Q.103

$$\sigma = \frac{ne^2\tau}{m^*}$$, where m^* is effective mass

for Na

$$n = \frac{6.02\times10^{26}\times 970}{23} \approx 2.54\times10^{28}$$

$$\tau = \frac{\sigma m^*}{ne^2} = \frac{2.17\times10^{7}\times 1.2\times 9.1\times10^{-31}}{2.54\times10^{28}\times(1.6\times10^{-19})^2} \approx 3.64\times10^{-14}\ s$$

$l = 1.15\times10^{5}\times 3.64\times10^{-14} \approx 4.19\ nm$, in Drude Model. Otherwise find v_F using given m^*

$$v_D = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{2.17\times10^{7}\times100}{2.54\times10^{28}\times1.6\times10^{-19}} \approx 0.53\ m/s$$

Q.104

$$\langle\varepsilon\rangle = \frac{\int_0^{\varepsilon_F}\varepsilon N(\varepsilon)\,d\varepsilon}{\int_0^{\varepsilon_F}N(\varepsilon)\,d\varepsilon}$$

$$= \frac{\int_0^{\varepsilon_F}\varepsilon_F^{3/2}\,d\varepsilon}{\int_0^{\varepsilon_F}\varepsilon^{1/2}\,d\varepsilon}$$

$$= \frac{3}{5}\varepsilon_F$$

For N electrons, multiply by N

Q.107

(a) 107.87 kg of Ag shall have 6.02×10^{26} atoms

$\therefore$ 10,500 kg will have $\frac{6.02\times10^{26}}{107.87}\times 10{,}500 \approx 5.86\times10^{28}$ atoms

As 10,500 kg of A shall occupy $1\ m^3$ vol

$\therefore$ No of atoms in $1\ m^3$ vol $= 5.86\times10^{28}$ atoms

Assume each one of the Ag atom contribute one free electron

$\therefore\ n = 5.86 \times 10^{28}\ m^{-3}$

$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \approx 5.51\ eV$

$k T_F = 5.51 \times 1.6 \times 10^{-19}$

$\Rightarrow T_F = \frac{5.5 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} \simeq 63,884\ K$

(b) Radius of Fermi sphere k_F is to be given in k space

$\therefore\ \frac{\hbar^2 k_F^2}{2m} = \varepsilon_F \Rightarrow$

$k_F = (3\pi^2 n)^{1/3} \approx 1.20 \times 10^{10}\ m^{-1}$

(c) Fermi Velocity v_F is given as follows

$\frac{1}{2} m v_F^2 = \varepsilon_F$

$\Rightarrow v_F^2 = \frac{2 \times 5.51 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$

$\Rightarrow v_F \approx 1.39 \times 10^{6}\ m/s$

(d) $3/5\ \varepsilon_F \approx 3.31\ eV$

(e) $\frac{3}{2} kT = 3.31\ eV$

$\therefore T = \frac{2}{3} \frac{3.31 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} \approx 25,584$

(f) $\frac{1}{2} m v^2 = 3.31\ eV$

$v^2 = \frac{2 \times 3.31 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$

$\Rightarrow v \approx 1.08 \times 10^{6}\ m/s$

(g) At Room Temperature

$$\rho = \frac{m}{ne^2\tau} \quad \therefore \quad \tau = \frac{m}{ne^2\rho} = \frac{9.1\times10^{-31}}{5.86\times10^{28}\times e^2\times1.61\times10^{-8}}$$

$$\approx 3.77\times10^{-14}\ m$$

$$\therefore \ell = v_F \times \tau \approx 524\ A^0$$

At low temperature

$$\tau \approx 1.60\times10^{-11}\ s \Rightarrow \ell \approx 2.22\times10^{-5}\ m$$

To emphasize

(i) ℓ is very large in comparison of inter atomic distance

(ii) changes by 3 orders of magnitude between 20K and RT.

If cores are responsible for scattering this can not be understood

(h) $$v_D = \frac{E}{\rho n e} \qquad \left[\because |\vec{J}| = nev_D = \sigma E = \frac{E}{\rho}\right]$$

$$\therefore v_D = \frac{100}{1.61\times10^{-8}\times5.86\times10^{28}\times e}$$

$$\approx 0.66\ m/s$$

$$\therefore \frac{v_F}{v_D} = \frac{1.39\times10^6}{0.66} \approx 2.1\times10^6$$

To emphasize that drift velocity is many orders of magnitude small than even the average speed of electron.

Q108

The dispersion relation for free electrons is

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$

The periodic boundary conditions would quantize k_x and k_y as follows

$$k_x = \frac{2n_x\pi}{a}, \quad k_y = \frac{2n_y\pi}{a}$$

Here n_x and n_y are integers (both +ve and -ve)

$$g(k)\,dk = 2 \times \left(\frac{a}{2\pi}\right)^2 2\pi k\,dk$$

The factor '2' is for spin degeneracy. Note in 2-d constant energy surface is circle.

$$k = \left(\frac{2m}{\hbar^2}\right)^{1/2} \varepsilon^{1/2}$$

$$\therefore \quad dk = \left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{1}{2} \varepsilon^{-1/2}$$

$$\Rightarrow \quad g(\varepsilon)\,d\varepsilon = \frac{A}{2\pi}\left(\frac{2m}{\hbar^2}\right) d\varepsilon$$

Here A is the area