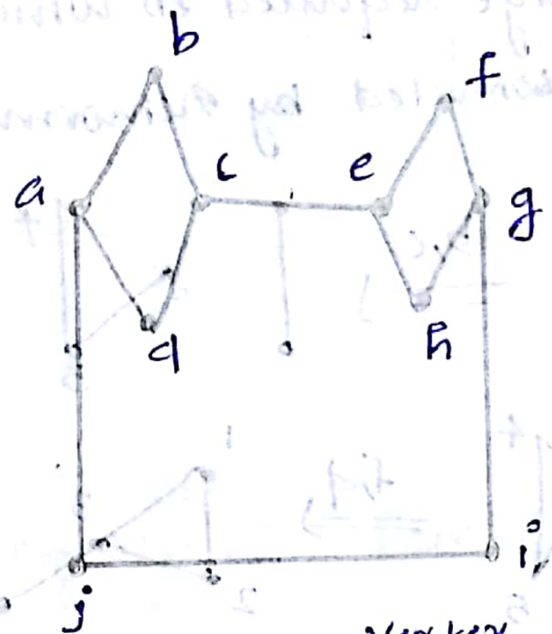
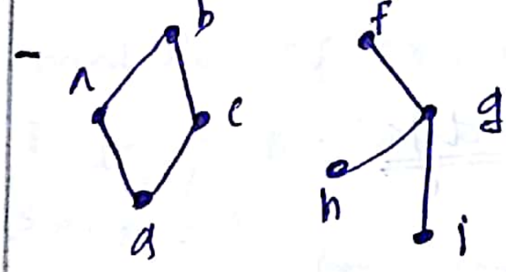
11/10/17

4 Vertex Connectivity

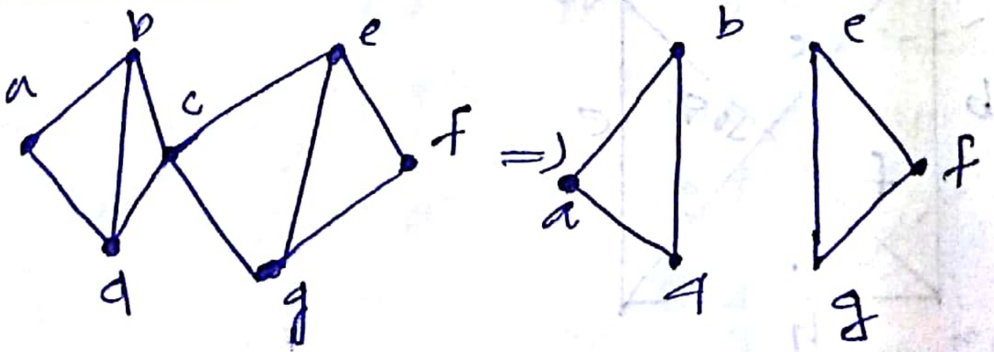


The min no:of ~~vertex~~ ~~edges~~ Vertex required when we remove from the graph makes the graph disconnected is called vertex connectivity. K(G)

• when we remove e and j



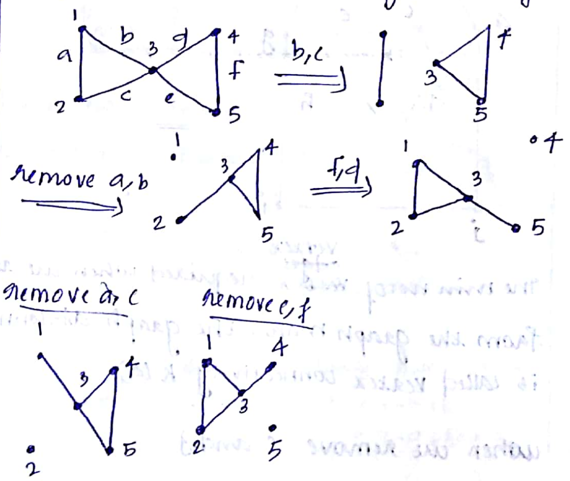
Here K(G) = 2



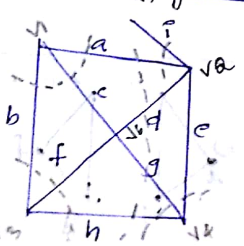
K(G) = 1

Edge Connectivity ($\lambda(G)$)

Min no: of edge required in which a graph becomes disconnected by removing.



Cut Sets / cut edges / Bridges.



ia de } X
adge } X
~~adgh~~

acb
bfh
hge
acde
adgh
acfh

cut set

Set of edges removed from the graph makes the graph disconnected when there is no proper subset of for the cut set
b

a cut set reduces rank by 1

Here $r = n - k = 6 - 1 = 5$

Theorems:-

Every edge of a tree is a cut set.

✓ Every cut set in a connected graph G must contain atleast one branch of every spanning tree of G.

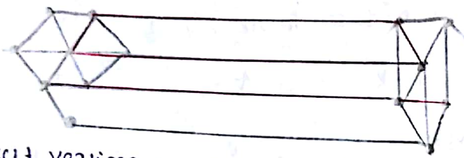
✓ Converse Theorem:

In a connected graph G any minimal set of edges containing atleast one branch of every spanning tree of G is a cut set.

Spanning tree	Cut set
~~a b h g d i~~ X	{a, b, c}
~~i e b h e d~~ X	{b, f, h}
{d a b h e}	{h, g, e}
{c h b a d}	{a, d, e}
{a b h g d}	

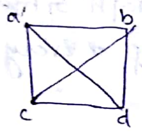
THEOREM:

Every circuit has an even no:of edges in common with any cutset.

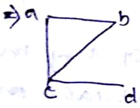


cut vertices

Cut vertex is a vertex which remove in a graph make disconnected.

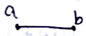


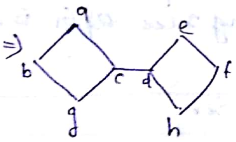
no cut vertex.



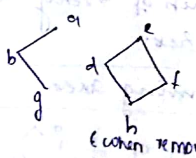
here cut vertex = c

$\Longleftrightarrow$





$\Longleftrightarrow$



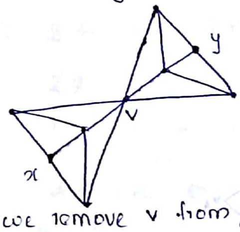
cut vertex = c or d.

(when remove c)

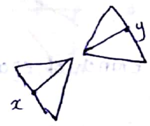
if $k(G) = 1$ then it is seperable graph.

Theorem

a vertex v in a connected graph G is a cutvertex if and only if there exist 2 vertices x & y in G such that every path b/w x & y passes through v.



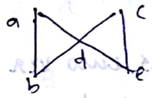
if we remove v from graph,



Theorem.

The edge connectivity of graph G cannot exceed the degree of the vertex with smallest degree.

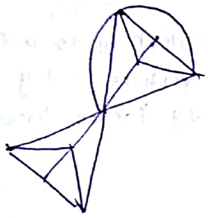
$\lambda(G) \le \min d(v)$



Theorem

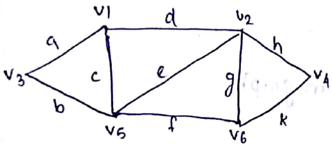
The vertex connectivity of a graph G can never exceeds the edge connectivity of graph G.

$k(G) \le \lambda(G) \le \min$ ~~deg(G)~~ $d(v)$.



$\lambda(G) = 3$

Fundamental cutset.



Is a cutset those exist only one branch of spanning tree. Others are ~~codes~~. chords

cut set $\rightarrow$ { a (chord) c (branch) d (chord) }

Fundamental ckt (Γ)

circuit in the graph there exist only one chord and others are branches.

17/10/19 {c, b, a}, {c e d} {h, g, k} {e h k f}

Q Suppose that a tree has two ver of degree 2 and 4 vertices of degree 3. and 3 vertices of degree 4. Find the no: of Pendant vertices in 'T'.

A

$n = 2+4+3+P = 9+P$

$e = n-1 = 8+P$

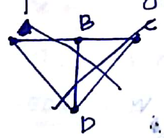
$\Sigma d(v) = 2e$

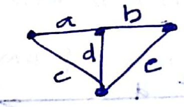
$2\times2 + 4\times3 + 3\times4 + P = 2(8+P)$

$28 + P = 16 + 2P$

$P = \underline{\underline{12}}$

Q All spanning tree of following graph



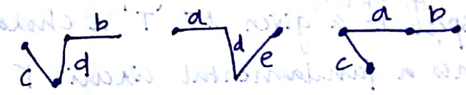


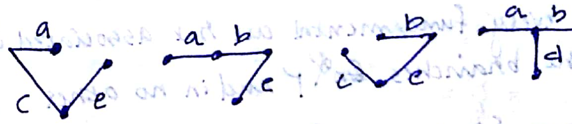
A $n = n-c$ No: of branches ST = 3.

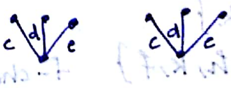
$= 4-1$

$= 3$//

abe, ceb, bac, bdc, ace







a) prove that a pendant edge in a connected graph 'G' is contained in every spanning tree of 'G'. Show that

b) Show that hamiltonian path is ST.

c) prove that subgraph of a connected graph 'G' is contained in some spanning tree of 'G' if and only if 'G' contains no circuit.

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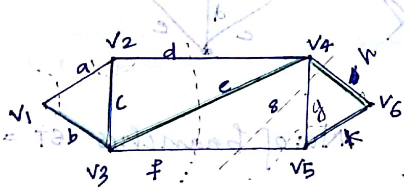


$\Gamma = \{c_i, b_1, b_2 .. b_k\}$

$S = \{b_1, c_1, c_2, .. c_q$

THEOREM

With respect to a given ST 'T' a chord c_i determines a fundamental circuit Γ occurs in every fundamental cut set associated with the branches ~~in~~ of Γ and in no other

eg: $ST = \{b, c, e, h, k\}$

circuit $= \Gamma = \{e, h, k, f\}$

cut set of $e \{d, e, f\}$ − f, e

$h \to \{h, g, f\}$ − f, h

$k \to \{k, f\}$ − f, k

- In circuit f is the chord
- f is member of other cut set formed by the other member of circuit. Also f will not be the member of any other cut set other than i
- From the above theorem we can say that

$T \cap S =$ even and $\{c_i, b_1/b_2/b_3/b_k\}$

THEOREM-2

with respect to a spanning tree T a branch T_i that determines fundamental cut set 'S' is contained in every fundamental circuit associated with chords in S and in no other set.

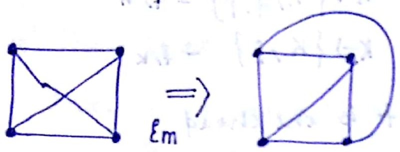
~~S=~~ $ST = \{b, c, e, h, k\}$

$S = \{a, \underset{\text{branch}}{c}, d\}$

fundamental circuit of $a = \{a\ b\ c\}$ (chord)

" " " $d = \{d, c, e\}$.

Planar Graphs K_4

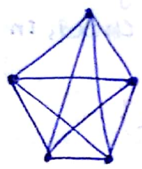


complete graph ⟹ plane graph

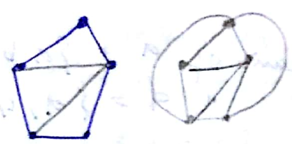
A graph whose geometric representation has edge crossing which can be converted to a plane graph.

→ **Kuratowski's Graph** : are non-planar (K_5 & $K_{3,3}$)

The complete graph of 5 vertices is non-planar

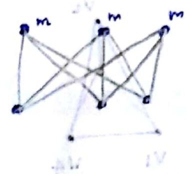


It can't ~~pe~~ be drawn as plane graph. ∴ It is non-planar.



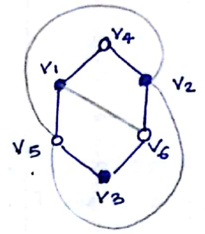
Bipartite graph.

The vertex set is divided into 2 set ~~in~~ there will not be vertex ~~betwee~~ from a set of vertex n to other set of vertex m



Kuratowski's $K_{3,3}$

- non planar.
- Bipartite graph.
- Each m vertex is connected to all n



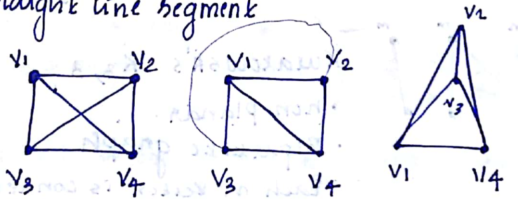
- Connect all • vertex with o
- connectⁿ from $V_3 \to V_4$ is not possible
- Non-planar graph.

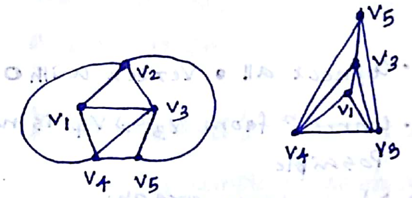
→ K_5 & $K_{3,3}$:- [Properties]

- Regular graph (equal degree)
- Removal of one vertex or edge it becomes a planar graph
- Non-planar.

25/10/17 **Different Representation of a Planar graph.**

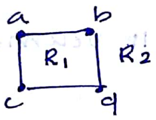
Any simple planar graph can be embedded in a plane. such that every edge is drawn as a straight line segment





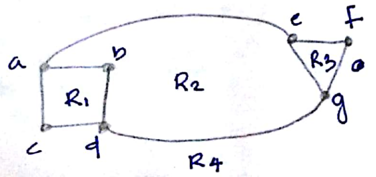
Region / Faces

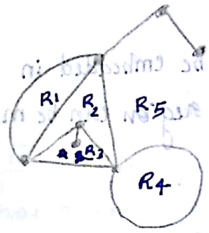
Set of edges | ~~boundar~~ vertices that form it's boundary



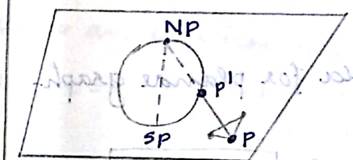
Region inside — R_1

" outside — R_2.





Embedding On a Sphere.



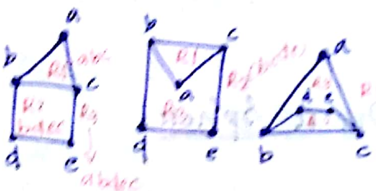
- Place a sphere on a plane
- There is a point of intersection b/w them ie sp.
- SP is perpendicular to NP in sphere.
- Straight line from p to Np, touches a pnt P'
- Each point on the plane have a corresponding point in sphere. vice verse ie Projecting point.

Theorem:

A graph can be embedded on the surface of a sphere if and only if it can be embedded in a plane

Theorem:-

A planar graph may be embedded in a plane, such that any specified region can be made infinite region.



Euler's formula for planar graph.

$\boxed{e-n+2=r}$ or $\boxed{n-e+r=2}$

e - no: of edges

n - " " vertices

r - " " region

Induction method: on edges.

Case I : $e=0$ (isolated)

$n=1, e=0, r=1$

$e-n+2=r$

$0-1+2=1$

Case - II $e \geq 1$

G is a tree.

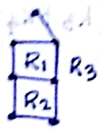
$n=5$

$e=4$ $\quad e-n+2=r$

$r=$ $\quad 4-5+2=1$

Case - III $e \geq 1$

G is not a tree.



$n=7, e=8$

$e-n+2 = 8-7+2$

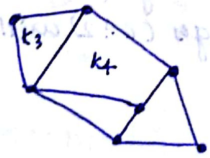
$= 3$

Remove an edge from here.

$n=7, e=7$ $\quad e-n+2 = 7-7+2$

$= 2$

Method II Sum of angles.



Kp - no. p

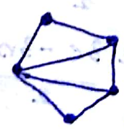
3-sided - 2 polygon

4 sided - 1 polygon

$3K_3+4K_4+\dots n.K_n = 2e$ — (1)

Each edge is counted twice

$K_3+K_4+\dots K_r = r$ — (2)



n - Vertices

n-2 P sided polygon.

5, 3 poly

~~Interior~~

Sum of interior angles

$\pi(3-2)\cdot k_3 + \pi(4-2)\cdot k_4 + \pi(5-2)k_5 + \dots$

$\pi(n-2)k_n + 4\pi = 2\pi n.$

$\pi[3k_3 + 4k_4 + 5k_5 + \dots + n k_n] - \cancel{\pi}2[k_3 + k_4 + \dots k_n] + 4\pi = 2\pi n$

$\pi[2e - 2n] + 4\pi = 2\pi n$

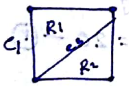
$2e\pi - 2n\pi + 4\pi = 2\pi n$

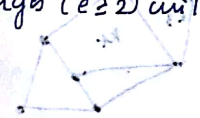
$\cancel{2\pi}[e - n + 2] = \cancel{2\pi} n$

$e - n + 2 = n$

Theorem:-

In any simple connected planar graph with 'R' regions with n vertices and edges ($e \geq 2$) will follow.





$2e \geq 3R$

~~$2e \geq 3e - 3n + 6$~~

~~$3n - 6 \leq e$~~

~~or $e \leq 3n - 6$.~~

$e \geq \frac{3}{2} R$

$e \geq \frac{3}{2}(e - n + 2)$

$e \geq \frac{3e - 3n + 6}{2}$

~~$e \geq \frac{3}{2} R$~~

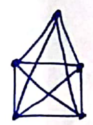
$e \geq \frac{3}{2}(e - n + 2)$

$2e \geq 3e - 3n + 6$

$3n - 6 \leq e$

or $e \leq 3n - 6$.

Q Draw K_5 graph and check the condition $e \leq 3n - 6$. Check planar or not



$e \leq 3n - 6$

$\leq 3 \times 5 - 6$

$10 \leq 9$ not satisfy so not a planar.

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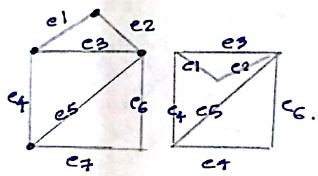
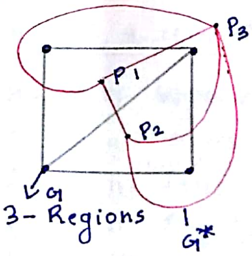
Unique embedding

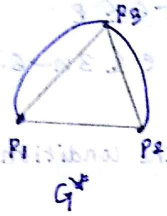


This two are ~~not~~ isomorphic but dont coincide similar ∴ distinct.

Geometric Dual of a graph:

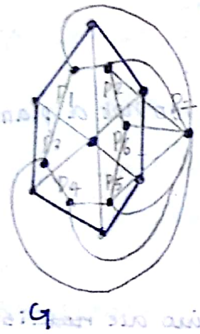


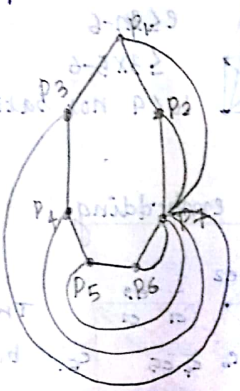
Draw a line from one region to another region, that line should intersect with a edge that seperates the region. all seperating edges should be intersected.

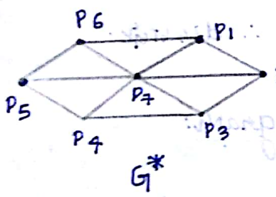


Series edges in G will be parallel in G^* and vice versa

Q Draw a dual graph

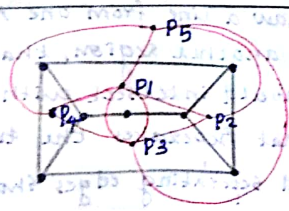


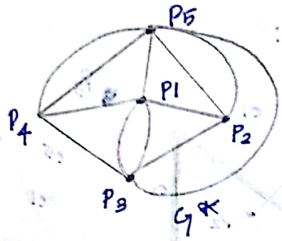




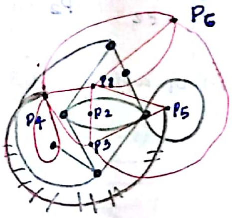
This is self dual.

Q.





- P_4 has 3 boundary edge then in G^* P_4 has degree 3
- P_3 has 5 boundary edges then P_3 has degree 5 in G^*.



If a edge completely lies inside a region then draw a self loop ~~by draw~~ from the region to the edge in such a way that intersect exactly once.

- n, e, f, r, μ & $n^*, e^*, f^*, r^*, \mu^*$

n - no: of ~~edges~~ vertices

e - " " edges.

f - " " faces.

r - " " region

μ - nullity

$n^* = f$

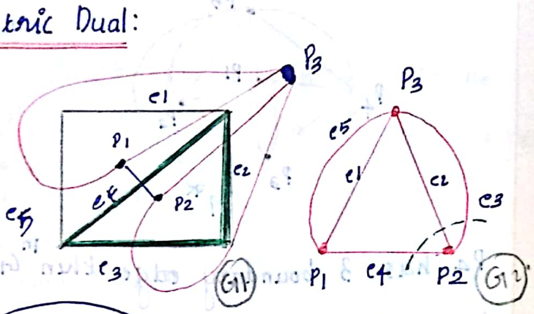
$e^* = e$

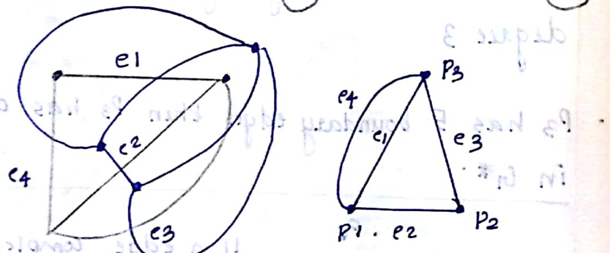
$f^* = n$

$r^* = \mu$

$\mu^* = r$.

Geometric Dual:





e2 and ~~e~~ e3 are parallel in G and series in G^*

Combinatorial Dual

Theorem:

A necessary and sufficient condition for two planar graph G_1 and G_2 to be duals of each other. If there is one to one correspondance b/w edges in G_1 & edges in G_2 such that a set of edges in G_1 forms a circuit if and only if the corresponding set G_2 forms a cutset.

- Consider a circuit in G_1 ($e_2 e_3 e_4$) are cut set in G_2

CKT $\Gamma = \{e_2, e_3, e_4\}$

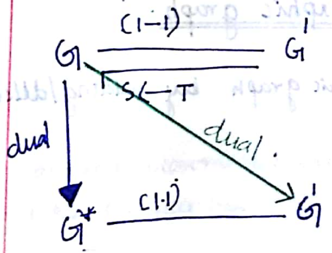
$e_2 e_3 e_4$ contains P_2 then removing $e_2 e_3 e_4$ from G_2 forms a set of subset

$\underbrace{e_2 e_3 e_4 \Rightarrow P_2}_{\text{cut set}}$

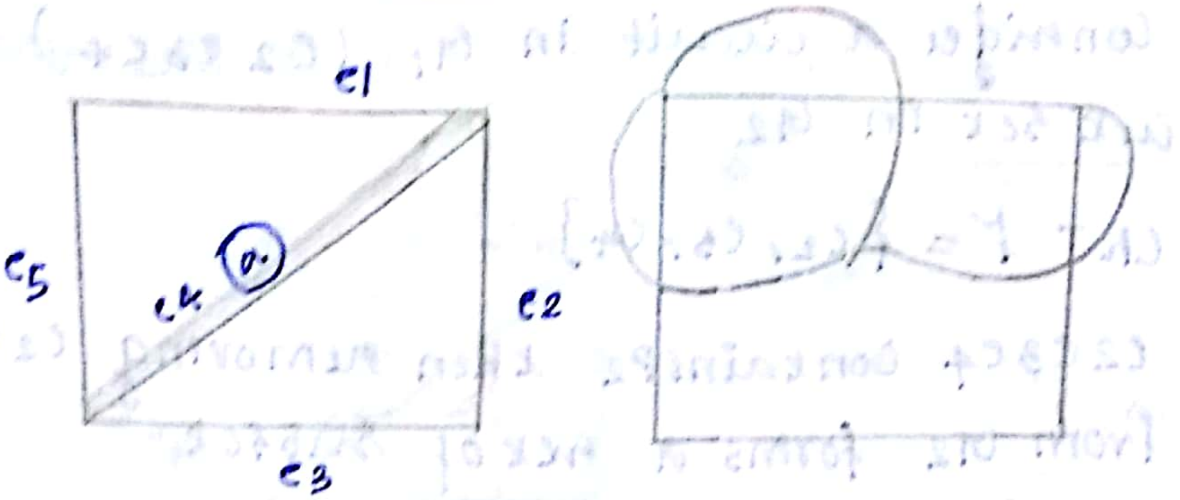
- Consider circuit $e_1 e_5$ in G_2, which forms a cutset in G_1

$\Gamma^* = \{e_1, e_5\} \rightarrow G_2$

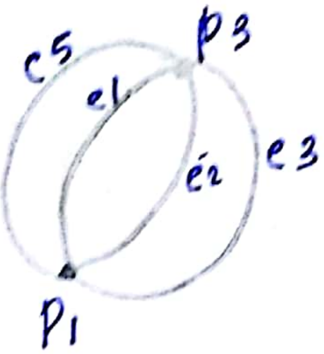
$S := \{e_1 e_5\} \rightarrow G_1$



Dual of a Subgraph



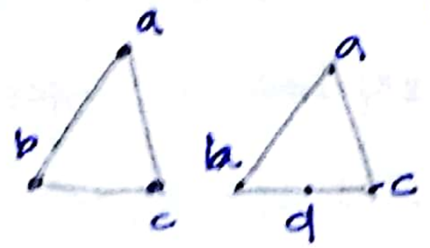
Remove e4 (eg edge b/w two region)
Remove e4 from dual graph then join I* edge end Vertices.



we get this from G_2 ie dual

Dual of a Homeomorphic graph.

we get a homeomorphic graph by adding/deleting Vertex on an edge.



By adding a series edge in G then a parallel edge is added in dual.

Refer text for