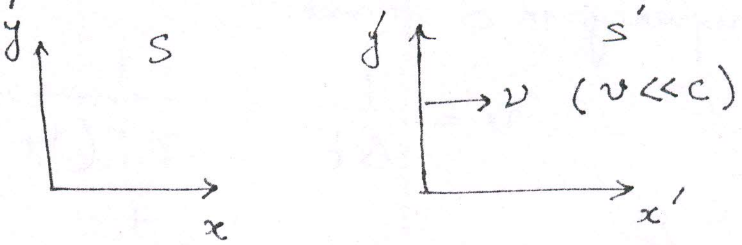
# TUTORIAL SHEET · 2

Q. 2

(a) A source of photon of frequency '$\nu$' is moving with a speed '$v$' in laboratory frame of reference. Show that in the limit $v \ll c$, the frequency of photon $\nu'$, as observed in laboratory frame of reference is given by the following expression; $\nu' = \nu\left(1 - \frac{v}{c}\right)$

(b) What is the value of the required speed in case the energy of photons of energy 14.4 KeV is to be increased by $10^{-6}$ eV.

<u>Solution:</u>




Let the source be considered in moving frame $S'$, which is moving with velocity $v$.

$T$ be the time between two crests in frame $S'$ at $x' = 0$

Basically two events happening at $x' = 0$ after a time interval '$T$'.

$$S' \Rightarrow \left.\begin{array}{ll} x_1' = 0, & t_1' = 0 \\ x_2' = 0, & t_2' = T \end{array}\right\} \text{proper time } T$$

Look from the $S$ frame now; these two events will appear at two diff points and time in frame $S$.

Do the LT; $x_1 = \gamma(x_1' + v t_1') \Rightarrow x_1 = 0$

$$x_2 = \gamma(x_2' + v t_2') \Rightarrow x_2 = \gamma v t_2' = \gamma v T$$

Suppose the observer is 'd' distance from $x_1'$

The first crest will take time $= d/c$

The II$^{nd}$ crest has moved to $\gamma v T$ distance in S.

Its obvious that it will take more time to cover this time.

2$^{nd}$ crest comes after time 'T' in S' frame. This appears to be dialated.

$$\text{Total time} = \frac{d}{c} + \frac{\gamma v T}{c} + \gamma T \text{ in S frame}$$

$$\text{Time difference} = \Delta t = \frac{d}{c} + \frac{\gamma v T}{c} + \gamma T - \frac{d}{c}$$

$$= \gamma T (1 + v/c)$$

$\therefore$ Frequency in S frame

$$\nu' = \frac{1}{\Delta t} = \frac{1}{\gamma T (1 + v/c)}$$

$$\nu' = \frac{1}{T} (1 + v/c)^{-1}$$

$$\nu' = \nu (1 - v/c)$$

(b)

$$\nu' = \nu \left(1 - v/c\right)$$

$$h\nu' = h\nu (1 - v/c)$$

$$E' = E - E\, v/c \quad , \quad E = 14.4 \text{ keV}$$

$$E' = 14.4 \text{ keV} + 10^{-6} \text{ eV}$$

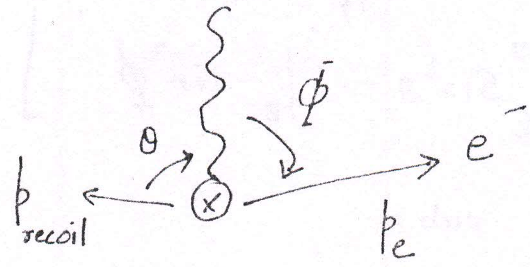
$$\Rightarrow 10^{-6} \text{ eV} = 14.4 \text{ keV} \times v/c$$

$$\therefore \quad v = \frac{10^{-6} \text{ eV} \times 3 \times 10^{8} \text{ m/s}}{14.4 \times 10^{3} \text{ eV}}$$

$$= 0.0208 \text{ m/s}.$$

Q.31 In a photoionization experiment, let E be the binding energy and let the photon with frequency $\nu$ be incident. Assuming that the recoil energy of the atom is small, show that the electrons come out with a momentum approximately equal to $[2m(h\nu - E)]^{1/2}$. Obtain an expression for the recoil energy of the atom when the electron comes out at an angle $\phi$ with respect to the momentum of the photon.

Solution:




photon energy $= h\nu$

B.E. of the electron $= E$

If $h\nu$ is greater than E, the rest energy goes to the KE of the atom and electron;

$$h\nu - E = (KE)_e + (KE)_{atom\ (recoil)}$$

If you neglect the recoil momentum of atom

$$\Rightarrow (K.E)_e = h\nu - E$$

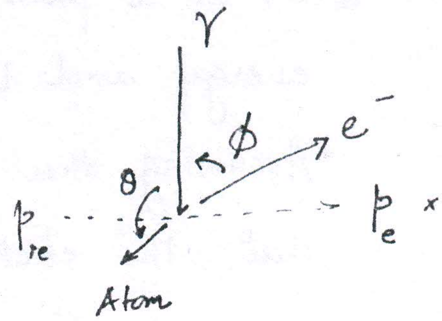
In classical approxim$^n$ $(v \ll c)$

$$KE = p_e^2/2m$$

$$\Rightarrow p_e = [2m(h\nu - E)]^{1/2}$$

Conservation of momentum

For simplicity take the photon direction to be y-axis




$$\left.\begin{aligned} p_\gamma &= p_e \cos\phi + p_{re} \cos\theta \\ p_e \sin\phi &= p_{re} \sin\theta \end{aligned}\right\} \text{—(1)}$$

$$p_{re}\cos\theta = p_\gamma - p_e \cos\phi$$

$$\left.\begin{aligned} \therefore\ p_{re}^2 \cos^2\theta &= (p_\gamma - p_e\cos\phi)^2 \\ \text{S'lly}\quad p_{re}^2 \sin^2\theta &= p_e^2 \sin^2\phi \end{aligned}\right\} \cdots (2)$$

Add (2) subs

$$p_{re}^2 = (p_\gamma - p_e\cos\phi)^2 + p_e^2\sin^2\phi$$

$$= p_\gamma^2 + p_e^2\cos^2\phi - 2p_\gamma p_e \cos\phi + p_e^2 \sin^2\phi$$

$$\boxed{\begin{aligned} E_\gamma &= p_\gamma c \\ \Rightarrow p_\gamma &= h\nu/c = h/\lambda \end{aligned}}$$

$$= p_\gamma^2 + p_e^2 - 2p_e p_\gamma \cos\phi$$

$$= \left(\frac{h\nu}{c}\right)^2 + 2m(h\nu - E) - 2\frac{h\nu}{c}\sqrt{2m(h\nu - E)}\cos\phi$$

$$\frac{p_{re}^2}{2M} = \frac{1}{2M}\left[\frac{(h\nu)^2}{c^2} + 2m(h\nu - E) - 2\frac{h\nu}{c}\sqrt{2m(h\nu - E)}\cos\phi\right]$$

$$\therefore E_{recoil} = \frac{1}{2Mc^2}\left[(h\nu)^2 + 2mc^2(h\nu - E) - 2h\nu\cos\phi\sqrt{2mc^2(h\nu - E)}\right]$$

34. Light of wavelength 2000 Å falls on a metal surface. If the work function of the metal is $4.2eV$, find the kinetic energy of the fastest and the slowest emitted photoelectrons. Also find the stopping potential and cutoff wavelength for the metal.

[Ans: 2.0 eV, 0, 2V, 2960 Å]

Solution: $\lambda = 2000 \text{ Å}$ , $\phi = 4.2 \text{ eV}$

Photoelectric Equation says

$$h\nu - \phi = K.E.$$

$\therefore (K.E.)_{max}$ for fastest $e^-$ will be

$$= \frac{hc}{\lambda} - \phi$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2000 \times 10^{-10}} - 4.2 \text{ eV}$$

$$= 1.57 \text{ eV}$$

$\Rightarrow$ Slowest photoelectron should have $K.E. = 0$

If $V_s$ is the stopping potential to stop this electron with max$^m$ KE, we have

$$(KE)_{max} = eV_s$$

$$\Rightarrow V_s = 1.57 \text{ Volts.}$$

Cutoff wavelength: The wavelength of light which matches with the work function.

$$\therefore \phi = h\nu_{cutoff}$$

$$\therefore \frac{hc}{\lambda_{cut}} = \phi \Rightarrow \lambda_{cut} = \frac{hc}{\phi}$$

$$\therefore \lambda_{cutoff} = 3 \times 10^{-7} \text{ m.}$$

35. An experiment on photoelectric effect of a metal gives the result that the stopping potential for $\lambda = 1850\text{Å}$ and $5460\text{Å}$ are $4.62V$ and $0.18V$ respectively. Find the value of Planck's constant, the threshold frequency and the work function.

**[Ans: 6.64 x $10^{-34}$ Js, 0.5 x $10^{15}$ Hz, and 2.1 eV]**

Solution: $\lambda_1 = 1580\ \text{Å}$ , $V_s^1 = 4.62\ V$

$\lambda_2 = 5460\ \text{Å}$ , $V_s^2 = 0.18\ V$

According to PE eq$^n$ $h\nu - \phi = KE =$ Stopping potential

$\because$ Material is same, $\phi$ for the mat is same all time

$\therefore$ we can write

$$\left.\begin{aligned} \frac{hc}{\lambda_1} - \phi &= eV_s^1 \\ \frac{h\nu}{\lambda_2} - \phi &= eV_s^2 \end{aligned}\right\} \Rightarrow hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = (4.62 - 0.18)$$

Solving this

$h = 6.626 \times 10^{-34}$ Js

Take one of the equations,

$$\frac{hc}{\lambda_1} - \phi = eV_s^1$$

$$\Rightarrow \phi = \frac{hc}{\lambda_1} - eV_s^1$$

$$= 2.07\ eV$$

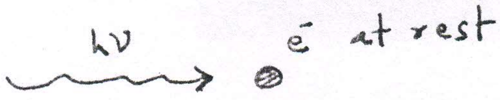
S/ly $h\nu_{cutoff} = \phi$

$$\Rightarrow \nu_{cutoff} = \phi/h$$

$$= 0.5 \times 10^{15}\ Hz.$$

38. Show that a free electron can not absorb a photon. Hence photoelectron requires bound electron. In Compton effect, however, the electron can be free.

Solution :




First check will be conservation of momentum

$$\frac{h\nu}{c} + 0 = p_{e^-}$$

$$\Rightarrow h\nu = p_{e^-} c$$

→ Energy Conservation

$$h\nu + m_e c^2 = \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$\Rightarrow p_e c + m_e c^2 = \sqrt{p_e^2 c^2 + m_e^2 c^4}, \quad \text{square it}$$

$$\Rightarrow p_e^2 c^2 + m_e^2 c^4 + 2 m_e p_e c^3 = p_e^2 c^2 + m_e^2 c^4$$

$$\Rightarrow 2 p_e m_e c^3 = 0$$

and this is not possible.

39. **Find the angle $\theta$ for which the energy of the Compton scattered photon would be $2m_oc^2$, if the energy of the incident photon is $4m_oc^2$. Show that if $\theta$ is greater than $60^o$, the scattered photon will never have an energy greater than $2m_oc^2$, irrespective of the energy of the incident photon.**

Solution: We have $\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$

$E = 4 m_o c^2$, $E' = 2 m_o c^2$ $\qquad m_o = e^-$ mass

$$\frac{1}{2} - \frac{1}{4} = (1 - \cos\theta)$$

$$\therefore \cos\theta = 3/4$$

For scattered photon,

$$\frac{1}{E'} = \frac{1}{E} + \frac{1}{m_e c^2}(1 - \cos\theta)$$

$$\therefore E' = \frac{E m_e c^2}{m_e c^2 + E(1 - \cos\theta)}$$

$$= \frac{2 m_o c^2}{\left(\frac{2 m_o c^2}{E} + 2(1 - \cos\theta)\right)}$$

Hence for $\theta > 60^0$, $2(1 - \cos\theta) > 1$

$\Rightarrow$ For any value of E, the denominator is $> 1$

$$\therefore E' < 2 m_o c^2$$

44. In Compton scattering of radiation by an angle $\theta$, by a scatterer of rest mass $m$ if the scatterer gets excited (or de-excited) consequent to scattering so that its final rest mass becomes $M \neq m$, show that the change in the wavelength will become $\Delta\lambda = \lambda_c(1-\cos\theta)+(\lambda\lambda' c/2hm)(M^2-m^2)$. Show that the additional term is consistent with energy conservation.

Solution:


The particle of mass 'm' is at rest

After scattering the mass becomes M

→ Conserv$^n$ of Momentum

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\phi \quad \text{— along X-}$$

$$\frac{h\nu'}{c}\sin\theta = p\sin\phi \quad \text{— along Y-axis}$$

$$\Rightarrow h\nu'\sin\theta = pc\sin\phi$$

$$\text{X-axis} \Rightarrow h\nu - h\nu'\cos\theta = pc\cos\phi$$

Square and add

$$h^2(\nu-\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2 = p^2c^2$$

$$(h\nu)^2 + (h\nu'\cos\theta)^2 - 2h^2\nu\nu'\cos\theta + (h\nu'\sin\theta)^2 = p^2c^2$$

$$\Rightarrow (h\nu)^2 + (h\nu')^2 - 2h\nu h\nu'\cos\theta = p^2c^2$$

→ Energy Conservation

$$h\nu + mc^2 = h\nu' + \sqrt{p^2c^2 + M^2c^4}$$

Squaring and balancing

$$(h\nu - h\nu' + mc^2)^2 = \left(\sqrt{p^2c^2 + M^2c^4}\right)^2$$

$$(h\nu)^2 + (h\nu')^2 + m^2c^4 - 2h\nu h\nu' - 2h\nu' mc^2 + 2h\nu\, mc^2$$

$$= p^2c^2 + M^2c^4$$

$$=$$

$$\cancel{(h\nu)^2} + \cancel{(h\nu')^2} + m^2c^4 - 2h\nu h\nu' - 2h\nu' mc^4 + 2h\nu mc^2$$

$$= \cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2h\nu h\nu' \cos\theta + M^2c^4$$

$$2hmc^2(\nu - \nu') = 2h\nu h\nu' \cancel{\cos\theta} (1 - \cos\theta) + M^2c^4 - m^2c^4$$

$$2hmc^2c\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = 2h\frac{c}{\lambda}\, h\frac{c}{\lambda'}\,(1-\cos\theta) + (M^2 - m^2)c^4$$

$$2hmc^3\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{2h^2c^2}{\lambda\lambda'}(1-\cos\theta) + (M^2-m^2)c^4.$$

$$\Delta\lambda = \frac{h^2c^2}{hmc^3}(1-\cos\theta) + \frac{(M^2-m^2)c}{2hm}\lambda\lambda'$$

$$= \frac{h}{mc}(1-\cos\theta) + \frac{(M^2-m^2)c}{2hm}\lambda\lambda'$$

$$\therefore \Delta\lambda = \lambda_c(1-\cos\theta) + \frac{(M^2-m^2)c}{2hm}\lambda\lambda'$$

where $\lambda_c \equiv$ Compton wavelength $= \dfrac{h}{m_e c}$