Computer Programming

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Session: Merge Sort in C++ and Its Analysis

Quick Recap of Relevant Topics

- The sorting problem
- Selection sort
- Merge sort
  - Intuition
  - Divide-and-conquer strategy
  - Key role of merging sorted sub-arrays

Overview of This Lecture

- Merge sort
  - C++ implementation
  - Analysis of performance
    - Counting “basic” steps to sort an array of size n

Merge Sort: Basic Idea

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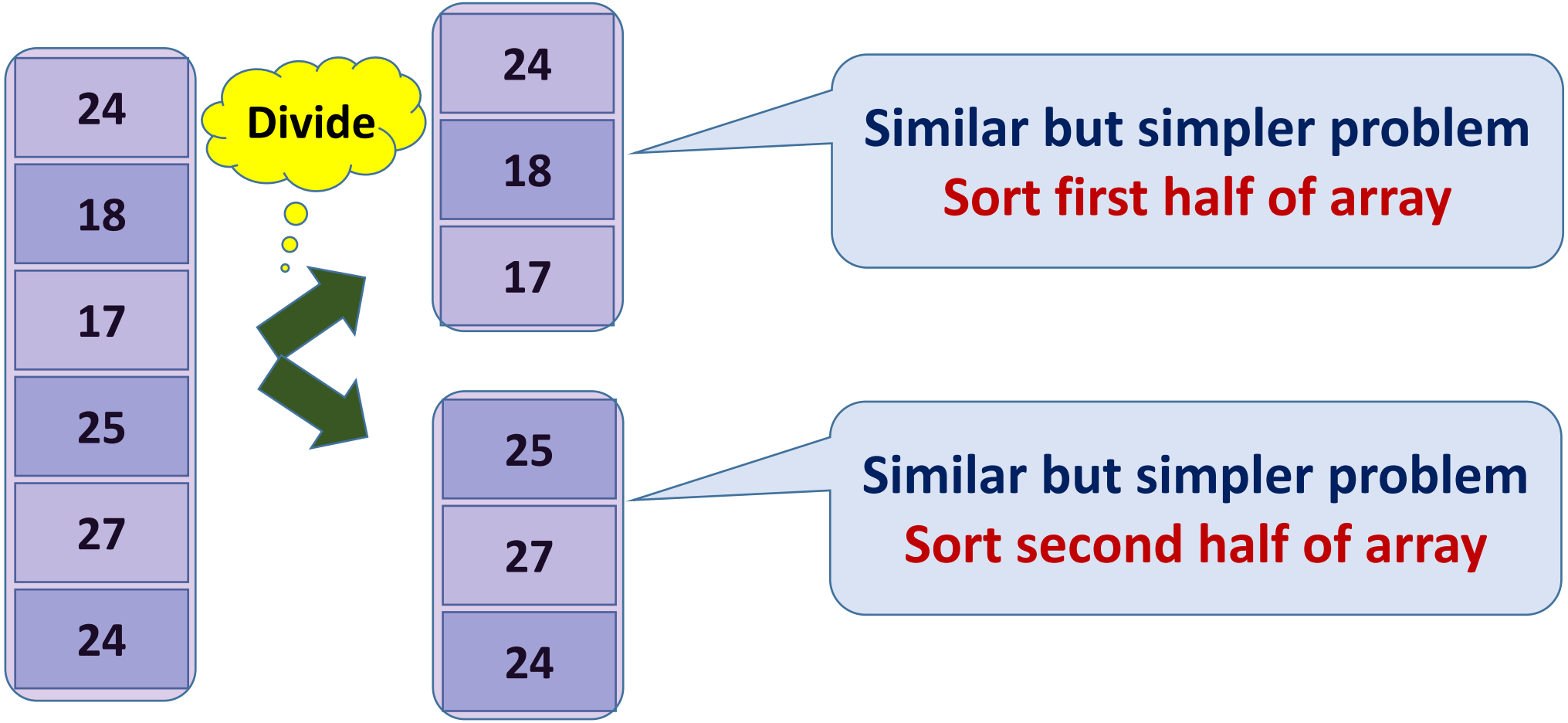

Sorting by Divide-and-Conquer

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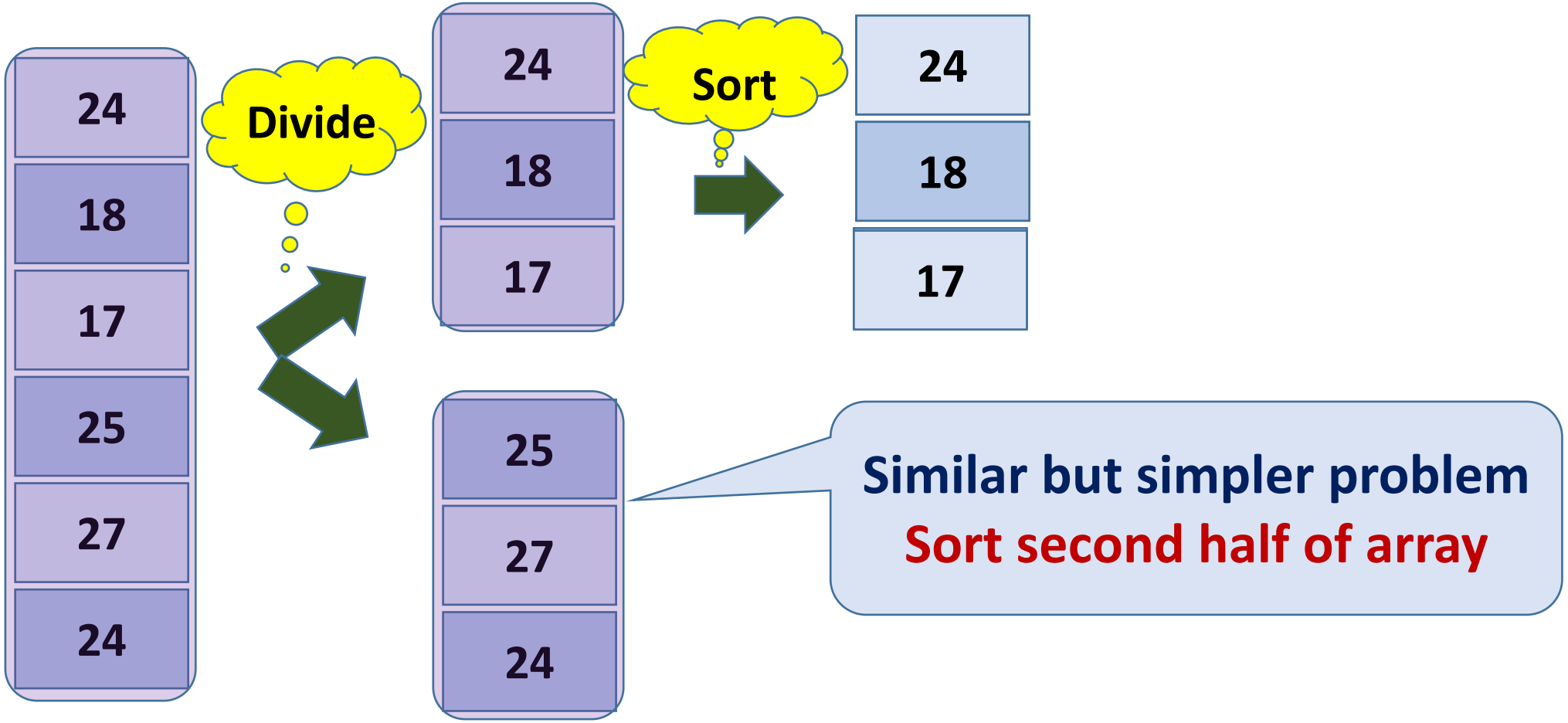

Sorting by Divide-and-Conquer

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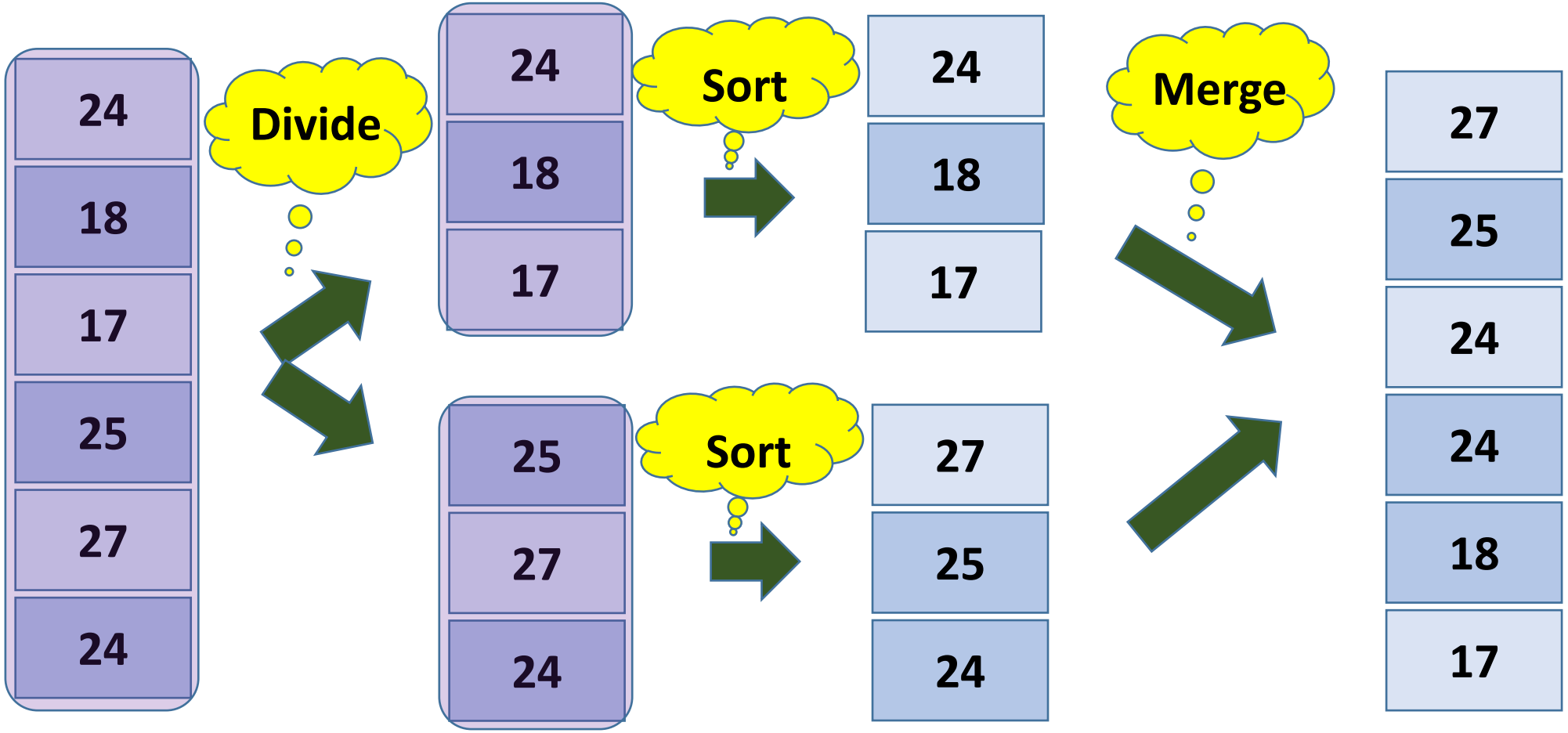

What Were The Steps?

- Divide an array of size n into two sub-arrays of size ≈ n/2
  - Sub-array sizes may differ by 1 if n is odd
  - Easy!
- Sort each sub-array of size n/2
  - Use same technique as for sorting original array (recurse !!!)
  - Termination case of recursion: arrays of size 1
- Merge sorted sub-arrays, each of size n/2
  - One pass over each sorted sub-array

Merge Sort In C++

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```
int main() {
   int i, n, A[100];    // Variable declarations
   cout << "Give count of integers: ";  cin >> n;// Read and validate inputs
   if ((n <= 0) || (n > 100)) { cout << "Invalid input!" << endl; return -1; }
   cout << "Give " << n << " integers to sort." << endl;
   for (i = 0; i < n; i++) { cin >> A[i]; }  // Read elements of array A
   mergeSort(A, 0, n);  // Sort elements A[0] … A[n-1]
    … Rest of code …
   return 0;
}
```

Merge Sort In C++

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```
int main() {
   int i, n, A[100];    // Variable declarations
   cout << "Give count of integers: ";  cin >> n;// Read and validate inputs
   if ((n <= 0) || (n > 100)) { cout << "Invalid input!" << endl; return -1; }
   cout << "Give " << n << " integers to sort." << endl;
   for (i = 0; i < n; i++) { cin >> A[i]; }  // Read elements of array A
   mergeSort(A, 0, n);  // Sort elements A[0] … A[n-1]
    … Rest of code …
   return 0;
}
```

Merge Sort In C++

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```
int main() {
   int i, n, A[100];    // Variable declarations
   cout << “Give count of integers: “;  cin >> n;// Read and validate inputs
   if ((n <= 0) || (n > 100)) { cout << “Invalid input!” << endl; return -1; }
   cout << “Give “ << n << “ integers to sort.” << endl;
   for (i = 0; i < n; i++) { cin >> A[i]; }  // Read elements of array A
   mergeSort(A, 0, n);  // Sort elements A[0] … A[n-1]
    … Rest of code …
   return 0;
}
```

Merge Sort In C++

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```
// PRECONDITION: start < end,  both within array bounds of A
void mergeSort(int A[], int start, int end) {
  if (end == start +1) { return; }  //Subarray of interest has 1 element
  int mid = (start + end)/2;  //Get mid-index of subarray of interest
  mergeSort(A, start, mid);  // Sort subarray A[start] … A[mid-1]
  mergeSort(A, mid, end);   // Sort subarray A[mid] … A[end-1]
  // Merge sorted subarrays A[start] … A[mid-1]  and A[mid] … A[end-1]
  mergeSortedSubarrays(A, start, mid, end);
  return;
}
// POSTCONDITION: A[start] … A[end-1] sorted in decreasing order
```

Merge Sort In C++

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```
// PRECONDITION: start < end,  both within array bounds of A
void mergeSort(int A[], int start, int end) {
  if (end == start +1) { return; }  //Subarray of interest has 1 element
  int mid = (start + end)/2;  //Get mid-index of subarray of interest
  mergeSort(A, start, mid);  // Sort subarray A[start] … A[mid-1]
  mergeSort(A, mid, end);  // Sort subarray A[mid] … A[end-1]
  // Merge sorted subarrays A[start] … A[mid-1] and A[mid] … A[end-1]
  mergeSortedSubarrays(A, start, mid, end);
  return;
}
// POSTCONDITION: A[start] … A[end-1] sorted in decreasing order
```

Termination case of recursion

Merge Sort In C++

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```
// PRECONDITION: start < end,  both within array bounds of A
void mergeSort(int A[], int start, int end) {
  if (end == start +1) { return; }  //Subarray of interest has 1 element
  int mid = (start + end)/2;  //Get mid-index of subarray of interest
  mergeSort(A, start, mid);  // Sort subarray A[start] … A[mid-1]
  mergeSort(A, mid, end);   // Sort subarray A[mid] … A[end-1]
  // Merge sorted subarrays A[start] … A[mid-1]  and A[mid] … A[end-1]
  mergeSortedSubarrays(A, start, mid, end);
  return;
}
// POSTCONDITION: A[start] … A[end-1] sorted in decreasing order
```

Merge Sort In C++

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```
// PRECONDITION: start < end,  both within array bounds of A
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  if (end == start +1) { return; }  //Subarray of interest has 1 element
  int mid = (start + end)/2;  //Get mid-index of subarray of interest
  mergeSort(A, start, mid);  // Sort subarray A[start] … A[mid-1]
  mergeSort(A, mid, end);   // Sort subarray A[mid] … A[end-1]
  // Merge sorted subarrays A[start] … A[mid-1]  and A[mid] … A[end-1]
  mergeSortedSubarrays(A, start, mid, end);
  return;
}
// POSTCONDITION: A[start] … A[end-1] sorted in decreasing order
```

Merge Sort In C++


```
// PRECONDITION: A[start] … A[mid-1]  and A[mid] … A[end-1] sorted in
//                          decreasing order
void mergeSortedSubarrays(int A[], int start, int mid, int end) {
   int i,   j;  int tempA[100],  index = start;
   for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
      // Determine whether A[i] or A[j] should appear next in sorted order
      // Update tempA[index] accordingly
     index ++;
  } // end of merging loop
  // Copy tempA[start] … tempA[end-1]  to  A[start] … A[end-1]
  return;
}
// POSTCONDITION:  A[start] … A[end-1] sorted in decreasing order
```

Merge Sort In C++

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```
// PRECONDITION: A[start] … A[mid-1]  and A[mid] … A[end-1] sorted in
//                          decreasing order
void mergeSortedSubarrays(int A[], int start, int mid, int end) {
  int  i,   j; int tempA[100],  index = start;
  for (i = start, j = mid; //i
    // Det      e whether
    // Upda      [inde
    index ++;
  } //end of me
  // C
  ret
}
// POSTCONDITION:  A[start] … A[end-1] sorted in decreasing order
```

Running index for second subarray A[mid] … A[end-1]

Running index for first subarray A[start] … A[mid-1]

Merge Sort In C++

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```
// PRECONDITION: A[start] … A[mid-1]  and A[mid] … A[end-1] sorted in
//                          decreasing order
void mergeSortedSubarrays(int A[], int start, int mid, int end) {
   int i,   j;  int tempA[100],   index = start;
   for (i = start, j = mid; ((i < mid) && (j < end)); ) { // Merging loop
      // Determine the
      // Update t
     index ++;
   } // end of m
}
// POSTCONDITION:  A[start] … A[end-1] sorted in decreasing order
```

Index in tempA where next element of merged-and-sorted subarray is stored

Merged-and-sorted subarray temporarily stored in tempA[start] … tempA[end-1]

Merge Sort In C++


```
// PRECONDITION: A[start] … A[mid-1]  and A[mid] … A[end-1] sorted in
//                          decreasing order
void mergeSortedSubarrays(int A[], int start, int mid, int end) {
  int i,   j; int tempA[100],  index = start;
  for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    // Determine whether A[i] or A[j] should appear next in sorted order
    // Update tempA[index] accordingly
    index ++;
  } // end of merging loop
  // Copy tempA[start] … tempA[end-1]  to  A[start] … A[end-1]
  return;
}
// POSTCONDITION:  A[start] … A[end-1] sorted in decreasing order
```

Merging Loop In C++

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```
for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    if ((i < mid) && (j < end)) {  // None of the two subarrays fully seen yet



    }
    else {  // One of the two subarrays fully seen.

    }
    index ++;
} // end of merging loop
```

Merging Loop In C++

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```
for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    if ((i < mid) && (j < end)) {  // None of the two subarrays fully seen yet



    }
    else {  // One of the two subarrays fully seen.
            // Copy elements from the other subarray to tempA.
    }
    index ++;
} // end of merging loop
```

Merging Loop In C++

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```
for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    if ((i < mid) && (j < end)) {  // None of the two subarrays fully seen yet



   }
   else { if (i < mid) {tempA[index] = A[i]; i++;}   // A[mid] … A[end-1] seen
          else         {tempA[index] = A[j]; j++;}  // A[start] … A[mid-1] seen
   }
   index ++;
} // end of merging loop
```

Merging Loop In C++

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```
for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    if ((i < mid) && (j < end)) {  // None of the two subarrays fully seen yet
       // Find which of A[i]  (from subarray A[start] … A[mid-1])  or
       //  A[j] (from subarray A[mid] … A[end-1]) is the next sorted element,
       //  copy that element to tempA[index]
    }
    else { if (i < mid) {tempA[index] = A[i]; i++;}  // A[mid] … A[end-1] seen
           else         {tempA[index] = A[j]; j++;} // A[start] … A[mid-1] seen
    }
    index ++;
} // end of merging loop
```

Merging Loop In C++


```
for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    if ((i < mid) && (j < end)) {  // None of the two subarrays fully seen yet
            if (A[j] > A[i])     {tempA[index] = A[j];   j++;}
            else                 {tempA[index] = A[i];   i++;}
    }
    else { if (i < mid) {tempA[index] = A[i]; i++;}  // A[mid] … A[end-1] seen
            else        {tempA[index] = A[j]; j++;} // A[start] … A[mid-1] seen
    }
    index ++;
} // end of merging loop
```

Merging Loop In C++: The Whole Story

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```
for (i = start, j = mid; ((i < mid) || (j < end)); ) {  // Merging loop
    if ((i < mid) && (j < end)) {  // None of the two subarrays fully seen yet
            if (A[j] > A[i])     {tempA[index] = A[j];   j++;}
            else                 {tempA[index] = A[i];   i++;}
    }
    else { if (i < mid) {tempA[index] = A[i]; i++;}  // A[mid] … A[end-1] seen
            else         {tempA[index] = A[j]; j++;} // A[start] … A[mid-1] seen
    }
    index ++;
} // end of merging loop
```

Merge Sort In C++


```
// PRECONDITION: A[start] … A[mid-1]  and A[mid] … A[end-1] sorted in
//                          decreasing order
void mergeSortedSubarrays(int A[], int start, int mid, int end) {
   int i,   j; int tempA[100],  index = start;
   for (i = s
      // Def                                                    order
      // Update temp                    dingly
     index ++;
  } // end of merging loop
  // Copy tempA[start] … tempA[end-1]  to  A[start] … A[end-1]
  return;
}
// POSTCONDITION:  A[start] … A[end-1] sorted in decreasing order
```

```
for (i = start; i < end; i++) {A[i] = tempA[i];}
```

“Basic” Steps in mergeSortedSubarrays

- Reading two array elements, comparing them, writing an element in tempA, incrementing indices
- Copying an element from tempA to A

Counting "Basic" Steps In mergeSortedSubarrays

- Suppose mergeSortedSubarrays called with sub-arrays A[start] … A[mid-1] and A[mid] … A[end-1], each of size **n/2**
  - At most **n/2** "basic" steps to iterate over A[start] … A[mid-1]
  - At most **n/2** "basic" steps to iterate over A[mid] … A[end-1]
  - In all, at most **(n/2 + n/2)** or **n** "basic" steps to get merged sorted subarray in tempA
  - **n** "basic" steps to copy tempA[start] … tempA[end-1] to A[start] … A[end-1]

Overall, at most 2n "basic" steps

Counting “Basic” Steps in Merge Sort

- Let T_n denote maximum count of “basic” steps to merge sort an array of size n

Merge Sort: Looking Back

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```
// PRECONDITION: start < end,  both within array bounds of A
void mergeSort(int A[], int start, int end) {
  if (end == start +1) { return; }  //Subarray of interest has 1 element
  int mid = (start + end)/2;  //Get mid-index of subarray of interest
  mergeSort(A, start, mid);  // Sort subarray A[start] … A[mid-1]
  mergeSort(A, mid, end);   // Sort subarray A[mid] … A[end-1]
  // Merge sorted subarrays A[start] … A[mid-1]  and A[mid] … A[end-1]
  mergeSortedSubarrays(A, start, mid, end);
  return;
}
// POSTCONDITION: A[start] … A[end-1] sorted in decreasing order
```

Counting "Basic" Steps in Merge Sort

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- Let Tn denote maximum count of "basic" steps to merge sort an array of size n
- $T_n = T_{n/2} + T_{n/2} + 2n$
- $T_1 = 1$

mergeSortedSubarrays

Second recursive call of mergeSort

First recursive call of mergeSort

Solution of recurrence:

$$T_n \approx \lceil 2n.\ \log_2 n + n \rceil$$

Comparing "Basic" Steps With Selection Sort

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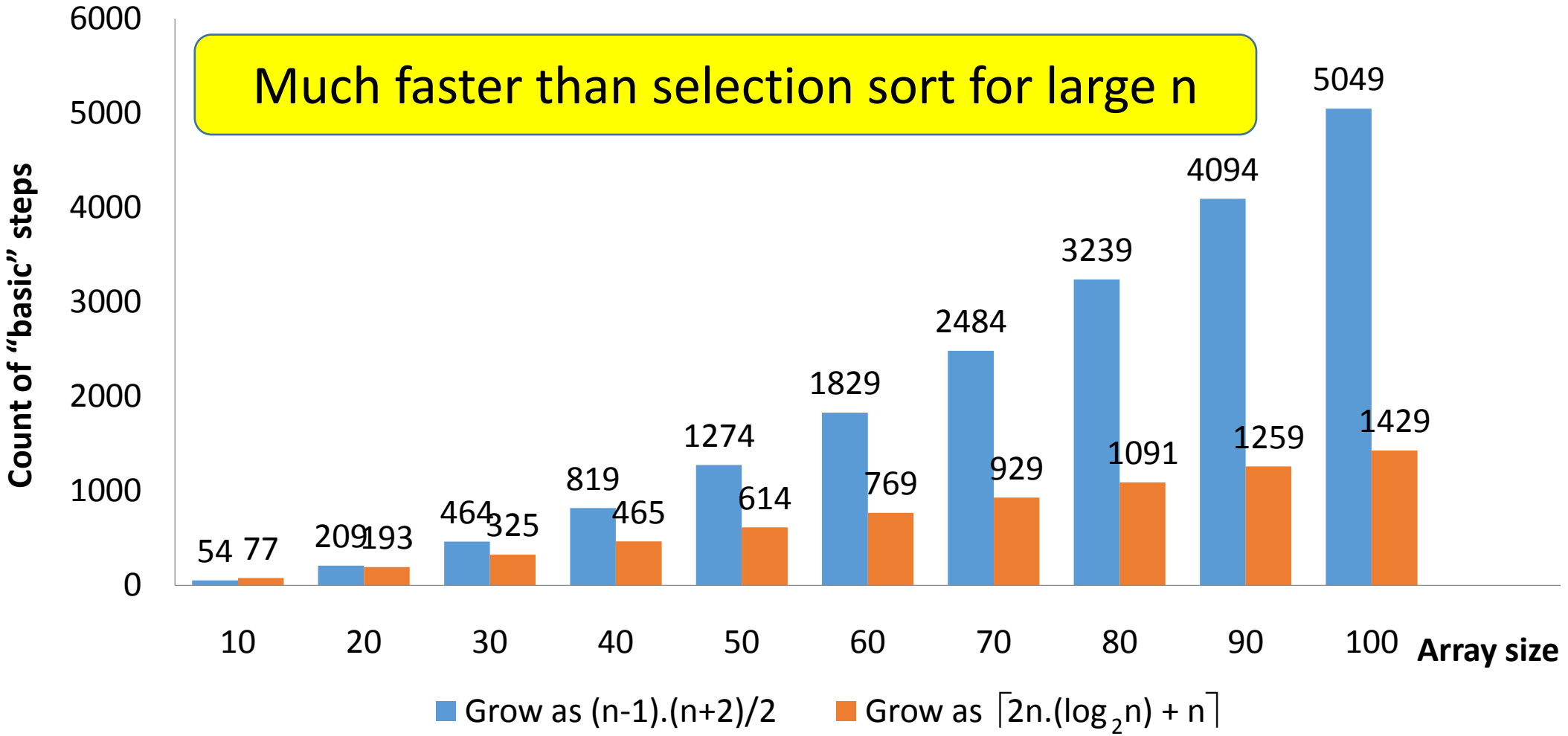

Summary

- Merge sort
  - C++ implementation using recursion
  - Analysis of performance
    - Counting “basic” steps in sorting an array of size n
  - Clear winner performance-wise over selection sort