

Solutions to Tut 6

P115

$$\begin{aligned}
 v &= \frac{1}{\hbar} \frac{dE}{dk} \Rightarrow \\
 \frac{dx}{dt} &= \frac{1}{\hbar} \frac{dE}{dk} \\
 &= \frac{1}{\hbar} \frac{dE}{dt} \frac{dt}{dk} \\
 &= \frac{1}{F} \frac{dE}{dt} \\
 \therefore x - x_0 &= \frac{1}{F} (E(k_0) - E(0))
 \end{aligned}$$

$$J = \frac{E}{\rho}$$

$$E = 100 \times 10^{-8} \times \frac{1}{1 \times 10^{-6}} = 1 \text{ V/m}$$

$$F = \hbar \frac{dk}{dt} \Rightarrow$$

$$t = \frac{\hbar \times \pi \times 10^{10}}{1 \times 1.6 \times 10^{-19}} \approx 2.07 \times 10^{-5} \text{ s}$$

Energy at $k = \pi(\text{\AA})^{-1}$ is

$$\frac{\hbar^2 k^2}{2m} \approx 37.6 \text{ eV}$$

$$\therefore x_2 - x_1 = 37.6 \text{ m}$$

P116

$$E = A - B \cos ka$$

$$v = \frac{1}{\hbar} \frac{dE}{dk} = \frac{1}{\hbar} B a \sin ka$$

Therefore, v is maximum when $k = \frac{\pi}{2a}$

$$m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}} = \frac{\hbar^2}{B a^2 \cos ka}$$

P117

$$\epsilon = \epsilon_0 (1 - \cos ka)$$

$$\frac{d\epsilon}{dk} = \epsilon_0 a \sin(ka) \text{ and } v_g = \frac{1}{\hbar} \frac{d\epsilon}{dk} = \frac{1}{\hbar} \epsilon_0 a \sin(ka)$$

speed corresponding to $k = \frac{10\pi}{3} \times 10^9 \text{m}^{-1}$ is $2.6 \times 10^5 \frac{\text{m}}{\text{s}}$

This is the speed of the missing electron (hole with the positive charge). That is the current is $I = nev_g = 1 \times 1.6 \times 10^{-19} \times 2.6 \times 10^5 = 4.2 \times 10^{-14} \text{A}$, along the + x direction.

effective mass of the electron, $m^* = \frac{\hbar^2}{\epsilon_0 a^2 \cos ka} = -3.4 \times 10^{-30} \text{kg}$.

Therefore, the effective mass of the hole $= 3.4 \times 10^{-30} \text{kg}$

Acceleration is $a = F/m^*$

Force on the hole $= 1.6 \times 10^{-19} \times 0.5 = 0.8 \times 10^{-19} \text{N}$ along the – ve x axis

Therefore, a of the hole $= -\frac{0.8 \times 10^{-19}}{3.4 \times 10^{-30}} = -2.3 \times 10^{10} \text{m/s}^2$

Hence the k value of the hole decreases with time. Or equivalently, the k value of the electron increases. Acceleration is dependent on m^* , which depends on the value of k.

Therefore, the speed of the electron becomes zero when $ka = \pi$ or when $k = \frac{\pi}{a} = \frac{\pi}{2} \times 10^{10}$

Therefore, change in k, $\Delta k = \left(\frac{\pi}{2} - \frac{\pi}{3}\right) \times 10^{10} \text{m}^{-1}$

Therefore, $\Delta t = \frac{\hbar \Delta k}{F} = 6.9 \times 10^{-6} \text{s}$

P 121

Before the application of B,

$$J_x = \frac{2}{2 \times 10^{-2} \times 10^{-4}} = 10^6 \text{A/m}^2$$

$$E_x = \frac{100}{8 \times 10^{-2}} = 1.25 \times 10^3 \text{V/m}$$

$$\therefore \sigma = \frac{J_x}{E_x} = 800 (\text{ohm.m})^{-1}$$

After the application of the magnetic field, $E_y = \frac{R_H}{J_x B_z}$

$$\therefore R_H = \frac{1.5}{2 \times 10^{-2}} \frac{1}{10^6 \times 0.2} = 3.75 \times 10^{-4}$$

$$\sigma = ne(\mu_n + \mu_p) \text{ with } n = p$$

$$R_H = \frac{1}{ne} \frac{\mu_p^2 - \mu_n^2}{(\mu_p + \mu_n)^2} = \frac{1}{ne} \frac{\mu_p - \mu_n}{(\mu_p + \mu_n)}$$

$$\therefore \sigma R_H = \mu_p - \mu_n = 800 \times 3.75 \times 10^{-4} = \frac{0.3m^2}{V.s}$$

$$\mu_n + \mu_p = \frac{800}{10^{22} \times 1.6 \times 10^{-19}} = 0.5$$

$$\therefore \mu_n = 0.1 \frac{m^2}{V.s} \text{ and } \mu_p = 0.4 \frac{m^2}{V.s}$$

$$\mu = \frac{e\tau}{m} \Rightarrow \tau = \mu \frac{m}{e} = 5.687 \times 10^{-12}$$

$$\tau_p = 2.275 \times 10^{-12} s \text{ and}$$

$$\tau_n = 0.5687 \times 10^{-12} s$$

P 122

$$\sigma = ne(\mu_p + \mu_n)$$

$$\sigma = 2 \left(\frac{kT}{2\pi\hbar^2} \right)^{\frac{3}{2}} (m_e * m_h)^{\frac{3}{4}} \exp\left(\frac{-E_g}{2kT}\right) e(\mu_p + \mu_n)$$

solving this, $E_g = 0.68 \text{ eV}$

Using this, the effective mass of the electron can be calculated.

P123

Use the relation

$$n = \left(\frac{kT}{2\pi\hbar^2} \right)^{\frac{3}{2}} (m_e * m_h)^{\frac{3}{4}} \exp\left(\frac{-E_g}{2kT}\right) \text{ to calculate } m_h^*$$

P124

$$n_i = 2.5 \times 10^{19} / m^3$$

$$n_d = 10^{20}$$

$$n_i^2 = np = (p + N_d)p \text{ .Solve this to get p and n/p.}$$

P 126

$$p + N_d = n + N_a$$

Using the equation given in problem P 123, calculate n and p (n=p) with the given information.

This works out to be 2.499×10^{25}

$$\therefore 2.499 \times 10^{25} \exp\{-(E_F - E_V)/kT\} + 5 \times 10^{19} = 2.499 \times 10^{25} \exp\{-(E_C - E_F)/kT\} + 10^{20}$$

Take $E_V=0$, $E_C=0.7$ eV, Put

$$\exp\left(\frac{E_F}{kT}\right) = x, \text{ which will give a quadratic equation in } x. \text{ Solving this gives } E_F = 0.332 \text{ eV}$$

P 129

From the given intrinsic conductivities at two different temperatures, calculate band gap and n_i values at the two temperatures. Then find $\mu_p + \mu_n$. Taking $n=(N_d+p)$ and $n_i^2=np$, calculate n. From this, calculate E_F . Use the expression of the extrinsic semiconductor expression for the conductivity, calculate μ_n