PH-105 Assignment Sheet - 1

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9. An observer O sees another observer A pass by him with a velocity v_1. At this instant, the watches of O and A read zero. After time t_1, O sees another observer passing by him with velocity v_2. Sometime later, B catches A. At this instant watch of O reads 245 μs and watch of A reads 173 μs. According to B, the time difference between passing of O and catching of A is 100 μs. Assume that the observers O, A and B are at the origins of their respective frames, calculate v_1, v_2 and t_1. Also calculate the relative velocity of B and A and the time in A's frame when B passes O.
Solution :
Let us denote the 3 events (of the three persons passing each other) by subscripts OA,OB,AB. Also let the three quantities in each frame be denoted by superscripts O,A,B. Lets denote γs of v_1 and v_2 by γ_1 and γ_2

Now we know $t^O_{OA} = t^A_{OA} = x^O_{OA} = x^A_{OA}$, which means lorentz tranform is applicablt to space-time coordinates of any event happening in these two frames. There is no need for taking difference in coordinates.
Also $t^A_{AB} = 173\mu s$, $x^A_{AB} = 0$ and $t^O_{AB} = 245\mu s$.

Now by lorentz transformation between frames O and A on event AB, we have

$$t^O_{AB} = \gamma_1(t^A_{AB} + \frac{vx^A_{AB}}{c^2})$$

$$245\mu s = \gamma_1 173\mu s$$

$$\gamma_1 = 245/173$$

$$\gamma_1 = 1.416184971$$

Thus, $v_1 = 0.7080c$.

Now $\Delta t^O_{OB-AB} = t_1$. In frame O, event AB occurs. Distances travelled by both A and B is the same in O's frame.

$$(245\mu s - t_1)v_2 = 245\mu s v_1$$

$$(245\mu s - t_1)v_2 = 173.481c\mu s$$

Also now consider space-time interval between OB and AB in frames O and B. By lorentz transformation we have

$$\Delta t^O = \gamma_2(\Delta t^B - \frac{v_2\Delta x^B}{c^2})$$

$$(245\mu s - t_1) = \gamma_2 100\mu s)$$

$$173.481c/v_2 = \gamma_2 100$$

$$\gamma_2 v_2 = 1.73481c$$

$$v_2^2(1 + 1.72481^2) = 1.72481^2 c^2$$

$$v_2 = 0.8651c$$

Thus we have $t_1 = 44.47\mu s$

$$v_{AB} = \frac{v_A - v_B}{1 - (v_A v_B)/c^2}$$

$$v_{AB} = \frac{0.1571c}{0.39388}$$

$$v_{AB} = 0.4445$$

Consider the space-time interval between events OA and OB in frames O and A. By Lorentz transformation,

$$\Delta t^A = \gamma_1(\Delta t^O - v\Delta x^O/c^2)$$

$$\Delta t^A = 1.4161(44.47)\mu s$$

$$\Delta t^A = 62.97\mu s$$