53. One of the lines in the hydrogen spectrum has a wavelength 4861.32Å. It was later discovered that this line has a faint companion located at 4859.975Å. The explanation is that a heavier isotope deuterium is responsible for the faint line. Use this data to compute the ratio of the deuterium mass to the proton mass.

[Ans. 2]

Solution:

$$E_H = \mu_H \cdot \text{const}$$

$$= h\nu_H = \frac{hc}{\lambda_H}, \quad \lambda_H = 4861.32\,\text{Å}$$

$$E_D = \mu_D \cdot \text{const} = h\nu_D = \frac{hc}{\lambda_D}, \quad \lambda_D = 4859.975\,\text{Å}$$

Take the ratio

$$\frac{E_H}{E_D} = \frac{\mu_H}{\mu_D} = \frac{\lambda_D}{\lambda_H}$$

$$E_n = \frac{-\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2}$$

$$\Rightarrow \frac{\mu_H}{\mu_D} = \frac{\lambda_D}{\lambda_H}$$

$$\mu_H = \frac{M_p m_e}{M_p + m_e}, \quad M_p = M_n = 1836\, m_e$$

$$\Rightarrow \mu_H = \frac{1836\, m_e^2}{1837\, m_e}$$

Silly $\mu_D = \dfrac{M_D m_e}{M_D + m_e} = \dfrac{n \cdot 1836 m_e^2}{(n \cdot 1836 + 1) m_e}$ $\quad M_D = n \cdot x\, 1836\, m_e$

$$\therefore \frac{\mu_H}{\mu_D} = \frac{(n \cdot 1836 + 1)}{n \cdot 1837} = \frac{4859.975}{4861.32}$$

$$\Rightarrow n = 2.0336$$