PH-105 (2012)
Tutorial Sheet 1 (QM Portion)

* Problems to be done in tutorial. Rest are for practice.

P 1*: Show that the Einstein's expression of specific heat gives a value of 3R at high temperatures and tends to zero as temperature tends to zero.

P 2: Light of wavelength 2000 Å falls on a metal surface. If the work function of the metal is 4.2 eV, find the kinetic energy of the fastest and the slowest emitted photoelectrons. Also find the slopping potential and cutoff wavelength for the metal. [**Ans.:** 2.0 eV, 0, 2V, 2960 Å]

P 3*: An experiment on photoelectric effect of a metal gives the result that the stopping potential for λ = 1850 Å and 5460 Å are 4.62 and 0.18 V respectively. Find the value of Planck's constant, the threshold frequency and the work function. [**Ans.:** 6.64×10^{-34} Js, 0.5×10^{15} Hz, and 2.1 eV]

P 4: X-ray of wavelength λ = 0.1 Å is scattered by an electron. At what angle will the scattered photon have a wavelength of 0.11 Å?
[**Ans.:** 54.3^{o}]

P 5: A 200 MeV photon strikes a stationary proton. If the photon is back scattered, what is the kinetic energy of the recoiling proton?
[**Ans.:** 60 MeV]

P 6*: Consider an x-ray beam with λ = 1 Å and a γ ray beam with $\lambda = 1.88\times10^{-2}$ Å. The radiation scattered from free electron is viewed at 90^{o} to the incident beam. In each case

(a) What is the Compton shift?
(b) What is the kinetic energy of the recoiling electron?
(c) What percentage of initial energy is lost in collision?

[**Ans.:** 0.024 Å, 0.295 keV, 2.4% and 57%]

P 7*: Show that a free electron can not absorb a photon. Hence photoelectron requires bound electron. In Compton effect, however, the electron can be free.

P 8: A photon strikes an electron at rest and undergoes a pair production process, giving rise to an electron and a positron, i.e.

$$\gamma + e^- \rightarrow e^- + e^- + e^+$$

The two electrons and the positron move off with identical momenta in the same direction as the incident photon. Find the energy of the photon and the speed of the particles. [**Ans.:** 2MeV. 0.8c]

P 9: Find the smallest energy that a photon may have and still transfer one half of its energy to an electron initially at rest.
[**Ans.:** 0.256MeV]

P10: A photon of energy $h\nu$ is scattered through 90^o by an electron initially at rest. The scattered photon has a wavelength twice that of incident photon. Find the frequency of the incident photon and the recoil angle of the electron.
[**Ans.:** 1.23×10^{20} Hz, $\theta= 26.6^o$]

P11: Find the energy of the incident x-ray if the maximum kinetic energy of the Compton electron is $m_oc^2/2.5$.
[**Ans.:** 0.69 m_oc^2]

P12: Compare the momentum of an electron with that of a photon whose energy is same as the kinetic energy of the electron.

P13: A beam of mono-energetic x-ray gets scattering from a particle of mass m_o. It is found that the wavelength of the x-ray scattered at 60^o is half that scattered at 120^o. Find the incident energy of the photon.
[**Ans.:** 2 m_oc^2]

P14*: Consider Compton Scattering. Show that if the angle of scattering θ increases beyond a certain value θ_o, the scattered photon will never have energy larger than $2m_oc^2$, irrespective of the energy of the incident photon. Find the value of θ_o.
[**Ans.:** $\theta_o= 60^o$]

P15*: Show that if the nucleus in the Bohr atom is assumed to be of finite mass, the angular momentum of the system, the allowed radii and energies are all given by identical expressions except for replacement of m by the reduced mass μ.

P16*: Two similar particles of mass *m* are connected to each other by a spring of negligible natural length and mass and spring constant *k*. The particles are made to rotate in a circle about their common centre of mass, such that the distance between them is *R*. Assume that the only force between the particles is the one provided by the spring. Apply Bohr's quantization rule to this system and find the allowed value of *r* and the energies in terms of fundamental constants if any, the mass and the spring constant.

P17: Assume that the wavelengths λ of the hydrogen atom spectra are given by the following expression instead of the usual one.

$$\frac{1}{\lambda} = R\left(\frac{1}{m^3} - \frac{1}{n^3}\right)$$

Here R is a constant and m and n are integers with $n>m$. If we write the angular momentum (L) quantization condition as $L = a\,\hbar$, what values a should take so as to explain the above spectra. Construct a theory similar to Bohr's using this quantization condition and find out an expression of the energy and the Bohr's radius.

P18: A μ^- meson is an elementary particle of charge $-e$ and mass that is 207 times the mass of the electron. A muonic atom consists of a nucleus of charge Ze with μ^- meson circulating about it.

(a) Calculate the radius of the first Bohr orbit of a muonic atom with $Z=1$.

(b) Calculate the binding energy of a muonic atom with Z=1.

(c) Find the wavelength of the first line in the Lymann series for such an atom.

[**Ans.:** 2.85×10^{-3} Å, 2.53 keV, 6.55 Å]

P19*: One of the lines in the Hydrogen atom has a wavelength 4861.320 Å. It was later discovered this line has a faint companion located at 4859.975 Å. The explanation for this line was the presence of a small amount of heavier isotope deuterium in hydrogen. Use this data to compute the deuterium mass to the proton mass.

P20: A positronium atom (consisting of an electron and a positron revolving about common centre of mass; positron being a particle with mass equal to the mass of electron but charge plus e) is excited from a state with n=1 to n=4. Apply Bohr's theory with suitable modifications. (a) Calculate the energy that would have been absorbed by the atom. (b) Calculate minimum possible wavelength emitted when such an electron de-excites. (c) Calculate the recoil speed and recoil energy of the positronium atom, assumed initially at rest, after the excitation takes place.

P21: (a) Calculate the recoil speed of hydrogen atom assumed initially at rest, when it makes a transition from n=4 to n=1.

(b) What is the kinetic energy given to the hydrogen atom.

(c) At what temperature would the hydrogen atoms have this as the average speed assuming the gas to be classical?

(d) What would you expect if the hydrogen atoms were in motion approaching the photon?

(e) Can you think of a simple experiment that can be used to cool a gas?

[**Ans.:** 4.1 m/s, 8.65×10^{-8} eV, 0.7 mK]

P22*: (a) A source of photons of frequency ν is moving with a speed v in laboratory frame of reference. Show that in the limit $v<<c$, the frequency of photon ν', as

$$\nu' = \nu\left(1+\frac{v}{c}\right)$$

observed in laboratory frame of reference is given by the following expression.

(b) What is the value of the required speed in case the energy of photons of energy 14.4 keV is to be increased by 10^{-6} eV?
[**Ans.:** 2.1 cm/s]

P23*: A photon of energy *E* is emitted as a result of a particular transition. What would be the value of the recoil energy, assuming that the atom recoils with non relativistic speed. Let the lifetime of the state be of the order of 10^{-8} s. What would be the order of natural line width of the emitted line. For what value of *E*, would the recoil energy be of the same order of magnitude as the natural line width? For order of magnitude calculation take the mass number of the atom as 100. What conclusions would you draw from this regarding resonant absorption?
[**Ans.:** $E^2/(2mc^2)$, 6.6×10^{-8} eV, 111 eV]

P24: A beam containing photons of energy 10.21 eV approaches a hydrogen atom which is also moving along the same line as the incident beam. Assume that the beam is perfectly mono-energetic and the ground state of hydrogen atom is exactly equal to -13.6 eV. Also assume that the hydrogen atom levels are perfectly sharp. What is the smallest speed the hydrogen atom should have, so that the photon can be absorbed by it? What would be the change in the speed of the electron after the photon gets absorbed. You can make any suitable assumptions provided you justify it.

P25*: (a) A container contains monatomic hydrogen gas in thermal equilibrium at a temperature T, for which k_BT= 0.025 eV. Let E_1 is the difference between the ground state and the first excited state energy of the atom when at rest. Let E_2 be the energy of photon (in the frame of container) required to make this transition in an atom, which is traveling towards the photon (in an antiparallel direction) with the average energy at the above specified temperature.
(i) Find E_1-E_2.
(ii) After the absorption of photon what would be the final velocity of the hydrogen atom.
(iii) If the lifetime of the first excited state is 10^{-8} s, will the photon with energy E_2 be able to cause a transition, had the atom been at rest. Discuss quantitatively.
You are free to make any assumption, provided you justify it.

P26*: Show that the Bohr's condition of quantization of angular momentum leads to a condition of formation of standing wave of electron along the circumference in the Bohr model of hydrogen atom.

P27: Calculate the wavelength of the matter waves associated with the following. Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.
(a) A 2000 kg car moving with a speed of 100km/h.
(b) A 0.28 kg cricket ball moving with a speed of 40 m/s.

(c) An electron moving a speed of 10^7m/s. [**Ans.:** 1.2×10^{-38}m, 5.9×10^{-35}m, 0.73Å]

P28*: (a) Find the de Broglie wavelength of thermal neutrons. Can these neutrons be diffracted by solid?

(b) Find the energy of the photon, electron and a neutron for which they will have a wavelength of 1 Å.

(c) At what temperature would the classical kinetic energy of neutron would make its wavelength equal to 10^{-6} m. [**Ans.:** 1.48 Å, 12.431 keV, 151 eV, 0.082 eV, 6.34×10^{-6} K]

P29: The band structure of a solid in the low wave vector limit is approximately given by the following equation.

$$\hbar\omega=Ak^2-Bk^4$$

where A and B are constants.

(a) Show that group and phase velocities are same when

$$\omega=\frac{1}{\hbar}\left[\frac{2A^2}{9B}\right]$$

(b) Show that if the second term in the dispersion relation is neglected, the group velocity of electrons would be twice that phase velocity.

P30*: The dispersion relation for a lattice wave propagating in a one dimensional chain of atoms of mass *m* bound together by force constant β is given by the following equation.

$$\omega=\omega_o\sin(ka/2)$$

$$\omega_o=\sqrt{\frac{4\beta}{m}}$$

Here *a* is distance between atoms and β is given by the following expression.

(a) Show that in the long wavelength limit the medium is non-dispersive.

(b) Find the group and phase velocities at $k=\pi/a$.

[**Ans.:** 0, $\omega_o a/\pi$]

P31: The phase velocity v_p of ocean waves are given by

$$v_p=\sqrt{\frac{g\lambda}{2\pi}}$$

Find the ratio of group to phase velocities for such a wave.

[**Ans.:** 0.5]

P32: Find the group and the phase velocity of the matter wave associated with a free particle under the assumption the frequency is defined using (i) the kinetic energy (ii) total relativistic energy.

P33: The speed of a bullet of mass *50 g* and an electron is measured to be *300 m/s*, both with an uncertainty of *0.01%.* With what fundamental accuracy could we locate the position of each, if the position is measured simultaneously with the speed?

P34: The position and the momentum of a *1keV* electron are determined simultaneously. Its position is known to an accuracy of only *1 Å* along x-axis. Using uncertainty principle, what is the minimum permissible percentage uncertainty in its momentum along the x-axis? From the above data can you determine the uncertainty along y-axis?

P35: Out of the two electrons, the position uncertainty of the first electron equals its de Broglie wavelength, and that of the second electron equals its Compton wavelength. Both the electrons are non-relativistic, and move along the same line. If the velocity uncertainty were as low as allowed, which of the two electrons has its velocity better defined?

P36: An electron is moving in a parallel beam along the x-direction with momentum, $p=mv$. It encounters a slit of width w. Assuming that the electron gets diffracted somewhere within the central maximum of small angular magnitude $\Delta\theta$, estimate the uncertainty Δp in its momentum component transverse to the direction of motion. Check that uncertainty principle is satisfied in this experiment.

P37: An electron falls from a height of *10 m* and passes through a hole of radius *1 cm*. To study the motion of the electron afterwards, should we apply the wave aspect or the particle aspect?

P38*: A wave packet is constructed by superposing waves, their wavelengths varying continuously in the following way

$$y(x,t) = \int A(k)\cos(kx - \omega t)dk \quad .$$

Where $A(k) = A$ for $(k_o - \Delta k/2) \leq k_o \leq (k_o + \Delta k/2)$ and $= 0$ otherwise. Sketch approximately $y(x,t)$ and estimate Δx by taking the difference between two values of x for which the central maximum and nearest minimum is observed in the envelope. Verify uncertainty principle from this.

P39: In a world where h is large, the uncertainty principle makes it hard for a croquet player to hit a ball through the wicket. To help alleviate the problem, an inventor designs a device in which the ball is passed through a cylinder that is permitted to fluctuate position through a transverse distance Δy. The ball acquires a transverse uncertainty of velocity Δv_y. Influenced by both the uncertainties Δy and Δv_y, the ball misses the center of the wicket by a distance *Y*. If the wicket is at a distance *L* from the cylinder, and v_o is the horizontal velocity of the ball, estimate the value of Δy for the optimum design (i.e. minimum *Y*), in (i) a world where $h=0.01$Js (ii) our world.

P40*: A beam of electron of energy *0.025 eV* moving along x-direction, passes through a slit of variable width *w* placed along y-axis. Estimate the value of the width of the slit for which the spot size on a screen kept at a distance of *0.5 m* from slit would be minimum.

P41*: Assume that in case the average value of momentum p_x is zero, the uncertainty in the x-component of momentum Δp_x is related to the average of square of *x* component of momentum $\langle p_x^2 \rangle$ by the following relation.

$$\Delta p_x = \sqrt{\langle p_x{}^2 \rangle}$$

Using the above relation estimate the following.

(a) The minimum kinetic energy that proton and electron would have if they were confined to a nucleus of approximate diameter 10^{-14} *m*. This is an argument used against the existence of electron in nuclei.

(b) The ground state energy of a particle in an infinite square well potential of width *L*.

(c) The radial distance *r* for which the sum of kinetic and potential energies is minimum in hydrogen atom.

P42: Estimate the minimum possible energy consistent with uncertainty principle (i.e. ground state energy) for a particle of mass *'m'* bound in a potential $V=kx^4$. Use the following uncertainty principle for this problem $\Delta x\, \Delta p_x \geq \hbar/2$.

P43: In the Young's double slit experiment with electrons, the separation between the two slits is *d* along the *y*- direction. If we want to be sure that an electron goes through a particular slit, Δy should be smaller than *d*/2. Using uncertainity relation $\Delta y \Delta p_y \geq \hbar/2$, show that this will create enough uncertainity in the p_y, so that the electron would land on the screen over a width larger than the fringe width of the intereference pattern. Thus the interefernce pattern will be destroyed.

P44: An atom can radiate at anytime after it is excited. It is found that on the average the excited atom has a lifetime of 10^{-8} *s*. That is, during this period it emits a photon and is de excited.

(a) What is the minimum uncertainty $\Delta\nu$ in the frequency of the photon and $\Delta\nu/\nu$ for radiation of wavelength $\lambda = 5000Å$?

(b) What is the uncertainty ΔE in the energy of the excited state of the atom?

P45*: A charged pi-meson has a rest energy of 140 MeV and a lifetime of 26 ns, while a rho-meson has a rest energy of 765 MeV and a lifetime of 4.4×10^{-24}s. In each case find the absolute and fractional uncertainty in energy. Use the following uncertainty principle for this problem $\Delta E\ \Delta t \geq \hbar/2$.