## PH-105 (2012) Problem Sheet

- **P86:** Find the value of  $n\lambda^3$  for bosons at the critical temperature for Bose Einstein Condensation. Here n is number of particles per unit volume and  $\lambda$  is their de Broglie wavelength at critical temperature (assuming equipartition law). Using the above value, express the condition of B E condensation in the form of a relationship between the de Broglie wavelength and the average distance between particles.
- **P87:** In how many ways three electrons can occupy ten states (the states include spin degeneracy)? Is the number same as the way in which three persons can occupy ten chairs in a room? State the reason. In case the number is different, find the other number also.
- **P88:** A system has three energy states with energies  $\varepsilon_1, \varepsilon_2$ , and  $\varepsilon_3$ , with respective degeneracies of 1, 4 and 8. In how many ways a distribution of six indistinguishable particles obeying Pauli exclusion principle, can be obtained, where 1 particle has energy  $\varepsilon_1$ , 2 particles have energy  $\varepsilon_2$  and 3 particles have energy  $\varepsilon_3$  (call it 1,2,3). With an equal a priory probability, is this distribution more probable than (1,3,2). What are the ratios of the probability?
- **P89**: (a) Using the Fermi Dirac Statistics, find the probability that a state is occupied if its energy is higher than  $\varepsilon_F$  by 0.1 kT, 1.0 kT, 2.0 kT and 10.0 kT, where  $\varepsilon_F$  is the Fermi Energy. How good is the approximation of neglecting 1 in the denominator for an energy equal to 10 kT.(b) In the Fermi Dirac distribution substitute  $\varepsilon = \varepsilon_F + \delta$ . Compute  $\delta$  for the probability of occupancy equal to 0.25 and 0.75. (c) Show that for a distribution system governed by F.D. distribution, the probability of occupation of a state with energy higher than  $\varepsilon_F$  by an amount  $\Delta E$  is equal to the probability that a state with energy lower than  $\varepsilon_F$  by  $\Delta E$  is unoccupied.
- **P90\*:** A copper wire of cross-sectional area  $3.3x \ 10^{-6} \ m^2$  is carrying a current of  $25 \ A$ . One meter length of this wire has a resistance of  $5.8x \ 10^{-3}$  ohms. Calculate the conductivity, average drift velocity of the electrons, electron mobility and mean free path of the electrons in Drude and Sommerfeld models.
- **P91\*:** For Sodium the conductivity at 300 K is  $2.17x10^7 \text{ ohm}^{-1} \text{m}^{-1}$  and the effective mass of the electron is 1.2 times the mass of the free electron. Calculate the relaxation time and the mean free path. Calculate the drift velocity of the electrons in an electric field of 100V/m. [Density of Na =  $970 \text{ kg/m}^3$ , Atomic weight 23]
- **P92\*:** Show that the kinetic energy of a three dimensional gas of N free electrons at 0 K is  $(3/5)N\varepsilon_F$ .
- **P93\*:** Using the data given and any other constants, evaluate the Fermi energy of the alkali metals.

	Li	Na	K	Rb	Cs
Density (g/cc)	0.534	0.971	0.860	1.530	1.870
Atomic weight	6.939	22.99	39.102	85.47	132.905

- **P94:** The Fermi energy of Cu is 7.04 eV. Calculate the velocity and de Broglie wavelength of electrons at the Fermi energy of Cu. Can these electrons be diffracted by a crystal.
- **P95:** Assuming that Silver is a monovalent metal obeying Sommerfeld model, calculate the following quantities.(a) Fermi energy and Fermi temperature. (b) Radius of Fermi sphere.(c) Fermi velocity. (d) the average energy of free electrons at 0 K. (e) the temperature at which the average molecular energy in the ideal gas will have the same value as the average energy of free electrons at 0 K.(f) the speed of electron with this energy.(g) Mean free path of electrons at room temperature and near absolute zero.(h) the ratio of the Fermi velocity to drift velocity at room temperature in a field of 1 V/cm. [Given density of Ag = 10.5 g cc; Atomic wt. of Ag = 107.87; Resistivity of Ag at 295 K = 1.61 x 10<sup>-6</sup> ohm cm and at 20K = 3.8 x 10<sup>-9</sup> ohm cm.]
- **P96:** Consider a gas of electrons (mass m) confined to a dimensional square box of size a. Find an expression of density of state and Fermi energy at  $\theta^{o}K$ .
- **P97:** Find an expression of density of state for free electrons confined within a distance L in one dimension.
- **P98\*:** Show that the fraction of electrons within kT of the Fermi energy is  $1.5kT/\varepsilon_F$ , under the assumption that the temperature is so low that the probability of occupancy of levels is not altered from the one at  $0^{\circ}K$ . Calculate numerically the value of this fraction for copper ( $\varepsilon_F = 7.04 \text{ eV}$ ) at  $300^{\circ}K$  and  $1360^{\circ}K$  (approximate melting point of Cu). This fraction is of interest because it is a rough measure of the

- percentage of electrons excited to higher energy states at a temperature T. Find roughly the electronic contribution to specific heat of Cu using this expression.
- **P99\*:** In a Hall experiment on Silver, a current of 25 A passes through a long foil which is 0.1 mm thick (in the direction of **B**) and 3 cm wide. What is the Hall voltage for B = 1.4T, if the Hall coefficient is  $-0.84 \times 10^{-10}$  m<sup>3</sup> /C. Given the conductivity of Ag =  $6.8 \times 10^{7}$  mho/m, estimate the Hall angle and the mobility of electrons. What would be the values of the Hall voltage, Hall angle and the mobility if the width of the foil is doubled.
- **P100:** For Cu, the Hall coefficient is  $R_H = -0.55x10^{-10} \ m^3 / C$ . If the relaxation time  $\tau = 2.1x10^{-14} \ s$  and the conductivity  $\sigma = 6.0x10^7 \ mho/m$ , calculate the mobility of the electron and the ratio of the average effective mass of the electron to the electron mass.