Network Theorem

Presented By-Md. Shahiduzzaman Torun

2.25. Norton's Theorem

This theorem is an alternative to the Thevenin's theorem. In fact, it is the dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

This theorem may be stated as follows :–

(*i*) *Any two-terminal active network containing voltage sources and resistances when viewed from its output terminals, is equivalent to a constant-current source and a parallel resistance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances.*

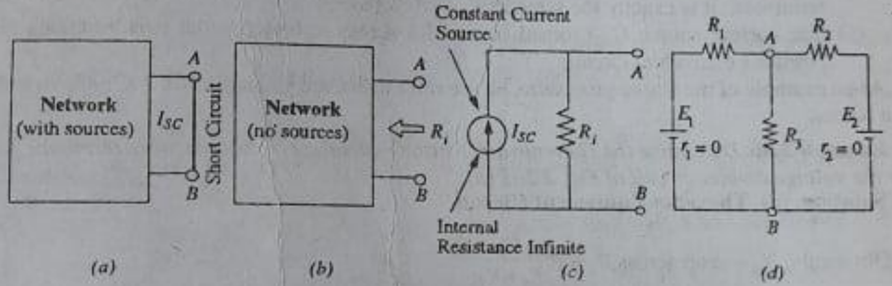


Fig. 2.202

Explanation

As seen from Fig. 2.202 (*a*), a short is placed across the terminals *A* and *B* of the network *with all its energy sources present.* The short-circuit current I_{SC} gives the value of constant-current source.

For finding R_i, all sources have been removed as shown in Fig. 2.202 (*b*). The resistance of the network when looked into from terminals *A* and *B* gives R_i.

The Norton's equivalent circuit is shown in Fig. 2.202 (*c*). It consists of an ideal constant-current source of infinite internal resistance (Art. 2.16) having a resistance of R_i connected in parallel with it. Solved Examples 2.96, 2.97 and 2.98 etc. illustrate this procedure.

(*ii*) Another useful generalized form of this theorem is as follows :

The voltage between any two points in a network is equal to $I_{SC} \cdot R_i$ where I_{SC} is the short-circuit current between the two points and R_i is the resistance of the network as viewed from these points with all voltage sources being replaced by their internal resistances (if any) and current sources replaced by open-circuits.

Suppose, it is required to find the voltage across resistance R_3 and hence current through it [Fig. 2.202 (*d*)]. If short-circuit is placed between *A* and *B*, then current in it due to battery of e.m.f. E_1 is E_1/R_1 and due to the other battery is E_2/R_2.

$$\therefore \qquad I_{SC} = \frac{E_1}{R_1} + \frac{E_2}{R_2} = E_1 G_1 + E_2 G_2$$

where G_1 and G_2 are branch conductances.

Now, the internal resistance of the network as viewed from *A* and *B* simply consists of three resistances R_1, R_2 and R_3 connected in parallel between *A* and *B*. Please note that here load resistance *R_3 has not been removed.* In the first method given above, it has to be removed.

$$\therefore \qquad \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = G_1 + G_2 + G_3 \qquad = \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore \qquad R_i = \frac{1}{G_1 + G_2 + G_3} \qquad \therefore \quad V_{AB} = I_{SC} \cdot R_i = \frac{E_1 G_1 + E_2 G_2}{G_1 + G_2 + G_3}$$

Current through R_2 is $I_3 = V_{AB}/R_3$.
Solved example No. 2.96 illustrates this approach.

2.26. How To Nortonize a Given Circuit ?

This procedure is based on the first statement of the theorem given above.

(1) Remove the resistance (if any) across the two given terminals and put a short-circuit across them.

(2) Compute the short-circuit current I_{SC}.

(3) Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open-circuits *i.e.* by infinite resistance.

(4) Next, find the resistance R_i (also called R_N) of the network as looked into from the given terminals. It is exactly the same as R_{th} (Art. 2.16)

(5) The current source (I_{SC}) joined in parallel across R_i between the two terminals gives Norton's equivalent circuit.

As an example of the above procedure, please refer to Solved Example No. 2.87, 88, 90 and 91 given below.

Example 2.96. *Determine the Thevenin and Norton equivalent circuits between terminals A and B for the voltage divider circuit of Fig. 2.203 (a).*

Solution. (*a*) **Thevenin Equivalent Circuit**

Obviosuly, V_{th} = drop across $R_2 = E\dfrac{R_2}{R_1 + R_2}$

When battery is replaced by a short-circuit.

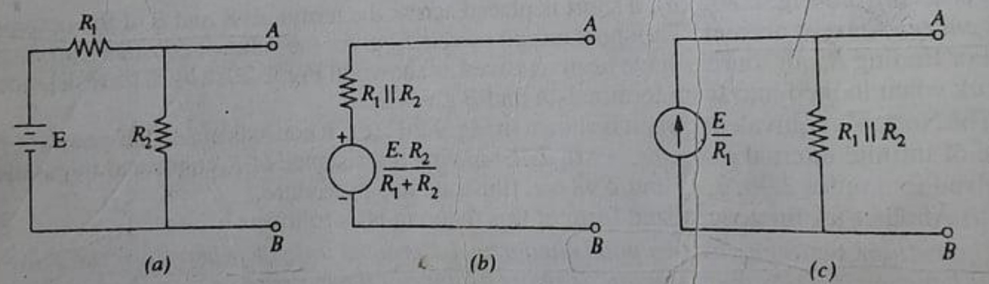


Fig. 2.203

$$R_i = R_1 \| R_2 = R_1 R_2 / (R_1 + R_2)$$

Hence, Thevenin equivalent circuit is as shown in Fig. 2.203 (*b*).

(*b*) **Norton Equivalent Circuit**

A short placed across terminals *A* and *B* will short out R_2 as well. Hence, $I_{sc} = E/R_1$. The Norton equivalent resistance is exactly the same as Thevenin resistance except that it is connected in parallel with the current source as shown in Fig. 2.203 (*c*).

Example 2.98. *Apply Norton's theorem to calculate current flowing through 5 – Ω resistor of Fig. 2.205 (a)*

Solution. (*i*) Remove 5 – Ω resistor and put a short across terminals *A* and *B* as shown in Fig. 2.205 (*b*). As seen, 10 – Ω resistor also becomes short-circuited.

(*ii*) Let us now find I_{SC}. The battery sees a parallel combination of 4 Ω and 8 Ω in series with a 4 Ω resistance. Total resistance seen by the battery = 4 + 4 || 8 = 20/3 Ω. Hence, $I = 20 \div 20/3 = 3$ A. This current divides at point *C* of Fig. 2.205 (*b*). Current going along path *CAB* gives I_{SC}. Its value = 3 × 4/12 = 1 A.

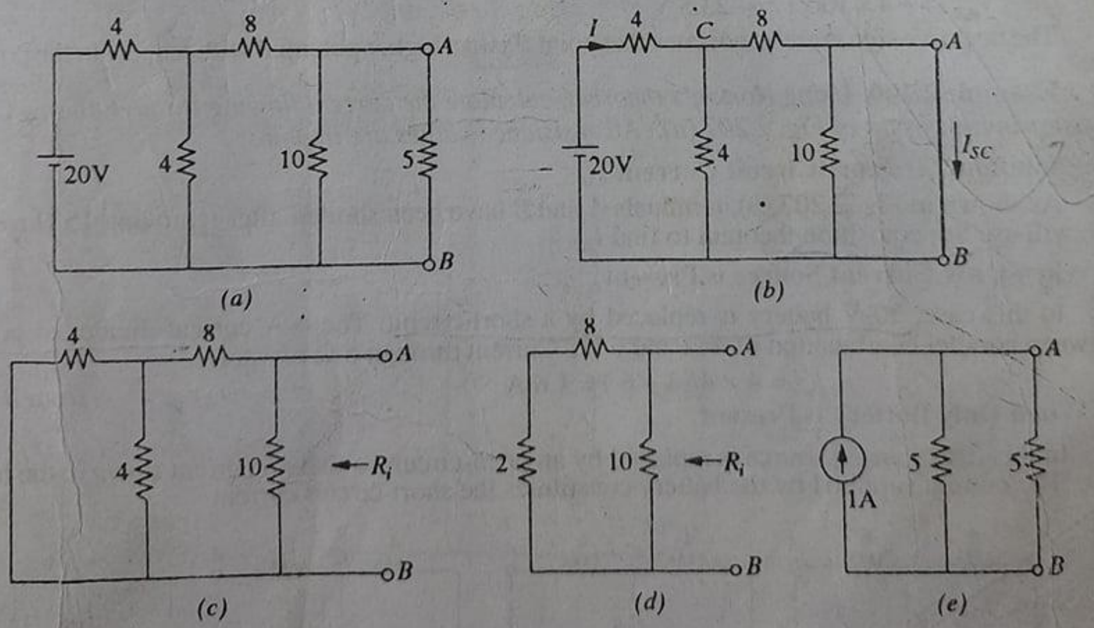


Fig. 2.205

(*iii*) In Fig. 2.205 (*c*), battery has been removed leaving behind its internal resistance which, in this case, is zero.

Resistance of the network looking into the terminals *A* and *B* in fig. 2.205 (d) is

$$R_i = 10 \parallel 10 = 5\ \Omega$$

(*iv*) Hence, Fig. 2.205 (*e*), gives the Norton's equivalent circuit.

(*v*) Now, join the 5 – Ω resistance back across terminals *A* and *B*. The current flowing through it, obviously, is $I_{AB} = 1 \times 5/10 =$ **0.5 A**

Example 2.100. *Using Norton's theorem, calculate the current flowing through the 15 Ω load resistor in the circuit of Fig. 2.207 (a). All resistance values are in ohm.*

Solution. (*a*) **Short-Circuit Current** I_{sc}

As shown in Fig. 2.207 (*b*), terminals *A* and *B* have been shorted after removing 15 Ω resistor. We will use Superposition theorem to find I_{SC}.

(*i*) **When Only Current Source is Present**

In this case, 30-V battery is replaced by a short-circuit. The 4 A current divides at point *D* between parallel combination of 4 Ω and 6 Ω. Current through 6 Ω resistor is

$$I_{sc}' = 4 \times 4/(4+6) = 1.6 \text{ A} \qquad \text{—from } B \text{ to } A$$

(*ii*) **When Only Battery is Present**

In this case, current source is replaced by an open-circuit so that no current flows in the branch *CD*. The current supplied by the battery constitutes the short-circuit current

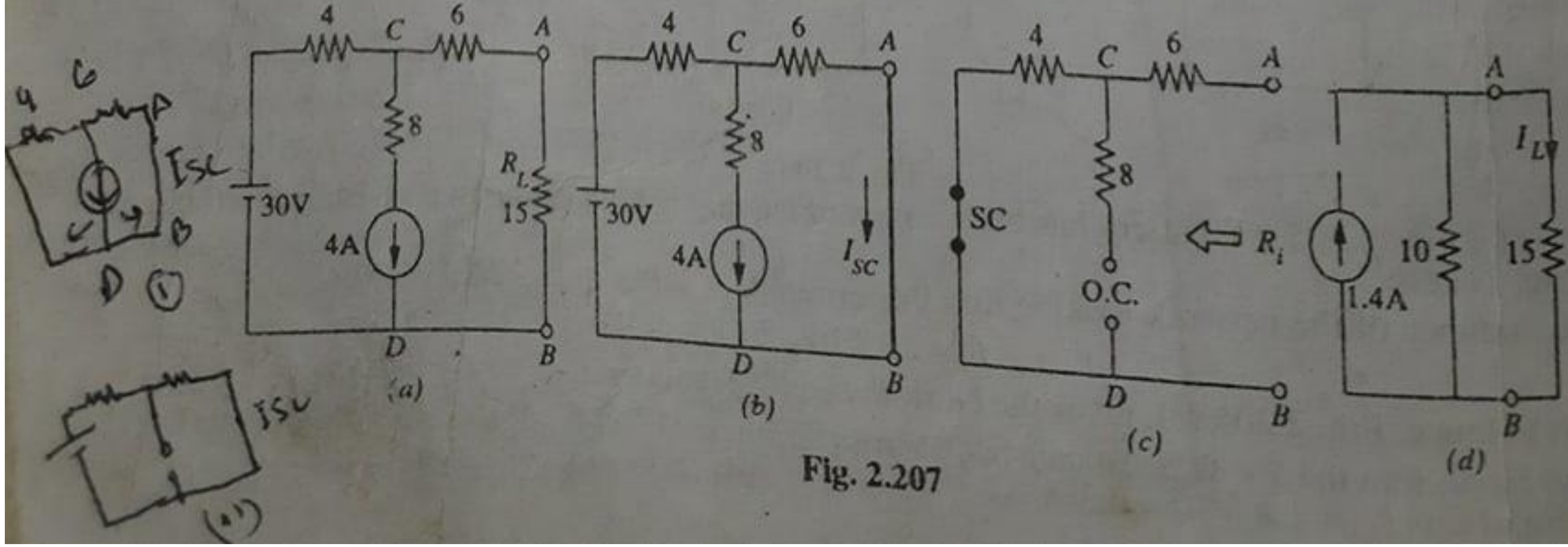


Fig. 2.207

$$\therefore \quad I_{sc}'' = 30/(4+6) = 3 \text{ A} \qquad \text{—from A to B}$$

$$\therefore \quad I_{sc} = I_{sc}'' - I_{sc}' = 3 - 1.6 = 1.4 \text{ A} \qquad \text{—from A to B}$$

(b) Norton's Parallel Resistance

As seen from Fig. 2.207 (c), $R_i = 4 + 6 = 10\ \Omega$. The 8 Ω resistance does not come into the picture because of an 'open' in the branch *CD*.

Fig. 2.207 (*d*) shows the Norton's equivalent circuit along with the load resistor.

$$I_L = 1.4 \times 10/(10 + 15) = 0.56 \text{ A}$$

2.30. Maximum Power Transfer Theorem

Although applicable to all branches of electrical engineering, this theorem is particularly useful for analysing communication networks. The overall efficiency of a network supplying maximum power to any branch is 50 per cent. For this reason, the application of this theorem to power transmission and distribution networks is limited because, in their case, the goal is high efficiency and not maximum power transfer.

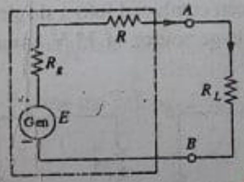
However, in the case of electronic and communication networks, very often, the goal is either to receive or transmit maximum power (through at reduced efficiency) specially when power involved is only a few milliwatts or microwatts. Frequently, the problem of maximum power transfer is of crucial significance in the operation of transmission lines and antennas.

As applied to d.c. networks, this theorem may be stated as follows :

A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances.

In Fig. 2.230 (a), a load resistance of R_L is connected across the terminals A and B of a network which consists of a generator of e.m.f. E and internal resistance R_g and a series resistance R which, in fact, represents the lumped resistance of the connecting wires. Let $R_i = R_g + R$ = internal resistance of the netowrk as viewed from A and B.

According to this theorem, R_L will abstract maximum power from the network when $R_L = R_i$.



$\approx$ R_{th} V_{th} R_L

here, $V_{th} = E$

$R_{th} = R + R_g = R_i$

Here $V_{th} = E$ but have to do V_{th}

Fig. 2.230

Proof. Circuit current $I = \dfrac{V_{th}}{R_L + R_i} = \dfrac{V_{th}}{R_{th} + R_L}$

Power consumed by the load is

$$P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + Ri)^2} \qquad \text{...(i)}$$

$$\boxed{\text{For } P_L \text{ to be maximum, } \frac{dP_L}{dR_L} = 0.}$$

Differentiating Eq. (i) above, we have

$$\frac{dP_L}{dR_L} = E^2 \left[\frac{1}{(R_L + R_i)^2} + R_L \left(\frac{-2}{(R_L + R_i)^3} \right) \right] = E^2 \left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3} \right]$$

$$\therefore \quad 0 = E^2 \left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3} \right] \quad \text{or} \quad 2R_L = R_L + R_i \quad \text{or} \quad R_L = R_i$$

It is worth noting that under these conditions, the voltage across the load is half the open-circuit voltage at the terminals A and B.

$$\therefore \quad \text{(i) Max. power is } P_{L\,max.} = \frac{E^2 R_L}{4R_L^2} = \frac{E^2}{4R_L} = \frac{E^2}{4R_i} \qquad I^2 R_L \quad \frac{E}{2R_L} \cdot R_L = \frac{V_{th}^2}{4R_L}$$

Let us consider an a.c. source of internal impedance $(R_1 + j\,X_1)$ supplying power to a load impedance $(R_L + jX_L)$. It can be proved that maximum power transfer will take place when the modules of the load impedance is equal to the modulus of the source impedance i.e. $|Z_L| = |Z_1|$

Where there is a completely free choice about the load, the maximum power transfer is obtained when load impedance is the complex conjugate of the source impedance. For example, if source impedance is $(R_1 + jX_1)$, then maximum transfer power occurs, when load impedance is $(R_1 - jX_1)$. It can be shown that under this condition, the load power is $= E^2/4R_1$.

when load impedance is the complex conjugate of the source impedance. For example, if source impedance is $(R_1 + jX_1)$, then maximum transfer power occurs, when load impedance is $(R_1 - jX_1)$. It can be shown that under this condition, the load power is $= E^2/4R_1$.

Example 2.115. *In the network shown in Fig. 2.231 (a), find the value of R_L such that maximum possible power will be transferred to R_L. Find also the value of the maximum power and the power supplied by source under these conditions.* **(Elect. Engg. Paper I Indian Engg. Services 1989)**

Solution. We will remove R_L and find the equivalent Thevenin's source for the circuit to the left of terminals A and B. As seen from Fig. 2.231 (b) V_{th} equals the drop across the vertical resistor of 3 Ω because no current flows through 2 Ω and 1 Ω resistors. Since 15 V drops across two series resistors of 3 Ω each, $V_{th} = 15/2 = 7.5$ V. Thevenin's resistance can be found by replacing 15 V source with a short-circuit. As seen from Fig. 2.231 (b), $R_{th} = 2 + (3||3) + 1 = 4.5\ \Omega$. Maximum power transfer to the load will take place when $R_L = R_{th} = 4.5\ \Omega$. $= R_l$

$\frac{15}{6} = 2.5$

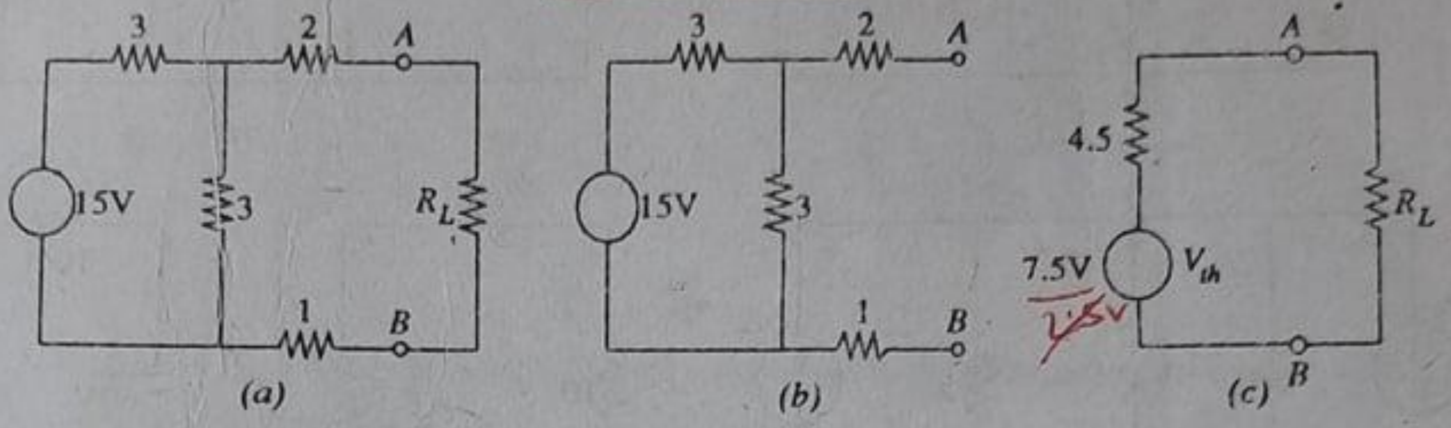


Fig. 2.231

Maximum power drawn by $R_L = V_{th}^2/4 \times R_L = 7.5^2/4 \times 4.5 = 3.125$ W.

Since same power is developed is R_{th}, power supplied by the source $= 2 \times 3.125 = 6.250$ W.

Example 2.116. *In the circuit shown in Fig. 2.232 (a) obtain the condition from maximum power transfer to the load R_L. Hence determine the maximum power transferred.*

(Elect. Science-I Allahabad Univ. 1992)

Solution. We will find Thevenin's equivalent circuit to the left of terminals A and B for which purpose we will convert the battery source into a current source as shown in Fig. 2.232 (*b*). By combining the two current sources, we get the circuit of Fig. 2.232 (*c*). It would be seen that open-circuit voltage V_{AB} equals the drop over 3 Ω resistance because there is no drop on the 5 Ω resistance connected to terminal A. Now, there are two parallel paths across the current source each of resistance 5 Ω. Hence, current through 3 Ω resistance equals 1.5/2 = 0.75 A. Therefore, $V_{AB} = V_{th} = 3 \times 0.75 =$ 2.25 V with point A positive with respect to point B.

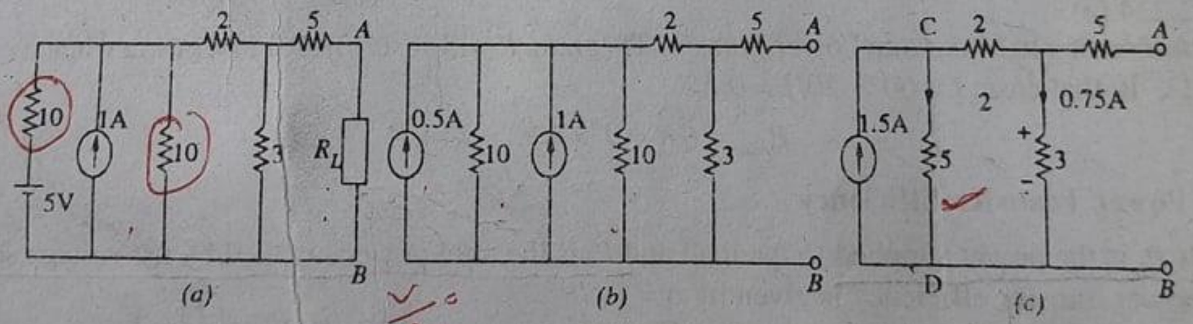


Fig. 2.232

For finding R_{AB}, current source is replaced by an infinite resistance.

$$\therefore \qquad R_{AB} = R_{th} = 5 + 3 \parallel (2+5) = 7.1\ \Omega$$

The Thevenin's equivalent circuit alongwith R_L is shown in Fig. 2.233. As per Art. 2.30, the condition for MPT is that $R_L = 7.1\ \Omega$.

Maximum power transferred $= V_{th}^2/4R_L = 2.25^2/4 \times 7.1 = 0.178\ \text{W} = 178\ \text{mW}$.

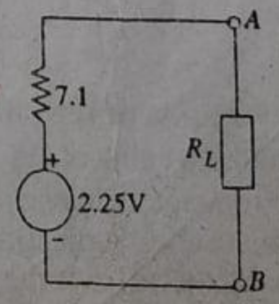


Fig. 2.233

Example 2.117. *Calculate the value of R which will absorb maximum power from the circuit of Fig. 2.234 (a). Also, compute the value of maximum power.*

Solution. For finding power, it is essential to know both I and R. Hence, it is essential to find an equation relating I to R.

2.31. Power Transfer Efficiency

If P_L is the power supplied to the load and P_T is the total power supplied by the voltage source, then power transfer efficiency is given by $\eta = P_L / P_T$.

Now, the generator or voltage source E supplies power to both the load resistance R_L and to the internal resistance $R_i = (R_g + R)$.

$$P_T = P_L + P_i \quad \text{or} \quad E \times I = I^2R_L + I^2R_i$$

$$\therefore \qquad \eta = \frac{P_L}{P_T} = \frac{I^2R_L}{I^2R_L + I^2R_i} = \frac{R_L}{R_L + R_i} = \frac{1}{1 + (R_i/R_L)}$$

The variation of η with R_L is shown in Fig. 2.235 (a). The maximum value of η is unity when $R_L = \infty$ and has a value of 0.5 when $R_L = R_i$. It means that under maximum power transfer conditions, the power transfer efficiency is only 50%. As mentioned above, maximum power transfer condition is important in communication applications but in most power systems applications, a 50% efficiency is undesirable because of the wasted energy. Often, a compromise has to be made between the load power and the power transfer efficiency. For example, if we make $R_L = 2\, R_i$, then

$$P_L = 0.222\, E^2/R_i \quad \text{and} \quad \eta = 0.667.$$

It is seen that the load power is only 11% less than its maximum possible value, whereas the power transfer efficiency has improved from 0.5 to 0.667 *i.e.* by 33%.

vol

S