PH-107 (2014)
Tutorial Sheet 6
(Band Theory and Semiconductors)

Q: Electron in Periodic Potential:

P114: (a) The wave function of the free electron can be written as $\phi(x) = Ae^{ikx}$. Use the momentum operator to show that $\langle p_x \rangle = \hbar k$.

(b) The wave function of an electron subjected to a periodic force in a one-dimensional lattice can be written in the form $\phi(x) = u(x)e^{ikx}$, where $u(x)$ has the same periodicity as that of crystal. Use momentum operator to show that $\langle p_x \rangle \neq \hbar k$. $\hbar k$ is called the crystal momentum of solid and is different from the usual momentum.

P115*: (a) In a one-dimensional crystal an electric field of magnitude E is applied to an electron at time *t=0* in negative *x*- direction, when it was at the bottom of the conduction band at k=0. Show that, in the absence of any scattering, the position of the electron in real space at time t is given by $x = x_o + \frac{\varepsilon(k_o)}{F}; k_o = \frac{Ft}{\hbar}$ and x_o is the original position and $F = eE$ is the magnitude of electric force. Is the motion periodic in real space? Explain.

(b) Assume that there is a current of 1 A in a wire of 1 mm area of cross-section. Let the wire be highly resistive with resistivity equal to 100 μ ohm cm. Find the electric field in the wire. If this electric field is applied to an electron in the part (a) of the problem, find the time taken by this electron to reach k= π $Å^{-1}$ starting from k=0, assuming no scattering. Also find the distance over which the electron will execute oscillations in the absence of scattering. Calculate the energy of the electrons at k= π $Å^{-1}$ by assuming a free electron dispersion relation.

P116*: Assume that the energy of an electron in a band of one-dimensional solid is given by

$$E = A - B\cos(ka)$$

Here *A* and *B* are constants and '*a'* is the distance between atoms. Find the velocity and the effective mass of the electron as a function of *k*. Find the value of *k* for which the velocity of the electron is maximum.

P117*: The energy of an electron in a band of an one-dimensional infinite solid is given as

$$E = E_o\left(1 - \cos ka\right)$$

where E_o =1 eV and '*a*', the distance between atoms is 0.2 nm. Assume the solid to be of unit length. This band is full except for a single state at $k = \left[10\pi/3\right]$(nm)$^{-1}$. An electric field of 0.5 V/m is applied in the negative x-direction at time *t=0*. Find the following:

(i) The initial current in the solid.
(ii) The acceleration of the hole.
(iii) Is this acceleration constant? Explain.
(iv) Assuming no scattering, find the shortest time when the current is zero.

P118: For a certain one-dimensional crystal, energies are given by

$$E = \left(\frac{\hbar^2 k^2{}_o}{2m}\right)\left[\left(\frac{k}{k_o}\right)^2 + 0.61\left(\frac{k}{k_o}\right)^4 - 0.74\left(\frac{k}{k_o}\right)^6\right]$$

where k_o=5.4x10^{9}m^{-1}. Find the velocity of the electron with wave vector 0.5 k_o. If an electric field of 100 V/m is applied in the direction of negative k, find the acceleration of the electron at k=0.5 k_o. How long will this electron take to reach a value of k= 0.75k_o, assuming no scattering. Was the acceleration of the electron constant during this transit? What was the acceleration at k= 0.75k_o? How much distance would the electron actually travel in reaching k=0.75k_o.

R: Hall Effect with both Negative and Positive Charge Carriers:

P119: For a material, the number of electrons per unit volume is same as the number of holes and is equal to *10^{20} m^{-3}*. If the mobility of the electron is *0.05 m^2/Vs*, while that of holes is *0.15 m^2/Vs*, calculate the conductivity and the Hall coefficient of the material.

P120: A conductor has *5x10^{19}* electrons and *8x10^{20}* holes per cubic meter. If electron mobility is *0.05* and the hole mobility is *0.09 m^2/Vs* respectively, calculate the conductivity and the Hall coefficient.

P121*: A Hall Effect experiment is planned on a solid in which there are 10^{22} electrons in the conduction band per m^3 and equal the number of holes in the valence band. For this purpose a strip of the solid is made, the length of which is 8 cm, the width is 2 cm and thickness is 0.1 mm. Initially a current of 2 A is passed lengthwise through this metal strip (along +x direction) in the absence of the magnetic field. The potential drop across the length is found to be 100 V. Now a magnetic field of 0.2 T is applied along the thickness of the foil (along +z direction), keeping the current same. A positive potential drop (creating a field in positive y direction) of 1.5 volts is measured across the width of the strip. Find the electron and the hole mobilities for this solid. Also find the relaxation time of holes and electrons assuming their effective masses to be equal to the mass of free electron.

S: Semiconductors:

P122*: The conductivity of an intrinsic semiconductor at *20 ^oC* is three times its conductivity value at *0 ^oC*. Assuming that the mobilities do not change between these two temperatures, find the band gap of the semiconductor. If the conductivity at *0 ^oC* is *0.114 mho/m* and the hole density of state effective mass is *0.29 m_e*, find the electron effective mass. [m_e is the mass of free electron; μ_e= 0.18 &μ_p=0.36 m^2/V.s]

P123: For a hypothetical intrinsic semiconductor (*Band gap =1.1 eV*) there are *3.1 x10^{15}* electrons in the conduction band at 300^o*K*. If the electron effective mass is 0.3 times the mass of the free electron find the effective mass of the hole and the position of Fermi level.

P124*: A *Ge* crystal has intrinsic carrier density equal to *$2.5\times10^{19}/m^3$*. It is doped with donors to a concentration of *10^{20}* atoms /m^3. Assuming all the donor atoms to be ionized, find the minority carrier concentration. What is the ratio of majority to minority carrier concentration?

P125: Compute and compare the conductivity of pure *Si* and that of an extrinsic *Si* containing 0.1 ppm of phosphorus. [*μ_e = 1300 cm^2/V,s μ_h=500 cm^2/Vs at 20^o C; Number of carriers/cm^3 for pure Si = 2x10^{10}*] . Further assume that at *20^o C* essentially all P atoms are ionized.

P126: A sample of *Ge* at *300 ^oK* has *10^{20}* acceptors and *5×10^{19}* donors per meter cube. Assuming that all the donors and acceptors are ionized at *300 ^oK* and taking the effective mass of electrons and holes to be the same as mass of free electrons, calculate the Fermi energy and electron and hole densities. (Take band gap equal to *0.7 eV*)

P127*: Find the position of Fermi level *at* 300^oK for *Ge* doped with phosphorous to a concentration of $10^{22}/m^3$. Find also the level in case the material was doped with gallium instead of phosphorous to the same concentration. Assume that all the impurity atoms are ionized and the minority carrier concentration is negligible at this temperature.

P128: (a) For an intrinsic semiconductor ($Eg = 1.1\ eV$), the room temperature conductivity is $2.12x10^{-4}\ (ohm\text{-}m)^{-1}$ and the Hall coefficient is $-425\ m^3/C$. Find the mobility of electrons and holes in *SI* system of units. Assume that hole and electron effective masses are same and are equal to the rest mass of free electron. (b) The above semiconductor is doped with 10^{21} acceptor impurities per m^3. Assuming all the acceptor atoms to be ionized find (i) the ratio of majority to minority carrier concentration, (ii) the conductivity and Hall coefficient of the material and (iii) the position of Fermi level at room temperature. Assume that mobilities do not change as a result of impurity addition.

P129: The conductivity of an intrinsic semiconductor is 0.47 $(ohm\text{-}m)^{-1}$ and 1.8 $(ohm\text{-}m)^{-1}$ for kT= 0.023 and 0.025 *eV* respectively. When the same semiconductor is doped with donor impurities to a concentration of 10^{20} atoms/m^3, the conductivity at kT=0.025 *eV* is found to become 6.38 $(ohm\text{-}m)^{-1}$. Assume the electron and hole effective masses to be same as that of free electron, all donor atoms to be ionized at both the temperatures of interest and mobility independent of temperature and doping. Find the electron and hole concentration and the Fermi energy in the doped semiconductor at kT=0.025 *eV*, and the electron and hole mobilities.