PH-105 Assignment Sheet - 1

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7. **Two observers A and B, sitting at the origin of their respective inertial frames are moving with respect to each other with a relative speed of 0.6c. The observer A sends a light signal to observer B, 3×10^{-6}s after B passes A, as measured in A's frame. The observer B receives this light signal, He waits for 2×10^{-6}s after receiving the light signal as per his watch and then sends a light signal back to A.**

   (a) **Find the time when the light signal is received by B as per the watches of A and B.**

   (b) **Find the time when the light signal is received back by A as per the watches of A and B.**

   (c) **What was the distance of B from A when B had sent the light signal according to A and B?**

   (d) **What was the time taken by light to reach A after it was emitted from B, according to A and B?**

   Solution :
   Relative speed, v=0.6c. Hence γ=1.25.
   The following events can be identified in the above problem :

   E1 : A sends light signal.
   E2 : B receives light signal.
   E3 : B sends light signal.
   E4 : A receives light signal.

   Let (x_1, t_1), (x_2, t_2), (x_3, t_3) and (x_4, t_4) be the coordinates of the events in A's frame.
   Similarly, let (x'_1, t'_1), (x'_2, t'_2), (x'_3, t'_3) and (x'_4, t'_4) be the corresponding coordinates in B's frame.

   Now, we find all unknown values, one event at a time.

   E1:
   $x_1 = 0$ (A is at origin of 1^{st} frame)
   $t_1 = 3 \times 10^{-6}s$ (from given data)
   $x'_1 = \gamma(x_1 - vt_1) = -675m$
   $t'_1 = \gamma(t_1 - \frac{vx_1}{c^2}) = 3.75 \times 10^{-6}s$

   E2:
   $x'_2 = 0$ (B is at origin of 2^{nd} frame)
   $t'_2 = t'_1 + \frac{|x'_1|}{c} = 6 \times 10^{-6}s$ (Light was emitted at a distance $|x'_1|$ away and travelled at speed c)
   $x_2 = \gamma(x'_2 + vt'_2) = 1350m$
   $t_2 = \gamma(t'_2 + \frac{vx'_2}{c^2}) = 7.5 \times 10^{-6}s$

   E3:
   $x'_3 = 0$ (B is at origin of 2^{nd} frame)
   $t'_3 = t'_2 + 2 \times 10^{-6}s = 8 \times 10^{-6}s$ (wait-time of $2 \times 10^{-6}s$ between receiving and sending)
   $x_3 = \gamma(x'_3 + vt'_3) = 1800m$
   $t_3 = \gamma(t'_3 + \frac{vx'_3}{c^2}) = 10 \times 10^{-6}s$

   E4:
   $x_4 = 0$ (A is at origin of 1^{st} frame)
   $t_4 = t_3 + \frac{|x_3|}{c} = 16 \times 10^{-6}s$ (Light was emitted at $|x_3|$ distance away at time t_3 and moved at speed c)
   $x'_4 = \gamma(x_4 - vt_4) = -3600m$
   $t'_4 = \gamma(t_4 - \frac{vx_4}{c^2}) = 20 \times 10^{-6}s$

So, the solutions are as follows:

(a) Time when the light signal is received by B as per the watch of A $= t_2 = 7.5 \times 10^{-6}s$
Time when the light signal is received by B as per the watch of B $= t'_2 = 6 \times 10^{-6}s$

(b) Time when the light signal is received back by A as per the watch of A $= t_4 = 16 \times 10^{-6}s$
Time when the light signal is received back by A as per the watch of B $= t'_4 = 20 \times 10^{-6}s$

(c) Distance of B from A when B had sent the light signal according to A $= x_3 = 1800m$
Distance of B from A when B had sent the light signal according to B $= vt'_3 = 1440m$

(d) Time taken by light to reach A after it was emitted from B, according to A $= t_4 - t_3 = 6 \times 10^{-6}s$
Time taken by light to reach A after it was emitted from B, according to B $= t'_4 - t'_3 = 12 \times 10^{-6}s$