

# PH-105 QM Sheet 1

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30. The dispersion relation for a lattice wave propagating in a one dimensional chain of atoms of mass  $m$  bound together by force constant  $\beta$  is given by the following equation.

$$\omega = \omega_0 \sin(ka/2)$$
$$\omega_0 = \sqrt{\frac{4\beta}{m}}$$

Here  $a$  is distance between atoms.

(a) Show that in the long wavelength limit the medium is non-dispersive.

(b) Find the group and phase velocities at  $k = \pi/a$ .

Solution :

In the long wavelength limit, we have  $k \rightarrow 0$ . So,  $\omega \rightarrow \omega_0(ka/2)$ . Hence phase speed  $= \omega/k = \omega_0(a/2)$  which is independent of  $k$  and hence, the medium is non-dispersive.

At  $k = \pi/a$ ,  $\sin(ka/2) = 1$  while  $\cos(ka/2) = 0$ . Hence, phase speed  $= \omega/k = \omega_0 a/\pi$  and group speed  $= d\omega/dk = 0$ .