

$$V = \{x_1, \dots, x_n\}$$

$$G(V) = \{G = (V, E) / E \subseteq V^{(2)}\}$$

$$|G(V)| = 2^{\binom{n}{2}} = 2^{\frac{n(n-1)}{2}}$$

$G(V), t^*, t^*_{\sim}$ relatie de echivalență

$$G_1 \stackrel{t^*}{\sim} G_2$$

$$(T_1)V = \{x_1, \dots, x_n\}, G(V), G_1, G_2 \in G(V)$$

$$[d_{G_1}(x) = d_{G_2}(x), \forall x \in V] \Leftrightarrow G_1 \stackrel{t^*}{\sim} G_2$$

Dem:

“ \Leftarrow ” evident

“ \Rightarrow ”

Lema 2

Fie $V = \{x_1, \dots, x_n\}, G(V), G \in G(V)$

$$d_i := d_G(x_i), i \leq i \leq n$$

$$d_1 \geq \dots \geq d_n$$

$\exists G' \in G(V)$ cu următoarele proprietăți:

$$G' \stackrel{t^*}{\sim} G \quad (\Rightarrow d_{G'}(x) = d_G(x), \forall x \in V)$$

$$x_1x_2, x_1x_3, \dots, x_1x_{d_1+1} \in E'$$

Dem Lema 2

$$\text{Cazul 1 } x_1x_2, \dots, x_1x_{d_1+1} \in E \Rightarrow G' = G$$

$$\text{Cazul 2 } \exists i \in \{2, \dots, d_1 + 1\} \text{ cu } \left. \begin{matrix} x_1x_2, x_1x_3, \dots, x_1x_{i-1} \in E \\ x_1x_i \notin E \end{matrix} \right\} \Rightarrow \exists j, i < j \leq n : x_i, x_j \in E$$

$$G_1 = G - x_ix_k - x_1x_j + x_1x_i + x_jx_k$$

$$G \stackrel{t^*}{\sim} G_1 \stackrel{t^*}{\sim} G_2 \stackrel{t^*}{\sim} \dots \stackrel{t^*}{\sim} G_p = G'$$

Lema 3

$$V = \{x_1, \dots, x_n\}$$

$$G \in G(V)$$

$$x \in V$$

$$\exists G' \in G(V) : G' \stackrel{t^*}{\sim} G$$

x este adiacent în G' cu $d(x)$ vârfuri cu grade maxime diferite de d .

T4 (Havel-Hakimi)

$$s_0 = (d_1 \geq \dots \geq d_n) \in N_{\geq 0}^*, d_1 \leq n - 1$$

$$s_0 \text{ multisetgrafic} \Leftrightarrow s_0^1 \text{ (este multiset grafic a unui graf simplu).}$$

$$\text{unde: } \underbrace{s_0^1 = d_2 - 1, d_3 - 1, \dots, d_{1+i} - 1}_{d_1}, \underbrace{d_{1+2}, \dots, d_n}_{n-1-d_1}$$

Dem: “ \Rightarrow ” este multiset grafic atunci

$$\exists G' \text{ cu } s(G') = s_0^1$$

$$V' = \{x_1, \dots, x_n\} \text{ a.}\hat{\text{a.}} \ d_i(x_i) = \begin{cases} d_{i-1}, 2 \leq i \leq d_1 + 1 \\ d_i, d_1 + 2 \leq i \leq n \end{cases} \ G = G^1 + [x_1x_2] + [x_1x_3] + \dots + [x_1x_n]$$

G graf simplu $s(G) = s_0 \Rightarrow s_0$ multiset grafic.

“ \Rightarrow ” s_0 multiset grafic $\Rightarrow \exists G, s(G) = s_0$

Fie ...