PH-105 QM Sheet 2

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49. **Show that the expectation value of momentum for any well-behaved function is always real.**

<u>Solution</u> :
The quantum mechanical operator for momentum is given by $p_x = -i\hbar\frac{\partial}{\partial x}$.
Hence, the expected value of momentum for a wave-function ψ is given by

$$\langle p_x \rangle = -i\hbar \int_{-\infty}^{+\infty} \psi^* \frac{\partial \psi}{\partial x} dx$$

Using integration by parts with the partial derivative as second function, we get

$$\langle p_x \rangle = -i\hbar[\psi^*\psi]_{-\infty}^{+\infty} + i\hbar \int_{-\infty}^{+\infty} \psi \frac{\partial \psi^*}{\partial x} dx$$

Since ψ is well-behaved, it vanishes as $|x| \to \infty$ and hence the first term above is zero. The second term remains and it is equal to the complex conjugate of $\langle p_x \rangle$. So, we see that $\langle p_x \rangle = \langle p_x \rangle^*$. Hence we conclude that the expected value of momentum must be real.