

The Babylonian square-root algorithm

The iterative method is called [the Babylonian method for finding square roots](#), or sometimes Hero's method. It was known to the ancient Babylonians (1500 BC) and Greeks (100 AD) long before Newton invented his general procedure.

Here's how it works. Suppose you are given any positive number S . To find the square root of S , do the following:

1. **Make an initial guess.** Guess any positive number x_0 .
2. **Improve the guess.** Apply the formula $x_1 = (x_0 + S / x_0) / 2$. The number x_1 is a better approximation to \sqrt{S} .
3. **Iterate until convergence.** Apply the formula $x_{n+1} = (x_n + S / x_n) / 2$ until the process converges. Convergence is achieved when the digits of x_{n+1} and x_n agree to as many decimal places as you desire.

Let's use this algorithm to compute the square root of $S = 20$ to at least two decimal places.

1. An initial guess is $x_0 = 10$.
2. Apply the formula: $x_1 = (10 + 20/10)/2 = 6$. The number 6 is a better approximation to $\sqrt{20}$.
3. Apply the formula again to obtain $x_2 = (6 + 20/6)/2 = 4.66667$. The next iterations are $x_3 = 4.47619$ and $x_4 = 4.47214$.