PH-105 Assignment Sheet - 1

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25. A containder contains a monoatomic hydrogen gas in thermal equilibrium at a temperature T for which $k_BT = 0.025$ eV. Let E_1 be the difference between the ground state and the first excited state energy of the atom when at rest. Let E_2 be the energy of photon (in frame of container) required to make this transition in an atom, which is travelling towards the photon (in an antiparallel direction) with the average energy at the above specified temperature.
(i) Find $E_1 - E_2$
(ii) After the absorption of photon what would be the final velocity of the hydrogen atom
(iii) If the lifetime of the first exited state is 10^{-8}, will the photon with energy E_2 be able to cause a transition, had the atom been at rest. Discuss quantitatively.
Solution :
Using Relativistic Doppler effect, we have

$$\nu' = \nu\sqrt{\frac{1+v/c}{1-v/c}}$$

This formula can he used to calculate frequency of photon in rest frame of Hydrogen atom. As v 2000m/s¡¡c, hence

$$\nu' = \nu(1+v/c)$$

Now, in rest frame of Hydrogen (S')

$$p'_p = h\nu'/c$$

$$p_{H(aftercollision)} = p'_p = h\nu'/c$$

$$E_{H(aftercollision)} = h^2\nu'^2 + m_H^2c^4$$

Here $h^2\nu'^2 << m_H^2c^4$, hence we can assume that almost all energy of incoming photon is used for electron transition. Thus,

$$E_p = h\nu' = E_1$$

$$E_2(1+v/c) = E_1$$

$$v_{rms} = \sqrt{\frac{3K_BT}{M}} = 2.73 * 10^3 m/s$$

$$E_1 - E_2 \approx E_1 v/c = 10.2 * 9.1 * 10^{-4} eV = 9.28 * 10^-3$$

In the frame of Hydrogen atom, its recoil speed (non-relativistic) is given by

$$h\nu'/c = m_H v_H$$

$$v_H = \frac{10.2eV * c}{m_H c^2} \approx (10.2eV/1GeV) * c = 3.06m/s$$

By uncertainity principle, the uncertainity in energy of photon which causes the transition is given by,

$$\delta_E \delta_t \leq 4.13 * 10^-15$$

$$\delta_E \leq 4.13 * 10^-7$$

Thus only photons of energy $10.2 \pm 4.13 * 10^-7$ can excite the electron. Hence the electron with energy E_2, will not be able to excite the electron had the atom been at rest.