Q.95

There are only two ways in which the given conditions can be reached.

(1) 1 e in ϵ, 1 e in 2ϵ and 1 e in 2ϵ

(2) All three electrons in ϵ.

In first case total number of ways to achieve the configuration.

$$\frac{\lfloor 2}{\lfloor 1 \lfloor 1} \times \frac{\lfloor 10}{\lfloor 1 \lfloor 9} \times \frac{\lfloor 20}{\lfloor 1 \lfloor 19} = 2 \times 10 \times 20 = 400$$

In the second case no. of ways are

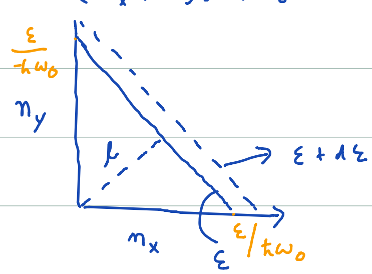
$$\frac{\lfloor 10}{\lfloor 3 \lfloor 7} = \frac{10 \times 9 \times 8}{6} = 120$$

The probabilities of the two distributions are

$$P(1) = \frac{400}{400+120} = \frac{10}{13}$$

$$P(2) = \frac{120}{400+120} = \frac{3}{13}$$

Q.97 $\varepsilon = (n_x + n_y)\hbar\omega_0$



The constant energy curves are str. lines.

We draw two lines corresponding to energies ε & $\varepsilon + d\varepsilon$. Find the area enclosed by these lines.

The length of line is $\sqrt{2}\,\dfrac{\varepsilon}{\hbar\omega_0}$

The perpendicular gap between two lines is dl, where l is $\dfrac{\varepsilon}{\hbar\omega_0}\dfrac{1}{\sqrt{2}}$

$\therefore$ Area enclosed between lines is

$$\frac{\sqrt{2}\,\varepsilon}{\hbar\omega_0}\;\frac{1}{\sqrt{2}}\;\frac{d\varepsilon}{\hbar\omega_0}$$

As there is one point per unit area, we get

$$\mathcal{D}(\varepsilon)\,d\varepsilon = \frac{\varepsilon}{(\hbar\omega_0)^2}\,d\varepsilon$$

Q. 101

Assume a cubic BBR Cavity

$\varepsilon = \hbar c k$, The Boundary conditions are similar

$\Rightarrow k_x L = n_x\pi,\ k_y L = n_y\pi,\ k_z L = n_z\pi$

$$\therefore \varepsilon = \hbar c\sqrt{k_x^2+k_y^2+k_z^2}$$

$$= \frac{\hbar c\pi}{L}\sqrt{n_x^2+n_y^2+n_z^2}$$

$$= \frac{\hbar c\pi}{L}\times n$$

This expression is different from particle in a box

However, density of state in 'n' space is same.

$$g(n)\,dn = 2\times\frac{1}{8}\times 4\pi n^2\,dn$$

factor '2' takes care of two possible polarizations

$$\therefore g(n)\,dn = \pi n^2\,dn$$

$\varepsilon = \dfrac{\hbar c\pi}{L}\times n \Rightarrow d\varepsilon = \dfrac{\hbar c\pi}{L}\,dn$. Changing variable

$$g(\varepsilon)\,d\varepsilon = \pi\left(\frac{L}{\hbar c\pi}\right)^3\varepsilon^2\,d\varepsilon$$

Note α has been taken as zero, as no. of particles are not fixed

To put in standard form use

$\hbar = \frac{h}{2\pi}$, $\varepsilon = \frac{hc}{\lambda}$, $d\varepsilon = \frac{hc}{\lambda^2}|-d\lambda|$

$$N(\lambda)d\lambda = \pi V\left(\frac{2\pi}{hc\lambda}\right)^3 \frac{\left(\frac{hc}{\lambda}\right)^2\left(\frac{hc}{\lambda^2}\right)d\lambda}{e^{hc/\lambda kT}-1}$$

$$= \frac{8\pi V}{\lambda^4}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

For energy density, multiply by $\frac{hc}{\lambda}$

Further multiply by $c/4$ to convert to $e(\lambda)d\lambda$. [See this derivation in appendix of Richtmyer & Kennard]

Q.102

Resistivity $\rho = \frac{5.8\times10^{-3}}{1}\times 3.3\times10^{-6} = 1.914\times10^{-8}$ ohm m

$\therefore$ Conductivity $\sigma = 5.22\times10^{7}\ (\text{ohm}-\text{m})^{-1}$

Current density $J = \frac{25}{3.3\times10^{-6}} = 7.58\times10^{6}\ A/m^2$

Now $J = n e v_D$. We have to know 'n' to find drift velocity.

63.5 kg of Cu has 6.02×10^{26} atoms

$\therefore$ 8940 kg of Cu shall have $\frac{6.02\times10^{26}\times 8940}{63.5} = 8.475\times10^{28}$

As 8940 kg of Cu shall occupy $1m^3$, volume, assuming each atom contributes one free electron, we have

$n = 8.475\times10^{28}\ m^{-3}$

Thus $v_D = \frac{7.58\times10^{6}}{8.475\times10^{28}\times1.6\times10^{-19}} = 5.6\times10^{-4}$ m/s

Note: Drift velocity is very small in comparison to even classical rms speed.

To calculate mean free path in Drude model, one has to calculate rms speed.

$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$

Take $kT \approx .025$ eV at R.T.

$\therefore v_{rms} = \sqrt{\frac{3 \times .025 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \approx 1.15 \times 10^5$ m/s

We should know relaxation time τ

$$\tau = \frac{\sigma m}{n e^2} = \frac{5.22 \times 10^7 \times 9.1 \times 10^{-31}}{8.475 \times 10^{28} \times (1.6 \times 10^{-19})^2} \approx 2.19 \times 10^{-14} \text{ s}$$

Mean free path in Drude Model

$$l = v_{rms} \times \tau = 1.15 \times 10^5 \times 2.19 \times 10^{-14} \approx 2.52 \text{ nm}$$

Sommerfeld model $l = v_F \times \tau$

E_F can be calculated for Cu $= 7.06$ eV $\Rightarrow v_F = 1.58 \times 10^6$ m/s

$\Rightarrow l = 34.7$ nm