Special Theory of Relativity

Entirely a new concept of Energy.

$$E = mc^2$$

$$m = \frac{m_o}{\sqrt{1 - u^2 / c^2}}$$

New Definitions from STR

$$m = \frac{m_o}{\sqrt{1 - u^2 / c^2}}$$

$$\vec{p} = m\vec{u}$$

$$E = mc^2$$

$$K = mc^2 - m_o c^2$$

$$\boxed{E^2 = p^2c^2 + m_0^2c^4}$$

Conservation Law

In every process both energy and momentum have to be conserved.

Zero Rest Mass Particle

If rest mass is zero

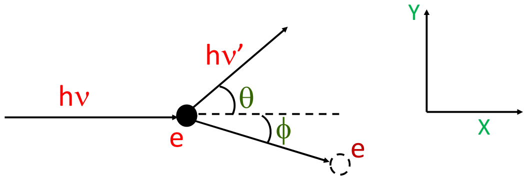
$$E = \frac{m_o c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \quad \text{and} \quad \vec{p} = \frac{m_o \vec{u}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

are zero, unless u=c. In such cases

$$E = K = pc$$

Compton Effect

Recoil of a photon by a free electron.
Going ahead with an idea that light shows a particle nature



Conservation Equations

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e\cos\phi$$

$$\frac{h\nu'}{c}\sin\theta = p_e\sin\phi$$

$$m_o c^2 + h\nu = h\nu' + E_e$$

$$\boxed{E_e^{\,2} = p_e^{\,2}c^2 + m_o^{\,2}c^4}$$

Eliminate ϕ

$$p_e^{\,2}(\cos^2\phi + \sin^2\phi) = p_e^{\,2}$$

$$= \left(\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta\right)^2 + \left(\frac{h\nu'}{c}\sin\theta\right)^2$$

Eliminate p_e

$$p_e^{\,2}c^2 = E_e^{\,2} - m_o^{\,2}c^4$$

$$= (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta$$

$$(h\nu - h\nu' + m_o c^2)^2 - m_o^{\,2}c^4$$

$$= (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta$$

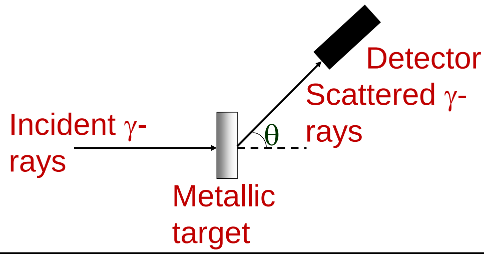
$$(h\nu - h\nu')m_o c^2 = (h\nu)(h\nu')(1-\cos\theta)$$

$$\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)m_o c^2 = \frac{hc}{\lambda}\frac{hc}{\lambda'}(1-\cos\theta)$$

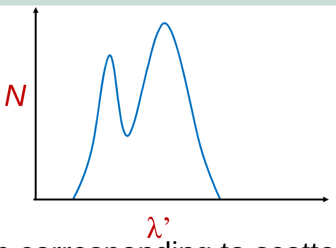
$$\boxed{\lambda' - \lambda = \frac{h}{m_o c}(1-\cos\theta)}$$

The Experimental Set up

Free Electrons within the metals are used for this experiment

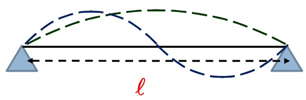


Experimental Results



Linewidth corresponding to scattered photons is more due to a distribution of speeds of ‘free’ electron.

Vibration of strings



$$\ell = \frac{n\lambda}{2} \qquad \nu = \frac{n}{2\ell}\sqrt{\frac{T}{m}}$$

In a way the frequencies that can be excited in this string are 'quantized'. The boundary condition has limited the frequencies in the string to discrete values.

De Broglie Wavelength (1924)

For a photon

$$p = \frac{h\upsilon}{c} = \frac{h}{\lambda}$$

For a particle, photon or massive

$$\lambda = \frac{h}{p}$$

Discussion

- The de Broglie wave length would make the wave along the Bohr circumference stationary.
- The wavelength of a 200 g ball moving with speed 100 m/s is 3.32×10^{-35} m.
- A thermal electron would have a wavelength of 6.25×10^{-9} m.

Experimental Verification

- Davisson and Germer (1925) and later G.P. Thomson succeeded in diffracting electrons.
- The crystal lattice was used to diffract electrons. The same formula used by Bragg for X-ray can also explain the observed diffraction pattern.

$$2d\sin\theta = n\lambda$$