T. I. Schrödinger Equation

If the potential *V(x)* is independent of time, it is possible to separate out the spatial and time part of the Schrödinger Equation by a method called separation of variables. Let

$$\psi(x,t) = \phi(x) f(t)$$

Substitute in the Schrödinger Equation and write separately the spatial and the time part.

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

$$i\hbar \frac{\partial}{\partial t}\left(\phi(x) f(t)\right) =$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\left(\phi(x) f(t)\right) + V(x)\left(\phi(x) f(t)\right)$$

$$i\hbar \phi(x) \frac{df(t)}{dt} =$$

$$-\frac{\hbar^2}{2m} f(t) \frac{d^2\phi(x)}{dx^2} + V(x)\left(\phi(x) f(t)\right)$$

Divide both sided by $\phi(x) f(t)$

$$i\hbar \frac{1}{f(t)} \frac{df(t)}{dt} = -\frac{\hbar^2}{2m} \frac{1}{\phi(x)} \frac{d^2\phi(x)}{dx^2} + V(x)$$

$$i\hbar \frac{1}{f(t)} \frac{df(t)}{dt} = E$$

$$-\frac{\hbar^2}{2m} \frac{1}{\phi(x)} \frac{d^2\phi(x)}{dx^2} + V(x) = E$$

The solution of the first equation is given as follows.

$$\frac{df(t)}{f(t)} = -\frac{iE}{\hbar}dt$$

$$f(t) = Ae^{-\frac{iE}{\hbar}t}$$

Comparing with the standard wave equation we interpret E as the energy of the particle.

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} + V(x)\phi(x) = E\phi(x)$$

E is the total energy of the particle.

$$\boxed{\frac{d^2\phi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]\phi(x) = 0}$$

Remember $\psi(x,t) = \phi(x)e^{-\frac{iE}{\hbar}t}$

For time independent V(X)

- Consider a solution of time independent Schrödinger Equation $\phi(x)$.
- Let us try to find expected value of *f(x)*.

The expectation of f(x)

$$\langle f(x)\rangle = \int_{-\infty}^{+\infty} \psi^*(x,t) f(x) \psi(x,t)\, dx$$

$$= \int_{-\infty}^{+\infty} \left(\phi^*(x)e^{+\frac{iE}{\hbar}t}\right) f(x) \left(\phi(x)e^{-\frac{iE}{\hbar}t}\right) dx$$

$$= \int_{-\infty}^{+\infty} \phi^*(x)\, f(x)\, \phi(x) dx$$

This is time independent. These are called Stationary States.