What happens to the solution of the equation below T_c? In such a case, it was predicted that the excess molecules go to ground state. So the integral is to be equated to $\frac{(N-N_0)}{V}$. Here N_o is the number of particles in ground state.

At T<T_c (taking α=0)

$$\frac{N_1}{V} = 0.3265 \times \left(\frac{2mkT}{\pi\hbar^2}\right)^{3/2}$$

$$\frac{N}{V} = \frac{N_o + N_1}{V} = \frac{N_o}{V} + CT^{3/2}$$

$$C \equiv 0.3265 \times \left(\frac{2mk}{\pi\hbar^2}\right)^{3/2}$$

$$\frac{N}{V} = \frac{N_o + N_1}{V} = \frac{N_o}{V} + CT^{3/2}$$

$$\therefore \frac{N_o}{V} = \frac{N}{V} - CT^{3/2} = CT_c^{3/2} - CT^{3/2}$$

$$\frac{N_o}{V} = \frac{N}{V}\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]$$

With the lowering of temperature, it could be predicted that larger and larger fraction of molecules will occupy the ground state. This is known as Bose-Einstein Condensation.

In 1995 Cornell and Wieman demonstrated BE condensation in a gas of ^{87}Rb atoms.

Noble Prize in 2001 to Cornell, Ketterle and Wieman for "Achievement of Bose-Einstein Condensation in dilute gases of alkali atoms…"

Liquid ^{4}He

- Was liquefied by Kamerlingh Onnes, for which he got Noble Prize in 1913.
- Boiling point ~4.2 K.
- Can not be solidified under normal pressure.
- Opened up possibility of making experimental measurements at low temperatures.

Superfluidity in ^{4}He

- At 2.17K, the liquid become superfluid.
- Below this temperature liquid does not show boiling and calms down, though evaporation continues.
- Creeping motion of fluid. (Eisberg and Resnick)
- Shows a high degree of order.

Bose Condensation in ^{4}He?

- It is believed that superfluid state in ^{4}He is caused by BE Condensation.
- The critical temperature may not exactly match as we have treated atoms as non-interacting gas rather than liquid.

Black Body Radiation

What changes do we need?

$$E = \hbar c k = \frac{\hbar c \pi}{L}\sqrt{n_x^2 + n_y^2 + n_z^2}$$

Density of State is calculated in a similar way. Use BE Statistics. Put α=0, f=2.

Metals

- Metals have always been considered important.
- Ages have been named after metals
- Have High Electrical and Thermal Conductivity.

Free Electron Theory

- 1897 J.J. Thomson discovered electron.
- 1900 Drude gave the first theory of metals, known as Free Electron Theory.
- Drude essentially used assumptions of Kinetic Theory of Gases with slight modifications.

Drude Model

- Positive charges immobile and 'valence' electrons get detached and wander freely in metals.
- Neglect e-e interaction.
- Neglect electron-core interaction.
- Only way electron 'sees' core is by scattering.

Relaxation Time

- The probability of scattering in time dt is dt/τ.
- τ is called relaxation time.
- Relaxation time is independent of position and velocity of electron.

Electrical Conductivity

Consider an electron, whose velocity immediately after collision (say at $t=0$) is $\vec{v}_o$. Let the electric field be $\vec{E}$. Its velocity at time t would be given as

$$\vec{v}(t) = \vec{v}_o - \frac{e\vec{E}t}{m}$$

provided electron does not suffer a collision in between.

The average (drift) velocity of all the electrons would be given by.

$$\vec{v}_D = \langle \vec{v} \rangle = -\frac{e\vec{E}\tau}{m}$$

If n is the total number of electrons per unit volume, then current density is given as follows.

$$\vec{J} = -ne\vec{v}_D = \frac{ne^2\tau}{m}\vec{E}$$

Ohm's Law

$$\vec{J} = \sigma\vec{E}$$

$$\sigma \equiv \frac{ne^2\tau}{m}$$

$$\rho = \frac{1}{\sigma}$$

σ is called conductivity, while ρ is called resistivity of the metals.

$$\rho_{Cu} \sim 0.3\mu\ \Omega \text{ cm at 77K}$$
$$\sim 1.56\mu\ \Omega \text{ cm at 273K}$$

We define mean free path as follows.

$$\ell = v_{rms} \times \tau$$
$$\frac{1}{2}m\left(v_{rms}\right)^2 = \frac{3}{2}kT$$

$$\tau \sim 10^{-14}\text{ s}$$
$$v_{rms} \sim 10^{5}\text{ m/s}$$
$$\ell \sim 10^{-9}\text{ m}$$

The mean free path gives an expected number.

Problems

- Conductivity depends on temperature, purity, crystalline quality.
- Much larger mean free paths have been observed experimentally in many carefully prepared systems.
- Observed small contribution to specific heat.

Equipartition Law for Metals

$$\varepsilon = 3N_A \times kT + 3N_A \times \frac{1}{2}kT$$
$$C_v = \frac{d\varepsilon}{dT} = 4.5N_A k = 4.5R$$