

PH-105 Assignment Sheet - 3

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56. For a particle in one-dimensional box of side L , show that the probability of finding the particle between $x = B$ and $x = B + b$ approaches the classical value b/L , if the energy of the particle is very high.

Solution :

Wavefunction for particle in a box is given by

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \text{ for } 0 < x < L$$
$$= 0 \text{ elsewhere}$$

Now probability of finding the particle between B and $B + b$ is given by

$$P = \int_B^{B+b} \frac{2}{L} \sin^2 \left(\frac{n\pi x}{L} \right) dx$$

$$P = \frac{2}{L} \int_B^{B+b} \frac{1 - \cos \left(\frac{2n\pi x}{L} \right)}{2} dx$$

$$P = \frac{1}{L} \left(b - \frac{L}{2n\pi} \left(\sin \left(\frac{2n\pi(B+b)}{L} \right) - \sin \left(\frac{2n\pi(B)}{L} \right) \right) \right)$$

$$P = \frac{1}{L} \left(b - \frac{L}{n\pi} \cos \left(\frac{2n\pi(2B+b)}{L} \right) \sin \left(\frac{2n\pi b}{L} \right) \right)$$

$$\lim_{n \rightarrow \infty} P = \frac{b}{L}$$