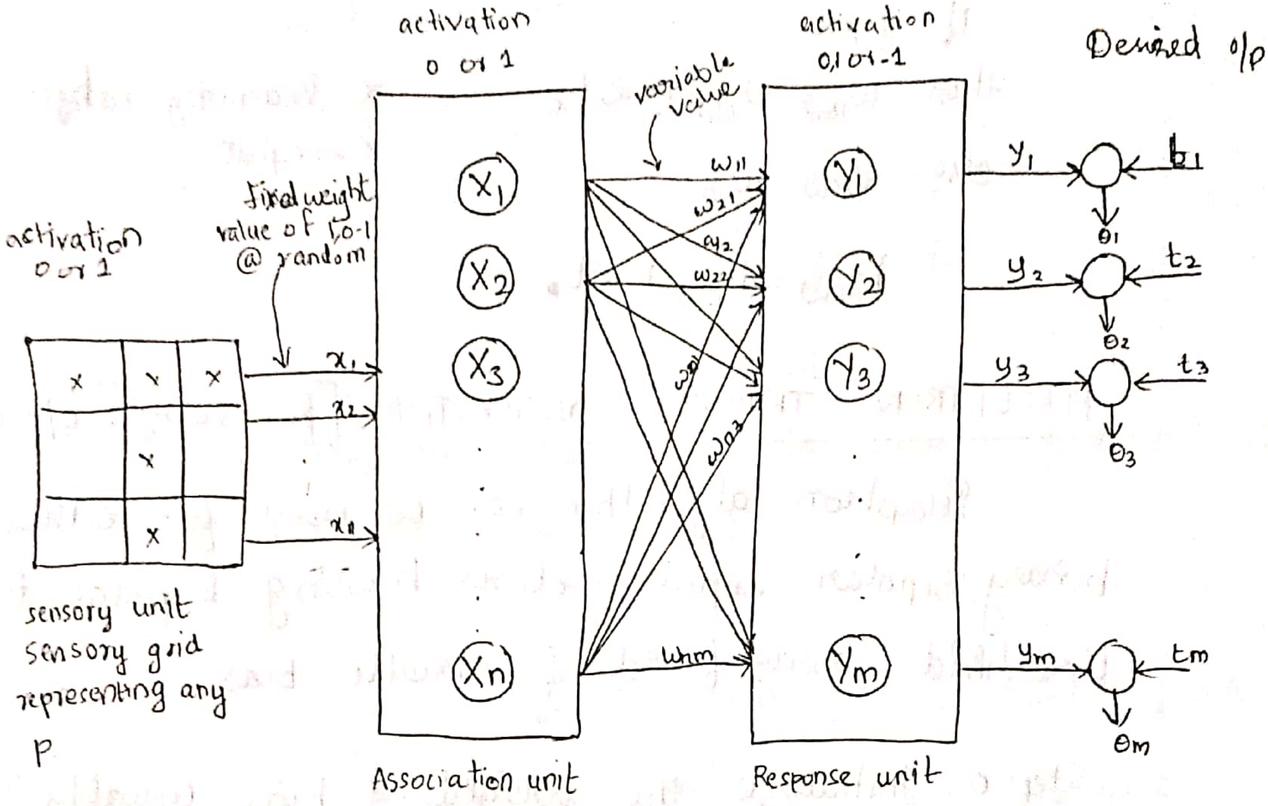
MODULE 2

PERCEPTRON NETWORK



Three important units of Perceptron Networks are:

1) Sensory unit (0 or 1) } fixed (0,1,-1)
2) Associatory unit (0 or 1) } variable
3) Response unit (0,1 or -1)

Perceptron Learning Rule

Consider a finite n no. of i/p training vectors with their associated target values as x(n) and t(n) where n ranges from 1 to N. The target is either +1 or -1. O/p y is obtained on the basis of net i/p calculated & activation fn. being applied over net i/p.

$$y = f(y_{in}) = \begin{cases} 1 & y_n > \theta \\ 0 & -\theta \le y_n \le \theta \\ -1 & y_n < -\theta \end{cases}$$

weight updation

if $y \neq t$

then $w_{new} = w_{old} + \alpha t x$ α = learning rate

else $w_{new} = w_{old}$ x = input

$b_{new} = b_{old} + \alpha t$

PERCEPTRON TRAINING ALGORITHM [for single o/p class]

Perceptron algorithm can be used for either binary/bipolar input vectors, having bipolar targets threshold being fixed & variable bias

Step 0: Initialize the weight & bias. Usually it is set to 0. Also initialize learning rate α $(0 < \alpha \le 1)$ for simplicity it is set to 1.

Step 1: Perform steps 2 to 6 until final stopping condition is false.

Step 2: Perform steps 3 to 5 for each training pair s:t. s → i/p & t → o/p.

Step 3: The i/p layer containing input unit is applied with identity activation fn: i.e $x_i = s_i$

Step 4: Calculate the o/p of the network.

To calculate the o/p first obtain net i/p.

$$y_{in} = \sum_{i=1}^{n} x_i w_i + bias$$

Then apply activation fn. over net i/p & calculate the o/p.

i.e $y = f(y_{in}) = \begin{cases} 1 & y_{in} > \theta \\ 0 & -\theta \leq y_{in} \leq \theta \\ -1 & y_{n} < -\theta \end{cases}$

Step 5: Weight & bias adjustment:

Compare the value of the actual o/p (calculated) and target o/p (desired)

If $y \neq t$ then

$W_{inew} = W_{iold} + \alpha t x_i$ $\quad$ $W_{inew} - W_{iold} = \alpha t x_i = \Delta w_i$

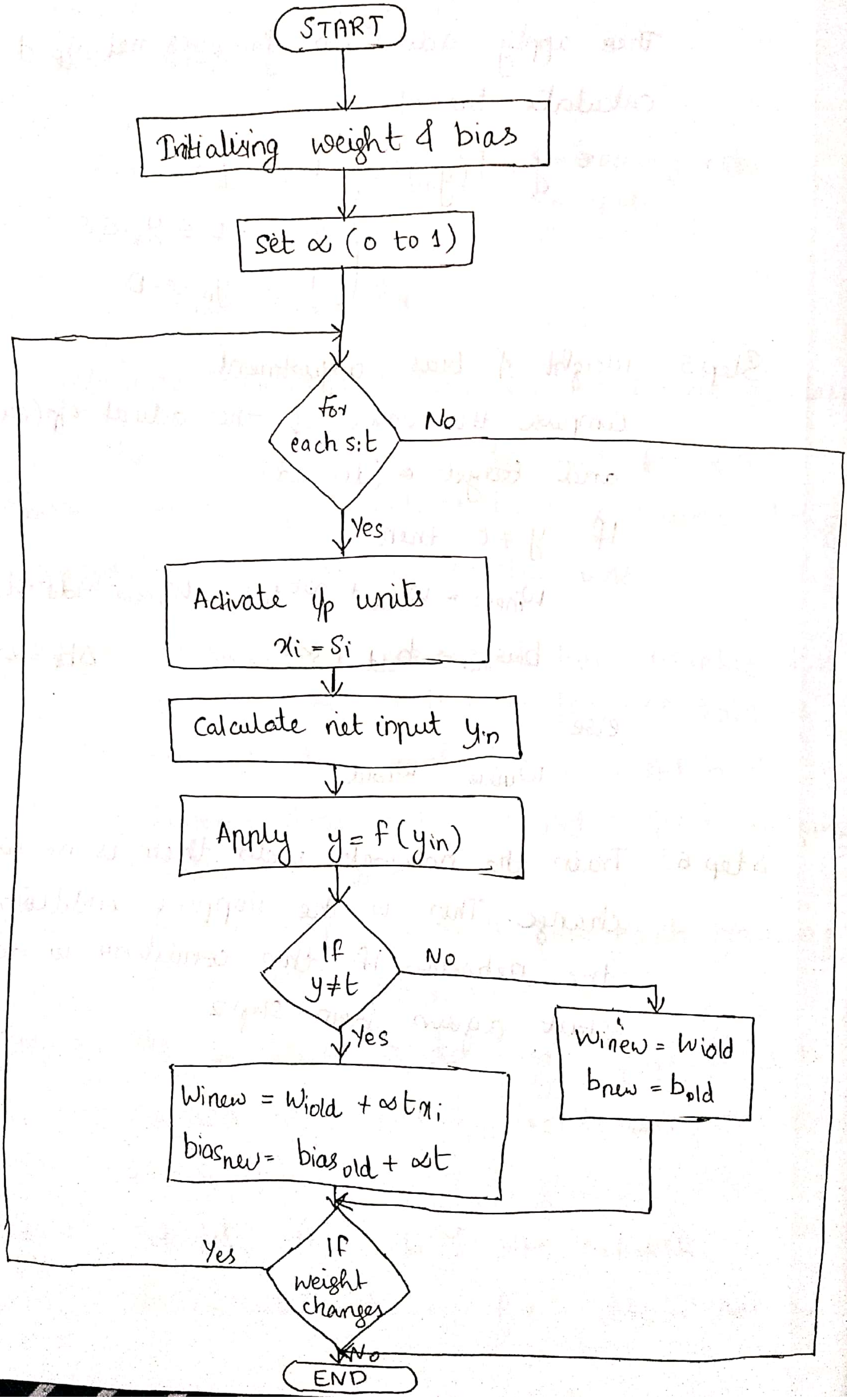
$bias_{new} = b_{old} + \alpha t$ $\quad$ $\Delta b = \alpha t$

else:

$W_{inew} = W_{iold}$

Step 6: Train the network until there is no weight change. This is the stopping condition for the network. If this condition is not met, start again from step 2

FLOWCHART FOR PERCEPTRON NETWORK [single o/p]



PERCEPTRON TESTING ALGORITHM

Step 0 : The initial weights to be used here are taken from training algorithm (The final weights obtained during training)

Step 1 : For each input vector x, perform steps 2 to 3.

Step 2 : Set Activation of the i/p unit.

Step 3 : Obtain response of o/p unit.

$$y_{in} = b + \sum_{i=1}^{n} x_i w_i$$

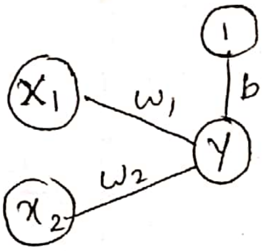
$$y = f(y_{in}) = \begin{cases} 1 & y_{in} > \theta \\ 0 & -\theta \leq y_{in} \leq \theta \\ -1 & y_{in} < -\theta \end{cases}$$

MODULE 2

Q. Implement AND function using perceptron network for bipolar i/ps & targets. Assume ~~the~~ θ is 0.

Epoch 1

x_1	x_2	t	y_{in}	y	$\Delta w_1 = \alpha t x_1$	$\Delta w_2 = \alpha t x_2$	$\Delta b = \alpha t$	w_1 (0)	w_2 (0)	b (0)
1	1	1	0	0	1	1	1	1	1	1
-1	-1	-1	1	1	-1	1	-1	0	2	0
-1	+1	-1	2	1	1	-1	-1	1	1	-1
-1	-1	-1	-3	-1	0	0	0	1	1	-1



init: $w_1 = 0, w_2 = 0, b = 0, \alpha = 1$, x

$x_1 = 1, x_2 = 1$

$$y_{in} = \sum_{i=1}^{n} x_i w_i + b$$

$$y_{in_1} = x_1 \cdot w_1 + x_2 w_2 + b = 1 \times 0 + 1 \times 0 + 0 = 0$$

$$y = f(y_{in}) = \begin{cases} 1, & y_{in} > \theta \\ 0, & -\theta \le y_{in} \le \theta \\ -1, & y_{in} < -\theta \end{cases}$$

$$f(y_{in}) = \begin{cases} 1, & y_{in} > 0 \\ 0, & y_{in} = 0 \\ -1, & y_{in} < 0 \end{cases}$$

$\underline{y = 0}$

$y_{in_2} = 1 \times 1 + -1 \times 1 + 1 = 1$

$\therefore y = 1$

$y_{in_3} = -1 \times 0 + 1 \times 2 + 0 = 2$

$y = 1$

$y_{in_4} = -1 \times 1 + -1 \times 1 + -1 = -3$

$y = -1$

Epoch 2

x_1	x_2	t	y_{in}	y	Δw_1 $\alpha t x_1$	Δw_2 $\alpha t x_2$	Δb αt	w_1	w_2	b
								1	1	-1
1	1	1	1	1	0	0	0	1	1	-1
1	-1	-1	-1	-1	0	0	0	1	1	-1
-1	1	-1	-1	-1	0	0	0	1	1	-1
-1	-1	-1	-3	-1	0	0	0	1	1	-1

$y_{in_5} = x_1 w_1 + x_2 w_2 + b = 1\times1 + 1\times1 + -1$

$y = 1$

$y_{in_6} = 1\times-1 + 1\times1 - 1 = -1$

$y = -1$

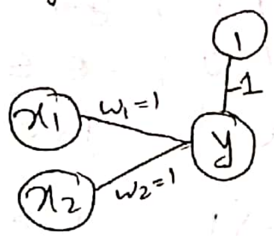
$y_{in_7} = -1\times1 + 1\times1 + -1 = -1$

$y = -1$

$y_{in_8} = -1\times1 + -1\times1 - 1 = -3$

$y = -1$

Final weights : $w_1 = 1$, $w_2 = 1$, $b = -1$



When all the 4 i/p patterns are presented then 1 Epoch is completed.

Testing.

$w_1 = 1 \quad w_2 = 1 \quad b = -1$

$y_{in_1} = 1\times1 + 1\times1 + -1 = 1$

$y = 1 = t$ ✓

$y_{in_2} = 1\times1 + -1\times1 + -1 = -1$

$y = -1 = t$ ✓

$y_{in_3} = -1\times1 + 1\times1 - 1 = -1$

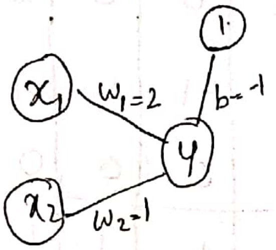
$y = -1 = t$ ✓

$y_{in_4} = -1\times1 + -1\times1 - 1 = -3$

$y = -1 = t$ ✓

Q2. Implement OR fn with binary i/p & bipolar target using perceptron training algorithm upto 3 epochs. Assume value of θ as 0.2.

x_1	x_2	t	y_{in}	y	Δw_1 $\alpha t x_1$	Δw_2 $\alpha t x_2$	Δb αt	w_1 0	w_2 0	b 0
1	1	1	0	0	1	1	1	1	1	1
1	0	1	2	1	0	0	0	1	1	1
0	1	1	2	1	0	0	0	1	1	1
0	0	-1	1	1	0	0	-1	1	1	0
1	1	1	2	1	0	0	0	1	1	0
1	0	1	1	1	0	0	0	1	1	0
0	1	1	1	1	0	0	0	1	1	0
0	0	-1	0	0	0	0	-1	1	1	-1
1	1	1	2	1	0	0	0	1	1	-1
1	0	1	0	0	1	0	1	2	1	0
0	1	1	1	1	0	0	0	2	1	0
0	0	-1	0	0	0	0	-1	2	1	-1



$$y = f(y_{in}) = \begin{cases} 1 & y_{in} > 0.2 \\ 0 & -0.2 \le y_{in} \le 0.2 \\ -1 & y_{in} < -0.2 \end{cases}$$

$\alpha = 1$

$y_{in} = w_1 x_1 + w_2 x_2 + b$

$y_{in1} = 0$
$y = 0$

$y_{in2} = 1\times1 + 1\times0 + 1 = 2$
$y = 1$

$y_{in3} = 0\times1 + 1\times1 + 1 = 2$
$y = 1$

$y_{in4} = 0 + 0 + 1 = 1$
$y = 1$

$y_{in5} = 1\times1 + 1\times1 + 0 = 2$
$y = 1$

$y_{in6} = 1\times1 + 0 + 0 = 1$

$y_{in7} = 0 + 1 + 0 = 1$

$y_{in8} = 0 + 0 + 0$

$y_{in9} = 1\times1 + 1\times1 + -1 = 2$

$y_{in10} = 1\times1 + 0 - 1 = 0$

$y_{in11} = 0\times2 + 1\times1 + 0 = 3$

$y_{in12} = 0 + 0 + 0 = 0$

Final weights: $w_1 = 2$
$w_2 = 1$
$b = -1$

Find the weight required to perform the following classification using perceptron network. The vectors (1,1,1,1) & (-1,1,-1,-1) are belonging to the class so have target value 1. Vectors (1,1,1,-1) & (1,-1,-1,1) are not belonging to the class so have target value -1. Assume learning rate is 1. & $\theta = 0.2$

~~x_1~~	x_2	x_3	x_4	t	Δw_1	~~Δw_2~~	~~Δw_3~~	~~Δw_4~~	y	y_{in}	~~w_1~~	~~w_2~~	~~b~~
1	1	1	1	1									
-1	1	-1	-1	1									
1	1	1	-1	-1									
1	-1	-1	1	-1									

y_{in}	y	Δw_1 $\alpha t x_1$	Δw_2 $\alpha t x_2$	Δw_3 $\alpha t x_3$	Δw_4 $\alpha t x_4$	Δb αt	w_1 0	w_2 0	w_3 0	w_4 0	b 0
0	0	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	1	-1	-1	1	0	2	0	0	2
4	1	-1	-1	-1	1	-1	-1	1	-1	1	1
1	1	-1	1	1	-1	-1	-2	2	0	0	0
0	0	1	1	1	1	1	-1	3	1	1	1
3	1	0	0	0	0	0	-1	3	1	1	1
3	1	-1	-1	-1	1	-1	-2	2	0	2	0
-2	-1	0	0	0	0	0	-2	2	0	2	0
2	1	0	0	0	0	0	-2	2	0	2	0
2	1	0	0	0	0	0	-2	2	0	2	0
-2	-1	0	0	0	0	0	-2	2	0	2	0
-2	-1	0	0	0	0	0	-2	2	0	2	0

$$y = f(y_{in}) = \begin{cases} 1 & y_{in} > 0.2 \\ 0 & -0.2 \le y_{in} \le 0.2 \\ -1 & y_{in} < -0.2 \end{cases} \qquad \alpha = 1$$

$\cancel{y_{in_1} =}$ $y_{in} = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + b$

$y_{in_1} = 0$

$y = 0$

$y_{in_2} = \cancel{4}\ -1 + 1 + -1 + -1 + 1 = -1$

$y = -1$

$y_{in3} = 0 + 2 + 0 + 0 + 2 = 4$

$y = \cancel{4}\ 1$

$y_{in4} = -1 + -1 + 1 + 1 + 1 = 1$

$y = 1$

$y_{in5} = -2 + 2 + 0 + 0 + 0 = \cancel{4}\ 0$

$y = \cancel{8}\ 0$

$y_{in6} = 1 \times 3 - 1 - 1 + 1 = 3$

$y = 1$

$y_{in7} = -1 + 3 + 1 - 1 + 1 = 3$

$y = 1$

$y_{in8} = -2 + -2 + 0 + 2 + 0 = -2$

$y = -1$

$y_{in9} = -2 + 2 + 0 + 2 + 0 = 2$

$y = 1$

$y_{in10} = 2 + 2 + 0 + -2 + 0 = 2$

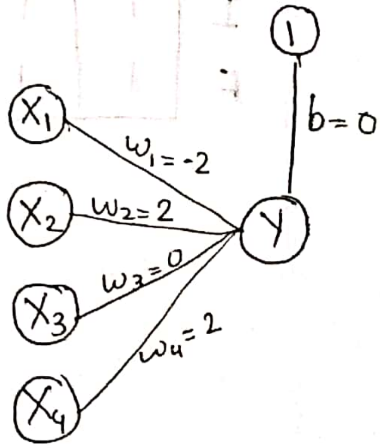
$y = 1$

$y_{in11} = -2 + 2 + 0 + -2 + 0 = -2$

$y = -1$

$y_{in12} = -2 - 2 + 0 + 2 + 0 = -2$

$y = -1$



Classify 2-D i/p pattern shown in the fig using Perceptron n/w. The symbol * indicate data to be 1 & · → −1. The patterns & ~~I~~ are I, F. For pattern I, target is +1 & for F target is −1.

x_1 *	x_2 *	x_3 *
x_4 ·	x_5 *	x_6 ·
x_7 *	x_8 *	x_9 *

I

*	*	*
*	*	*
*	·	·

F

$\alpha = 1$

$\theta = 0$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	y_{in}	y	t
I	1	1	1	−1	1	−1	1	1	1			1
F	1	1	1	1	1	1	1	−1	−1			−1