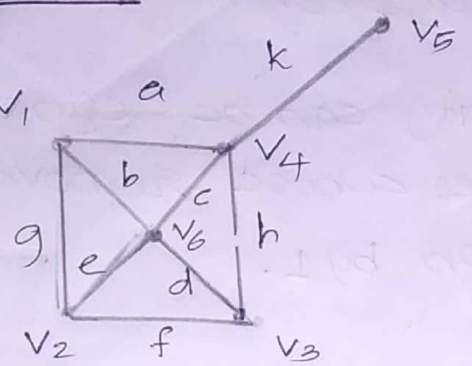
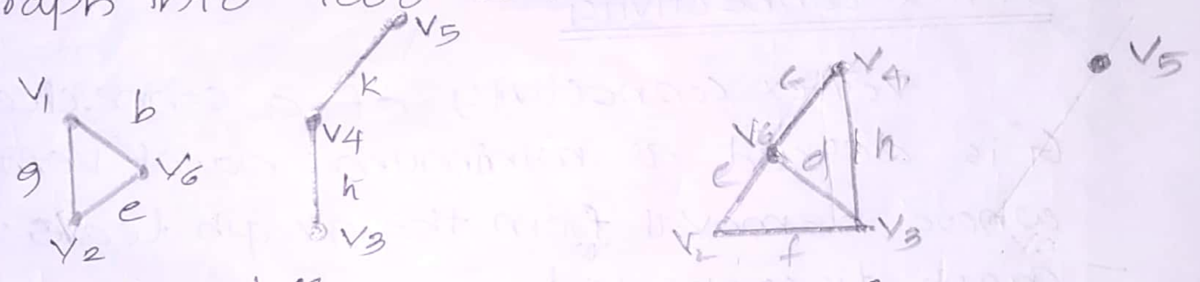
Module : 4

## Cut-Set

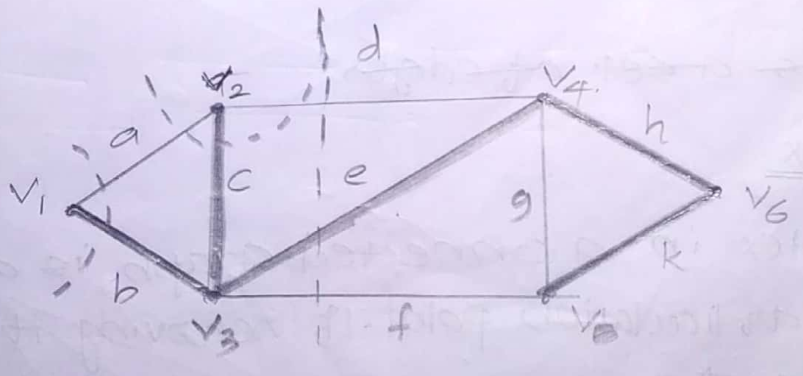



In a ~~graph~~ connected graph 'G', a cut-set is a set of edges or a subgraph whose removal leaves the graph disconnected ie, cut-set is always ~~a graph~~ cuts the graph into two connected components.




## Fundamental cut-set (Basic cut-set)

A cut-set 'S' is containing exactly one branch of tree 'T' is called fundamental cut-set.




{a, b} , {a, c, d} {d, e, f}

## Edge connectivity

The no. of edges in the smallest cut-set is defined as the edge connectivity of G.

edge connectivity can be defined as the minimum no. of edges whose removal reduces the rank of the graph by 1.

*Edge connectivity of a tree is 1

In previous qn:

Minimum no. of edges to disconnect the graph = edge connectivity = 2

## Vertex connectivity

Vertex connectivity of a connected graph 'G' is defined as minimum no. of vertices whose removal from the graph leaves the graph disconnected.

In previous qn:

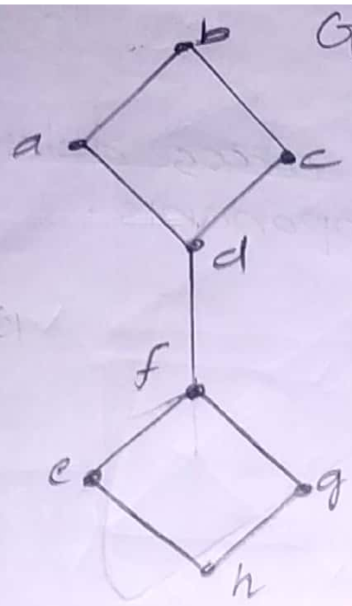
Vertex connectivity = Mini. no. of Vertices to disconnect the graph.

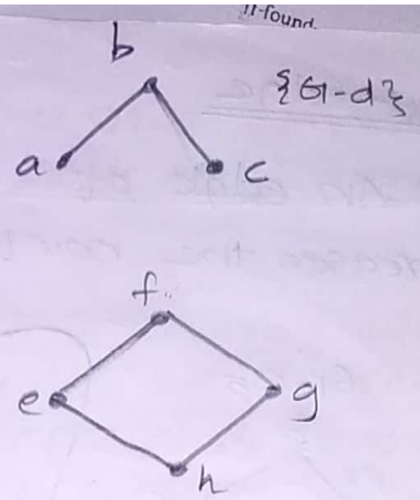
= 2

~~It is a set of edges~~

## Cut - vertex

A vertex in a connected graph is a cut-vertex/an articulation point. If removing it disconnecting the graph.



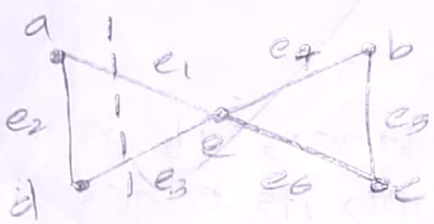




$\omega(G)=1$
$\omega(G-d)=2$

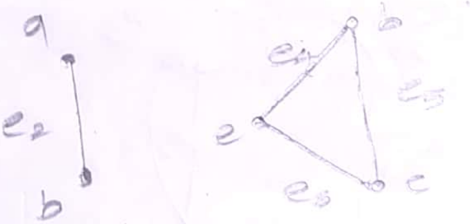
We can denote an edge connectivity as '$\lambda$'. ie, $\lambda(G)$.

vertex connectivity is $\kappa(G)$.

?

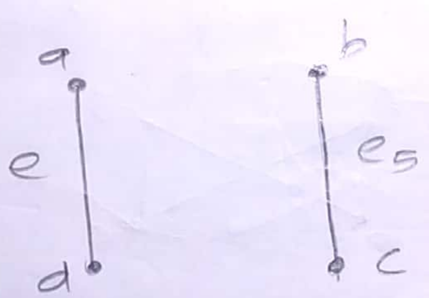



G




$G-\{e_1, e_3\}$

$\lambda(G)=2$

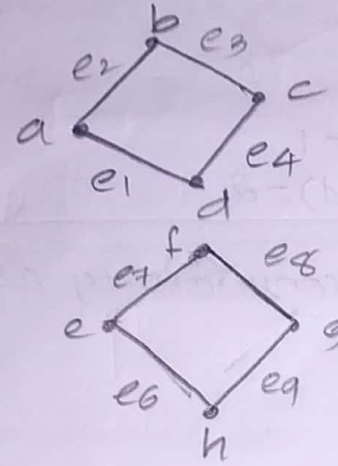



$\{G-e\}$

## Cut edge

An edge of a graph whose deletion increases the no. of its components.

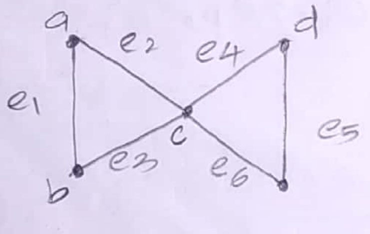
$G - e_5$

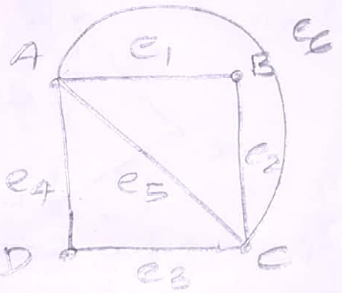



## Planar Graph.

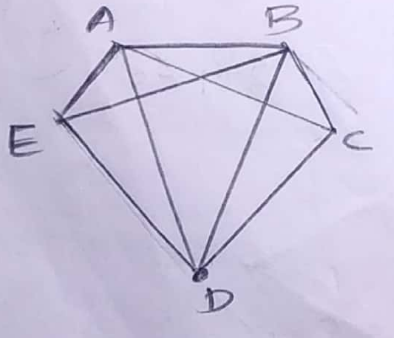
A graph 'G' is planar, if it can be drawn in the plane with its edges intersecting at their vertices only.

Planar graph



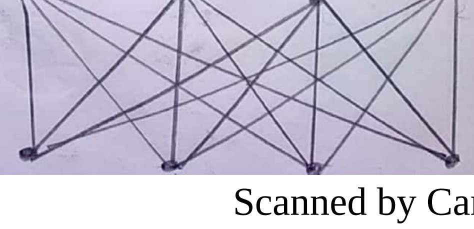




## Non planar graph:





Complete bipartite Graph:



Theorem:

Edge connectivity of a graph G cannot exceed the degree of a vertex having the smallest degree in G.

Proof:

Let vertex '$v_i$' be the vertex with smallest degree in G. Let $d(v_i)$ be the degree of $v_i$. The vertex $v_i$ can be seperated from G by removing the $d(v_i)$ edges incident on vertex $v_i$. Hence the theorem.

Theorem:

The vertex connectivity of a graph G cannot exceed the edge connectivity of G.

Proof:

Let '$\alpha$' denote the edge connectivity of G. ∴ there exist a ~~co~~ cut set 's' with $\alpha$ edges. Let s partition the graph into 2 subsets $V_1$ & $V_2$. By removing atmost '$\alpha$' vertices from $V_1$ or $V_2$ on which the edges in s are incident, we can remove s from G.

Properties of cut-set

- Every cut-set in a connected graph 'G' was contains atleast one branch of every spanning tree of G.

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- Every ckt has even no. of edges in common with any cut set.
- Edge connectivity of a graph cannot exceed the degree of the vertex having the smallest degree in 'G.'
- The vertex connectivity of any graph cannot

exceed the edge connectivity of G.

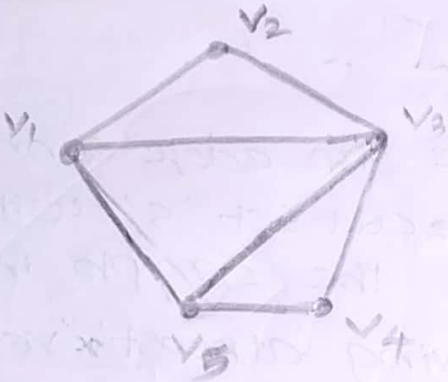
## Different representations of planar graph

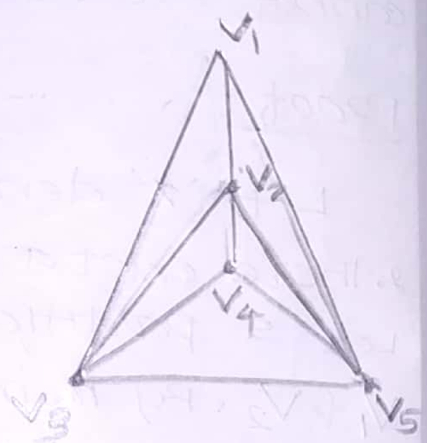
3 ways:

1) Straight line representation
2) Plane representation
3) Embedding on a sphere

### 1) Straight line

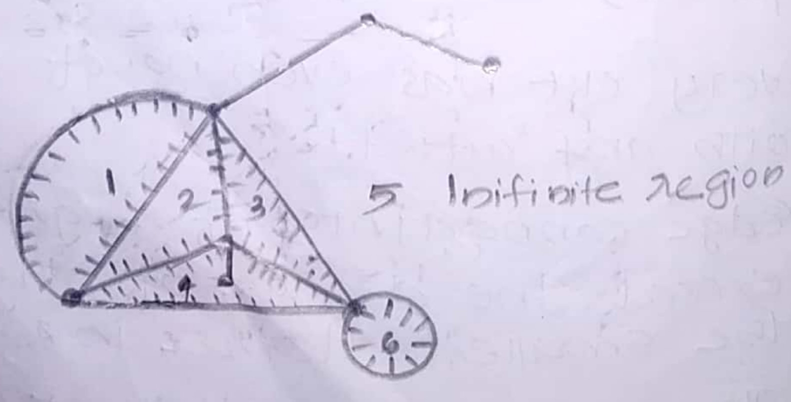
Any simple planar graph can be embedded in a plane such that every edge is drawn as straightline.




### 2) Plane representation

In plane representation the graph is divided the planes into regions. A region is characterized by a set of edges or set of vertices that from its boundary.

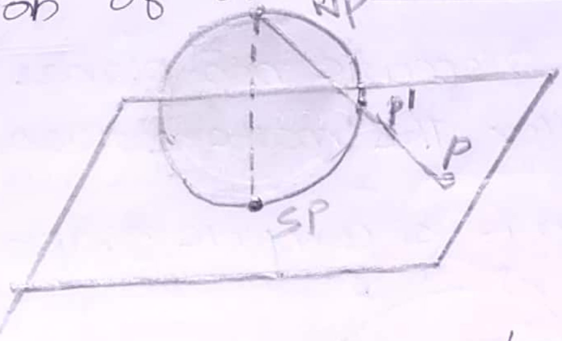


## 2) Infinite graph

The portion of the plane lying outside the graph is called an infinite region / unbounded region / exterior region / outer region. Regions are also called windows / meshes / phases.

## 3) Embedding on a sphere

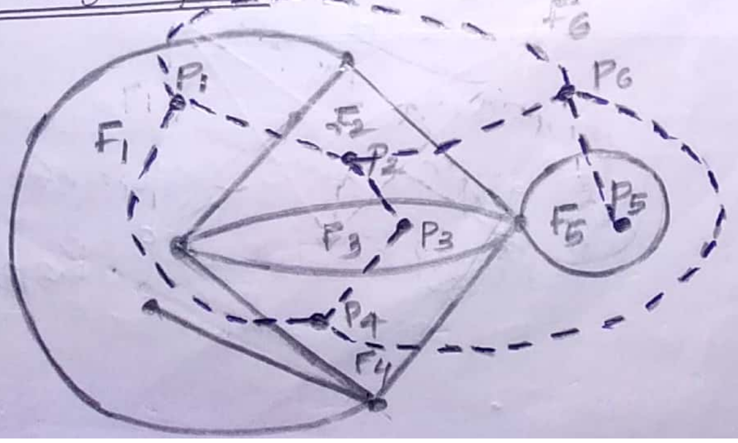
This is similar to stereographic projection of a sphere on to a plane.




Let 'sp' (south pole) be the point at which the sphere to

Where P' is the point at which the straightline from P to NP intersects the surface of the sphere.

## Dual of a graph



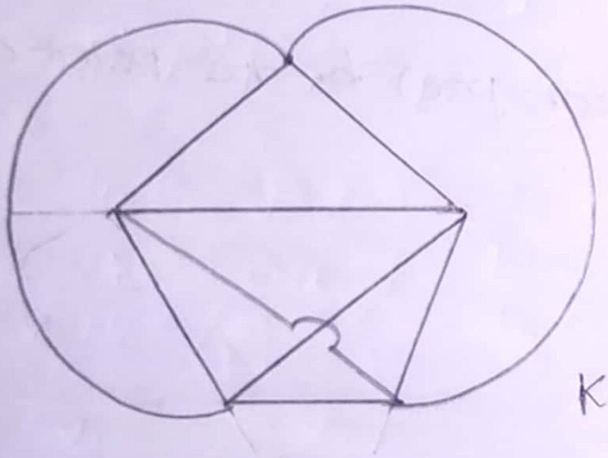
If 2 regions $F_i$ & $F_j$ are adjacent, then draw a line joining the pts $P_i$ & $P_j$ that intersects the common edge b/w $F_i$ & $F_j$ exactly once.

If there is more than 1 edge common b/w $F_i$ & $F_j$ draw one line b/w the points $P_i$ & $P_j$ for each of the common edges.

03/10/19 Kuratowski's Graphs

These are 2 specific non-planar graphs which was named after the mathematician Kuratowski.

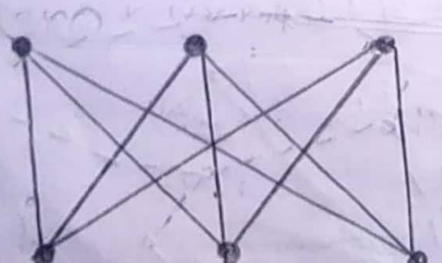
The first graph is a complete graph with 5 vertices.



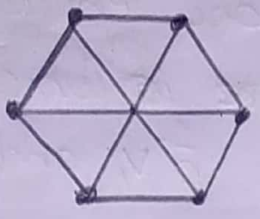
$K_5$

Second graph is a regular connected graph with 6 vertices & 9 edges.

A regular graph is a graph where each vertex has the same degree.



$K_{3,3}$



## Kuratowski's Theorem

A graph is planar iff it doesnot have any subdivision of $K_5$ (or) $K_{3,3}$.

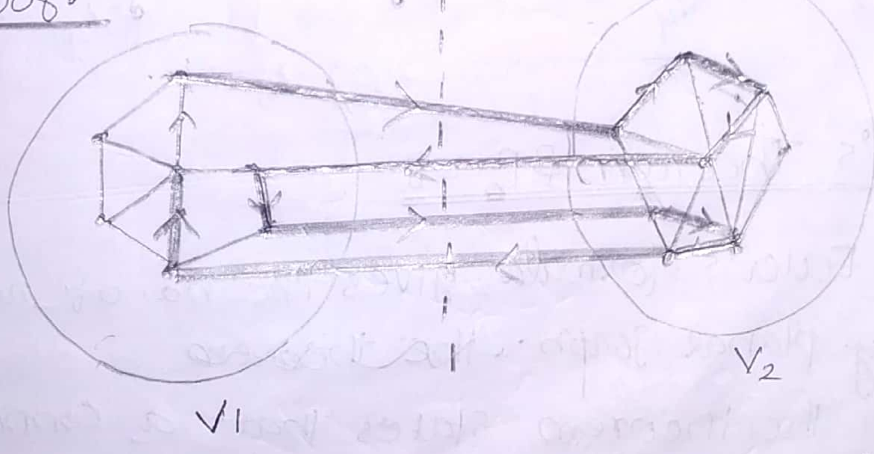
properties of Kuratowski's graph

- → Both of them are non-planar graphs.
- → Both are regular graphs
- → removal of one edge or vertex makes them a planar graph.
- → Kuratowski's 1st graph is a non-planar graph with smallest no. of vertices & 2nd graph is a non-planar graph with smallest no. of edges

## Theorem:

Every circuit has even no. of edges in common with any ~~concept~~ cut-set.

proof:



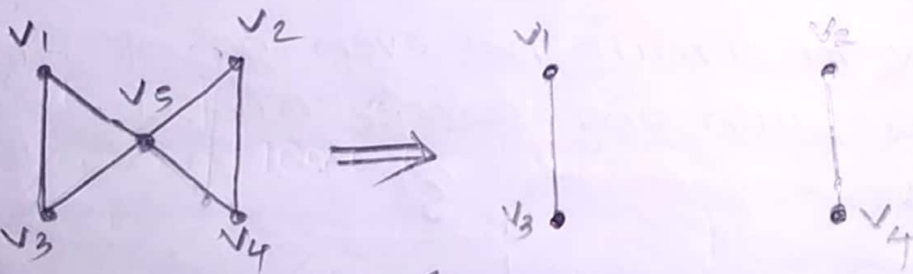

Consider a cut-set 'S' in a graph 'G'. Let removal of 'S' partition the vertices of 'G' into

2 disjoined subsets $V_1$ & $V_2$. Consider a ckt 'R' in G. If all the vertices in R are entirely in set $V_1$ or in set $V_2$ then $S \cap R = \phi$. ~~edge~~ which is even. else, if some vertices are in $V_1$ and some are in $V_2$ then, we traverse back & forth in $V_1$ & $V_2$, and because of the closed nature of the circuit, the no. of edges which we traversed must be even. Since, every edge in 'S' has one edge end in $V_1$ and other end in $V_2$, the no. of edges common to 'S' & 'R' is even. 16/10

## Seperable graphs

A connected graph is said to be seperable if its vertex connectivity is '1'. All other connected graphs are called non-seperable graphs.

In a seperable graph the vertex whose removal disconnects the graph is called ~~cut~~ cut-vertex / a cut-node / an articulation point.




Removing $v_5$

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## Euler's Theorem & proof

Euler's formula gives the no. of regions in any planar graph. ~~The theorem~~

The theorem states that a connected planar graph with 'n' vertices & 'e' edges has $e-n+2$ regions. (faces).

16/10 ~~Prove that~~

PROOF

Let the no. of regions /faces ~~be~~ 'f'. Here we shall use mathematical induction to prove this theorem.

Proof by induction; basic step

Let $n=1$. We have no. of vertices as 1


$f=e-n+2$

$n-e+f=2$

$e=3$.

$f=$ region $=3+1=4$

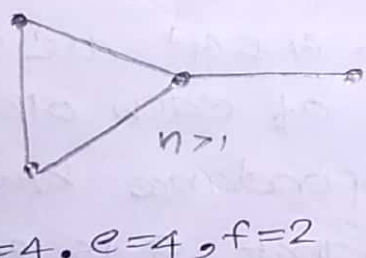
$n=1, e=3, f=4$

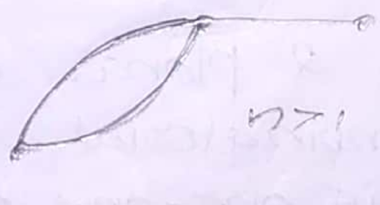
$n=1, e=0, f=1$

If $e=0, n=1$ then $f=1$ and the formula holds. that is the formula holds for $n=1$ for any no. of edges. we add a self loop or parallel edge it increases 'e' and 'f' by 1.

∵ it ~~cu~~ cuts the region into 2.

Induction Step (When $n>1$)




$n=4, e=4, f=2$ $n=3, e=3, f=2$

If we remove an edge which is not a loop then the no. of regions remains same and 'e' and 'n' decreases by 1.

$n' = n-1$, $e' = e-1$, $f' = f$

$n = n'+1$, $e = e'+1$

Applying Induction hypothesis,

we have

$$n - e + f = 2$$

$$\Rightarrow (n'+1) - (e'+1) + f = 2$$

$$\Rightarrow n'+1-e'-1+f = 2$$

$$\Rightarrow n'-e'+f = 2$$

NOTE:

→ We can have geometric dual as well as combinational dual.

~~Note~~ Combinational dual

Two planar graphs 'G' & (G') are said to be combinatorial duals of each other if there is one-one correspondance b/w the edges in G and G* such that a set of edges in G forms a ckt iff the corresponding set in G' forms a cutset.

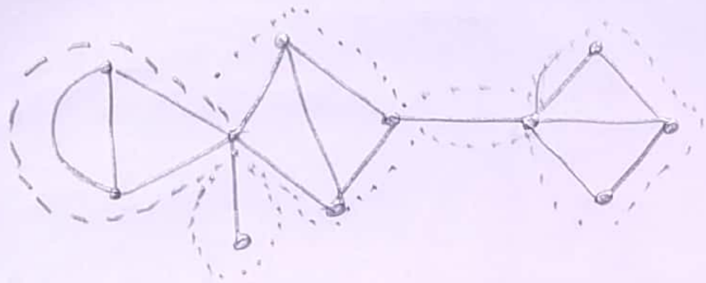
2 planar graphs G & G' are said to be combinatorial duals of each other if there is one-one correspondance b/w the edges in G and G' such that g is any sub graph of G and h is the corresponding subgraph

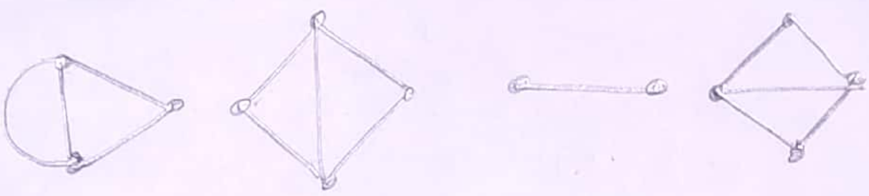
in G' then

rank of $(G'-h)$ = rank of $G'$ - nullity of g


ISOMORPHISM

A seperable graph consists of 2/more non-seperable subgraphs. Each of the largest non-seperable subgraphs is called BLOCK.





Two graphs $G_1$ & $G_2$ are said to be 1-isomorphic. If they become isomorphic to each other under the following operations. ie, ~~The~~ split the cut-vertex in to 2 vertices to produce 2 disjoint subgraphs.