54. **A wave packet is constructed by superposing waves, their wavelengths varying continuously in the following way**

$$y(x,t) = \int A(k)\cos(kx - \omega t)dk$$

where $A(k) = A$ **for** $(k_o - \Delta k/2) \le k_o \le (k_o + \Delta k/2)$ **and** $= 0$ **otherwise. Treat** ω **as constant. Sketch approximately** $y(x,t)$ **and estimate** Δx **by taking the difference between two values of** x **for which central maximum and nearest minimum is observed in the envelope. Verify uncertainty principle from this.**

Solution: $y(x,t) = \int A(k)\cos(kx - \omega t)\,dk$

$A(k) = A$ for $(k_o - \Delta k/2) \le k_o \le (k_o + \Delta k/2)$

$= 0$ otherwise,

$$\Rightarrow y(x,t) = \int_{k_o - \Delta k/2}^{k_o + \Delta k/2} A\cos(kx - \omega t)\,dk$$

$$= A\frac{\sin kx - \omega t}{x}\Bigg|_{k_o - \Delta k/2}^{k_o + \Delta k/2}$$

$$= \frac{A}{x}\left[\sin\{(k_o + \Delta k/2)x - \omega t\} - \sin\{(k_o - \Delta k/2)x - \omega t\}\right]$$

$$= \frac{2A}{x}\sin\left(\frac{\Delta k x}{2}\right)\cos(k_o x - \omega t)$$

Now take $t = 0$,

(i) For $x = 0$

$$y(0,0) = \frac{2A}{x}\frac{\Delta k x}{2}\cos(k_o x)$$

$$= \frac{2A}{2}\Delta k\cos(k_o x)$$

$$y(0) = A\Delta k, \quad \text{maximum}$$