Module: 4

9/10/19

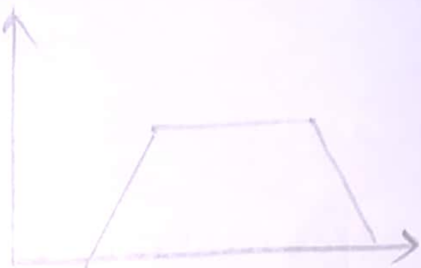
MEMBERSHIP FUNCTIONS

Membership Function (MF)

It defines the fuzziness in a FS irrespective of the elements in the set.

- Membership fns are generally represented in graphical form.
- The commonly used MF are:

i) Triangular MF ii) Trapezoidal MF



Features of MF

A FS $\underset{\sim}{A}$ can be defined as the set of ordered pairs.

i.e,

$$\underset{\sim}{A} = \{(x_i, \mu_{\underset{\sim}{A}}(x_i)) \mid x \in X\}$$

eg: $A = \left\{\frac{.8}{1} + \frac{.2}{2} + \frac{.4}{3}\right\}$ or

$$\underset{\sim}{A} = \{(1,8), (2,2), (3,4)\}$$

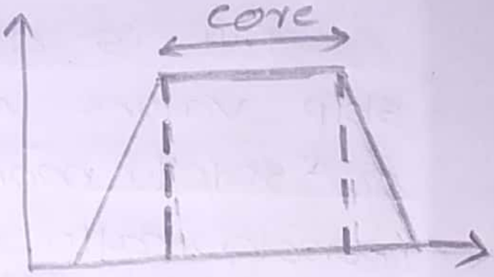
where, $\mu_{\underset{\sim}{A}}(x_i)$ — Degree of membership

The features of MFs are:

1) Core
2) Support
3) Boundary

1) Core

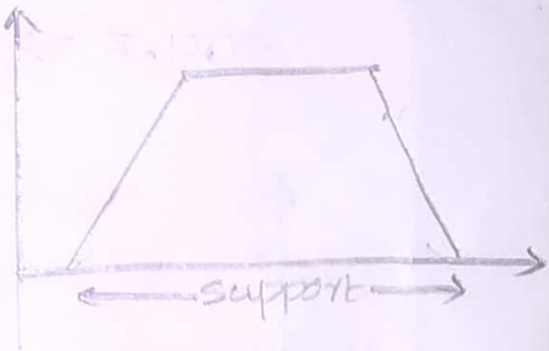
Core of a MF is defined as the region of universe characterised by complete and full membership in FS $\underset{\sim}{A}$.



$\mu_{\underset{\sim}{A}}(x) = 1$

2) Support

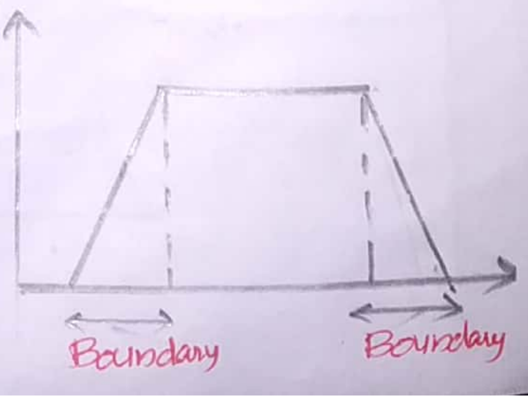
Support of a MF is defined as the region of universe characterised by non-zero membership in FS $\underset{\sim}{A}$.



$\mu_{\underset{\sim}{A}}(x) > 0$

3) Boundary

Boundary of a MF is defined as the region of universe ~~Characterised by~~ containing elements that have non-zero membership but not complete membership.



$0 < \mu_{\underset{\sim}{A}}(x) < 1$

Normal Fuzzy Set

A set whose MF has atleast one element x_i whose degree of membership is unity or 1.

Fuzzy set which is not a Fuzzy set is called a SUBNORMAL Fuzzy set

Convex Fuzzy Set

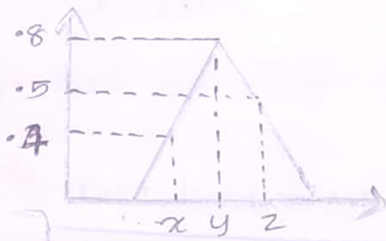
It is describe by a MF whose membership values are * strictly monotonically increasing SMI or * strictly monotonically decreasing SMD or * strictly monotonically increasing then strictly monotonically decreasing SMI then SMD with increasing value of elts in the universe.

consider 3 elts x, y, z where $x < y < z$

$$x, y, z \Rightarrow \mu_{\underset{\sim}{A}}(y) \geq \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{A}}(z)]$$

SMD

SMI

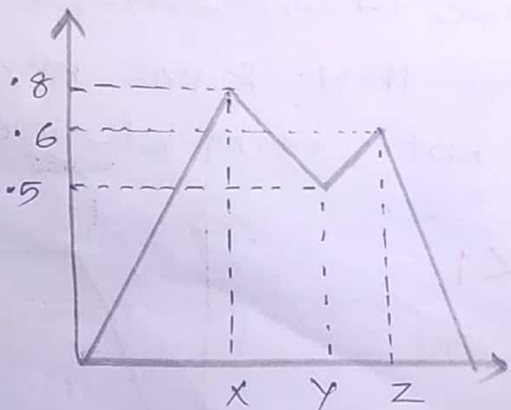


SMI then SMD

$\cdot 8 \geq \min[\cdot 3, \cdot 5]$

$\cdot 8 \geq \cdot 3$

$\therefore$ It is a convex FS.



$\cdot 5 \geq \min[\cdot 8, \cdot 6]$

$\cdot 5 \ngeq \cdot 6$

$\therefore$ It is not a convex FS

ie, it is a non-convex FS.

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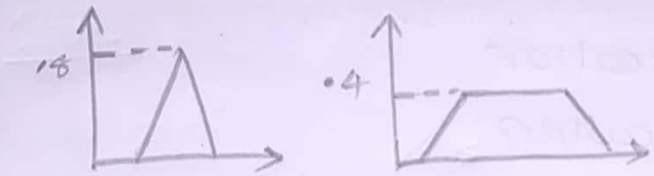
Height of a FS

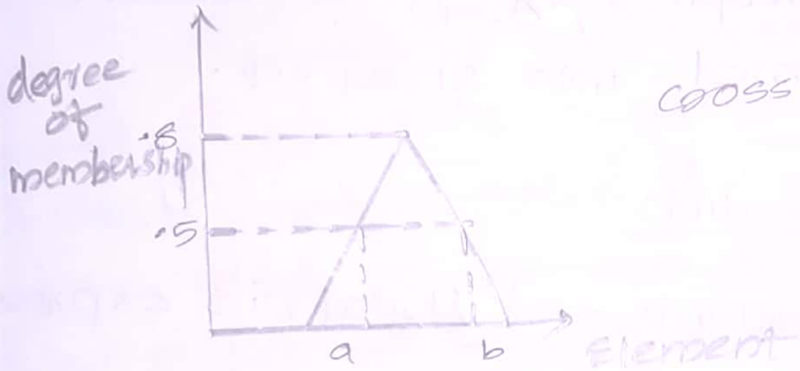
It is the maxi. value of the degree of membership. Denoted by:

$$hgt(\underset{\sim}{A}) = max\left[\mu_{\underset{\sim}{A}}(x)\right]$$



Crossover point of a MF

It is defined as the elt in the universe for which a particular FS $\underset{\sim}{A}$ as the degree of membership .5.



Cross over points are a,b

? Consider a FS $\underset{\sim}{A} = \{ 1/1 + .5/1.5 + .3/2 + .5/2.5 + 0/3 \}$ Check whether the FS is normal. Find

i) cross-over point

ii) height of the FS

i) Cross-over points are 1.5 & 2.5. They have the degree of membership ~~as~~ .5.

ii) Height of the FS is 1.
ie, maximum degree of membership is 1.

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Fuzzification

11/10

The process of converting crisp set/crisp data to fuzzy set or fuzzy data.

~~Fuzzy set~~

fuzzification can be classified into

1) Support fuzzification
2) Grade fuzzification

1) Support fuzzification

For a FS $\underset{\sim}{A} = \left\{ \frac{\mu_{\underset{\sim}{A}}(x_i)}{x_i} \right\}$

A common fuzzification algorithm is performed by keeping $\mu_{\underset{\sim}{A}}(x_i)$ as constant and x_i is transformed into fuzzy set.

2) Grade fuzzification

x_i is constant and $\mu_{\underset{\sim}{A}}(x_i)$ is expressed as fuzzy set.

Different Methods of fuzzification

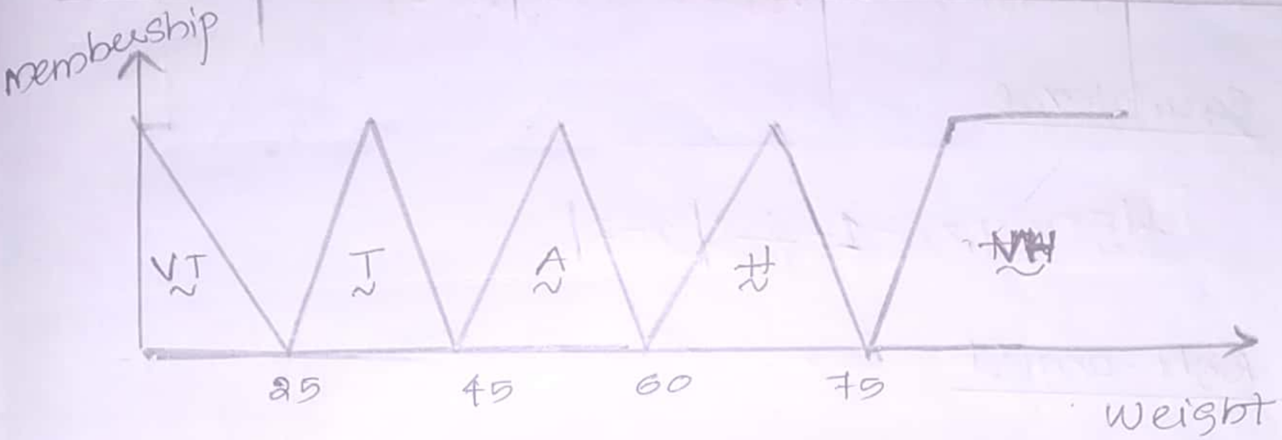
1) Intuition
2) Inferrence
3) Rank order

1) Intuition

It's based on the common intelligence of human. i.e, Human can develop MF on their own intelligence.

Eg: Using your own intuition plot the fuzzy MF for "Weight of people".

Very thin	Thin	Average	Heavy	Very Heavy
$W \leq 25$	$25 < W \leq 45$	$45 < W \leq 60$	$60 < W \leq 75$	$W > 75$



2) Inferrence

This method uses knowledge to perform detective reasoning. There are ~~variab~~ various methods for detective reasoning. Here the knowledge of geometrical shapes is used for defining membership values. The inferrence method, we are discussing here is triangular.

Consider a triangle where ~~as~~ x, y, z are the angles and the condition is

i) $x \geq y \geq z \geq 0$

ii) $x + y + z = 180$

Different types of triangles & membership values are:

~~Isoscles~~

Isosceles

$$\mu_{\underset{\sim}{I}}(x,y,z) = 1 - \frac{1}{60} \min[x-y, y-z]$$

Equilateral

$$\mu_{\underset{\sim}{E}}(x,y,z) = 1 - \frac{1}{180} |x-z|$$

Right-angled

$$\mu_{\underset{\sim}{R}}(x,y,z) = 1 - \frac{1}{90} |x-90|$$

Isosceles-right angled

$$\mu_{IR}(x,y,z) = \min[\mu_{\underset{\sim}{I}}(x,y,z), \mu_{\underset{\sim}{R}}(x,y,z)]$$

Others:

$$\mu_{\underset{\sim}{T}}(x,y,z) = \min[1-\mu_{\underset{\sim}{I}}(x,y,z), 1-\mu_{\underset{\sim}{E}}(x,y,z), 1-\mu_{\underset{\sim}{R}}(x,y,z)]$$

14/10/19 3) Rank Ordering

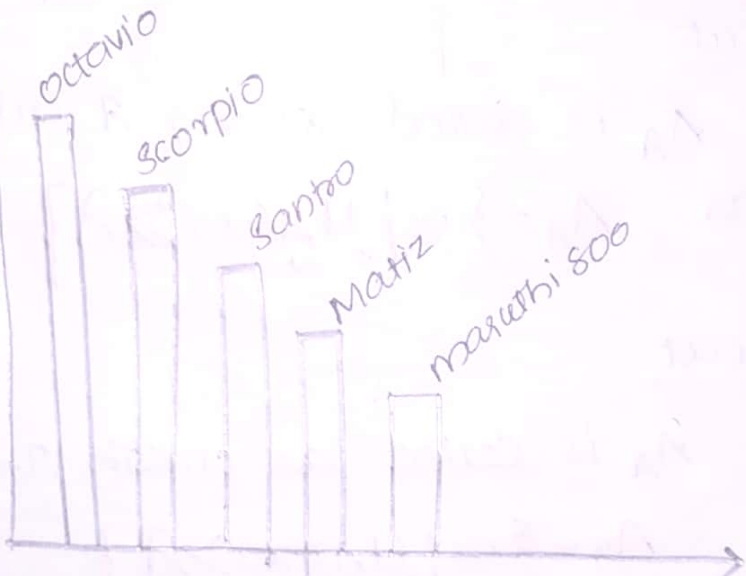
On the basis of the ~~~~ rank ordering is performed on the basis of preferences made by an individual, a committee, a poll and other opinion methods. This methodology can be accepted to assign membership to a fuzzy variable.

Pairwise comparison enable as to determine the preferences and this results in determine the order of membership.

Eg: suppose 1000 people respond to a questionnaire about their pairwise preferences among the 5 cars S={Maruthi 800, Scorpio, Matiz, Santro, Octavia}. Define a fuzzyset $\underset{\sim}{A}$ on the universe of cars "BEST CARS".

Number who preferred.

	Maruthi 800	Scorpio	Matiz	Santro	Octavia	Total	Percentage	Rank order
Maruthi 800	—	192	246	596	621	1651	16.5	5
Scorpio	403	—	621	540	391	1955	19.6	2
Matiz	235	336	—	797	492	1860	18.6	4
Santro	523	364	417	—	608	1912	19.1	3
Octavio	616	534	746	726	—	2622	26.2	1
Total						10000		



Defuzzification

The process of converting fuzzy data into crisp data.

→ fuzzy data can be represented as fuzzy set, Relation matrix & graph (MF)

Defuzzification Methods

i) λ-cut for fuzzy sets

consider a FS $\underset{\sim}{A}$, the λ-cut for the FS can be

1. strong λ-cut
2. weak λ-cut

1. strong λ-cut

A set A_λ is called strong λ-cut if its ~~sto~~ defined as $A_\lambda = \{x \mid \mu_{\underset{\sim}{A}}(x) \geq \lambda\}$

2. weak λ-cut

A set A_λ is called as weak λ-cut if its defined as $A_\lambda = \{x \mid \mu_{\underset{\sim}{A}}(x) > \lambda\}$

Properties of λ-cut

* 1. $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = \underset{\sim}{A}_\lambda \cup \underset{\sim}{B}_\lambda$
* 2. $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = \underset{\sim}{A}_\lambda \cap \underset{\sim}{B}_\lambda$
* 3. $(\overline{A})_\lambda \neq \overline{\underset{\sim}{A}_\lambda}$; Except for $\lambda = .5$
* 4. $\underset{\sim}{A}_{0+} = \{x \mid \mu_{\underset{\sim}{A}}(x) > 0\}$

ii) λ-cut for fuzzy Relation

λ-cut for a relation is defined as

$$\underset{\sim}{R}_\lambda = \{(x, y) \mid \mu_{\underset{\sim}{R}}(x, y) \geq \lambda\}.$$

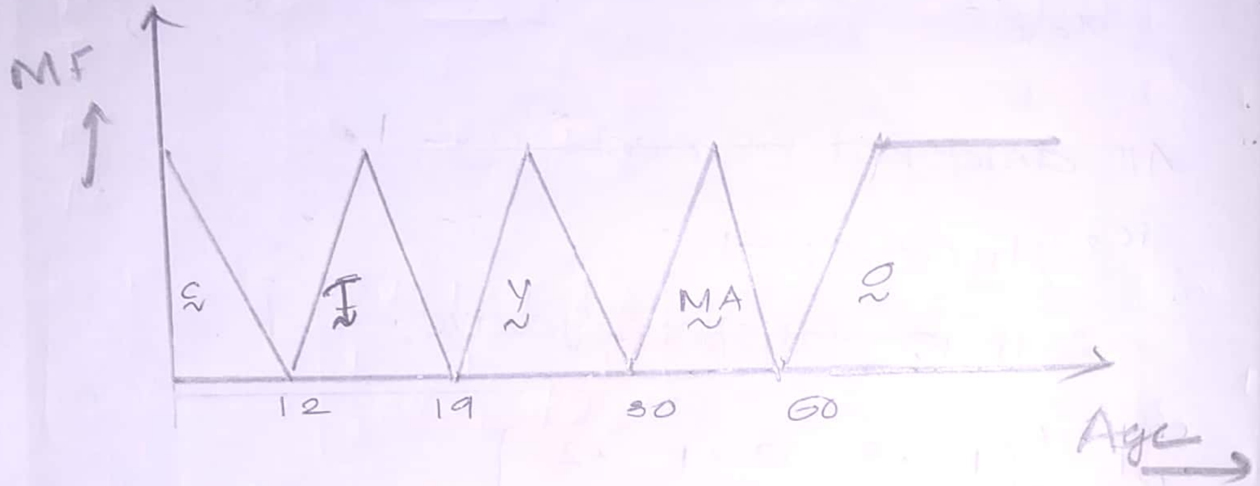
$$= \{1, \text{ if } \mu_{\underset{\sim}{R}}(x, y) \geq \lambda \mid 0, \mu_{\underset{\sim}{R}}(x, y) < \lambda\}$$

Module: 4

1) Using your own intuition plot the FMF for a "Age of people".

11/10

Child	Teen	Young	Middleage	old
$A \leq 12$	$12 < A \leq 19$	$19 < A \leq 30$	$30 < A \leq 60$	$A > 60$



2) Using the inference approach find the membership values for the triangular shapes $\underset{\sim}{I}$, $\underset{\sim}{R}$, $\underset{\sim}{IR}$, $\underset{\sim}{E}$ and $\underset{\sim}{T}$ for a triangle with angles $45°, 55°$ & $80°$.

10/10

$x > y > z$

$x = 80$

$y = 55$

$z = 45$

$$\mu_{\underset{\sim}{I}}(x, y, z) = 1 - \frac{1}{60} \min[x-y, y-z]$$

$$= \cancel{1 - 1/60 \min [80-55, 55-80]}$$

$$= 1 - 1/60 \min[80-55, 55-45]$$

$$= 1 - 1/60 \min[25, 10]$$

$$= 1 - 1/6\cancel{0} \times 1\cancel{0}$$

$$= 5/6$$

$$= .8333$$

$$\mu_{R}(x,y,z) = 1 - \tfrac{1}{90}|x-90|$$
$$= 1 - \tfrac{1}{90} \times |10|$$
$$= 8/9 = .888$$

$$\mu_{IR}(x,y,z) = \min[\mu_{I}(x,y,z), \mu_{R}(x,y,z)]$$
$$= \min[.833, .888]$$
$$= .833$$

$$\mu_{E}(x,y,z) = 1 - \frac{1}{180}|x-z|$$
$$= 1 - \tfrac{1}{180}|80-45|$$
$$= 1 - \tfrac{1}{180} \times 35$$
$$= .8055$$

$$\mu_{T}(x,y,z) = \min[1-\mu_{I}(x,y,z), 1-\mu_{E}(x,y,z), 1-\mu_{R}(x,y,z)]$$
$$= \min[1-.8333, 1-.8055, 1-.888]$$
$$= \min[.167, .1945, .112]$$
$$= .112$$

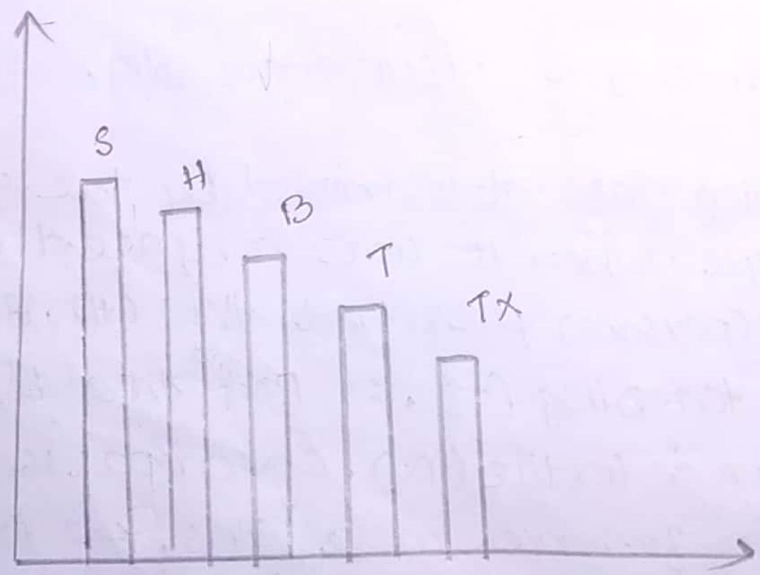
More chance for right angle △le.

(3) The following data determined by the work preference of 100 people. When it was compared with S/W (S), 72 peoples(persons) preferred H/W (H), 65 peoples preferred teaching (T), 55 preferred business (B) & 25 preferred textile (Tx). On comparisons with H, the ~~pere~~ preferences were 64 S, 42 for T, 66 for B & 35 for Tx. When compared with teaching, the preferrences were 70 S, 64 H, 38 B, 20 Tx

on comparison with B, 54 for S, 47 for H, 35 for T & 25 for Tx. when compared with T, 70 for S, 65 for H, 44 for T, 40 for B. Using rank ordering plot the MF for the most preferred work.

	S	H	T	B	Tx	Total	%	Rank order
S	—	72	65	55	25	217		
H	72	—	42	66	35	207		
T								
B								
Tx								

	S	H	T	B	TX	Total	%	Rank order
S	—	64	70	54	70	258	25.9	1
H	72	—	64	47	65	248	24.8	2
T	65	42	—	35	44	186	18.67	4
B	55	66	38	—	40	199	19.97	3
TX	25	35	25	20	—	110	11.0	5
Total						996		



Not given the St

16/1

(14) Consider 2 FS $\underset{\sim}{A}$ & $\underset{\sim}{B}$. Both defined as

$$\underset{\sim}{A} = \left\{ \frac{.2}{x_1} + \frac{.3}{x_2} + \frac{.4}{x_3} + \frac{.7}{x_4} + \frac{.1}{x_5} \right\},$$

$$\underset{\sim}{B} = \left\{ \frac{.4}{x_1} + \frac{.5}{x_2} + \frac{.6}{x_3} + \frac{.8}{x_4} + \frac{.9}{x_5} \right\}.$$ Find λ-cut

Not given w.k/strng then find strong

set for (i) $(\overline{\underset{\sim}{A}})_{.7}$ (ii) $(\underset{\sim}{B})_{.2}$ (iii) $(\underset{\sim}{A} \cup \underset{\sim}{B})_{.6}$

(iv) $(\underset{\sim}{A} \cap \underset{\sim}{B})_{.5}$ (v) $(\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}})_{.8}$

Ans $A_\lambda = \{x \mid \mu_{\underset{\sim}{A}}(x) \geq \lambda\}$

16/10

(i) $$\overline{\underset{\sim}{A}} = \left\{ \frac{.8}{x_1} + \frac{.7}{x_2} + \frac{.6}{x_3} + \frac{.3}{x_4} + \frac{.9}{x_5} \right\}$$

$(\overline{\underset{\sim}{A}})_{.7} = \{x_2, x_1, x_5\}$

(ii) $(\underset{\sim}{B})_{.2} = \{x_1, x_2, x_3, x_4, x_5\}$

(iii) $$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{.4}{x_1} + \frac{.5}{x_2} + \frac{.6}{x_3} + \frac{.8}{x_4} + \frac{.9}{x_5} \right\}$$

$(\underset{\sim}{A} \cup \underset{\sim}{B})_{.6} = \{x_3, x_4, x_5\}$

(iv) $\underset{\sim}{A} \cap \underset{\sim}{B} = \{ .2/x_1 + .3/x_2 + .4/x_3 + .7/x_4 + .1/x_5 \}$

$(\underset{\sim}{A} \cap \underset{\sim}{B})_{.5} = \{x_4\}$

(v) $$\overline{\underset{\sim}{B}} = \left\{ \frac{.6}{x_1} + \frac{.5}{x_2} + \frac{.4}{x_3} + \frac{.2}{x_4} + \frac{.1}{x_5} \right\}$$

$$(\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}) = \left\{ \frac{.8}{x_1} + \frac{.7}{x_2} + \frac{.6}{x_3} + \frac{.3}{x_4} + \frac{.9}{x_5} \right\}$$

$(\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}})_{.8} = \{x_1, x_5\}$

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(5) Determine the crisp λ cut relation when $\lambda = 0.1, 0+, 0.3, \& .9$ for the following relation $\underset{\sim}{R}$.

$$\underset{\sim}{R} = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.3 & 0.7 & 0.1 \\ 0.8 & .9 & 1.0 \end{bmatrix}$$

i) $\underset{\sim}{R}_{0.1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ iii) $\underset{\sim}{R}_{.3} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

ii) $\underset{\sim}{R}_{0+} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ iv) $\underset{\sim}{R}_{.9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(6) Show that any λ-cut relation of a fuzzy tolerance relation results in a crisp tolerance relation

$$\underset{\sim}{R} = \begin{bmatrix} 1 & .8 & 0 & .1 & .2 \\ .8 & 1 & .4 & 0 & .9 \\ 0 & .4 & 1 & 0 & 0 \\ .1 & 0 & 0 & 1 & .5 \\ .2 & .9 & 0 & .5 & 1 \end{bmatrix}$$

The diagonal elts are 1. $\therefore \underset{\sim}{R}$ is reflexive relation.

$$\underset{\sim}{R}^T = \begin{bmatrix} 1 & .8 & 0 & .1 & .2 \\ .8 & 1 & .4 & 0 & .9 \\ 0 & .4 & 1 & 0 & 0 \\ .1 & 0 & 0 & 1 & .5 \\ .2 & .9 & 0 & .5 & 1 \end{bmatrix}$$

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$\because \underset{\sim}{R} = \underset{\sim}{R}^T$, $\therefore \underset{\sim}{R}$ is a symmetric relation.

$\therefore \underset{\sim}{R}$ is a tolerance relation.

Apply $\lambda = .8$

$$(\underset{\sim}{R})_{.8} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$(\underset{\sim}{R})_{.8}$ is a reflexive relation. Because diagonal elements are 1.

$$(\underset{\sim}{R})_{.8}^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$(\underset{\sim}{R})_{.8} = (\underset{\sim}{R})_{.8}^T$. $\therefore (\underset{\sim}{R})_{.8}^T$ is a symmetric relation.

$\therefore (\underset{\sim}{R})_{.8}^T$ is a tolerance relation.