5/08/19

Module : 1

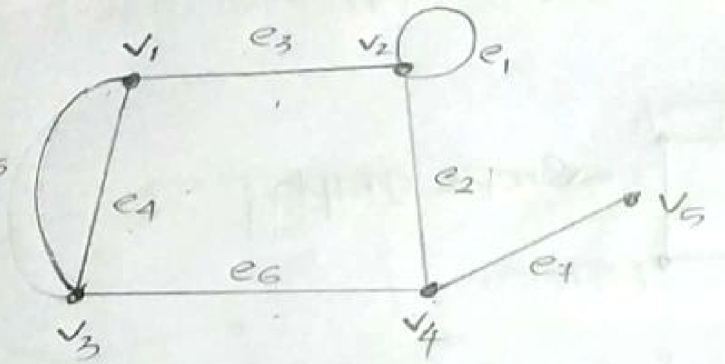
Introduction

GRAPH

— It consist of vertices & edges.

A Graph, $G = \{V, E\}$ consist of a set of objects $V = \{v_1, v_2, v_3, \ldots\}$ called vertices and another set of objects $E = \{e_1, e_2, e_3 \ldots\}$ called edges such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.

→ Represent the graph mathematically:



$G = \{V, E\}$

$V = \{v_1, v_2, v_3, v_4, v_5\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

- The vertices v_i, v_j associated with edge e_k are called the end vertices of e_k.

- An edge having the same vertex as both its end vertices is called a self-loop

- When more than one edge is associated with a given pair of vertices, the edges are referred as parallel edges.

- A graph that has neither self-loops nor parallel edges is called a simple graph.

Q Identify the end vertices of e_2, e_6:

$e_2 - v_2, v_4$

$e_6 - v_3, v_4$

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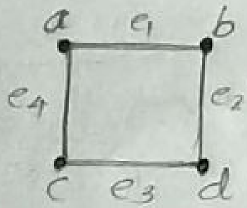
Q Identify the self loop and parallel edges:

self loop $- e_1$

parallel edges $- e_4, e_5$

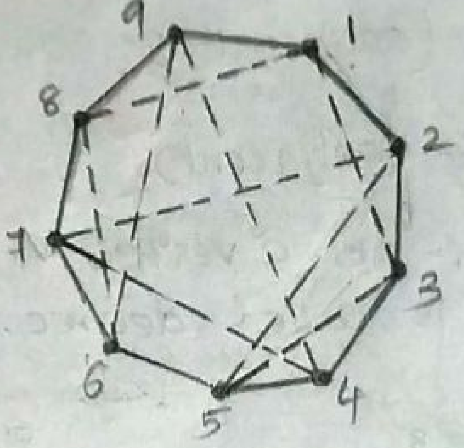


→ Simple graph.

Applications of graph

1. Seven bridges of Konigsberg problem.
2. Utilities Problem
3. Electrical Networks Problem.
4. Seating problem.

9 members of a club decide to meet everyday for lunch at a round table. They decide to sit such a way that each member has different neighbours at each lunch. For how many days can this arrangement last?



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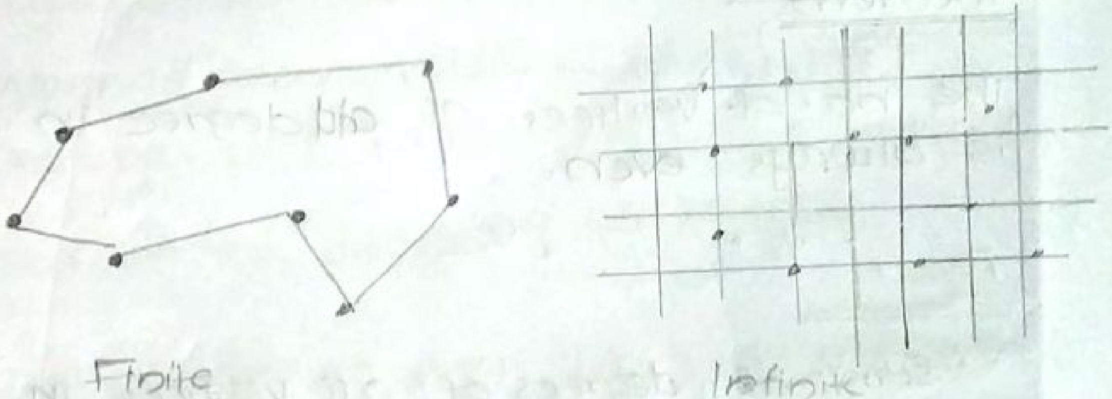
Finite graph & Infinite graph

Finite graph

The graph with finite no. of vertices and finite no. of edges.

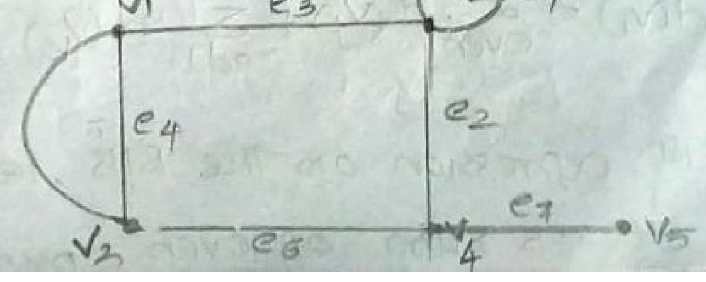
Infinite graph

The graph with infinite no. of vertices and edges.



Finite Infinite

- When vertex v_i, it is an end vertex of some edge e_j, v_i and e_j are said to be incident with each other.
- Two non-parallel edges are said to be adjacent if they are incident on common vertex



e_4 & e_6 — v_3 (adjacent)

e_2, e_7 & e_6 — v_4 (adjacent)

- No. of edges incident on a vertex v_i with self loop rounded twice is called degree. (valency)

degree of $v_1 = 3$

Proof

Sum of degrees of all vertices in graph 'G' is twice the no. of edges in G.

$$\boxed{\sum_{i=1}^{n} d(v_i) = 2e}$$

e = No. of edges.

Theorem

The no. of vertices of odd degree in a graph is always even.

Proof

Sum of degrees of all vertices in 'G' is given by

$$\sum_{i=1}^{n} d(v_i) = 2 \times e$$

If we consider vertices with odd degrees & even degrees respectively. Above qn can be written as

$$\sum_{i=1}^{n} d(v_i) = \sum_{even} d(v_j) + \sum_{odd} d(v_k)$$

The 1st expression on the RHS, ie $\sum_{even} d(v_j)$ is even. since it is sum of even numbers.

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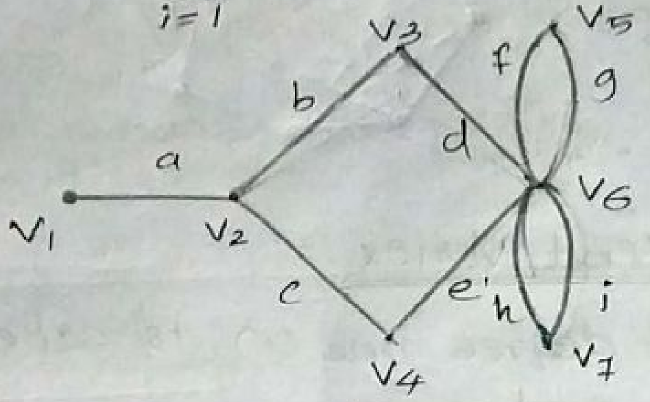
But the 2nd term on the RHS ie, $\sum_{odd} d(v_k)$ should also be even.

$\because \sum_{i=1}^{n} d(v_i) = 2 \times e.$



i) Edges incident with v_4 : c, e

ii) Edges incident with v_6 : f, g, h, i, d, e

iii) Write any pair of adjacent edges :- d & e, b & c

iv) Example of non-adjacent edge :- b & g, d & a

v) Adjacent vertices : v_1 & v_2, v_2 & v_3

vi) Non adjacent vertices : v_1 & v_4, v_2 & v_6

vii) Any parallel edge : ~~a & d~~ f & g.

viii) Findout the degree of all vertices.

$d(v_1) = 1$, $d(v_2) = 3$, $d(v_3) = 2$, $d(v_4) = 2$

$d(v_5) = 2$, $d(v_6) = 6$, $d(v_7) = 2$

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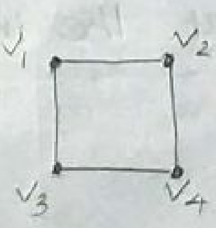
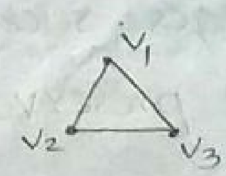
Regular graph

A graph in which all vertices have the same degree is called Regular graph.

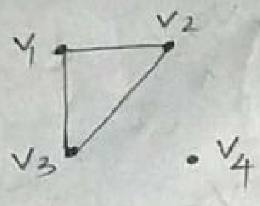
eg:





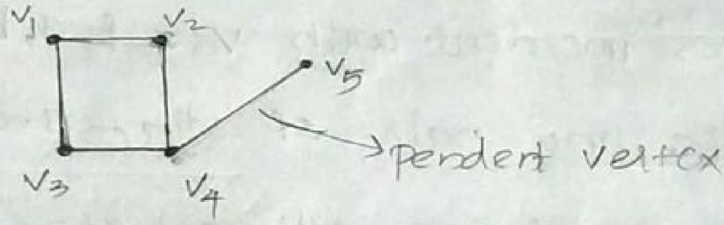
Isolated vertex

Vertex having no incident edge is called isolated vertex.



Pendent vertex / End Vertex

A vertex of degree one (1) is called pendent vertex or end vertex.



Null graph

Graph without any edges is called Null graph.

Every vertex in a null graph is a isolated vertex.

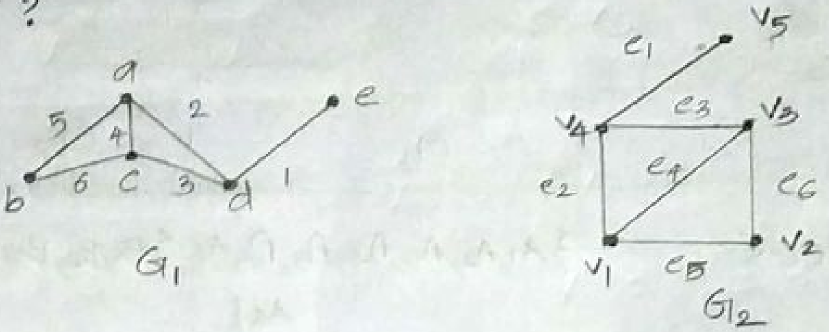
ISOMORPHISM

Two graphs G, G' are said to be isomorphic, if there is one to one correspondance b/w their vertices and edges, such that the incident relationship is preserved.

isomorphic graph must have

* same no. of edges
* same no. of vertices.
* equal no. of vertices with given degree.

? Check whether the graph is isomorphic or not ?



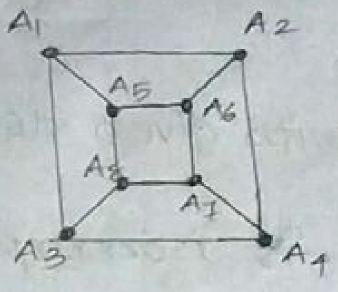
Vertices of G_1 {a, b, c, d, e}

	G_1	G_2
Vertices	{a, b, c, d, e}	{v_1, v_2, v_3, v_4, v_5}
No. of vertices	5	5
Vertices with degree arranged in descending order	a, c, d, b, e 3 3 3 2 1	v_1, v_3, v_4, v_2, v_1 3 3 3 2 1
Total no. of edges = (Sum of degrees of all vertices) / 2	$\frac{3+2+3+3+1}{2} = 6$	$\frac{3+2+3+3+1}{2} = 6$

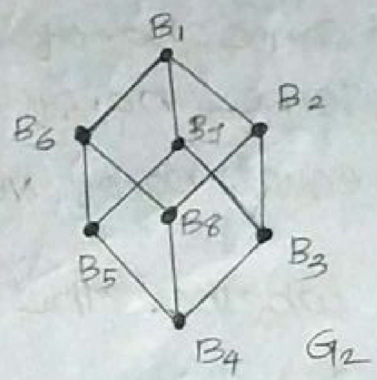
G_1 & G_2 have same no. of vertices & edges.

So G_1 & G_2 are isomorphic.

? Check whether the 2 graphs are isomorphic or not.



G_1



G_2

	G_1	G_2
Vertices	$\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$	$\{B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8\}$
No. of Vertices	8	8
Vertices with degree arranged in descending order	$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$	$B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8$
Total no. of edges	$\frac{(3+3+3+3+3+3+3+3)}{2} = 12$	$\frac{(3+3+3+3+3+3+3+3)}{2} = 12$

G_1 & G_2 have same no. of edges & vertices.

So G_1 & G_2 are isomorphic.

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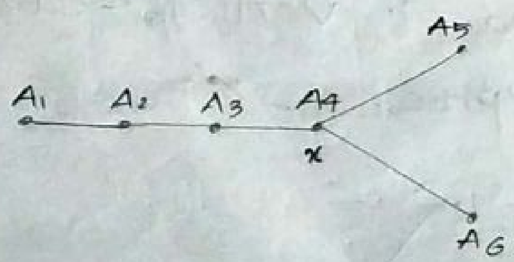
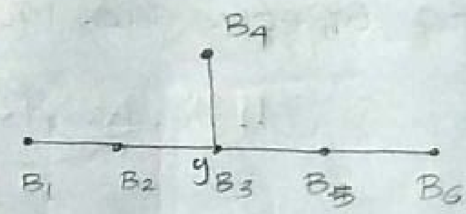
? Check whether the 2 graphs are isomorphic or not.



G_1



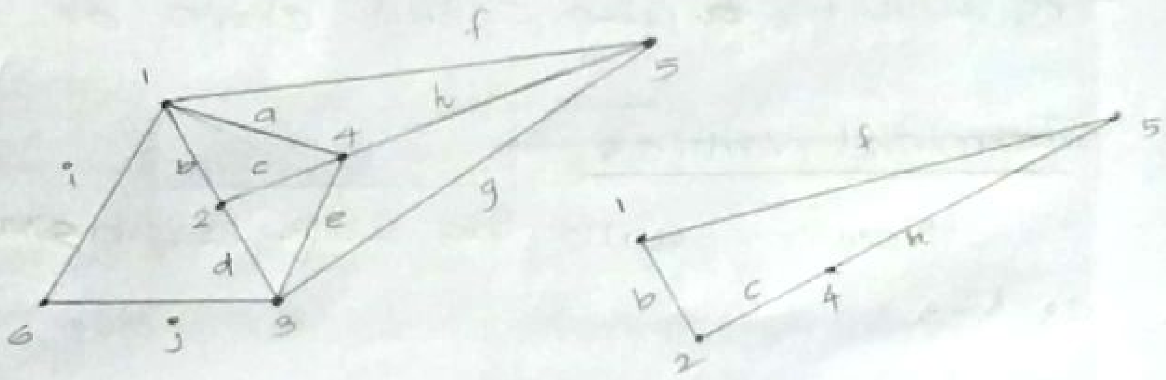
G_2

If the 2 graphs G_1 & G_2 are isomorphic the vertex 'x' must corresponds to vertex 'y'. Because there are no other vertices having degree 3.

So G_1 & G_2 are not isomorphic.

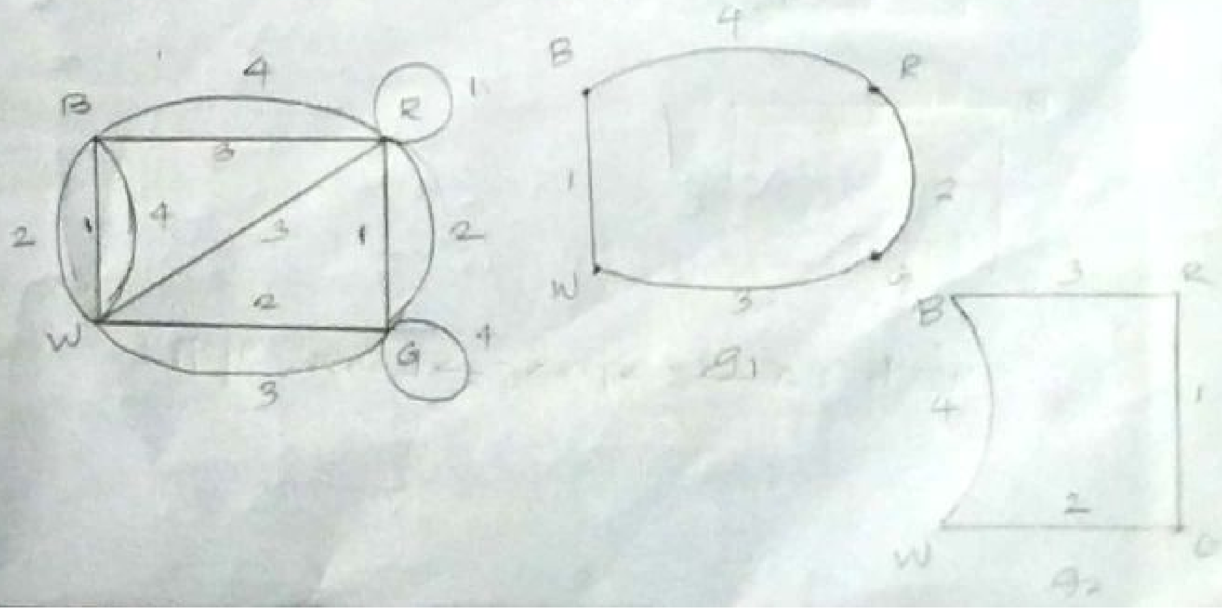
Subgraph

The graph 'g' is said to a subgraph of 'G' if all vertices and all edges of 'g' are in 'G'. And each edge of 'g' has the same end vertices ~~as~~ has in 'G'.



Edge disjoin subgraphs

Two subgraphs 'g_1' & 'g_2' of a graph 'G' are said to be edge disjoin if g_1 & g_2 do not have any edge in common.



Walk

Walk is defined as ~~no~~ a finite alternating sequence of ~~edges~~ vertices-edges, beginning & ~~ed~~ ending vertices such that each edges incident with preciding vertex as well as the vertex following it.

- No edge should appear more than once in a walk.
- A Vertex may appear more than once in a walk 19/0
- A walk is ~~a~~ also called chain or train.

Terminal vertices

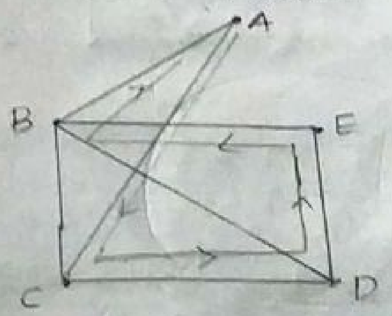
Vertices with the walk begins and ends.

closed walk

If a walk begins and ents with same vertex.

open walk

If a walk is not closed.



walk : A → C → D → E → B → A

? Identify whether the following statements is walk or not.

1) B → B → E → B → C → walk

2) E → C → D → E → Not walk

3) A → B → C → D → B → A → Not walk

Path

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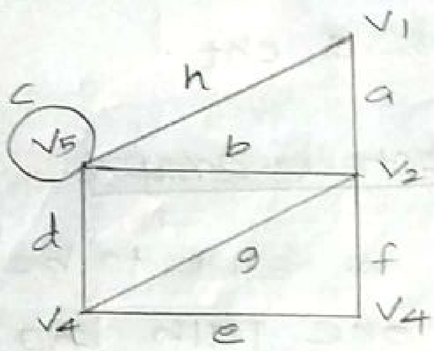
A path is defined as an open walk in which no vertex appears more than once.



$v_1 \to v_2 \to v_5 \to v_4$: It is a path

Path donot contains cycle.

$v_1 \to v_5 \to v_2 \to v_4$

length of path

- No. of edges in path

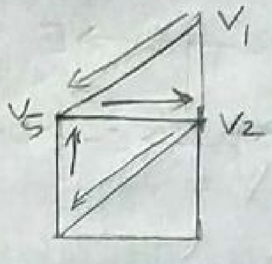
* Self loop can be included in a walk but not in a path

* Terminal vertices of a path are of degree 1 and rest of the vertices ~~have~~ are of degree 2

Circuit

A closed walk in which no vertex appears more than one. is called circuit.

$v_1 \to v_3 \to v_2 \to v_4 \to v_5$



- A ckt is also called a cycle, elementary cycle, circular path & polygon.
- Every self loop is a ckt.

Connected & disconnected graph

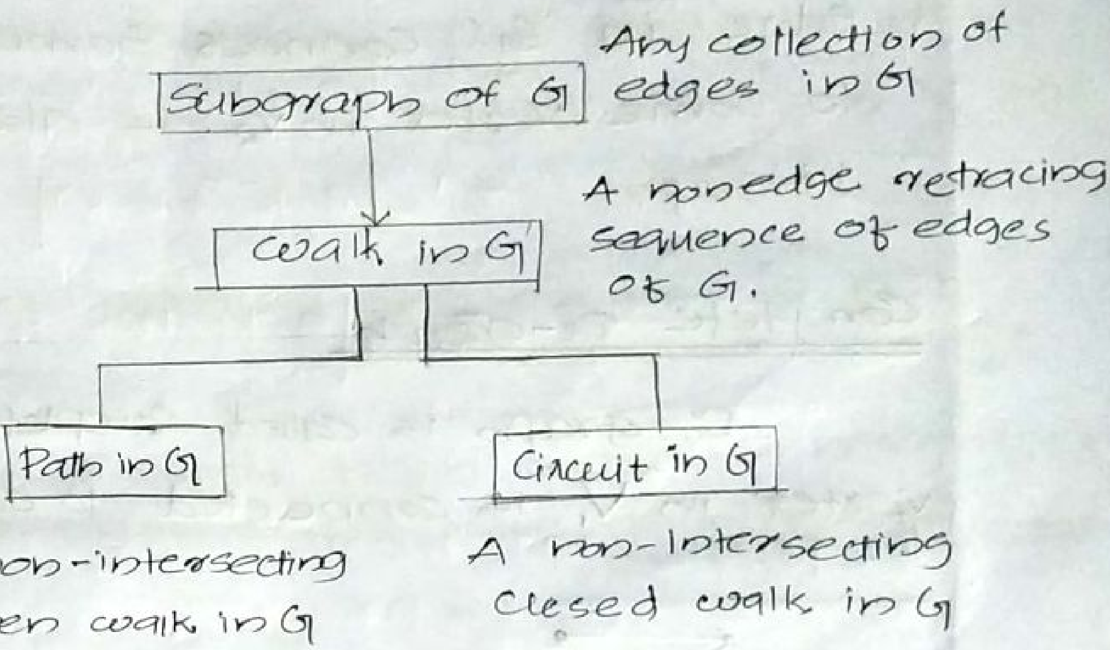
A graph 'G' is said to be connected if there is atleast one path b/w every pair of vertices in 'g'

A graph which is not a connected then called disconnected graph.

- disconnected graph consists of two or more connected graphs.
- Each of these connected subgraphs are called component.

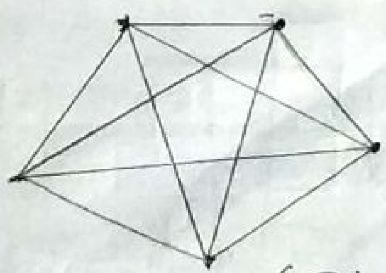
Theorem

A graph 'G' is disconnected if and only if its vertex set 'V' can be partitioned into 2 non-empty disjoint subsets V_1 & V_2 such that there exists no edge in 'G' whose one end vertex is in subset V_1 and another is in V_2.



Complete Graph

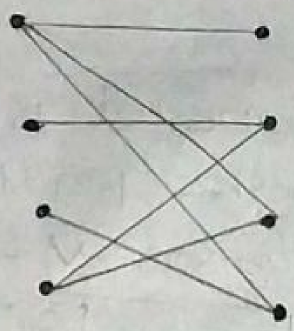
It is a simple graph in which each pair of distinct ~~edges~~ vertices is joined by an edge.



Bipartite Graph (Bi graph)

It is a set of vertices decomposed into 2 disjoint sets such that no 2 graph vertices within the same set are adjacent.

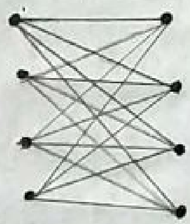
Bi-graph



* A graph 'G' in which 'V' can be partioned into 2 subsets V_1 & V_2. so that each edge in 'G' connects some vertex in V_1 to some vertex in V_2. is also called bi-graph.

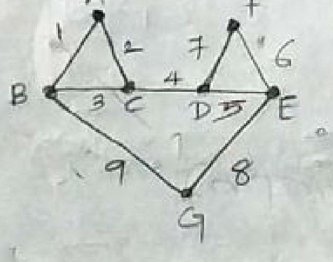
Complete Bi-graph

Bi-graph is called complete, if each vertex in 'V_1' is connected to each vertex in 'V_2' 20



Weighted graph

A graph in which weights are assigned in every edge.



A graph 'G' is disconne

Theorem

A graph 'G' is disconneted [previous pg]

Proof :

Suppose that such a partitioning exist.

consider a arbitrary vertices 'a & b' such that $a \in V_1$ and $b \in V_2$. No path can exist b/w the vertices a&b. Otherwise they would be atleast one edge whose one end vertex is in V_1 and the other in V_2.

Hence, partition exist or 'G' is not connected.

20/08/19 ? Prove that a simple graph with ~~each~~ 'n' vertices and 'k' components can have $\frac{(n-k)(n-k+1)}{2}$ edges.

To proove this we use the algebraic inequality. $\sum_{i=1}^{k} n_i^2 \leq n^2-(k-1)(2n-k)$.

Maximum no. of edges in a graph with 'n' vertices is $\frac{n \times (n-1)}{2}$.

Maximum no. of edges in the i^{th} component is $\frac{n_i(n_i-1)}{2}$.

$\therefore$ Total no. of edges in each of the 'k' components.

$$= \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}$$

$$= \sum_{i=1}^{k} \frac{n_i^2-n_i}{2}$$

$$= \tfrac{1}{2} \sum_{i=1}^{k} n_i^2 - \sum_{i=1}^{k} n_i$$

$$= \tfrac{1}{2}\left[\sum_{i=1}^{k} n_i^2 - n \right]$$

$$= \tfrac{1}{2}\left[n^2 - (k-1)(2n-k) - n\right]$$

$$= \tfrac{1}{2}\left[n^2 - 2nk + k^2 + 2n - k - n\right]$$

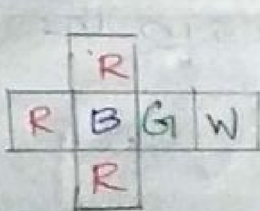
$$= \tfrac{1}{2}\left[n^2 + k^2 - 2nk - k + n\right]$$

$$= \tfrac{1}{2}\left[(n-k)^2 + (n-k)\right]$$

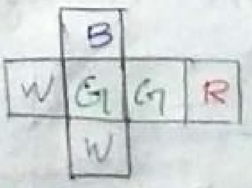
$$= \tfrac{1}{2}\left[(n-k)((n-k)+1)\right]$$

? The puzzle with multicoloured cubes. We are given 4 cubes. 6 phases of every cube is coloured blue, green, Red or white.

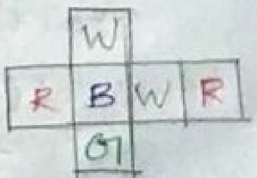
i) is it possible to stack the cubes one on top of the other to form a coloumn such that no colour appears twice on any of the 4 side of the column.



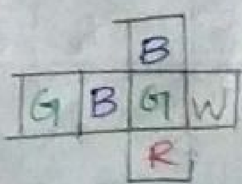
Cube 1



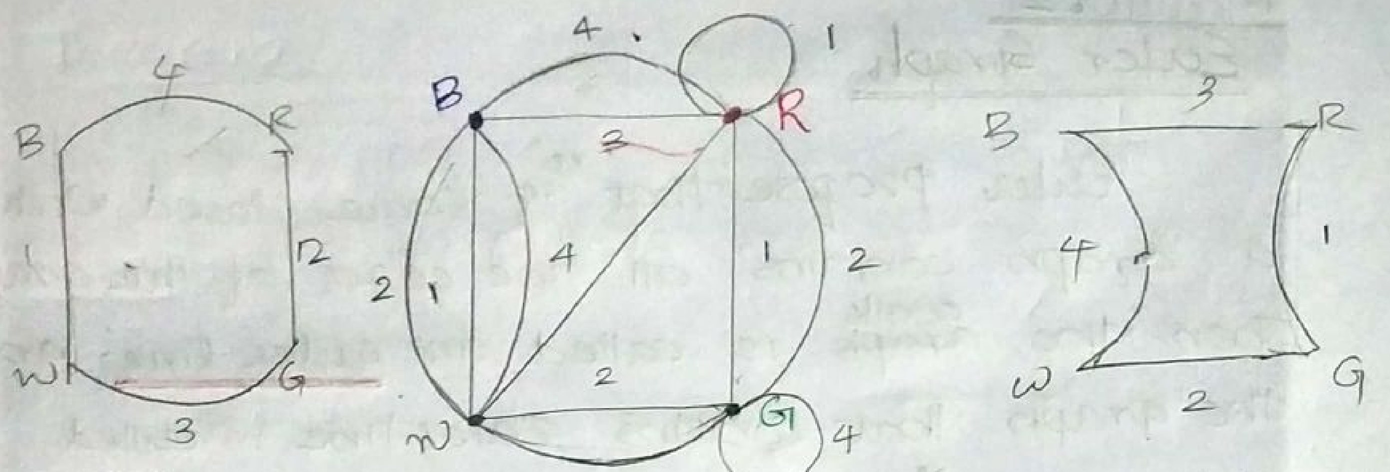
Cube 2



Cube 3



Cube 4



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Q? Prove that Maximum no. of edges in a graph 'G' having small end vertices $\frac{n(n-1)}{2}$.

→ The 4 cubes can be ~~[illegible]~~ ~~[illegible]~~ arrange iff their exist 2 edge disjoint subgraph each with 4 edges and each if the edges label differently and such that each vertex is of degree 2.