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| **PA1 Individual Report** |

**Zhenrui Yue**

Computer Science & Engineering

UC San Diego

La Jolla, CA 92093

*yuezrhb@gmail.com*

**1 Problems from Bishop (20 points)**

* 1. **Problem 1.1**

Given the equations involving the transformation from Cartesian to polar coordinates:

Let , we transform the first equation into:

Plug into the left side of the second equation:

By gamma function definition, we substitute with :

Plug into the right side of the second equation and rearrange the equation, we proved:

For :

For :

* 1. **Problem 1.2**

Given the surface of a unit hypersphere , the surface of a hypersphere with radius in dimensions could be written as:

The corresponding volume of hypersphere is its integration of surface area from to :

Therefore, the volume of a hypersphere with radius divided by a hypercube with side length :

As , substitute with ,

The above equation could be seen as the product of two different parts:

Therefore,

The distance of hypercube (side length 2a, from center to one of its corners) and hypersphere (radius a, from center to surface) could be written as:

Thus, we proved the ratio would be as ,

* 1. **Problem 1.3**

The volume of hypersphere with radius in dimension:

The volume of hypersphere with radius in dimension:

Hence,

For ,

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Fraction of the sphere volume which lies inside the radius of :

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* 1. **Problem 1.4**

Given the probability density function :

The probability mass of thin shell with radius equals the product of sphere surface and probability density with :

Hence the probability mass of this thin shell with thickness is

The first and second order derivative of is:

Let , for large we could approximate to :

Plug this value into the second order derivative:

equals for and is negative for , therefore we proved the function has a single maximum for large at , thus,

With Taylor expansion of at :

The above equation could be further simplified:

**2 Logistic Regression**

Given the Cross-Entropy and parametrized model,

Note is the sigmoid function and thus:

The negated partial derivative could be simplified correspondingly:

Hence, we proved the partial gradient of the Cross-Entropy.