Solution to Relativity tutorial Q.6

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6. Given that a rod of length 3m proper in frame S is angled at 60° to the x-axis. A bullet with speed $u = 0.8c$ is launched at it from the lower end of the rod (which we assume as the origin of S), parallel to the rod. Another frame S' travels at 0.6c relative to S along the x-axis.

For bullet, u_x= x-component of its velocity in S frame=$u\cos(60°) = 0.4c$
u_y = y-component of its velocity in S frame= $u\sin 60° = 0.4\sqrt{3}c$

Let E be the event of the bullet reaching the end of the rod in S frame.
If (x, y, t) and (x', y', t') are its coordinates in the frames S and S' respectively, then
$x = 3\cos 60° = 1.5m, y = 3\sin 60° = 1.5\sqrt{3}m$,
Speed of S' w.r.t. s = $v = 0.6c$ in the positive x direction, so
$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.25$
$\beta = \frac{v}{c} = 0.6$

a) In S, the time taken by the bullet to reach the stopper will simply be $\frac{3}{0.8c} = 1.25 \times 10^{-8}$ seconds since the bullet travels along the rod.
The time coordinates of this event in S' $=t' = \gamma(t - \frac{vx}{c^2}) = 1.19 \times 10^{-8}$ seconds.

b) By velocity transformation across frames S and S' ($v = 0.6c, u_x = 0.4c, u_y = 0.4\sqrt{3}c$), we have

x' component of bullet's velocity = $u'_x = \frac{u_x - v}{1-\frac{u_x v}{c^2}}$ = -0.26c
($i.e.$0.26c *along* -x'*direction*)
y' component of bullet's velocity = $u'_y = \frac{u_y}{\gamma(1-\frac{u_x v}{c^2})} = 0.73c$

c) To check if the bullet reaches the stopper or not, we first calculate the space coordinates of event E (bullet hitting the stopper in S) in S' by Lorentz transformation.
$x' = \gamma(x - vt) = -0.94$ metres ,
$y' = y = 1.5\sqrt{3}$ metres
Now if the bullet does hit the stopper in S', it must be at $((x', y')$ after time t'.

So,($u'_y t' = 0.73c \times 1.19 \times 10^{-8} = 2.61m = y'$
and $u'_x t' = -0.26c \times 1.19 \times 10^{-8} = -0.94m = x'$
This confirms that the bullet is indeed at the stopper's location at the moment they are supposed to collide. Hence, **the bullet does hit the stopper.**