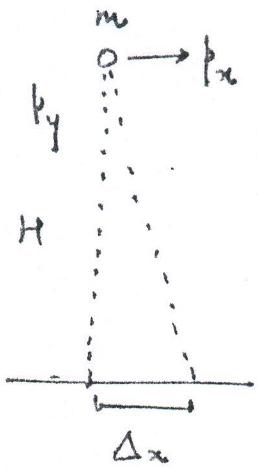
57. A boy on the top of a ladder of height H is dropping marbles of mass m to the floor and is trying to hit a crack on the floor. To aim, he uses equipment of highest possible precision. Show that, despite his great care, the marbles miss the crack by an average distance of the order of $(h/2m)^{1/2}(2H/g)^{1/4}$.



Solution: When ball is dropped, it has uncertainty in the x-component of the momentum. If it has no uncertainty in p_x, then according to uncertainty relationship, Δx would have been infinite. In order to make Δx finite, you will have uncertainty in p_x.

$$\therefore \quad \Delta p_x \, \Delta x \sim \hbar$$

$$m \Delta v_x \Delta x \sim \hbar$$, Δv_x is the uncertainty in the x-component of the velocity.

We can substitute

$$\frac{\Delta x}{t} = \Delta v_x$$, $t \rightarrow$ time taken by the ball to reach the ground.

$$\therefore \Rightarrow \frac{m(\Delta x)^2}{t} \sim \hbar$$

$$(\Delta x)^2 \sim \frac{\hbar t}{m}$$

$\because$ we have $H = ut + \frac{1}{2}gt^2$, $u = 0$

$$\Rightarrow t^2 = 2H/g \Rightarrow t = \sqrt{\frac{2H}{g}}$$

$$\therefore (\Delta x)^2 \sim \frac{\hbar}{m}\left(\sqrt{\frac{2H}{g}}\right)$$

$$\Rightarrow \Delta x \sim \sqrt{\frac{\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}$$