PH-107 (2014)
Tutorial Sheet 2
(de Broglie Wavelengh, Uncertainty Principle)

* Problems to be done in tutorial

E: De Broglie Wavelength:

P22*: Show that the Bohr's condition of quantization of angular momentum leads to a condition of formation of standing wave of electron along the circumference in the Bohr model of hydrogen atom.

P23: Calculate the wavelength of the matter waves associated with the following. Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.

(i) A 2000 kg car moving with a speed of 100km/h.
(ii) A 0.28 kg cricket ball moving with a speed of 40 m/s.
(iii) An electron moving a speed of 10^7m/s.

[**Ans.:** 1.2×10^{-38}m, 5.9×10^{-35}m, 0.73Å]

P24*: A photon having a wavelength equal to half the Compton wavelength gets scattered by an electron at rest. After the scattering the electron is found to have a kinetic energy same as its rest mass energy. Find

(i) The energy of the scattered photon in MeV and the angle at which it was scattered.
(ii) The angle at which the electron was scattered.
(iii) The de-Broglie wavelength of the scattered electron in nm.

F: Group Velocity:

P25*: (a) Find the de Broglie wavelength of thermal neutrons. Can these neutrons be diffracted by solid?

(b) Find the energy of the photon, electron and a neutron for which they will have a wavelength of 1 Å.

(c) At what temperature would the classical kinetic energy of neutron would make its wavelength equal to 10^{-6} m.

[**Ans.:** 1.48 Å, 12.431 keV, 151 eV, 0.082 eV, 6.34 $\times10^{-6}$K]

P26: The band structure of a solid in the low wave vector limit is approximately given by the following equation.

$$\hbar\omega = Ak^2 - Bk^4$$

where A and B are constants.

(a) Show that group and phase velocities are same when

$$\omega = \frac{1}{\hbar}\left[\frac{2A^2}{9B}\right]$$

(b) Show that if the second term in the dispersion relation is neglected, the group velocity of electrons would be twice that phase velocity.

P27*: The dispersion relation for a lattice wave propagating in a one dimensional chain of atoms of mass *m* bound together by force constant β is given by the following equation.

$$\omega = \omega_o \sin(ka/2)$$

$$\omega_o = \sqrt{\frac{4\beta}{m}}$$

Here *a* is distance between atoms and β is given by the following expression.

(a) Show that in the long wavelength limit the medium is non-dispersive.

(b) Find the group and phase velocities at $k=\pi/a$.

[**Ans.:** 0, $\omega_o a/\pi$]

P28: The phase velocity v_p of ocean waves are given by

$$v_p = \sqrt{\frac{g\lambda}{2\pi}}$$

Find the ratio of group to phase velocities for such a wave.

[**Ans.:** 0.5]

P29*: Find the group and the phase velocity of the matter wave associated with a free particle under the assumption the frequency is defined using (i) the kinetic energy (ii) total relativistic energy.

P30: The phase speed V_p of light in a certain wavelength range in a dispersive medium is given by the following expression:

$$V_p = \frac{c}{A + \frac{4\pi^2}{\lambda^2}B}$$

Here λ is the wavelength, *c* is speed of light, and *A* and *B* are constants.

Find an expression of the group speed in terms of the wave vector k.

Taking $A = 1.7$ and $B = \left(\frac{0.01}{4\pi^2}\right)(\mu m)^2$, calculate the group and phase speed of light for a wavelength of 400 *nm*. Which of the two speeds is physically important and why?

G: Uncertainty Principle:

P31: The speed of a bullet of mass *50 g* and an electron is measured to be *300 m/s*, both with an uncertainty of *0.01%.* With what fundamental accuracy could we locate the position of each, if the position is measured simultaneously with the speed?

P32: The position and the momentum of a *1keV* electron are determined simultaneously. Its position is known to an accuracy of only *1 Å* along x-axis. Using uncertainty principle, what is the minimum permissible percentage uncertainty in its momentum along the x-axis? From the above data can you determine the uncertainty along y-axis?

P33: Out of the two electrons, the position uncertainty of the first electron equals its de Broglie wavelength, and that of the second electron equals its Compton wavelength. Both the electrons are non-relativistic, and move along the same line. If the velocity uncertainty were as low as allowed, which of the two electrons has its velocity better defined?

P34: An electron is moving in a parallel beam along the x-direction with momentum, $p=mv$. It encounters a slit of width w. Assuming that the electron gets diffracted somewhere within the central maximum of small angular magnitude $\Delta\theta$, estimate the uncertainty Δp in its momentum component transverse to the direction of motion. Check that uncertainty principle is satisfied in this experiment.

P35: An electron falls from a height of *10 m* and passes through a hole of radius *1 cm*. To study the motion of the electron afterwards, should we apply the wave aspect or the particle aspect?

P36*: A wave packet is constructed by superposing waves, their wavelengths varying continuously in the following way

$$y(x,t) = \int A(k)\cos(kx - \omega t)dk \quad .$$

Where $A(k) = A$ for $(k_o - \Delta k/2) \le k_o \le (k_o + \Delta k/2)$ and $= 0$ otherwise. Sketch approximately $y(x,t)$ and estimate Δx by taking the difference between two values of x for which the central maximum and nearest minimum is observed in the envelope. Verify uncertainty principle from this.

P37: In a world where h is large, the uncertainty principle makes it hard for a croquet player to hit a ball through the wicket. To help alleviate the problem, an inventor designs a device in which the ball is passed through a cylinder that is permitted to fluctuate position through a transverse distance Δy. The ball acquires a transverse uncertainty of velocity Δv_y. Influenced by both the uncertainties Δy and Δv_y, the ball misses the center of the wicket by a distance Y. If the wicket is at a distance L from the cylinder, and v_o is the horizontal velocity of the ball, estimate the value of

Δy for the optimum design (i.e. minimum Y), in (i) a world where $h=0.01$Js (ii) our world.

P38*: A beam of electron of energy *0.025 eV* moving along x-direction, passes through a slit of variable width w placed along y-axis. Estimate the value of the width of the slit for which the spot size on a screen kept at a distance of *0.5 m* from slit would be minimum.

P39: In an imaginary world the value of Planck's constant is very large and is given by $\hbar = 0.01$ J.s. In this world stones of mass 40 g are horizontally being fired in positive x-direction with a constant velocity of 5 m/s. Assume that the source of the stone gun is very far away in negative x-direction such that at x=0, the speed of the stones is purely in +x-direction. At this point, there are two vertical walls with a small gap 'd' in y-direction, in such a way that the stones can cross to the other side only after passing through this gap. At x= 10 m, there is a large wall without any gap, which stops all the stones. Neglect gravity in this problem and take uncertainty product equal to $\hbar$. Find the optimum value of 'd' for which the spread of stone-hits along y-direction on the wall (at x=10) is the least. Also estimate the width of this spread.

P40*: Assume that in case the average value of momentum p_x is zero, the uncertainty in the x-component of momentum Δp_x is related to the average of square of x component of momentum $\langle p_x^2 \rangle$ by the following relation.

$$\Delta p_x = \sqrt{\langle p_x^2 \rangle}$$

Using the above relation estimate the following. [Use the following uncertainty principle for this problem $\Delta x\, \Delta p_x \geq \hbar/2$]

(a) The minimum kinetic energy that proton and electron would have if they were confined to a nucleus of approximate diameter 10^{-14} *m*. This is an argument used against the existence of electron in nuclei.

(b) The ground state energy of a particle of mass m bound by a potential $V = \frac{1}{2}kx^2$.

(c) The radial distance r for which the sum of kinetic and potential energies is minimum in hydrogen atom.

P41: Estimate the minimum possible energy consistent with uncertainty principle (i.e. ground state energy) for a particle of mass *'m'* bound in a potential $V=kx^4$. Use the following uncertainty principle for this problem $\Delta x\, \Delta p_x \geq \hbar/2$.

P42: In the Young's double slit experiment with electrons, the separation between the two slits is d along the y- direction. If we want to be sure that an electron goes through a particular slit, Δy should be smaller than $d/2$. Using uncertainity relation $\Delta y \Delta p_y \geq \hbar/2$, show that this will create enough uncertainity in the p_y, so that the electron would land on the screen over a width larger than the fringe width of the intereference pattern. Thus the interefernce pattern will be destroyed.

P43: An atom can radiate at anytime after it is excited. It is found that on the average the excited atom has a lifetime of 10^{-8} *s*. That is, during this period it emits a photon and is de excited.

(a) What is the minimum uncertainty $\Delta\nu$ in the frequency of the photon and $\Delta\nu/\nu$ for radiation of wavelength $\lambda = 5000Å$?

(b) What is the uncertainty ΔE in the energy of the excited state of the atom?

P44*: A photon of energy E is emitted as a result of a particular transition. What would be the value of the recoil energy, assuming that the atom recoils with non relativistic speed. Let the lifetime of the state be of the order of 10^{-8} s. What would be the order of natural line width of the emitted line. For what value of E, would the recoil energy be of the same order of magnitude as the natural line width? For order of magnitude calculation take the mass number of the atom as 100. What conclusions would you draw from this regarding resonant absorption?
[**Ans.:** $E^2/(2mc^2)$, 6.6×10^{-8} eV, 111 eV]

P45*: (a) A container contains monatomic hydrogen gas in thermal equilibrium at a temperature T, for which k_BT= 0.025 eV. Let E_1 is the difference between the ground state and the first excited state energy of the atom when at rest. Let E_2 be the energy of photon (in the frame of container) required to make this transition in an atom, which is traveling towards the photon (in an antiparallel direction) with the average energy at the above specified temperature.

(i) Find E_1-E_2.

(ii) After the absorption of photon what would be the final velocity of the hydrogen atom.

(iii) If the lifetime of the first excited state is 10^{-8} s, will the photon with energy E_2 be able to cause a transition, had the atom been at rest. Discuss quantitatively.

You are free to make any assumption, provided you justify it.