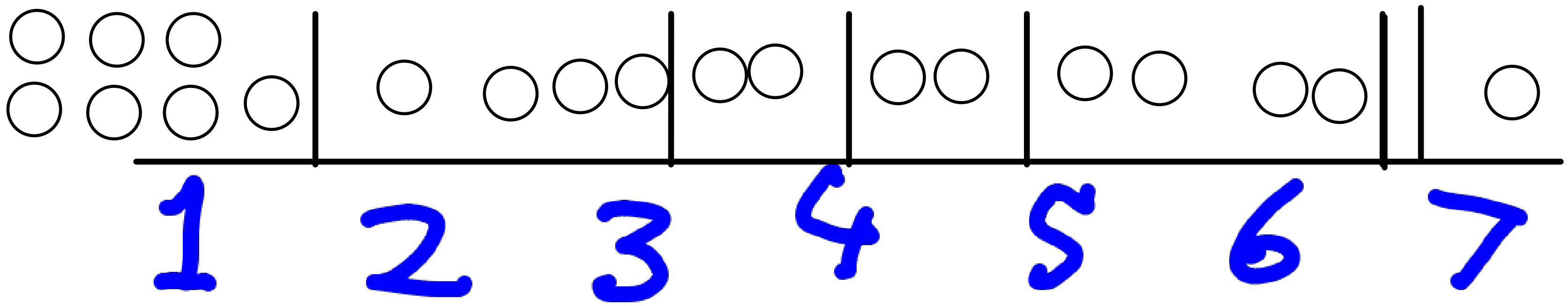
Combinatorics 3
poker hands
and Some general probability

How many different ways to place 12 circles into 7 cells?
(Each bin can hold any number of circles)

$C(m+n, n)$

$\bigcirc = C(m+n, m)$

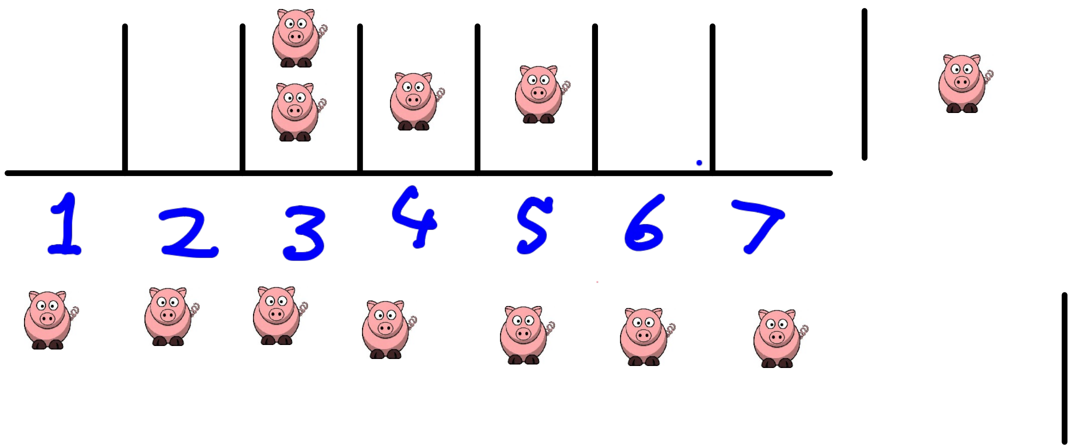


12 0 6 1

$C(12+6, 6)$

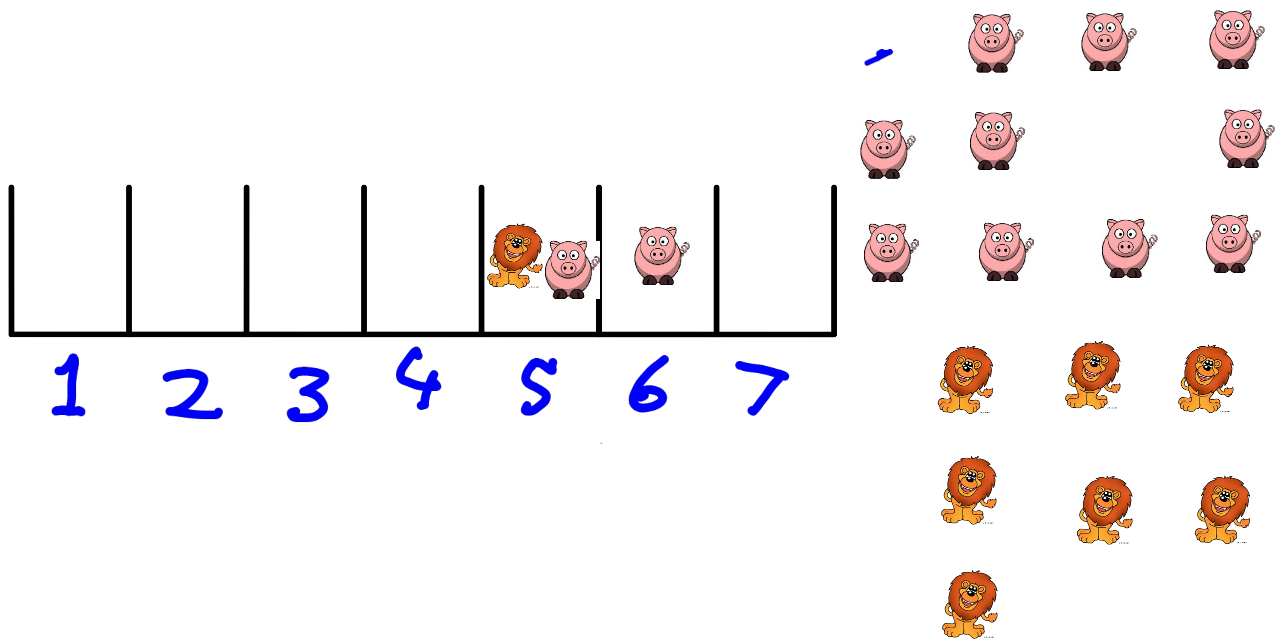
How many different ways to place 12 pigs into 7 bins such that each bin contains at least one pig ?

$C(5+6, 5)$



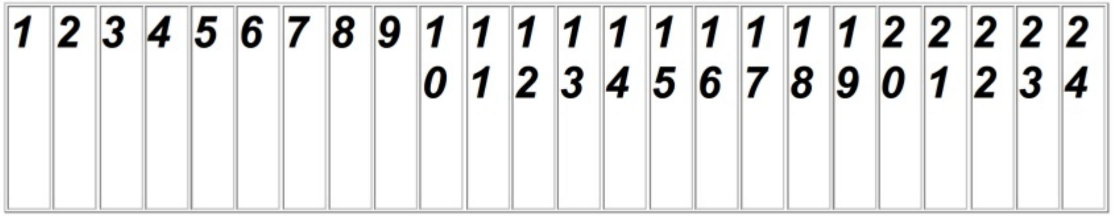
$C(18,6) \times C(13,6)$

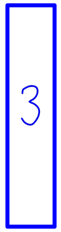
How many different ways to place 12 pigs and 7 lions into into 7 bins, where each bin can contain any number of pigs and lions.?



You have a shelf with 24 books on it. You can pick any 3 books but no two picked books can be right next to each other. How many choices do you have?

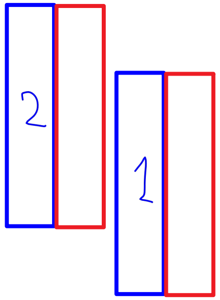
24 books:





BLUE: chosen Book

RED: place Holder



Equivalent to choosing 3 out of 24-2=22 books:
If we care about order of chosen books: P(24-2,3)
If we don't care about order of chosen books: C(24-2,3)

If you choose 3 out of 24 books at random, what is the probability that at least 2 would be next to each other?

Size of sample space (Omega):
number of way to choose 3 out of 24 books:
If we care about order of chosen books: P(24,3)
If we don't care about order of chosen books: C(24,3)

$$P(A) = 1 - \frac{|A^c|}{|\Omega|} = 1 - \frac{P(22,3)}{P(24,3)} = 1 - \frac{C(22,3)}{C(24,3)}$$

Play cards

Total: 4X13=52 cards

You pick one card from a shuffled deck.
What is the probability that it is the Ace of Spades?
1/52

You pick one card from a shuffled deck.
What is the probability that it is a spade or a diamond?
2/4 = 1/2

You pick one card from a shuffled deck.
What is the probability that it's rank is higher than 5?

Assuming that Ace is the highest we get 9/13

Basic Poker Rules

1. Each player has two private cards
2. There are 5 shared cards
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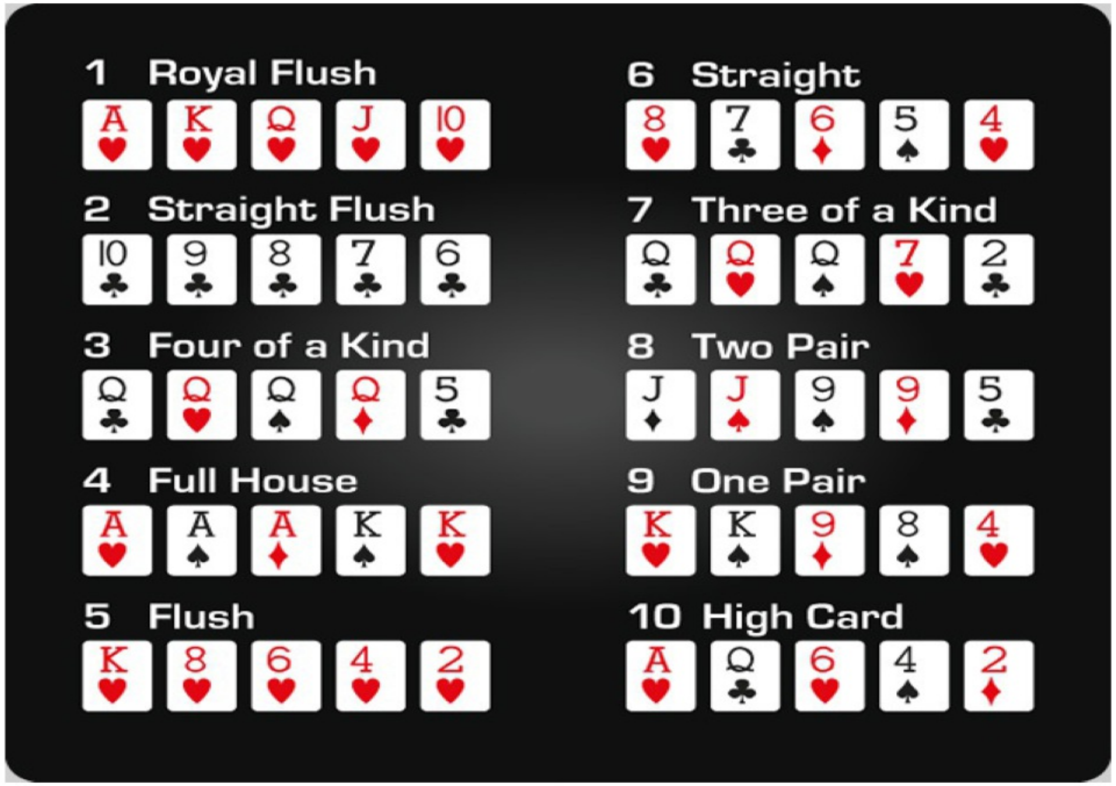
High Rank = Low Probability

The rank of hands in poker



The basic rules of texas hold'm poker

1. Each player is dealt 2 cards (the hole)
2. A round of betting
3. Three cards are revealed (the flop)
4. A round of betting
5. Fourth card is revealed (the turn)
6. A round of betting
7. Fifth card is revealed (the river)
8. Final round of betting
9. The highest ranked hand wins.

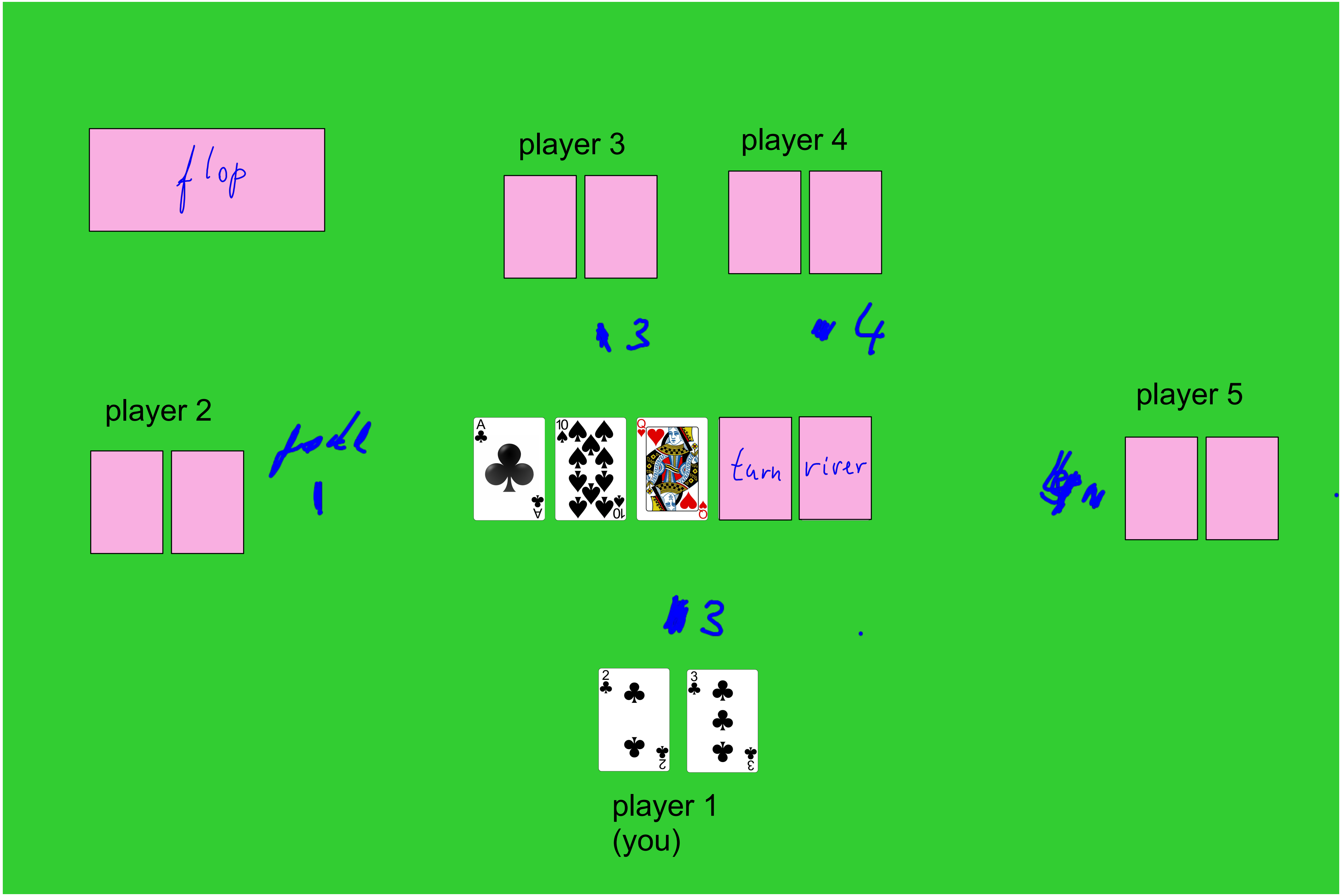
(unless everybody folds and the leader bluffs)

Betting rounds

1. Proceed clockwise.
2. Bets start with a minimal amount and can only increase
3. Each person has to either:
- *Check: Match the current bet.*
- *Raise: bet a larger amount*
- *fold: quit the game (losing the money already put in)*

4. A round of betting repeats circling until a round where all players either checked or folded = a round in which the bet has not increased.

5. In the final round:
- if only one player remains, they win all of the bets (the pot).
- if more than one player is checked, there is a "showdown", the checked players show their cards and the one with the stronger hand wins.



flop
player 3
player 4
$3
$4
player 2
fold
player 5
turn
river
$3
player 1
(you)

Poker is a game of talent, not of chance

Each player tries to estimate the chances that theirs is a winning hand from the revealed cards and from the betting actions of the others.

At the high levels of the game, familiarity with the betting styles of other players is critical.

Winning or losing a single game is of little importance, it is the long term average that matters.

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At a minimum, a player has to have an intuitive knowledge of the probabilities of different hands.

Which is what we will now do.

Calculating the probabilities of different hands

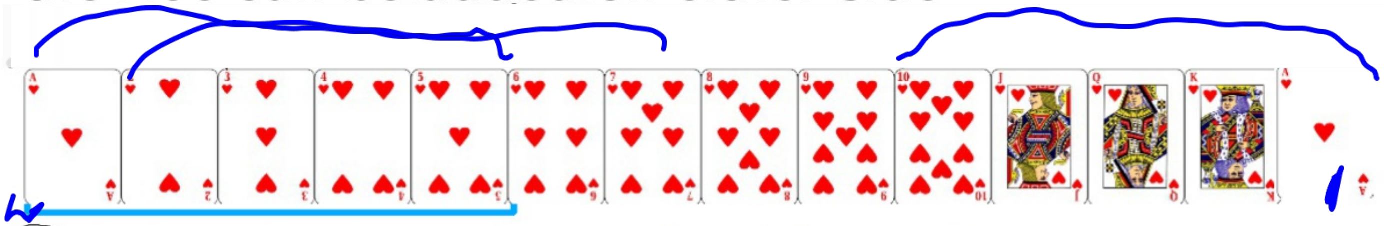
What is the sample space?

The sets of 5 cards out of 52.
Order does not matter
C(52,5) = 2,598,960

$$P(A) = \frac{|A|}{|\Omega|}$$

1 choice for the card ranks
4 choices for the suit
Prob = 4/C(52,5)

How many choices for the ranks?
the Ace can be added on either side



9 choices for the card ranks (can't be royal)
4 choices for the suit
Prob = 4*9/C(52,5)=36/C(52,5)

Number of choices for the rank of the 4 cards?
13

choices for the rank of the single?
12

choices for the suit of the single?
4

Prob = (13*12*4)/C(52,5)=624/C(52,5)

Number of choices for the rank of the trip

13

Number of choices for the rank of the pair:

12

Number of choices for the suits of the triple:

C(4,3)=4

Number of choices for the suits of the pair :

C(4,2)

Prob = (13*12*C(4,3)*C(4,2))/C(52,5) = 3744/C(52,5)

Number of choices for the ranks of the cards?
C(13,5) = 10

choices for the suit of the cards?
4

Prob =
(C(13,5)*4)/C(52,5)=5148/C(52,5)

How many choices for the card ranks?
10

How many choices for the card suits (cannot be royal flush or straight flush) ?
4^5-4

Prob =
10*(4^5-4)/C(52,5)=10,200/C(52,5)

Number of choices for the rank of the triple:
13

Number of choices for the suits of the triple:
C(4,3)=4

Number of choices for the ranks of the other 2 cards
C(12,2)

Number of choices for the suits of the other 2 cards:
4*4

Prob = (13*4*C(12,2)*4*4)/C(52,5) = 54,912/C(52,5)

Unlike full house (2,3) the two pairs are indistinguishable
Number of choices for the ranks of the pairs?
C(13,2)

Number of choices for the rank of the single:
11

Number of choices for the suits of the pairs:
C(4,2)^2

Number of choices for the suit of the single:
4

Prob = (C(13,2)*11*(C(4,2)^2)*4)/C(52,5) = 123,552/C(52,5)

The lowest ranked hand = The hand with highest probability

Number of choices for the pair:
13*C(4,2)

The other 3 cards must not form a pair, else the hand will be two pairs or full house.

Number of possible ranks for the 3 cards:
C(12,3)

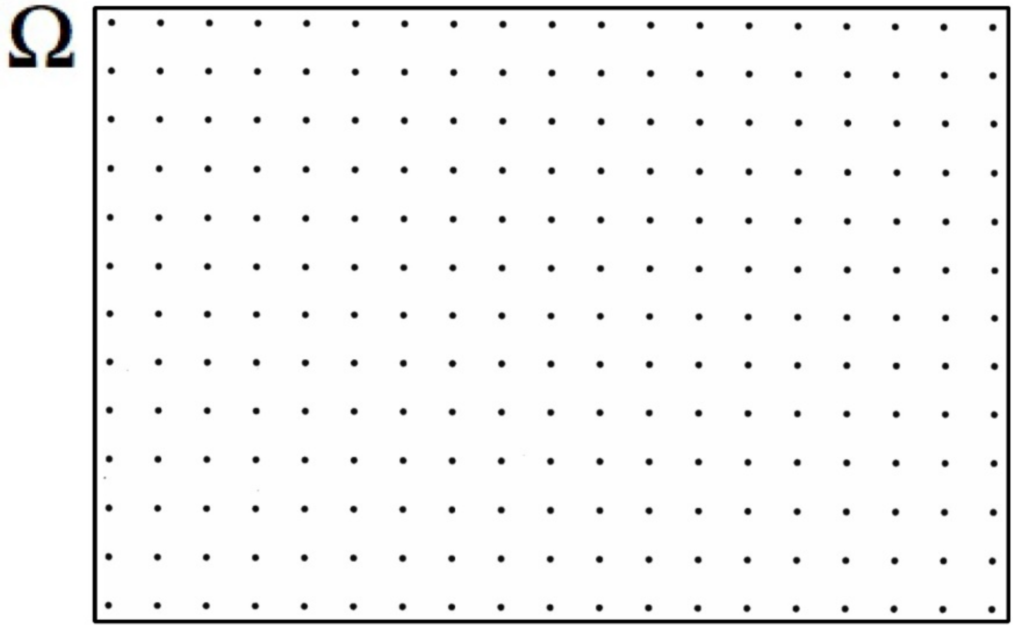
Number of possible suites for the 3 cards:
4**3

Prob=
(13*C(4,2)*C(12,3)*4**3)/C(52,5) = 1,098,240/C(52,5)

Hand	Distinct Hands	Frequency	Probability	Cumulative probability	Odds	Mathematical expression of absolute frequency
Royal flush 	1	4	0.000154%	0.000154%	649,739 : 1	$\binom{4}{1}$
Straight flush (excluding royal flush) 	9	36	0.00139%	0.00154%	72,192 : 1	$\binom{10}{1}\binom{4}{1}-\binom{4}{1}$
Four of a kind 	156	624	0.0240%	0.0256%	4,164 : 1	$\binom{13}{1}\binom{12}{1}\binom{4}{1}$
Full house 	156	3,744	0.144%	0.17%	693 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$
Flush (excluding royal flush and straight flush) 	1,287	5,148	0.198%	0.367%	508 : 1	$\binom{13}{5}\binom{4}{1}-\binom{10}{1}\binom{4}{1}$
Straight (excluding royal flush and straight flush) 	10	10,200	0.392%	0.76%	254 : 1	$\binom{10}{1}\binom{4}{1}^5-\binom{10}{1}\binom{4}{1}$
Three of a kind 	858	54,912	2.11%	2.87%	46.3 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2$
Two pair 	858	123,552	4.75%	7.62%	20.0 : 1	$\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}$
One pair 	2,860	1,098,240	42.3%	49.9%	1.36 : 1	$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$
No pair / High card 	1,277	1,302,540	50.1%	100%	0.995 : 1	$\left[\binom{13}{5}-10\right]\left[\binom{4}{1}^5-4\right]$
Total	**7,462**	**2,598,960**	**100%**	**---**	**1 : 1**	$\binom{52}{5}$

General Probability Spaces

Discrete, finite, uniform probability spaces



So Far, we considered
finite sample spaces and
uniform distributions.

a	b	c	d	e
0.2	0.2	0.2	0.2	0.2

$$P(\{a,c,d\}) = \frac{|\{a,c,d\}|}{|\{a,b,c,d,e\}|} = \frac{3}{5} = 0.6$$

We now consider
finite sample spaces and
non-uniform distributions.

a	b	c	d	e
0.1	*0.2*	*0.5*	*0.1*	*0.1*

$$P(\{a,c,d\}) = p(a) + p(c) + p(d)$$
$$= 0.1 + 0.5 + 0.1 = 0.7$$

Properties of general probability distributions

$$\forall \ A \subseteq \Omega, \ 0 \leq P(A) \leq 1 \quad P(\Omega) = 1$$

$$\forall \ A, B \subseteq \Omega, \ P(A \cup B) \leq P(A) + P(B)$$

$$\forall \ A, B \subseteq \Omega, \ A \cap B = \varnothing \quad \textbf{\textit{Disjoint sets}}$$

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

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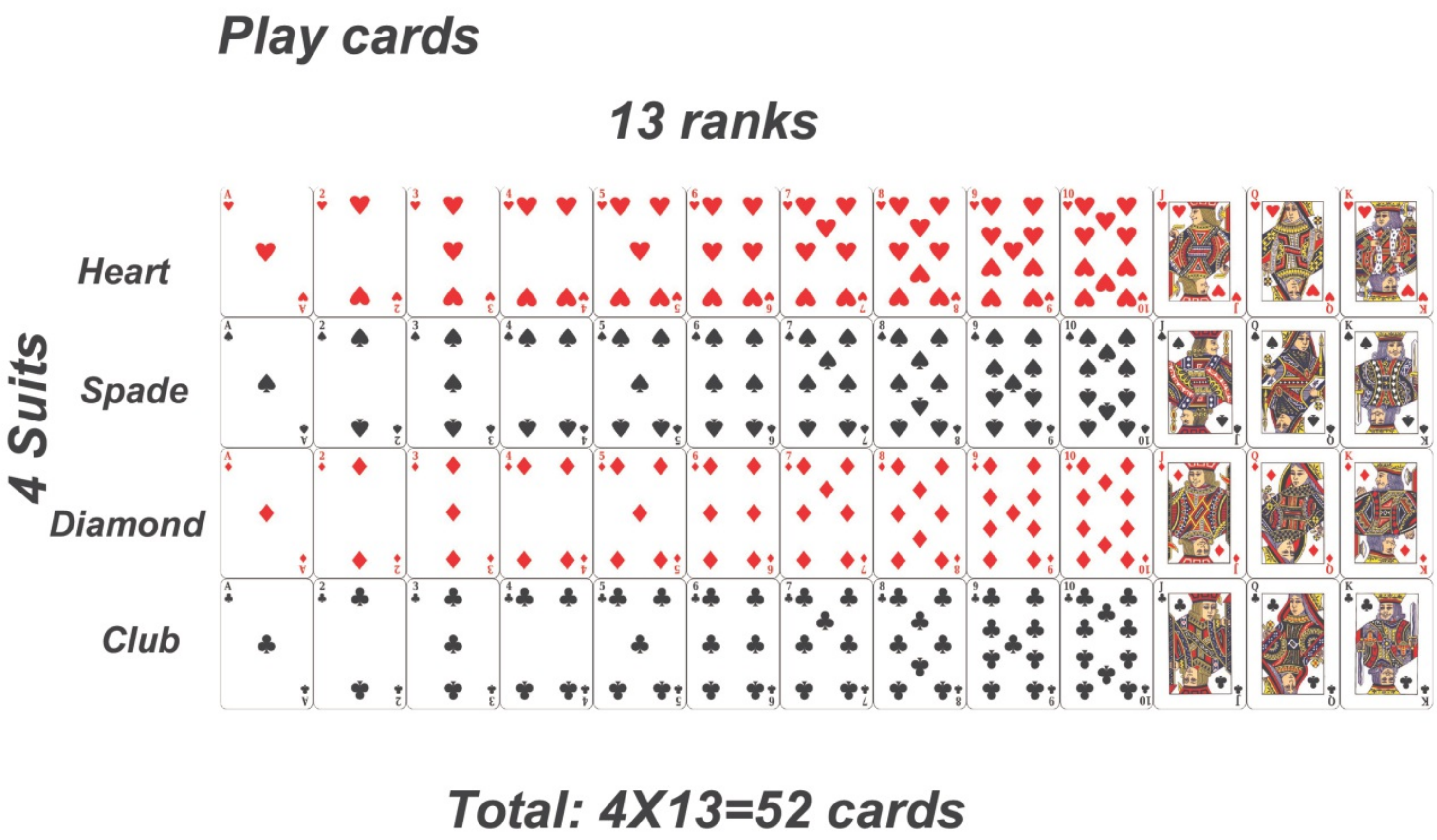
Play cards
13 ranks
4 Suits
Heart
Spade
Diamond
Club
Total: 4X13=52 cards

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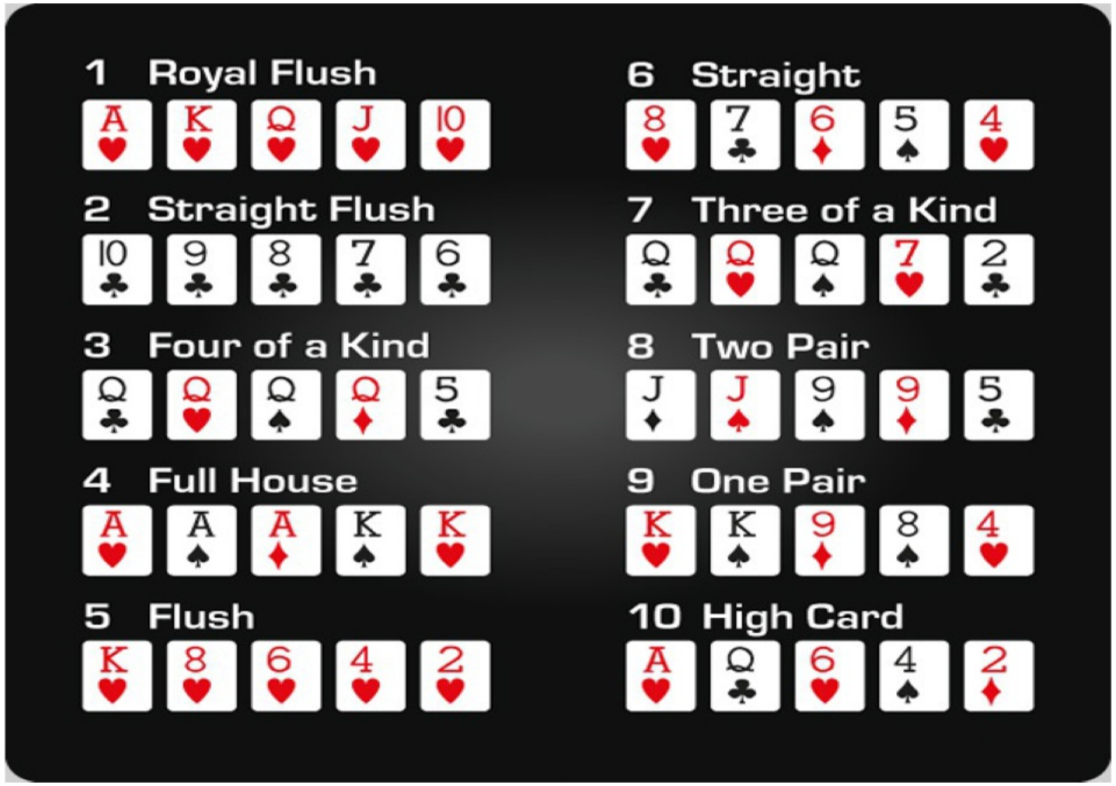

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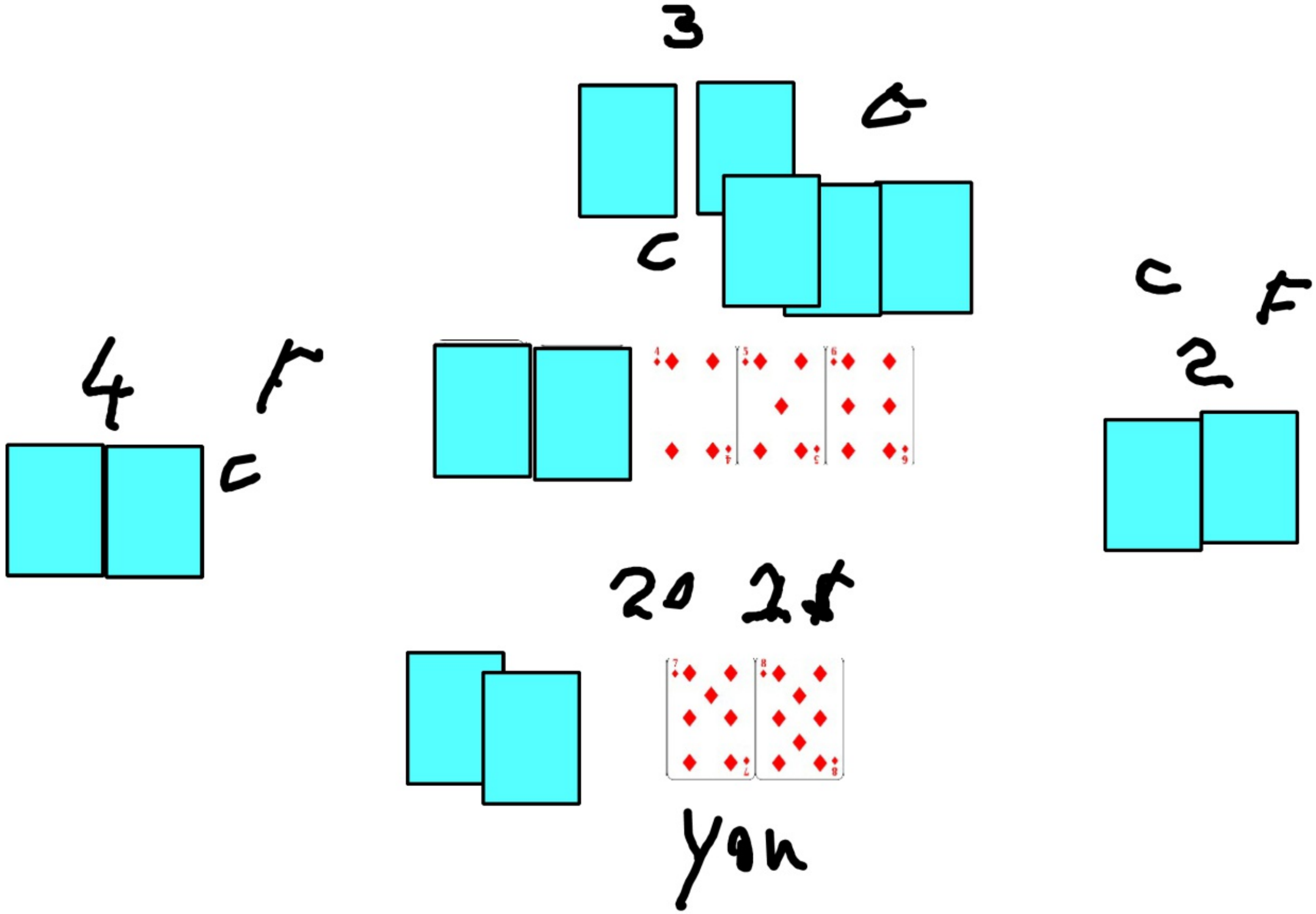
3
c
4
r
c
c
F
2
20
25
you

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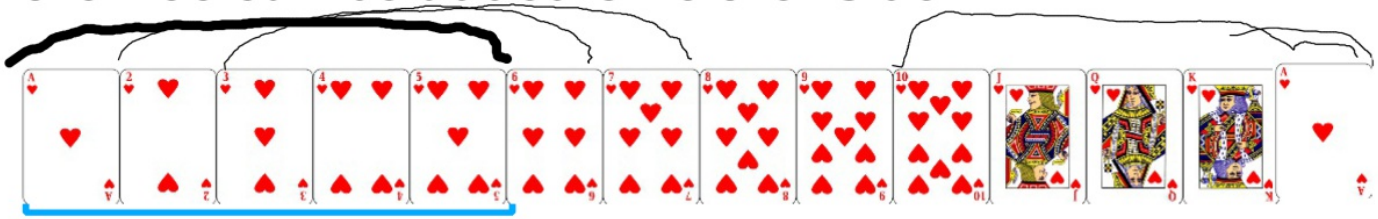

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Number of choices for the ranks of the cards?
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choices for the suit of the cards?
4

Prob =
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Number of choices for the ranks of the cards?
(excluding straight flush and royal flush)
C(13,5)-10

choices for the suit of the cards?
4

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$C(44,2)$

~ 10

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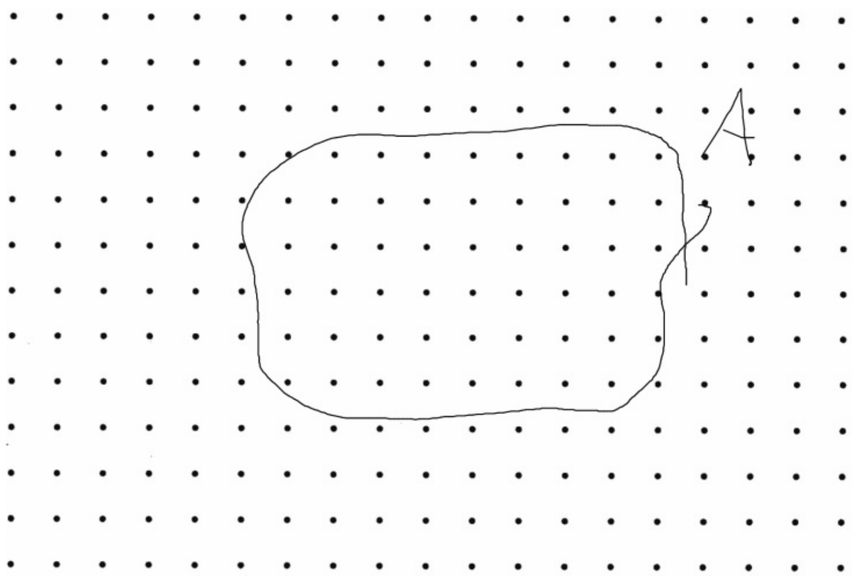
Counting probability distributions

Until now, we considered finite outcome spaces
where all outcomes the same probability.



Ω

$$P(A) = \frac{|A|}{|\Omega|}$$

All events have rational probabilities: n/m
In general, probabilities can be irrational.

Properties of general probability distributions

every event has probability between 0 and 1.

$$\forall \; A \subseteq \Omega, \; 0 \leq P(A) \leq 1$$

The outcome space has probability 1.

$$P(\Omega) = 1$$

The probability of a union is at most the sum of the probabilities

$$\forall \; A, B \subseteq \Omega, \; P(A \cup B) \leq P(A) + P(B)$$

The probability of a union of disjoint sets is equal to the sum of the probabilities

$$\forall \; A, B \subseteq \Omega, \; A \cap B = \varnothing \quad \textbf{Disjoint sets}$$

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

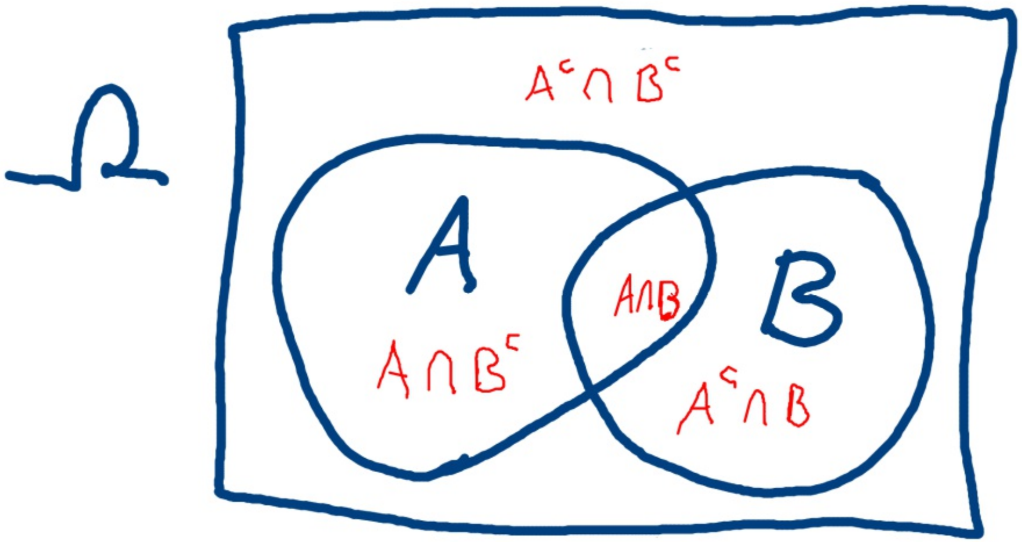
Implies that: $P(A^c) = 1 - P(A)$

$$A \cup A^c = \Omega, P(\Omega) = 1, A \cap A^c = \varnothing$$

$$\Rightarrow P(A) + P(A^c) = 1$$

The total probability equation

Partitioning a union



$$A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$$

$$P(A \cup B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$$

$$= P(A) + P(B) - P(A \cap B)$$

If $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{6}$, What is $P(A\cup B)=$?

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$$

A few simple questions:

If $P(A) = \dfrac{1}{2},\ P(B) = \dfrac{2}{3}$, What can be said about $P(A \cap B)$?

<u>at most</u> $A \cap B \subseteq A \qquad P(A \cap B) \leq P(A) = 1/2$

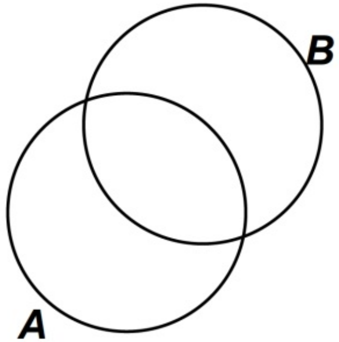
<u>at least</u> $1 \geq P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$1/2 + 2/3$$

$$P(A \cap B) \geq \frac{1}{6}$$

General Formula:

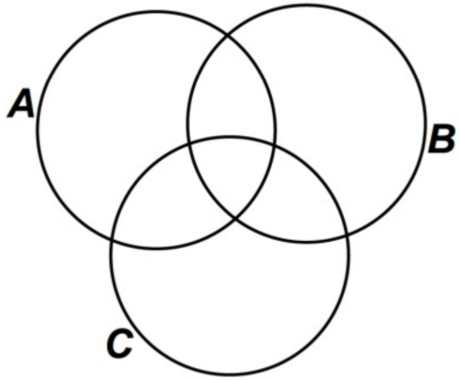
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



How about:

$$P(A \cup B \cup C) = ?$$

$P(A) + P(B) + P(C)$
$- P($



For Tue.

1. Finish Week2 homework.
2. Read class notes:
   - Section 4.5 (Poker)
   - Chapter 5

The total probability equation for (countably) infinite sets

$$A_1, A_2, A_3, \ldots \subseteq \Omega$$

$$\forall i \neq j,\ A_i \cap A_j = \varnothing$$

$$A_1 \cup A_2 \cup A_3 \cup \cdots \doteq \bigcup_{i=1}^{\infty} A_i = \Omega$$

$$\text{Then } \sum_{i=1}^{\infty} P(A_i) = 1$$

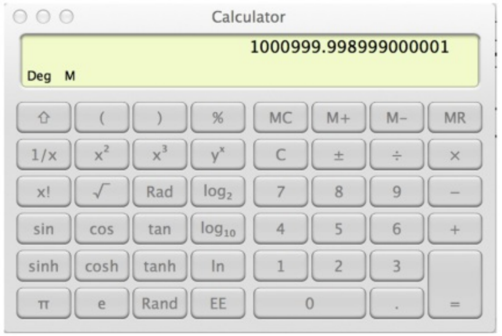
Consider the natural numbers: 1,2,3,...
Is it possible to define a uniform distribution over them?

1st possibility: 0=P(1)=P(2)=... $P(\Omega) =$

2nd possibility: 0<P(1)=P(2)=.... $P(\Omega) =$

General Probability Spaces

$$\frac{1}{\frac{1}{1000}-\frac{1}{1001}}=\frac{1}{\frac{1001-1000}{1000\times 1001}}=1001000$$

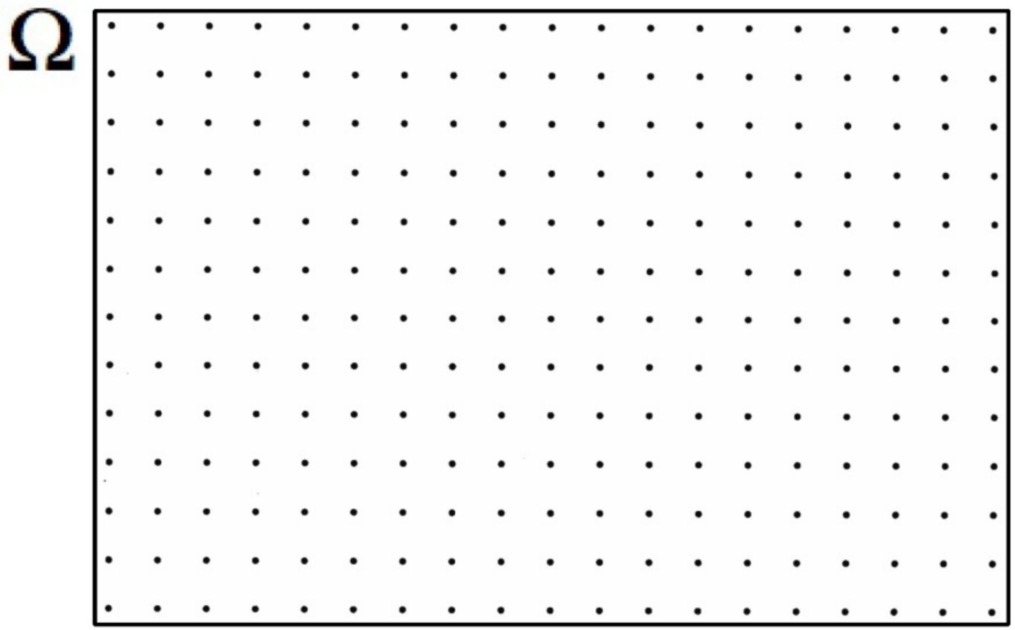


1/10000 = 1e-4 not 9.9E-5

WebWork checks your answers against the correct answers within some tolerance.
If you use a calculator your mistake might be masked and reappear at a later point in the problem.

Write complete expressions, don't use a calculator!

Discrete, finite, uniform probability spaces



So Far, we considered finite sample spaces and uniform distributions.

a	***b***	***c***	***d***	***e***
0.2	*0.2*	*0.2*	*0.2*	*0.2*

$$P(\{a,c,d\}) = \frac{|\{a,c,d\}|}{|\{a,b,c,d,e\}|} = \frac{3}{5} = 0.6$$

We now consider
finite sample spaces and
non-uniform distributions.

a	b	c	d	e
0.1	*0.2*	*0.5*	*0.1*	*0.1*

$$P(\{a,c,d\}) = p(a) + p(c) + p(d)$$
$$= 0.1 + 0.5 + 0.1 = 0.7$$

Properties of general probability distributions

$$\forall \ A \subseteq \Omega, \ 0 \leq P(A) \leq 1 \quad P(\Omega) = 1$$

$$\forall A, B \subseteq \Omega, \ P(A \cup B) \leq P(A) + P(B)$$

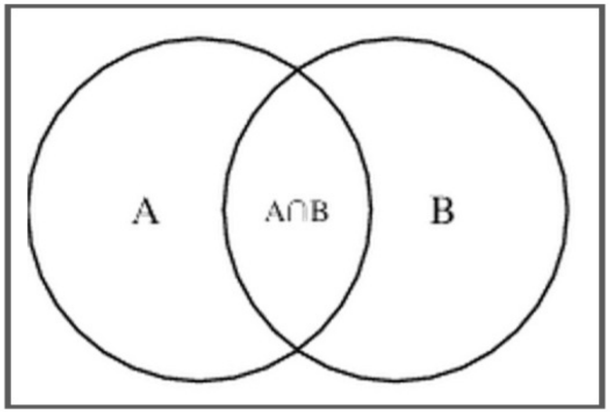
$$\forall A, B \subseteq \Omega, \ A \cap B = \varnothing$$ ***Disjoint sets***

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

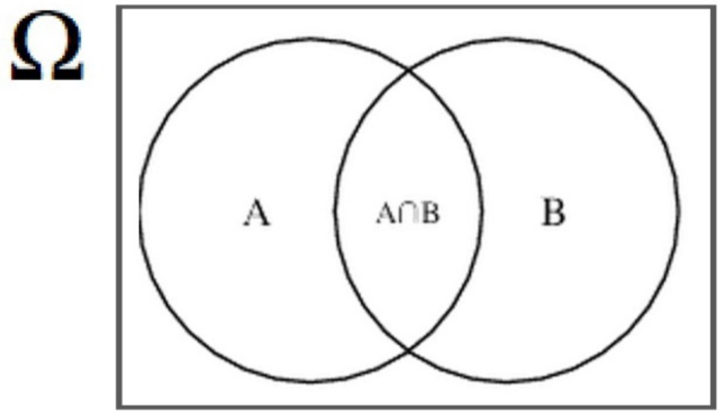
A few simple questions:

If $P(A)=\frac{1}{2}$, $P(B)=\frac{2}{3}$, What can be said about $P(A\cap B)$?

Ω



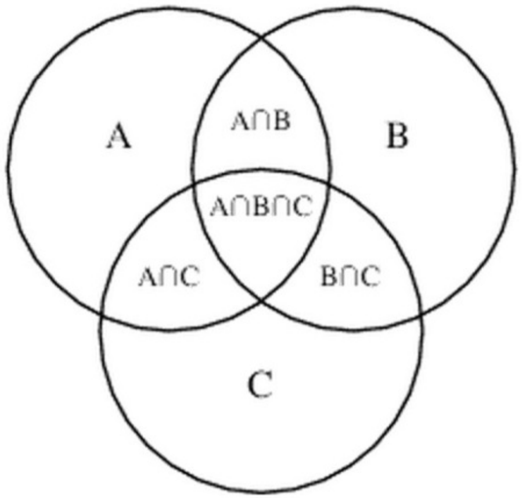
If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{6}$, What is $P(A \cup B) =$?



General Formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

How about: $P(A \cup B \cup C) = ?$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$



The inclusion/exclusion principle

Countably infinite sets

The natural numbers: 1,2,3,4,5....
-- an infinite set
-- represents counting
-- A set is infinitely countable if each element can be given an integer index.
-- Equivalently, if the elements can be put in a list

The total probability equation for (countably) infinite sets

$$A_1, A_2, A_3, \ldots \subseteq \Omega$$

$$\forall i \neq j,\ A_i \cap A_j = \varnothing$$

$$A_1 \cup A_2 \cup A_3 \cup \cdots \doteq \bigcup_{i=1}^{\infty} A_i = \Omega$$

$$\text{Then } \sum_{i=1}^{\infty} P(A_i) = 1$$

Consider the natural numbers: 1,2,3,...
Is it possible to define a uniform distribution over them?

1st possibility: 0=P(1)=P(2)=... $P(\Omega) =$

2nd possibility: 0<P(1)=P(2)=.... $P(\Omega) =$

What is the meaning of $\sum_{i=1}^{\infty} p_i$?

$$p_i \geq 0, \quad \sum_{i=1}^{\infty} p_i \doteq \lim_{n\to\infty} \sum_{i=1}^{n} p_i$$

This is a non-decreasing sequence

it can either converge to

some real number or to infinity (∞)

p_i = c (constant) $\sum_{i=1}^{\infty} c = \lim_{n\to\infty} nc,$

$$If \;\; c = 0: \;\; 0,0,0,0,0,0 \to 0$$

$$If \;\; c > 0: \;\; c, 2c, 3c, 4c \to \infty$$

Is it enough if $p_i \xrightarrow{i\to\infty} 0$?

Can we define a distribution of the form P(i)c/i ?

No, because $\sum_{i=1}^{\infty}(1/i) = \infty$

$$\left(\frac{1}{1}\right), \left(\frac{1}{2}\right), \left(\frac{1}{3}, \frac{1}{4}\right), \left(\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right), \ldots$$

$$\sum_{i=1}^{n} \frac{1}{i} \approx \ln n$$

Can we define a distribution of the form $P[X(\omega) = i] = \frac{c}{i^2}$?

Yes, because $$\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6} \approx 1.6449\ldots$$

If we define the distribution to be $P(X = i) = \frac{6}{\pi^2 i^2}$

Then the sum of the probabilities over all natural numbers is 1

If the series is finite then we can define a distribution by dividing each term by the sum of the series = the normalization factor

$$\text{if } \alpha > 1 \qquad \sum_{i=1}^{\infty} \frac{1}{i^{\alpha}} < \infty$$

- **Geometric Series**: Let r be a number in the range $[0, 1]$, i.e. $0 \leq r \leq 1$. Then:

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$$

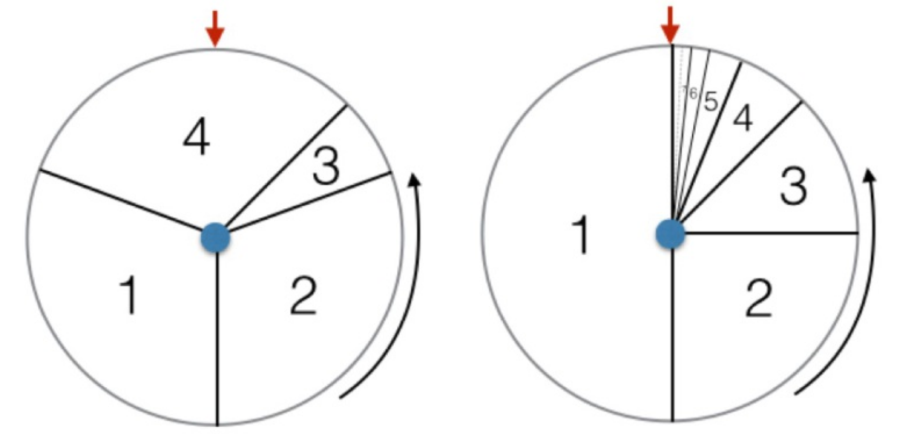
$$\sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$$

and

$$\sum_{i=1}^{\infty} i r^i = \frac{r}{(1-r)^2}$$

Note that if r=1 the sums are infinite.

Probabilities over uncountable sets



(a) A wheel with four outcomes

(b) A wheel with Infinitely many outcomes

How can we define the uniform distribution over angles?
Each angle has probability 0
Summing over all angle still gives 0

It seems like we can represent the points on the line using a countable set

Numbers that can be written as i/j, where i,j are natural numbers

Each element corresponds to a pair of natural numbers. Therefor the

The rational numbers in [0,1]:

The distance between i/n and (i+1)/n is 1/n
As n increases the distance decreases to zero
---> the rationals are dense on the line
= there is a rational number arbitrarily close to any positive real number

Does that mean that the all real number are rational? NO! (sqrt(2))
Does that mean that the reals are countable? NO!

The real number 0 <=x <= 1 are uncountable

Proof by contradiction:
1. suppose they are countable.
2. write the list of all of the numbers in binary expansion

0.000001101001100011100010001000...
0.000101101001100011100010001000...
0.000000101001100011100010001000 ...
0.000001001001100011100010001000 ...
0.000001100001100011100010001000 ...
0.000001101000000011100010001000 ...
0.000001101001111011100010001000 ...
0.000001101001100111100010001000 ...
0.000001101001100011000010001000 ..

Construct a number that differes from the 1st element in the 1st position, from the 2nd in the 2nd position ...

0.11111001011...
This number is not in the list: contradiction

The uniform distribution over [0,1]

$$0 \le x_1, x_2, \ldots \le 1$$

$$P(\{x_1\}) = 0$$

$$P(\{x_1, x_2\}) = 0$$

$$P(\{x_1, x_2, x_3, \ldots\}) = 0$$

But, if $0 \le a < b \le 1$

$$P([a,b]) = b - a$$

This is called a density distribution.
General density distributions - on monday.

For Friday

1. Read Chapter 5 (it is updated)
2. Start working on the homework (Get going, it is harder than previous)
3. Akshay will replace me on Friday lecture.