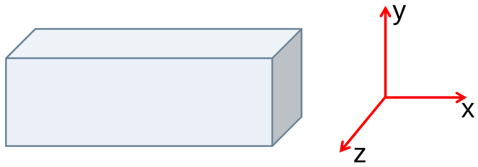
Hall Effect

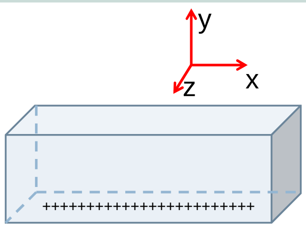
- Performed in 1879 by E. H. Hall, much before the discovery of electron.
- Very important research tool even today.
- Hall probe is used to measure the magnetic field.
- Can be used to determine the sign of charge carrier.



Conventional Current Flow in +x-direction.
Magnetic Field in +z-direction.
What will happen in y-direction?

Force Assuming (+) carriers

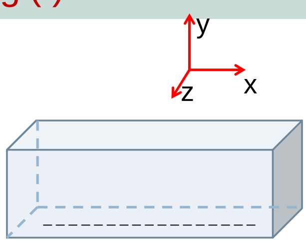
$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$
$$= qv_x B_z \left(\hat{i} \times \hat{k}\right)$$
$$= -qv_x B_z \hat{j}$$



Would result in an electric field in +y-direction within material.

Force Assuming (-) carriers

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$
$$= -|q| v_x B_z \left(-\hat{i} \times \hat{k}\right)$$
$$= -|q| v_x B_z \hat{j}$$



Would result in an electric field in -y direction within material.

In Equilibrium

A field would develop along *y* direction, which would balance the force due to magnetic field. The current would continue to flow in *+x* direction.

$$\pm|q|E_y\hat{j} + \left(-|q|v_xB_z\hat{j}\right) = 0$$

$$E_y = \pm v_xB_z$$

Hall Coefficient

Defined in terms of experimentally measures quantities.

$$R_H \equiv \pm\frac{E_y}{J_xB_z}$$

It is positive for positive and negative for negative charge carriers.

The value of Hall Coefficient

$$|R_H| \equiv \left|\frac{E_y}{J_xB_z}\right| = \left|\frac{v_xB_z}{nqv_xB_z}\right|$$

$$= \left|\frac{1}{nq}\right|$$

Hall coefficient depends on the number of charges per unit volume.

Experimental Data

Metal	- $(1/R_Hne)$	Valency
Na	1.2	1
K	1.1	1
Rb	1.0	1
Cs	0.9	1
Cu	1.5	1
Ag	1.3	1
Au	1.5	1

Metal	- ($1/R_H ne$)	Valency
Be	-0.2	2
Mg	-0.4	2
In	-0.3	3
Al	-0.3	3

Source of both Tables:
"Solid State Physics" - N.W. Ashcroft and N.D. Mermin, Holt Rinehart and Winston (1976)

Failures of Free Electron Theory

- Can still not explain the variation of resistivity with impurities, crystalline purity, temperature.
- Can not explain positive Hall coefficients and number of charge carriers in some metals.

Electron in a Periodic Potential

- We have to discard the assumption that electrons in solid are free.
- Next step is to consider that electron experiences a periodic potential inside a solid.
- Let us consider one dimensional case for simplicity.

Bloch Theorem

- Consider a periodic potential.

$$V(x+na)=V(x)$$

- The wave function of an electron in such a potential can always be written in the following form.

$$\phi(x)=u(x)e^{ikx}$$

- The actual form of *u(x)* shall depend on *V(x)*, but following periodicity condition will be obeyed.

$$u(x+na)=u(x)$$

Justification

$$\phi(x+na)=u(x+na)e^{ik(x+na)}$$
$$=u(x)e^{ikx}e^{ikna}$$
$$=\phi(x)e^{ikna}$$

This implies

$$\left|\phi(x+na)\right|^2=\left|\phi(x)\right|^2$$

This is expected.

Implications

The value of *k (wave vector)* is not uniquely defined. The following is also equally valid wave vector.

$$k'=k+\frac{2m\pi}{a}$$

Let us try this out.

$$\phi(x)=u(x)e^{ikx}$$
$$=u(x)e^{ikx}e^{i\frac{2m\pi}{a}x}e^{-i\frac{2m\pi}{a}x}$$
$$=u(x)e^{ik'x}e^{-i\frac{2m\pi}{a}x}$$
$$=U(x)e^{ik'x}$$
$$U(x)\equiv u(x)e^{-i\frac{2m\pi}{a}x}$$

The above is a valid Bloch wave function if *U(x)* also shows the desired periodicity.

$$
\begin{aligned}
U(x+na) &= u(x+na)e^{-i\frac{2m\pi}{a}(x+na)} \\
&= u(x)e^{-i\frac{2m\pi}{a}x}e^{-i\frac{2m\pi}{a}na} \\
&= U(x)e^{-i2mn\pi} \\
&= U(x)
\end{aligned}
$$

Wave Vector

- In FET, wave vector is unique and is related to momentum.
- Here wave vector is not unique and obviously not related to the momentum.

$$\vec{p} \neq \hbar\vec{k}$$

- $\hbar\vec{k}$ is called crystal momentum of electron.

Speed of electron

The speed can be shown to be given by the following expression valid for one – dimension only.

$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

Resembles Group Velocity Expression.

Speed of electron

If the state of electron is given by a particular value of k, one can evaluate its speed by evaluating the differential at that value of k. If k changes the velocity may also change.

This implies

- In spite of the presence of core potentials the speed of electron for a particular wave vector is constant and time independent.
- Can explain large mean free paths that were experimentally observed.

Semi-Classical Theory of Motion

In the presence of an external force $\vec{F}$ in the form of an electric or magnetic field, the following equation gives the motion.

$$\vec{F} = \hbar \frac{d\vec{k}}{dt}$$

$\hbar\vec{k}$ is called Crystal momentum of electron.

- This equation resembles Newton's second law. But note $\vec{F}$ is not the only force on electron and $\vec{p} \neq \hbar\vec{k}$.
- The effect of external force in the form of electric and magnetic field is only to change the wave vector of the electron.
- The speed may change due to the changed wave vector.