Module: 3

27/09/18

FUZZY LOGIC

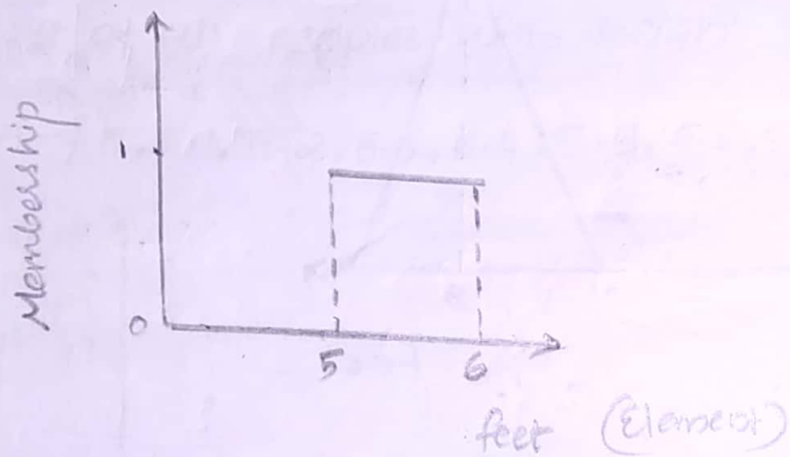
Fuzzy Logic (FL) are represented using fuzzy set. Fuzzy sets ~~are~~ provides a mathematical way to represent a and

Classic set (Regular/Normal/Crisp set) & Fuzzy Set

- Classic set

Eg: Set of all peoples with height b/w 5 to 6 feet.

$A = \{5, 5.1, 5.2, 5.3 \ldots \ldots 6\}$

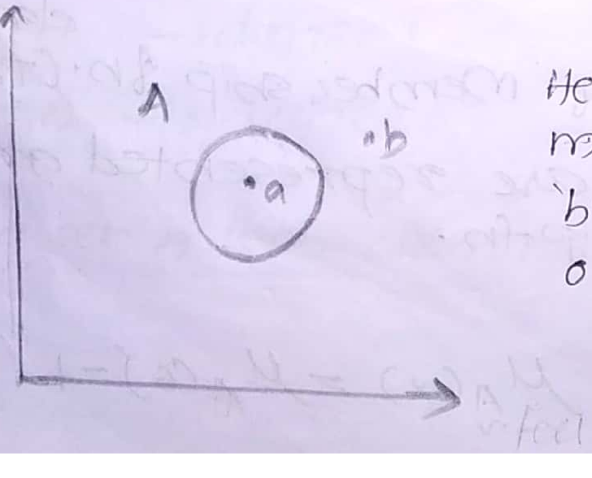


* For a crisp set, an element in Universe 'x' either a member of the set or not a member of the set.

ie, $x \in A \longrightarrow$ degree 1

$x \notin A \longrightarrow$ degree 0

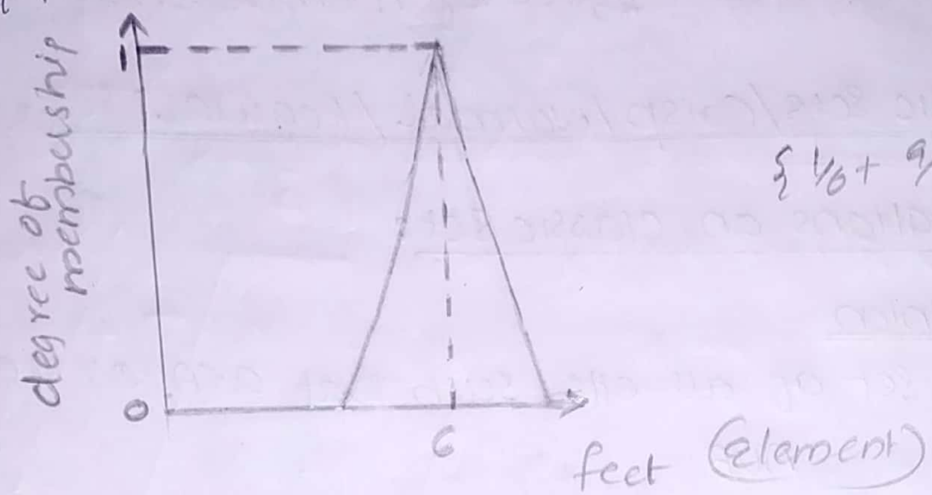
* Consider a set A with elts 'a' & 'b'



Here, 'a' have membership 1 and 'b' have membership 0.

• Fuzzy Set

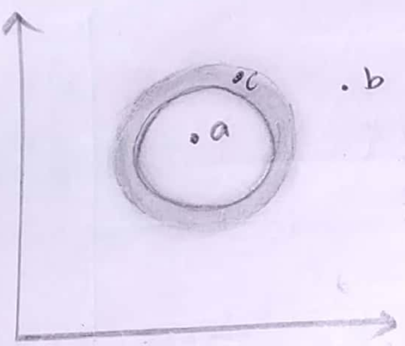
* Consider a fuzzy set $\underset{\sim}{A}$ (A tild) ie, height near 6 feet.



$\{ 1/6 + 9/5.9 + 8/5.8 \}$..

* Fuzzy set doesnot have boundary.

* Consider a set with elts. a, b & c.



degree of a = 1
degree of b = 0

C is partialy belongs to the given set. C has partial membership.

∴ degree of a = 0 & 1

Properties of Membership function (Triangular)

Fuzzy sets are represented graphically as membership function.

i) Normality

μ – degree of membership

$\mu_{\underset{\sim}{A}}(x) =$

In pre. graph, $\mu_{\underset{\sim}{A}}(6) = 1$

ii) Monotonocity

The closer the height is to 6, the closer the degree of membership to 1.

iii) Symmetry

The number equi distance from 6 should have a same degree of membership.

Classic Sets/Crisp/Normal/Regular set

Operations on classic set:

i) Union

set of all elts such that $a \in A$ or $a \in B$

$A \cup B$

ii) Intersection

$a \in A$ and $a \in B$

iii) Complement

A'

iv) Difference

$A - B$

Properties:

i) Commutative

$A \cup B = B \cup A$

$A \cap B = B \cap A$

ii) Associative

$(A \cup B) \cup C = \cancel{(A \cup B)}\, A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

iii) Distributive

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

iv) idempotent

$A \cup A = A$

$A \cap A = A$

v) Transitivity

If $A \subseteq B$ & $\underset{\sim}{B} \subset \underset{\sim}{C}$, then $\underset{\sim}{A} \subset \underset{\sim}{C}$

vi) identity

$A \cup \phi = A$

$A \cap \phi = \phi$

$A \cup X = X$

$A \cap X = A$

vii) Double negation / envolution

$\overline{(\overline{A})} = A$

viii) Demorgan's law

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

ix) Law of excluded middle

$A \cup A' = X$

x) Law of contradiction

$A \cap A' = \phi$

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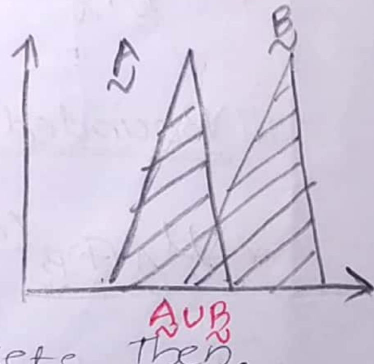
Operations on fuzzy set



i) Union

Suppose $\underset{\sim}{A}$ & $\underset{\sim}{B}$ are 2 fuzzy sets then,

* $\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \max\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$

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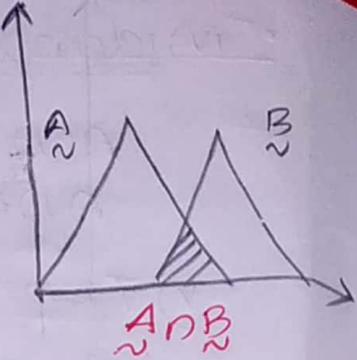
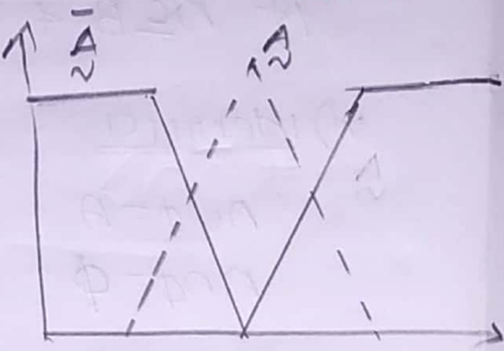
ii) Intersection

* $\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$



iii) complement

* $\mu_{\bar{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$



iv) difference:

* $\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$

$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}}$

More operations on fuzzy set

i) Algebric Sum

* $\mu_{\underset{\sim}{A} + \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x) * \mu_{\underset{\sim}{B}}(x)$

ii) Algebric Product

* $\mu_{\underset{\sim}{A} \cdot \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$

iii) Bounded Sum

* $\mu_{\underset{\sim}{A} \oplus \underset{\sim}{B}}(x) = \min\left[1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)\right]$

iv) Bounded difference

* $\mu_{\underset{\sim}{A} \ominus \underset{\sim}{B}}(x) = \max\left[0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)\right]$

Properties of fuzzy set

i) Commutative

$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$$

$$\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

ii) Associative

$$(\tilde{A} \cup \tilde{B}) \cup \tilde{C} = \tilde{A} \cup (\tilde{B} \cup \tilde{C})$$

$$(\tilde{A} \cap \tilde{B}) \cap \tilde{C} = \tilde{A} \cap (\tilde{B} \cap \tilde{C})$$

iii) Distributive

* $\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$
* $\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$

iv) Idempotent

$$\tilde{A} \cup \tilde{A} = \tilde{A}$$

v) Transitive

If $\tilde{A} \subset \tilde{B}$ & $\tilde{B} \subset \tilde{C}$, then $\tilde{A} \subset \tilde{C}$

vi) Identity

$\tilde{A} \cup \phi = \tilde{A}$ $\qquad$ $\tilde{A} \cap X = \tilde{A}$

$\tilde{A} \cup X = X$

$\tilde{A} \cap \phi = \phi$

vii) Double negation

$$\overline{\overline{\tilde{A}}} = \tilde{A}$$

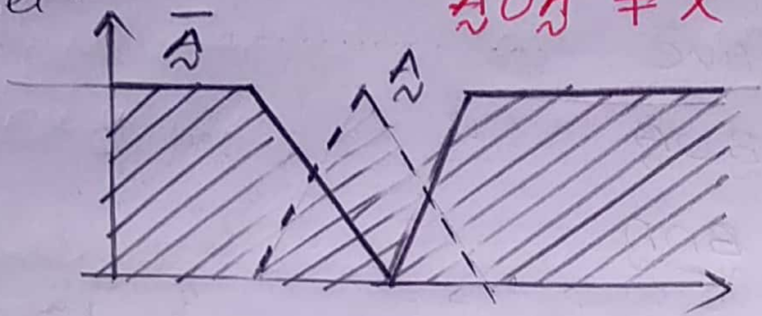
viii) Demorgan's law

$$\overline{\tilde{A} \cap \tilde{B}} = \overline{\tilde{A}} \cup \overline{\tilde{B}}$$

$$\overline{\tilde{A} \cup \tilde{B}} = \overline{\tilde{A}} \cap \overline{\tilde{B}}$$

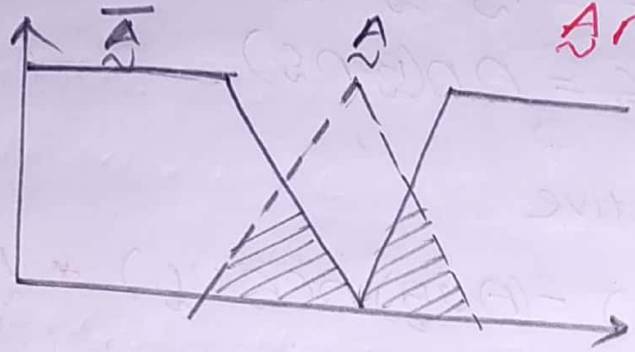
• Law of excluded middle is not applicable to fuzzy set.

$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X$



• Law of contradiction is also not applicable to FS.

$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \phi$



Fuzzy Set Representation

Fuzzy sets are set containing elts that have varying degree of membership. Fuzzy sets are representing using $\underset{\sim}{A}$.

$$\underset{\sim}{A} = \left\{ \frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{x_2} + \cdots\cdots + \frac{\mu_{\underset{\sim}{A}}(x_n)}{x_n} \right\}$$

'+' is not addition

$A = \{x_1, x_2, x_3 \cdots x_n\}$

$\boxed{0 \leq Nr \leq 1}$

Numerator Values

Fuzzy Relation & Normal Relations

Normal/Crisp/Classic Relations

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Consider 2 sets $X = \{2, 4, 6\}$, $Y = \{p, q, r\}$.

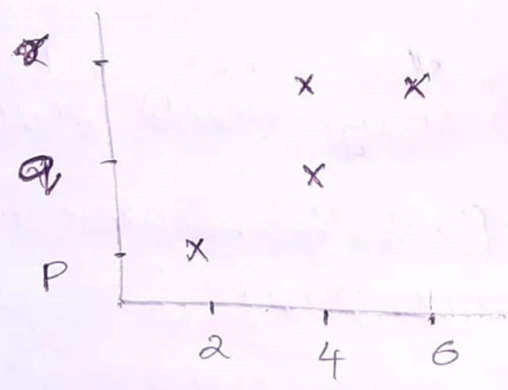
$X \times Y = \{(2,p), (2,q), (2,r), (4,p), (4,q), (4,r), (6,p), (6,q), (6,r)\}$.

Consider a relation $R = \{(2,p), (4,q), (4,r), (6,r)\}$

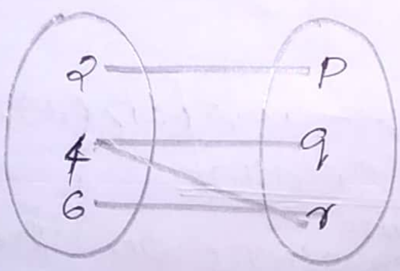
Now, represent R in the i) matrix form :

$$\begin{array}{c|ccc} & p & q & r \\ \hline 2 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 \\ 6 & 0 & 0 & 1 \end{array}$$

ii) Coordinate Representation :



iii) Mapping :



Fuzzy Relation

A fuzzy relation b/w a set 'x' & 'y' is called **Binary** fuzzy relation and it is denoted as:

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$$X=\{x_1, x_2 \ldots x_m\} \quad Y=\{y_1, y_2 \cdots y_n\}$$

$$\underset{\sim}{R}(\cancel{x}, X) = \mu_{\underset{\sim}{R}}(x_1, y_1), \mu_{\underset{\sim}{R}}(x_1, y_2$$

$$\underset{\sim}{R}(x,Y) = \begin{bmatrix} \mu_{\underset{\sim}{R}}(x_1,y_1) & \mu_{\underset{\sim}{R}}(x_1,y_2) & \cdots & \mu_{\underset{\sim}{R}}(x_1,y_n) \\ \mu_{\underset{\sim}{R}}(x_2,y_1) & \mu_{\underset{\sim}{R}}(x_2,y_2) & \cdots & \mu_{\underset{\sim}{R}}(x_2,y_n) \\ \vdots & & & \\ \mu_{\underset{\sim}{R}}(x_m,y_1) & \mu_{\underset{\sim}{R}}(x_m,y_2) & \cdots & \mu_{\underset{\sim}{R}}(x_m,y_n) \end{bmatrix}$$

→ Let R be a relation $x \to y$ as,

$$\underset{\sim}{R} = \frac{.2}{(x_1,y_3)} + \frac{.4}{(x_1,y_2)} + \frac{.1}{(x_2,y_2)} + \frac{.6}{(x_2,y_3)} + \frac{1}{(x_3,y_3)} + \frac{.5}{(x_3,y_1)}$$

$X=\{x_1, x_2, x_3\}$ & $Y=\{y_1, y_2, y_3\}$. Represent the above relation in the matrix form.

$$\begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0 & .4 & .1 \\ 0 & .1 & .6 \\ .5 & 0 & 1 \end{bmatrix} \end{array}$$

Operations on fuzzy relation

Let $\underset{\sim}{R}$ & $\underset{\sim}{S}$ be 2 fuzzy relations on the cartesian space on x&y then the diff. operations are:

i) Union

$$\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x,y) = \max[\mu_{\underset{\sim}{R}}(x,y); \mu_{\underset{\sim}{S}}(x,y)]$$

ii) Intersection

$$\mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x,y) = \text{Min}\left[\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(x,y)\right]$$

iii) Complement

$$\mu_{\bar{\underset{\sim}{R}}}(x,y) = 1 - \mu_{\underset{\sim}{R}}(x,y)$$

iv) Containment

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$$\underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x,y) \leq \mu_{\underset{\sim}{S}}(x,y)$$

v) Inverse

Inverse of a fuzzy relation R on $X \times Y$ is denoted by, R^{-1} ~~Rela~~.

It is a relation on $X \times Y$ defined by $R^{-1}(y,x) = R(x,y)$

For all pairs $\boxed{(y,x) \in Y \times X}$

vi) Projection

For a fuzzy relation, $R(x,y)$.

Projection on

Let $R \downarrow Y$ denotes the projection of R on to Y and it is denoted using $\mu_{(R \downarrow Y)}(x,y) = \max\limits_{x}\left[\mu_{\underset{\sim}{R}}(x,y)\right]$

Projection on x

$$\mu_{(R \downarrow X)}(x,y) = \max_{y}\left[\mu_{\underset{\sim}{R}}(x,y)\right]$$

Properties of FR

i) Commutative

ii) Associative

iii) Distributive

iv) Idempotent

v) Transitive

vi) Identity

vii) Double negation (Involution)

viii) Demorgan's law.

Same as fuzzy set

• 2 properties not statisfied by FR are law of excluded middle & law of contradiction. Because FS doesnot statisfies these 2 properties.

Cartesian Product of Relation

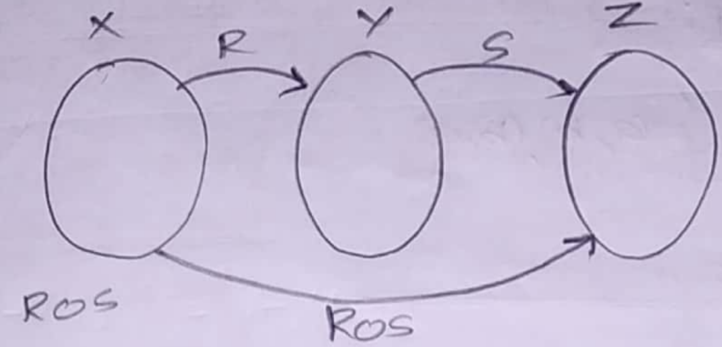
Let $\underset{\sim}{A}$ be a FS ~~on~~ on the universe x & $\underset{\sim}{B}$ be a FS on the universe Y. Then the cartesian pdt over $\underset{\sim}{A}$ & $\underset{\sim}{B}$ results in a FR $\underset{\sim}{R}$. Then the membership of the fuzzy relation is given by,

$$\mu_{\underset{\sim}{R}}(x,y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x,y) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)\right]$$

Fuzzy Composition

There are 3 types of Composition:

1. Max-min composition
2. Min-max
3. Max-Product



i) max-min composition

max-min composition of $R(x,y)$ & $S(y,z)$ is denoted by $R(x,y) \circ S(y,z)$ is defined by $T(x,z)$ as

$$\mu_{\underset{\sim}{T}}(x,z) = \max\{\min[\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(y,z)]\}$$

ii) min-max composition

Min-max composition of $R(x,y)$ & $S(y,z)$ is denoted by $R(x,y) \circ S(y,z)$ is denoted by $T(x,z)$ as

$$\mu_{\underset{\sim}{T}}(x,z) = \min\{\max[\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(y,z)]\}$$

iii) Max-product

$$\mu_{\underset{\sim}{T}}(x,z) = \max[\mu_{\underset{\sim}{R}}(x,y) \times \mu_{\underset{\sim}{S}}(y,z)]$$

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FUZZY TOLERANCE & EQUIVALENCE RELATION

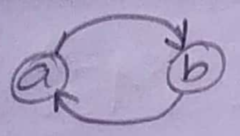
Normal equivalence & tolerance relation

A relation is said to be equivalence relation if it statisfies the below 3 properties:

i) Reflexive

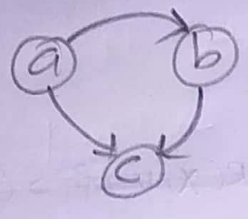
$\}$ (a,a)

ii) Symmetry:-



(a, b), (b, a)

iii) Transitive:



(a, b) (b, c) (a, c)

→ If the 1st 2 properties are only statisfied then it is called TOLERANCE Relation.

Fuzzy tolerance & equivalence Relation

i) Reflexive:

CONDITION:

$$\mu_{\underset{\sim}{R}}(x_i, x_i) = 1$$

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 1 & & \\ x_2 & & 1 & \\ x_3 & & & 1 \end{array}$$

* If the diagonal elts of a matrix are 1, then it is reflexive.

ii) Symmetry:

$$\mu_{\underset{\sim}{R}}(x_i, x_j) = \mu_{\underset{\sim}{R}}(x_j, x_i)$$

* if the transpose of the matrix is equal to the matrix, then it is symmetric $(A = A^T)$

iii) Transitivity:

$$\mu_{\underset{\sim}{R}}(x_i, x_j) = \lambda_1$$

$$\mu_{\underset{\sim}{R}}(x_j, x_k) = \lambda_2$$

then,

$$\mu_{\underset{\sim}{R}}(x_i, x_k) = \lambda \; ; \text{ where } \lambda \geq \min[\lambda_1, \lambda_2]$$

For a fuzzy relation to be equivalence, it must be statisfies the above 3 properties.

If a FR said to be tolerance, it must be statisfies the 1st 2 properties.

Module : 3

1) Find the power set and cardinality of a given set $X = \{2, 4, 6\}$. Also find the cardinality of the power set.

power set $= P(X) = \{\{\phi, \{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}, \{2,4,6\}\}$

cardinality of $X = 3$

Cardinality of $P(X) = 2^n = 2^3 = 8$

2) Consider 2 fuzzy sets are given below

$$\underset{\sim}{A} = \{ 1/2 + \frac{.3}{4} + \frac{.5}{6} + \frac{.2}{8} \}$$

$$\underset{\sim}{B} = \{ \frac{.5}{2} + \frac{.4}{4} + \frac{.1}{6} + \frac{1}{8} \}$$

Perform union, intersection, complement & difference.

<u>Union</u>

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \max [\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)]$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \{ \frac{1}{2} + \frac{.4}{4} + \frac{.5}{6} + \frac{1}{8} \}$$

<u>Intersection</u>

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \{ \frac{.5}{2} + \frac{.3}{4} + \frac{.1}{6} + \frac{.2}{8} \}$$

<u>Complement</u>

$$\overline{\underset{\sim}{A}} = \{ \frac{1-1}{2} + \frac{1-.3}{4} + \frac{1-.5}{6} + \frac{1-.2}{8} \}$$

$$= \{ \frac{0}{2} + \frac{.7}{4} + \frac{.5}{6} + \frac{.8}{8} \}$$

$$\overline{\underset{\sim}{B}} = \{ \frac{1-.5}{2} + \frac{1-.4}{4} + \frac{1-.1}{6} + \frac{1-1}{8} \}$$

$$= \{ \frac{.5}{2} + \frac{.6}{4} + \frac{.9}{6} + \frac{0}{8} \}$$

Difference

$$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$$
$$= \left\{ \frac{.5}{2} + \frac{.3}{4} + \frac{.5}{6} + \frac{0}{8} \right\}$$

$$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}}$$
$$= \left\{ \frac{0}{2} + \frac{.4}{4} + \frac{.1}{6} + \frac{.8}{8} \right\}$$

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(3) Given a FS $\underset{\sim}{B_1} = \left\{ \frac{1}{1.0} + \frac{.75}{1.5} + \frac{.3}{2.0} + \frac{.15}{2.5} + \frac{0}{3.0} \right\}$

$\underset{\sim}{B_2} = \left\{ \frac{1}{1.0} + \frac{.6}{1.5} + \frac{.2}{2.0} + \frac{.1}{2.5} + \frac{0}{3.0} \right\}$ find the following:

a) $\overline{\underset{\sim}{B_1} \cup \underset{\sim}{B_2}}$ b) $\overline{\underset{\sim}{B_1} \cap \underset{\sim}{B_2}}$ c) $\underset{\sim}{B_1} \cap \overline{\underset{\sim}{B_1}}$

d) $\underset{\sim}{B_2} \cup \overline{\underset{\sim}{B_2}}$

a) $$\underset{\sim}{B_1} \cup \underset{\sim}{B_2} = \left\{ \frac{1}{1.0} + \frac{.75}{1.5} + \frac{.3}{2.0} + \frac{.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\overline{\underset{\sim}{B_1} \cup \underset{\sim}{B_2}} = \left\{ \frac{0}{1.0} + \frac{.25}{1.5} + \frac{.7}{2.0} + \frac{.85}{2.5} + \frac{1}{3.0} \right\}$$

b) $$\underset{\sim}{B_1} \cap \underset{\sim}{B_2} = \left\{ \frac{1}{1.0} + \frac{.6}{1.5} + \frac{.2}{2.0} + \frac{.1}{2.5} + \frac{0}{3.0} \right\}$$

$$\overline{\underset{\sim}{B_1} \cap \underset{\sim}{B_2}} = \left\{ \frac{0}{1.0} + \frac{.4}{1.5} + \frac{.8}{2.0} + \frac{.9}{2.5} + \frac{1}{3.0} \right\}$$

c) $$\overline{\underset{\sim}{B_1}} = \left\{ \frac{0}{1.0} + \frac{.25}{1.5} + \frac{.7}{2.0} + \frac{.85}{2.5} + \frac{1}{3.0} \right\}$$

$$\underset{\sim}{B_1} \cap \overline{\underset{\sim}{B_1}} = \left\{ \frac{0}{1.0} + \frac{.25}{1.5} + \frac{.3}{2.0} + \frac{.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\bar{B_2} = \left\{\frac{.0}{1.0} + \frac{.4}{1.5} + \frac{.8}{2.0} + \frac{.9}{2.5} + \frac{1}{3.0}\right\}$$

$$B_2 \cup \bar{B_2} = \left\{\frac{1}{1.0} + \frac{.6}{1.5} + \frac{.8}{2.0} + \frac{.9}{2.5} + \frac{1}{3.0}\right\}$$

(4) consider 2 FS $A = \left\{\frac{.2}{1} + \frac{.3}{2} + \frac{.4}{3} + \frac{.5}{4}\right\}$

$B = \left\{\frac{.1}{1} + \frac{.2}{2} + \frac{.2}{3} + \frac{1}{4}\right\}$. Find the algebraic sum, algebric pdt, bounded sum & bounded ~~product~~ difference.

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x)$$

Algebric pdt

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

$.30 - .02 = 28$

$$\mu_{A \cdot B} = \left\{\frac{.02}{1} + \frac{.06}{2} + \frac{.08}{3} + \frac{.5}{4}\right\}$$

Algebric sum

$$\mu_{A+B} = \left\{\frac{.3 - .02}{1} + \frac{.5 - .06}{2} + \frac{.6 - .08}{3} + \frac{1.5 - .5}{4}\right\}$$

$$= \left\{\frac{.28}{1} + \frac{.44}{2} + \frac{.52}{3} + \frac{1}{4}\right\}$$

Bounded sum

$$\mu_{A \oplus B}(x) = \min\left\{1, \mu_A(x) + \mu_B(x)\right\}$$

Nr, greater than 1 = 1

$$\mu_A(x) + \mu_B(x) = \left\{\frac{.3}{1} + \frac{.5}{2} + \frac{.6}{3} + \frac{1.5}{4}\right\}$$

$$\mu_{A \oplus B}(x) = \min\left\{1, \frac{.3}{1} + \frac{.5}{2} + \frac{.6}{3} + \frac{1.5}{4}\right\}$$

$$= \left\{.3/_1 + .5/_2 + .6/_3 + 1/_4\right\}$$

Bounded difference

$$\mu_{\underset{\sim}{A} \ominus \underset{\sim}{B}}(x) = \max[0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)]$$

$$\mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x) = \left\{\frac{.1}{1} + \frac{.1}{2} + \frac{.2}{3} + \frac{-.5}{4}\right\}$$

Nr, less than 0 = 0

$$\mu_{\underset{\sim}{A} \ominus \underset{\sim}{B}}(x) = \max\left[0, \frac{.1}{1} + \frac{.1}{2} + \frac{.2}{3} + \frac{-.5}{4}\right]$$

$$= \left\{\frac{.1}{1} + \frac{.1}{2} + \frac{.2}{3} + \frac{0}{4}\right\}$$

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(5) Consider the following a FS $\underset{\sim}{A} = \left\{\frac{.3}{x_1} + \frac{.7}{x_2} + \frac{1}{x_3}\right\}$ $\underset{\sim}{B} = \left\{\frac{.4}{y_1} + \frac{.9}{y_2}\right\}$. Perform the cartessian pdt over these fs.

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & .3 & .3 \\ x_2 & .4 & .7 \\ x_3 & .4 & .9 \end{array}$$

(6) Two fuzzy relations are given by,

$$\underset{\sim}{R} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & .6 & .3 \\ x_2 & .2 & .9 \end{array} \quad \& \quad \underset{\sim}{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 1 & .5 & .3 \\ y_2 & .8 & .4 & .7 \end{array}$$

. obtained the FR $\underset{\sim}{T}$ as a composition b/w the FRs. Find max-min composition & max-pdt composition.

Max-min

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & .6 & .5 & .3 \\ x_2 & .8 & .4 & .7 \end{array}_{2\times 3}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_1) = \max\{\min[\mu_{\underset{\sim}{R}}(x_1, y_1), \mu_{\underset{\sim}{S}}(y_1, z_1)], \min[\mu_{\underset{\sim}{R}}(x_1, y_2), \mu_{\underset{\sim}{S}}(y_2, z_1)]\}$$

$$= \max\{\min[\cdot 6, 1], \min[\cdot 3, 8]\}$$

$$= \max\{\cdot 6, \cdot 3\}$$

$$= \underline{\underline{\cdot 6}}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_2) = \max\{\min[\mu_{\underset{\sim}{R}}(x_1, y_1), \mu_{\underset{\sim}{S}}(y_1, z_2)], \min[\mu_{\underset{\sim}{R}}(x_1, y_2), \mu_{\underset{\sim}{S}}(y_2, z_2)]\}$$

$$= \max\{\min[\cdot 6, \cdot 5], \min[\cdot 3, \cdot 4]\}$$

$$= \max\{\cdot 5, \cdot 3\}$$

$$= \underline{\underline{\cdot 5}}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_3) = \max\{\min[\mu_{\underset{\sim}{R}}(x_1, y_1), \mu_{\underset{\sim}{S}}(y_1, z_3)], \min[\mu_{\underset{\sim}{R}}(x_1, y_2), \mu_{\underset{\sim}{S}}(y_2, z_3)]\}$$

$$= \max\{\min[\cdot 6, \cdot 3], \min[\cdot 3, \cdot 7]\}$$

$$= \max\{\cdot 3, \cdot 3\}$$

$$= \underline{\underline{\cdot 3}}$$

$$\mu_{\underset{\sim}{T}}(x_2, z_1) = \max\{\cdot 2, \cdot 8\} = \underline{\cdot 8}$$

$$\mu_{\underset{\sim}{T}}(x_2, z_2) = \max\{\cdot 2, \cdot 4\} = \underline{\cdot 4}$$

$$\mu_{\underset{\sim}{T}}(x_2, z_3) = \max\{\cdot 2, \cdot 7\} = \underline{\cdot 7}$$

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} \cdot 6 & \cdot 5 & \cdot 3 \\ \cdot 8 & \cdot 4 & \cdot 7 \end{bmatrix} \end{array}$$

max-pdt composition

$$\underset{\sim}{T} = \underset{\sim}{R} o \underset{\sim}{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & & & \\ x_2 & & & \end{array}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_1) = \max\{[\mu_{\underset{\sim}{R}}(x_1, y_1) \times \mu_{\underset{\sim}{S}}(y_1, z_1)], \; \cancel{\max} \; [\mu_{\underset{\sim}{R}}(x_1, y_2) \times \mu_{\underset{\sim}{S}}(y_2, z_1)]\}$$

$$= \max[\cdot 6 \times 1, \cdot 3 \times \cdot 8]$$

$$= \max[\cdot 6, \cdot 24]$$

$$= \underline{\underline{\cdot 6}}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_2) = \max\{\cdot 30, \cdot 12\}$$

$$= \underline{\underline{\cdot 3}}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_3) = \max\{\cdot 18, \cdot 21\} = \underline{\underline{\cdot 21}}$$

$$\mu_{\underset{\sim}{T}}(x_2, z_1) = \max\{\cdot 2, \cdot 72\} = \underline{\underline{\cdot 72}}$$

$$\mu_{\underset{\sim}{T}}(x_2, z_2) = \max\{\cdot 1, \cdot 36\} = \underline{\underline{\cdot 36}}$$

$$\mu_{\underset{\sim}{T}}(x_2, z_3) = \max\{\cdot 6, \cdot 63\} = \underline{\underline{\cdot 63}}$$

$$\underset{\sim}{T} = \underset{\sim}{R} o \underset{\sim}{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & \cdot 6 & \cdot 3 & \cdot 21 \\ x_2 & \cdot 72 & \cdot 36 & \cdot 63 \end{array}$$

(7) For a speed controller of a DC motor. The membership fns of series resistance, armature current & speed are given as follows:

10 marks

$$\underset{\sim}{R} = \left\{ \frac{.4}{30} + \frac{.6}{60} + \frac{1}{100} + \frac{.1}{120} \right\}$$

$$\underset{\sim}{I} = \left\{ \frac{.2}{20} + \frac{.3}{40} + \frac{.6}{60} + \frac{.8}{80} + \frac{1}{100} + \frac{.2}{120} \right\}$$

$$\underset{\sim}{N} = \left\{ \frac{.35}{500} + \frac{.67}{1000} + \frac{.97}{1500} + \frac{.25}{1800} \right\}$$

Compute $\underset{\sim}{T}$ for relating series resistance to motor speed ie, $\underset{\sim}{R}$ to $\underset{\sim}{N}$. Perform max-min composition.

$$\underset{\sim}{A} = \underset{\sim}{R} \times \underset{\sim}{I}$$

$$= \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccccc} 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{bmatrix} .2 & .3 & .4 & .4 & .4 & .2 \\ .2 & .3 & .6 & .6 & .6 & .2 \\ .2 & .3 & .6 & .8 & 1 & .2 \\ .1 & .1 & .1 & .1 & .1 & .1 \end{bmatrix} \end{array}$$

.2
.3
.35
.35
.35

$$\underset{\sim}{B} = \underset{\sim}{I} \times \underset{\sim}{N}$$

$$= \begin{array}{c} \\ 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} .2 & .2 & .2 & .2 \\ .3 & .3 & .3 & .25 \\ .35 & .6 & .6 & .25 \\ .35 & .67 & .8 & .25 \\ .35 & .67 & .97 & .25 \\ .2 & .2 & .2 & .2 \end{bmatrix} \end{array}$$

$$\underset{\sim}{T} = \underset{\sim}{A} \circ \underset{\sim}{B} = \begin{array}{c|cccc} & 500 & 1000 & 1500 & 1800 \\ \hline 30 & .35 & .4 & .4 & .25 \\ 60 & .35 & .6 & .6 & .25 \\ 100 & .35 & .67 & .97 & .25 \\ 120 & .1 & .1 & .1 & .1 \end{array}$$

(8) Consider 2 fuzzy sets given by

$$\underset{\sim}{A} = \left\{ \frac{1}{low} + \frac{.2}{medium} + \frac{.5}{high} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{.9}{Positive} + \frac{.4}{zero} + \frac{.9}{Negative} \right\}$$

a) Find fuzzy relation for the CP of $\underset{\sim}{A}$ & $\underset{\sim}{B}$
ie, $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$

b) Introduce a FS, $\underset{\sim}{C}$ given by

$$\underset{\sim}{C} = \left\{ \frac{.1}{low} + \frac{.2}{medium} + \frac{.7}{high} \right\}$$

Find the relation b/w $\underset{\sim}{C}$ & $\underset{\sim}{B}$ using CP
ie, find $\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B}$

c) Find $\underset{\sim}{C} \circ \underset{\sim}{R}$ using max-min composition

d) Find $\underset{\sim}{C} \circ \underset{\sim}{S}$ using ,, ,,

a) $$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{array}{c|ccc} & Positive & zero & negative \\ \hline low & .9 & .4 & .9 \\ medium & .2 & .2 & .2 \\ high & .5 & .4 & .5 \end{array}$$

b) $$\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B} = \begin{array}{c|ccc} & Positive & zero & negative \\ \hline low & .1 & .1 & .1 \\ medium & .2 & .2 & .2 \\ high & .7 & .4 & .7 \end{array}$$

c) $\tilde{S} \circ \tilde{R} = \begin{bmatrix} .1, & .2, & .7 \end{bmatrix}_{1\times 3} \circ \begin{bmatrix} .9 & .4 & .9 \\ .2 & .2 & .2 \\ .5 & .4 & .5 \end{bmatrix}_{3\times 3}$

$= \begin{bmatrix} .5 & .4 & .5 \end{bmatrix}_{1\times 3}$

d) $\tilde{S} \circ \tilde{S} = \begin{bmatrix} .1, & .2, & .7 \end{bmatrix}_{1\times 3} \circ \begin{bmatrix} .1 & .1 & .1 \\ .2 & .2 & .2 \\ .7 & .4 & .7 \end{bmatrix}_{3\times 3}$

$= \begin{bmatrix} .7 & .4 & .7 \end{bmatrix}_{1\times 3}$

(9) 3 fuzzy sets are given as follows

$\tilde{P} = \left\{ \frac{.1}{2} + \frac{.3}{4} + \frac{.7}{6} + \frac{.4}{8} + \frac{.2}{10} \right\}$

$\tilde{Q} = \left\{ \frac{.1}{.1} + \frac{.3}{.2} + \frac{.3}{.3} + \frac{.4}{.4} + \frac{.5}{.5} + \frac{.2}{.6} \right\}$

$\tilde{T} = \left\{ \frac{.1}{0} + \frac{.7}{.5} + \frac{.3}{1} \right\}$

Find the following operations

i) $\tilde{R} = \tilde{P} \times \tilde{Q}$ iii) $\tilde{M} = \tilde{R} \circ \tilde{S}$ (max-min)

ii) $\tilde{S} = \tilde{Q} \times \tilde{T}$ iv) $\tilde{M} = \tilde{R} \circ \tilde{S}$ (max-pdt)

$\cancel{\tilde{R} = \tilde{P} \times \tilde{Q} =}$

i) $\underset{\sim}{R} = \underset{\sim}{P} \times \underset{\sim}{Q} = \begin{array}{c|cccccc} & .1 & .2 & .3 & .4 & .5 & .6 \\ \hline 2 & .1 & .1 & .1 & .1 & .1 & .1 \\ 4 & .1 & .3 & .3 & .3 & .3 & .2 \\ 6 & .1 & .3 & .3 & .4 & .5 & .2 \\ 8 & .1 & .3 & .3 & .4 & .4 & .2 \\ 10 & .1 & .2 & .2 & .2 & .2 & .2 \end{array}$

ii) $\underset{\sim}{S} = \underset{\sim}{Q} \times \underset{\sim}{T} = \begin{array}{c|ccc} & 0 & .5 & 1 \\ \hline .1 & .1 & .1 & .1 \\ .2 & .1 & .3 & .3 \\ .3 & .1 & .3 & .3 \\ .4 & & & \\ .5 & & & \\ .6 & & & \end{array}$

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⑩ Consider the FR, $\underset{\sim}{R}$. Check whether it is fuzzy tolerance/equivalence relation.

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$$\underset{\sim}{R} = \begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 1 & .8 & 0 & .1 & .2 \\ x_2 & .8 & 1 & .4 & 0 & .9 \\ x_3 & 0 & .4 & 1 & 0 & 0 \\ x_4 & .1 & 0 & 0 & 1 & .5 \\ x_5 & .2 & .9 & 0 & .5 & 1 \end{array}$$

All diagonal elements are 1.

ie, $\mu_{\underset{\sim}{R}}(x_i, x_i) = 1$

∴ It statisfies reflexive.

$$\underset{\sim}{R}^T = \begin{bmatrix} 1 & .8 & 0 & .1 & .2 \\ .8 & 1 & .4 & 0 & .9 \\ 0 & .4 & 1 & 0 & 0 \\ .1 & 0 & 0 & 1 & .5 \\ .2 & .9 & 0 & .5 & 1 \end{bmatrix}$$

$$\underset{\sim}{R} = \underset{\sim}{R}^T$$

∴ It statisfies symmetry.

It statisfies both reflexive & symmetric. ∴ It is tolerance.

$\mu_{\underset{\sim}{R}}(x_1, x_2) = \lambda_1 = .8$

$\mu_{\underset{\sim}{R}}(x_2, x_3) = \lambda_2 = .4$

$\mu_{\underset{\sim}{R}}(x_1, x_3) = \lambda = 0$

$\lambda \geq \min[\lambda_1, \lambda_2]$

$0 \geq \min[.8, .4]$

$0 \ngeq .4$ ∴ It not statisfies transitive property.

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