

# Recapitulate

- Role of Bragg reflection.

$$\frac{2\pi}{\lambda} = \frac{n\pi}{a}$$

$$\Rightarrow 2a = n\lambda$$

- Discussed the electron transport in Kronig Penny model.

# Recapitulate

- Filled Bands do not conduct.
- Transport at the bottom of band can be described in terms of **electrons** while at the top of the band in terms of **holes**.
- Difference between metals, insulators and semiconductors.

# Conductivity

The electron and hole currents add and so does the conductivity

$$\sigma = ne\mu_n + pe\mu_p$$

$$\mu \equiv \left| \frac{e\tau}{m^*} \right|$$

# Hall Effect

The electron and hole oppose each other in Hall effect as both the type of charge carriers get deflected to the same face.

$$R_H = \frac{1}{e} \frac{p\mu_p^2 - n\mu_n^2}{(p\mu_p + n\mu_n)^2}$$

# Electrons in CB per Unit Volume

$$n = \int_{\varepsilon_c}^{\varepsilon_{ct}} g(\varepsilon) f(\varepsilon) d\varepsilon$$

$$g(\varepsilon) = \frac{1}{2\pi^2} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} (\varepsilon - \varepsilon_c)^{1/2}$$

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon - \varepsilon_F)/kT} + 1}$$

## Note

- It is per unit volume, so  $V$  is not therein the density of state expression.
- An effective mass ( $d.o.s$ ) has been used rather than the actual mass of electron.
- The zero of the energy has been shifted to the bottom of conduction band.

# Two Approximations

1. Put  $\varepsilon_{ct} = \infty$ .
2. Neglect 1 in the denominator of  $f(\varepsilon)$ .

## After the Approximations

$$n = A \int_{\varepsilon_c}^{\infty} (\varepsilon - \varepsilon_c)^{1/2} e^{-(\varepsilon - \varepsilon_F)/kT} d\varepsilon$$

$$= A e^{-(\varepsilon_c - \varepsilon_F)/kT} \int_{\varepsilon_c}^{\infty} (\varepsilon - \varepsilon_c)^{1/2} e^{-(\varepsilon - \varepsilon_c)/kT} d\varepsilon$$

$$A \equiv \frac{1}{2\pi^2} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2}$$



$$n = Ae^{-(\varepsilon_c - \varepsilon_F)/kT} \int_{\varepsilon_c}^{\infty} (\varepsilon - \varepsilon_c)^{1/2} e^{-(\varepsilon - \varepsilon_c)/kT} d\varepsilon$$

Put  $(\varepsilon - \varepsilon_c) / kT = x; d\varepsilon = (kT)dx$

$$n = Ae^{-(\varepsilon_c - \varepsilon_F)/kT} (kT)^{3/2} \int_0^{\infty} x^{1/2} e^{-x} dx$$

$$= \frac{\sqrt{\pi}}{2} Ae^{-(\varepsilon_c - \varepsilon_F)/kT} (kT)^{3/2}$$

## Final Expression

$$n = 2 \left( \frac{m_e^* kT}{2\pi \hbar^2} \right)^{3/2} e^{-(\varepsilon_c - \varepsilon_F)/kT}$$

For  $m_e^* = m_e$

$$n \approx 4.83 \times 10^{21} T^{3/2} e^{-(\varepsilon_c - \varepsilon_F)/kT} m^{-3}$$

# Hole Statistics

$$\begin{aligned}f_h(\varepsilon) &= 1 - f(\varepsilon) \\&= 1 - \frac{1}{e^{(\varepsilon - \varepsilon_F)/kT} + 1} \\&= \frac{e^{(\varepsilon - \varepsilon_F)/kT}}{e^{(\varepsilon - \varepsilon_F)/kT} + 1} \\&= \frac{1}{e^{(\varepsilon_F - \varepsilon)/kT} + 1}\end{aligned}$$

# Hole Density of States

$$g_h(\varepsilon) = \frac{1}{2\pi^2} \left( \frac{2m_h^*}{\hbar^2} \right)^{3/2} (\varepsilon_v - \varepsilon)^{1/2}$$

# Holes in VB per Unit Volume

$$p = \int_{-\infty}^{\varepsilon_v} g_h(\varepsilon) f_h(\varepsilon) d\varepsilon$$

$$p = B \int_{-\infty}^{\varepsilon_v} (\varepsilon_v - \varepsilon)^{1/2} e^{-(\varepsilon_F - \varepsilon)/kT} d\varepsilon$$

$$B \equiv \frac{1}{2\pi^2} \left( \frac{2m_h^*}{\hbar^2} \right)^{3/2}$$

## Final Expression

$$p = B \int_{-\infty}^{\varepsilon_v} (\varepsilon_v - \varepsilon)^{1/2} e^{-(\varepsilon_F - \varepsilon)/kT} d\varepsilon$$

Put  $(\varepsilon_v - \varepsilon) / kT = x; d\varepsilon = -(kT)dx$

$$p = 2 \left( \frac{m_h^* kT}{2\pi \hbar^2} \right)^{3/2} e^{-(\varepsilon_F - \varepsilon_v)/kT}$$

# Fermi Energy

## Charge Neutrality Condition

$$n = p$$

$$2 \left( \frac{m_e^* kT}{2\pi \hbar^2} \right)^{3/2} e^{-(\varepsilon_c - \varepsilon_F)/kT}$$
$$= 2 \left( \frac{m_h^* kT}{2\pi \hbar^2} \right)^{3/2} e^{-(\varepsilon_F - \varepsilon_v)/kT}$$

$$\left(m_e^*\right)^{3/2} e^{-(\varepsilon_c - \varepsilon_F)/kT}$$

$$= \left(m_h^*\right)^{3/2} e^{-(\varepsilon_F - \varepsilon_v)/kT}$$

$$\frac{3}{2} kT \ln \left( \frac{m_e^*}{m_h^*} \right) = (\varepsilon_c - \varepsilon_F + \varepsilon_v - \varepsilon_F)$$

$$\varepsilon_F = \frac{\varepsilon_c + \varepsilon_v}{2} - \frac{3}{4} kT \ln \left( \frac{m_e^*}{m_h^*} \right)$$



## Note

- To a good approximation the Fermi energy is half way in the band gap.
- This justifies neglect of **1** in the denominator in F.D. Statistics.
- It also has a very weak temperature dependence.

## Order of Number of $n$ at 300 K

$$n = 2 \left( \frac{m_e^* kT}{2\pi \hbar^2} \right)^{3/2} e^{-(\varepsilon_c - \varepsilon_F)/kT}$$
$$\approx 2.5 \times 10^{25} e^{-(\varepsilon_c - \varepsilon_F)/kT} ; \text{for } m_e^* = m_e$$

At 300K

$$kT \approx 0.026 \text{ eV}$$

## $n$ for Si and Ge at 300 K

$$\begin{aligned} n &\approx 2.5 \times 10^{25} e^{-(1.1/2 \times 0.026)} \\ &\approx 1.63 \times 10^{16} m^{-3} \text{ for Si} \end{aligned}$$

$$\begin{aligned} n &\approx 2.5 \times 10^{25} e^{-(0.72/2 \times 0.026)} \\ &\approx 2.42 \times 10^{19} m^{-3} \text{ for Ge} \end{aligned}$$

## Note

- There is an exponential dependence of  $n$  on temperature.
- Hence the conductivity increases with temperature, even though mobility may show decrease.

## Extrinsic Semiconductor

- Doping of group 3 or group 5 elements.
- Group 3 doping creates electron states in the gap close to the top of valence band.
- Group 5 doping creates the same in the gap close to bottom of conduction band.

# Donor Energy Levels

$$\varepsilon_c - \varepsilon_d$$

	P	As	Sb
Ge	0.012	0.0127	0.0096
Si	0.045	0.049	0.039

# Acceptor Energy Levels

$$\varepsilon_a - \varepsilon_v$$

	B	Al	Ga
Ge	0.0104	0.0102	0.0108
Si	0.045	0.057	0.065

## Charge Carriers for Doped SC

- The old expressions are valid. Only position of Fermi level changes.
- Fermi Level has a strong dependence on temperature.



# Fermi Energy

## General Charge Neutrality Condition

$$n + N_a^- = p + N_d^+$$

## Two Approximations

1. All impurity atoms are ionized. Quite valid at R.T.
2. Minority carrier concentration is negligible. Valid for reasonable dopings.