PH 105 Tutorial Solution

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18)

$$\pi + \rho \rightarrow \Delta$$

ρ is at rest

$\underrightarrow{p_\pi} = (p_\pi , iE_\pi/c)$ $\underrightarrow{p_\rho} = (0, iE_\rho/c)$ $\underrightarrow{p_\Delta} = (p_\Delta , iE_\Delta/c)$

Applying conservation of momentum 4 vectors,

$\underrightarrow{p_\pi} + \underrightarrow{p_\rho} = \underrightarrow{p_\Delta}$ --(1)

i.e.

$(p_\pi , i(E_\pi+E_\rho)/c) = (p_\Delta , iE_\Delta/c)$

$(p_\pi , i(E_\pi+E_\rho)/c)^2 = (p_\Delta , iE_\Delta/c)^2$

$p_\pi^2 - (E_\pi+E_\rho)^2/c^2 = p_\Delta^2 - E_\Delta^2/c^2$ --(2)

We are free to compute the momentum 4 vector in any frame. In order to simply calculations, compute the RHS in the rest frame of the Δ particle.

In this frame $p_\Delta = 0$ and $E_\Delta = m_\Delta c^2$ (rest mass energy)

Simplify (2) to get

$-m_\pi^2c^4 - m_\rho^2c^4 - 2E_\pi m_\rho c^2 = -m_\Delta^2c^4$

$E_\pi = (m_\Delta^2c^4 - m_\pi^2c^4 - m_\rho^2c^4)/2 m_\rho c^2$

On solving

$\underline{E_\pi = 0.34\ GeV}$

$E_\pi^2 = p_\pi^2c^2 + m_\pi^2c^4$

Solve to get

$\underline{p_\pi = 0.31\ (GeV/c)}$

ii) Speed of Δ particle in lab frame.

$\underrightarrow{p_\pi} + \underrightarrow{p_\rho} = \underrightarrow{p_\Delta}$

$(p_\pi , i(E_\pi+E_\rho)/c) = (p_\Delta , iE_\Delta/c)$

This time solve RHS in Lab frame. Simply, equate the individual components.

$p_\pi = p_\Delta = 0.31$ GeV/c

$i(E_\pi+E_\rho)/c = iE_\Delta/c$

$0.34+0.94 =$ ***$1.28 = E_\Delta$***

Therefore, $\gamma_\Delta = E_\Delta / m_\Delta c^2 = 1.28/1.24 = 1.032$

$V_\Delta = 0.25c$

iii) Energy and momentum of pions in the frame in which the Δ particle is at rest.

The Δ particle is at rest in a frame moving at v = 0.25c (frame attached to the particle)
Apply the transformation equation to get

$p_\pi' = \gamma(p_\pi + i(v/c)*i\, E_\pi/c)$

$p_\pi' = \gamma(p_\pi - E_\pi v/c^2)$

$\gamma = 1.067$

$p_\pi' = 0.24$ (GeV/c)

$iE_\pi'/c = \gamma(i\, E_\pi/c - i(v/c)*\, p_\pi)$

$E_\pi' = \gamma(E_\pi - p_\pi v)$

$E_\pi' = 0.28$ GeV