MODULE - 2

F.A with o/p

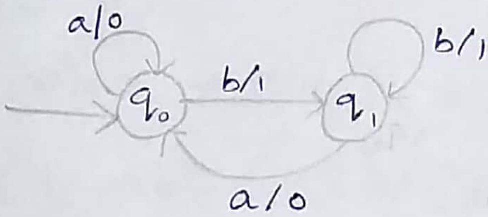
Moore and Mealy machine.

$M = \{Q, q_0, \delta, \Sigma, \lambda, \Delta\}$

$\lambda \rightarrow$ fn which gives o/p. $\Delta \rightarrow$ o/p symbol.

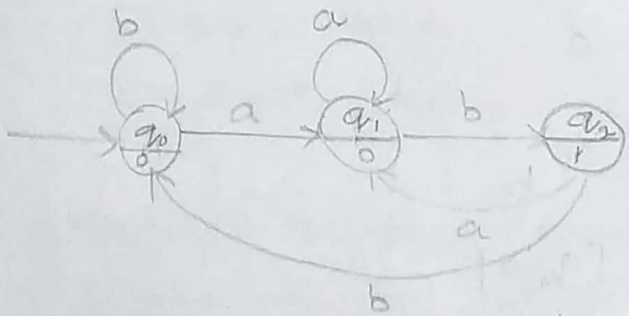
Moore :- $\lambda : Q \longrightarrow \Delta$

Mealy :- $\lambda : Q \times \Sigma \longrightarrow \Delta$



Q 1. Design a moore machine to count the occurance of substring 'ab' over $\{a, b\}$.

DFA :-



$\delta(q_0, abaab)$

$M = \{\{q_0, q_1, q_2\}, \{q_0\}, \delta, \{a, b\}, \lambda, \Delta\}$

$M = \{\{q_0, q_1, q_2\}, \{q_0\}, \delta, \{a, b\}, \lambda, \{0, 1\}\}$

a b a a b

$q_0 \; q_1 \; q_2 \; q_1 \; q_1 \; q_2$

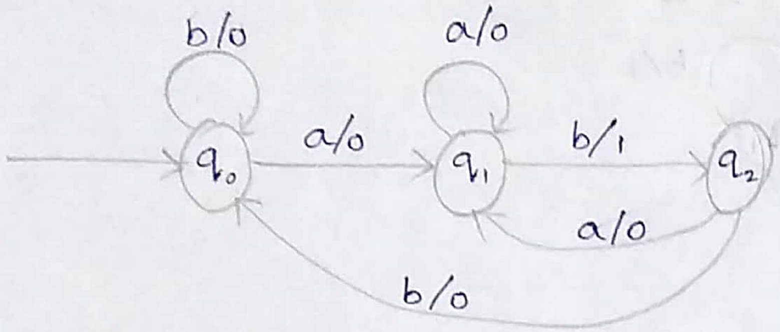
$\downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow$

0 0 1 0 0 1

two 1's are occuring. So, two times 'ab' is occuring in the string.

$Q \setminus \Sigma\ \Delta$	a	b	Δ
q_0	q_1	q_0	0
q_1	q_1	q_2	0
q_2	q_1	q_0	1

Q2. Design a mealy machine to count the occurence of substring 'ab' over $\{a, b\}$



a b a a b a ϵ

q_0 q_1 q_2 q_1 q_1 q_2 q_1

0 1 0 0 1 0

$Q \setminus \Sigma\ \lambda$	a	b
q_0	$[q_1, 0]$	$[q_0, 0]$
q_1	$[q_1, 0]$	$[q_2, 1]$
q_2	$[q_1, 0]$	$[q_0, 0]$

FA without o/p

1. DFA :-

$\delta : Q \times \Sigma \rightarrow Q$

$\delta(q_0, xa) = \delta(\delta(q_0, x), a)$

2. NFA :-

$\delta : Q \times \Sigma \rightarrow P(Q)$

$\delta(q_0, xa) = \delta(\delta(q_0, x), a)$

3. NFA - ϵ :-

$Q \times \Sigma \rightarrow P(Q)$ & $Q \times \epsilon \rightarrow P(Q)$

$\delta(q_0, xa) = \epsilon\text{-}clos(\delta(\epsilon\text{-}clos(q_0), x), a)$

FA with o/p

1. Moore :- $\delta : Q \times \Sigma \rightarrow Q$

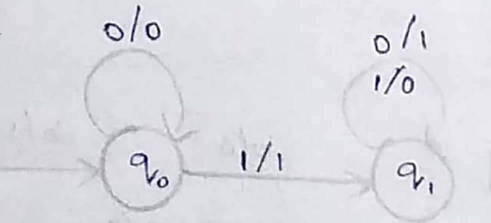
$\lambda : Q \rightarrow \Delta$

2. Mealy :- $\delta : Q \times \Sigma \rightarrow Q$

$\lambda : Q \times \Sigma \rightarrow \Delta$

Q2. Draw a mealy machine to compute 2's complement of binary numbers & convert to moore.

mealy:-



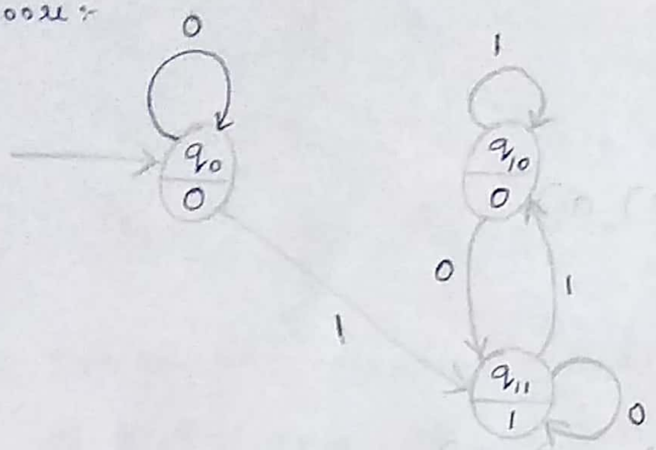
$Q \backslash \Sigma \lambda$	0	1
q_0	$[q_0, 0]$	$[q_1, 1]$
q_1	$[q_1, 1]$	$[q_1, 0]$

i/p associated with $q_0 = 0$

" with $q_1 = 1, 0$

$q_0 = 0$; $q_1 = 1, 0$

moore :-



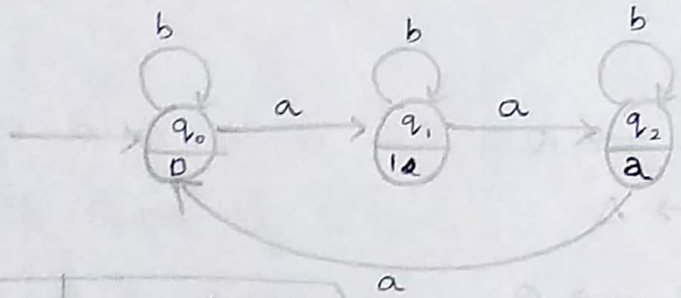
	0	1	Δ
q_0	q_0	q_{11}	0
q_{10}	q_{11}	q_{10}	0
q_{11}	q_{11}	q_{10}	1

$M = \{ \{q_0, q_{10}, q_{11}\},$

Q3. Design moore machine to find no. of a's mod 3 & convert to mealy over $\{a, b\}$

Mod3
0
1
2

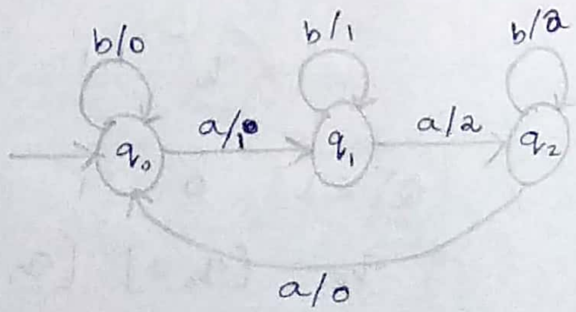
	mod3
0 a	0
1 a	1
2 a	2
3 a	0
4 a	1
5 a	2



	a	b	Δ
q_0	q_1	q_0	0
q_1	q_2	q_1	1
q_2	q_0	q_2	2

mealy :-

Q Σ λ	a	b
q_0	$[q_1, 1]$	$[q_0, 0]$
q_1	$[q_2, 2]$	$[q_1, 1]$
q_2	$[q_0, 0]$	$[q_2, 2]$



$M = \{ \{q_0, q_1, q_2\}, \{q_0\}, \delta, \{a, b\}, \lambda, \{0, 1, 2\} \}$

moore :-

eg :- a a b b a a

q_0 q_1 q_2 q_2 q_2 q_0 q_1

0 1 2 2 2 0 1

mealy :- a a b b a a

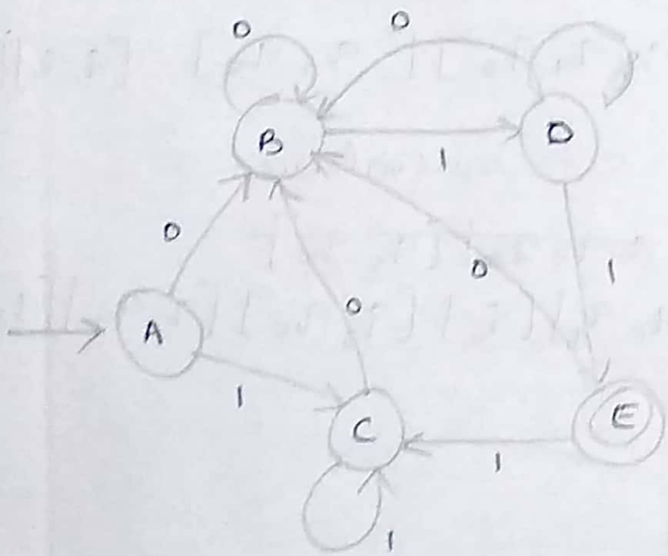
q_0 q_1 q_2 q_2 q_2 q_0 q_1

1 2 2 2 0 1

Minimization of DFA

Reducing / finding the equivalent DFA without changing the logic.
[Reduce the no of states]

Q1.



	0	1
A	B	C
B	B	D
C	B	C
D	B	E
E	B	C

0 equivalence

[A B C D] [E]

'1' equivalence

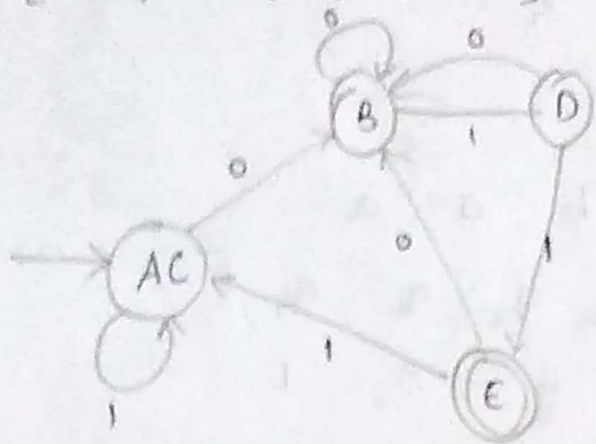
[A B C] [D] [E]

'2' equivalence

[A C] [B] [D] [E]

3 equivalence

[AC] [B] [D] [E]



Q2.

	0	1
→ q_0	q_1	q_5
q_1	q_6	q_2
* q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

0 equivalence

$[q_0\ q_1\ q_3\ q_4\ q_5\ q_6\ q_7]\ [q_2]$

1 equivalence

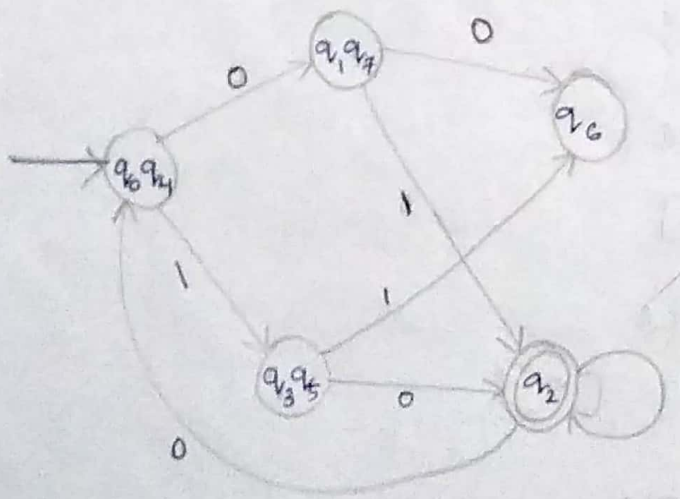
$[q_0\ q_4\ q_6]\ [q_3\ q_5]\ [q_1\ q_7][q_2]$

2 equivalence

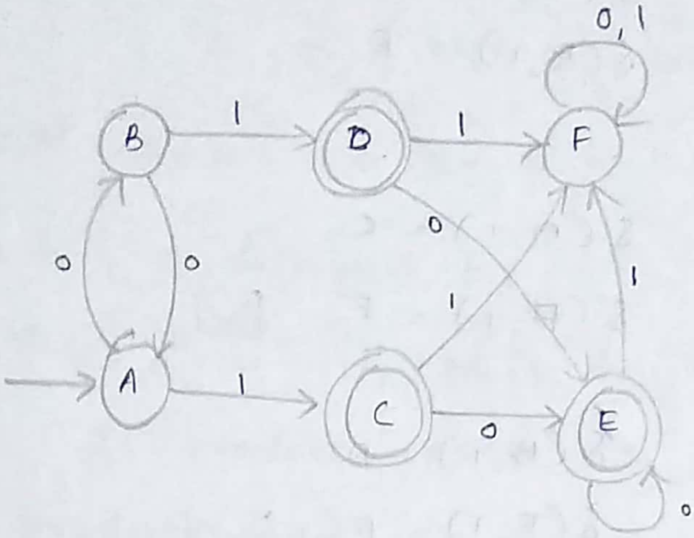
~~$[q_0\ q_4\ q_6]\ [q_3\ q_5]$~~

$[q_0\ q_4][q_6]\ [q_3\ q_5]\ [q_1, q_7][q_2]$

it cannot be separated



Myhill Nerode Theorem for minimization



	A	B	C	D	E	F
A						
B	1					
C	X	X				
D	X	X	2			
E	X	X	3	4		
F	5	6	X	X	X	

~~AB~~, AC, AD, AE, ~~AF~~
BC, BD, ~~BE~~, ~~BF~~
~~CD~~, ~~CE~~, CF
~~DE~~, DF
EF

✓ → X

<u>AB</u>

$\delta(A, 0) = B$ $\quad\quad$ $\delta(A, 1) = C$

$\delta(B, 0) = A$ $\quad\quad$ $\delta(B, 1) = D$

<u>CD</u>

$\delta(C, 0) = E$ $\quad\quad$ $\delta(C, 1) = F$

$\delta(D, 0) = E$ $\quad\quad$ $\delta(D, 1) = F$

It cannot (never) be mark CD

& also AB cannot mark.

<u>CE</u>

$\delta(C, 0) = E$ $\quad\quad$ $\delta(C, 1) = F$

$\delta(E, 0) = E$ $\quad\quad$ $\delta(E, 1) = F$

DE

$\delta(D,0) = E$ $\quad$ $\delta(D,1) = F$

$\delta(E,0) = E$ $\quad$ $\delta(E,1) = F$

AF

$\delta(A,0) = B$ $\quad$ $\delta(A,1) = C$

$\delta(F,0) = F$ $\quad$ $\delta(F,1) = F$ $\boxtimes$

BF

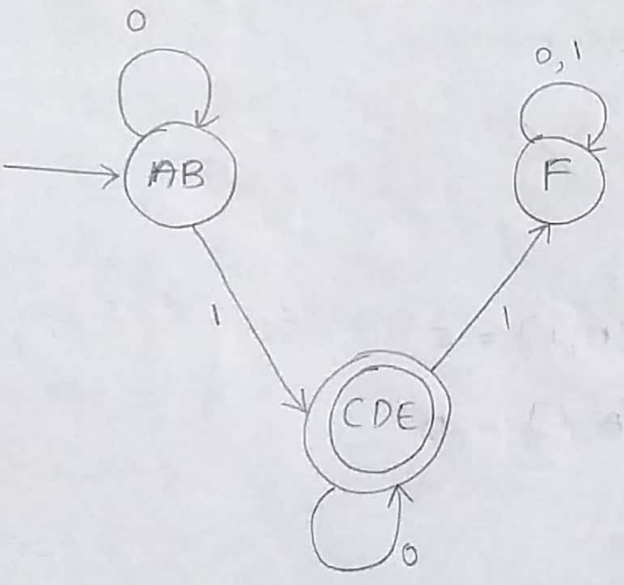
$\delta(B,0) = A$ $\quad$ $\delta(B,1) = D$

$\delta(F,0) = F$ $\quad$ $\delta(F,1) = F$ $\boxtimes$

AB $\quad$ CD $\rightarrow$ DE

CE

$\therefore$ AB & CDE & F $\longrightarrow$ states.



Regular expression

Representation of Σ

ϵ - string is empty

ϕ - whole string is" language

$\{a\} \longrightarrow a$

$\{a,b\} \longrightarrow a+b$

- $\Sigma^* = \Sigma^0$ to Σ^α ; $\Sigma^+ = \Sigma^1$ to Σ^α.

$(a+b)^* = (a+b)^0 . (a+b)^1 , (a+b)^2 . . . . . (a+b)^n$

$= \epsilon , a , b , aa , ab , ba , bb . . . . .$

$(a+b)^+ =$ a or b or combination of a & b.

$a \cdot b$ = a followed b.

$(11)^* = \epsilon , 11 , 1111 , 111111 . . . . .$ ($(11)^0, (11)^1, (11)^2, (11)^3 \ldots (11)^\alpha$)

∴ It produce even no of 1's.

→ To produce odd no of 1's.

$(11)^* . 1$

- $(a+b)^*$ = any combination of a & b including ϵ
- $(a+b)^* . abb$ = any combination of a & b ending with abb.
- $(a+b)^1 (a+b)^*$ = It should be start with either a or b and followed by any combination of a&b.

  $= (a+b)^+$
- $ab (a+b)^*$ = ab followed by any combinations of a & b.
- any no of a's followed by b's (any) followed by c's (any) = $a^* . b^* . c^*$

Q1. Find string of a's & b's of even length.

$((a+b).(a+b)+\epsilon)^*$ ϵ, aa, ab, ba, bb - -

$= ((a+b)^2+\epsilon)^*$ ∴ $(aa, ab, ba, bb+\epsilon)^*$

Q2. String over {a, b, c} with atleast 1 a & atleast one b.

$((a^*+b^*+c^*) a (a^*+b^*+c^*) b (a^*+b^*+c^*)) +$

$((a^*+b^*+c^*) b (a^*+b^*+c^*) a (a^*+b^*+c^*))$

3. String over $\{0,1\}$ with 10^{th} symbol from the right is 1.

4. $L = \{a^n b^m \mid n \geq 4, m \leq 3\}$.

$$a^4 . a^* (\epsilon + b + bb + bbb) = a^4 . a^* (\epsilon + b + b^2 + b^3)$$

5. $\{0, 1, 2\} = 0 + 1 + 2$

$(0+1)^1 = 0$ or 1

$(0+1)^2 = (0+1)(0+1)$

$= 0(0+1) + 1(0+1)$

$= 00 + 01 + 10 + 11$

6. $\{\epsilon, ab\} = \epsilon + ab$

7. $\{abb, a, b, bba\} = abb + a + b + bba$

8. $\{\epsilon, 0, 00, 000 \ldots\} = 0^*$

9. $\{1, 11, 111, 1111 \ldots\} = 1^+$

10. 10^{th} symbol from right is 1. $\leftarrow$ $1\overbrace{\longrightarrow}^{9}$

$$1.(1+0)^9$$

11. $\{a^{2n} b^{2n+1} \mid n \geq 0\}$

$$(a^2)^* (bb)^* . b = (a^2)^* . (b^2)^* b$$

$$= (a^2 b^2)^* . b$$

Identities

- $\phi + R = R$
- $\phi R + R\phi = \phi$
- $\epsilon R = R\epsilon = R$
- $\phi^* = \epsilon$ & $\epsilon^* = \epsilon$
- $R + R = R$
- $R^* . R^* = R^*$
- $R^* . R = R . R^* = R^+$
- $(R^*)^* = R^*$
- $\epsilon + RR^* = R^*$
- $(PQ)^* P = P(QP)^*$
- $(P+Q)^* = (P^* + Q^*)^* = (P^* . Q^*)^*$
- $(P+Q)R = PR + QR$

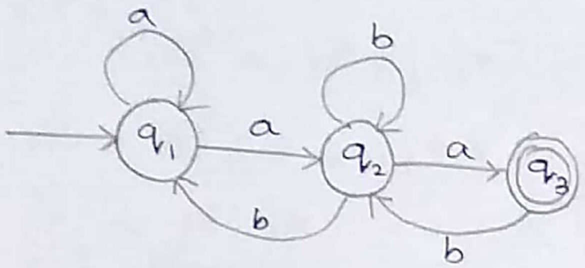
Arden's theorem

If P, Q are regular expression (RE), over I & if P doesnot have ϵ.

then, following eqn in R is given by,

$\boxed{R = Q + RP}$ has a solution $\boxed{R = QP^*}$

Q.



$$q_3 = q_2 a \text{ ——— } (1)$$

$$q_2 = q_1 a + q_2 b + q_3 b \text{ ——— } (2)$$

$$q_1 = \epsilon + q_1 a + q_2 b \text{ ——— } (3)$$

(2) in (1),

$$q_3 = (q_1 a + q_2 b + q_3 b) a$$

$$q_3 = q_1 aa + q_2 ba + q_3 ba \text{ ——— } (4)$$

(1) in (2),

$$q_2 = q_1 a + q_2 b + (q_2 a) b$$

$$q_2 = q_1 a + q_2 (b + ab)$$

in the form $R = Q + RP$ $\therefore$ $R = QP^*$

$$q_2 = (q_1 a)(b + ab)^* \text{ ——— } (5)$$

(5) in (3),

$$q_1 = \epsilon + q_1 a + ((q_1 a)(b + ab)^*) b$$

$$q_1 = \epsilon + q_1 (a + a(b + ab)^* b)$$

$R = Q + R \; P$ $\therefore$ $R = QP^*$

$$q_1 = \epsilon \, (a + a(b + ab)^* b)^*$$

$$q_1 = (a + a(b + ab)^* b)^* \text{ ——— } (6)$$

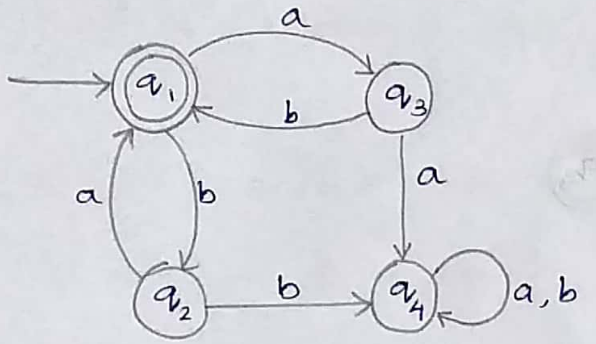
(6) in (5),

$$q_2 = (a + a(b+ab)^* b)^* a \, (b+ab)^* \quad \text{——} (7)$$

(7) in (1),

$$q_3 = (a + a(b+ab)^* b)^* \, a(b+ab)^* . a$$

Q2.



$$q_1 = \epsilon + q_2 a + q_3 b \quad \text{——} (1)$$

$$q_2 = q_1 b \quad \text{——} (2)$$

$$q_3 = q_1 a \quad \text{——} (3)$$

$$q_4 = q_2 b + q_3 a + q_4 a + q_4 b \quad \text{——} (4)$$

(2) & (3) in (1),

$$q_1 = \epsilon + q_1 ba + q_1 ab$$

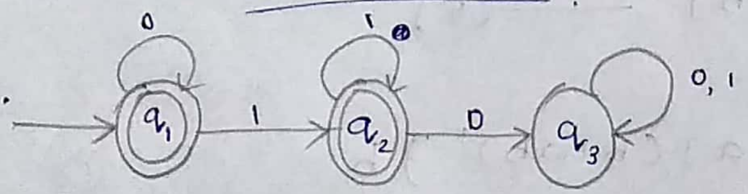
$$q_1 = \epsilon + q_1 (ba + ab)$$

$$R = Q + RP$$

$$q_1 = \epsilon . (ba + ab)^*$$

$$q_1 = (ba + ab)^*$$

Q3.



$$q_1 = \epsilon + q_1 0 \quad \text{——} (1)$$

$$R = Q + RP$$

$$q_2 = q_1 1 + q_2 1 \quad \text{——} (2)$$

$$q_3 = q_2 0 + q_3 0 + q_3 1 \quad \text{——} (3)$$

(1) $\rightarrow$ $q_1 = \epsilon . 0^* = 0^*$

$q_1 = 0^*$ ——— (4)

(4) in (2), $q_2 = 0^*.1 + q_2.1$

$R = Q + RP$

$q_2 = (0^*1).1^*$ ——— (5)

$R.E = q_1 + q_2$

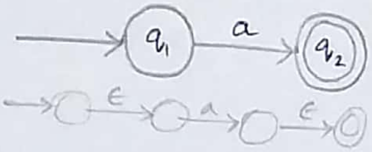
$= 0^* + (0^*.1).1^*$

$= 0^*(\epsilon + 1.1^*) = 0^*(\epsilon + 1^+)$

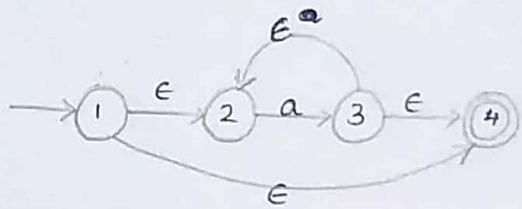
$= \underline{0^*.1^*}$

Thomson equivalence of NFA - ϵ

1) a

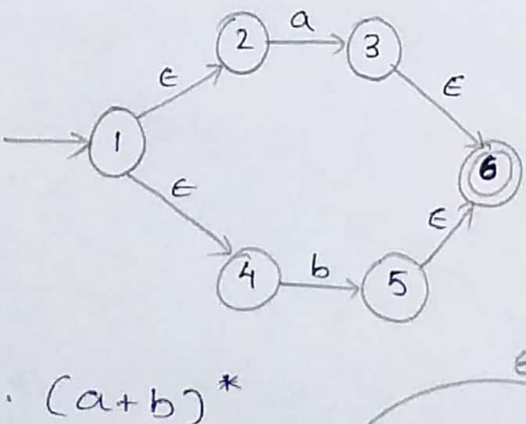


4) a^* - ϵ, a, aa - - -



2) ab

3) a+b



5) Language empty (ϕ)

6) ϵ

Q1. $(a+b)^*$