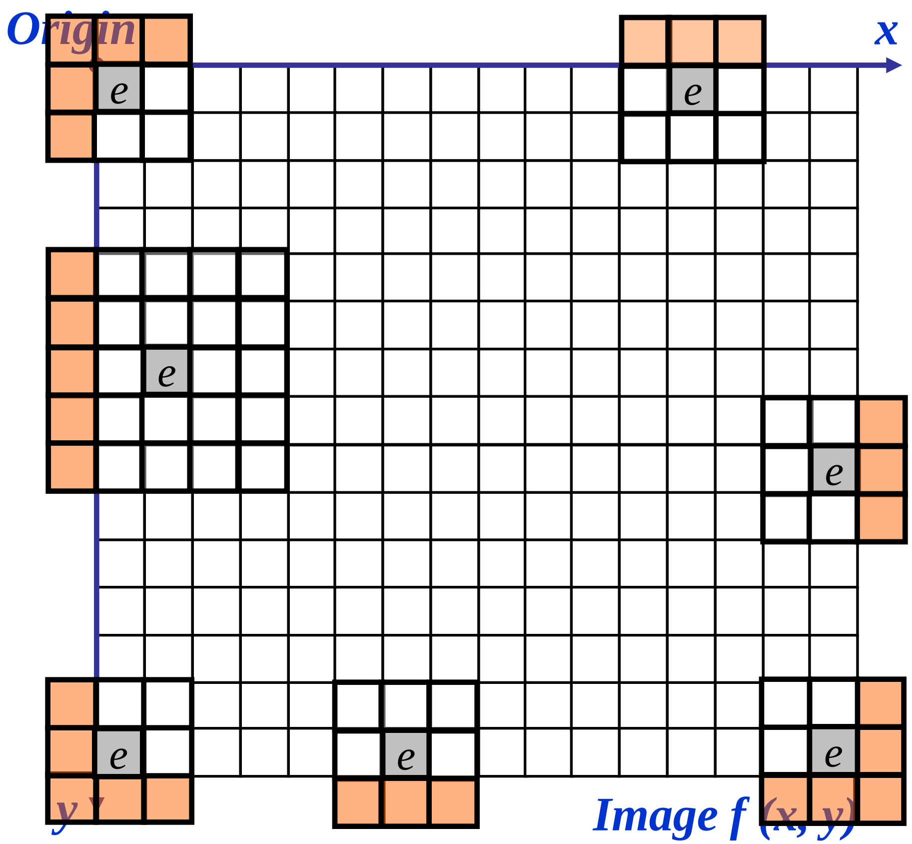
- Pixels closer to the central pixel are more important
- Often referred to as a *weighted averaging*

$^1/_{16}$	$^2/_{16}$	$^1/_{16}$
$^2/_{16}$	$^4/_{16}$	$^2/_{16}$
$^1/_{16}$	$^2/_{16}$	$^1/_{16}$

Weighted averaging filter

What Happens At The Edges!

At the edges of an image we are missing pixels to form a neighbourhood



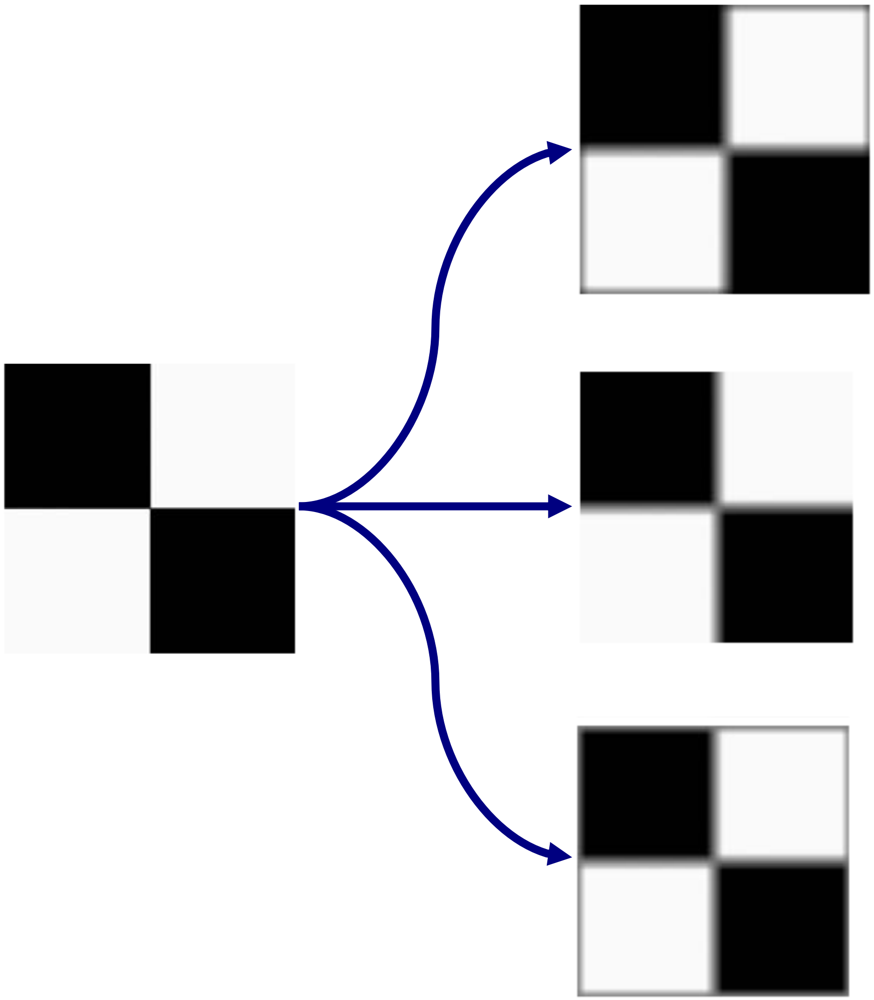
What Happens At The Edges! (cont…)

There are a few approaches to dealing with missing edge pixels:

- Omit missing pixels
  - Can add extra code and slow down processing
- Pad the image
  - Typically with either all white or all black pixels
- Replicate border pixels
- Truncate the image
- Allow pixels *wrap around* the image
  - Can cause some strange image artefacts

What Happens At The Edges!

Original Image



Averaged Image: Zero Padding

Averaged Image: Replicate Edge Pixels

Averaged Image: Wrap Around Edge Pixels

Image Smoothing Example

- The image at the top left is an original image of size 500*500 pixels
- The subsequent images show the image after filtering with an averaging filter of increasing sizes
  - – 3, 5, 9, 15 and 35
- Notice how detail begins to disappear

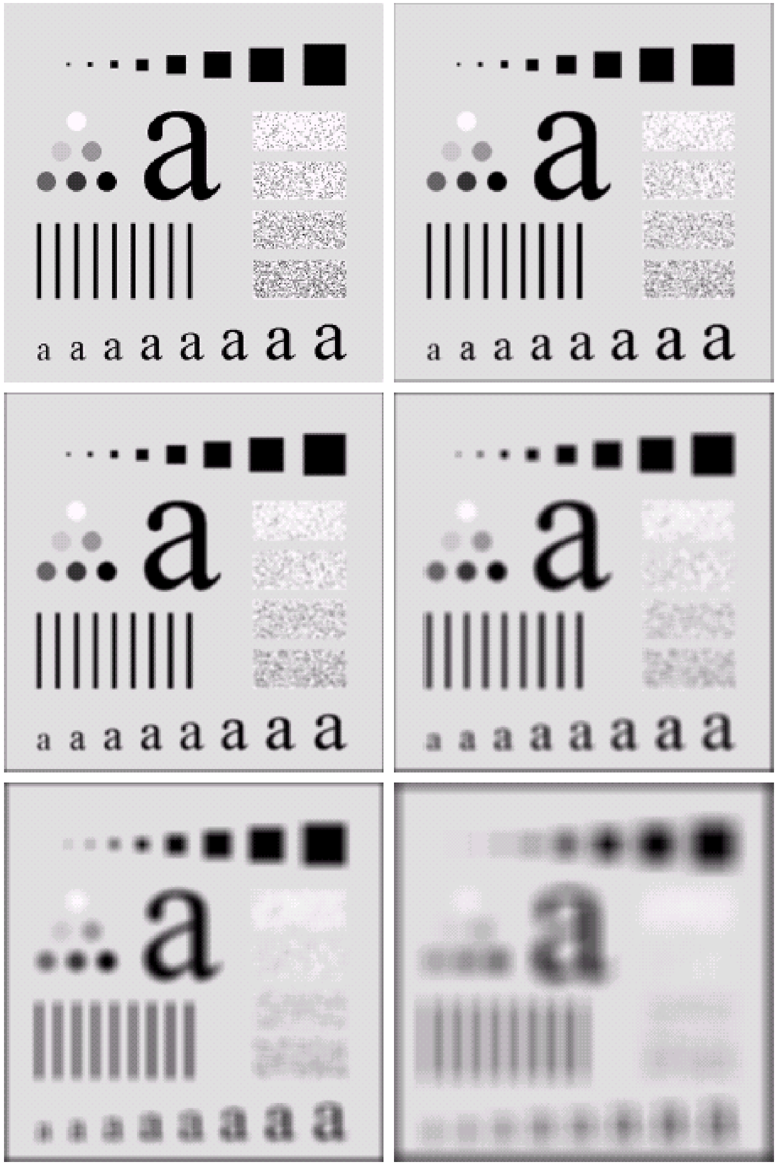


Image Smoothing Example

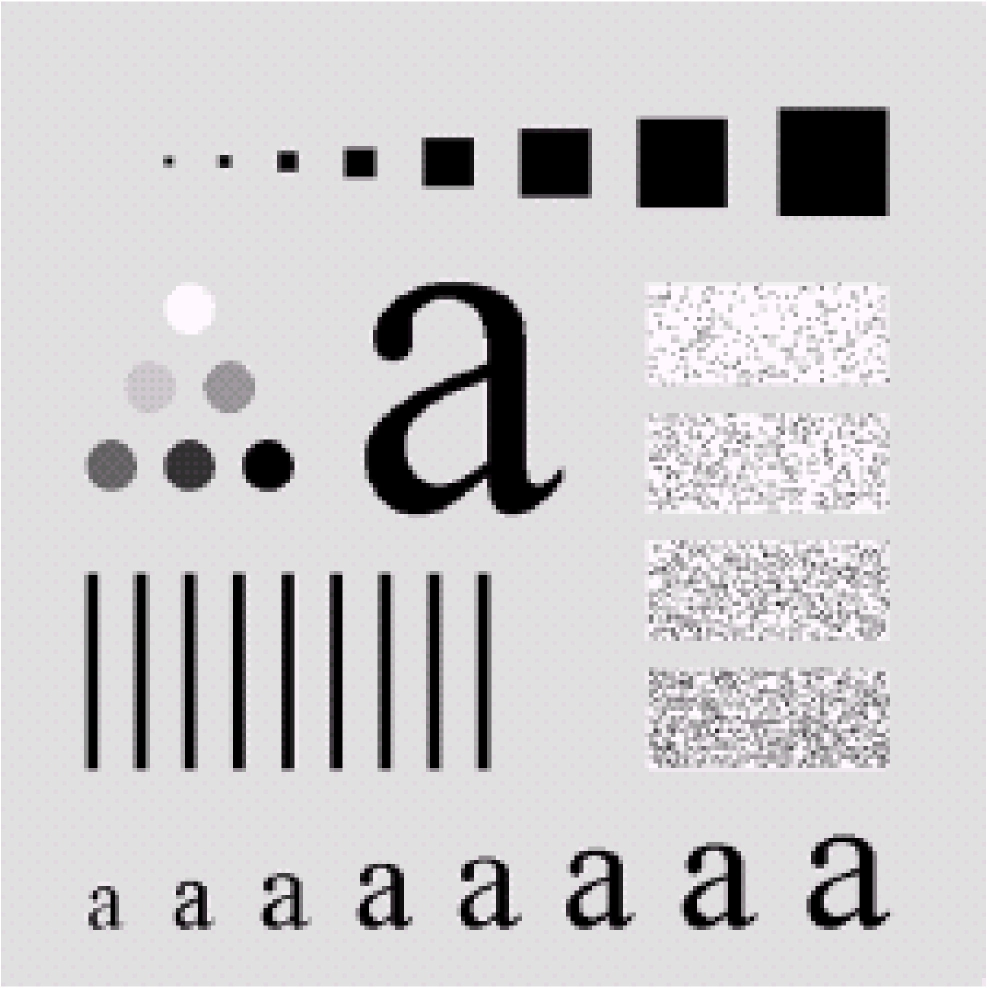
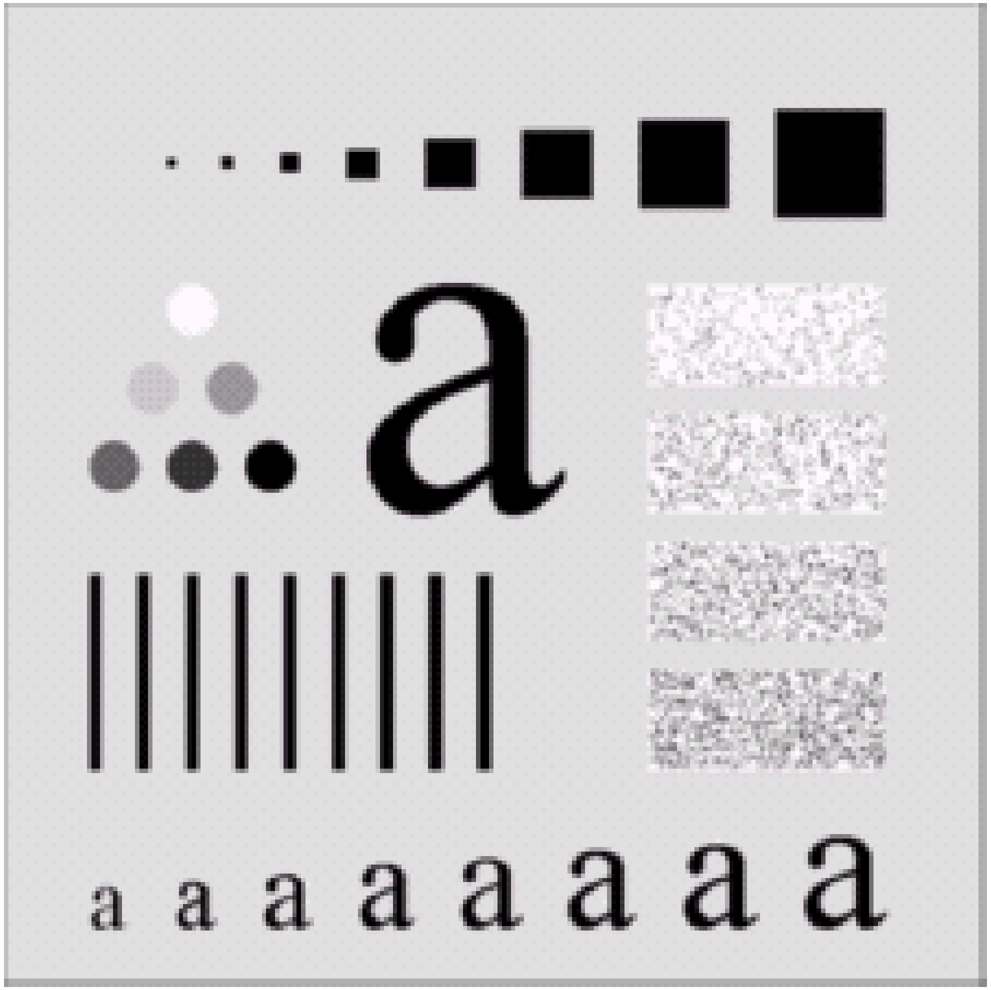


Image Smoothing Example



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Smoothing Example

Image Smoothing Example

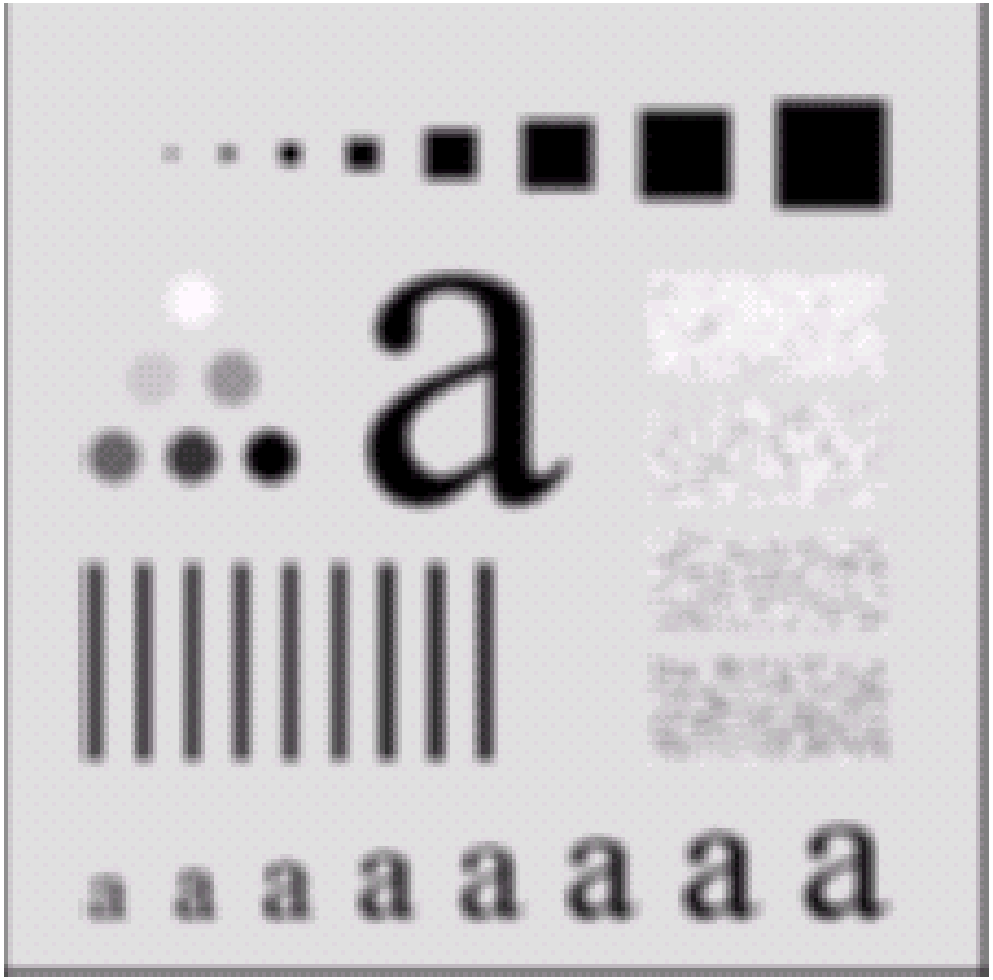


Image Smoothing Example

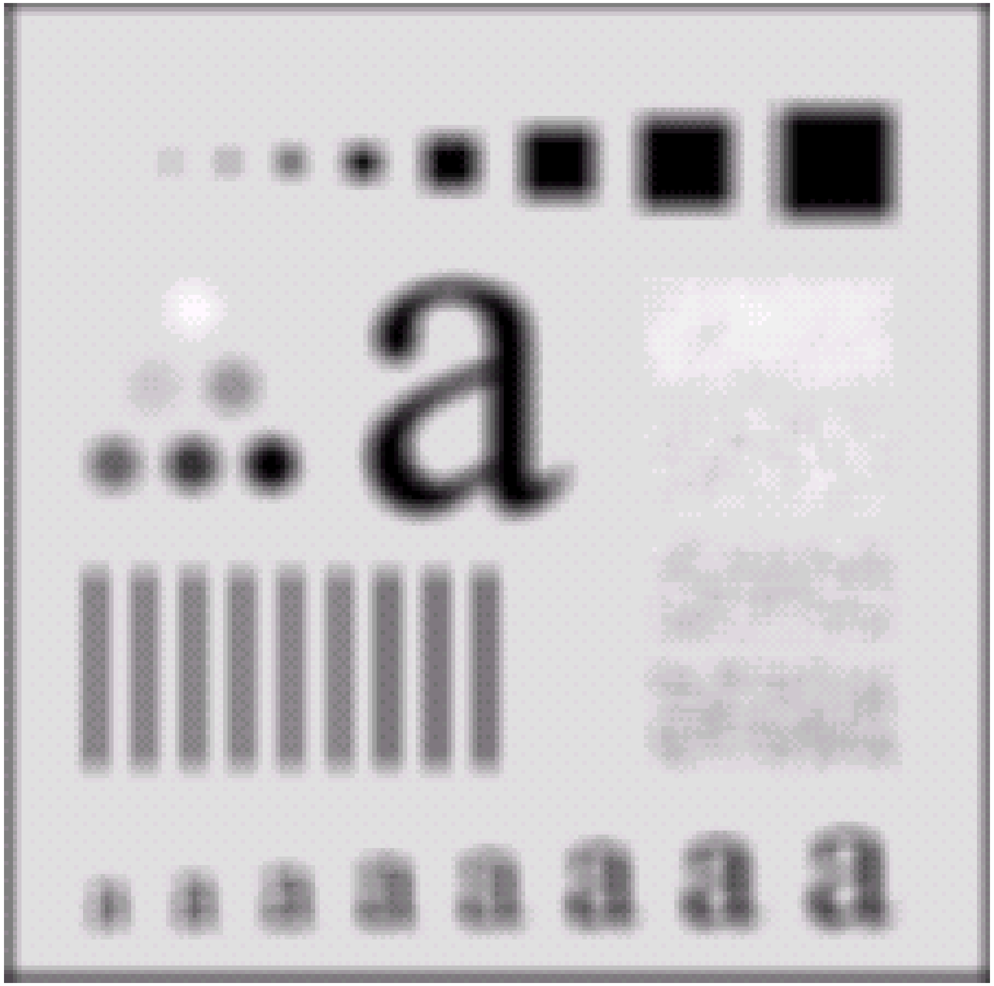
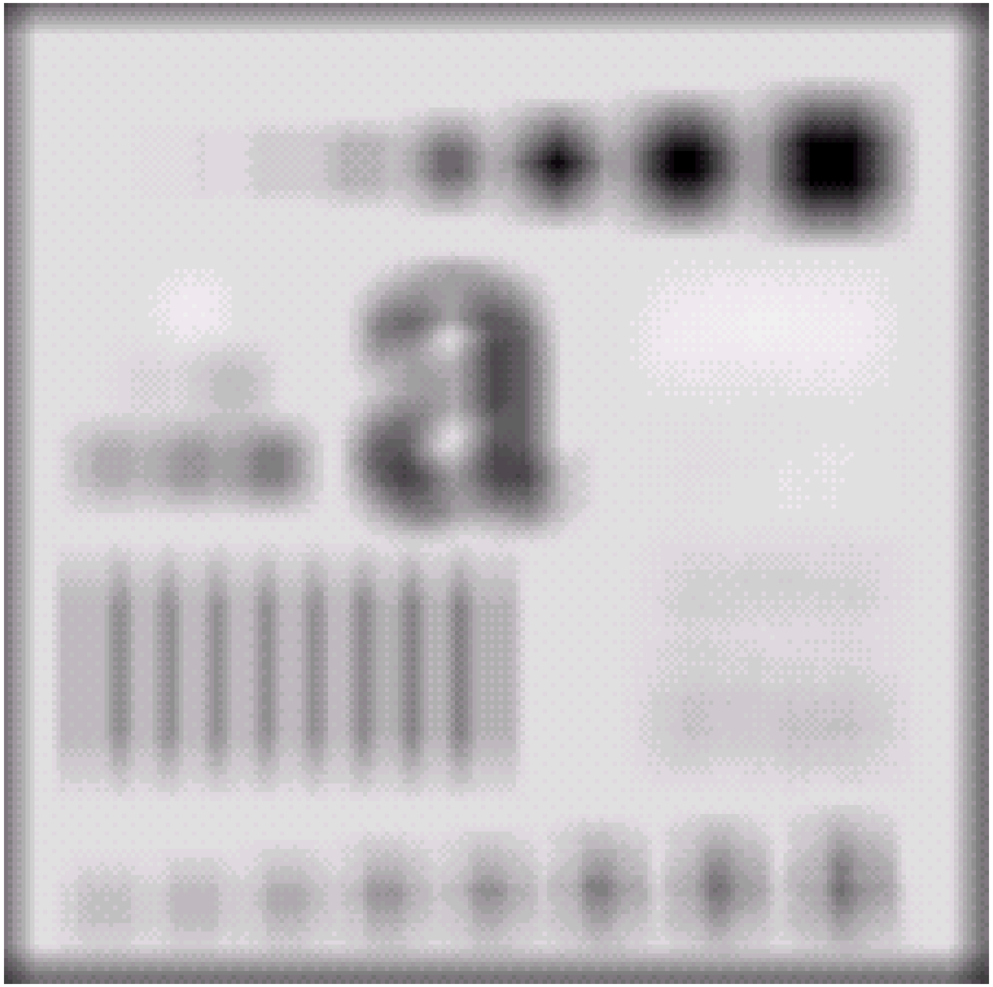


Image Smoothing Example

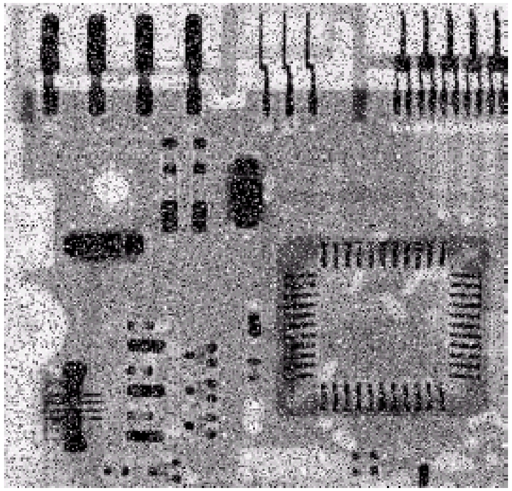


Order Statistics Filters

Some simple neighbourhood operations include:

- **Min:** Set the pixel value to the minimum in the neighbourhood
- **Max:** Set the pixel value to the maximum in the neighbourhood
- **Mean**: Set the pixel value to the mean in the neighbourhood
- **Median:** The median value of a set of numbers is the midpoint value in that set (e.g. from the set [1, 7, 15, 18, 24] 15 is the median).

Averaging Filter Vs. Median Filter Example



Original Image With Noise

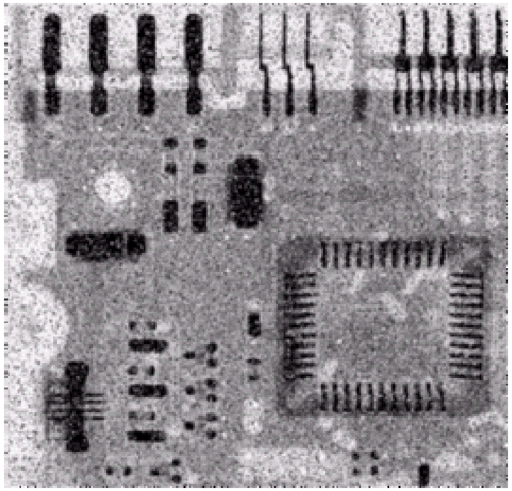


Image After Averaging Filter

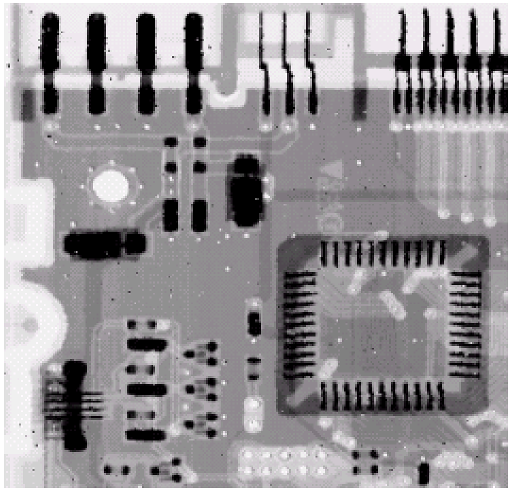
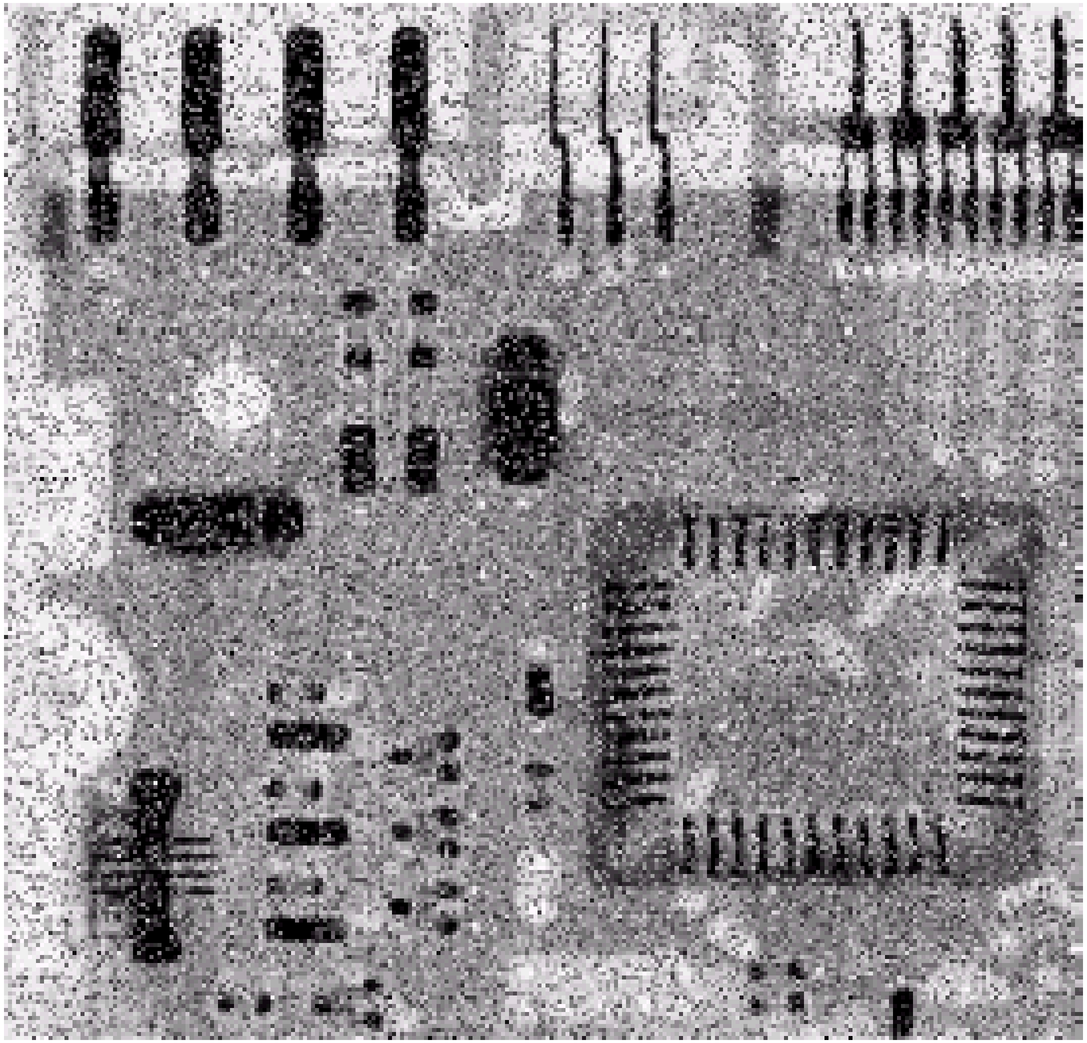


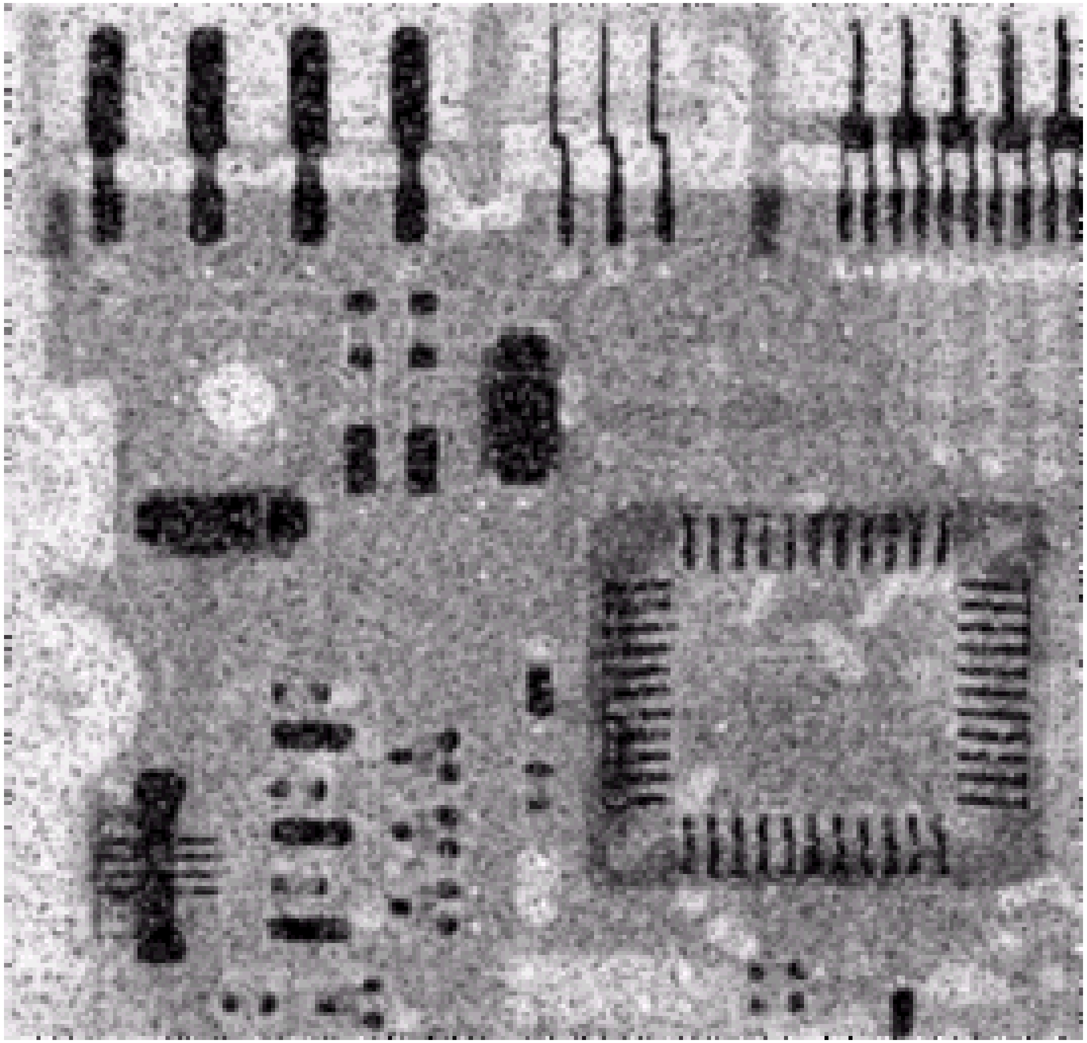
Image After Median Filter

Many times a median filter works better than an averaging filter for removing random noise

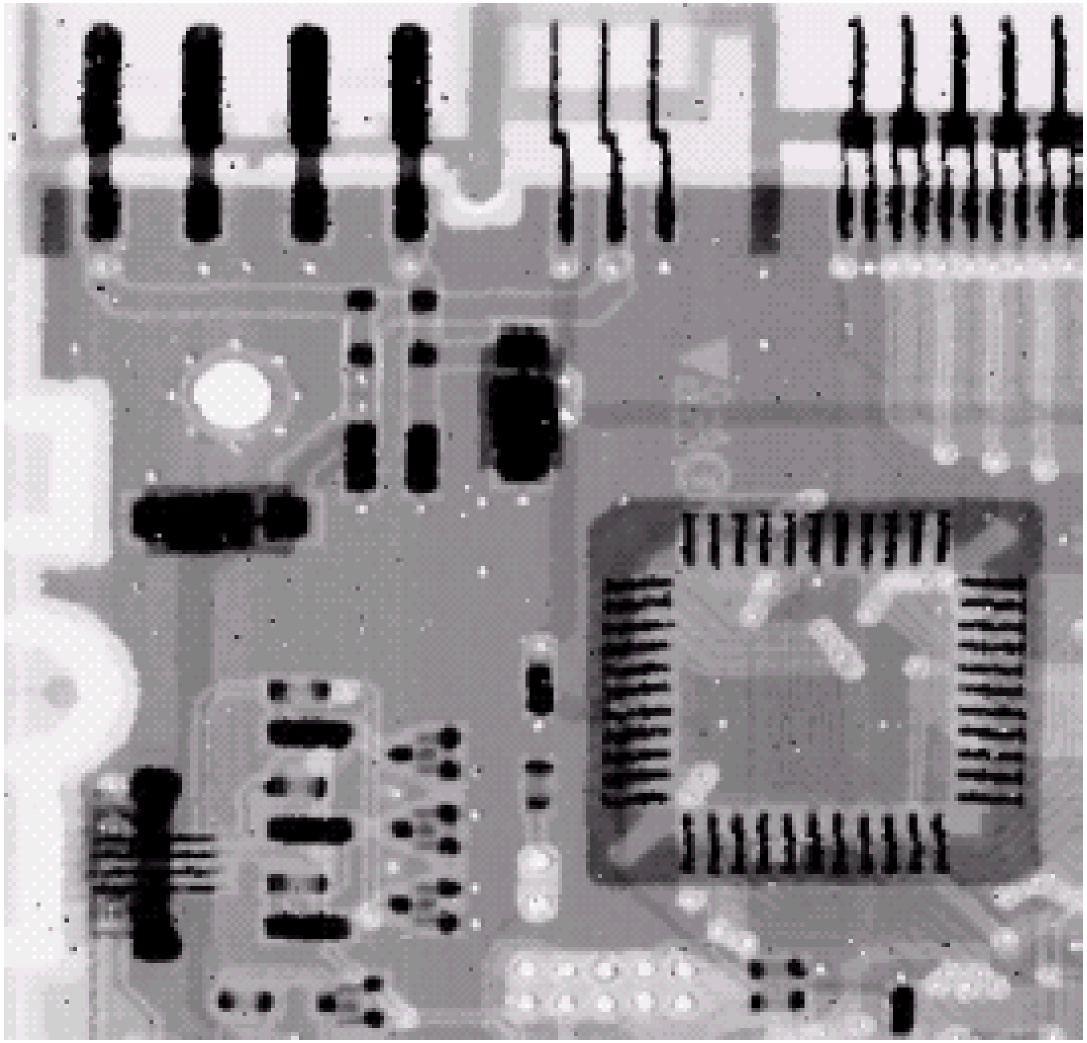
Averaging Filter Vs. Median Filter Example



Averaging Filter Vs. Median Filter Example



Averaging Filter Vs. Median Filter Example



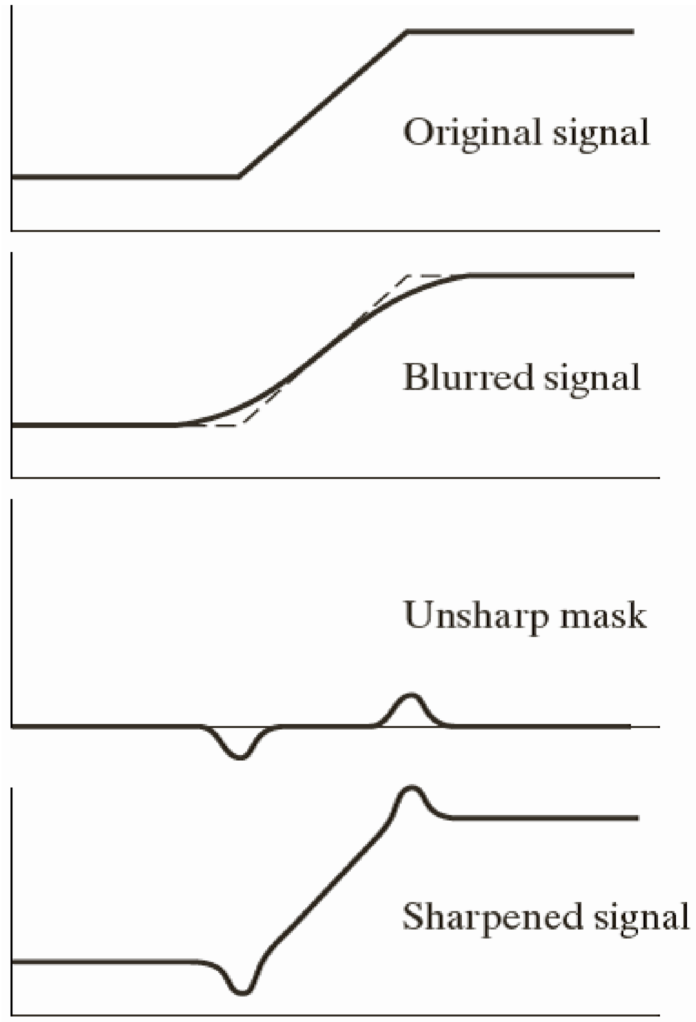
Sharpening Spatial Filters

Sharpening spatial filters seek to highlight fine detail

- Remove blurring from images
- Highlight edges

Sharpening filters are based on *spatial differentiation*

Unsharp masking & Highboost filtering



Spatial Differentiation

Differentiation measures the *rate of change* of a function. The formula for the **1st derivative** of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

It's just the difference between subsequent values and measures the rate of change of the function

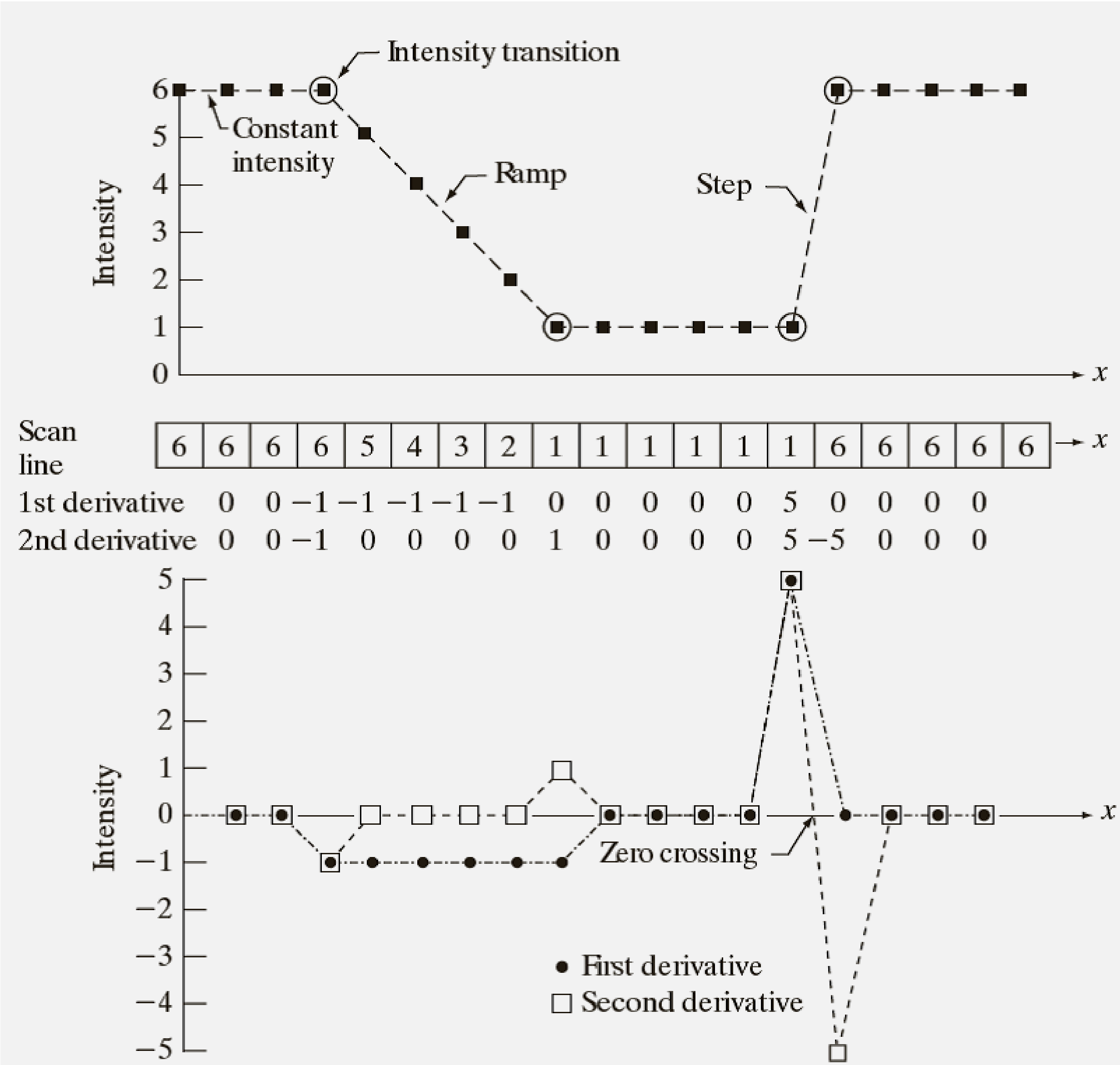
2^{nd} Derivative

The formula for the 2^{nd} derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Simply takes into account the values both before and after the current value

Let’s consider a simple 1 dimensional example



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Using Second Derivatives For Image Enhancement

- The 2nd derivative is more useful for image enhancement than the 1st derivative
  - Stronger response to fine detail
  - Simpler implementation
  - We will come back to the 1st order derivative later on
- The first sharpening filter we will look at is the *Laplacian*
  - Isotropic
  - One of the simplest sharpening filters
  - We will look at a digital implementation

The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 1st order derivative in the x direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the y direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

The Laplacian (cont…)

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

Variants On The Simple Laplacian

There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

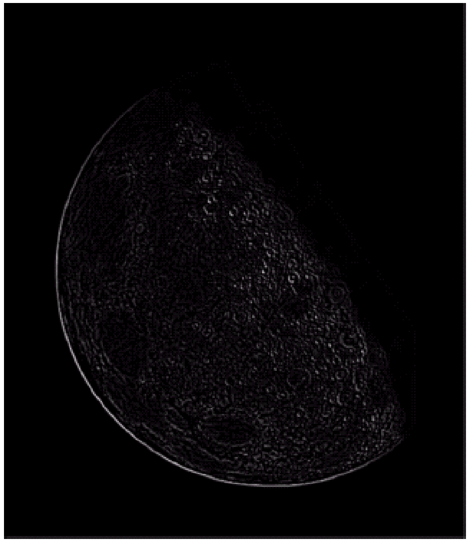
0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

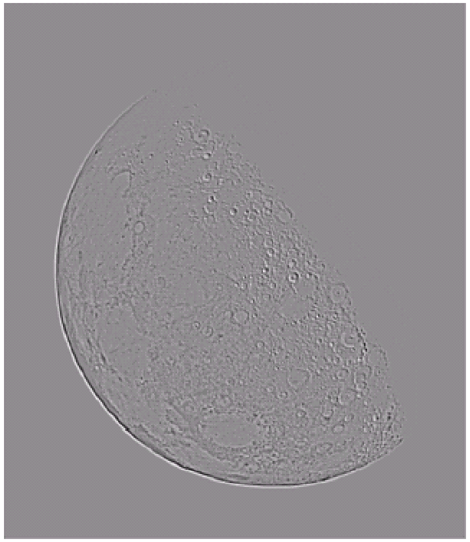
The Laplacian (cont…)

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original Image



Laplacian Filtered Image

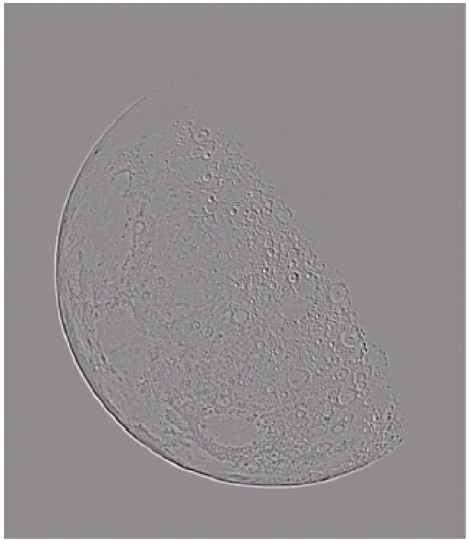


Laplacian Filtered Image Scaled for Display

But That Is Not Very Enhanced!

- The result of a Laplacian filtering is not an enhanced image
- We have to do more work in order to get our final image
- Subtract the Laplacian result from the original image to generate our final sharpened enhanced image

$$g(x, y) = f(x, y) - \nabla^2 f$$

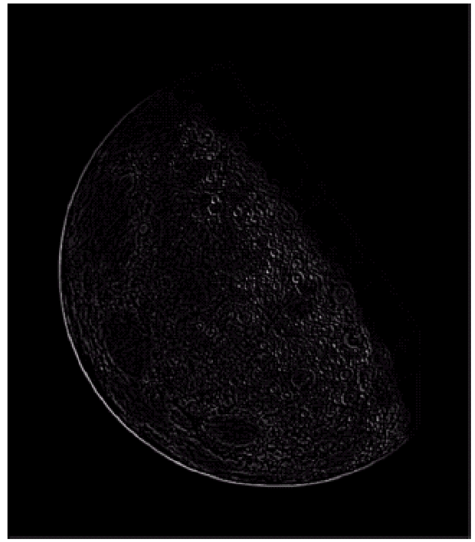


Laplacian Filtered Image Scaled for Display

Laplacian Image Enhancement

Original
Image

\-



Laplacian
Filtered Image

=

Sharpened
Image

Important to keep in mind which variant of Laplacian is used to decide whether to add or subtract

Laplacian Image Enhancement

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

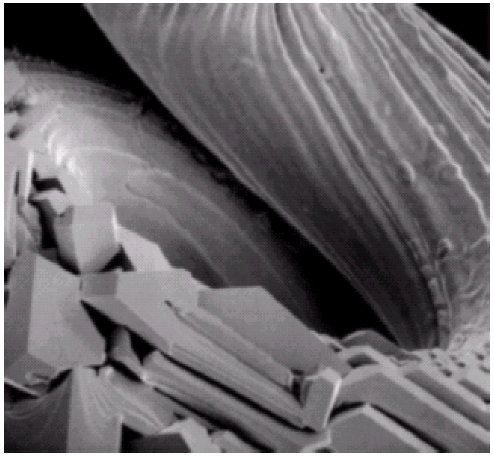
Simplified Image Enhancement

The entire enhancement can be combined into a single filtering operation

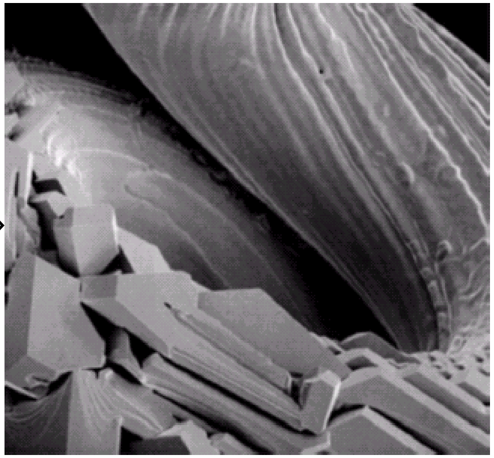
$$
\begin{aligned}
g(x, y) &= f(x, y) - \nabla^2 f \\
&= f(x, y) - [f(x+1, y) + f(x-1, y) \\
&\quad + f(x, y+1) + f(x, y-1) \\
&\quad - 4f(x, y)] \\
&= 5f(x, y) - f(x+1, y) - f(x-1, y) \\
&\quad - f(x, y+1) - f(x, y-1)
\end{aligned}
$$

Simplified Image Enhancement (cont…)

This gives us a new filter which does the whole job for us in one step

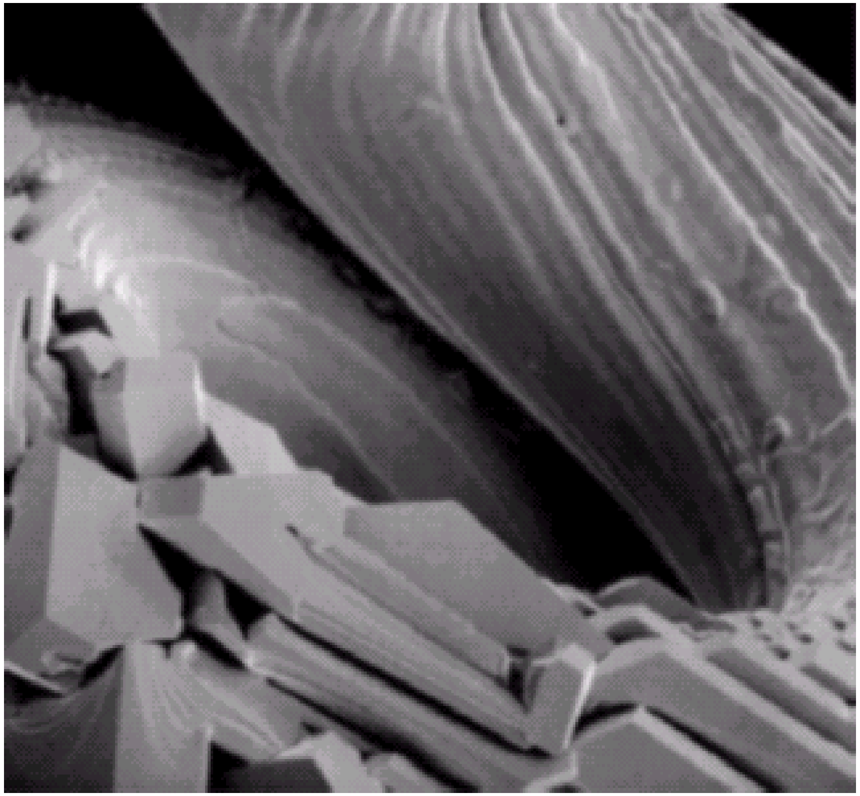


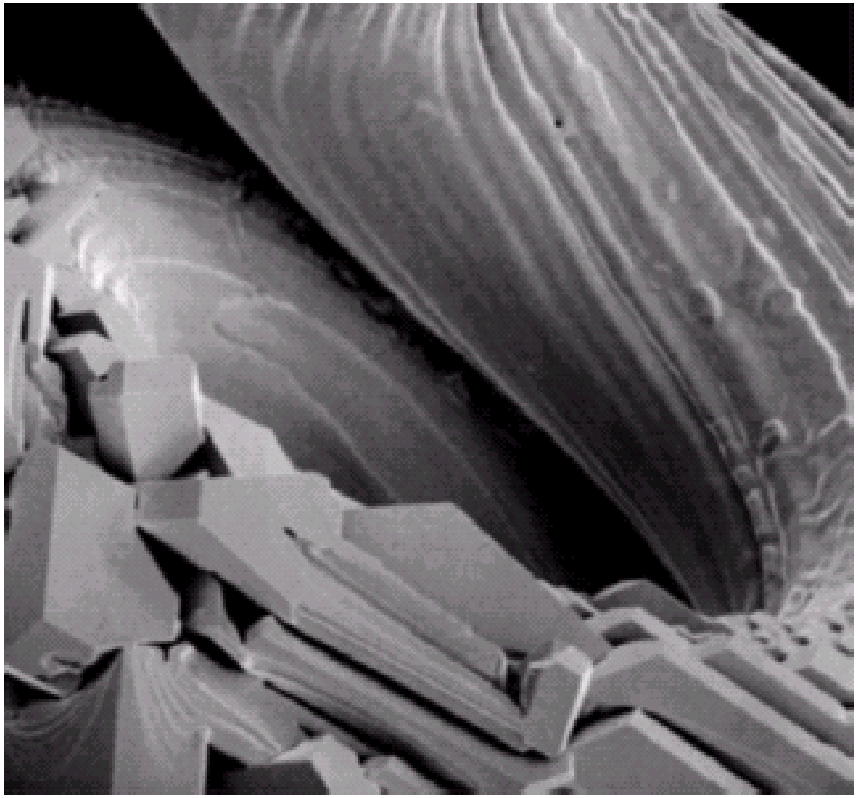
0	-1	0
-1	5	-1
0	-1	0



Q: What will be the combined filter if the diagonal directions also considered.

Simplified Image Enhancement (cont…)





1st Derivative Filtering

First derivative filtering is implemented using the magnitude of the gradient.

For a function *f(x, y)* the gradient of *f* at coordinates *(x, y)* is given as the column vector:

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

1^{st} Derivative Filtering (cont…)

The magnitude of this vector is given by:

$$\begin{aligned}\nabla f &= mag(\nabla \mathrm{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}\end{aligned}$$

For practical reasons this can be simplified as:

$$\nabla f \approx \left|G_x\right| + \left|G_y\right|$$

1^{st} Derivative Filtering (cont…)

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

−1	
1	

−1	1

−1	0		0	−1
0	1		1	0

Roberts

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

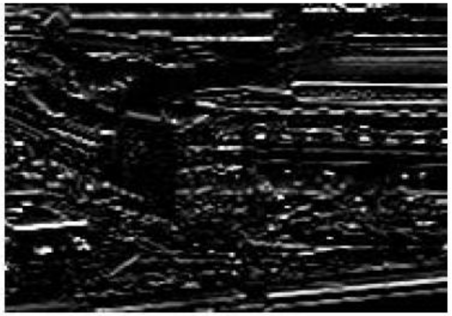
Prewitt

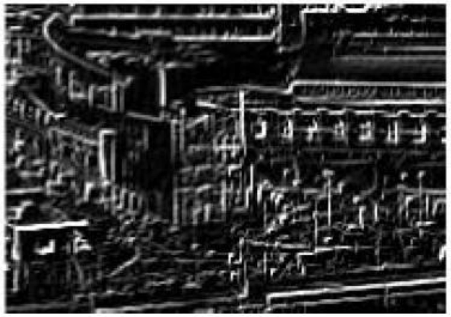
−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

Sobel

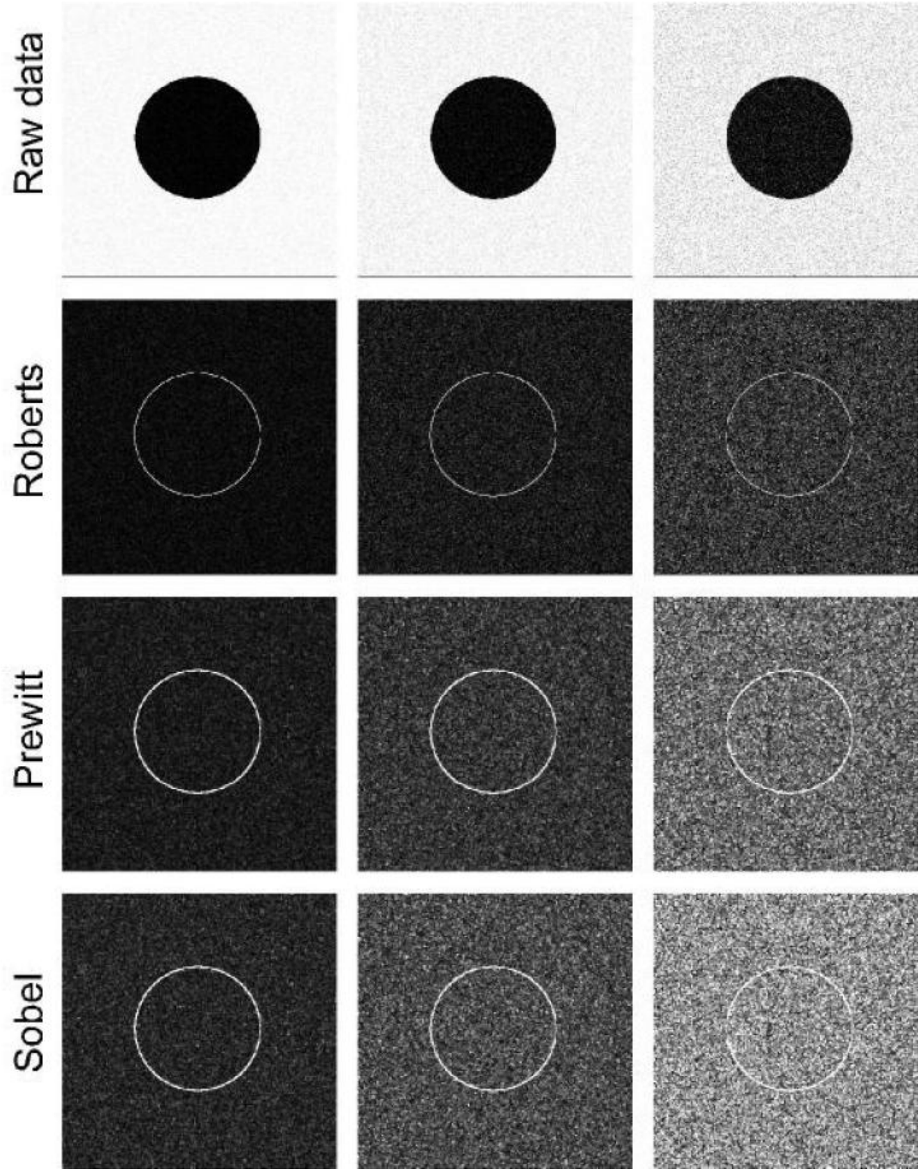
Given a 3*3 region of an image the following edge detection filters can be used

1^{st} Derivative Filtering (cont…)

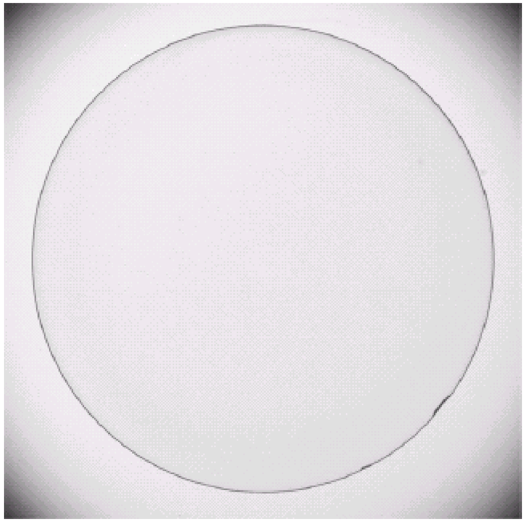


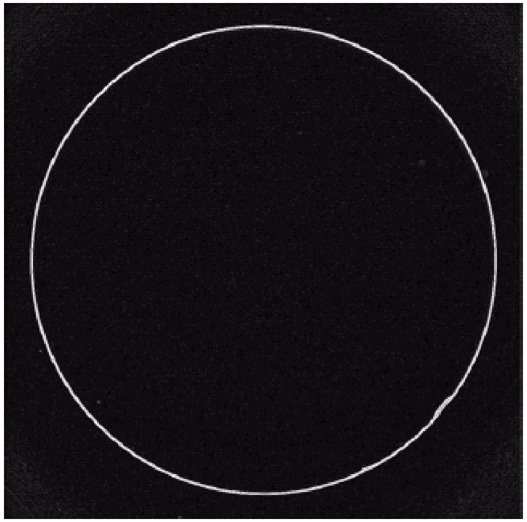


1^{st} Derivative Filtering (cont…)



Application example





An image of a contact lens which is enhanced in order to find defects (at four and five o'clock in the image) more obvious

Sobel filters are typically used for edge detection

Combining Spatial Enhancement Methods

- Successful image enhancement is typically not achieved using a single operation
- Rather we combine a range of techniques in order to achieve a final result
- This example will focus on enhancing the bone scan to the right

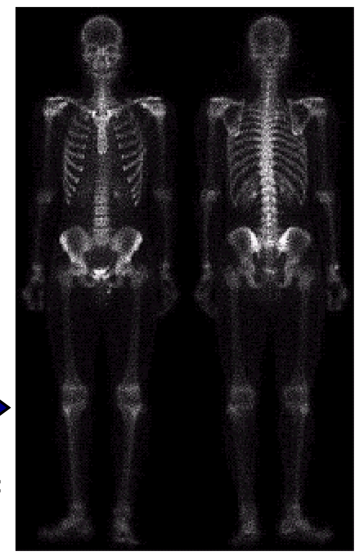
Combining Spatial Enhancement Methods (cont…)

(a)

Laplacian filter of bone scan (a)

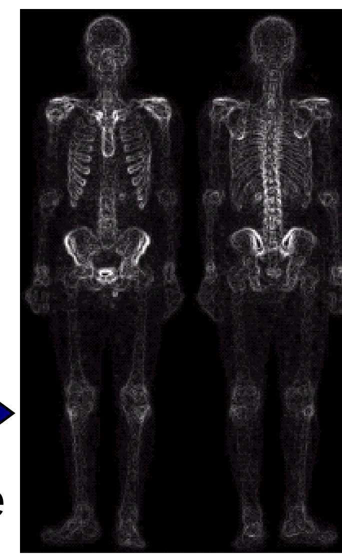
(b)

Sharpened version of bone scan achieved by subtracting (a) and (b)



(c)

Sobel filter of bone scan (a)



(d)

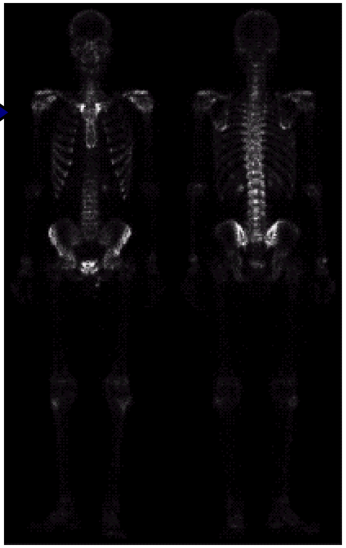
Combining Spatial Enhancement Methods (cont…)

(e)

Image (d) smoothed with a 5*5 averaging filter

The product of (c) and (e) which will be used as a mask

(f)



Sharpened image which is sum of (a) and (f)

(g)

Result of applying a power-law trans. to (g)

(h)

Combining Spatial Enhancement Methods (cont…)

Compare the original and final images