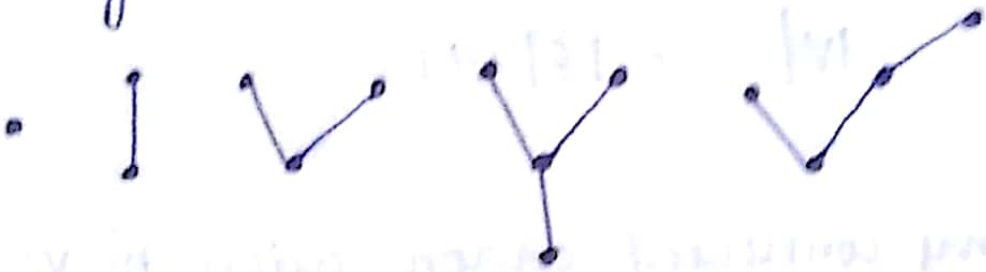
27/09/17

MODULE 3

Tree :- connected graph without any circuits b/w ~~bew~~ a pair of vertices there exist only one edge.



- A tree doesn't have self loop and parallel edges.

Properties

THEOREM - 1

There is one and only path b/w ~~Tree T~~ every pair of vertices in Tree T.

~~PROOF~~ THEOREM - 2

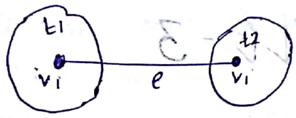
If in a graph G there is only one path b/w every pair of vertices, G is a tree.

THEOREM - 3

A tree with n vertices has n-1 edges.

$|V| = |V_1| + |V_2|$

$|E| = |E_1| + |E_2| + 1$.



$|E_1| = |V_1| - 1$ $|E_2| = |V_2| - 1$

$|V| = |V_1| + |V_2| = (E_1 + 1) + (E_2 + 1)$

$= (E_1 + E_2 + 1) + 1$

$|V| = |E| + 1$

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Any connected graph with n vertices and n-1 edges is a tree.

Suppose - T is not a tree

So, there exists circuits.

1 edge remove

remove about k edges.

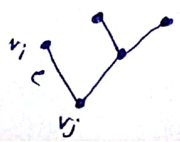
n vertices and n-1-k edges

contradiction

T is a tree.

THEOREM A Graph is a tree if and only if it is minimally connected.

Minimally connected : If we remove any edge it will be a disconnected graph.



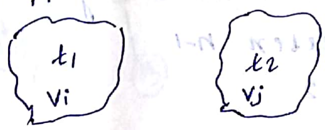
tree → only one path b/w (v_i v_j)

v_i (e) v_j remove

THEOREM A graph G with n vertices and n-1 edges and no circuits is connected.

To prove : It is connected

Suppose it is a disconnected graph.



Let us consider v_i in t_1 and v_j in t_2 add an edge between v_i and v_j.

n vertices

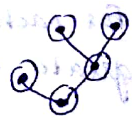
n edges.

Contradiction

→ Pendant Vertex

$$\sum_{i=1}^{n} d(v_i) = 2e$$

V	E	Sum(d)
n	n-1	2(n-1)
5	4	2(n-1) = 2×4 = 8



$2(n-1)$ is divided b/w n vertex
there will be atleast two pendant vertices.

IV In any tree with $n \geq 2$ there are atleast two pendant vertices

PROOF:

Suppose that one vertex is degree with one. If remaining vertex n-1

$d(n-1) = 1 \geq 2$

$\sum_{i=1}^{n} d(v_i) = d(v_1) + d(v_2) + \dots + d(v_n)$

$\geq 1 + 2 + 2 + \dots + 2$

$\geq 1 + 2(n-1)$

Contradiction

$\therefore$ atleast two vertex with degree one.

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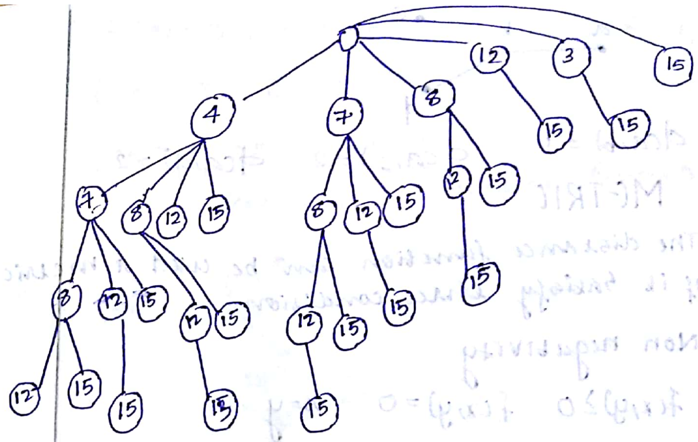
Monotonically Increasing Subsequence.

For each subsequence those number greater than the given number

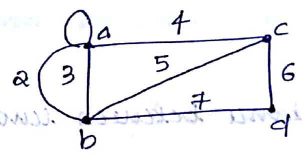
eg: $S = \{4, 7, 8, 12, 3, 15\}$



HW (4, 1, 13, 7, 0, 2, 8, 11, 13)

DISTANCE & CENTRES

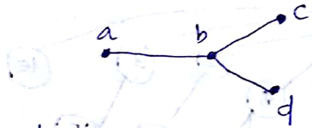
Shortest path between two vertices is called distance.



d(a,c) (4) – 1
(1,4) – 2
(3,5) – 2
(2,5) – 2
(3,7,6) – 3

distance = 1

eg:2



d(a,b) = 1   d(a,c) = 2   d(a,d) = 2

MGTRIC

The distance function can be used a metric if it satisfy three condition.

→ Non negativity
$f(x,y) \geq 0$ $f(x,y) = 0$ $x = y$

→ Symmetry
$f(x,y) = f(y,x)$

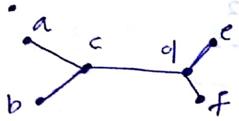
→ Triangle property
$f(x,z) + f(z,y) \geq f(x,y)$

ECCENTRICITY

The long distance between two vertex.

$E(V) = \max d(v, v_i)$

From the above eg:2 we get $E(c) = 2$



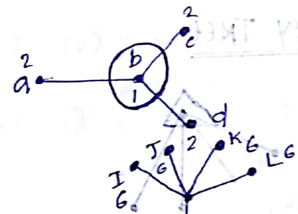
$E(a) = 3$, $E(b) = 3$, $E(c) = 2$, $E(d) = 2$

$E(e) = 3$ $E(f) = 3$

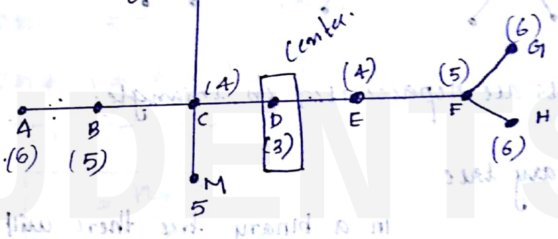
CENTER OF A GRAPH

Vertex with shortest Eccentricity

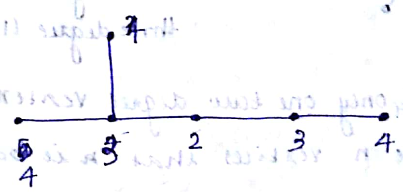
eg:1



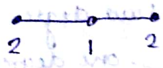
eg:2



1) Remove all pendant vertex



2) Repeat the process



3. Again remove

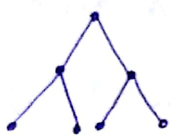
Radius: Eccentricity of centre vertex

Diameter: Max eccentricity of a graph

5/10/17 ROOTED AND BINARY TREE

Roots are represented in triangle.

binary tree



In a binary tree there will be vertex with one degree Pendants
two degree
three degree (Internal)

- There is only only one two degree vertex ie root
- If there are n vertices that n ie odd.

1 - two degree

(h-1) < one degree / three degree } even [Theorem in ~~the~~ first

∴ n is odd.

P - pendant vertex.

In a tree there is n vertices, n-1 edges.

$\Sigma d(v) = 2e$

$\Sigma d(v) = 1\times 2 + 1\times P + 3(n-1-P) = 2(n-1)$

$2(n-1) = 2 + P + 3n - 3 - 3P$

$= 2 - 2P + 3n - 3$

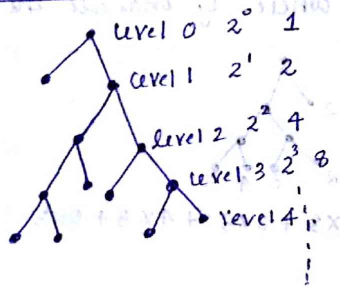
$2n - 2 = -1 - 2P + 3n -$

$2n - 3n = 2 - 1 - 2P$

$5n = 1 - 2P$

$$\boxed{p = \frac{n+1}{2}}$$

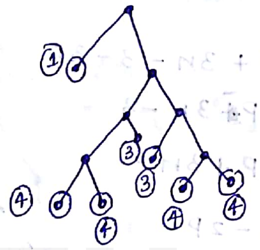
Internal Vertex



Each vertex v_i at level l_i emigrate that length of the vertex from root:

max r $2^0 + 2^1 + 2^2 + 2^3 + \dots 2^k \le n$

→ **Path length of a tree:-**

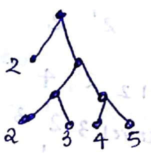


• Length from pendant vertex to root vertex is the Path length of a tree.

→ **weighted path length:-**

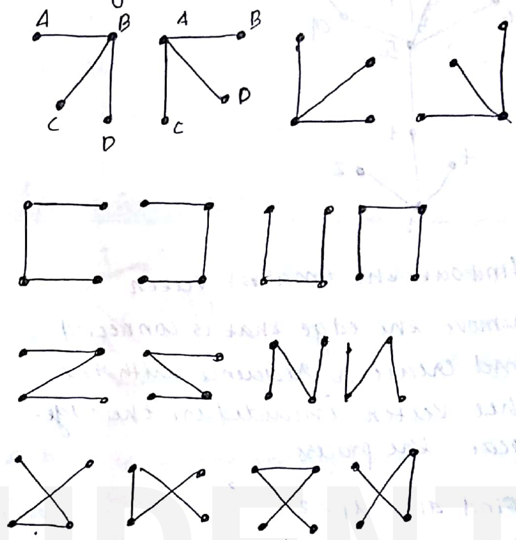
Each pendant vertex will be given a weight

$\sum W_j \cdot l_j$ where l_j indicate the level.



$2\times2 + 2\times3 + 3\times3 + 4\times3 + 5\times3$

9/10/17 Counting trees

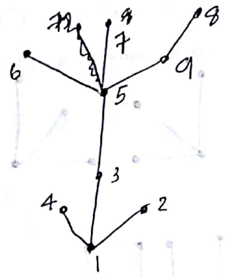


max No: of different trees possible with 'n' no: of labeled vertices

Labelled vertices: when we give a particular name to a vertex.

THEOREM: The no: of labelled trees with n vertices where $n \ge 2$ is n^{n-2}

we have to prove that the below tree has 9^7 possibility.



step 1: find out the smallest verten

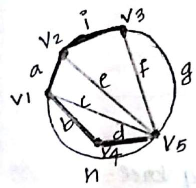
step 2: Remove the edge that is connected and create a sequence with the other verten included in the edge.

step 3: Repeat the process

- step 1: find a1 , $a_1 = 2$
  step 2: remove edge a
  add $b_1 = 1 \rightarrow$ sequence.
- $a_2 \rightarrow 4$ $a_2 b_2 \rightarrow 1, 4$
  add $b_2 = 1$
  $a_3 \rightarrow 1$ $a_3 \rightarrow 3$
- $a_4 - 3$ $b_4 - 5$
- $a_5 - 6$ $b_5 - 5$
- $a_6 - 7$ $b_6 - 5$
- $a_7 - 5$ $b_7 - 9$

sequence
[1, 1, 3, 5, 5, 5, 9]
there are n-2 ~~posn~~ sequence
$\therefore n^{n-2}$ possibilities

Spanning Trees



- It is a subgraph of G, it includes all the vertices, it should be a tree.

i a b d is a spanning tree
f i a b
g f i e c b
g i a b.

→ In every graph there is minimum one spanning tree

→ if no circuit then it is a spanning tree.

Branches & chords

- Edges of spanning tree is called branches
- Edges that don't form spanning tree is called chord.

In a spanning tree of a connected graph with n vertices and edges with $(n-1)$ trees, branches and $(e-n+1)$ chords.

Rank & Nullity

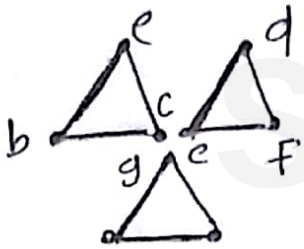
(R) Rank: no:of branches

(μ) Nullity: no:of chords

Disconnected graph:

Every components have a spanning tree.

eg:



$rank = n-k = 9-3 = 6$

$\mu = e-n+k$

$= 9-9+3 = \underline{\underline{3}}$