QM Tutorial Q.40

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A beam of electron of energy $0.025eV$ moving along x-direction, passes through a slit of variable width w placed along y-axis. Estimate the value of the width of the slit for which the spot size on a screen kept at a distance of $0.5m$ from slit would be minimum.

Initially, the particle moved along the x-direction i.e. $p_y = 0$. But when it passes through the slit, the uncertainty in its y-coordinate suddenly becomes finite, namely, Δy equals the slit width w. Thus, by the uncertainty principle, there must be some uncertainty in its y-momentum over and above its initial y-momentum (which is zero). By the uncertainty principle, we have

$\Delta y \Delta p_y \geq \frac{\hbar}{2} \Rightarrow \Delta p_y \geq \frac{\hbar}{2w}$

Thus, the maximum y-momentum a particle passing through the slit can possess (assuming maximum possible precision) is $|p_y| = \frac{\Delta p_y}{2} = \frac{\hbar}{4w}$ since this momentum arising due to Heisenberg uncertainty can be either in +y or -y direction.

Now, as the particle traverses $0.5m$ (slit-screen distance), it will traverse $\frac{0.5p_y}{p_x} = \frac{0.5p_y}{\sqrt{2mE}}$ metres in the y-direction (note that the energy of the electron is quite low, hence the classical momentum-energy relation nearly holds).

In addition, a particle may enter the slit at its top or bottom edge, hence the highest y-coordinate (w.r.t. the centre of the screen) that a particle can have upon reaching the screen is

$R = \frac{w}{2} + \frac{0.5p_y}{\sqrt{2mE}} = \frac{w}{2} + \frac{0.5\hbar}{4w\sqrt{2mE}}$.

For the minimum of this y-coordinate (same as the radius of the spot formed on the screen), minimize R w.r.t. w. Putting $\frac{dR}{dw} = 0$, we get
$w = \sqrt{\frac{\hbar}{4\sqrt{2mE}}} \approx 17.56\mu m$.

Do check that it indeed is a minimum by checking that $\frac{d^2R}{dw^2} > 0$ for this value of slit width w.