PH-105 QM Sheet 2

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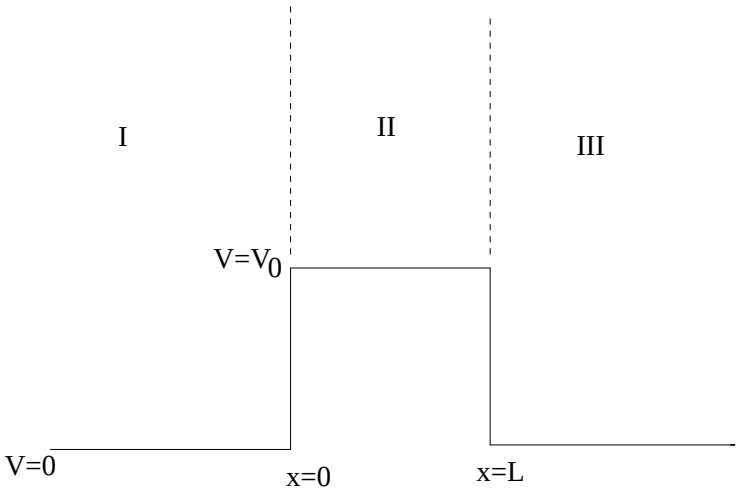
71. **A beam of particles of energy E and de-Broglie wavelength λ, travelling along the positive x-axis in potential free region, encounters a one-dimensional potential barrier of height V_0=E and width L.**
(a) Obtain an expression for the transmission coefficient.
(b) For what value of L (in terms of λ), will the reflection coefficient be half?

Solution :



Let $k^2 = \frac{2mV_0}{\hbar^2}$. Then, the wave-functions in the three regions are given by:
$\phi_I(x) = Ae^{ikx} + Be^{-ikx}$
$\phi_{II}(x) = Cx + D$
$\phi_{III}(x) = Fe^{ikx}$
At x=0, we have $A + B = D$ and $ik(A - B) = C$. At x=L, we have $CL + D = Fe^{ikL}$ and $C = ikFe^{ikL}$.
On solving, we get $\frac{F}{A} = \frac{2e^{-ikL}}{2-ikL}$
Transmission coefficient, $T = |\frac{F}{A}|^2 = \frac{4}{4+(kL)^2}$.
For $T = 0.5$, we get $kL = 2$ and hence $L = \frac{2}{k} = \frac{\lambda}{\pi}$.