Solution to Relativity tutorial Q.12

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Q.12

Define events E_1 through E_3 as follows

E_1 - Spaceships A, B and C coincide (i.e. the origins of the frames of A, B and C coincide)
E_2 - Spaceship B fires at C
E_3 - Spaceship C fires at B

Speed of B w.r.t. A = $0.6c$ in the positive-x direction, so

$$\gamma_{AB} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.25 \quad , \quad \beta_{AB} = \frac{v}{c} = 0.6$$

Speed of C w.r.t. A = $0.75c$ in the positive-y direction, so

$$\gamma_{AC} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.51 \quad , \quad \beta_{AC} = \frac{v}{c} = 0.75$$

The event table is as follows. Note that x, y and t are the space-time coordinates for the frame of A, x', y' and t' are the coordinates in the frame of B and x'', y'' and z'' correspond to the frame of C.

	$x(m)$	$y(m)$	$t(s)$	$x'(m)$	$t'(s)$	$y''(m)$	$t''(s)$
E_1	$x_1 = 0$	$y_1 = 0$	$t_1 = 0$	$x'_1 = 0$	$t'_1 = 0$	$y''_1 = 0$	$t''_1 = 0$
E_2	$x_2 = 3.6 \times 10^5$	$y_2 = 0$	$t_2 = 2 \times 10{-}3$	x'_2	t'_2	y''_2	t''_2
E_3	$x_3 = 0$	$y_3 = 4.5 \times 10^5$	$t_3 = 2 \times 10{-}3$	x'_3	t'_3	y''_3	t''_3

Table of Events

Note that $t_2 = \frac{x_2}{v_{AB}} = 2 \times 10^{-3}$ seconds $= \frac{y_3}{v_{AC}} = t_3$

a) The spaceship C travels at $u_y = 0.75c$ in the $+y$ direction and $u_x = 0$. So, by the velocity transformation between frames A and B,
Speed of C in B's frame =

$$u'_y = \frac{u_y}{\gamma_{AB}(1 - \frac{v_{AB}u_x}{c^2})} = 0.6c.$$

b) By Lorentz transformation, we calculate

$t_2' = \gamma_{AB}(t_2 - \frac{v_{AB}x_2}{c^2}) = 1.6 \times 10^{-3}$ seconds
$t_3' = \gamma_{AB}(t_3 - \frac{v_{AB}x_3}{c^2}) = 2.5 \times 10^{-3}$ seconds
$t_2'' = \gamma_{AC}(t_2 - \frac{v_{AC}y_2}{c^2}) = 3.02 \times 10^{-3}$ seconds
$t_3'' = \gamma_{AC}(t_3 - \frac{v_{AC}y_3}{c^2}) = 1.32 \times 10^{-3}$ seconds

So the time interval between spaceships B and C firing bullets in
B's frame= $t_3' - t_2' = 9 \times 10^{-4}$ **seconds (B fires before C)**
C's frame= $t_2'' - t_3'' = 1.7 \times 10^{-3}$ **seconds (C fires before B)**

c) Using the coordinates of E_2 and E_3 in the frame of spaceship A, **the proper time interval between E_2 and E_3 can be calculated as**

$$\Delta\tau = \sqrt{(\Delta t)^2 - \frac{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}{c^2}} = 1.92i \times 10^{-3} seconds$$