On the top right hand corner of your answerbook, please write the timings of your lab batch, e.g. "Thursday, 8:30-10:30".

Problem 1:(a)[5 marks] What is the output when the following code fragment is executed with the input being the last 5 digits of your roll number?

```
int n; cin >> n;  // assume last 5 digits of your roll number are typed
double m = 1 + n/10000;
char c = 'a' + m;
double q = 32 + 3/10*m;
int r = m*3/10 + 32;
cout << m << ' ' ' << c << ' ' ' << q << ' ' ' << r << endl;

say roll no digits are 50004.

m: 6 1 mark
c: g 2 marks
q: 32 1 mark
r: 33 1 mark</pre>
```

(b)[8 marks] Write a program that reads in 3 characters. If the three characters consist of two digits with a '.' between them, then your program should print the square of the decimal number represented by the characters. Otherwise your program should print a message saying that the input given is invalid.

```
#include <simplecpp>
main_program{
   char p, q, r; cin >> p >> q >> r;
   if(q == '.' && '0' <= p && p <= '9' && '0' <= r && r <= '9'){
      double num = p-'0' + (r-'0')/10.0;
      cout << num * num << endl;
   }
   else cout << "Invalid input.\n";
}

1 mark for declarations and reading in.
2 marks for being able to generate digits out of characters.
2 marks: generating square.
1 mark for error message.</pre>
```

(c)[12 marks] Given below is a series to compute $\cos(z)$. Write a program to evaluate it to n terms. Your program must take n and z as input. Do not use the function pow. Write the invariant for the loop.

 $\cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!} - \cdots$

```
#include <simplecpp>
main_program{
   double z; cin >> z;
   int n; cin >> n;

   double result=0, term = 1;

// Invariant: on ith entry, i=1 to n, term = ith term of the series.
// result = sum to i-1 terms.
// i+1th term = ith term * (-1) * z*z /(2i-1)/(2i)

for(int i=1; i<= n; i++){
   result += term;
   term = -term*z*z/(2*i-1)/(2*i);
}

cout << result << endl;
}</pre>
```

1 mark for declaring variables z and n correctly.

1 mark for stating what is result when entering the loop for the ith time.

1 mark for stating what is term when entering the loop correctly for the ith time.

1 mark for Alternating signs for the terms.

2 mark: multiplication of term by z*z.

2 mark: division by 2i-1, 2i or thereabouts.

1 mark: correct number of iterations, n if initialized to 0, n-1 if initialized to 1.

3: code is correct? Suggest: check by examining first 2 terms...

Problem 2: You are to write a program which multiplies an n digit number M by a 1 digit number d, where n could be large, e.g. 1000. The input will be given as follows. First the user gives d, then n and then the digits of M, starting from the least significant to the most significant. The program must print out the digits of the product one at a time, from the least significant to the most significant.

(a)[15 marks] State the idea of your program in 3-4 lines (at most). Give your program. Write the main loop invariant. You may get 0 marks if your program does not have comments and the main loop invariant.

```
#include <simplecpp>
/*
The algorithm mimics how we multiply digit by digit. We multiply the
current digit of M by d, then add the carry from the previous digit
into it. Then the least significant digit of the partial product is
output, the more significant digit is taken as the carry to the next digit.
*/
main_program{
  int d,n; // as in the problem statement.
  int mi, carry = 0; // multiplier digit, carry during the multiplication.
 cin >> d >> n;
 repeat(n){
    // on ith entry: i-1 digits of the product will have been printed,
   // carry will contain the carry into the next digit.
   cin >> mi;
    int prod = mi*d + carry; // partial product
   cout << prod % 10;
   carry = prod / 10;
 }
 cout << carry << endl;</pre>
}
/*
1 mark: algorithm outline
1 mark: declarations
1 mark: invariant: what is printed by i iterations
1 mark: invariant: what is the value of carry on ith iteration.
2 marks: generating prod, statement above.
2 marks: generating carry
2 marks: printing ith digit in loop
2 marks: printing last carry, outside loop.
3 marks: correctness
*/
```

(b)[5 marks] The program you wrote above will likely perform about n multiplication operations and a similar number of other operations. There is a more efficient way of writing this program, i.e. using fewer operations for multiplying the same numbers M, d. Describe it. You should not give a program but merely describe in English. Give as many details as possible. Hint: You may ask the user to give several digits of M at a time if you wish.

Work on 8 digits of M at a time. This will produce a product of 9 digits. Even after adding the carry sum could be about 2×10^9 , which will fit in an int.

2 marks: working on groups of digits.

2 marks: why 8 digits.

1 mark: Detail: n digits of M are partitioned into n/8 groups. Groups are given least significant to most significant. space between groups. Numbers within a group are given most significant to least significant.

Problem 3:[5 marks] Consider the problem of fitting a circle to a set of points in the plane. You are further given that the circle must be centered at the origin, so the only choice you have to make is that of the radius. For this define a suitable error function and minimize it. Based on it state what radius you will pick. No program is to be written.

5 marks: The error function is $\sum_{1}^{n}(r^2-x_i^2-y_i^2)^2$. Thus we get $\sum_{1}^{n}2r(r^2-x_i^2-y_i^2)^2=0$. Or in other words, $r^2=(\sum x_i^2+y_i^2)/n$, i.e. $r=\sqrt{(\sum x_i^2+y_i^2)/n}$.

Probably there cannot be partial marking on this.

If other error functions are written, judge them and grade. But cannot just say "take average".