

# CH107 - Physical Chemistry

- Atomic and molecular structure
  - Intermolecular forces
  - Molecular Spectroscopy

Instructor (D1):  
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# **CH107/ D1 Course Information**

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**Attendance, marks change and course related issues:**

**Course Secretary: Ms. Sangeeta Prasad  
Central Facility, Chemistry Department**

**Email: [sangeeta@chem.iitb.ac.in](mailto:sangeeta@chem.iitb.ac.in)**

**Phone: (022)2576 4159**

# **CH-107 / D1**

**15 lectures and 7 tutorials (Attendance is mandatory)**

**Lectures at Hall 1**

**Mondays 3.30pm and Thursdays 3.30 pm**

<b>15/09</b>	<b>18/09</b>	<b>22/09</b>	<b>25/09</b>	
<b>29/09</b>	<b>9/10</b>	<b>13/10</b>	<b>16/10</b>	<b>Quiz – 18/10</b>
<b>20/10</b>	<b>27/10</b>	<b>30/10</b>	<b>3/11</b>	
<b>5/11</b>	<b>10/11</b>	<b>13/11</b>		

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**Tutorials at LT (301, 302, 303, 202, 203) on Wednesday  
3-3.55 pm**

**17/09    24/09    1/10    8/10    15/10    22/10    29/11**

# **CH-107/D1**

## **Tutorial venue and Teaching Assistants**

**D1/T1**

**LT-301**

**Sandip Kar**

**D1/T2**

**LT-302**

**Avinash Kr. Singh**

**D1/T3**

**LT-303**

**Anindya Datta**

**D1/T4**

**LT-202**

**Sahidul Islam**

**D1/T5**

**LT-203**

**Srimukh Prasad**

**Emails and phone numbers to be provided**

**Office hours : 2 hrs/week and mutually convenient time**

# **CH-107**

## **Time Table**

<b>Duration</b>	<b>Half-semester (~8 weeks)</b>
<b>Quiz</b>	<b>18 October 2014</b>
<b>End-Semester Exam</b>	<b>Anywhere between 18-29 November 2014</b>

## **Evaluation Scheme**

<b>Total</b>	<b>50 Marks</b>
<b>Quiz</b>	<b>20 Marks</b>
<b>End-Semester Exam</b>	<b>30 Marks</b>
<b>Passing Marks</b>	<b>?</b>

# GRADING STASTICS

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<b>Course Code</b>	<b>CH 103 – <u>SEM1</u></b>	<b>Course Code</b>	<b>CH 103 – <u>SEM2</u></b>
<b>Course Name</b>	<b>Chemistry</b>	<b>Course Name</b>	<b>Chemistry</b>
<b>Total Grades Given are</b>	<b>416</b>	<b>Total Grades Given are</b>	<b>452</b>

**Out of Which**

<b>AA+AB</b>	<b>44</b>
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<b>BB</b>	<b>96</b>
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<b>BC</b>	<b>95</b>
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<b>CC</b>	<b>90</b>
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<b>CD</b>	<b>49</b>
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<b>DD</b>	<b>23</b>
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<b>FR</b>	<b>19</b>
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**Out of Which**

<b>AA +AB</b>	<b>22</b>
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<b>BB</b>	<b>49</b>
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<b>BC</b>	<b>71</b>
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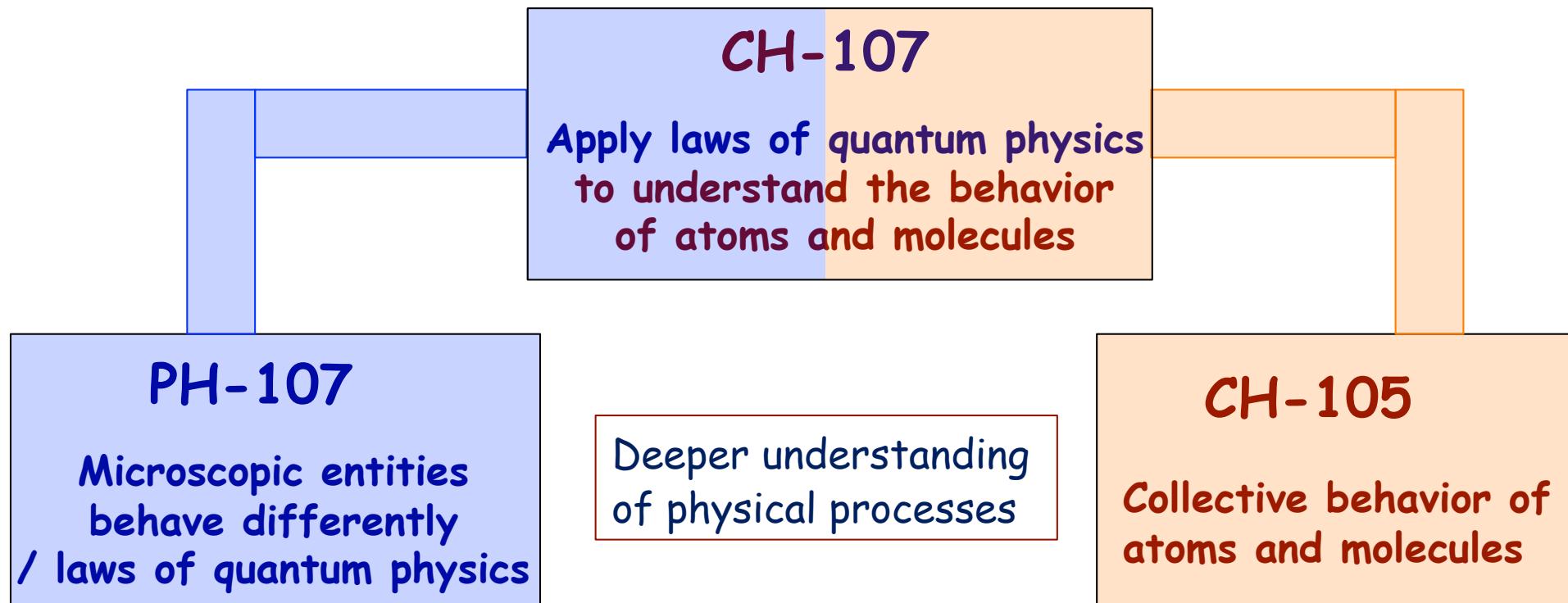
<b>CC</b>	<b>105</b>
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<b>CD</b>	<b>89</b>
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<b>DD</b>	<b>53</b>
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<b>FR</b>	<b>63</b>
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# Why should you study CH-107?



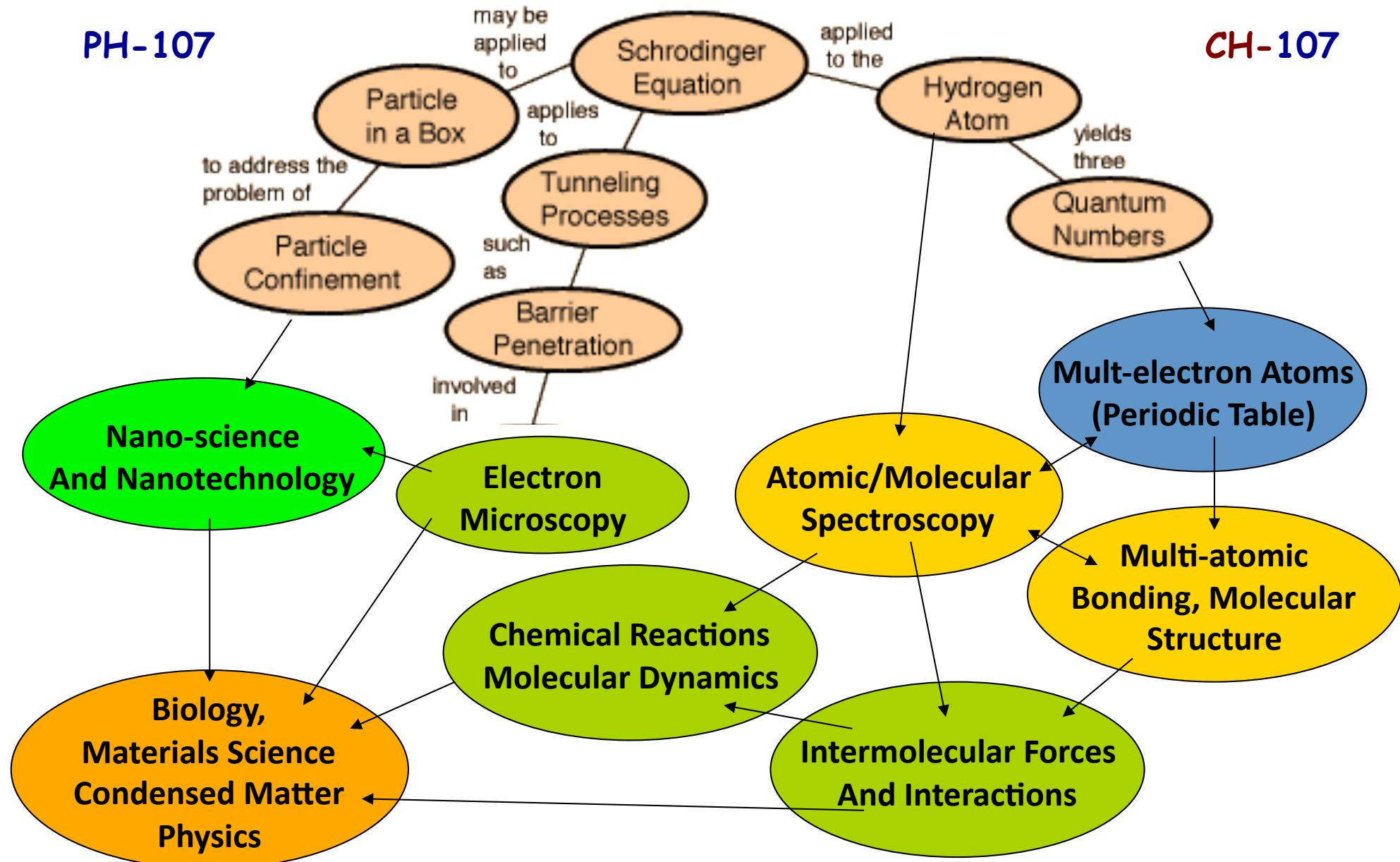
Bridge between novel molecules (materials) prepared  
by synthetic chemists and their potential applications,  
(often dealt with by engineers)

- Nanoelectronics/Nanotechnology: Molecular Electronics
- Energy Science - “Solar Energy” conversion
- BioTechnology - Disease cure, health, medicine
- Atmospheric Science - Go Green - “Save the World”

# Mechanics of Atoms and Molecules

PH-107

CH-107



# **Contents of Part-I**

- **Origin of Quantization:**

Need of a *new theory* for electrons, atoms and molecules  
Postulates of Quantum Mechanics  
Energy Quantization: Particle in a Potential Well

- **Electronic Structure in Atoms:**

Hydrogen Atom and Quantum Numbers  
Atomic Orbitals and Electron Densities  
Multi-electronic atoms and the implications of “*Spin*”

- **Chemical Bonding:**

Molecular Orbital Theory – Linear Combination of Atomic Orbitals  
Energetic and electronic structure of diatomic molecules

- **Molecular-Electronic structure:**

Bonding in poly-atomics using hybridization

# **Contents of Part-II**

- **Intermolecular Forces**
- **Molecular Spectroscopy**
- **Reaction Dynamics and Kinetic theories**
- **Driving Forces for Chemical reactions**

# **Recommended Text**

- **Physical Chemistry – I.N. Levine, 5<sup>th</sup> Ed.**
- **Physical Chemistry – P.W. Atkins 2<sup>nd</sup> Ed.**
- **Physical Chemistry: Molecular Approach - McQuarrie and Simon**

## **Important Websites:**

CH107 Course Material for 2013

And previous year's power-point slides:

[www.chem.iitb.ac.in/academics/menu.php](http://www.chem.iitb.ac.in/academics/menu.php)

<http://vod.cdeep.iitb.ac.in/> (Previous years lectures)

**and will be regularly updated in IITB Moodle**

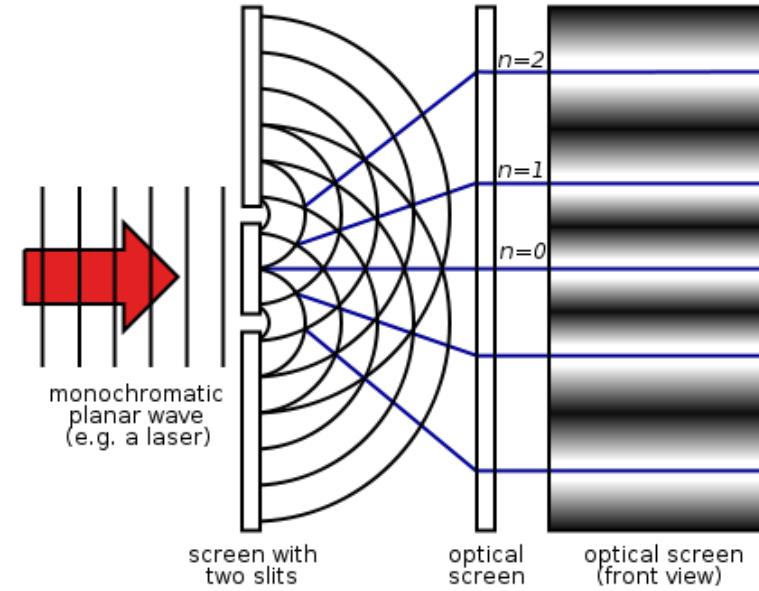
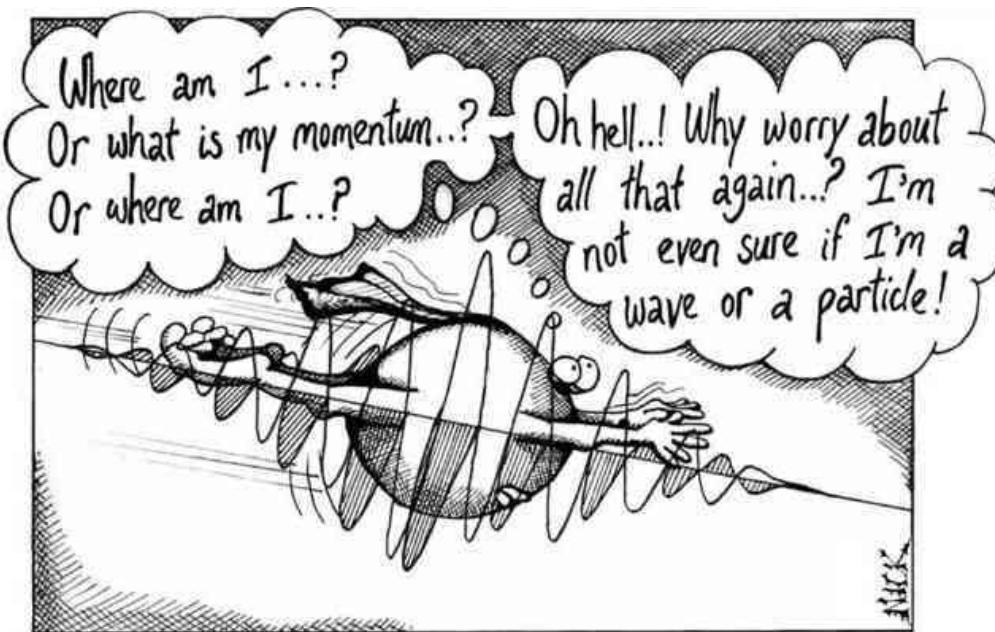
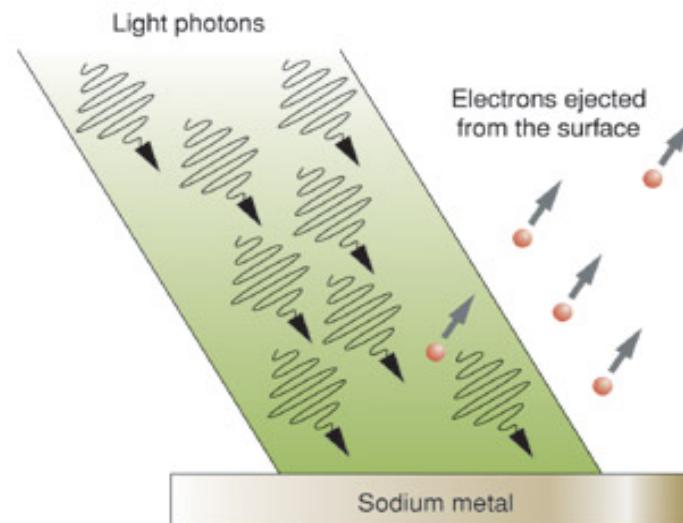
<http://moodle.iitb.ac.in>

# Landmark results in Quantum Physics

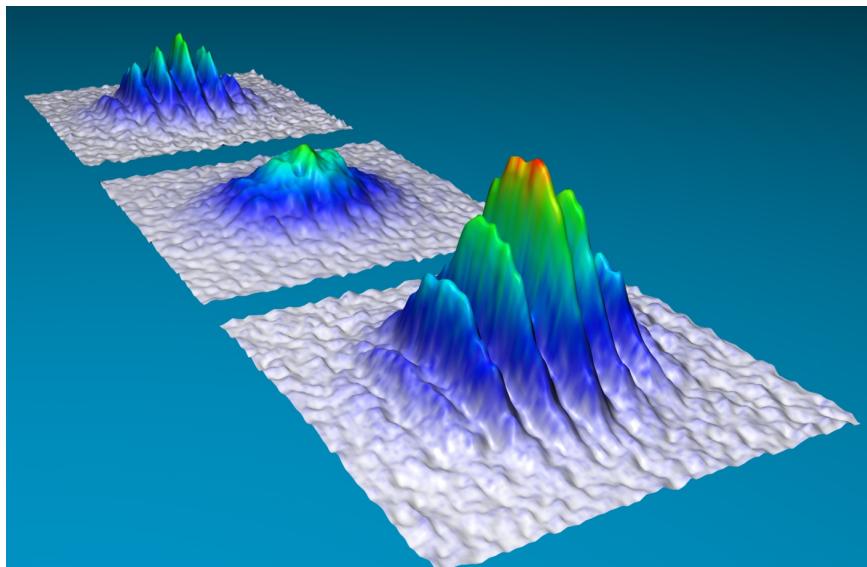


Einstein's relation

$$E = h\nu$$

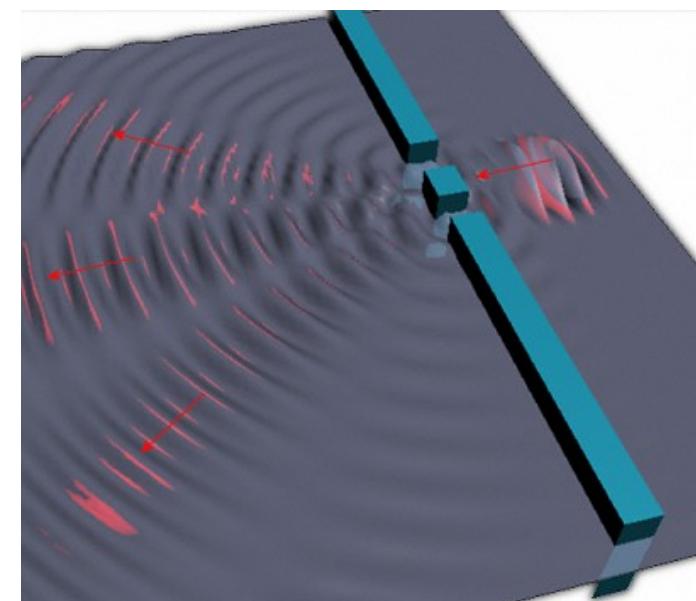
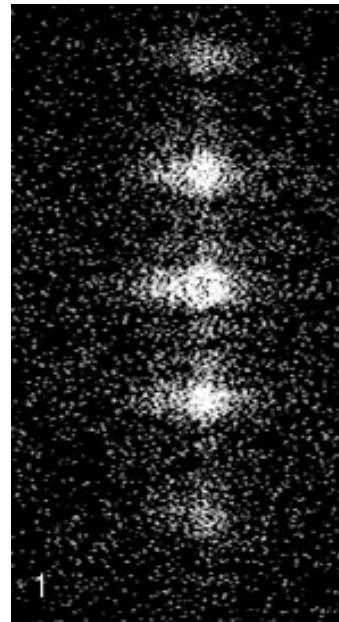
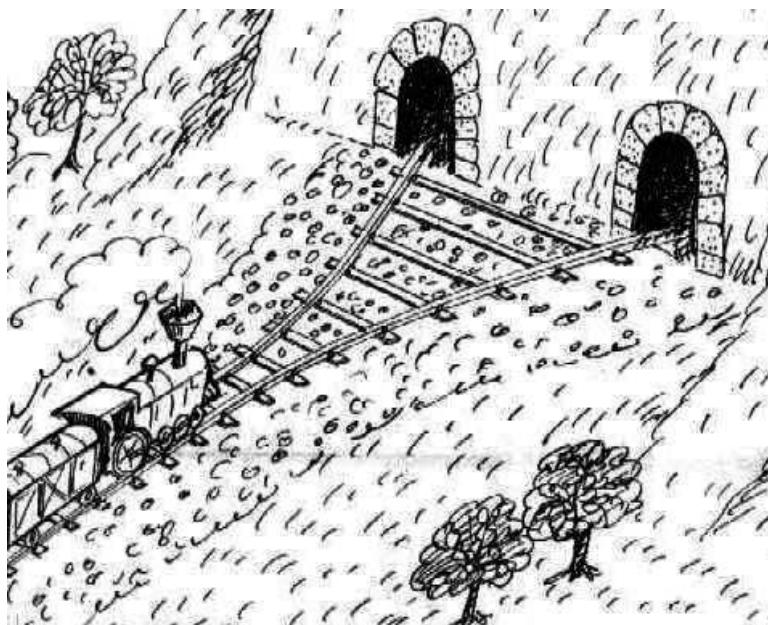
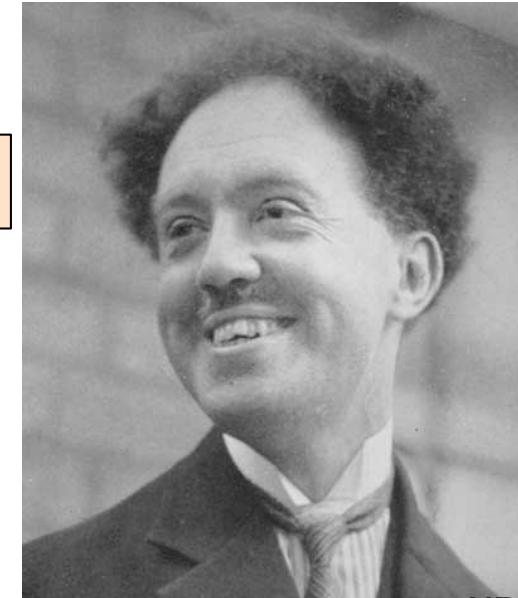


# Landmark results in Quantum Physics



De Broglie's relation

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$



# Wave-Particle Duality

Light can have "particle-like" and "wave-like" properties; Matter (electrons & atoms) have "particle-like" and "wave-like" properties,  
but we do not know what they really are!!

Best suited to be termed just "Energy"

Electrons or photons have both  
wave and particle nature: "WAVICLE"  
Their behaviors are identical!!

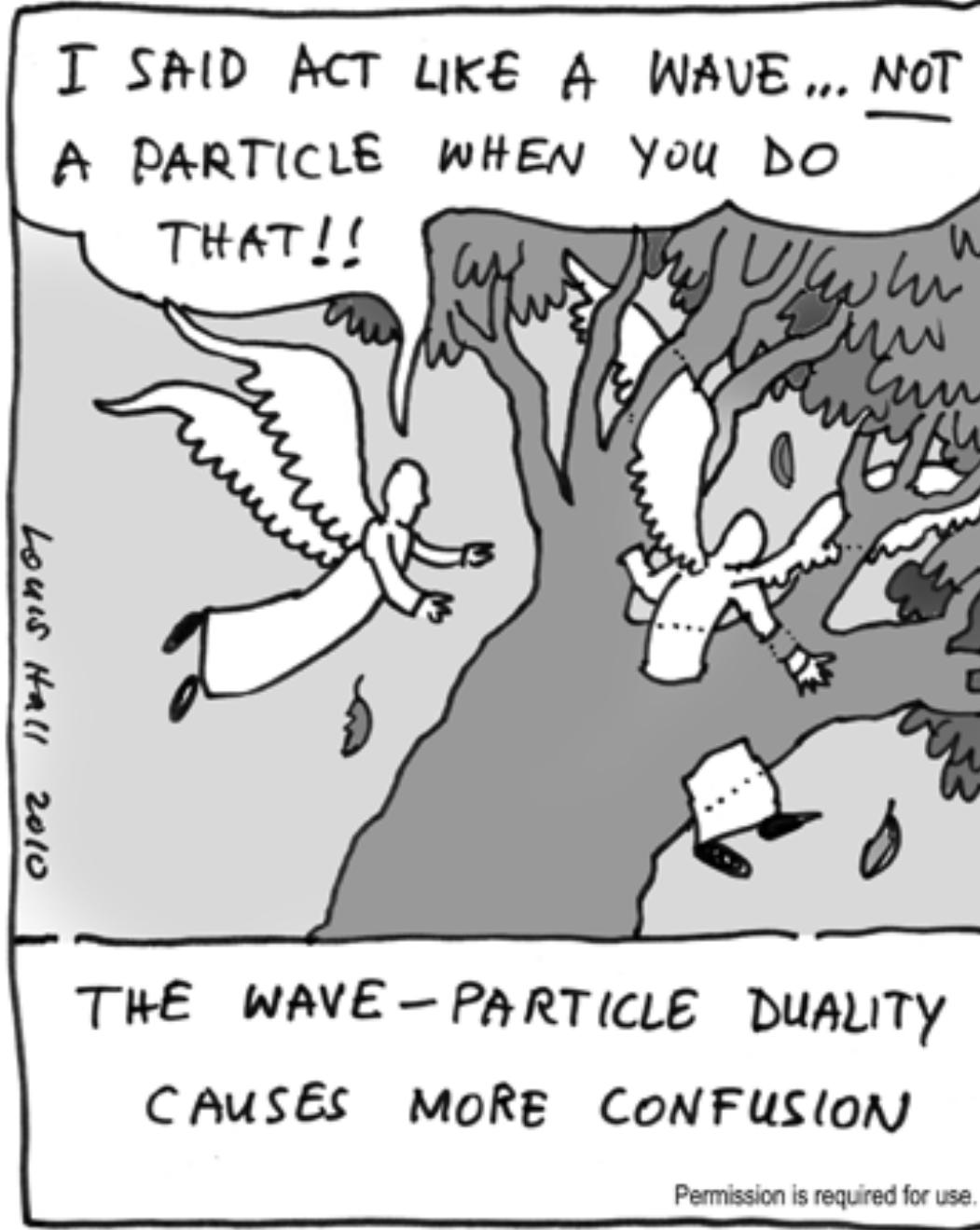
# Wave-Particle Duality

Light can have “particle-like” and “wave-like” properties; Matter (electrons & atoms) have “particle-like” and “wave-like” properties,  
but we do not know what they really are!!



When we refer to a (QM) particle,  
What we really mean is a “wave-particle”<sup>15</sup>

# Richard Feynman's Movie

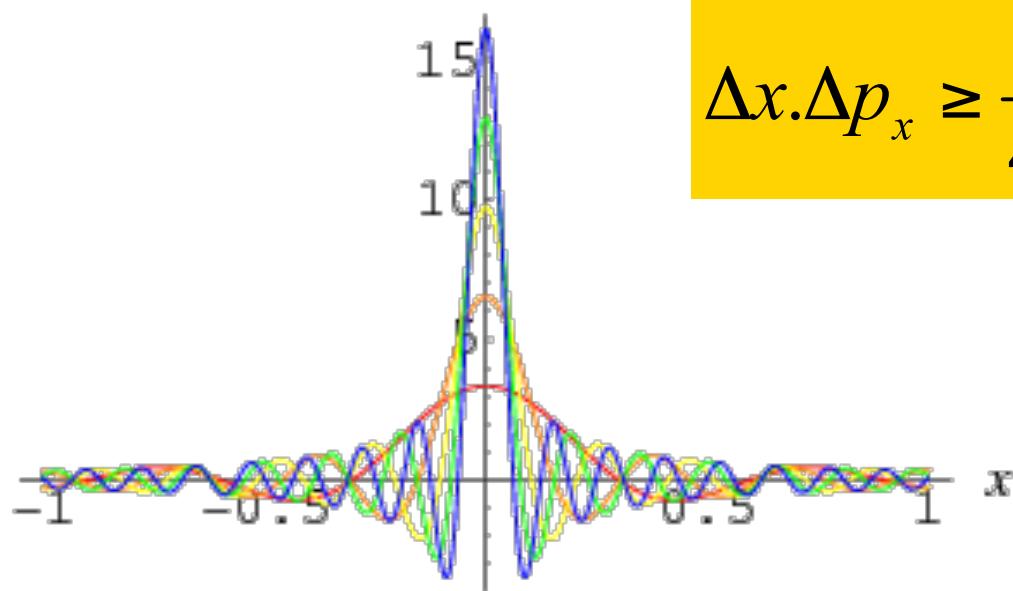
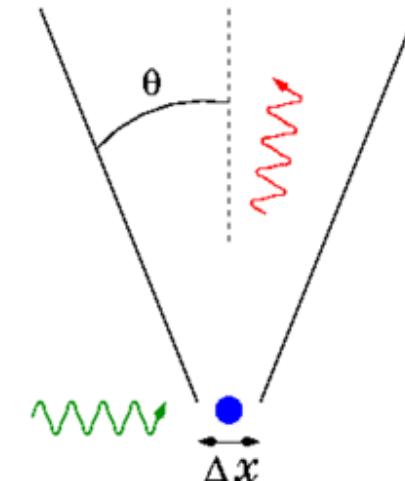


# Landmark results in Quantum Physics

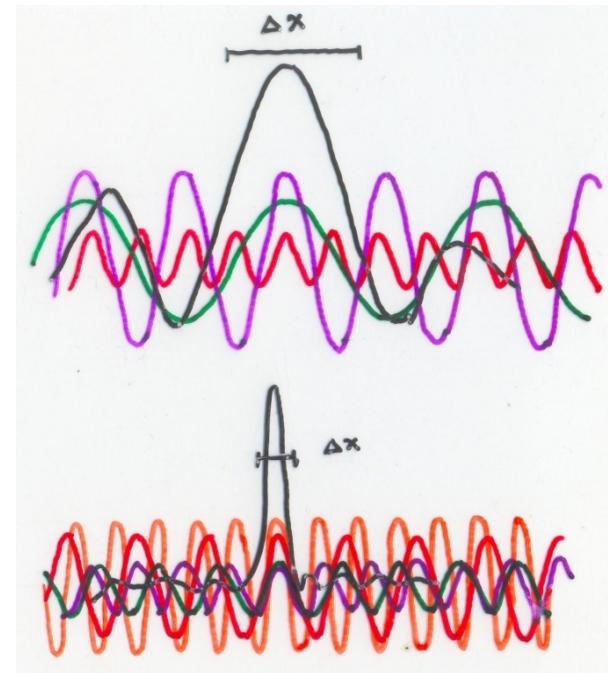


"The violent reaction on the recent development of modern physics can only be understood when one realizes that here the foundations of physics have started moving"

- Werner Heisenberg



$$\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}$$



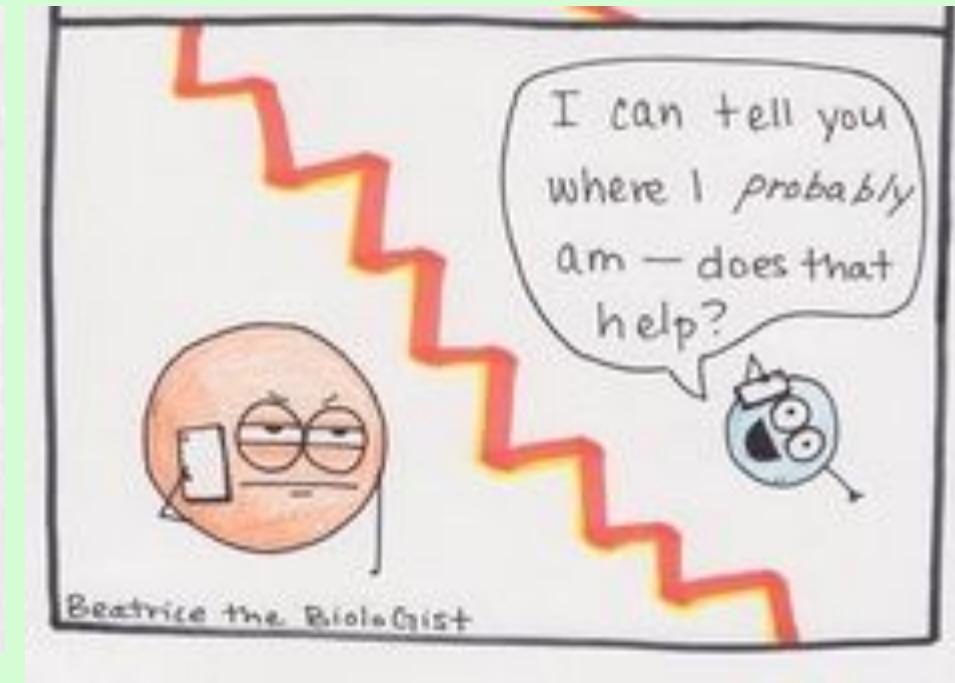
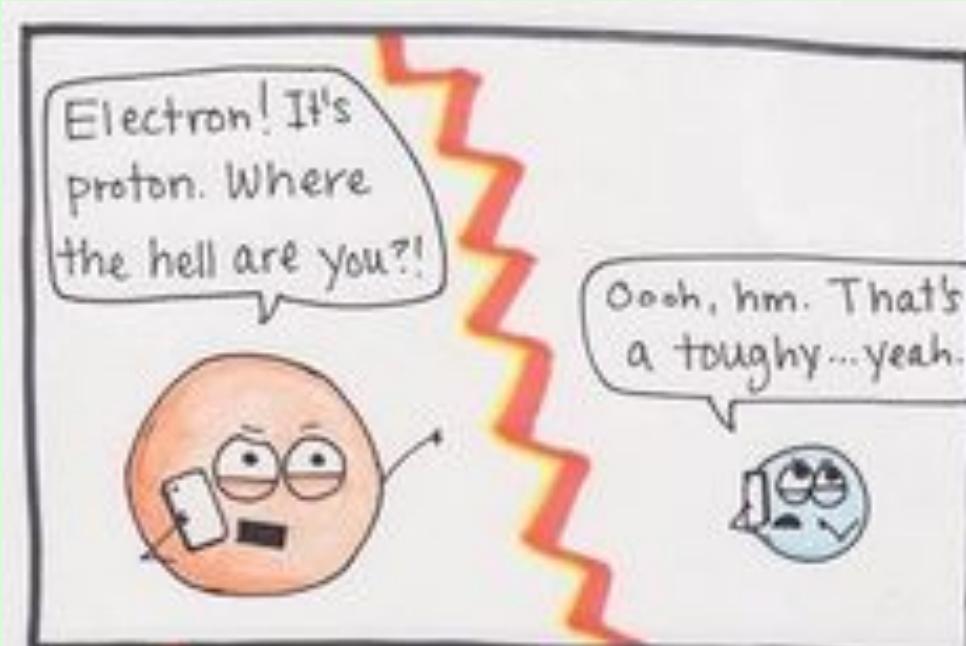
# Richard Feynman's Movie

The nature of the submicroscopic atoms is not directly observable and is beyond imagination!

We deduce it on the basis of highly indirect evidence, and then proceed to describe it...  
...in terms of quantities which we know from our experience of the macroscopic world  
(using analogy and contrast)

However, analogies may not always be good/ provide the correct picture (if there is one!)

Classical quantities such as position and momentum have decreased suitability (fuzzy!) for describing the subatomic world.



Inherently, "nature" at the atomic scale is not deterministic, and their exact behaviors can not be predicted - *Probabilistic!*

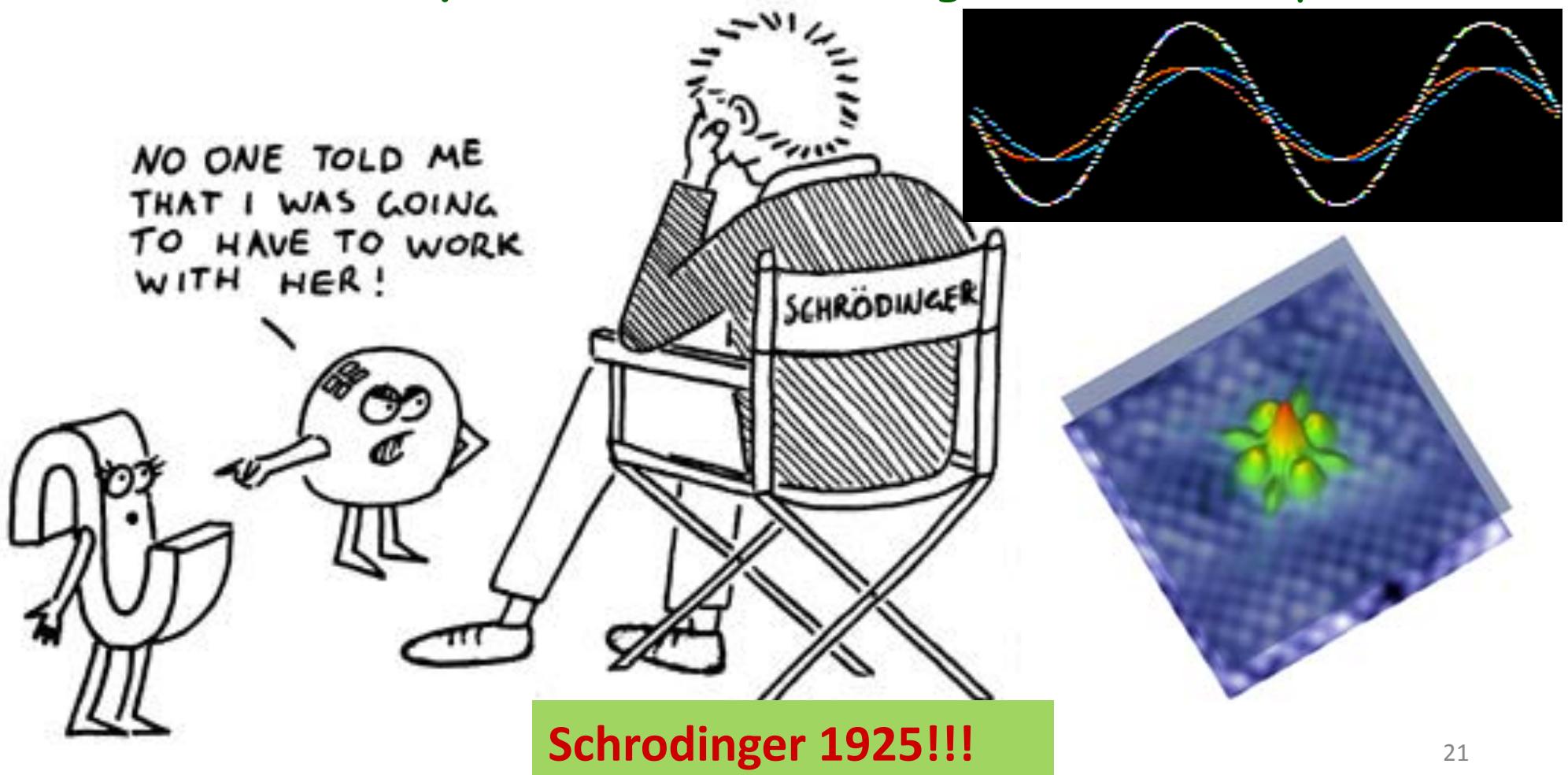
# Richard Feynman on Quantum Behavior

They behave in their own inimitable yet  
fascinating manner, which we call  
“Quantum Mechanical way”

Best to be described mathematically!

# Need a new theory for dynamics of electrons, atoms or molecules

- *Probabilistic, not deterministic (non-newtonian)*
- Wavelike equation for describing sub/atomic systems



# Schrödinger Equation (SE)

Predicts the probability of finding the particle at a specified volume element at future time..  
counterpart to Newton's Equations of motion.  
applicable for atomic/molecular  
and subatomic systems..



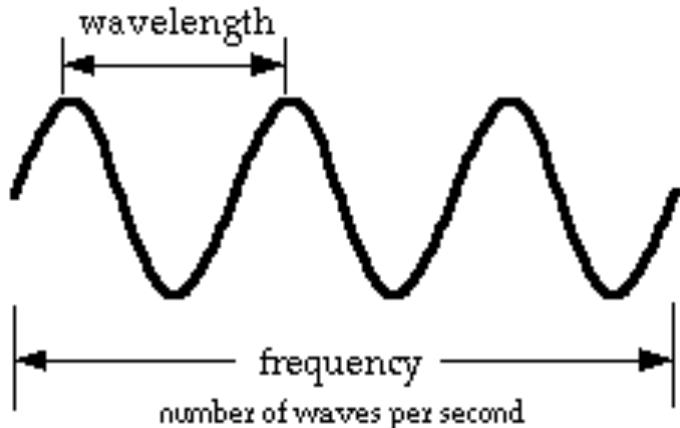
$$i\hbar \frac{\partial}{\partial t} \Psi(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) + V(x,t) \Psi(x,t)$$

"Where did we get that (equation) from? Nowhere. It is not possible to derive it from anything you know. It came out of the mind of Schrödinger."

Richard Feynman

Can we intuitively understand the origin of Schrödinger equation?

# Origin of Schrödinger equation



General form of a travelling wave

$$\psi(x,t) = A e^{i(Kx - \omega t)}$$

$$K = \frac{2\pi}{\lambda} = \text{Wave vector}$$

$$\omega = 2\pi\nu = \text{Angular frequency}$$

Relation between  $K$  and  $\omega$  distinguishes one traveling wave from the other

For light wave

$$\nu = \frac{c}{\lambda}$$

$$\omega = cK$$

Linear relationship

Highly related

Equation of light wave

$$\frac{\partial^2}{\partial t^2} \psi(x,t) = c^2 \frac{\partial^2}{\partial x^2} \psi(x,t)$$

Order of differentiation w r to  $t$  and  $x$  is also 1:1

# What about matter wave ?

General solution for matter wave

$$\psi(x,t) = A e^{i(Kx - \omega t)}$$

Do we know the relation between  
K and w for matter wave?

Einstein's relation

$$E = \hbar\omega$$

$\omega = 2\pi\nu$  = Angular frequency

Free Electron

$$E = \frac{P^2}{2m} = \frac{\hbar^2 K^2}{2m} = \hbar\omega$$

De Broglie's relation

$$P = \hbar K$$

$$K = \frac{2\pi}{\lambda} = \text{Wave vector}$$

$$\omega = \frac{\hbar K^2}{2m}$$

Quadratic not linear !!

Structure of the matter wave equation would be

$$\frac{\partial}{\partial t} \psi(x,t) \sim \frac{\partial^2}{\partial x^2} \psi(x,t)$$

Order of differentiation w r to t and x should be 1:2

# What about matter wave ?

General solution for matter wave

$$\psi(x,t) = A e^{i(Kx - \omega t)}$$

Do we know the relation between  
K and  $\omega$  for matter wave?

Einstein's relation

$$E = \hbar\omega$$

$\omega = 2\pi\nu$  = Angular frequency

Free Electron

$$E = \frac{P^2}{2m} = \frac{\hbar^2 K^2}{2m} = \hbar\omega$$

De Broglie's relation

$$P = \hbar K$$

$$K = \frac{2\pi}{\lambda} = \text{Wave vector}$$

$$\omega = \frac{\hbar K^2}{2m}$$

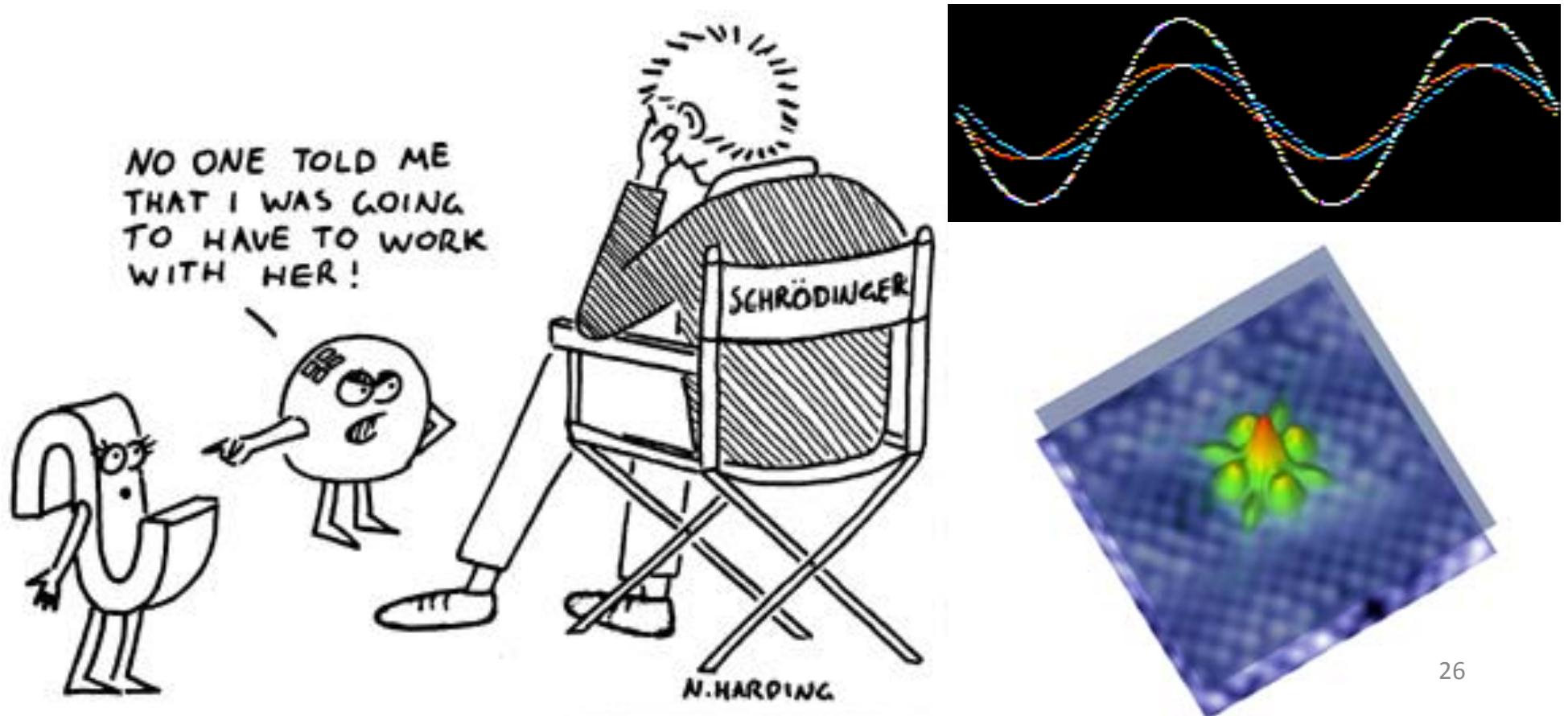
Quadratic not linear !!

Taking the general  $\psi(x,t)$  and differentiating with respect to space and time

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t)$$

# Why believe Schrödinger Equation?

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x, t) \Psi(x, t)$$



# Newton's LAWs of Motion

1<sup>st</sup> Law - inertia

2<sup>nd</sup> Law - momentum

3<sup>rd</sup> Law - energy

Did you ever ask "WHY?"

If yes, did you get an answer  
that has really satisfied you?

Laws → Based on Postulates or Axioms  
Not been proven wrong experimentally

# Postulates of Quantum Mechanics

The state of a system is completely specified by some function  $\psi(r,t)$ . Square of "wavefunction"  $\rightarrow$  probability density

To every observable in classical mechanics, there corresponds a linear operator in quantum mechanics

In measurement of observable associated with operator  $A$ , *only values that will be observed* are the eigenvalues of  $A$

The average value of the observable corresponding to  $A$  is

$\Psi(x,y,z,t)$  of a system evolves:

$$\langle a \rangle = \int \Psi^* \hat{A} \Psi d\nu$$

$$i\hbar \frac{\partial}{\partial t} \Psi(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) + V(x,t) \Psi(x,t)$$

# What can you get from solving the Schrodinger Equation?



$\Psi$  is a mathematical function; real or complex

What is the meaning of  $\psi(x,t)$ ?

# Born Interpretation of Wavefunction

The state of a QM system is *completely specified* by a wavef(n)  $\Psi(x,y,z,t)$  & all possible information can be derived from  $\Psi$

**$\Psi$  is a mathematical function: real or complex**

2.  $\Psi^* \cdot \Psi dV$  is the probability that the particle lies in volume element  $dV$ , so  $\Psi^* \Psi$  is the probability density at that point

$$\begin{aligned}\Psi \text{ can be complex: } \Psi &= f + ig \text{ and } \Psi^* = f - ig \\ \therefore \Psi^* \Psi &= (f - ig)(f + ig) = f^2 + g^2 = |\Psi|^2\end{aligned}$$



**Normalization of Wave function:** Probability of finding a particle somewhere in the entire space has to be unity.

$$\int_{\text{all space}} \int \int \Psi^*(x, y, z) \cdot \Psi(x, y, z) dx dy dz = \int_{\text{all space}} \Psi^* \Psi dV = \langle \Psi | \Psi \rangle = 1$$

# Physically acceptable wavefunctions

Restrictions on the wavefunction: Well-behaved wavefunctions

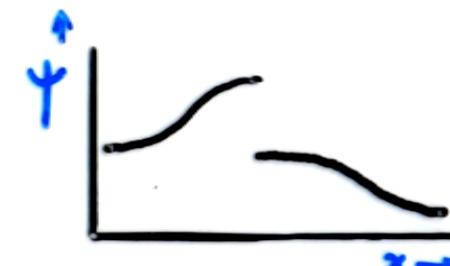
1.  $\Psi$  must be normalizable
  - $\Psi$  is finite and  $\Psi \rightarrow 0$  at boundaries/  $\pm \infty$
2.  $\Psi$  must be **single-valued** because  $\psi^*\psi$  can not have two values!



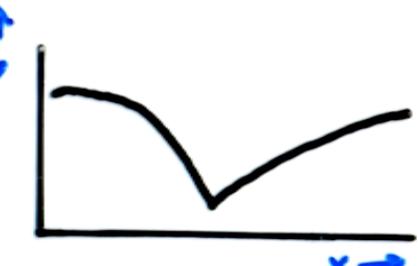
$\Psi$  must be finite  
everywhere



$\Psi$  must be  
single-valued



$\Psi$  must be  
continuous



$d\Psi/dx$  must be  
continuous

Single-point discontinuity sometimes acceptable "removable"<sup>31</sup>

# Mathematical Tools-1: Operators

## I. Operators: Does “something” to whatever follows it

- Takes an Action: Simple operations - addition, multiplication
- Can be real or complex; Can be represented as matrices.

$\hat{A}f(x) = g(x)$ :  $\div 4$ , Cube,  $\sqrt{\phantom{x}}$ ,  $(\phantom{x})^{-1}$ , Sin, log,  $\frac{d}{dx}$ ,  $\frac{\partial}{\partial y}$ ,  $\int_0^1$   $\Rightarrow$  new function / number

Symmetry operators - rotation, reflection, inversion!

Operation  $\rightarrow$  Changes/Perturbs function/system in some way!  
 $\rightarrow$  act of measurement or perform an experiment'

Linear Operators:  $\hat{A}[c_1 f_1(x) + c_2 f_2(x)] = c_1 \hat{A}f_1(x) + c_2 \hat{A}f_2(x)$ ; ex.  $\frac{d}{dx}$   
but  $(\phantom{x})^{-1}$ , SQR or  $\sqrt[3]{\phantom{x}}$  or Sin, Cos, exp, log: Nonlinear Operators

If  $\hat{A}[g(x)] = h(x)$ ;  $\hat{B}[f(x)] = g(x)$ ;  $\hat{A}\hat{B}[f(x)] = \hat{A}[\hat{B}[f(x)]] = \hat{A}[g(x)] = h(x)$

# Operator Formalism

To every observable in classical mechanics, there corresponds a *linear Hermitian operator* (real or complex) in quantum mechanics

<u>Classical quantity</u>	<u>Quantum Mechanical operator</u>
Position, $x$	$\hat{x}$
Momentum, $p_x$	$-i\hbar \frac{d}{dx}$
Kinetic energy in 1-D, $K_x = \frac{p_x^2}{2m}$	$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$
Kinetic energy in 3-D,	$-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$ (LAPLACIAN)
Potential Energy, $V$	Multiply by $V(x,y,z)$
Total Energy, $E = K + V$	$-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x,y,z)$ (HAMILTONIAN OPERATOR)

# SE: 2<sup>nd</sup> order *partial* differential equation

For  $\Psi(x,t)$ , Schrodinger Equation

$$\widehat{i\hbar} \frac{\partial}{\partial t} \Psi(x,t) = \boxed{-\frac{\hbar^2}{2m} \widehat{\nabla}_x^2} \Psi(x,t) + \boxed{\widehat{V}(x,t)} \Psi(x,t)$$

$$\widehat{i\hbar} \frac{\partial}{\partial t} \Psi(x,t) = \left[ -\frac{\hbar^2}{2m} \widehat{\nabla}_x^2 + \widehat{V}(x,t) \right] \Psi(x,t)$$

Hamiltonian (Total Energy) Operator

$$\Rightarrow \widehat{H} = -\frac{\hbar^2}{2m} \widehat{\nabla}_x^2 + \widehat{V}(x,t) \quad \text{where, } \widehat{\nabla}_x^2 = \frac{\partial^2}{\partial x^2} \text{ in 1D}$$

# Eigenfunction/Eigenvalue equations

Operator operating on a function re-generates the same function (which is called eigenfunction) multiplied by a number (eigenvalue)

Given  $\hat{A}$ , find  $\phi(x)$ , so that  $\hat{A}\phi(x) = a\phi(x); a \rightarrow (\#)$ : Eigenvalue Equation

$$\hat{A} = \hat{D}^n = \frac{d^n}{dx^n};$$

Can  $\phi(x)$  be  $e^{\alpha x}$  or  $\sin(\alpha x)$ ?

If so, Eigenvalues,  $a = ?$

If  $\Psi = \sin(\alpha x)$ ;  $\frac{d}{dx} = \hat{D}$

$$\hat{D}\Psi = \alpha \cos(\alpha x)$$

$$\hat{D}^2\Psi = -\alpha^2 \sin(\alpha x) = -\alpha^2 \Psi$$

$$\hat{D}^n \phi(x) = \frac{d^n}{dx^n} e^{\alpha x} = \alpha^n \phi(x) \Rightarrow \text{Yes! EV: } \alpha^n$$

$\hat{D}^n \sin(\alpha x) = (\#) \otimes \sin(\alpha x)$  for  $n = \text{even}$ , YES! Eigenfunction

$\hat{D}^n \sin(\alpha x) \neq (\#) \otimes \sin(\alpha x)$  for  $n = \text{odd}$ , NOT Eigenfunction

# Real (observable) eigenvalues

In a measurement of a classical variable (associated with  $\hat{A}$  satisfying the  
 $\hat{A}\Psi = a\Psi$ ) only real (not imaginary) eigenvalues ( $a_n$ ) will be ever observed

$$\hat{A}\Psi = a\Psi$$

$\Psi_n \rightarrow$  eigenfunctions or eigenstates or states

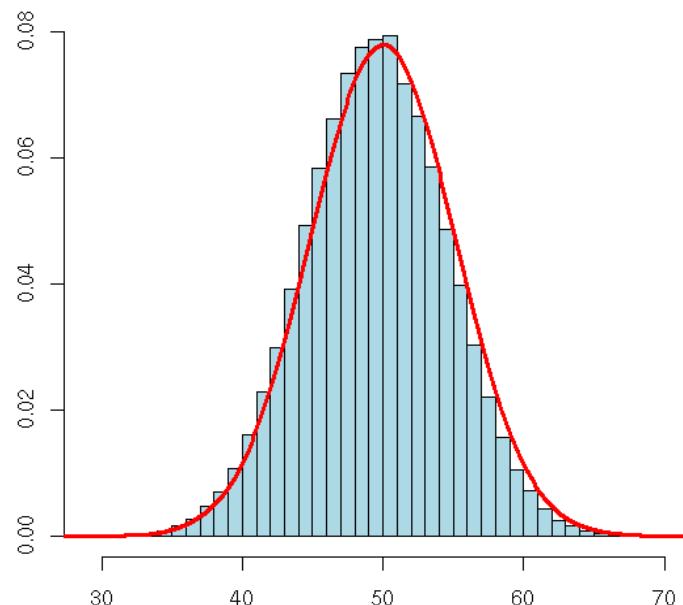
$a_n \rightarrow$  Eigenvalues (outcome of measurement)

If system is in state  $\Psi_k$ , a measurement  
on the system will yield a real eigenvalue  $a_k$

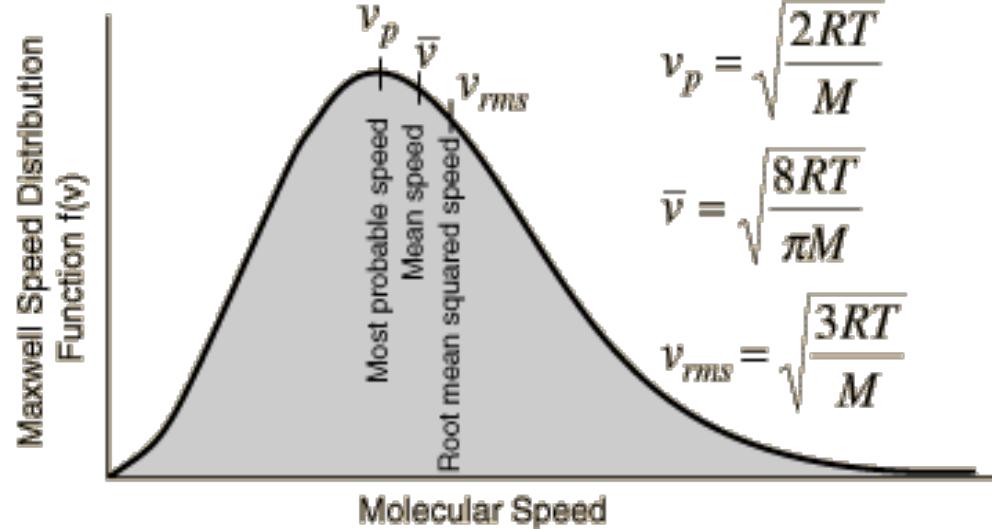
There are many Eigen-f(n)s (or eigenstates) for the same QM operator

Only real eigenvalues will ever be observed, which will specify a number  
corresponding to the classical variable for the particular eigenfunction  
**A HAS TO BE HERMITIAN OPERATOR IN QM**

# Probability Distributions: Expectation (or Average or Mean value) and Most-Probable Value



$$f(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$



If  $P(x)$  is the probability distribution

Most Probable (Modal) value at *Maxima*  $\rightarrow \frac{dP(x)}{dx} = 0$

*Mean:*  $E(x) = \langle x \rangle = \sum_{j=1}^n x_j P_j(x_j)$  and  $E(x^2) = \langle x^2 \rangle = \sum_{j=1}^n x_j^2 P_j(x_j)$

# Average or Expectation value

So, prescription needed for average value of a classical observable

$$P(x) \sim |\Psi|^2 dx = \Psi^* \cdot \Psi dx \text{ (in 1D)}$$

$\langle a \rangle$  corresponds to the mean value of a classical physical quantity (observable), and  $A$  represents the corresponding QM operator

$$\text{In 1D} \quad \langle a \rangle = \int_{-\infty}^{+\infty} \hat{A} \cdot P(x) dx = \int_{-\infty}^{+\infty} \hat{A} \cdot |\Psi|^2 dx \sim \int_{\text{all space}} \Psi^* \hat{A} \Psi dx$$

$$\langle a \rangle = \int_{\text{all space}} \Psi^* \hat{A} \Psi dv = \langle \Psi | \hat{A} | \Psi \rangle$$

$$\langle a \rangle = \frac{\int_{\text{all space}} \Psi^* \hat{A} \Psi dv}{\int_{\text{all space}} \Psi^* \Psi dv} = \frac{\langle \Psi | \hat{A} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

# Time-Dependent Schrödinger Equation

*Hamiltonian (Energy) Operator*

$$\Rightarrow \hat{H} = -\frac{\hbar^2}{2m} \hat{\nabla}_x^2 + \hat{V}(x, t) \quad \text{where, } \hat{\nabla}_x^2 = \frac{\partial^2}{\partial x^2} \text{ in 1D}$$

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \nabla_x^2 \Psi(x, t) + V(x, t) \Psi(x, t)$$

Very often,  $V(x, t) = V(x) \rightarrow$  solutions to TDSE has the form

$$\Psi(x, t) = \psi(x).f(t)$$

Special Solution where space part and time part can be separated out!

Separation of variables to solve linear 2<sup>nd</sup> order differential equations

## 2<sup>nd</sup> order Partial Differential Eqns.(multiple variables)

2<sup>nd</sup> order Normal Differential Equation (DE)  
With one variable (Simple Harmonic Motion)

$$\frac{d^2 f(x)}{dx^2} + kf(x) = C$$

Partial Differential Equation (PDE) contains two or more variables

$$\frac{\partial^2 F(x, y)}{\partial x^2} + \frac{\partial^2 F(x, y)}{\partial y^2} + \frac{\partial F(x, y)}{\partial x} + \frac{\partial F(x, y)}{\partial y} + kF(x, y) = 0$$

To Solve any PDE  $\rightarrow$  Separation of Variables (or else, can not solve)

$$\frac{d^2 f(x)}{dx^2} + \frac{df(x)}{dx} + mf(x) = \frac{d^2 f(y)}{dy^2} + \frac{df(y)}{dy} + nf(y)$$

$$LHS(x) = RHS(y) \Rightarrow F_1(x) = F_2(y) = \text{const.}$$

$$\frac{d^2 f(x)}{dx^2} + \frac{df(x)}{dx} + mf(x) = \text{const. (say } k\text{)}$$

$$-\frac{d^2 f(y)}{dy^2} - \frac{df(y)}{dy} - nf(y) = k \text{ (same const.)}$$

Two normal DEs  
with single variable

TIME- DEPENDENT PART of solution

$$-\frac{\hbar}{i} \frac{1}{f(t)} \frac{df(t)}{dt} = E \quad \Rightarrow \quad \frac{df(t)}{f(t)} = -\left(\frac{iE}{\hbar}\right) dt$$

$$f(t) \sim e^{-iEt/\hbar}$$

Rearranging time-INDEPENDENT SPACE part

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x) = E$$

Time-Independent Schrodinger Equation:

$$\hat{H}\psi(x) = E\psi(x)$$

$$\text{where } \hat{H} \equiv -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) = \hat{T} + \hat{V}$$

$$\begin{aligned} |\Psi|^2 &= |f\psi|^2 = (f\psi)^*(f\psi) \\ &= (e^{iEt/\hbar}\psi^*)(e^{-iEt/\hbar}\psi) \\ &= e^0 \cdot \psi^* \psi = |\psi|^2 \\ &\neq F(t) \end{aligned}$$

Does not mean that the system is at rest (recap Bohr's stationary state), but energies and the probability do not change with time.

Stationary states (i.e. a solution):  $\Psi^2$  and Energy  $\rightarrow$  Const. in time

MANY solutions to TISE possible  $\rightarrow$  different  $\Psi$  with different energies