# Infinite outcome spaces

# Why do we care about infinite sample spaces?

**Countable example:** suppose we are betting on the outcome of a coin. The odds are fair: At each round we can bet any positive amount $X,
  * if we predict correctly, we get back our bet and and additional $X, so our gain is $x
  * if we are wrong, we lose the $X that we bet, so our loss is $X

Here is a betting method that guarantees that we leave the casino with a gain of $1:
- On the first round we bet $1, if we win we stop.
- If we lose on the 1st round we bet $2 on the 2nd round, if we win we stop.
- If we lose on the 2nd round we bet $4 on the 3rd round, if we win we stop.
-.....
- If we lose on the n'th round we bet $\$2^n$ on the (n+1) round, if we win we stop.
- .......

Let $E_n$ be the event that we stop on round n. What is $P(E_n)$ ?

**UnCountable example:**
Suppose we pick a real number from the uniform distribution over the interval [0,1]

What is the probability that the number is 1/3?

What is the probability that the number is of the form 1/i for some integer i?

What is the probability that the number is rational?

What is the probability that the number is smaller than 0.1?

### *Properties of general probability distributions*

$$\forall \ A \subseteq \Omega,\ 0 \leq P(A) \leq 1 \quad P(\Omega) = 1$$

$$\forall A, B \subseteq \Omega,\ P(A \cup B) \leq P(A) + P(B)$$

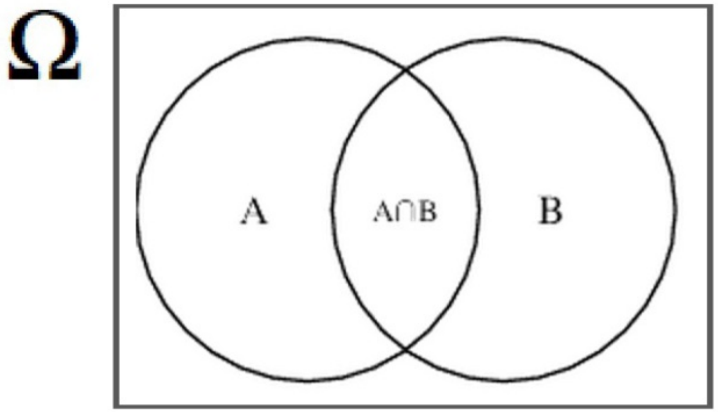
$$\forall A, B \subseteq \Omega,\ A \cap B = \varnothing \quad \textbf{\textit{Disjoint sets}}$$
$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

Suppose $A_1, A_2, \ldots, A_n$ are pairwise disjoint: $\forall i \neq j \quad A_i \cap A_j = \phi$

Then $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

***A few simple questions:***

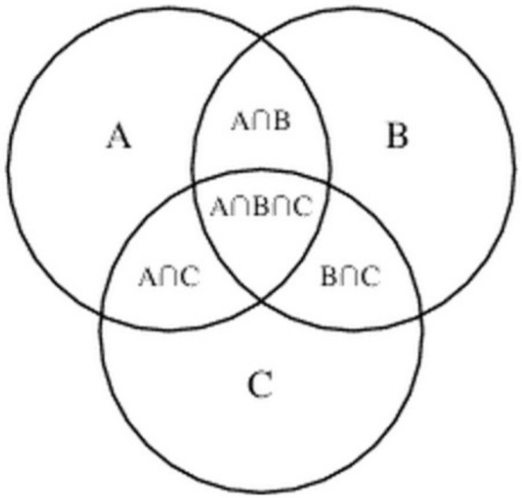
If $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, What can be said about $P(A \cap B)$ ?



**General Formula:** $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

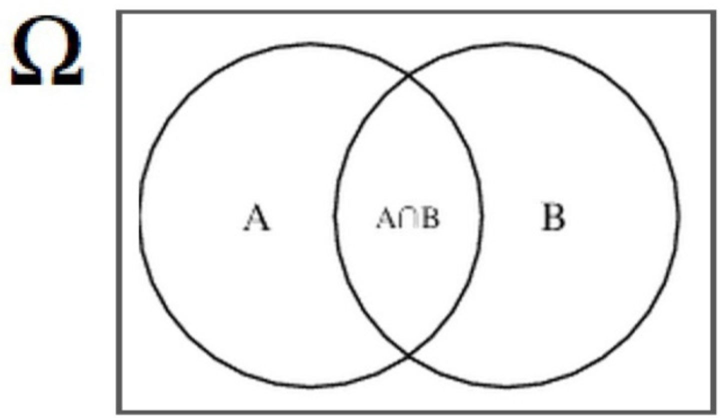
**How about:** $$P(A \cup B \cup C) = ?$$

$$\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & + P(A \cap B \cap C) \end{aligned}$$




***The inclusion/exclusion principle***

If $P(A)=\frac{1}{2},\ P(B)=\frac{1}{3},\ P(A\cap B)=\frac{1}{6}$, What is $P(A\cup B)=$ ?

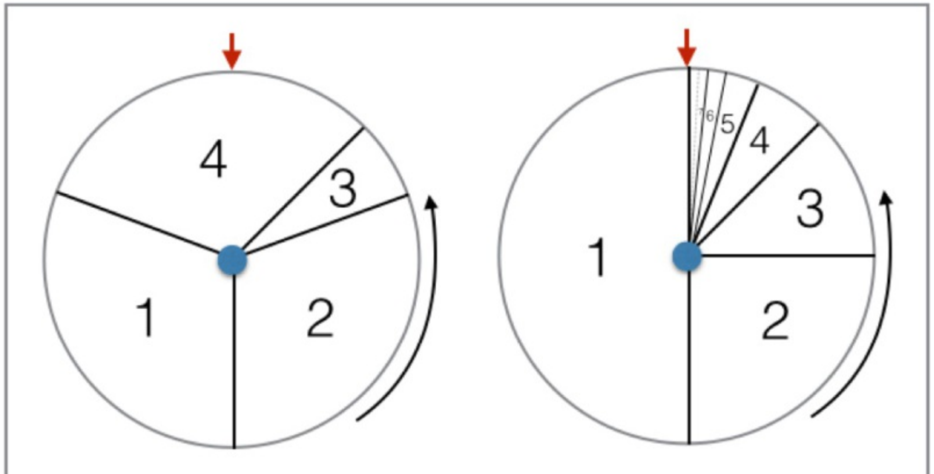


## *Countably infinite sets*

***The natural numbers: 1,2,3,4,5....***
***-- an infinite set***
***-- represents counting***
***-- A set is infinitely countable if each element can be given*** a natural number index 1,2,3,4,5,...
***-- Equivalently, if the elements can be put in a list***

Examples:

- The integers (pos and neg)
- Numbers of the form $1/i$ $i=1,2,3\ldots$
- Rational numbers: of the form $\frac{i}{j}$ $i,j$ are natural



(a) A wheel with four outcomes
(b) A wheel with Infinitely many outcomes

finite

Countably infinite

What is the meaning of $\sum_{i=1}^{\infty} p_i$ ?

$$p_i \geq 0, \quad \sum_{i=1}^{\infty} p_i \doteq \lim_{n\to\infty} \sum_{i=1}^{n} p_i$$

This is a non-decreasing sequence

it can either converge to

some real number or to infinity ( $\infty$ )

***p_i = c (constant)*** $\sum_{i=1}^{\infty} c = \lim_{n\to\infty} nc,$

$$If \quad c = 0: \quad 0,0,0,0,0,0 \to 0$$

$$If \quad c > 0: \quad c, 2c, 3c, 4c \to \infty$$

## *The total probability equation for (countably) infinite sets*

$$A_1, A_2, A_3, \ldots \subseteq \Omega$$

$$\forall i \neq j,\ A_i \cap A_j = \varnothing$$

$$A_1 \cup A_2 \cup A_3 \cup \cdots \doteq \bigcup_{i=1}^{\infty} A_i = \Omega$$

$$\text{Then } \sum_{i=1}^{\infty} P(A_i) = 1$$

***Consider the natural numbers: 1,2,3,...***
***Is it possible to define a uniform distribution over them?***

***1st possibility: 0=P(1)=P(2)=...*** $P(\Omega) =$

---

***2nd possibility: 0<P(1)=P(2)=....*** $P(\Omega) =$

# Analysis of the bet doubling scheme

Here is a betting method that guarantees that we leave the casino with a gain of $1:
- On the first round we bet $1, if we win we stop.
- If we lose on the 1st round we bet $2 on the 2nd round, if we win we stop.
- If we lose on the 2nd round we bet $4 on the 3rd round, if we win we stop.
-.....
- If we lose on the n'th round we bet $\$2^n$ on the (n+1) round, if we win we stop.
- .......

Let $E_n$ be the event that we stop on round n. What is $P(E_n)$ ?

$P(E_1) = \frac{1}{2}$ ; $P(E_2) =$ ; ... $P(E_i) =$

$$\sum_{i=1}^{\infty} P(E_i) =$$

- **Geometric Series**: Let $r$ be a number in the range $[0, 1]$, i.e. $0 \le r \le 1$. Then:

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$$

$$\sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$$

Doubling bets : $r = \frac{1}{2}$

$\sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i =$

and

$$\sum_{i=1}^{\infty} i r^i = \frac{r}{(1-r)^2}$$

***Note that if r=1 the sums are infinite.***

# Can we use any sequence converging to zero?

Is it enough if $p_i \xrightarrow{i \to \infty} 0$?

Can we define a distribution of the form P(i)c/i ?

***No, because*** $\sum_{i=1}^{\infty}(1/i) = \infty$

$$\left(\frac{1}{1}\right), \left(\frac{1}{2}\right), \left(\frac{1}{3}, \frac{1}{4}\right), \left(\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right), \ldots$$

$$\sum_{i=1}^{n} \frac{1}{i} \approx \ln n$$

Can we define a distribution of the form $p_i = \frac{1}{Z}\frac{1}{i^2}$ ?

***Yes, because*** $\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6} \approx 1.6449\ldots$

***If we define the distribution to be*** $P(X = i) = \frac{6}{\pi^2 i^2}$

***Then the sum of the probabilities over all natural numbers is 1***

***If the series is finite then we can define a distribution by dividing each term by the sum of the series***

$Z$ ***= the normalization factor***

$$\text{if } \alpha > 1 \qquad \sum_{i=1}^{\infty} \frac{1}{i^\alpha} < \infty$$

*It seems like we can represent the points on the line using a countable set*

Rational Numbers

*Numbers that can be written as i/j, where i,j are natural numbers*

*Each element corresponds to a pair of natural numbers. Therefor the*

*The rational numbers in [0,1]:*


*The distance between i/n and (i+1)/n is 1/n*
*As n increases the distance decreases to zero*
*---> the rationals are dense on the line*
*= there is a rational number arbitrarily close to any positive real number*

*Does that mean that the all real number are rational?  NO! (sqrt(2))*
*Does that mean that the reals are countable? NO!*

*The real number 0 <=x <= 1 are uncountable*

*Proof by contradiction:*
*1. suppose they are countable.*
*2. write the list of all of the numbers in binary expansion*

*0.000001101001100011100010001000...*
*0.000101101001100011100010001000...*
*0.000000101001100011100010001000 ...*
*0.000001001001100011100010001000 ...*
*0.000001100001100011100010001000 ...*
*0.000001101000000011100010001000 ...*
*0.000001101001111011100010001000 ...*
*0.000001101001100111100010001000 ...*
*0.000001101001100011000010001000 ..*

*Construct a number that differes from the 1st element in the 1st position, from the 2nd in the 2nd position ...*

*0.11111001011...*
*This number is not in the list: contradiction*

## *The Kolmogorov Axioms of probability theory*

1) $\Pr(\Omega) = 1$

2) If $V$ is a countable collection of disjoint events:

$$V = \{A_1, A_2, \ldots\}, \quad \forall i \neq j, \; A_i \cap A_j = \varnothing$$

Then Probability of the union is equal to the sum of the probabilities:

$$\Pr\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Pr(A_i)$$

We would like to define a uniform distribution over a range of reals [a,b].
Let Pr(x)=c if a <=x<=b

Don't we get a contradiction?

$$a < b, \quad \sum_{a \le x \le b} c = \begin{cases} 0 & \text{if } c = 0 \\ \infty & \text{if } c > 0 \end{cases}$$

No, because the sum is required to hold only over countable sets, and the set of points in [a,b] is uncountable

$$\sum_{a \le x \le b} 0 = 1$$

***The uniform distribution over [0,1]***

$$0 \le x_1, x_2, \ldots \le 1$$

$$P(\{x_1\}) = 0$$

$$P(\{x_1, x_2\}) = 0$$

$$P(\{x_1, x_2, x_3, \ldots\}) = 0$$

But, if $0 \le a < b \le 1$

$$P([a,b]) = b - a$$

Prob of other sets:

Construct from Countable

Unions & Intersections

of intervals

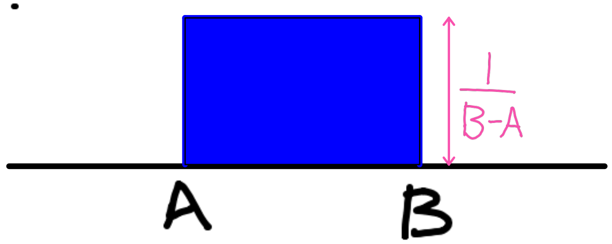
U(A,B) = The Uniform distribution over the segment [A,B]

$U(A,B)$ is defined by assigning probability

to every segment $[a,b]$ where $A \le a \le b \le B$ (A<B)

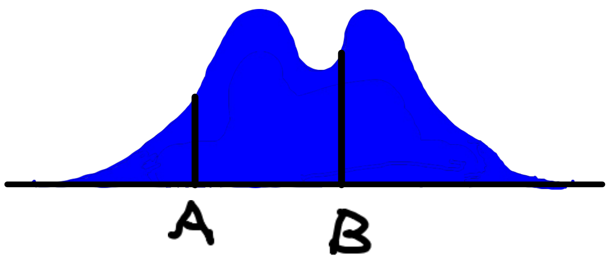
$$\Pr([a,b]) = \Pr((a,b)) = \frac{b-a}{B-A}$$

uniform density:




general density

$$P([A,B]) = \int_A^B f(x)\,dx$$




$\forall x \quad f(x) \geq 0$ ($f(x)$ can be larger than 1)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

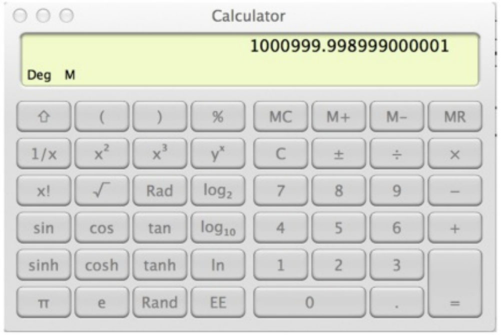
next class

PDF, CDF

& mixtures

## *General Probability Spaces*

$$\frac{1}{\frac{1}{1000}-\frac{1}{1001}}=\frac{1}{\frac{1001-1000}{1000\times 1001}}=1001000$$



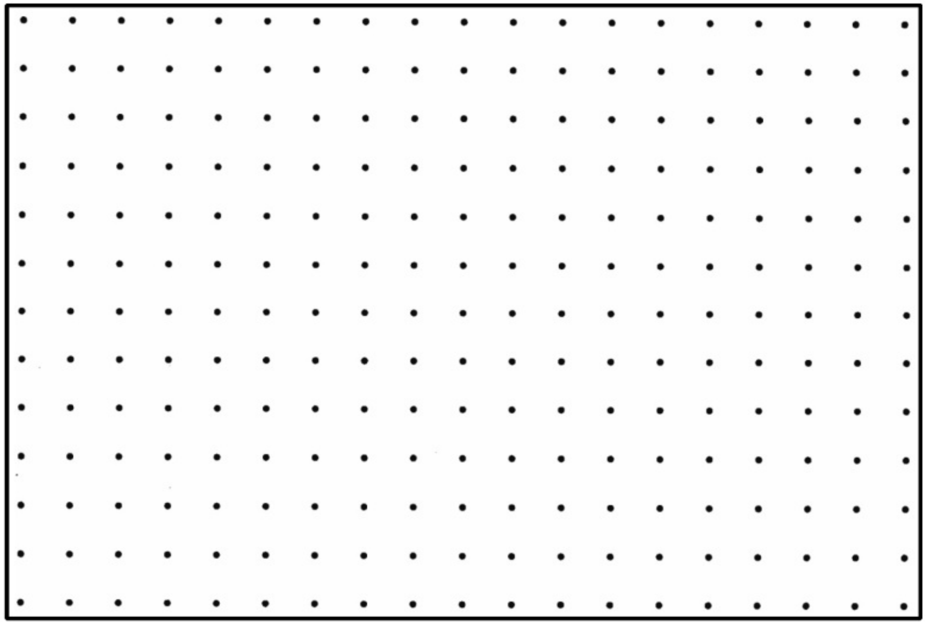

***1/10000 = 1e-4  not 9.9E-5***

***WebWork checks your answers against the correct answers within some tolerance.***
***If you use a calculator your mistake might be masked and reappear at a later point in the problem.***

***Write complete expressions, don't use a calculator!***

## *Discrete, finite, uniform probability spaces*

$\Omega$



***So Far, we considered***
***finite sample spaces and***
***uniform distributions.***

| a | b | c | d | e |
|---|---|---|---|---|
| 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |

$$P(\{a,c,d\}) = \frac{|\{a,c,d\}|}{|\{a,b,c,d,e\}|} = \frac{3}{5} = 0.6$$

***We now consider***
***finite sample spaces and***
***non-uniform distributions.***

| **a** | **b** | **c** | **d** | **e** |
|---|---|---|---|---|
| *0.1* | *0.2* | *0.5* | *0.1* | *0.1* |

$$P(\{a,c,d\}) = p(a) + p(c) + p(d)$$
$$= 0.1 + 0.5 + 0.1 = 0.7$$

***Properties of general probability distributions***

$$\forall\ A \subseteq \Omega,\ 0 \le P(A) \le 1 \quad P(\Omega) = 1$$

$$\forall\ A,B \subseteq \Omega,\ P(A \cup B) \le P(A) + P(B)$$

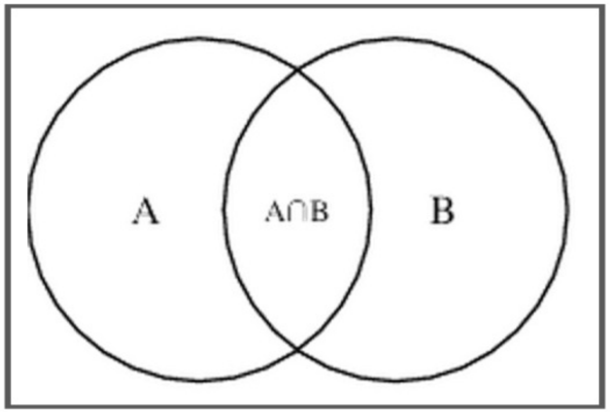
$$\forall\ A,B \subseteq \Omega,\ A \cap B = \varnothing$$ ***Disjoint sets***

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

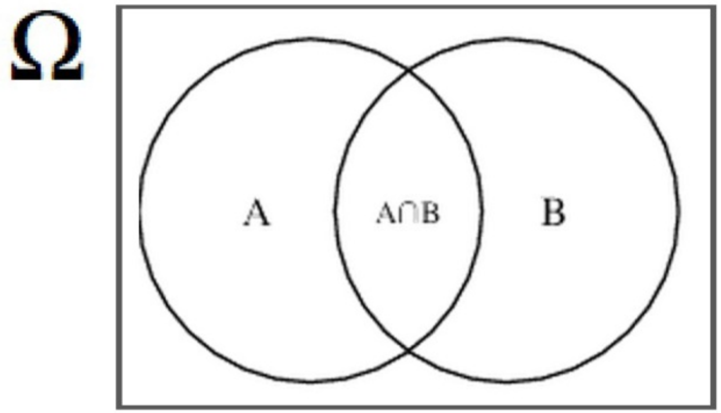
***A few simple questions:***

If $P(A)=\frac{1}{2}$, $P(B)=\frac{2}{3}$, What can be said about $P(A \cap B)$ ?

$\Omega$



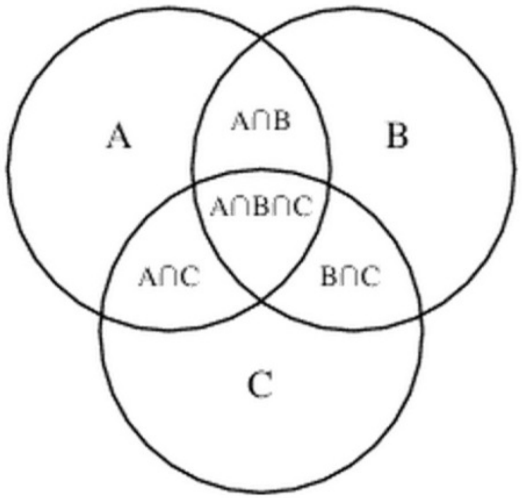
If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{6}$, What is $P(A \cup B) =$ ?



**General Formula:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**How about:** $P(A \cup B \cup C) = ?$

$$
\begin{aligned}
P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\
& - P(A \cap B) - P(A \cap C) - P(B \cap C) \\
& + P(A \cap B \cap C)
\end{aligned}
$$




***The inclusion/exclusion principle***

## *Countably infinite sets*

***The natural numbers: 1,2,3,4,5....***
***-- an infinite set***
***-- represents counting***
***-- A set is infinitely countable if each element can be given an integer index.***
***-- Equivalently, if the elements can be put in a list***

## *The total probability equation for (countably) infinite sets*

$$A_1, A_2, A_3, \ldots \subseteq \Omega$$

$$\forall i \neq j,\ A_i \cap A_j = \varnothing$$

$$A_1 \cup A_2 \cup A_3 \cup \cdots \doteq \bigcup_{i=1}^{\infty} A_i = \Omega$$

$$\text{Then } \sum_{i=1}^{\infty} P(A_i) = 1$$

***Consider the natural numbers: 1,2,3,...***
***Is it possible to define a uniform distribution over them?***

***1st possibility: 0=P(1)=P(2)=...*** $P(\Omega) =$

---

***2nd possibility: 0<P(1)=P(2)=....*** $P(\Omega) =$

What is the meaning of $\sum_{i=1}^{\infty} p_i$ ?

$$p_i \geq 0, \quad \sum_{i=1}^{\infty} p_i \doteq \lim_{n\to\infty} \sum_{i=1}^{n} p_i$$

This is a non-decreasing sequence

it can either converge to

some real number or to infinity ( $\infty$ )

***p_i = c (constant)*** $\quad \sum_{i=1}^{\infty} c = \lim_{n\to\infty} nc,$

$$\textit{If} \ \ c = 0: \ \ 0,0,0,0,0,0 \to 0$$

$$\textit{If} \ \ c > 0: \ c, 2c, 3c, 4c \to \infty$$

Is it enough if $p_i \xrightarrow{i \to \infty} 0$?

Can we define a distribution of the form P(i)c/i ?

***No, because*** $\sum_{i=1}^{\infty}(1/i) = \infty$

$$\left(\frac{1}{1}\right), \left(\frac{1}{2}\right), \left(\frac{1}{3}, \frac{1}{4}\right), \left(\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right), \ldots$$

$$\sum_{i=1}^{n} \frac{1}{i} \approx \ln n$$

Can we define a distribution of the form $P[X(\omega) = i] = \frac{c}{i^2}$ ?

***Yes, because*** $\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6} \approx 1.6449\ldots$

***If we define the distribution to be*** $P(X = i) = \frac{6}{\pi^2 i^2}$

***Then the sum of the probabilities over all natural numbers is 1***

***If the series is finite then we can define a distribution by dividing each term by the sum of the series = the normalization factor***

$$\text{if } \alpha > 1 \qquad \sum_{i=1}^{\infty} \frac{1}{i^\alpha} < \infty$$

- **Geometric Series**: Let $r$ be a number in the range $[0,1]$, i.e. $0 \leq r \leq 1$. Then:

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$$

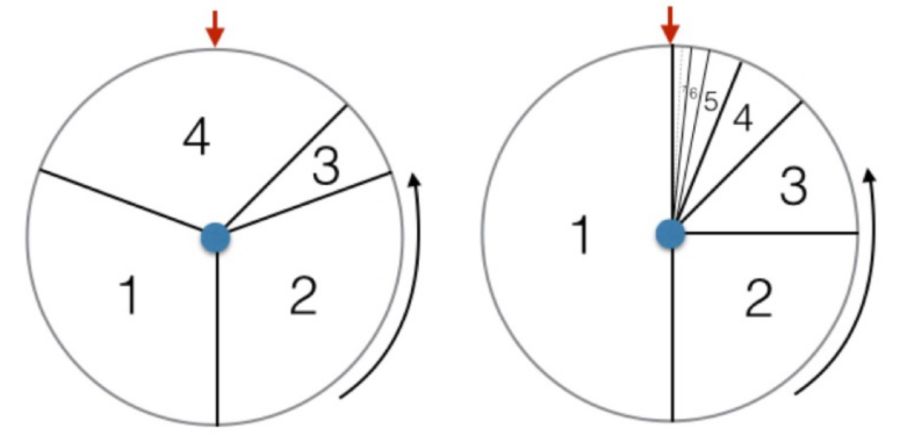
$$\sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$$

and

$$\sum_{i=1}^{\infty} i r^i = \frac{r}{(1-r)^2}$$

***Note that if r=1 the sums are infinite.***

# Probabilities over uncountable sets




(a) A wheel with four outcomes

(b) A wheel with Infinitely many outcomes

***How can we define the uniform distribution over angles?***
***Each angle has probability 0***
***Summing over all angle still gives 0***

***It seems like we can represent the points on the line using a countable set***

***Numbers that can be written as i/j, where i,j are natural numbers***

***Each element corresponds to a pair of natural numbers. Therefor the***

***The rational numbers in [0,1]:***


***The distance between i/n and (i+1)/n is 1/n***
***As n increases the distance decreases to zero***
***---> the rationals are dense on the line***
***= there is a rational number arbitrarily close to any positive real number***

***Does that mean that the all real number are rational? NO! (sqrt(2))***
***Does that mean that the reals are countable? NO!***

The real number 0 <=x <= 1 are uncountable

Proof by contradiction:
1. suppose they are countable.
2. write the list of all of the numbers in binary expansion

0.000001101001100011100010001000...
0.000101101001100011100010001000...
0.000000101001100011100010001000 ...
0.000001001001100011100010001000 ...
0.000001100001100011100010001000 ...
0.000001101000000011100010001000 ...
0.000001101001111011100010001000 ...
0.000001101001100111100010001000 ...
0.000001101001100011000010001000 ..

Construct a number that differes from the 1st element in the 1st position, from the 2nd in the 2nd position ...

0.11111001011...
This number is not in the list: contradiction

***The uniform distribution over [0,1]***

$$0 \le x_1, x_2, \ldots \le 1$$

$$P(\{x_1\}) = 0$$

$$P(\{x_1, x_2\}) = 0$$

$$P(\{x_1, x_2, x_3, \ldots\}) = 0$$

But, if $0 \le a < b \le 1$

$$P([a,b]) = b - a$$

***This is called a density distribution.***
***General density distributions - on monday.***

## *For Friday*

*1. Read Chapter 5 (it is updated)*
*2. Start working on the homework (Get going, it is harder than previous)*
*3. Akshay will replace me on Friday lecture.*

Densities vs. Point Mass distributions
Mixtures
Historgrams vs. CDFs

## *The Kolmogorov Axioms of probability theory*

1) $\Pr(\Omega) = 1$

2) If $V$ is a countable collection of disjoint events:

$$V = \{A_1, A_2, \ldots\}, \quad \forall i \neq j,\ A_i \cap A_j = \varnothing$$

Then Probability of the union is equal to the sum of the probabilities:

$$\Pr\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Pr(A_i)$$

lets consider the segment [0,1). Each point can be represented by an infinite digital expansion. i.e. 0.7345231323....
To say that the set [0,1] is uncountable means that it is impossible to create a list that contains all of these points.

## Proof by contradiction:

assume it is possible and
show that there is a point that is not in the list.

0.4491

diagonalization method

0.23324284902394839994839948391701...
0.33242659180129278501929388832938...
0.45231982375819828837829938271959...
0.64526481727366366277736281727367...
. . . .
. . . .

We would like to define a uniform distribution over a range of reals [a,b].
Let Pr(x)=c if a <=x<=b

Don't we get a contradiction?

$$a < b, \quad \sum_{a \le x \le b} c = \begin{cases} 0 & \text{if } c = 0 \\ \infty & \text{if } c > 0 \end{cases}$$

No, because the sum is required to hold only over countable sets, and the set of points in [a,b] is uncountable

$$\sum_{a \le x \le b} 0 = 1$$

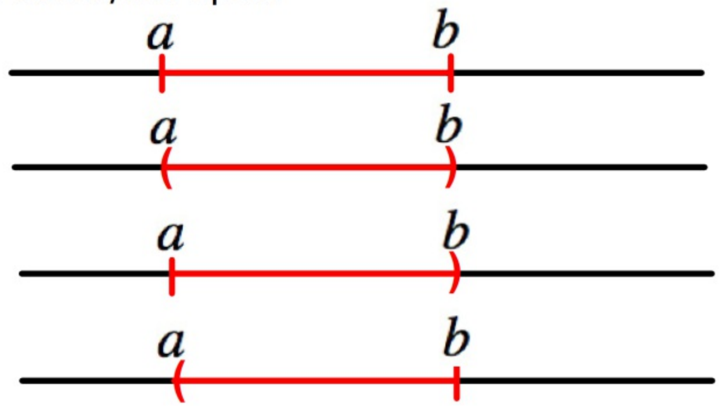
line segments: closed, open and half closed/half open:

$$[a,b] = \{x \mid a \le x \le b\}$$

$$(a,b) = \{x \mid a < x < b\}$$

$$[a,b) = \{x \mid a \le x < b\}$$

$$(a,b] = \{x \mid a < x \le b\}$$



U(A,B) = The Uniform distribution over the segment [A,B]

$U(A,B)$ is defined by assigning probability to every segment $[a,b]$ where $A \le a \le b \le B$ (A<B)

$$\Pr([a,b]) = \Pr((a,b)) = \frac{b-a}{B-A}$$

Lets calculate the probability of some sets with respect to the uniform distribution

Fix the probability distribution U(-1,1)

P([-1/3,1/3])=(1/3--1/3)/(1--1)=(2/3)/2=1/3

P([-1,0])=

P([-2,0])=

P([-3,2])=

P([0,2])=

P([-2,-1/2]U[1/2,2])=

## PDF - the Probability Density Function

When the density distribution is uniform, it is easy to de

$$U(a,b): \text{ for all } a \le x \le y \le b, \quad P([x,y]) = \frac{y-x}{b-a}$$

When the density distribution is not uniform,

we define a "probability density function" : $f(x) \doteq \lim_{\epsilon \to 0} \frac{Pr(}{}$

and the probability of a segment $[x,y]$ is: $\Pr([x,y]) =$

## *PDF and CDF*

***The Probability Density function is shortened to PDF***

***Another popular representation of a distribution on the real is the CDF: Cumulative Distribution function***

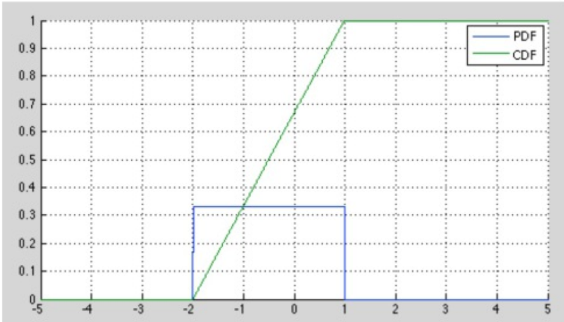
The CDF $F$ is defined as $F(a) \doteq \Pr(x \leq a)$

For density distributions, one can translate between PDF and CDF:

$$F(a) = \int_{-\infty}^{a} f(x)dx; \quad f(a) = \left.\frac{dF(x)}{dx}\right|_{x=a}$$

# CDF and PDF of the uniform distribution

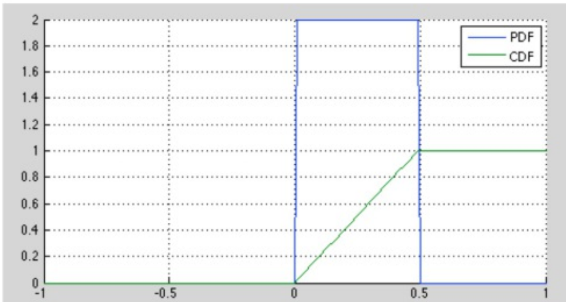
U(-2,1)




$f(x) =$ PDF=Probability Density Function

$F(x) =$ CDF=Cumulative Distribution function

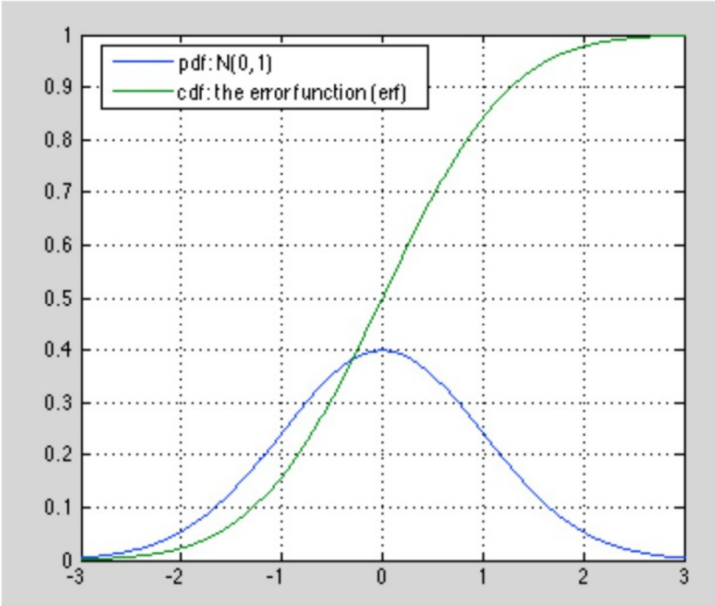
U(0,0.5)




$$F(x) = \int_{-\infty}^{x} f(s)ds$$

$$f(x) = \frac{d}{dx} F(x)$$

# The normal distribution

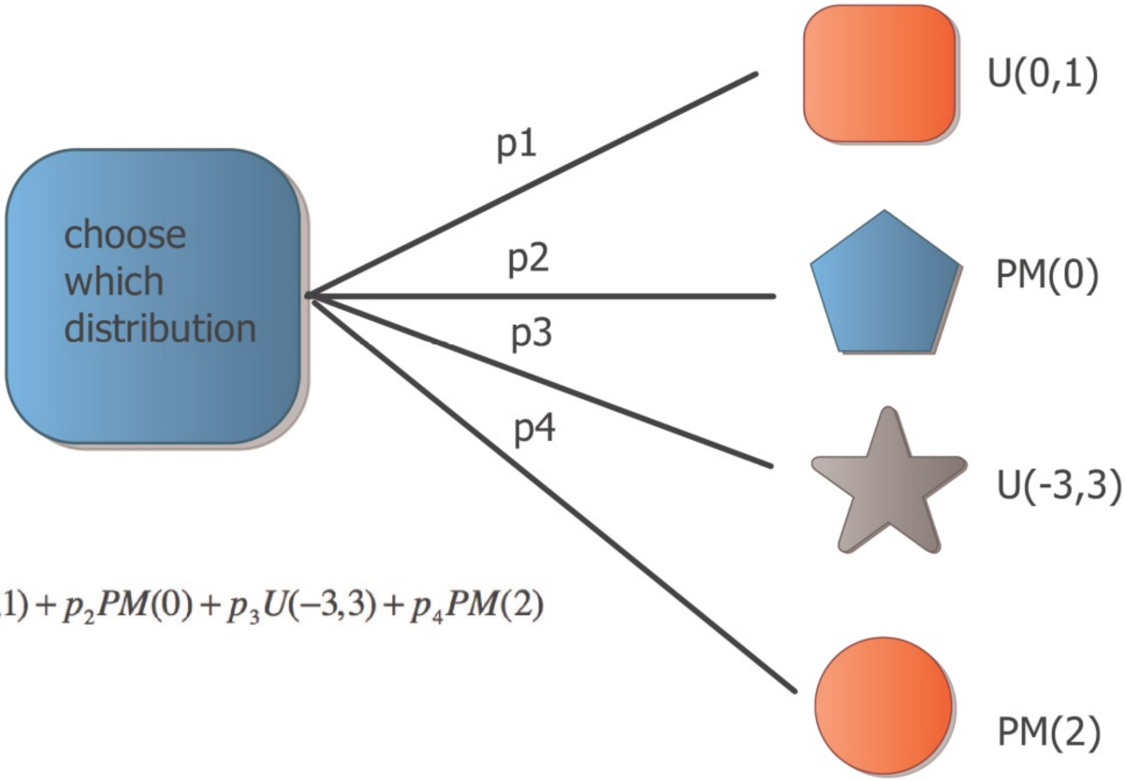



The normal distribution density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

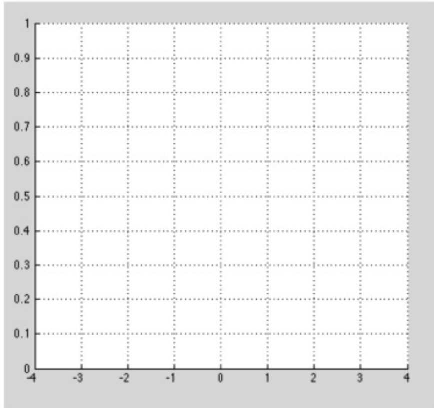
add exponential

# Mixtures distributions

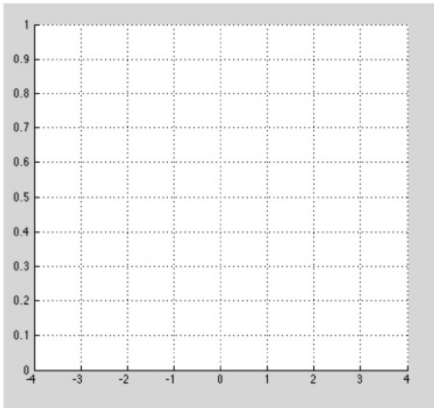



$$p_1 U(0,1) + p_2 PM(0) + p_3 U(-3,3) + p_4 PM(2)$$

**PM(1)**


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4

**U(-1,2)**


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4

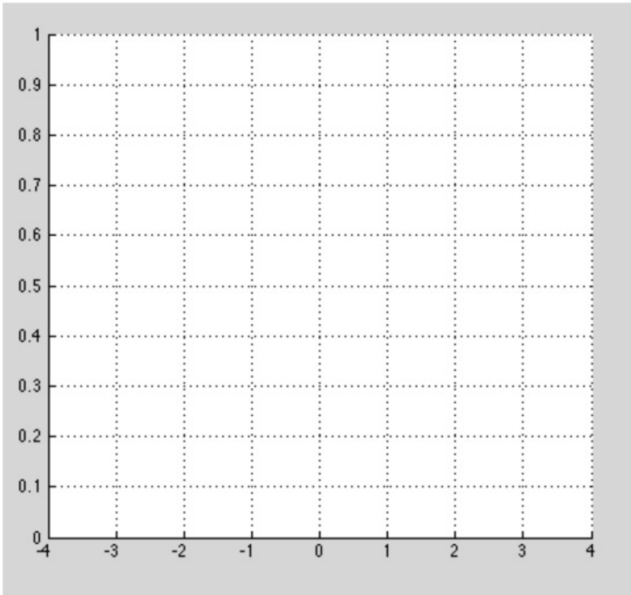
**Three PMs**

$$0.2PM(-1)+0.5PM(0)+0.3PM(3)$$

$$F(-1.01)=0;\quad F(-1)=0.2$$

$$F(-0.01)=0.2;\quad F(0)=0.7$$

$$F(2.99)=0.7;\quad F(3)=1.0$$

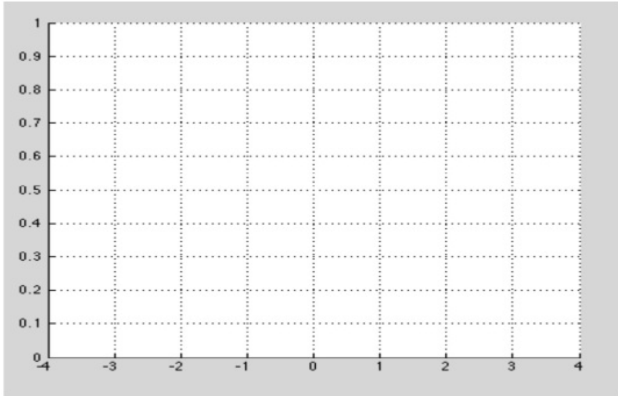


## Two Uniforms

$$0.4U(-2,0) + 0.6U(1,3)$$

$$F(-2) = 0; \quad F(0) = 0.4;$$

$$F(1) = 0.4; \quad F(3) = 1.0$$



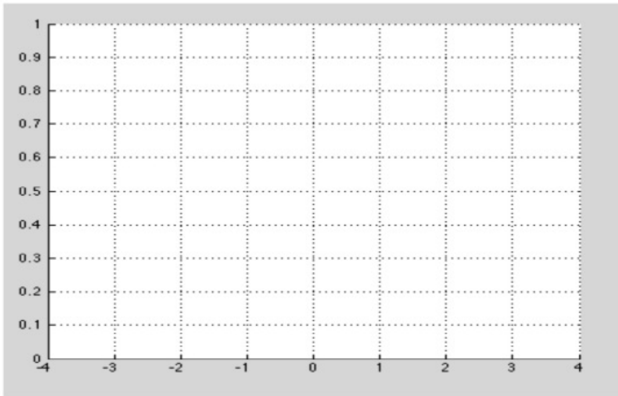

$$0.4U(0,2) + 0.6U(1,3)$$

$$F(0) = 0;$$

$$F(1) = 0.5 * 0.4 = 0.2;$$

$$F(2) = 0.4 + 0.5 * 0.6 = 0.7;$$

$$F(3) = 1.0$$



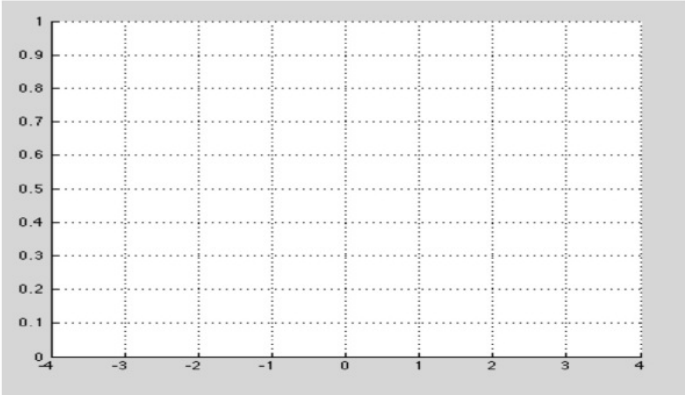
# Uniform and Point Mass

$$0.3PM(-2) + 0.7U(-1,1)$$

$$F(-2.01) = 0;\ \ F(-2) = 0.3;$$

$$F(-1) = 0.3;$$

$$F(1) = 1.0$$



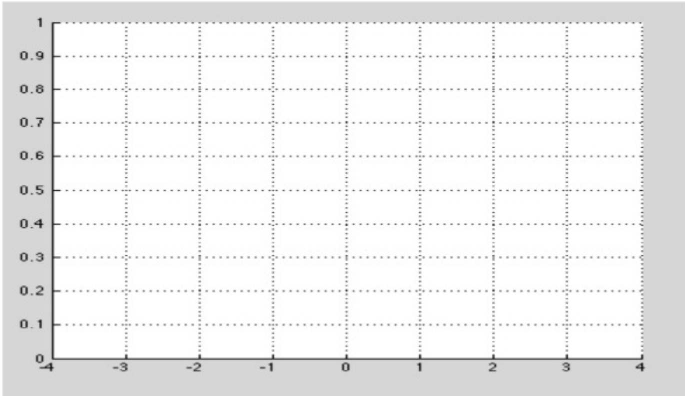

$$0.3PM(0) + 0.7U(-1,1)$$

$$F(-1) = 0;$$

$$F(-0.0001) = 0.34999$$

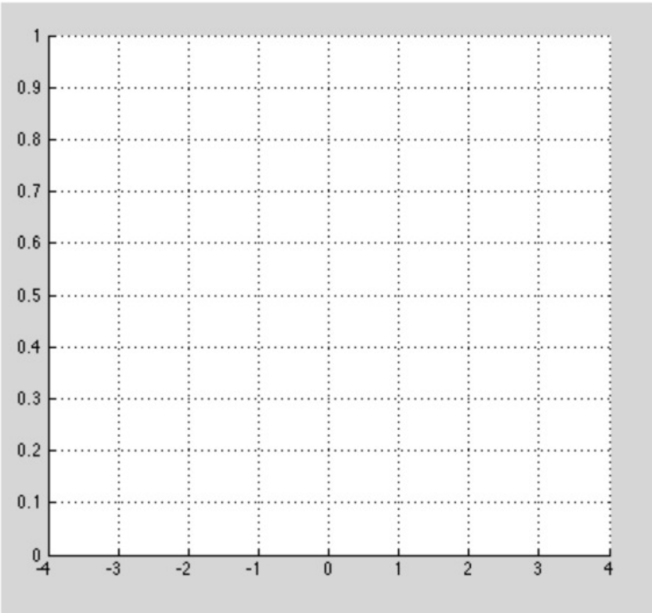
$$F(0) = 0.65$$

$$F(1) = 0$$

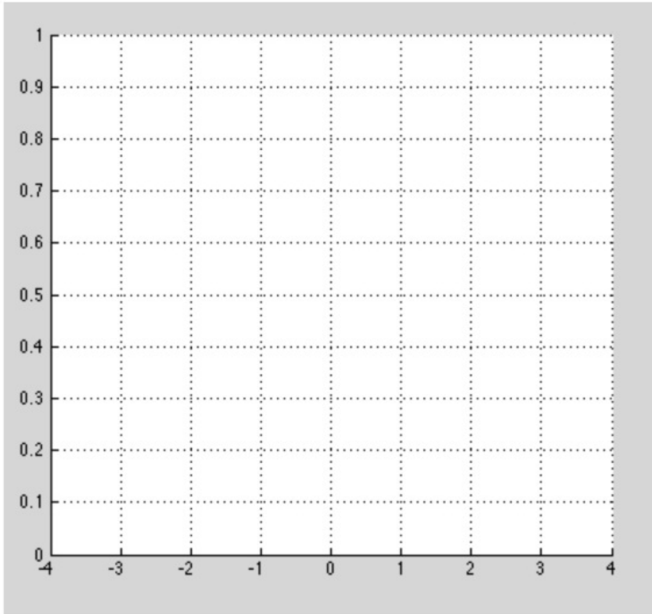


$$p_1 U(0,1) + p_2 PM(0) + p_3 U(-3,3) + p_4 PM(2)$$

Suppose P1=1/10, p2=2/10,p3=3/10,p4=4/10

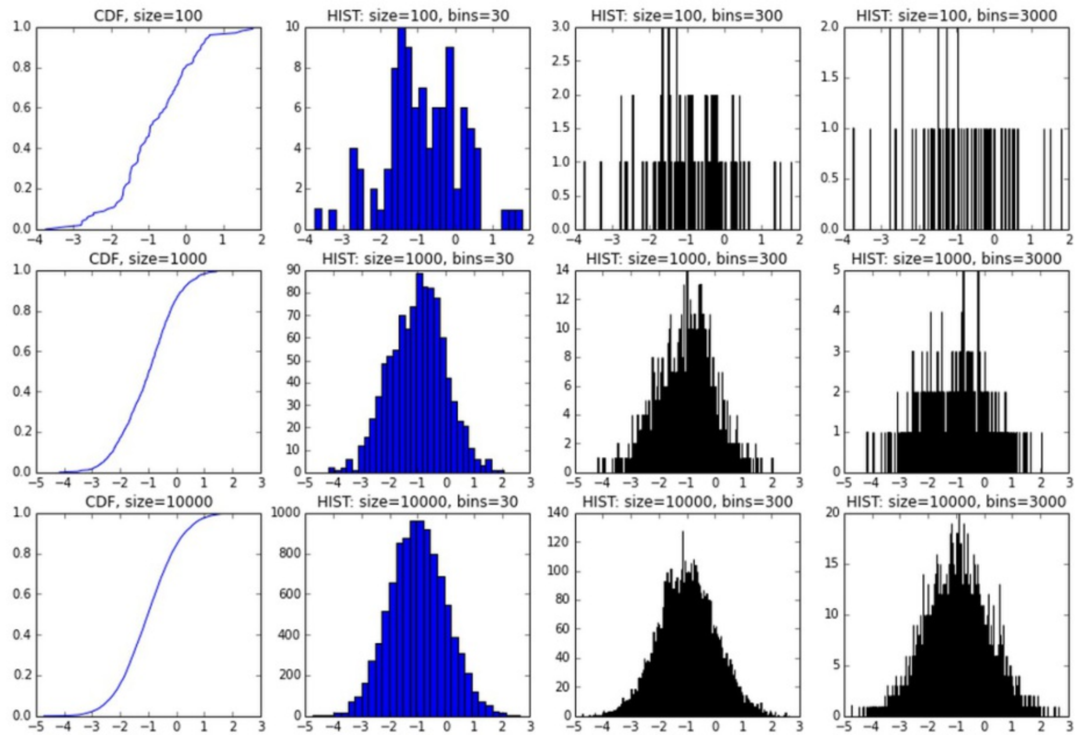



PDF+PM

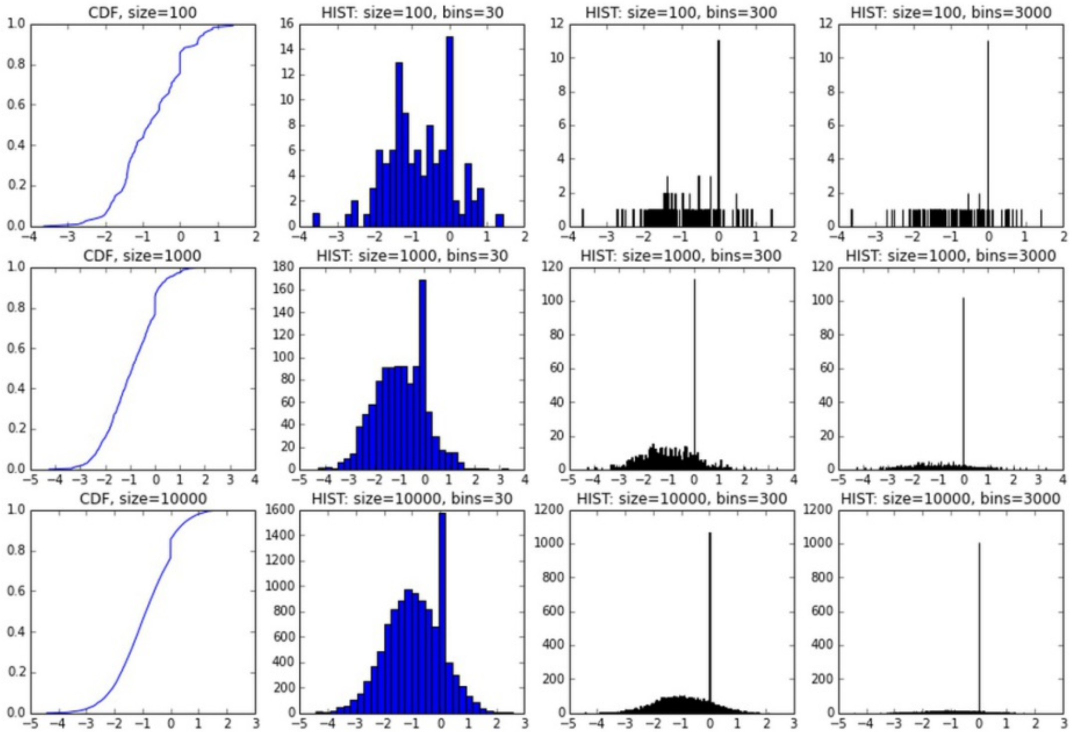



CDF

**_N(-1,1) = A normal distribution centered at -1, with width 1_**



## *A mixture of the normal and a point-mass (10*N(-1,1) + PM(0))*



1. *It is often hard to choose the number of bins in a histogram*
2. *When the distribution is a mixture of Point Masses and densities - there is no good choice.*
3. *Plotting CDFs does not require choosing a parameter.*
4. *Mixtures of PM and densities is not a problem.*

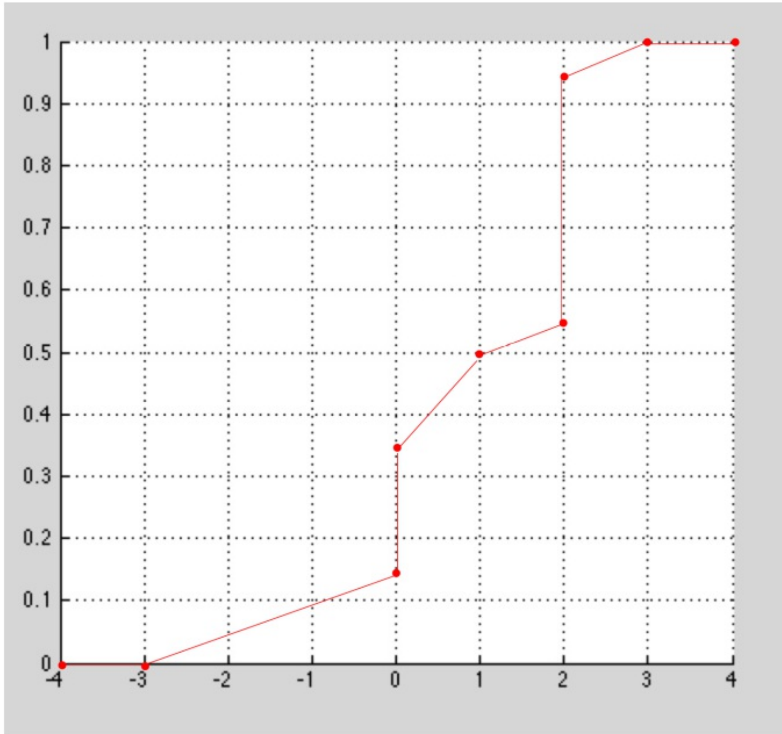
$$.1U(0,1)+.2PM(0)+.3U(-3,3)+.4PM(2)$$

$$F(-3)=0;\ F(-.01)\approx .5*.3=.15$$

$$F(0)=.35;\ F(1)=.35+.1+\frac{.3}{6}=0.5;$$

$$F(1.99)\approx 0.5+0.05=0.55;\quad F(2)=0.95$$

$$F(3)=0.95+\frac{.3}{6}=1.0$$



# density distributions vs. Point-Mass distribution

Point mass distributions assign non-zero probability to individual points.
PM(a) ---- $P(X=a)=1$

Density distributions assign non-zero probability to segments.

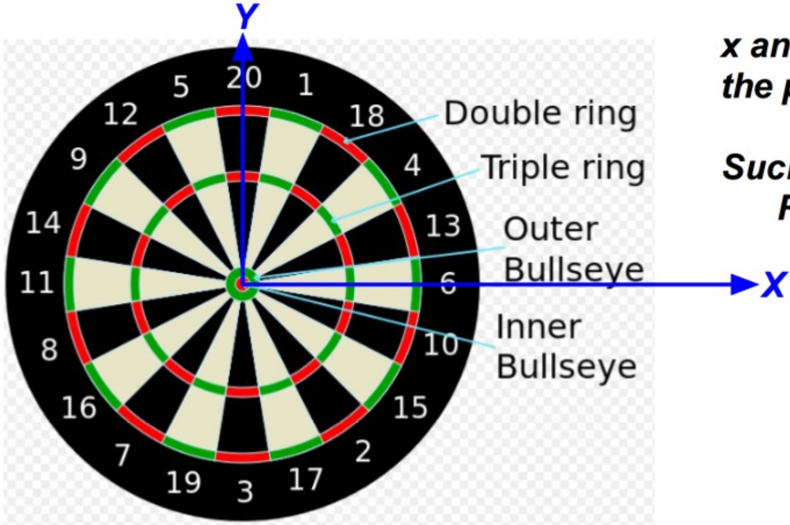
The probability of any single point under a density distribution is zero.
=> as a result $P([a,b])=P((a,b))=P([a,b))=P((a,b])$

=> the probability of any countable set is zero.

=> for example the probability of all rational numbers in [0,1], under the uniform distribution over [0,1] is zero!!!

In other words, if you pick a random number from U(0,1) the probability that it is a rational number is zero !!!

## *Densities over a 2D space*




***the sample space is the plane***

***x and y are mappings from the plane to R***

***Such mappings are called Random Variables***

***A natural assumption: the probability distribution of the location (x,y) that the dart falls is a density function that is highest at the bullseye and gets lower the further from the bullseye you get.***