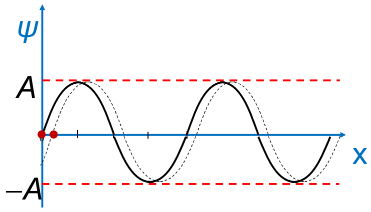
An ideal Wave (for all x and all t)



$$\psi = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}; \omega = 2\pi\upsilon$$

Phase Speed of a Wave

Take a point at $t = 0$ for which $\psi = 0$. Let time increase to Δt. What would be Δx to maintain $\psi = 0$.

$$k\Delta x - \omega\Delta t = 0$$

$$v_p = \frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

Phase Speed of Matter Wave

$$S = \frac{\omega}{k} = \lambda\upsilon = \frac{h}{p} \times \frac{E}{h} = \frac{E}{p}$$

What is E for matter wave?

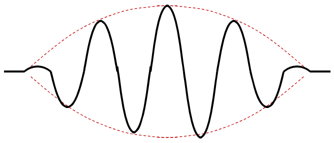
$$mc^2 \text{ or } \frac{1}{2}mv^2$$

Whatever it is, the phase speed is not v.

$$S = \frac{c^2}{v} \text{ or } \frac{v}{2}$$

Does it make sense?

Realist Wave Group



Questions?

1. What is the speed of the wave group?
2. How is a wave group formed?

Superposition of Two Waves

$$\psi = \psi_1 + \psi_2$$
$$= A\sin(kx - \omega\, t)$$
$$+ A\sin\left[(k+\Delta k)x - (\omega + \Delta\omega)t\right]$$

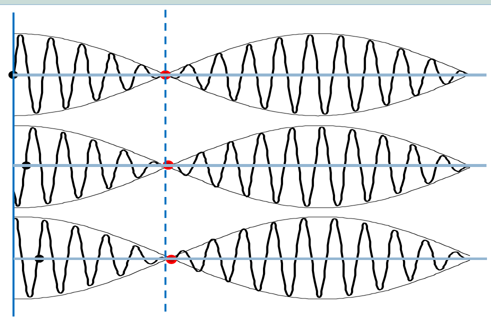
Use the following identity.

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\psi = \left[2A\cos\left(\frac{\Delta k\, x}{2} - \frac{\Delta\omega\, t}{2}\right)\right]\left(\sin(k'x - \omega' t)\right)$$

$$k' = \frac{2k+\Delta k}{2};\omega' = \frac{2\omega + \Delta\omega}{2}$$

Beats



Group Speed

What is the speed with which envelope moves?

$$v_g = \frac{d\omega}{dk}$$

Is it the same as phase speed. Yes if $\omega = vk$, with v constant. In other words the speed is not wavelength dependent.

Do it yourself

Show that group velocity of the matter wave is the speed of the particle. This is true if we take any of the two definition of frequency.

Localization

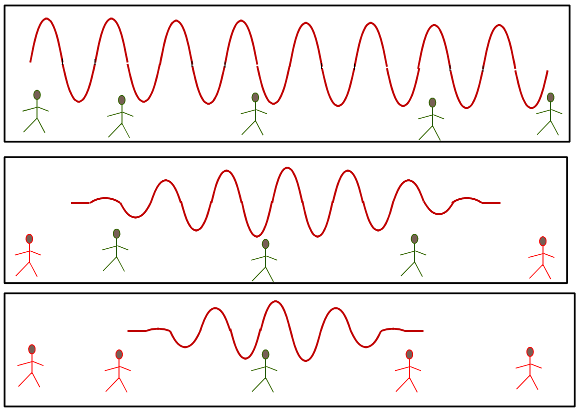
- A localized wave packet can be constructed by superposing infinite waves of varying wavelength.
- Using Fourier Transform it is possible to decompose any wave packet into their component ideal waves.

$$y(x,t) = \int_0^{\infty} A(k)\sin(kx - \omega t)dk$$

A Particle Behaving Like Wave

- A realistic wave is like a wave packet and hence somewhat localized.
- The wave associated with a particle is also expected to be of the wave packet form.

- Nevertheless, whatever is the extent of wave packet, the particle properties can be detected there.
- So the position of the particle is uncertain to the order of width of the wave packet.



Shorten the wave packet?

- We can shorten wave packet to any amount by superposing ideal waves in a larger and larger range of wavelengths.

Value of wavelength?

- But a unique value of wavelength can be assigned to only an ideal wave. Larger range of wavelengths means their wavelengths become more uncertain.
- As wave length is related to momentum by de Broglie relationship, the momentum becomes more and more uncertain for shorter wave packets.

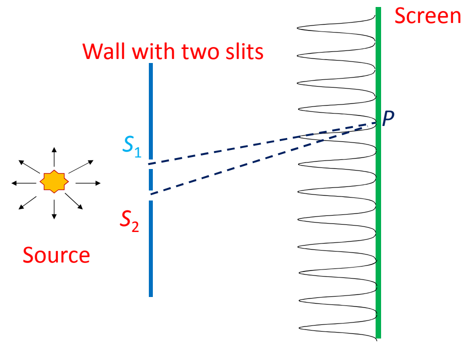
Basis of Uncertainty Principle

- An ideal wave has a precise wavelength, and hence its momentum is very precise. But then this wave extends from $+\infty$ to $-\infty$. Hence it has infinite uncertainty in its position.

- More we want to localize the wave packet, we make the position of the particle more precise, but then loose control on the precision of its momentum.
- Uncertainty principle protects wave-particle duality.

Young's double slit (Mysterious Particle)

- An interference experiment performed with light. Can also be performed with water waves etc.
- One obtains dark and bright fringes due to the destructive and constructive interference.



Experiment with Electrons

- Let us Assume that we perform this experiment with electrons, by choosing appropriate length scales, so that we can observe such an interference pattern. We know for sure, electrons can be diffracted like light waves.

Do electrons reaching screen pass through one of the two slits?

Since electron is a particle, which can not split, so is it not obvious that it should pass through one of the slits?

How silly is the question?

Had it been equally silly, if we were talking of interference of two classical waves?

Well it is not so silly as we shall soon see. If the statement is true. We can classify the electrons reaching the screen in two parts.

(1) Those which came through slit S_1.

(2) Those which came through slit S_2.

Can we experimentally verify this statement?

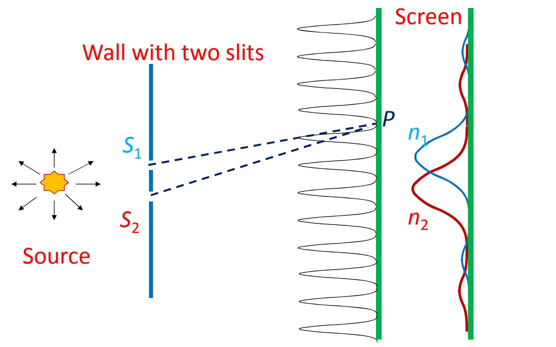
Perform Three Experiments

(1) Keep only slit S_1 open. Choose a point P on the screen, Count the number of electrons n_1 reaching that point in a given time. Keep on changing the position of point P and find n_1 as a function of position for the same time.

(2) Repeat the experiment with only slit S_2 open. Find n_2 as a function of the position.

(3) Keep both slits S_1 and S_2 open and find n_{12} as a function of the position.

Do we get $n_{12} = n_1 + n_2$?



The Answer is NO.

- With one slit open, one sees a diffraction pattern. There is no interference pattern due to the other slit.
- There are places on the screen, which receive much less number of electrons when both the slits are open, than when one slit is open.

Mysterious Particle

- Our 'impression' of smaller particle is based on the extrapolation of our ideas of bigger particles.
- These extrapolations clearly are not correct.
- The particle does behave mysteriously, when it is small.

Perturbing the Experiment 1

- Can we verify experimentally if the particles indeed go through one of the two slits, when both the slits are open.
- Put a light source to 'watch' the electron.

- We shall indeed find that electrons pass through one of the two slits. But now interference pattern would be lost.
- The photon interaction with electron could have changed their path.

Perturbing the Experiment 2

- We can reduce the perturbation by using low energy photons?
- If wavelength increases beyond a limit, the resolving power would be reduced and the two slits may appear as a one broad slit. Only in such circumstances, we shall be able to get back the interference pattern.