Applications of Wavelet Transform

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***Abstract*—Wavelet transform is a mathematical tool that can be used for time-frequency analysis of signals and images. It has gained popularity in signal and image processing applications over the past few decades due to its ability to provide multi-resolution analysis, noise reduction, compression, segmentation, classification, and medical image fusion. Wavelet transform is also used in various areas such as ECG and EEG signal processing, image compression, reinforcement, edge detection, and registration. Wavelet transform has several advantages over the classical Fourier transform or short-time Fourier analysis in signal and image processing. It is particularly useful for analyzing non-stationary signals such as those represented by traveling wave equations. The continuous wavelet transform (CWT) is used to map the behaviour of a signal in the x-t domain into the k- domain, which cannot be performed by the Fourier/Laplace transform. The wavelet transform has various implementations such as the complex discrete wavelet transform (CDWT), interval wavelet construction, and 3D-DWT techniques.**

***Index Terms*—Wavelet transform,image processing,denoising**

I. Introduction

Mathematical transformations occur when there is a change in position, shape, or size. In a transformation, the original figure is a preimage and the resulting figure is an image[1].
The Fourier or Laplace transform has been a useful tool in Mathematics for almost two centuries, but it has limitations when dealing with signals represented by traveling wave equations, which have existence in both time and spatial domains. A transformation is needed to map the behaviour of the signal in the $x-t$ domain into the $k-\omega$ domain, which requires a 2×2 mapping process that the Fourier/Laplace transform cannot perform[2].
Wavelet transforms can be used for time-frequency analysis of discrete signals as an alternative to the short-time Fourier transform, with minimal mathematical theory required for understanding and implementation .Wavelet theory was initially developed by a group of French mathematicians and physicists, but its application to digital signal processing was later defined by American researchers Daubechies and Mallat [3].
The continuous wavelet transform (CWT) is defined as[4]

$$\psi_{\alpha\beta}(t)= \frac{1}{\sqrt{\alpha}}\psi(\frac{t-\beta}{\alpha}).$$

The CWT maps a function $f(t)$ onto time scale space by[1].

$$W_f(\alpha,\beta) = \int_{-\infty}^{\infty} \psi_{\alpha\beta}(t) f(t)\, dt \ \ =< \psi_{\alpha\beta}(t), f(t) >$$

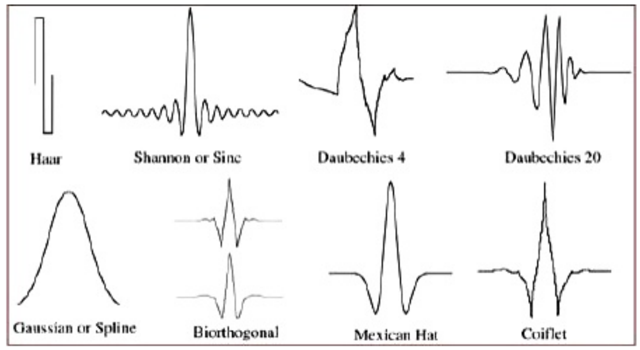

Fig. 1. Different Wavelets[5]

II. Applications of Wavelet Transform in Signal and Image Processing

Wavelet transforms have a lot of applications in the field of signal and image processing. In [6] the author reviews the increasing popularity of wavelet transform and multiresolution analysis in image processing over the past 25 years. The authors present over 190 recent papers on industrial applications of these techniques, highlighting their usefulness in various fields.

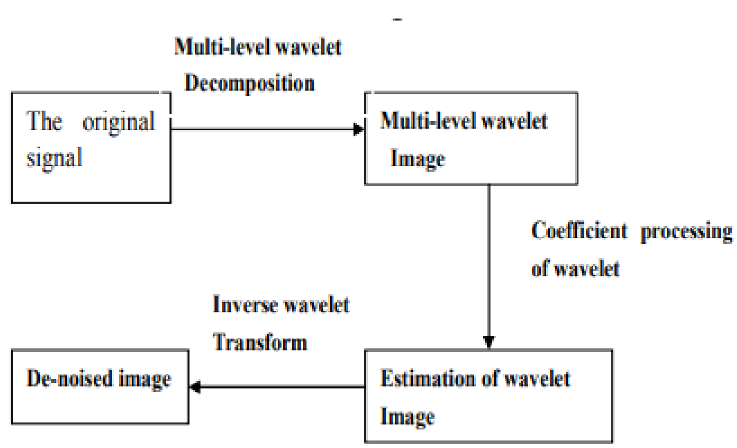

Fig. 2. The de-noising process based on wavelet transform[7]

In article [8] the authors review the basic properties of wavelet transforms in continuous and discrete versions both in one and two dimensions. Also some of the applications of wavelet transforms in signal and image processing were described. The extension of wavelet transform into higher dimensions and to the space time context, for the analysis of moving objects were also considered.

In [9] author presents a survey of the advantages and trends in medical image processing techniques based on multidimensional wavelet transforms and their combinations with other methods. These methods model each coefficient set as a tensor, which makes computing easier and useful for various medical processing methods, such as noise reduction, compression, segmentation, classification, and medical image fusion, particularly for multidimensional images such as MRI and CT data.

In article [10] the author provides a review of various applications of wavelet transforms in medical imaging, including ECG and EEG signal processing, image compression, reinforcement, edge detection, and registration. With the continued advancement of wavelet theory, the use of wavelet transforms is expected to become increasingly widespread in the medical imaging field.

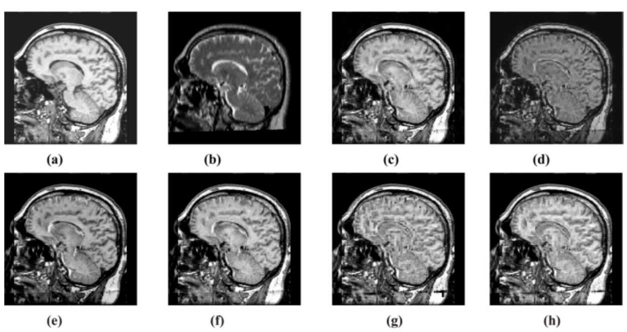

Fig. 3. (a) MRI-T1, (b) MRI-T2, image fusion results using (c) Curvelet, (d) Contourlet, (e) Laplacian pyramid, (f) NSCT, (g) DFRWT (p = 1), (h) DFRWT (p = 0.7)[11]

In [12] the authors present two new interval wavelet constructions that can be used for image compression and upscaling. By analyzing functions in a piecewise manner, the interval wavelet transform can generate sparser representations around discontinuities, avoiding Gibbs-like phenomena caused by quantization of wavelet coefficients. These new constructions offer an improved approach to signal approximation compared to classical wavelet transforms.

The paper [13] discusses the application of various discrete wavelet transform techniques such as Haar, Daubechies, and symlet 2 on images for analysis purposes. A comparative analysis is performed, and the responses of different filters are plotted using MATLAB 15, highlighting the advantages and disadvantages of each technique. The wavelet transform technique is emphasized as a useful tool for analyzing non-stationary signals.

In article [14] the author discusses the use of complex discrete wavelet transform (CDWT) in various signal and image processing techniques, particularly in image denoising. CDWT generates complex coefficients using a dual tree of wavelet filters, providing high shift-invariance and directionality compared to real DWT. The paper presents a theoretical analysis and verification of CDWT for simulated images, followed by its application in denoising magnetic resonance biomedicalimages.

The paper [15] discusses the advantages of wavelet transform over classical Fourier transform or short-time Fourier analysis in signal and image processing. The study explores the potential of wavelet transform in analyzing Acoustic Emission signals during quasi-static testing of composite materials, including optimal wavelet choice, decomposition, de-noising, and compressing of AE waveforms. The goal is to identify different damage mechanisms within the material structure contributing to the AE waveforms, and the results are evaluated and thoroughly discussed.

The article [16] proposes an image segmentation method based on wavelet transform to retain image details and automatically determine segmentation thresholds. The method decomposes the image based on wavelet transform theory to obtain detailed information of horizontal, vertical, and diagonal directions. Results show that the proposed method has better visual effect and advantages in medical image segmentation compared to traditional methods, providing a new direction for further research in this field.

In [17] the author proposes a DWT and Gabor technique for enhancing low resolution satellite images. The proposed method is compared to existing methods, such as DWT-based interpolation, SWT-based interpolation, and bicubic interpolation, and is shown to have advantages in terms of complexity, noise removal, and image sharpness, as well as higher PSNR and lower MSE.

The paper [18] focuses on image de-noising of natural images corrupted by Gaussian noise using wavelet techniques. The study investigates the suitability of different wavelet bases and neighborhood sizes on the performance of image de-noising algorithms in terms of PSNR. The proposed wavelet de-noising scheme thresholds the wavelet coefficients arising from the standard discrete wavelet transform.

In [19] the authors propose a contextual PolSAR image segmentation method that utilizes a channel-wise consistent feature set and the three-dimensional discrete wavelet transform (3D-DWT) technique to extract discriminative multi-scale features that are robust to speckle noise. Markov random field (MRF) is further applied to enforce label smoothness spatially during segmentation. Experimental results on three real benchmark PolSAR image datasets indicate that the proposed method achieves promising segmentation accuracy and spatial consistency using a minimal number of labeled pixels.

The article [20] proposes a new motor imagery classification scheme for brain-computer interface (BCI) systems using electroencephalography (EEG) data. The proposed method combines CWT with three mother wavelets and a convolutional neural network to improve classification performance and reduce computation complexity. Experimental results using two public BCI datasets demonstrate improved classification performance compared to existing methods, showing the feasibility of motor imagery BCI systems.

In [21] the authors propose an optical image watermarking method using singular value decomposition ghost imaging (SVDGI) and a blind watermarking algorithm. The encrypted watermark image is embedded in the host image using block DCT and integer LWT. The proposed method is shown to have good imperceptibility and robustness, and a secret key is needed for authorized users to decrypt and reconstruct the original watermark image. The feasibility of the proposed method is validated through both optical experiments and numerical simulations.

Article [22] provides a review on the use of fractional Fourier transform (FrFT) in the biomedical field. FrFT is a time-frequency analysis tool that can capture the non-stationary characteristics of biomedical signals, making it suitable for signal detection, filtering, and feature extraction. The review highlights the need for proper fractional order estimation and implementation criteria for discrete FrFT calculation in developing new biomedical tools.

In [23] the author explains the versatility of wavelet analysis in the separation sciences, particularly in detecting signals that vary in time or space and separating them from noise. The concepts of continuous and discrete wavelet transforms are explained, along with their applications in chromatography. An example of denoising a low signal-to-noise ratio chromatogram using wavelet analysis is provided, showing significant improvements in signal-to-noise ratio and the recovery of previously invisible peaks.

The article [24] explains a study which usesa redline laser source and a CMOS camera to capture speckle line images of specimens produced by electrical discharge machining (EDM). The images were then decomposed using a wavelet transform to obtain surface roughness parameters, and it was found that the RMS and variance of the 4th level decomposition correlated well with surface roughness parameters.

In [25] author proposes a new steganography technique based on a new 3D sine chaotic map to increase the security and key space of the proposed algorithm. The technique shows satisfactory performance, acceptable image distortion, and stronger robustness against some attacks, making it a good alternative to existing methods.

In [26] author proposes a method to enhance low-contrast images captured by side scan sonar, which is used to capture images of the seafloor. The method uses Stationary Wavelet Transform to decompose the input image and apply a Laplacian filter to sharpen the low frequency component. The enhanced image is obtained by adding a mask to the input image and then reconstructing the high-contrast image, and the proposed method outperforms state-of-the-art techniques in terms of contrast.

The article [27] proposes a reversible data hiding scheme in encrypted images using integer wavelet transform. The encrypted image is first decomposed into sub-bands, and location maps are generated to embed secret data into high frequency wavelet coefficients. The proposed algorithm improves the embedding capacity and reconstructed image quality compared to existing schemes, and allows for multi-level embedding and adaptive correction.

III. Conclusion

Wavelet transform is a powerful mathematical tool for time-frequency analysis of signals and images. It offers a multi-resolution analysis that can be used for noise reduction, compression, segmentation, classification, and medical image fusion. Wavelet transform has many advantages over classical Fourier transform or short-time Fourier analysis, particularly in analyzing non-stationary signals. The CWT can map the behaviour of a signal in the x-t domain into the k- domain, which is not possible with Fourier/Laplace transform. Various implementations of wavelet transform, such as the complex discrete wavelet transform (CDWT), interval wavelet construction, and 3D-DWT techniques, have been developed and applied in various fields such as ECG and EEG signal processing, image compression, reinforcement, edge detection, and registration. Overall, wavelet transform is a versatile and powerful tool for signal and image processing applications.

References

[1] Gilles, Jérôme. "Empirical Wavelet Transform." IEEE Transactions on Signal Processing 61, no. 16 (August 2013): 3999–4010. https://doi.org/10.1109/TSP.2013.2265222.

[2] Bakhoum EG, Toma C. Mathematical transform of traveling-wave equations and phase aspects of quantum interaction. Mathematical Problems in Engineering. 2010 Jan 1;2010.. https://doi.org/10.1137/130923774.

[3] Bentley PM, McDonnell JT. Wavelet transforms: an introduction. Electronics communication engineering journal. 1994 Aug 1;6(4):175-86.

[4] M. Kedadouche, M. Thomas, and A. Tahan, "A comparative study between empirical wavelet transforms and empirical mode decomposition methods: Application to bearing defect diagnosis," Mechanical Systems and Signal Processing, vol. 81, pp. 88–107, 2016.

[5] Bob McGinty, PhD, PE,Continuum Mechanics,Chapter VIII(B)

[6] Truchetet F, Laligant O. Review of industrial applications of wavelet and multiresolution-based signal and image processing. Journal of Electronic Imaging. 2008 Jul 25;17(3):031102.

[7] Xizhi Z. The application of wavelet transform in digital image processing. In2008 international conference on multimedia and information technology 2008 Dec 30 (pp. 326-329). IEEE.

[8] Antoine JP. Wavelet analysis of signals and images. A grand tour. Revista Ciencias Matematicas (La Habana). 2000 Jul 1;18:113-43.

[9] Georgieva V, Petrov P, Zlatareva D. Medical image processing based on multidimensional wavelet transforms-Advantages and trends. InAIP Conference Proceedings 2021 Mar 8 (Vol. 2333, No. 1, p. 020001). AIP Publishing LLC.

[10] Tian DZ, Ha MH. Applications of wavelet transform in medical image processing. InProceedings of 2004 international conference on machine learning and cybernetics (IEEE Cat. No. 04EX826) 2004 Aug 26 (Vol. 3, pp. 1816-1821). IEEE.

[11] Xu X, Wang Y, Chen S. Medical image fusion using discrete fractional wavelet transform. Biomedical signal processing and control. 2016 May 1;27:103-11.

[12] Lee WS, Kassim AA. Signal and image approximation using interval wavelet transform. IEEE Transactions on image processing. 2006 Dec 19;16(1):46-56.

[13] Thakral S, Manhas P. Image processing by using different types of discrete wavelet transform. InAdvanced Informatics for Computing Research: Second International Conference, ICAICR 2018, Shimla, India, July 14–15, 2018, Revised Selected Papers, Part I 2 2019 (pp. 499-507). Springer Singapore.

[14] Victor M, Alas P. Complex wavelet transform in signal and image analysis. Institute of Chemical Technology, Department of Computing and Control Engineering. Http://taco. poly. edu/WaveletSoftware. 2004.

[15] Loutas TH, Sotiriades G, Kostopoulos V. On the application of wavelet transform of AE signals from composite materials. Proc. of European WG on AE, Berlin. 2004;42:433-45.

[16] Gao J, Wang B, Wang Z, Wang Y, Kong F. A wavelet transform-based image segmentation method. Optik. 2020 Apr 1;208:164123.

[17] Muthukrishnan A, Kumar DV, Kanagaraj M. Internet of image things-discrete wavelet transform and Gabor wavelet transform based image enhancement resolution technique for IoT satellite applications. Cognitive Systems Research. 2019 Oct 1;57:46-53.

[18] Mohideen SK, Perumal SA, Sathik MM. Image de-noising using discrete wavelet transform. International Journal of Computer Science and Network Security. 2008 Jan;8(1):213-6

[19] Bi H, Xu L, Cao X, Xue Y, Xu Z. Polarimetric SAR image semantic segmentation with 3D discrete wavelet transform and Markov random field. IEEE transactions on image processing. 2020 Jun 2;29:6601-14

[20] Lee HK, Choi YS. Application of continuous wavelet transform and convolutional neural network in decoding motor imagery brain-computer interface. Entropy. 2019 Dec 5;21(12):1199.

[21] Wang S, Meng X, Yin Y, Wang Y, Yang X, Zhang X, Peng X, He W, Dong G, Chen H.Optical image watermarking based on singular value decomposition ghost imaging and lifting wavelet transform. Optics and Lasers in Engineering. 2019 Mar 1;114:76-82.

[22] Gómez-Echavarría A, Ugarte JP, Tobón C. The fractional Fourier transform as a biomedical signal and image processing tool: A review. Biocybernetics and Biomedical Engineering. 2020 Jul 1;40(3):1081-93.

[23] Wahab MF, O'Haver TC. Wavelet transforms in separation science for denoising and peak overlap detection. Journal of Separation Science. 2020 May;43(9-10):1998-2010.

[24] Ali JM, Jailani HS, Murugan M. Surface roughness evaluation of electrical discharge machined surfaces using wavelet transform of speckle line images. Measurement. 2020 Jan 1;149:107029.

[25] Valandar MY, Barani MJ, Ayubi P, Aghazadeh M. An integer wavelet transform image steganography method based on 3D sine chaotic map. Multimedia Tools and Applications. 2019 Apr;78(8):9971-89.

[26] Priyadharsini R, Sree Sharmila T, Rajendran V. A wavelet transform based contrast enhancement method for underwater acoustic images. Multidimensional Systems and Signal Processing. 2018 Oct;29:1845-59.

[27] Ma G, Wang J. Efficient reversible data hiding in encrypted images based on multi-stage integer wavelet transform. Signal Processing: Image Communication. 2019 Jul 1;75:55-63.