PH-105 Assignment Sheet - 2 (Quantum Mechanics)

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38. **A wave packet is constructed by superposing waves, their wavelengths varying continuously in the following way:**

$$y(x,t) = \int A(k)\cos(kx - \omega t)\,dk$$

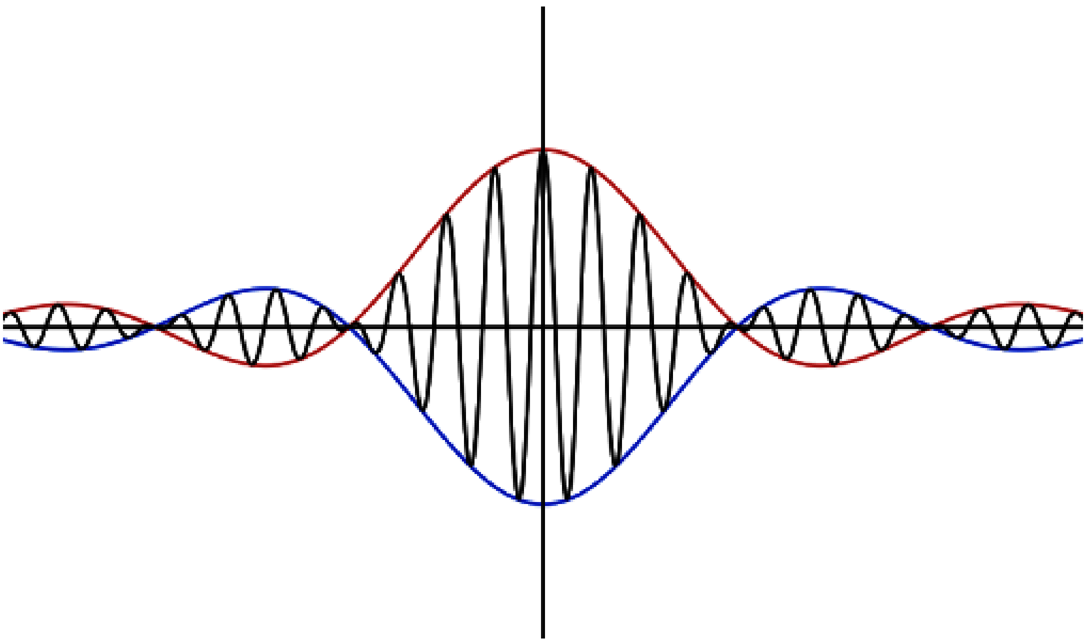
Where A(k) = A for $(k_o - \Delta k/2) \leq k_o \leq k_o + \Delta k/2)$ **and = 0 otherwise.**
Sketch approximately y(x,t) and estimate Δx **by taking the difference between two values of** x **for which the central maximum and nearest minimum is observed in the envelope. Verify uncertainty principle from this.**

<u>**Solution**</u> :

Integrating between $k = k_o - \Delta k/2$ and $k = k_o + \Delta k/2$, we have,

$$y(x,t) = \frac{A}{x}\sin(kx - \omega t)\Big|_{k_o-\Delta k/2}^{k_o+\Delta k/2} = \frac{2A}{x}\sin(\frac{\Delta k}{2}x)\cos(k_o x - \omega t)$$

The function can be plotted as follows:



The envelope curve is given by:-

$$\xi(x) = \frac{2A}{x}\sin(\frac{\Delta k}{2}x)$$

Central maxima occurs at $x = 0$. For neares minimum, we differentiate $\xi(x)$ to find the extremum:

$$\frac{\partial \xi(x)}{\partial x} = 0$$

$$2A(\frac{x\frac{\Delta k}{2}\cos(\frac{\Delta k}{2}x) - \sin(\frac{\Delta k}{2}x)}{x^2}) = 0$$

$$x\frac{\Delta k}{2} = \tan{(x\frac{\Delta k}{2})}$$

Let x_o be the solution of the above equation. Then $\Delta x = x_o - 0 = x_o$. The solution of the equation can be found using analytical methods. The value of x_o thus is $\frac{8.98682}{\Delta k}$.
Now, $\Delta p = \hbar \Delta k$.
Thus, the product $\Delta x \Delta p = 8.986\hbar > \hbar/2$.
This verifies the uncertainty principle.