PH-105 Assignment Sheet - 3 (Quantum Mechanics - 2)

Umang Mathur

48. Find the angular momentum operator in Cartesian co-ordinate system

Solution:

The angular momentum operator is defined as follows:

$$\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}}$$

where, $\hat{\mathbf{r}} = r$ and $\hat{\mathbf{p}} = -i\hbar\hat{\nabla}$. Hence,

$$\hat{\mathbf{L}} = -i\hbar(x\hat{i} + y\hat{j} + z\hat{k}) \times (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})$$

Therefore,
$$\hat{\mathbf{L}} = -i\hbar \Big((\mathbf{y} \frac{\partial}{\partial \mathbf{z}} - \mathbf{z} \frac{\partial}{\partial \mathbf{y}}) \hat{\mathbf{i}} + (\mathbf{z} \frac{\partial}{\partial \mathbf{x}} - \mathbf{x} \frac{\partial}{\partial \mathbf{z}}) \hat{\mathbf{j}} + (\mathbf{x} \frac{\partial}{\partial \mathbf{y}} - \mathbf{y} \frac{\partial}{\partial \mathbf{x}}) \hat{\mathbf{k}} \Big)$$