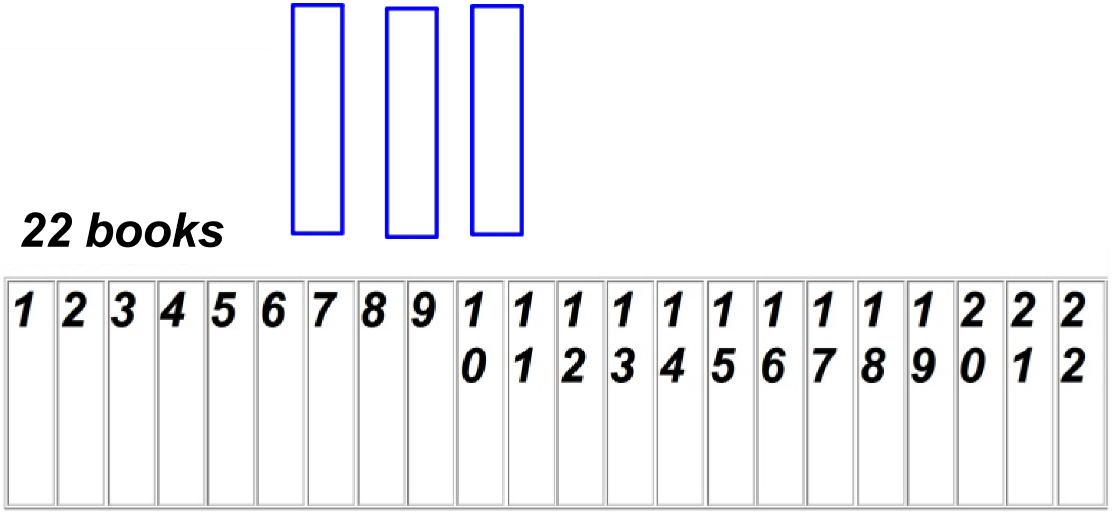
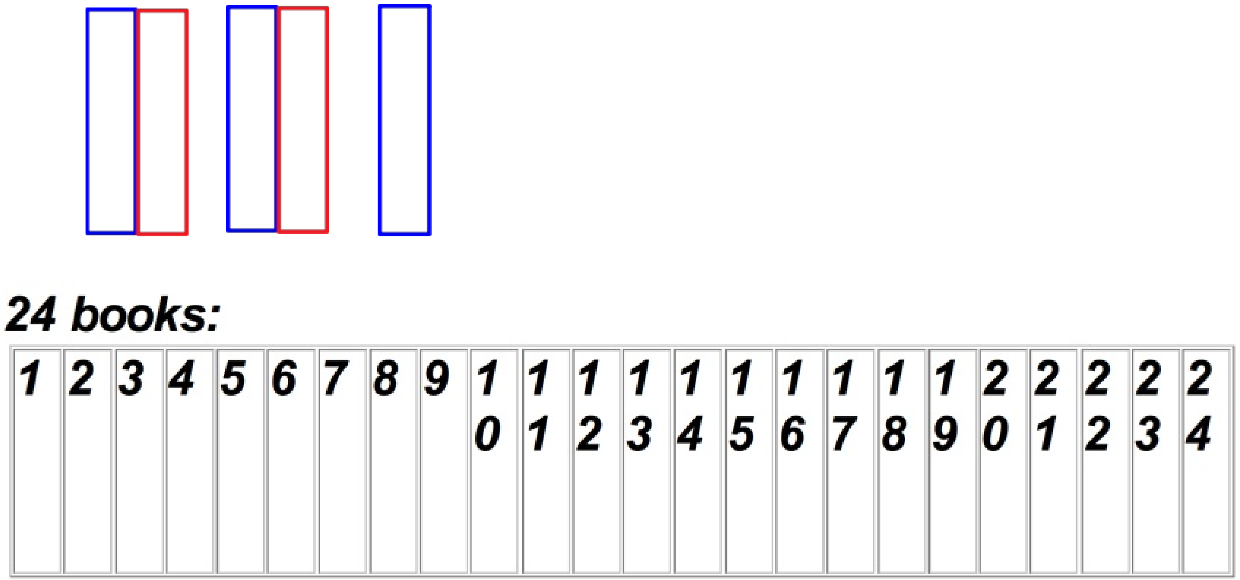
Conditioning and Bayes Formula

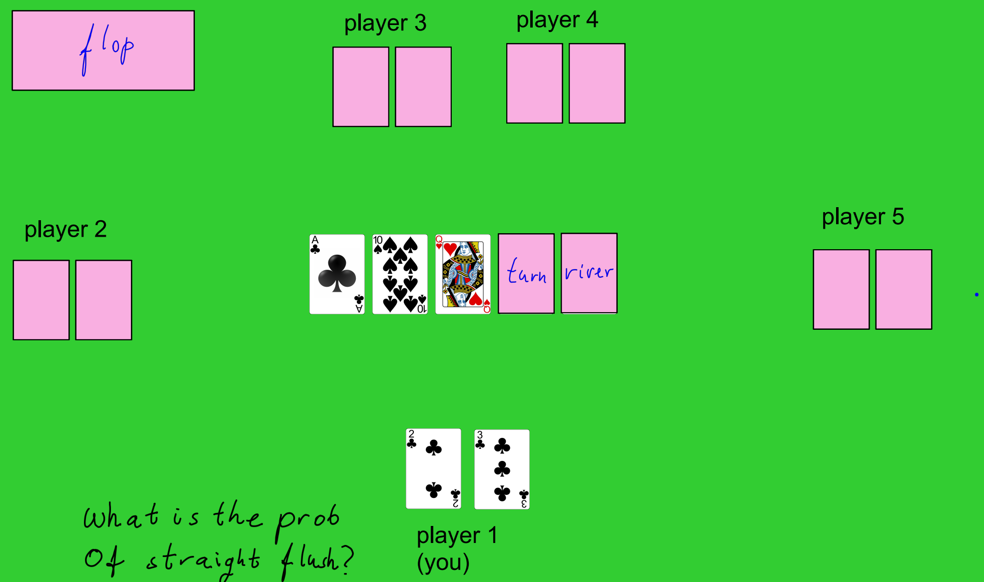
You have a shelf with 24 books on it. You can pick any 3 books but no two picked books can be right next to each other. How many choices do you have?

22 books



24 books:





What is the prob
Of straight flush?

Ω = any set of seven cards

A = a set of 7 cards that contains the 5 observed cards

B = a set of 7 cards that contain a straight flush.

$A \cap B$ = a set of 7 cards that contains the 5 observed cards and a straight flush.

we want to calc $P(B|A) = \frac{P(B,A)}{P(A)}$

$$= \frac{|B \cap A| / |\Omega|}{|A| / |\Omega|} = \frac{|B \cap A|}{|A|}$$

$|A|$ = number of ways to choose 2 more cards $\quad C(47,2)$

$|A \cap B|$ = number of ways to choose 2 more cards to make a straight flush $\quad 1$

Conditioning

Outcome space: $\Omega = \{A1, A2, B1, B2\}$

	$P(1)=$	$P(2)=$
	1	2
$P(A)=$ A	0.1	0.3
$P(B)=$ B	0.4	0.2

Events:

1 = outcome contains "1"

2 = —— " —— "2"

A = —— " —— "A"

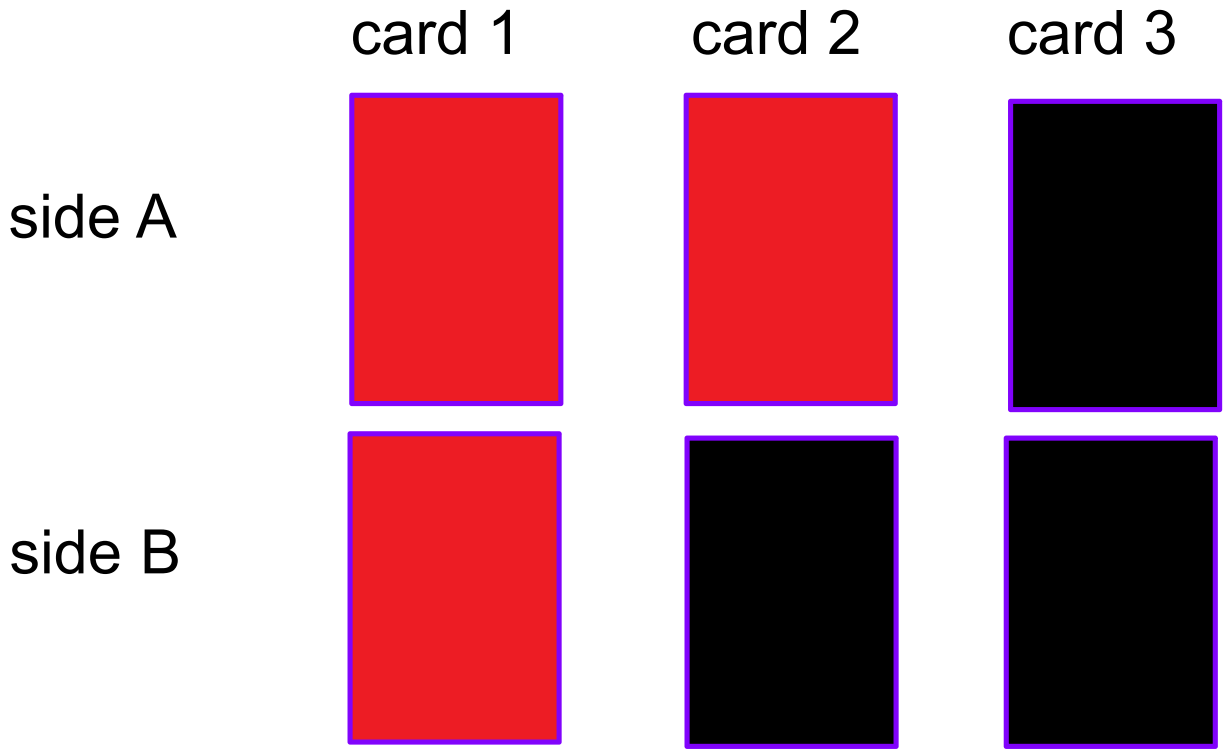
B = —— " —— "B"

$P(A1) =$

$P(A|1) =$

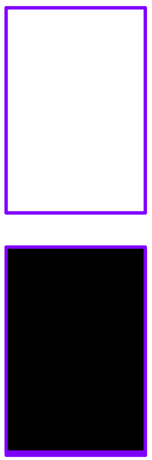
$P(1|A) =$

The three card game



The cards are in a hat, pick one at random and place it on one of the two sides at random

Choose one card at random, and put on random side.



Hidden (bottom) side

Exposed (top) side

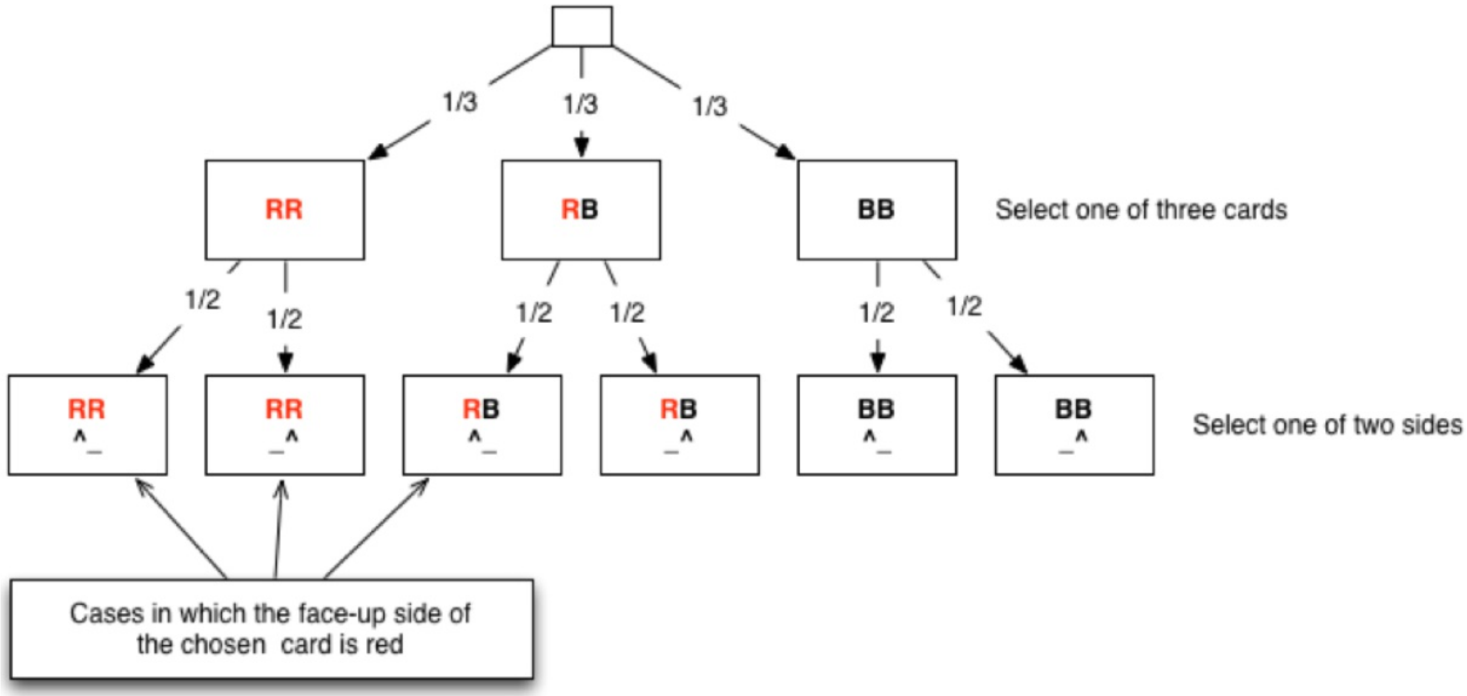
what is the color of the other side of the card?

What is the probability that the other side is red?

are the following odds fair?

Red - I give you 1$
Black - you give me 1$

Event tree for three cards



Conditional probability

- The probability that the seen color is R (B) is ½.
- The probability that the other side is R (B) given that the seen color is R(B) is 2/3.

$$P(R|R) = \frac{P(RR)}{P(R)} = \frac{1/3}{1/2} = \frac{2}{3}$$

Bayes Formula

A light burned out, we need a new light-bulb, we ask john to get a new light-bulb

with prob. 0.1 John gets it from Store

$p(\text{store}) = 0.1$

with prob 0.9 John Gets it from drawer

$p(\text{drawer}) = 0.9$

store bulbs are good with prob. 0.99

$p(\text{good} \mid \text{store}) = 0.99$

Drawer bulbs are good with prob 0.5

we test the light-bulb and it is bad.

what is the probability that john went to the store?

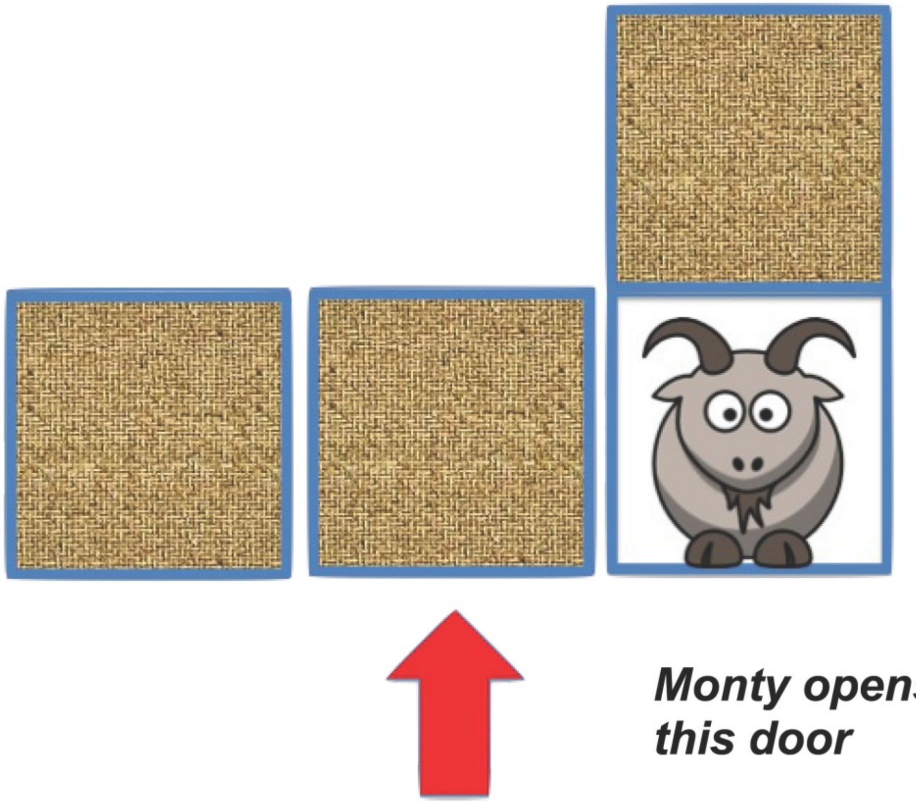
$$p(\text{store} \mid \text{bad}) = \frac{p(\text{store}, \text{bad})}{p(\text{bad})} = \frac{p(\text{bad} \mid \text{store})\, p(\text{store})}{p(\text{bad})}$$

$$p(\text{bad}) = p(\text{bad}, \text{store}) + p(\text{bad}, \text{drawer})$$

The Monty Hall Puzzle

- Monty Hall was a variety show on TV.
- In one of the games there are three doors, one hiding a treasure, two hiding goats.
- Your goal is to select the door with the treasure.


I am betting
on this door



Monty opens this door

I am betting on this door

I am allowed to switch, should I?

[Argument that it does not matter:](#)

The chance that the treasure is behind each of the doors 50%.

As the probabilities are equal, it does not matter whether we switch or not.

Argument for choosing one of the two unopen doors at random.

Before I had to choose between 3 doors - my probability of sucsses was 1/3

Now I am choosing between between two doors, my probability of success is 1/2
So random is better than staying on the same door.

Argument for Switching.

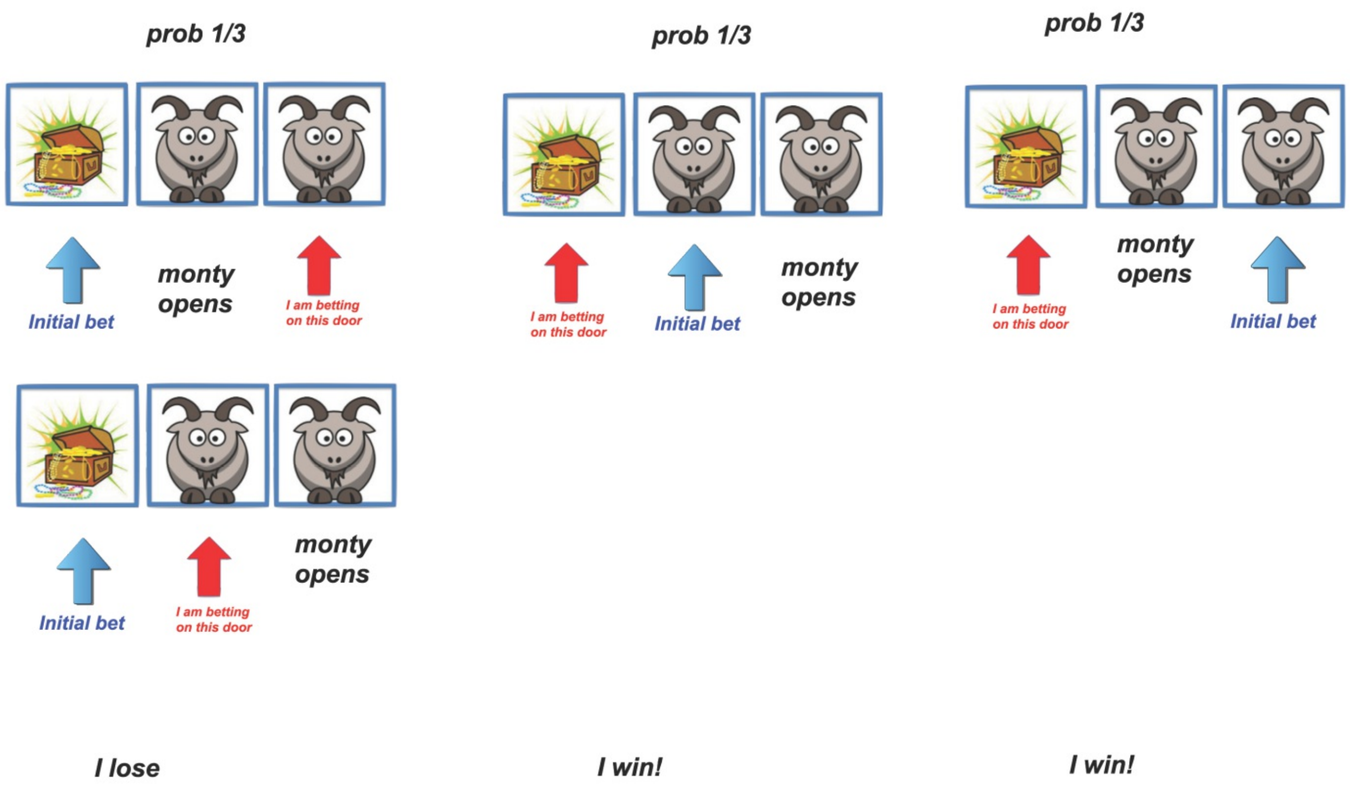
The probability that the treasure is behind the door I chose did not change.
Therefor the probability that switching will put me on the treasure must be 2/3:

1/2*1/3+1/2*2/3 = 1/2

Arguments against switching:

I know already that one of the other doors has a goat behind it. So getting the information does not tell me anything new.

Analysis for always switching



Hidden Assumption: *monty always opens a door to reveal a goat.*

In fact, he might have his own goals:

If Monty wants us to lose: *open door only when we choose the treasure door.*

If Monty wants us to win: *open door only when we choose a goat door.*

For us the only SAFE thing to do is not to switch.

This is called the "Min-Max" strategy.

Min-Max is the strategy the guarantees us the best outcome in the worst case.

More on that - game theory.