59. An atom can radiate at anytime after it is excited. It is found that on the average the excited atoms has a lifetime of $10^{-8}\,s$. That is, during this period it emits a photon and is de-excited.
(a) What is the minimum uncertainty $\Delta\nu$ in the frequency of the photon and $\Delta\nu/\nu$ for radiation of wavelength $\lambda = 5000\text{Å}$?
(b) What is the uncertainty ΔE in the energy of the excited state of the atom?

Solution: We fine the ΔE in the energy of the state from

b) $$\Delta E \geq \hbar/\Delta t = (1.055\times 10^{-34}\,J\cdot s)/10^{-8}\,s$$

$$= 1.1\times 10^{-26}\,J$$

$$= 6.6\times 10^{-8}\,eV$$

a.) $\Delta\nu = ?$ $\Delta\nu/\nu$ if $\lambda = 5000\,\text{Å}$

$\Delta t = 10^{-8}\,s$.

$$E = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \Delta E = -\frac{hc}{\lambda^2}\Delta\lambda$$

$$\Rightarrow |\Delta E| = \frac{hc}{\lambda^2}\Delta\lambda$$

Now $\frac{hc}{\lambda^2}(\Delta\lambda)(\Delta t) \sim \hbar$

$$\Delta\lambda \sim \frac{\hbar\lambda^2}{hc\Delta t} = \frac{\cancel{h}}{\cancel{h}\,2\pi}\frac{\lambda^2}{c\Delta t} = \frac{(5000\times 10^{-10}\,m)^2}{2\pi\times 3\times 10^8\,m/s\times 10^{-8}\,s}$$

$$\therefore \Delta\lambda = 13.262\times 10^{-15}\,m$$

$$\frac{\Delta\nu}{\nu} = \frac{c/\lambda^2\,\Delta\lambda}{c/\lambda} = \frac{\Delta\lambda}{\lambda} = \frac{13.262\times 10^{-15}\,m}{5000\times 10^{-10}\,m}$$

$$= 2.6524\times 10^{-2}$$

$$= 0.026524$$