PH 105 – Quantum Mechanics

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41)

(a) $\Delta p = [<p^2>]^{1/2}$

$\Delta p.\ \Delta x \geq \hbar$

$<p^2> = [\hbar/ \Delta x]^2$

For $\Delta x = 10^{-14}$

$<p^2> = 1.11 \times 10^{-40}$

$E^2 = <p^2>c^2 + m^2 c^4$

For an electron

$E = 20$ MeV

KE = E - mc² = 19.5 MeV

For a proton E= 950 MeV and **KE = 10 MeV**

The Binding Energy required to confine the electron within the nucleus would be too high.

(b) $\Delta p.\ \Delta x \geq \hbar$

$<p^2> = [\hbar/ \Delta x]^2$ with $\Delta x = L$

$E^2 = <p^2>c^2 + m^2c^4 = \hbar^2/ L^2 c^2 + m^2 c^4$

$\mathbf{KE = E - mc^2 = [\hbar^2/ L^2 c^2 + m^2 c^4]^{1/2} - mc^2}$

(c) The Bohr Model violates the uncertainty principle , since the angular momentum quantization assumes that we simultaneously know the momentum and the position with complete certainty .

$L = rp= n\, \hbar$

This is in clear violation of the uncertainty principle.

Overlooking this aspect and simply taking into account the Coulombic interaction,

$KE = <p^2>/2m = \hbar^2/2mr^2$

$E = \hbar^2/2mr^2 - Ze^2/4\pi\epsilon_0 r$

Differentiate and set the derivative = 0.

$- \hbar^2/mr^3 + Ze^2 /4\pi\epsilon_0 r^2 = 0$

$R_{min} = 4\pi\epsilon_0\, \hbar^2/m\, Ze^2 = 0.53$ Å

This matches the actual value observed. This validates the uncertainty principle but not the Bohr model as explained before.