ME119: Engineering Drawing & Graphics

1. Engineering Curves I - Conics

Department of Mechanical Engineering
Indian Institute of Technology Bombay

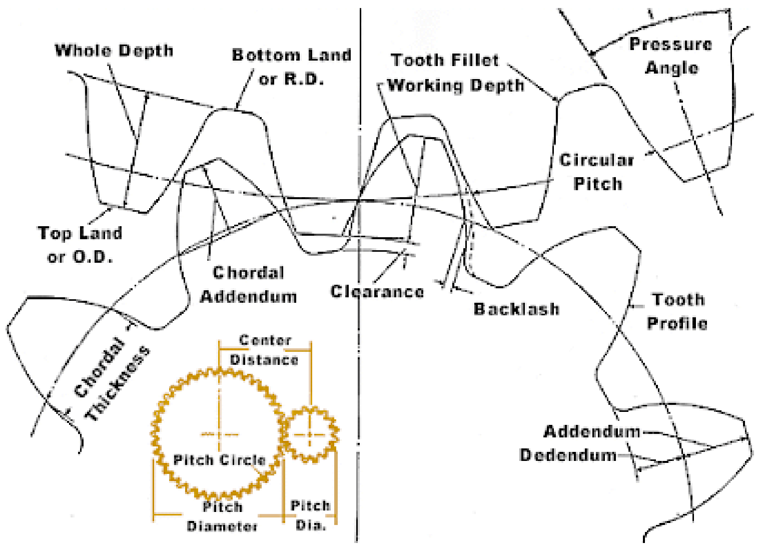
Engineering Curves

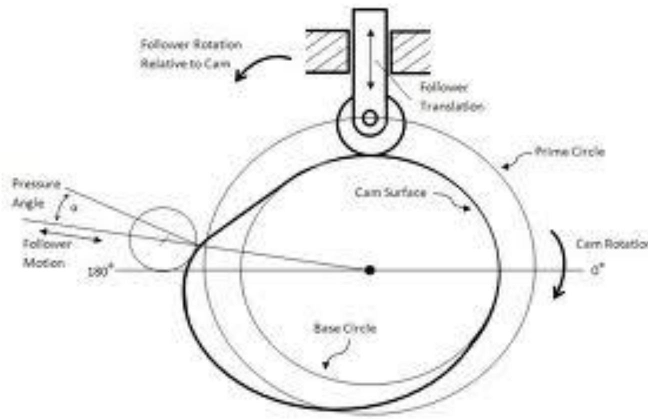
- Chapters 6 & 7 cover the details on Engineering Curves.
- Roughly work out all the problems given to you before you come to the drawing hall.

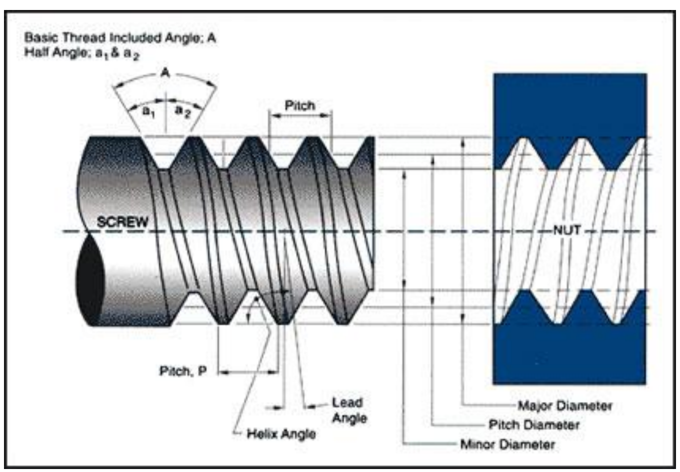
Types of Engineering Curves

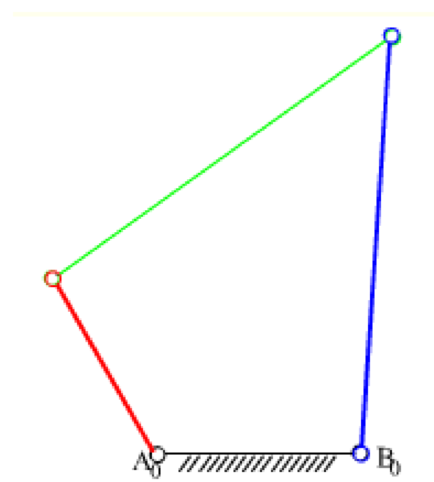
Conics	Line: … Circle: … Ellipse: Arches, bridges, dams, monuments, manholes, glands, stuffing boxes, Hawkins pressure cooker … Parabola: Arches, bridges, sound & light reflectors, … Hyperbola: Cooling towers, water channels, moda, horse seat, …
Cycloids & Trochoids	Trajectories, Gear profiles, …
Involutes & Evolutes	Gear profiles, …
Spirals	Springs, helical gear profile, cam profile …
Helices	3D curve. DNA, helicopter, springs, screws, tools, gears …
Freeform curves	Templates (Eg.: French curves), Mathematical: Ext. features
Cams	Mechanisms
Loci	Mechanisms

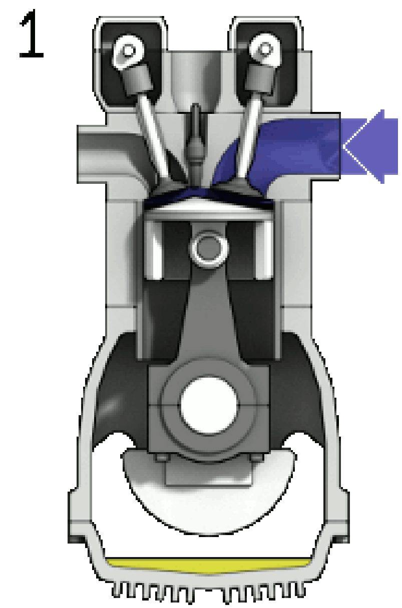
Applications of Engineering Curves





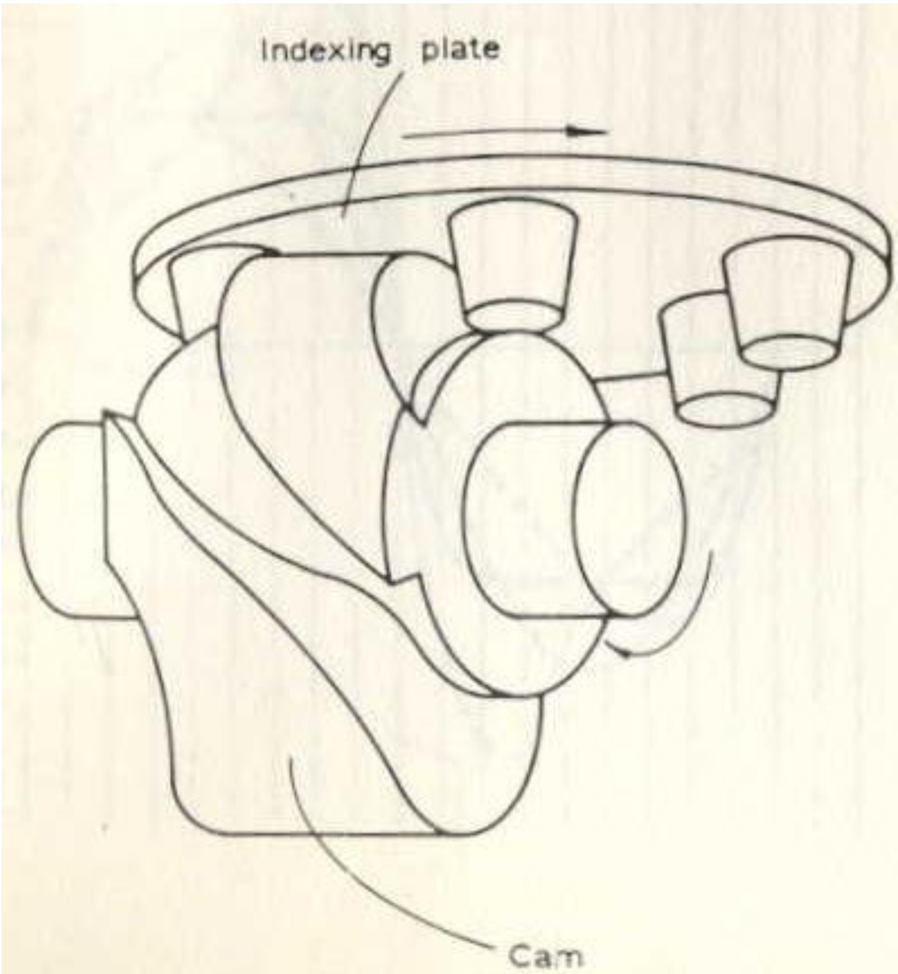


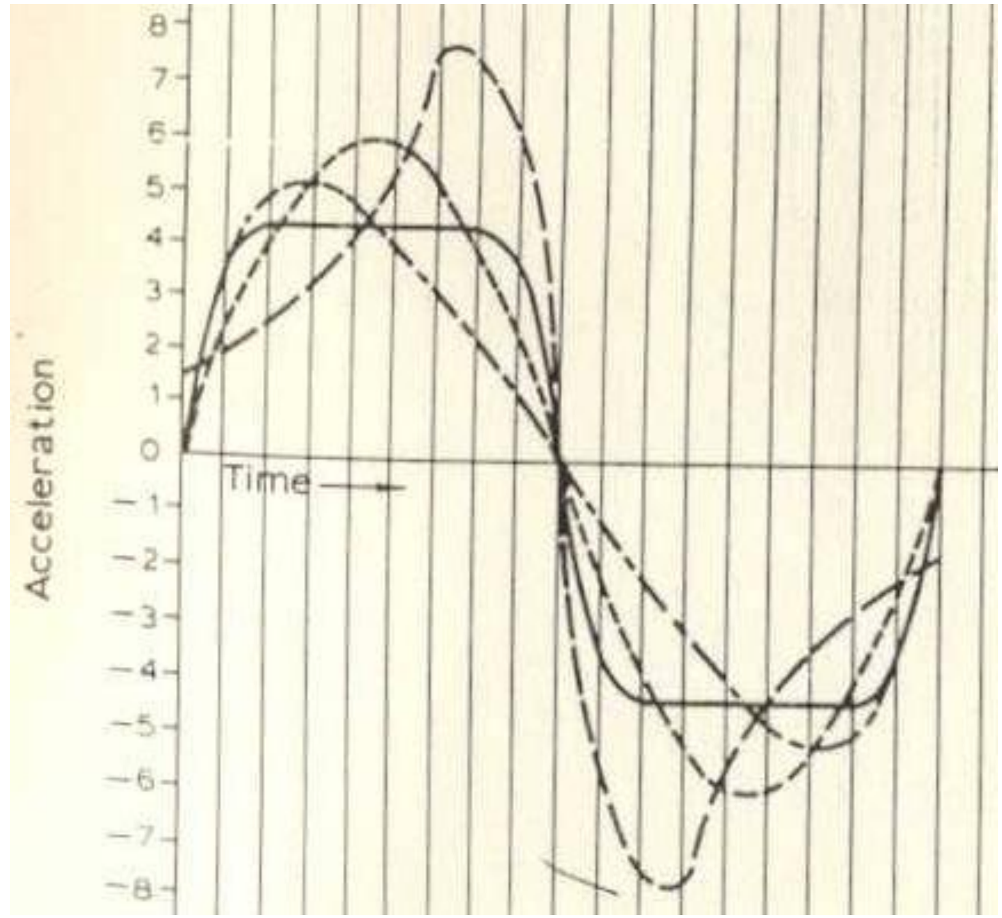




Applications of Engineering Curves …

3D cam





Engineering Curves

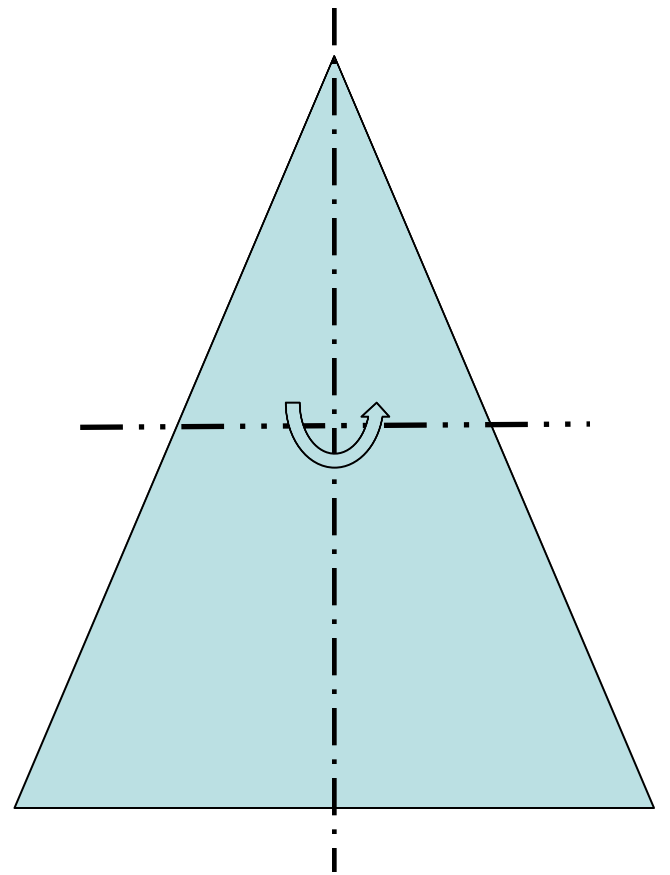
- Conic sections will be taught in this class and the rest of the curves in the next.

Conic Sections

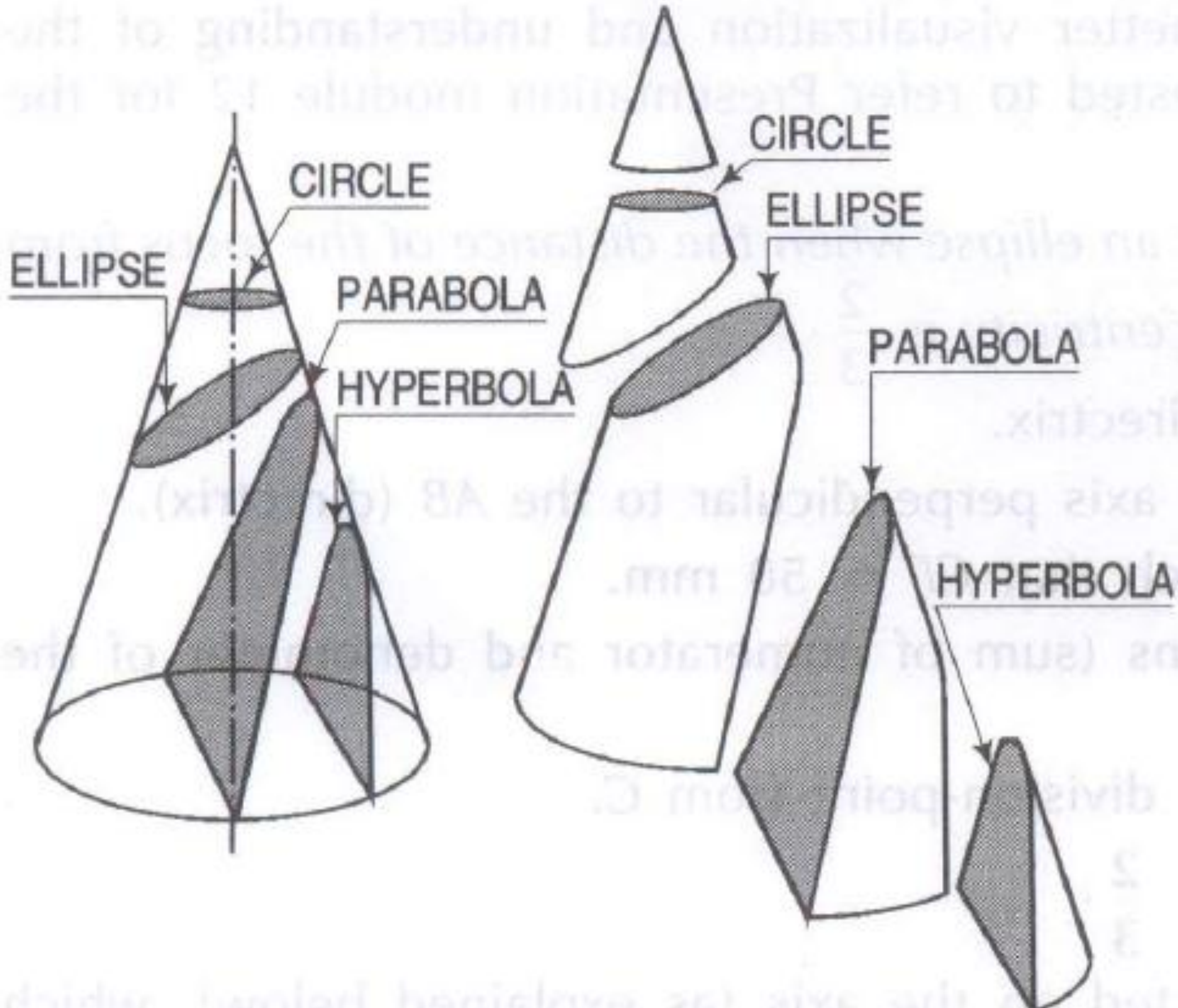
Conic Sections

Intersection of a (right circular) cone with a plane in various orientations.

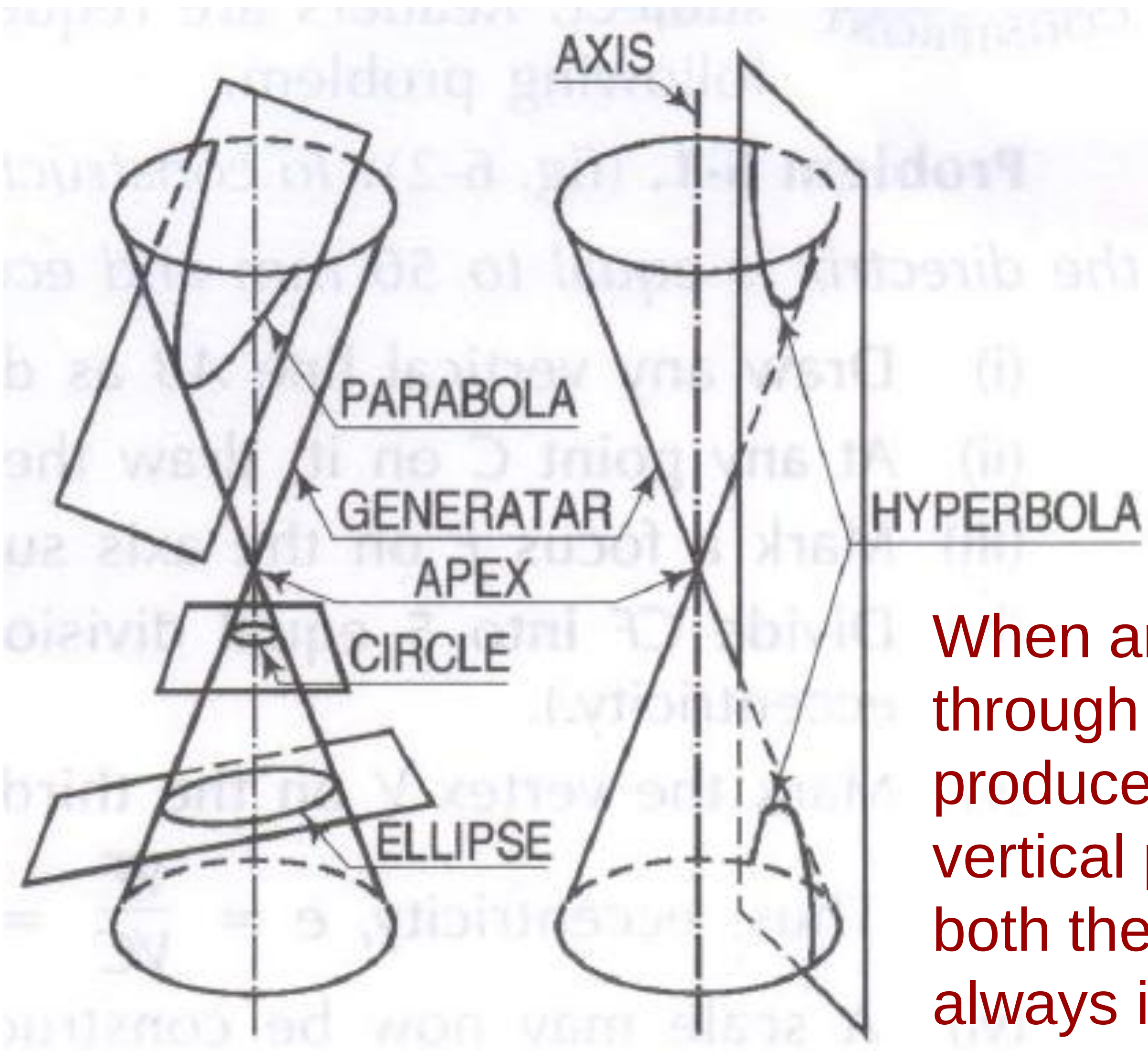
- Circle
- Ellipse
- Parabola
- Hyperbola
- Line (Cone is a ruled surface; line is extreme cases of both circle and hyperbola)



Conic Sections …



Conic Sections …



When an infinite rule passing through an apex is revolved, it produces 2 cones. But only a vertical plane (and beyond) cuts both the cones; hence hyperbola is always in pairs.

Conic Sections

Algebraic definition

You are familiar with conics as 2nd degree algebraic (quadratic) equations.

conic section	equation	eccentricity (e)	linear eccentricity (c)	semi-latus rectum (ℓ)	focal parameter (p)
circle	$x^2+y^2=a^2$	0	0	a	∞
ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\sqrt{1-\frac{b^2}{a^2}}$	$\sqrt{a^2-b^2}$	$\frac{b^2}{a}$	$\frac{b^2}{\sqrt{a^2-b^2}}$
parabola	$y^2=4ax$	1	a	$2a$	$2a$
hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\sqrt{1+\frac{b^2}{a^2}}$	$\sqrt{a^2+b^2}$	$\frac{b^2}{a}$	$\frac{b^2}{\sqrt{a^2+b^2}}$

Circle: $(a\cos\theta, a\sin\theta)$,

Ellipse: $(a\cos\theta, b\sin\theta)$,

Parabola: $(at^2, 2at)$,

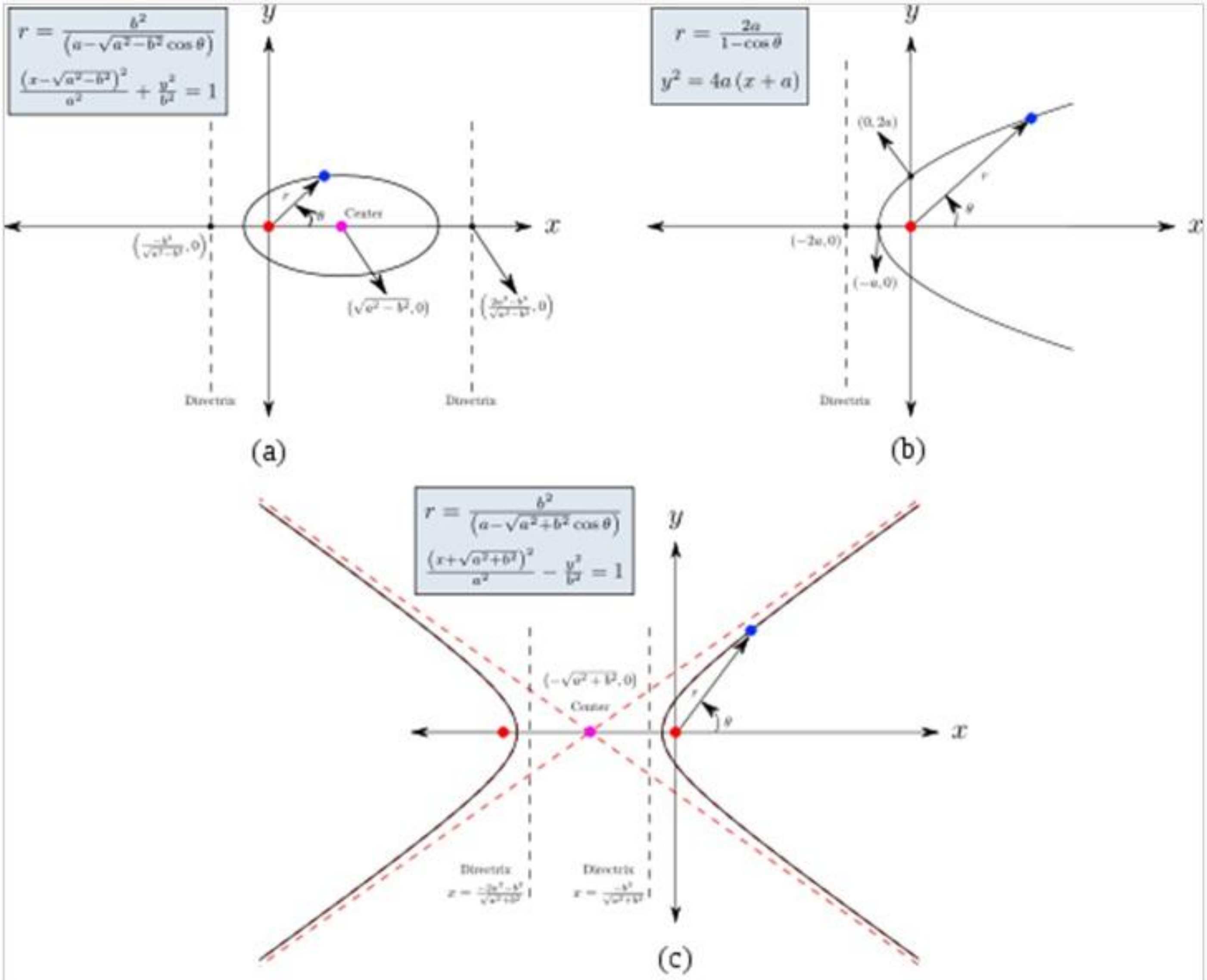
Hyperbola: $(a\sec\theta, b\tan\theta)$ or $(\pm a\cosh u, b\sinh u)$.

Generic definition in polar form:

$$r=\frac{l}{1-e\cos\theta}$$

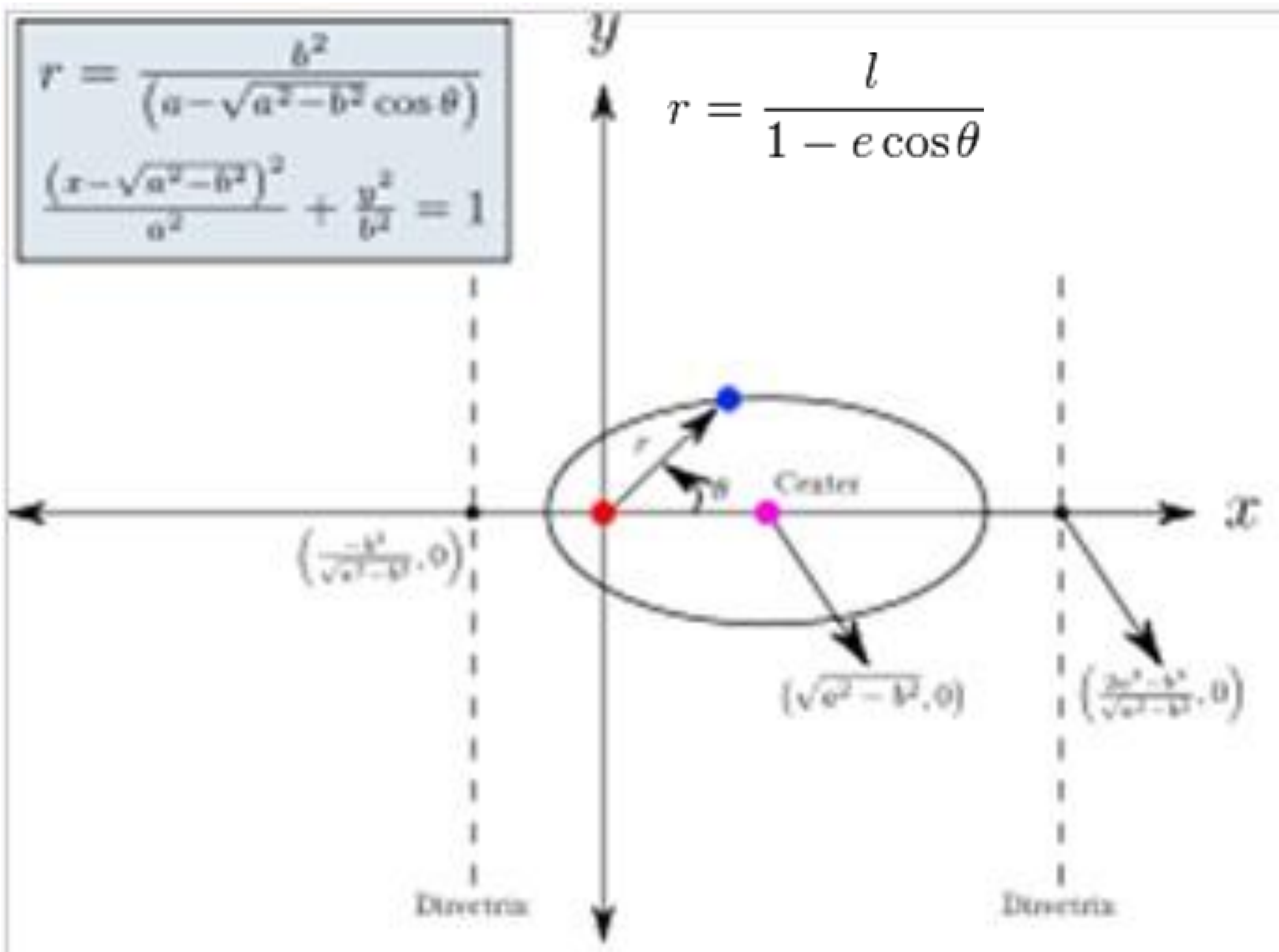
Conic Sections

Algebraic definition …



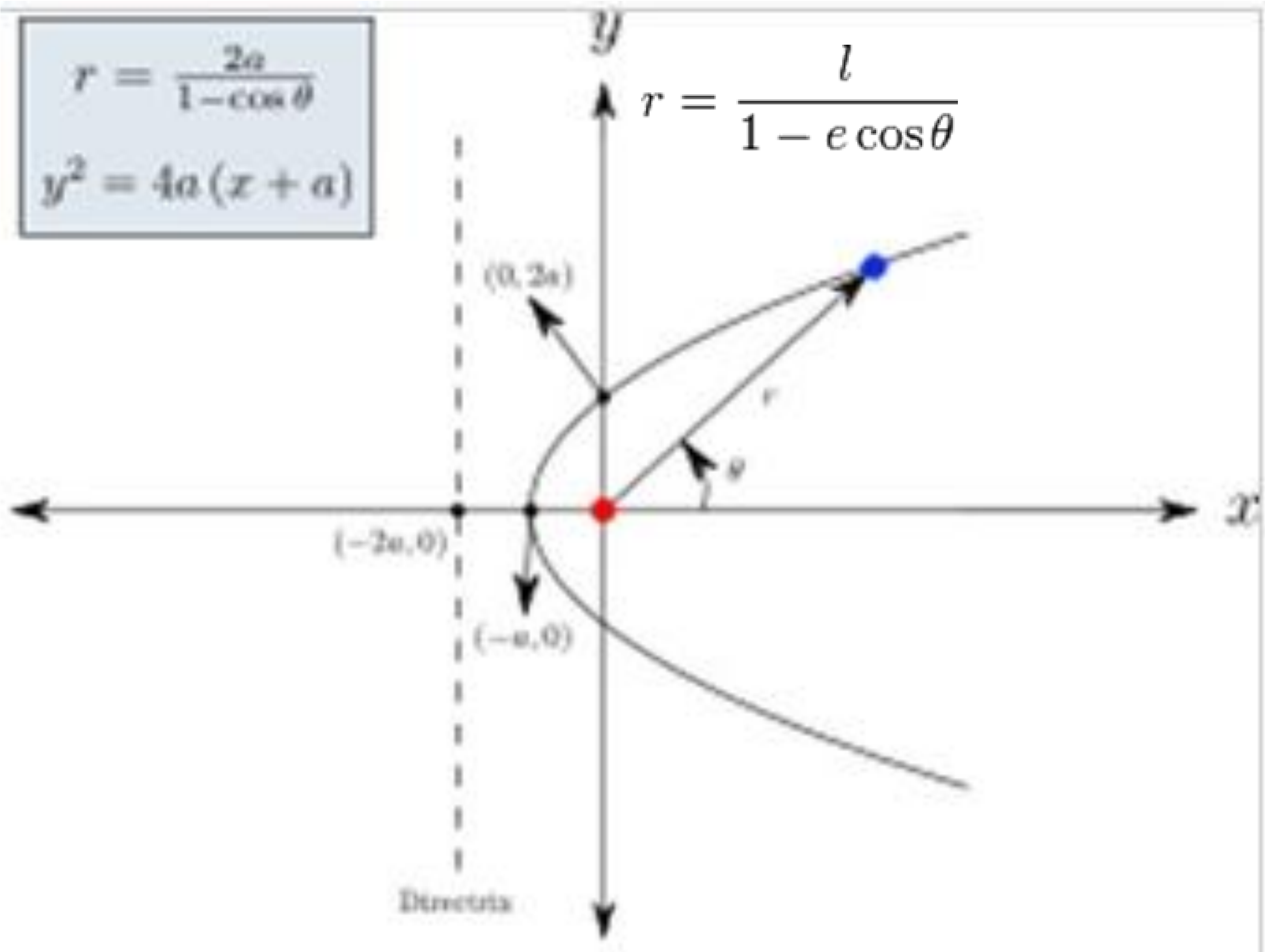
Conic Sections

Algebraic definition - Ellipse



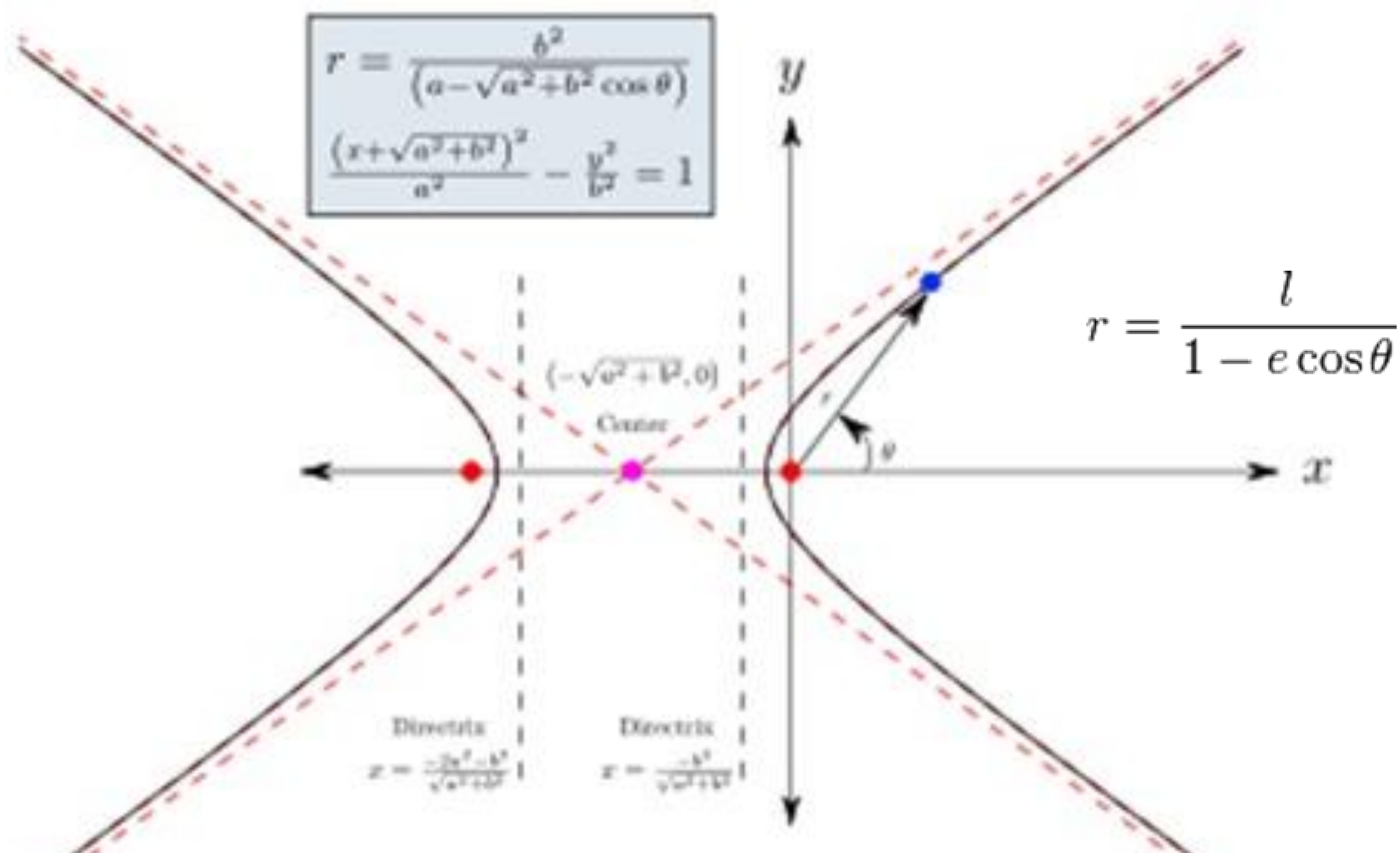
Conic Sections

Algebraic definition - Parabola



Conic Sections

Algebraic definition - Hyperbola



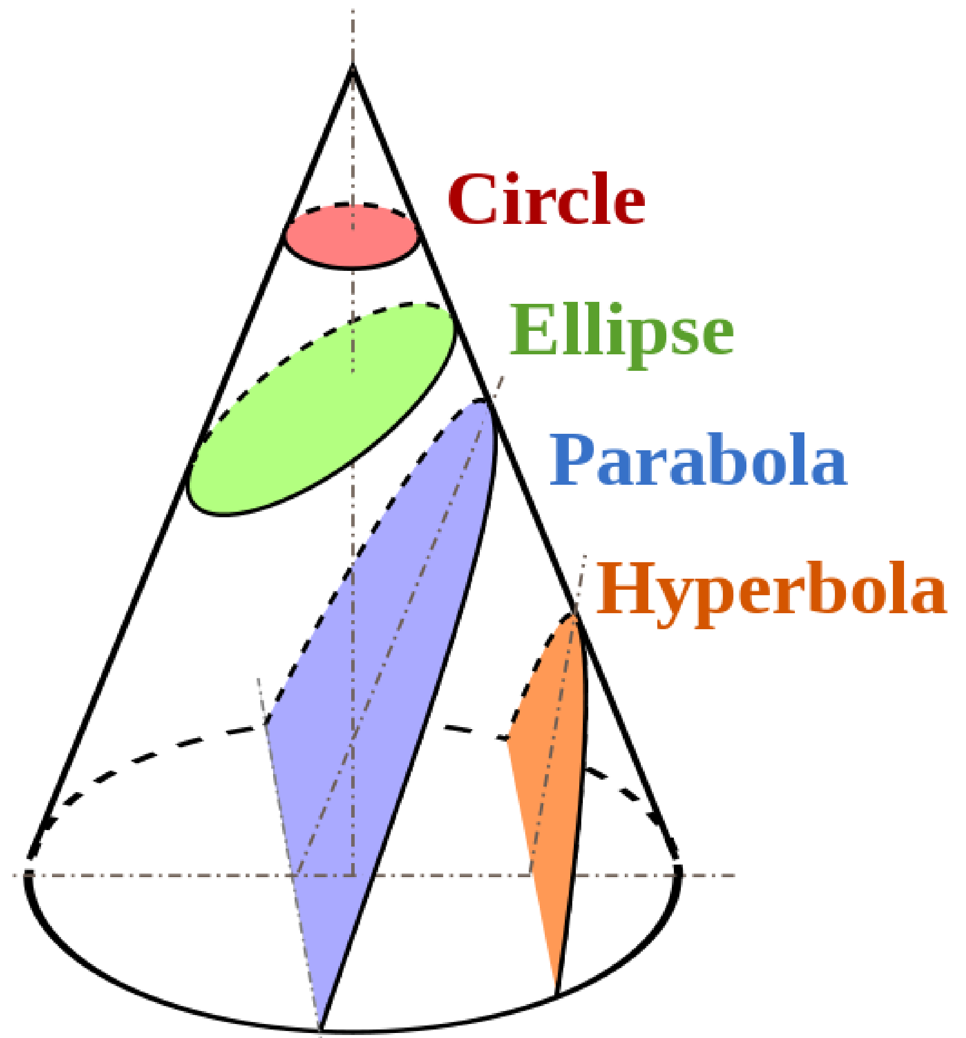
(c)

Conic Sections

Geometric/ graphical definition

A conic section (or just conic) is the intersection curve between a conical surface (more precisely, a right circular conical surface) with a plane.

Straight line is a conic. It is a hyperbola when the plane contains the cone axis. Alternately, it is a circle of infinite radius.

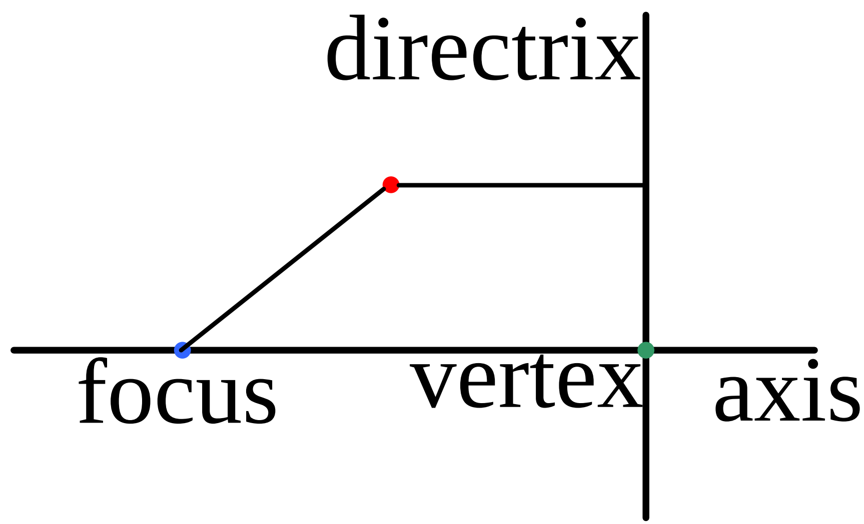


Conic Sections

Geometric/ graphical definition

A conic is the locus of a point whose distance to a fixed point, called focus (point F), and a fixed line, called directrix (line AB), are in a fixed ratio. This ratio is called eccentricity (e). The distance between the focus and directrix is called focal parameter (p).

- Circle (e=0)
- Ellipse (e<1)
- Parabola (e=1)
- Hyperbola (e>1)
- Line (e=0 or α?)



Conic Sections: Ellipse

Conic Sections: Ellipse

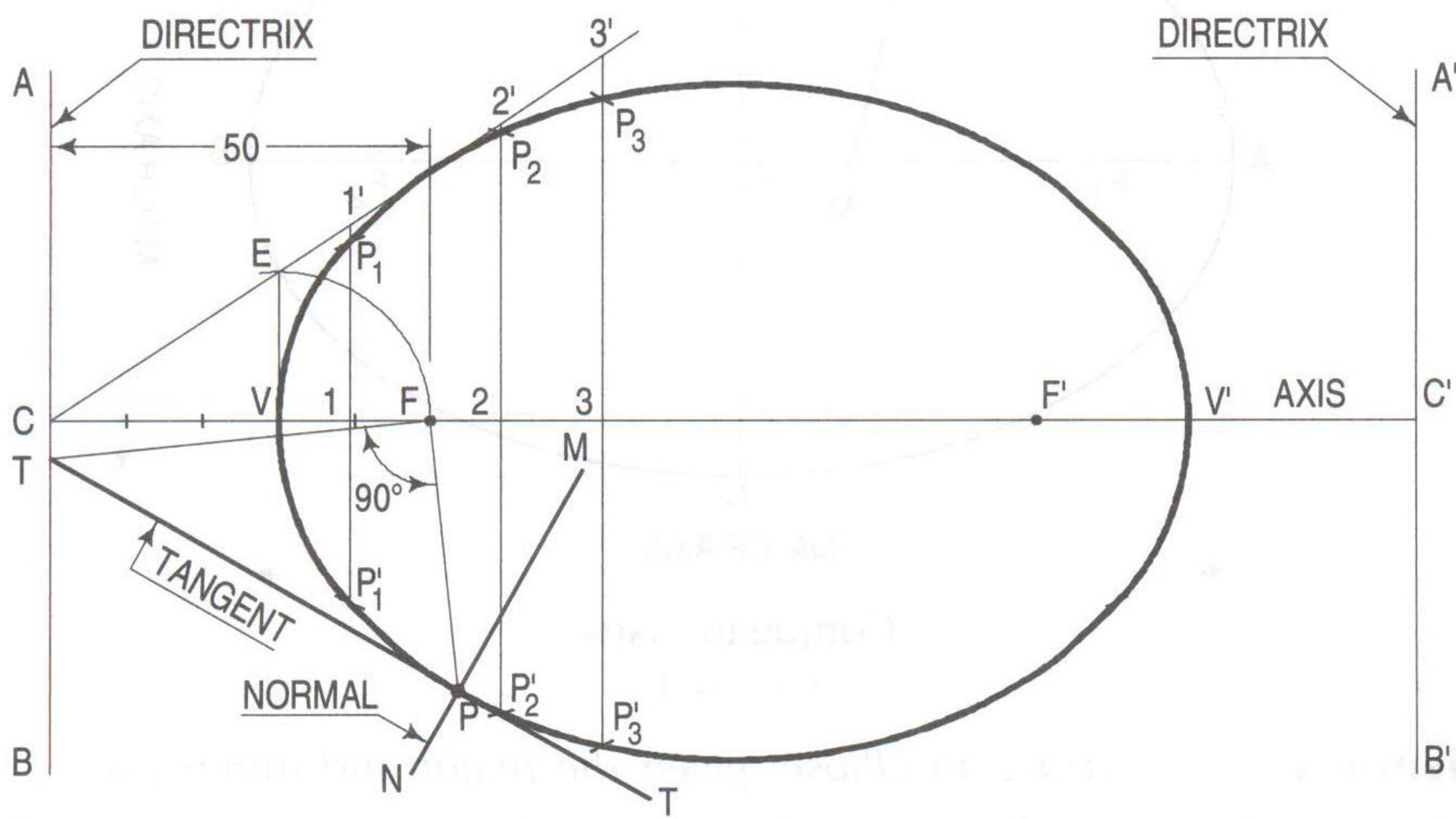
Methods of drawing

1. Basic method: From (i) distance of focus from the directrix (p) and (ii) eccentricity (e)
2. From bounding rectangle or parallelogram, i.e., (i) major axis (2a) and (ii) minor axis (2b):
   a. Arcs of circles method
   b. Loop of the thread method (construction using a thread and pin)
   c. Concentric circles method
   d. Oblong method (suitable for superscribing parallelogram also)
   e. Triangle method (suitable for superscribing parallelogram also)
   f. Trammel method (construction using two strips)
3. Four arc approximate method for a superscribing rhombus (useful in isometric views)

Conic Sections: Ellipse

Basic methods (from focus *p* & eccentricity *e*)

Construct an ellipse when the distance of the focus from the directrix is equal to 50mm and its eccentricity is 2/3.

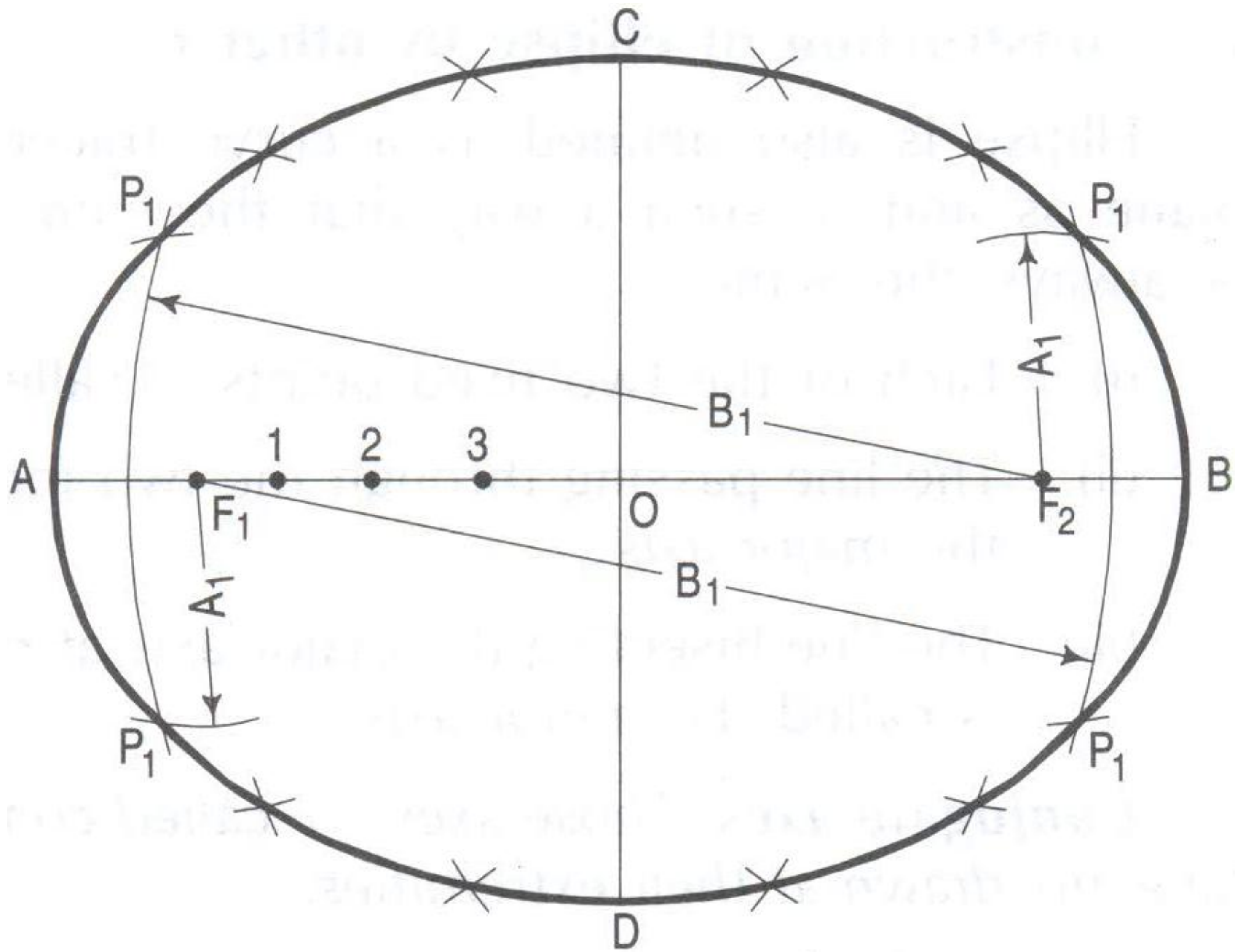


F’, V’ and RH directrix are not required. They require additional calculations.

Conic Sections: Ellipse

Arcs of circles method (from major & minor axes)

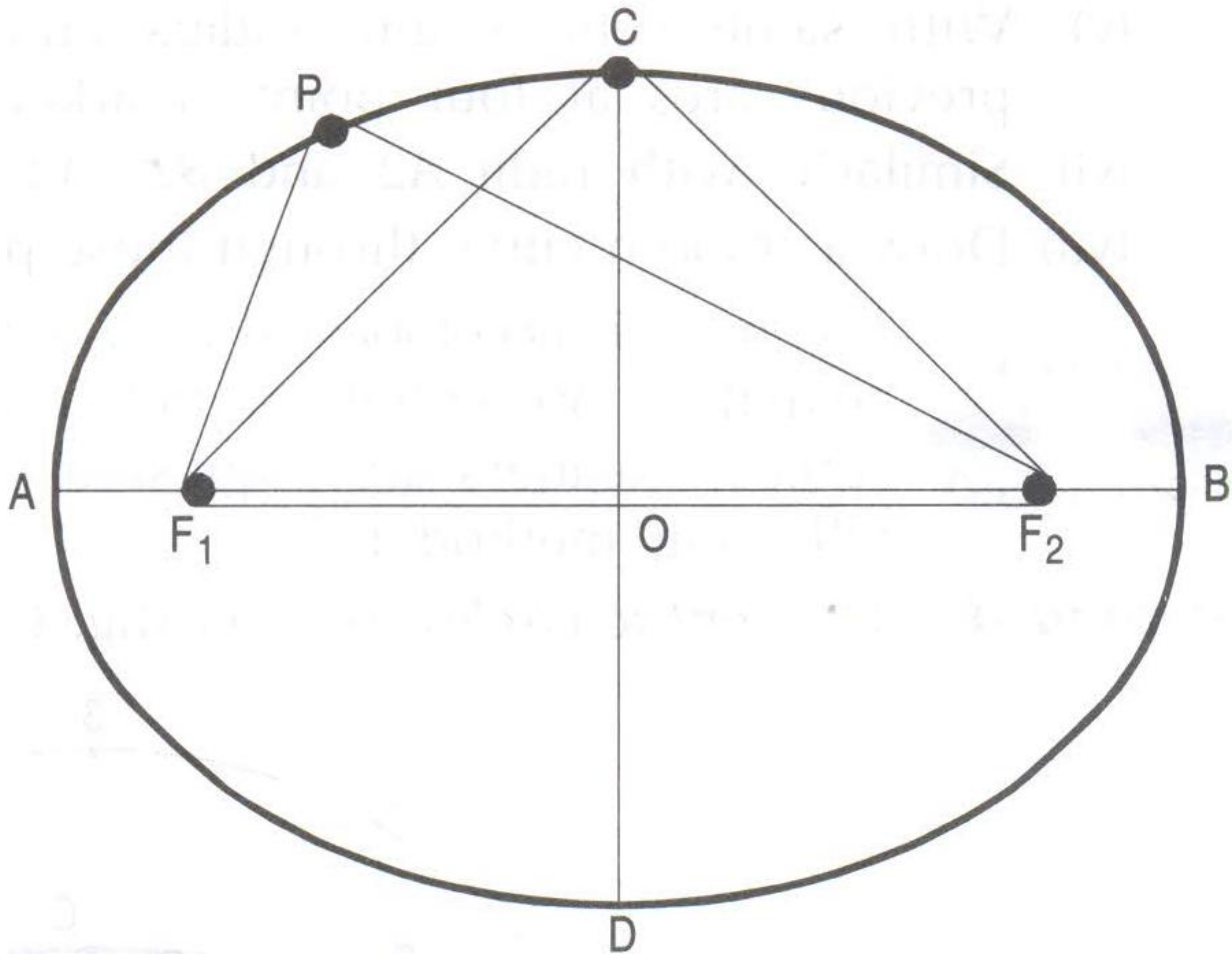
Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using arcs of circles method.



Conic Sections: Ellipse

Loop of the thread method (from major & minor axes)

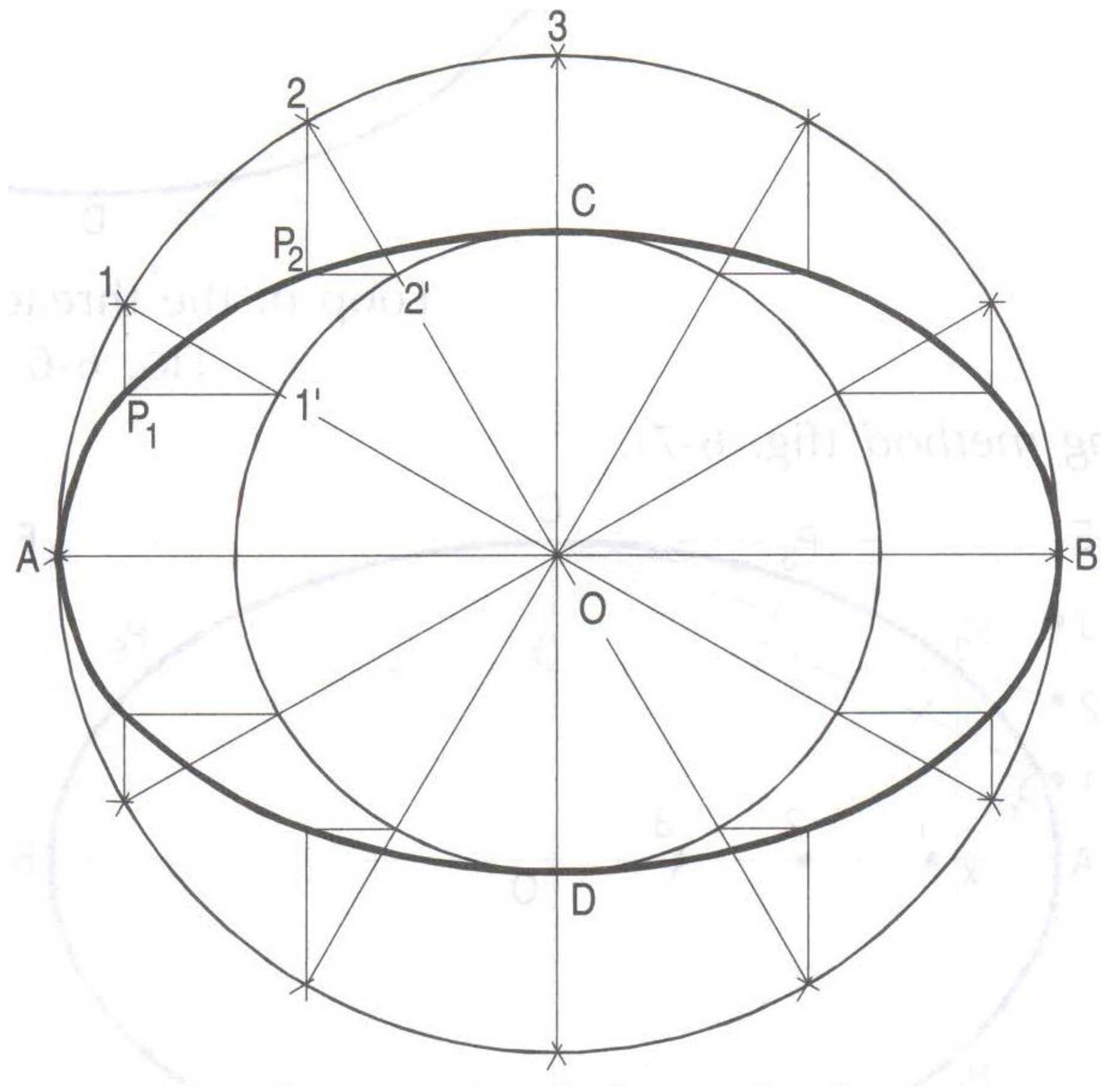
Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using loop of the thread method.



Conic Sections: Ellipse

Concentric circles method (from major & minor axes)

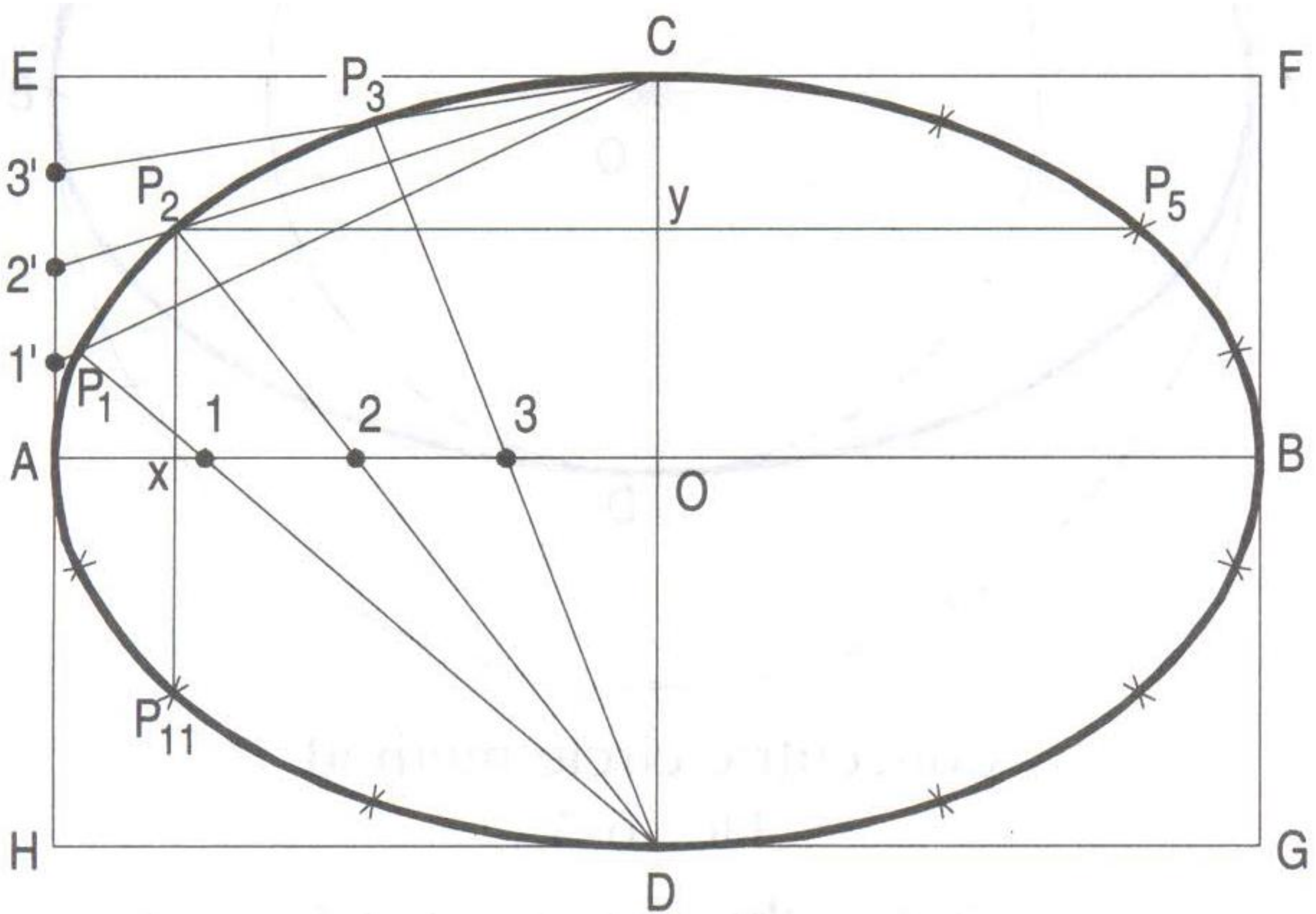
Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using concentric of circles method.



Conic Sections: Ellipse

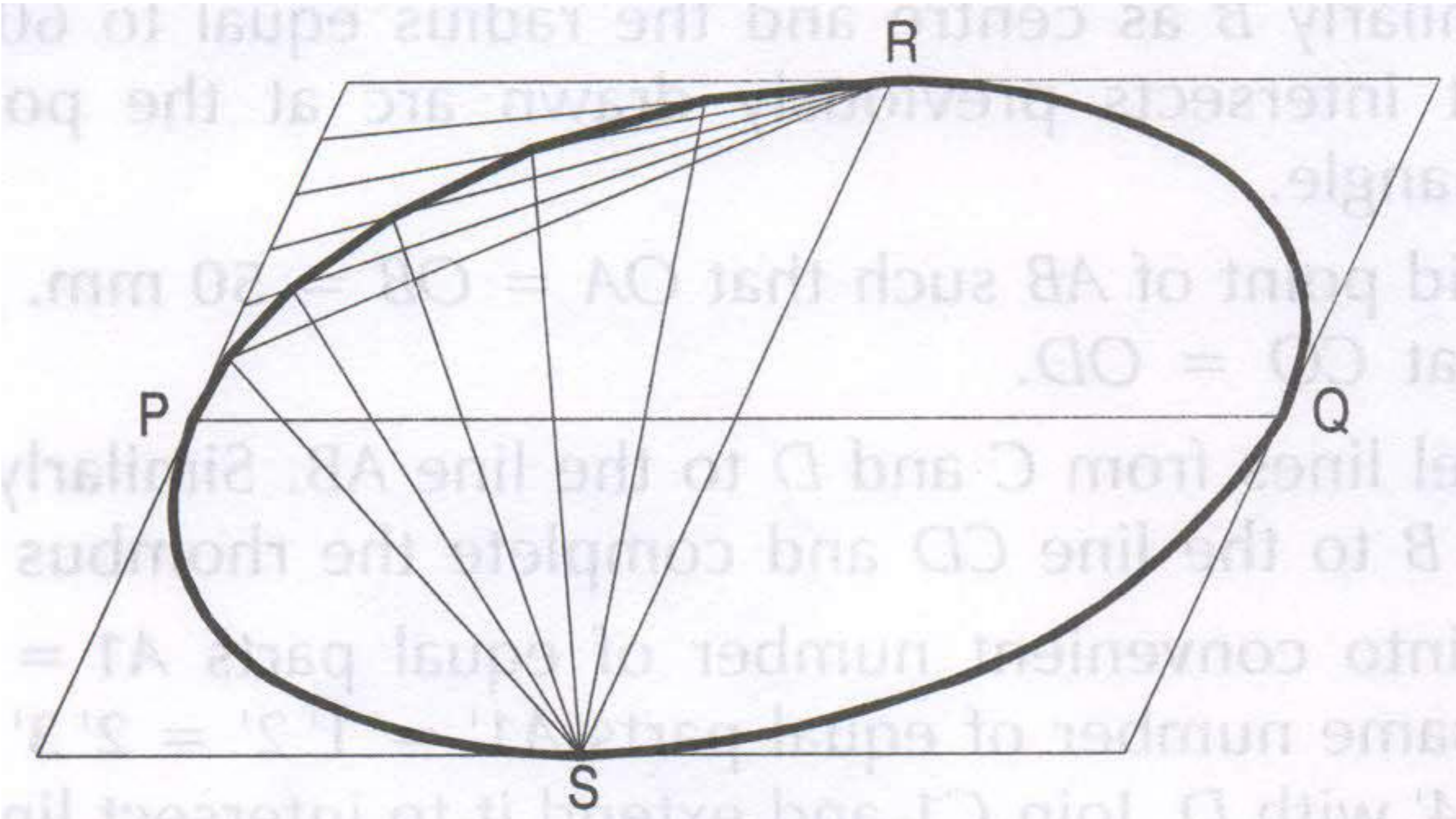
Oblong method (from major & minor axes)

Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using oblong method.



Conic Sections: Ellipse

Oblong method (from major & minor axes) …



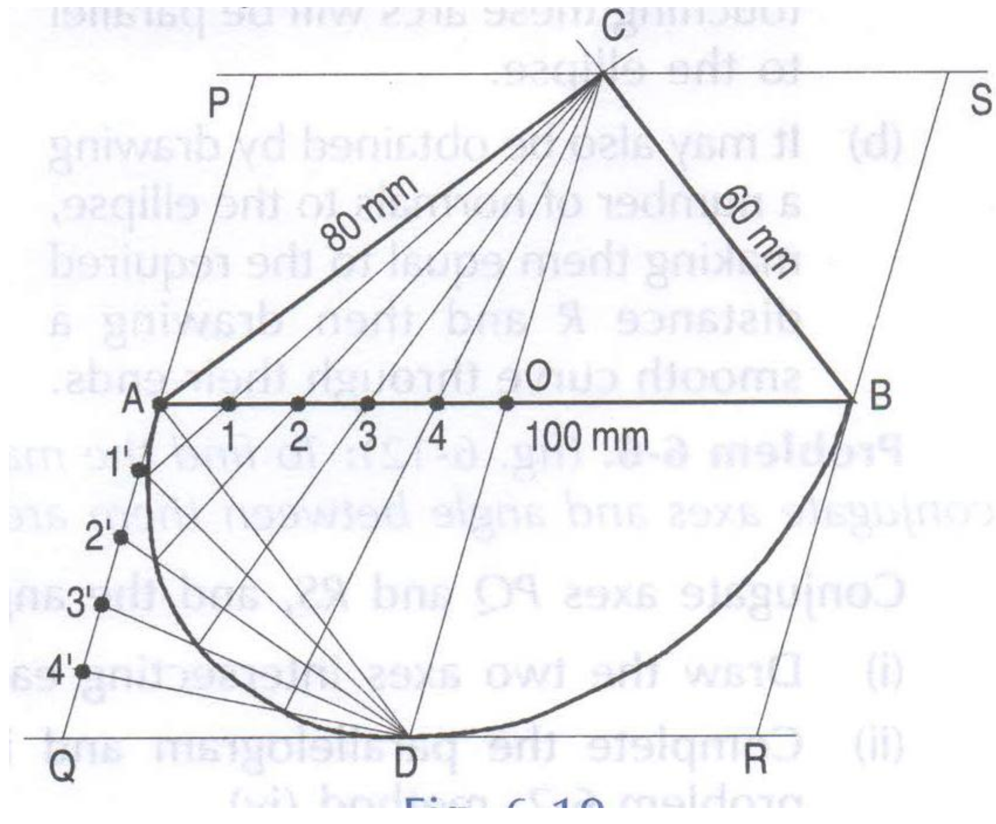
Inscribed in a parallelogram instead of a rectangle.

Conic Sections: Ellipse

Triangle method (from major & minor axes)

Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using trinagle method.

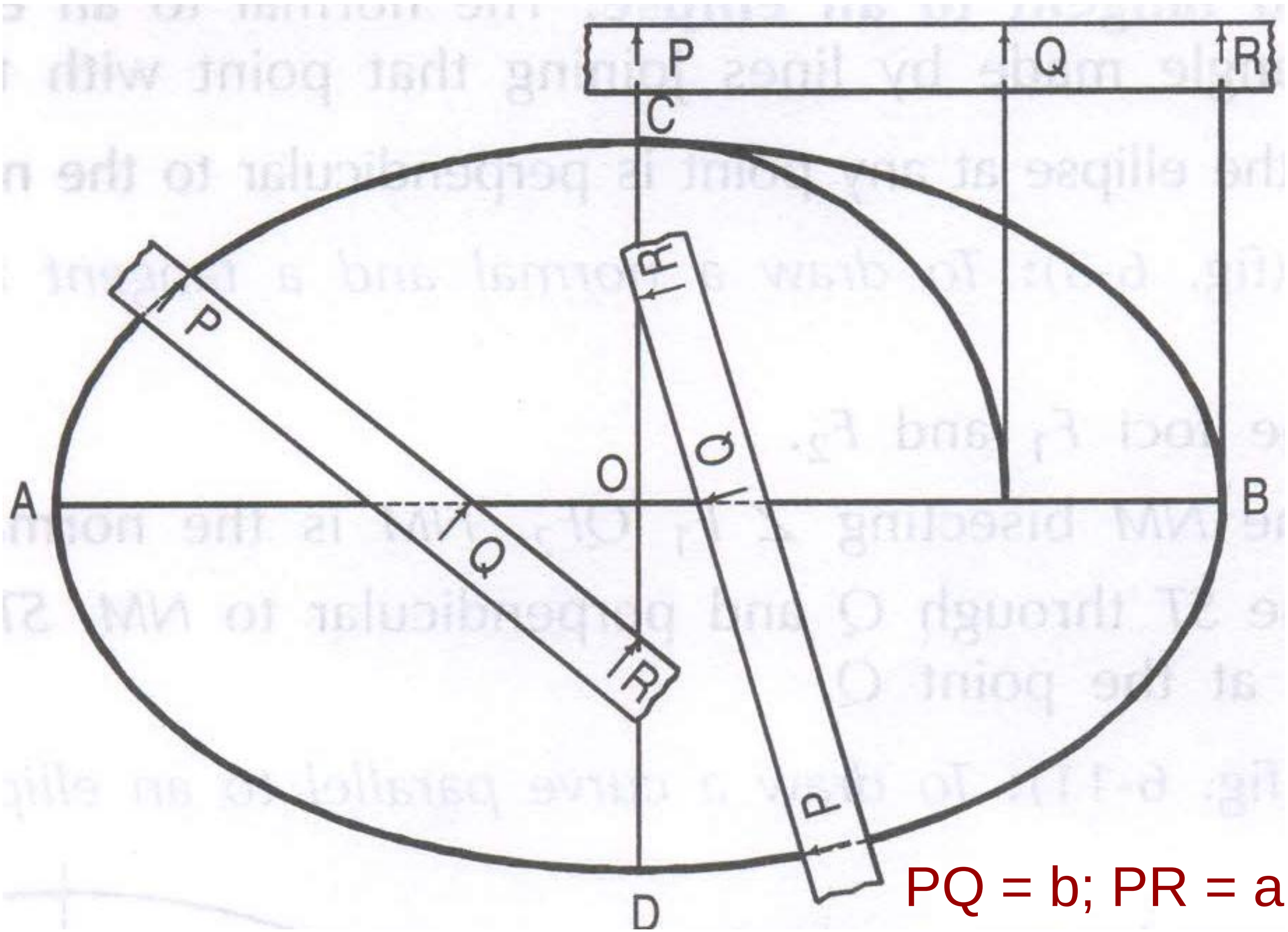
Obtain rectangle or parallelogram from the triangle. Then, it is same and oblong method.



Conic Sections: Ellipse

Trammel method (from major & minor axes)

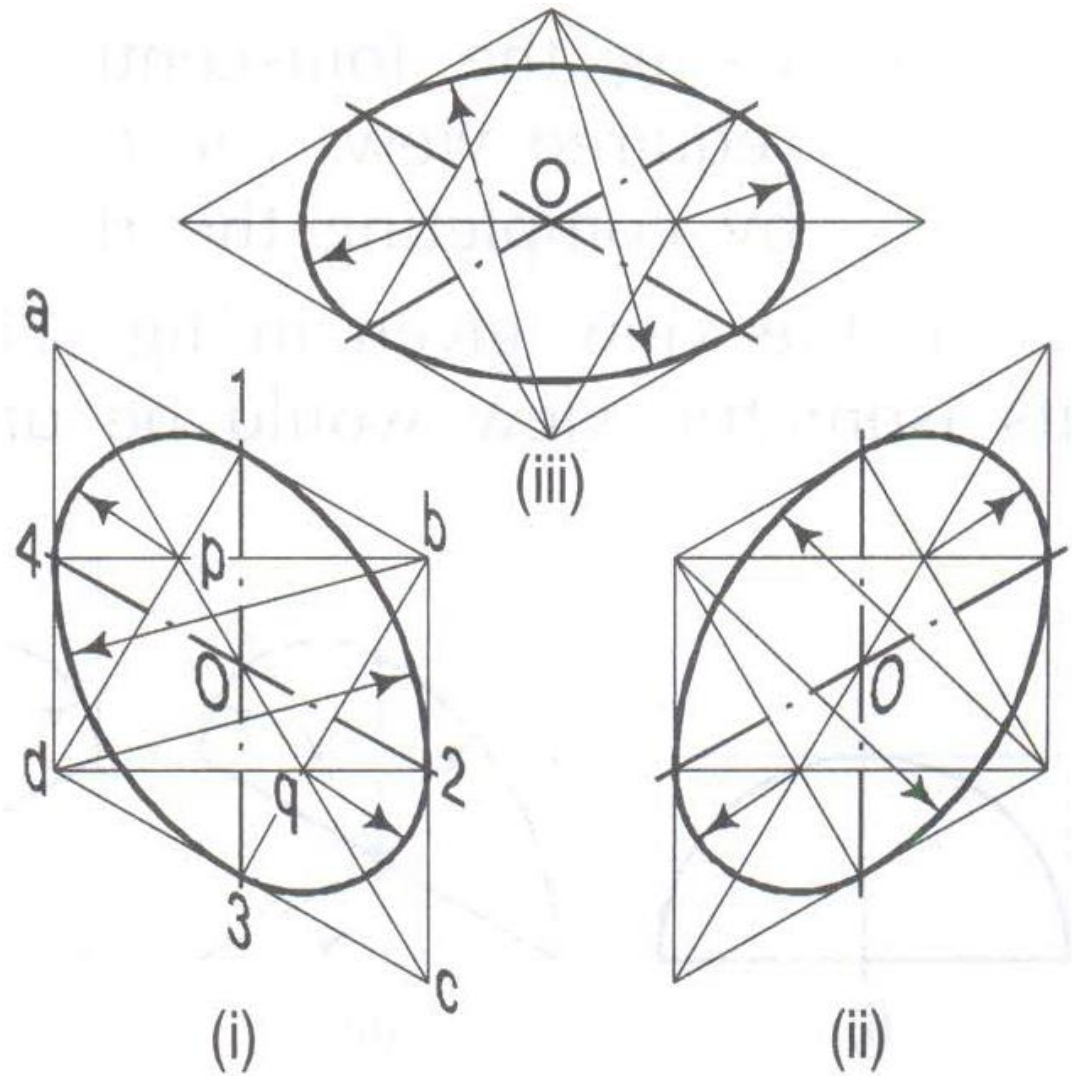
Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using trammel method.

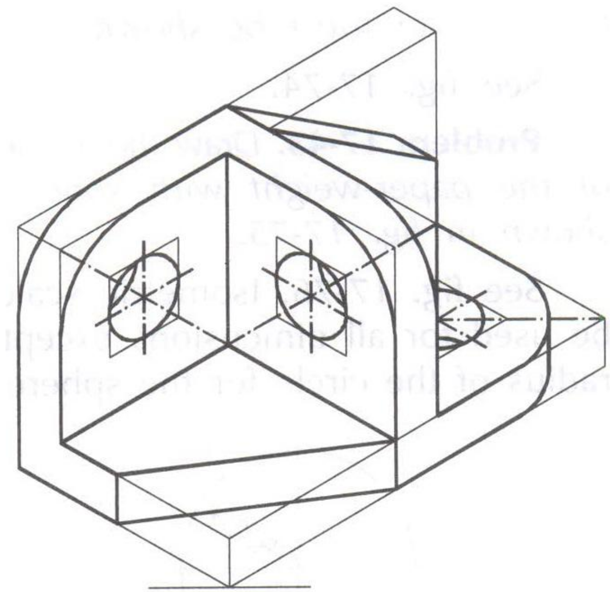


PQ = b; PR = a

Conic Sections: Ellipse

Approximate method for a superscribing rhombus





Conic Sections: Parabola

Conic Sections: Parabola

Methods of drawing

1. Basic method: From (i) distance of focus from the directrix (p) and (ii) eccentricity (e)
2. 2. From (i) base and (ii) axis
   a. Rectangle method
   b. Tangent (triangle) method

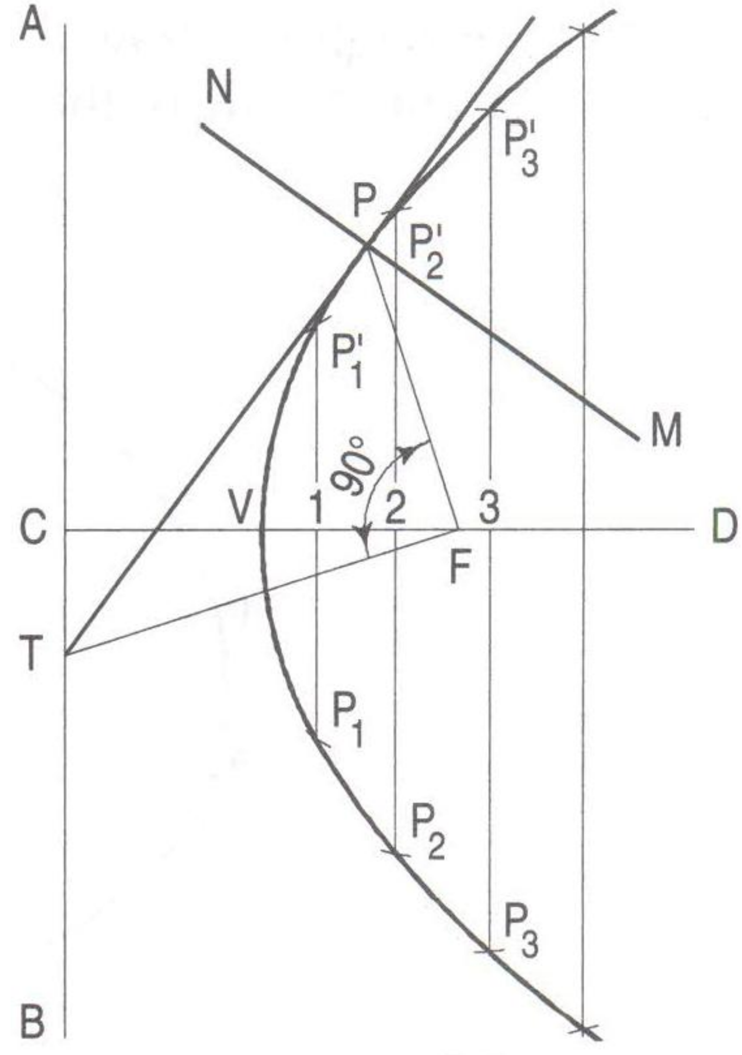
Note: The above can be used for non-rectangular parabolas too.

Conic Sections: Parabola

Basic method (from focus *f*)

Construct a parabola when the distance of the focus from the directrix is equal to 50mm.

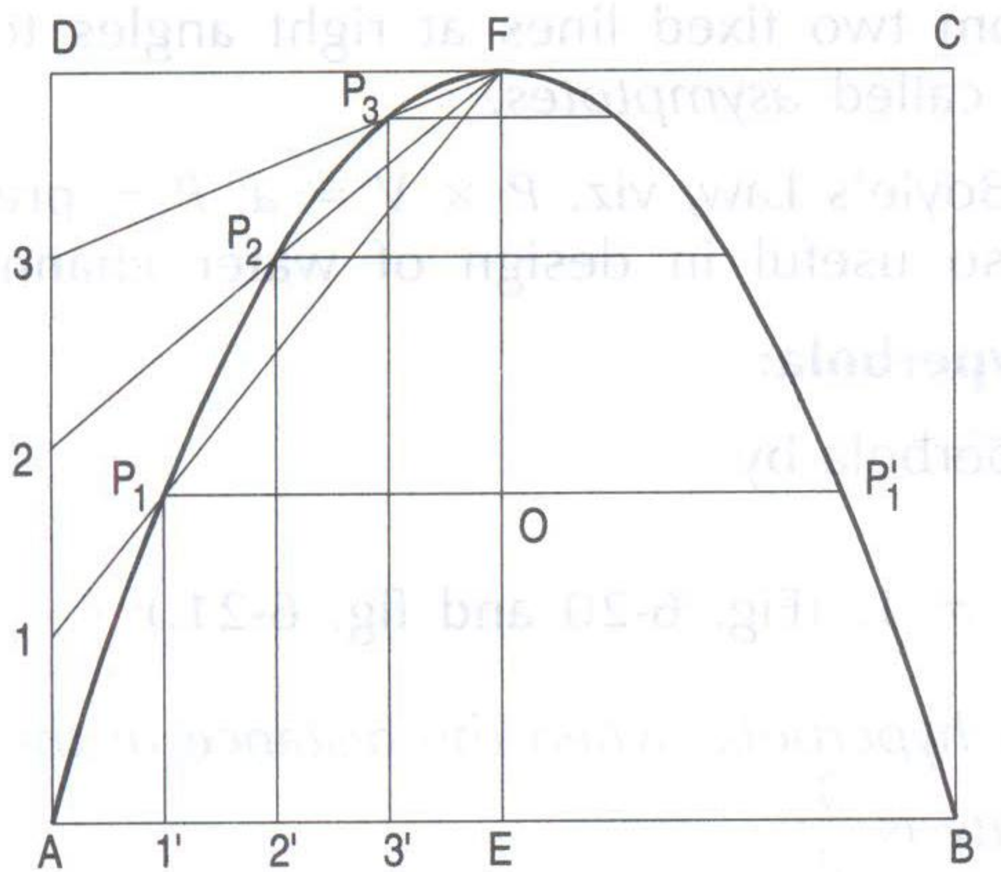
Same as ellipse but simpler as e=1. So, no need for the slant line as the horizontal distance of the axis point from the directrix can be directly used as radius.

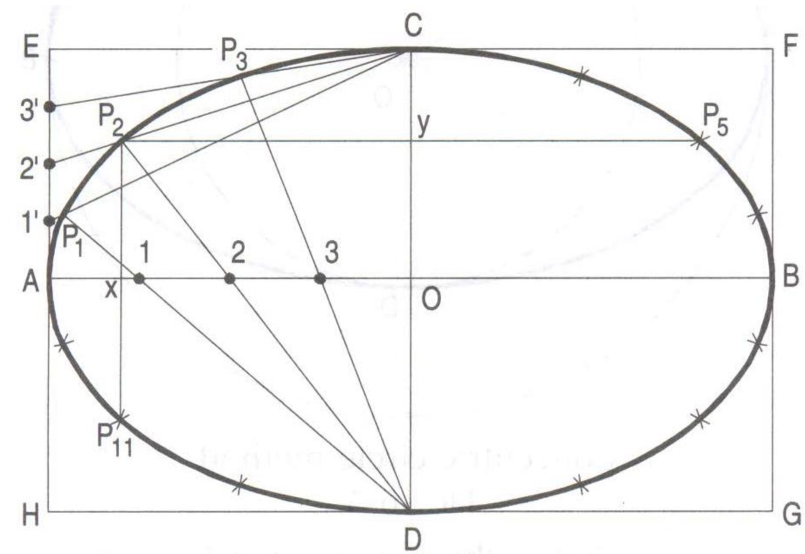


Conic Sections: Parabola

Rectangle method (from base & axis)

Construct a parabola inscribed in a rectangle of 120mm x 90mm using the Rectangle method.

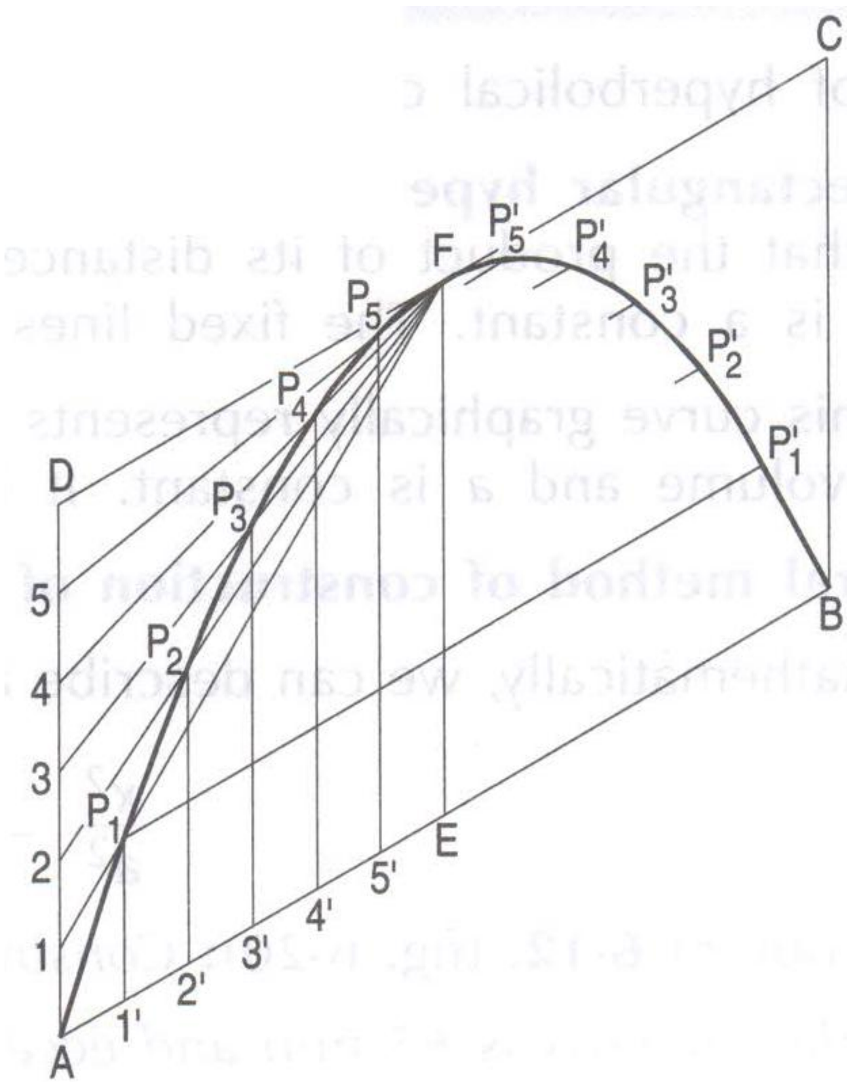




Similar to the oblong method of ellipse but for a minor change – one set of rays are parallel to the axis.

Conic Sections: Parabola

Rectangle method (from base & axis)

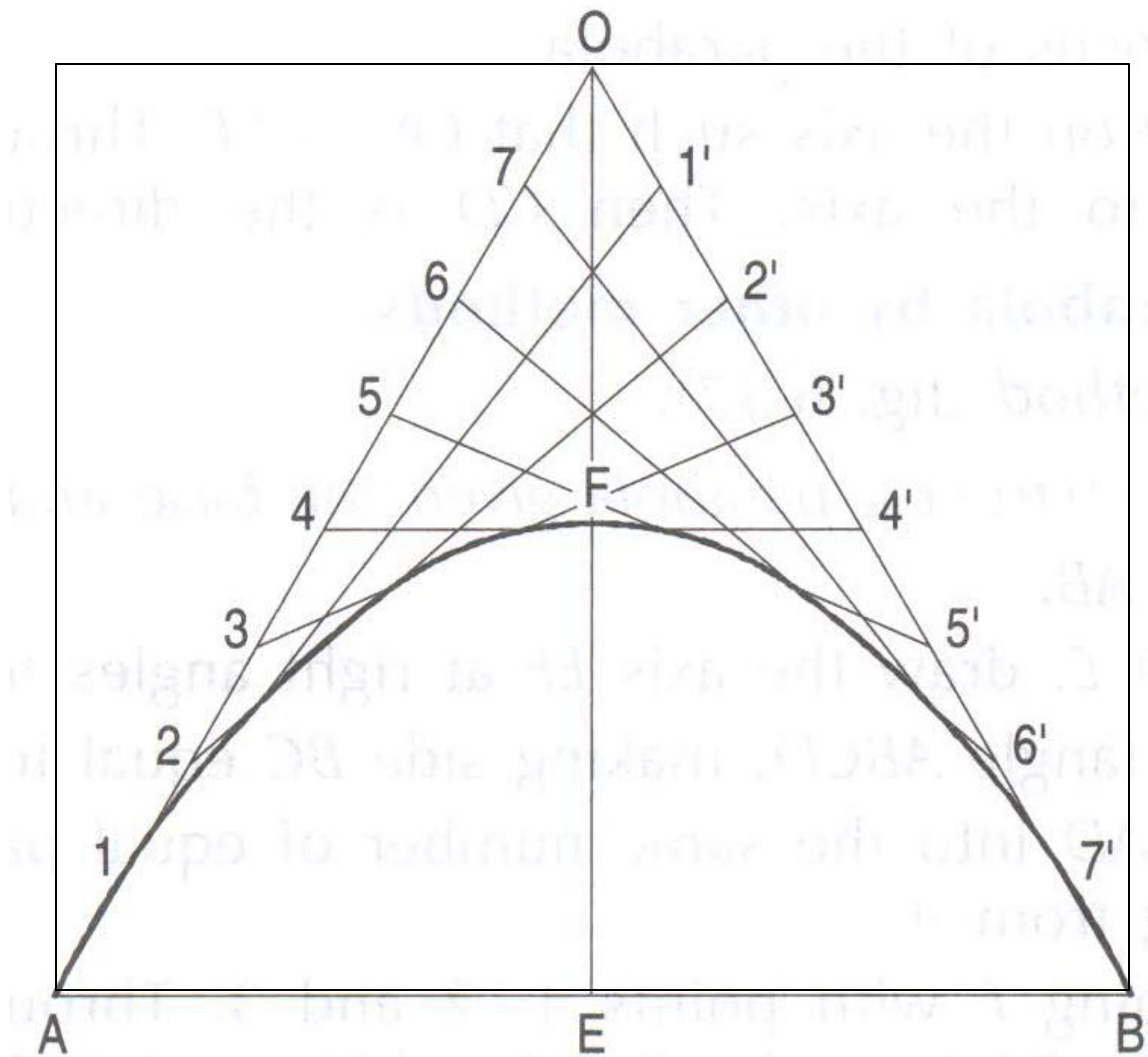


Applicable to oblique angles too.

Conic Sections: Parabola

Tangent method (from base & axis)

Construct a parabola inscribed in a rectangle of 120mm x 90mm using the tangent method.



Conic Sections: Hyperbola

Conic Sections: Hyperbola

Methods of drawing

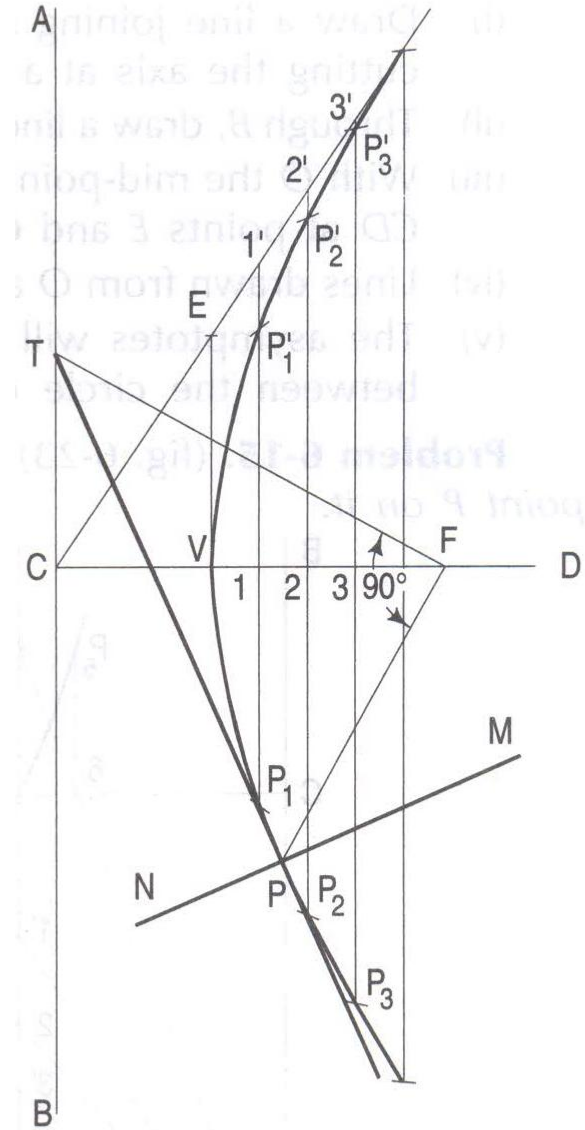
1. Basic method: From (i) distance of focus from the directrix (p) and (ii) eccentricity (e)
2. From (i) foci and (ii) vertices (This is similar to “arc of circle method” of ellipse)
3. From (a) asymptotes and (b) one point (This is similar to “concentric circle method” of ellipse)

Note: The above can be used for non-rectangular hyperbolas too.

Conic Sections: Hyperbola

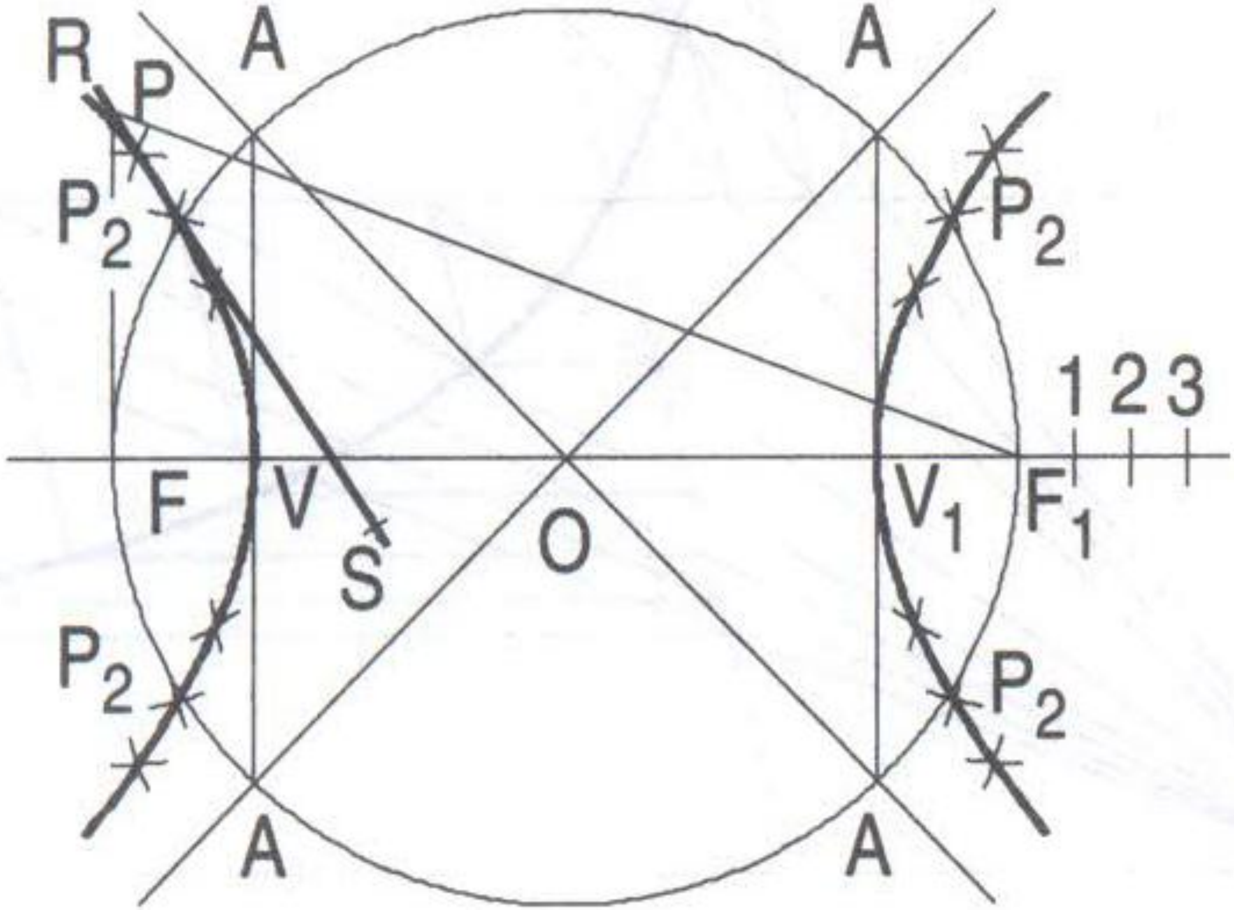
Basic method (from focus *f* & eccentricity *e*)

Construct a hyperbola when the distance of the focus from the directrix is 65mm and eccentricity is 3/2.

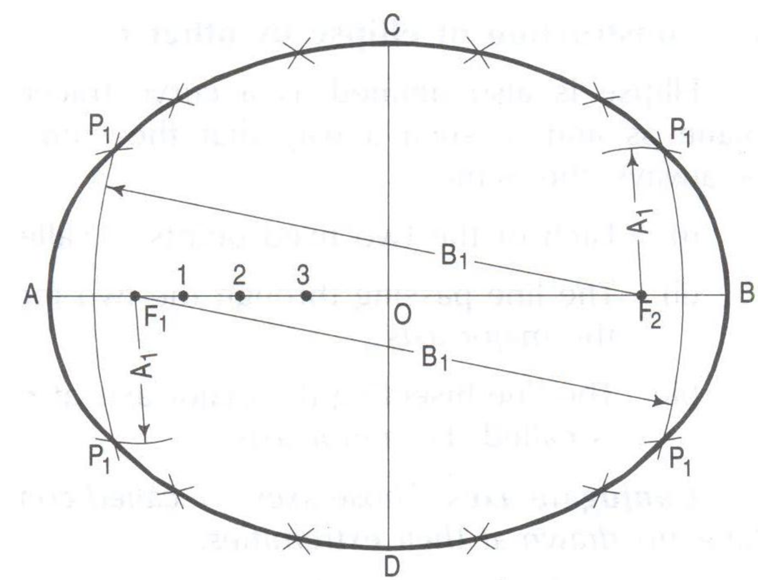


Conic Sections: Hyperbola

From (a) foci and (b) vertices

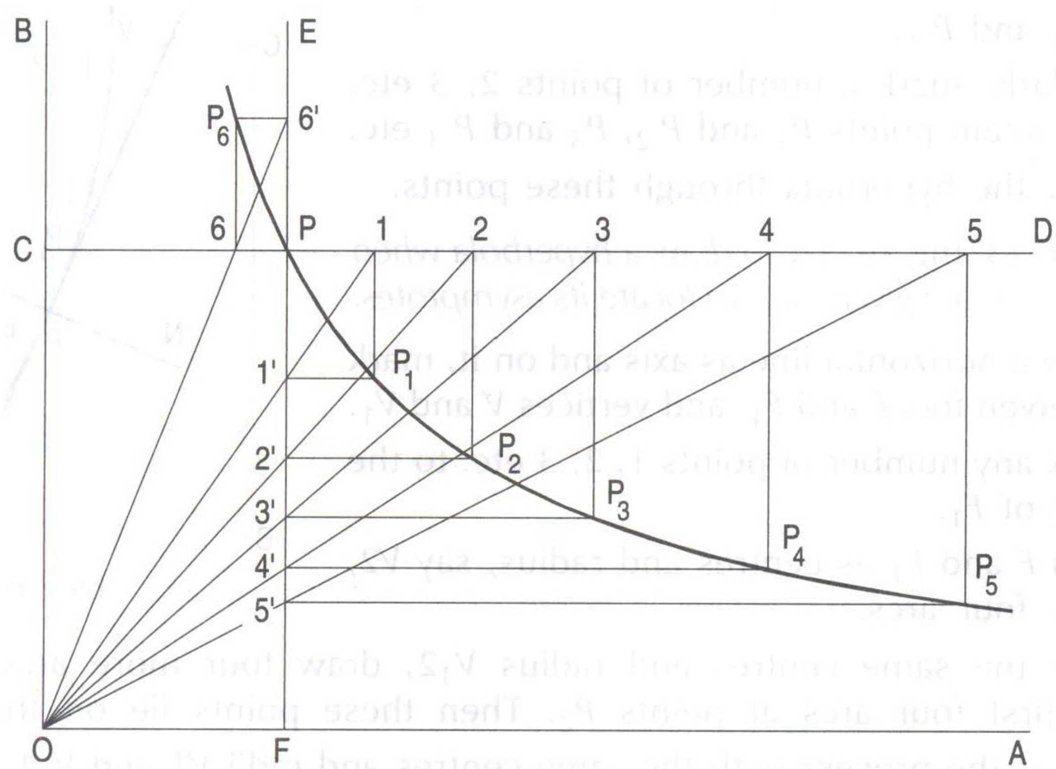


This is same as "arcs of circles method" used for ellipse.

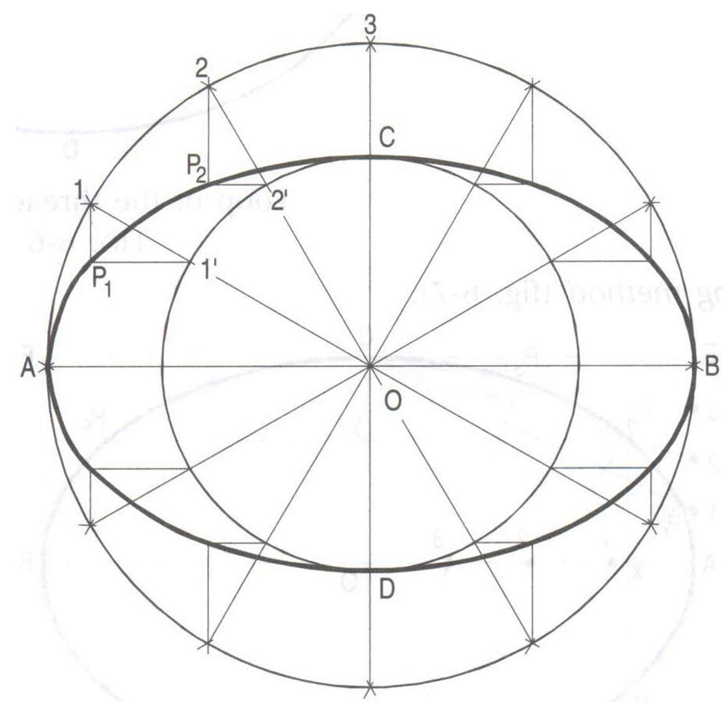


Conic Sections: Hyperbola

From (a) asymptotes and (b) one point



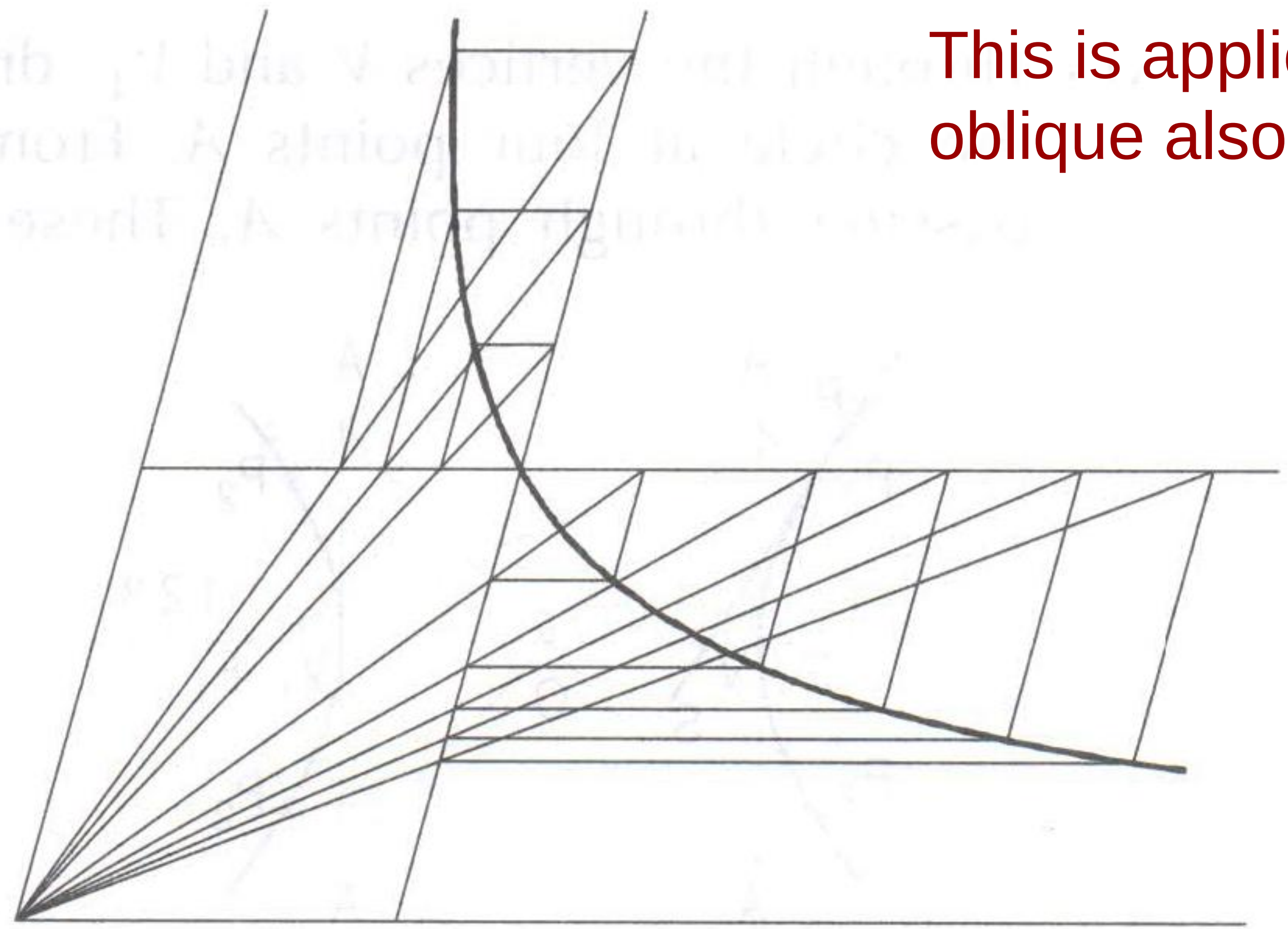
Construct a hyperbola from its asymptotes and one point.



This is analogous to "Concentric circles method" used for ellipse.

Conic Sections: Hyperbola

From (a) asymptotes and (b) one point



This is applicable for oblique also.

Conic Sections

Commonality of the methods

Ellipse	Parabola	Hyperbola
Basic method	Yes	Yes
Arcs of circles method/ Loop of the thread method		Yes
Concentric circles method		Yes
Oblong/Triangle method	Yes	

Involutes

Involutes

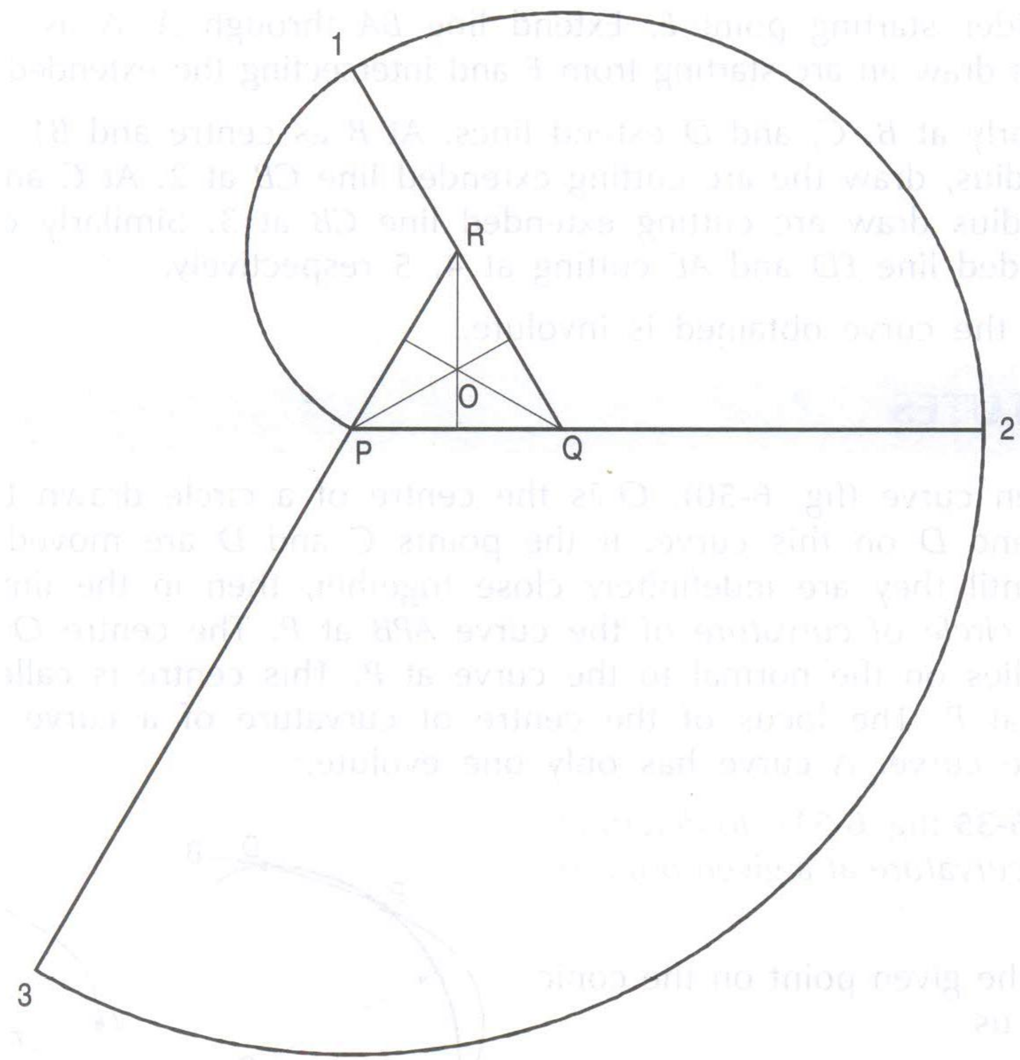
Involute: Curve traced by the end of a piece of thread unwound from a circle or polygon while keeping the thread tight. Alternately, it may be visualized as the curve traced by a point in a straight line when it is rolled without slip around a circle or polygon.

Evolute: Locus of the centre of curvature of a curve. This will not be covered in this course.

Involutes

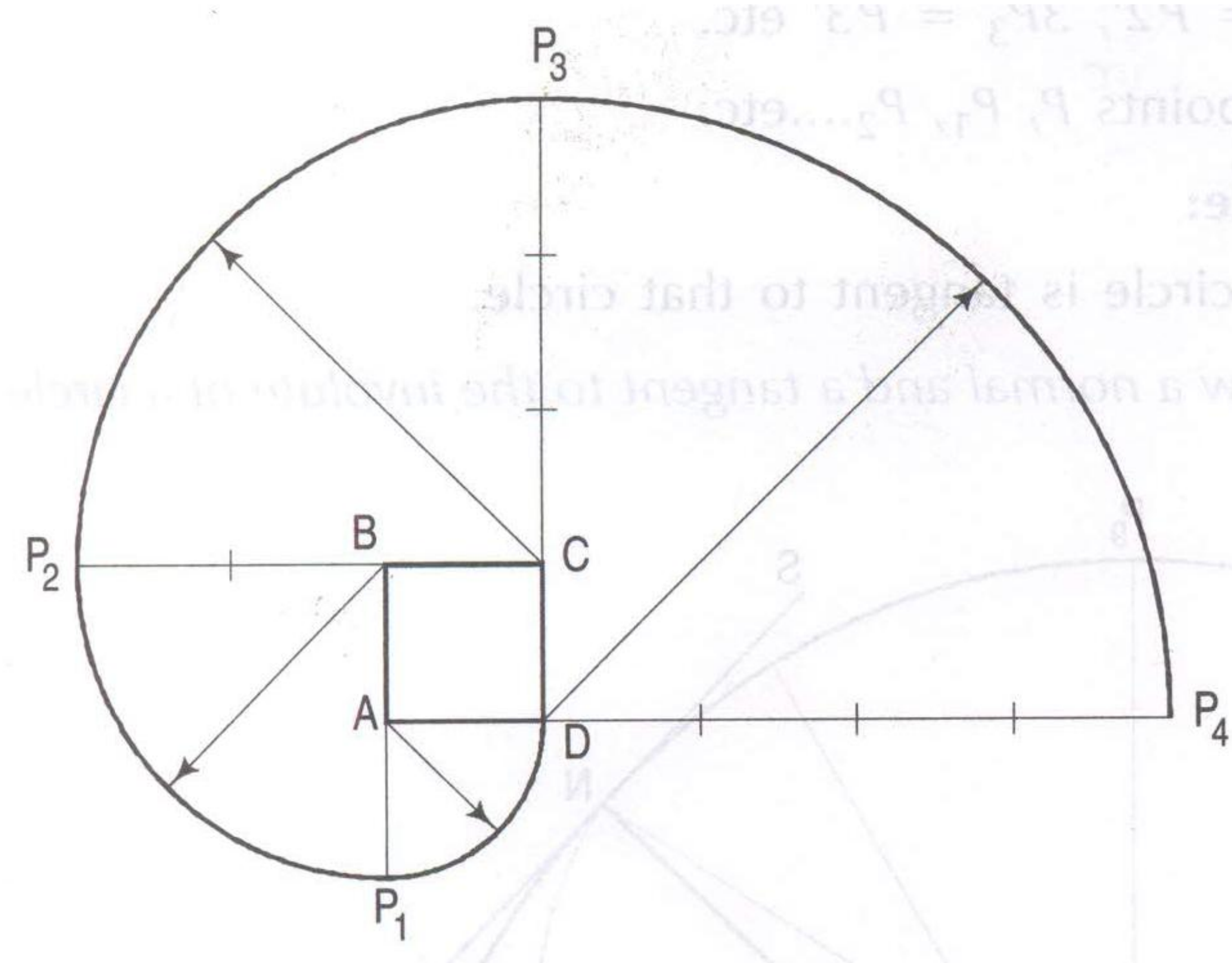
Construction of an involute of a triangle

Involute of a polygon will be a series of tangential arcs.



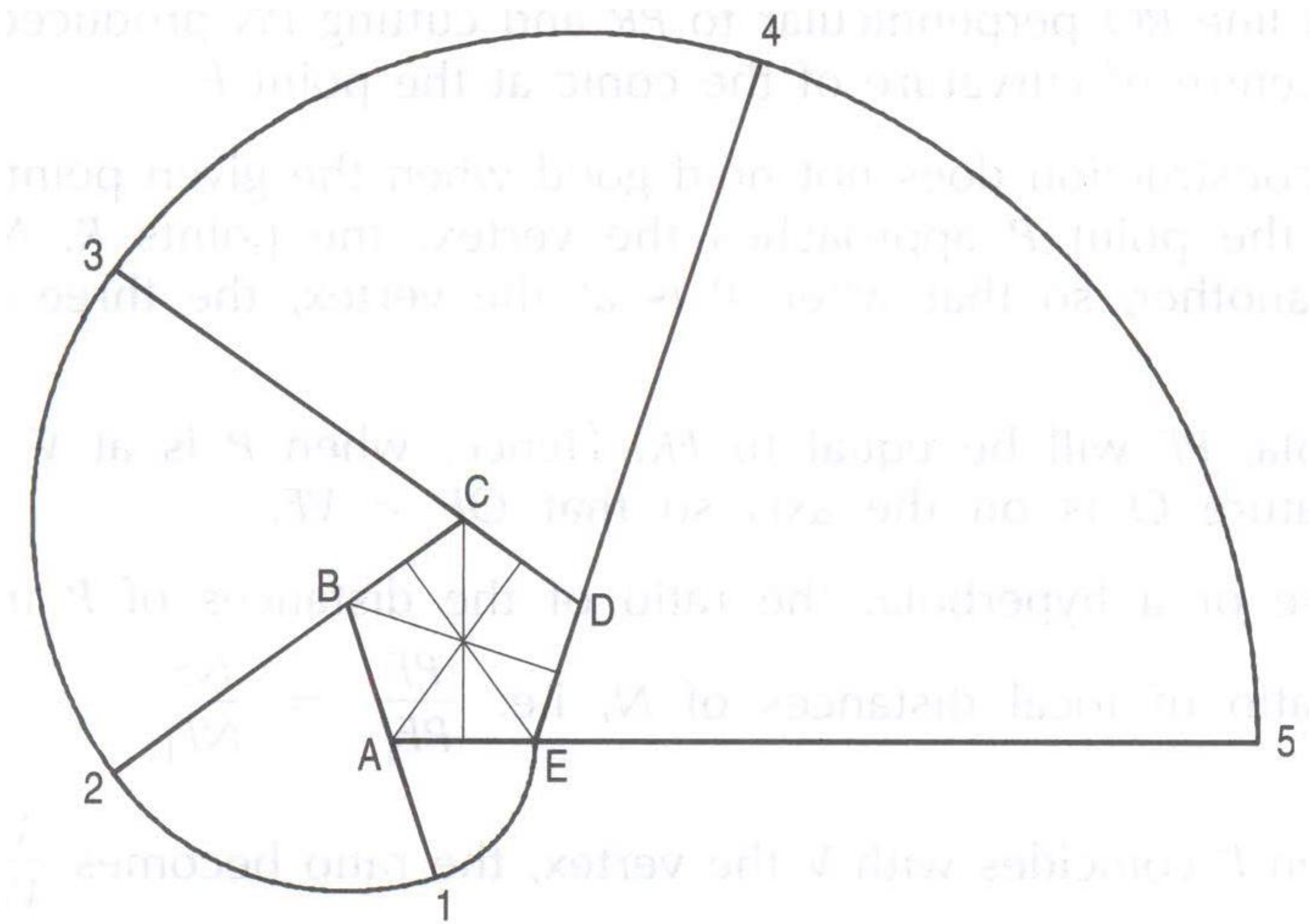
Involutes

Construction of an involute of a square



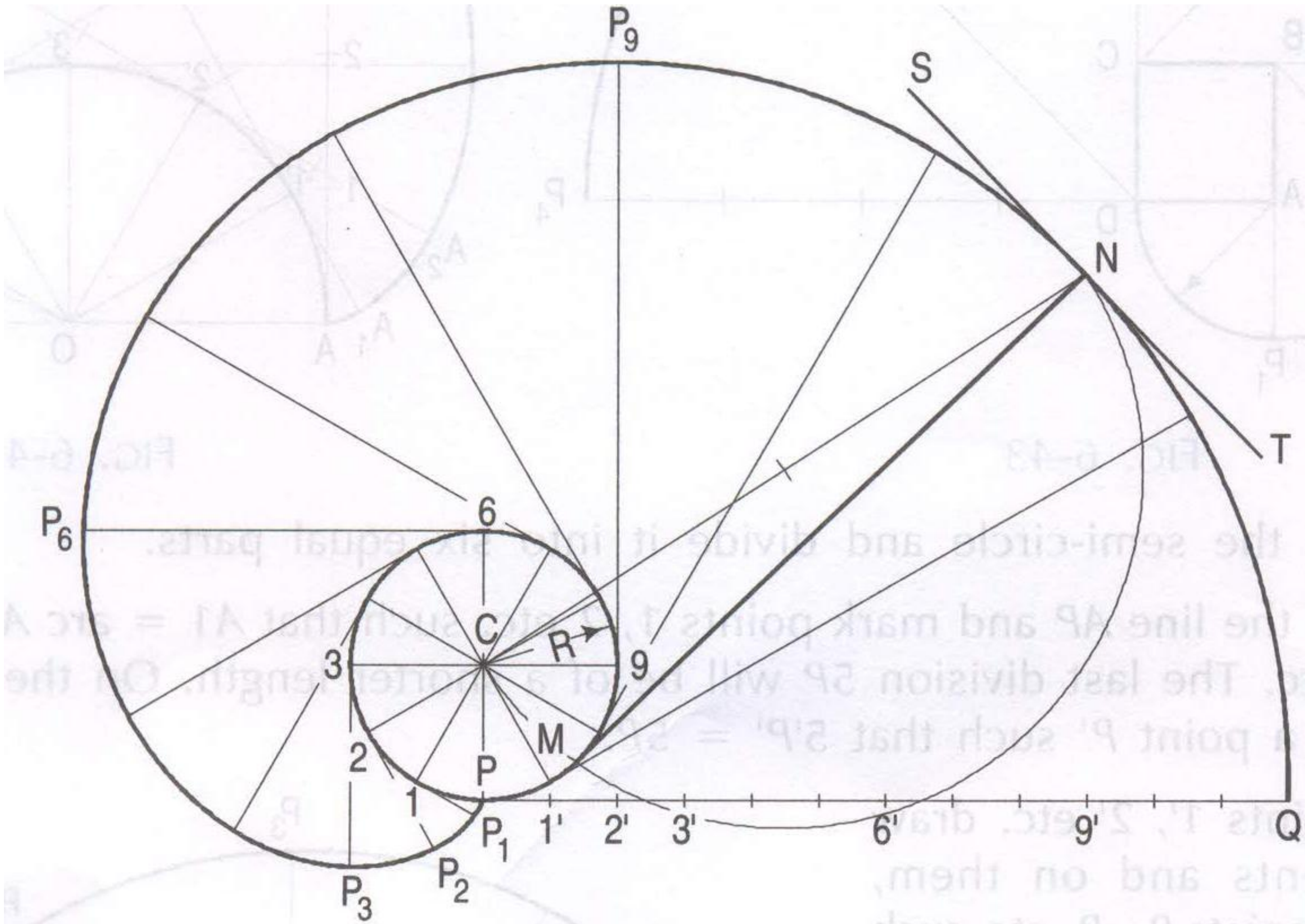
Involutes

Construction of an involute of a pentagon



Involutes

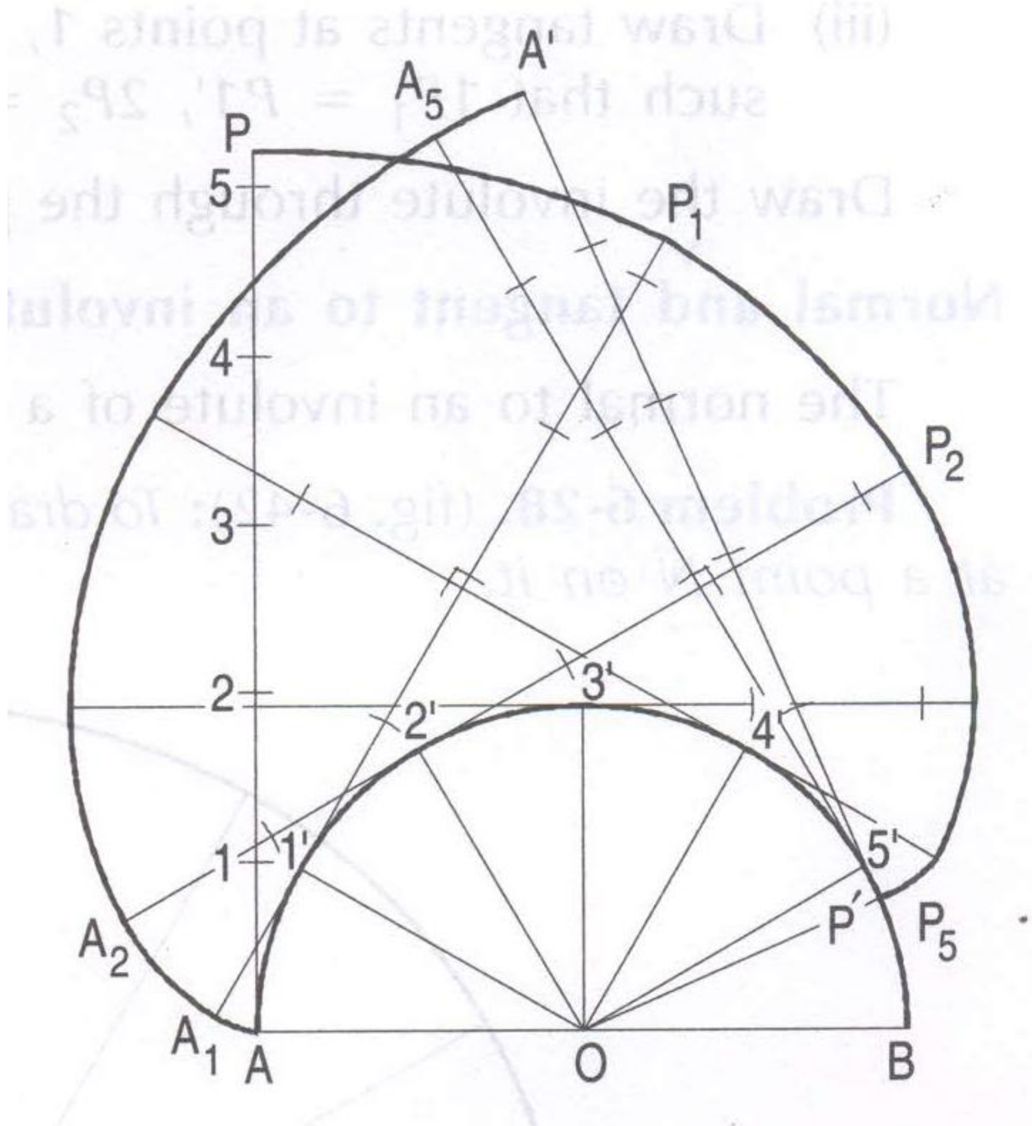
Construction of an involute of a circle



Involutes

Locus of the tip of a line rolling over a circle

Problem 6-30: A rod AP of 100mm long is perpendicular to the diameter AB=75mm of a semi-circle. The rod rolls over the circle clockwise without slip. Draw the path traced by the ends of the rod A and P.



Note: Locii of A and P correspond to the curves of unwinding and winding.

Spiral

Spiral

Curve traced by a point going around a point simultaneously moving towards it (spiral in) or away from it (spiral out). There could be a variety of spirals depending on the nature of relationship between the radial and peripheral velocities. However, the following two are the most popular:

1. Archimedean spiral: Radial displacement per revolution is constant.

1. Logarithmic or equi-angular spiral: Ratio of the lengths of consecutive radius vectors enclosing equal angles is always constant.

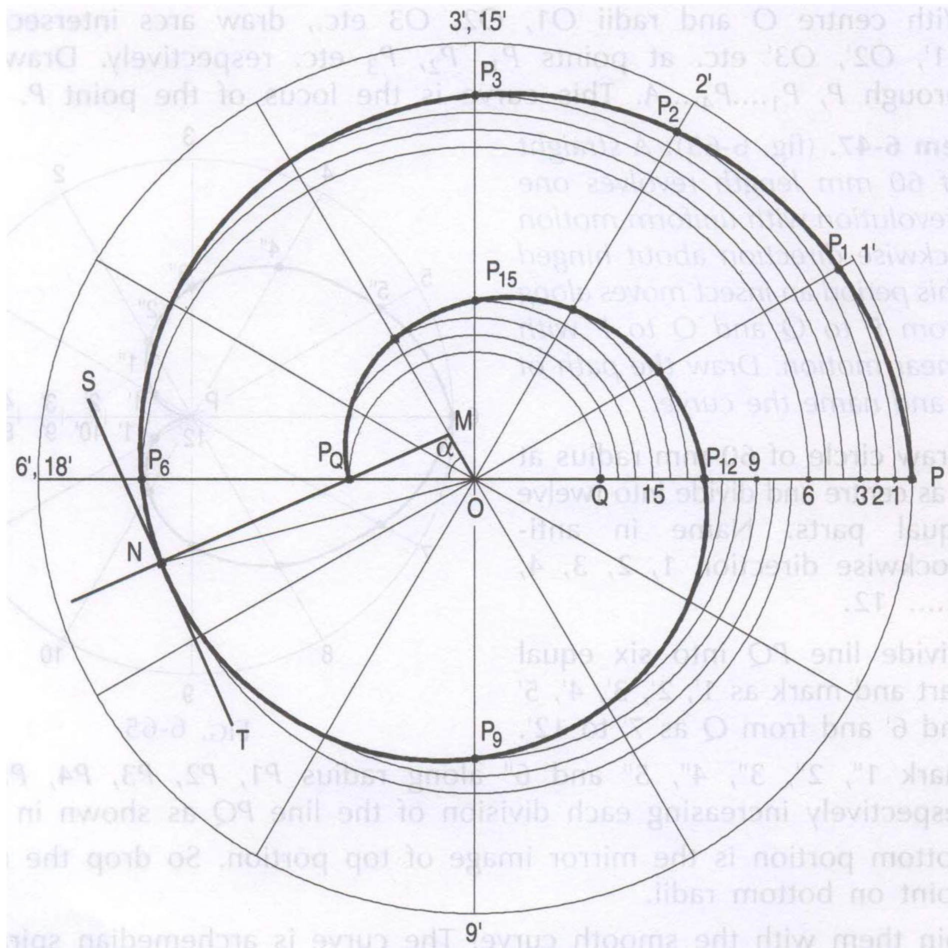
Archimedean Spiral

Problem 6-44: Construct an Archimedean spiral of 1.5 convolution for the greatest and shortest radii (say, r_i=15mm & r_o=60mm).

The constant of the curve is equal to the difference between the lengths of any two radii divided by the circular measure of the angle between them (in radians).

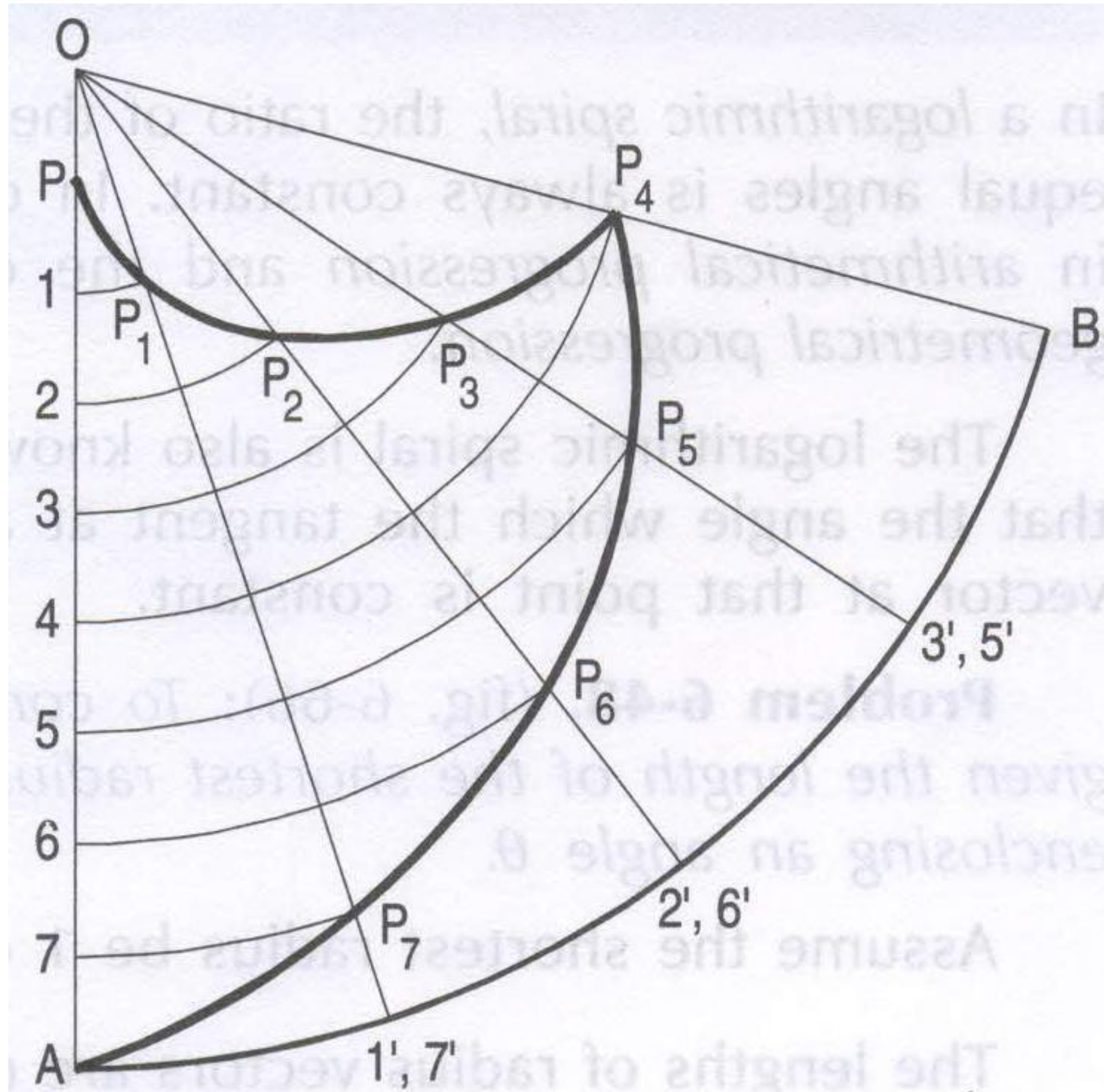
$$C = \frac{OP - OP_3}{90^\circ \text{ in rad}} = \frac{OP - OP_3}{\pi / 2}$$

OM (=C) is perpendicular to NO



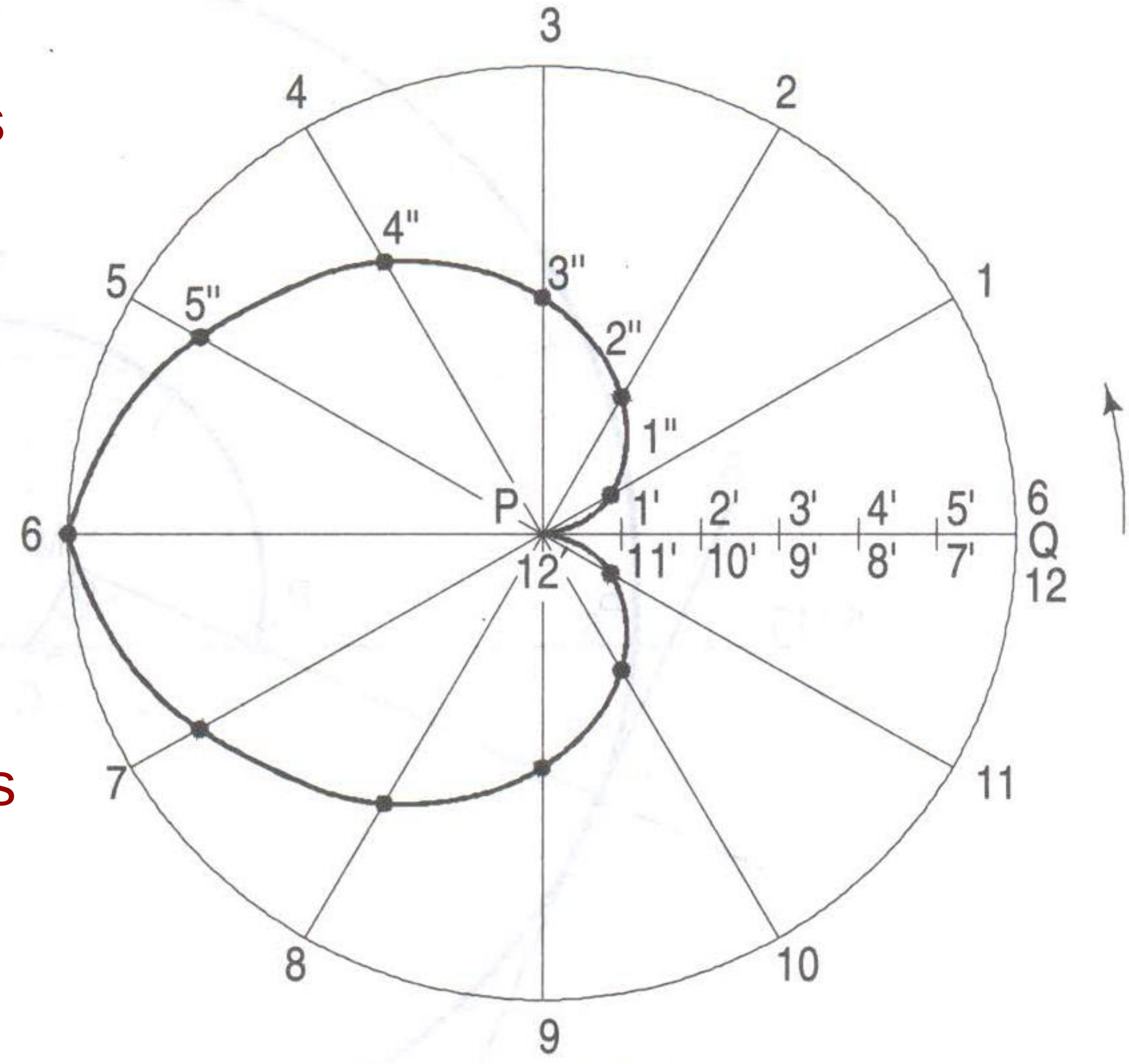
Archimedean Spiral

Problem 6-46: A link of 225mm long swings on a pivot O from its vertical position of rest to the right through an angle of 75° and returns to its initial position at uniform velocity. During this period, a sleeve approximated as a point P initially at a distance of 20mm from the pivot O moving at uniform speed along the link reaches its end. Draw the locus of P.



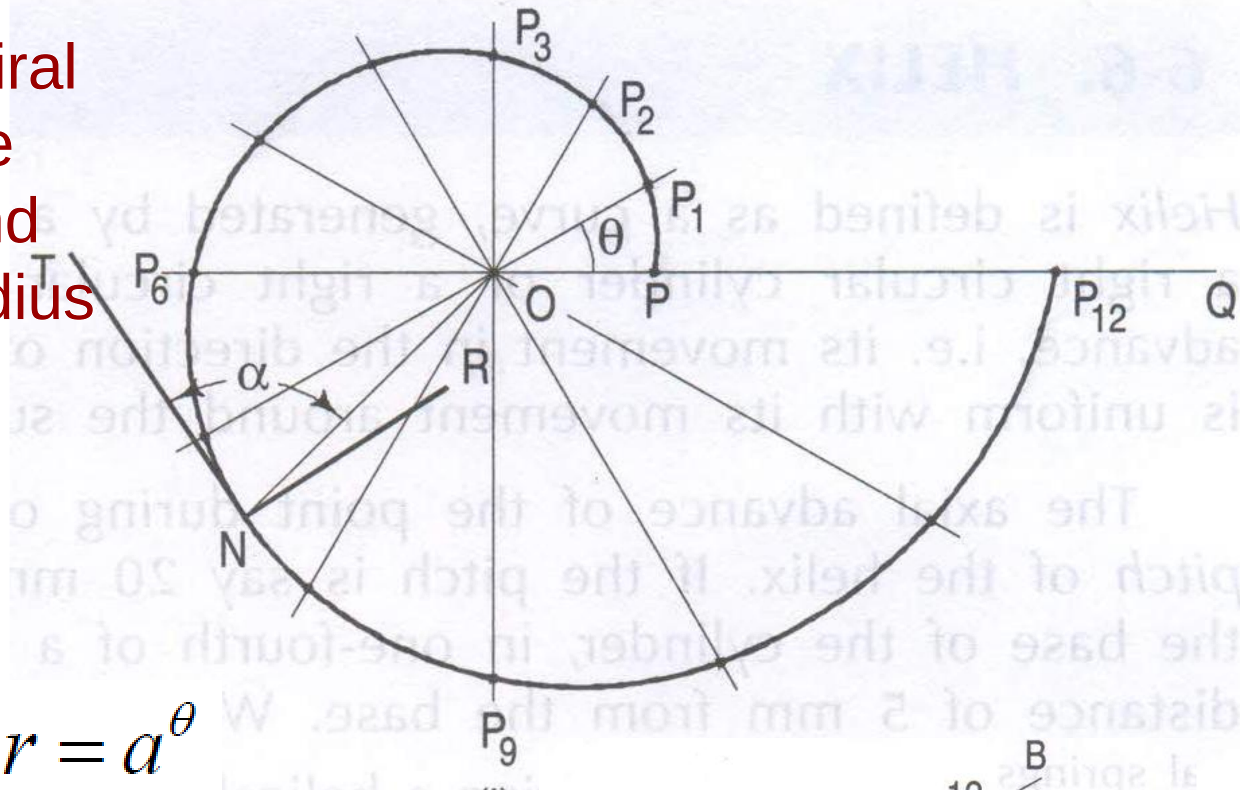
Archimedean Spiral

Problem 6-47: A straight link PQ of 60 mm length revolves one complete revolution with uniform motion in CCW direction about hinge P. During this period, an insect moves along the link from P to Q and back to P with uniform linear speed. Draw the path of the insect for a stationary viewer. Identify this curve.



Logarithmic or Equi-angular spiral

Construct the logarithmic spiral of one convolution, given the shortest radius r(=10mm) and the ratio of the lengths of radius vectors enclosing an angle θ(=30°) is k=10/9.

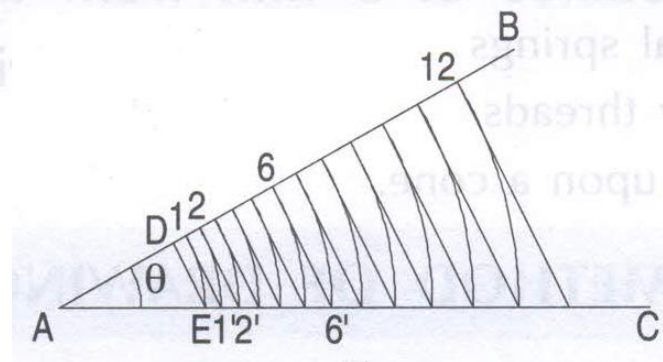


(i)

Equation of logarithmic spiral $$r = a^{\theta}$$

At θ = 0 and π/6: get r and *a*

$$\tan\alpha = \frac{\log e}{\log a}$$



(ii)

Helix

Helix

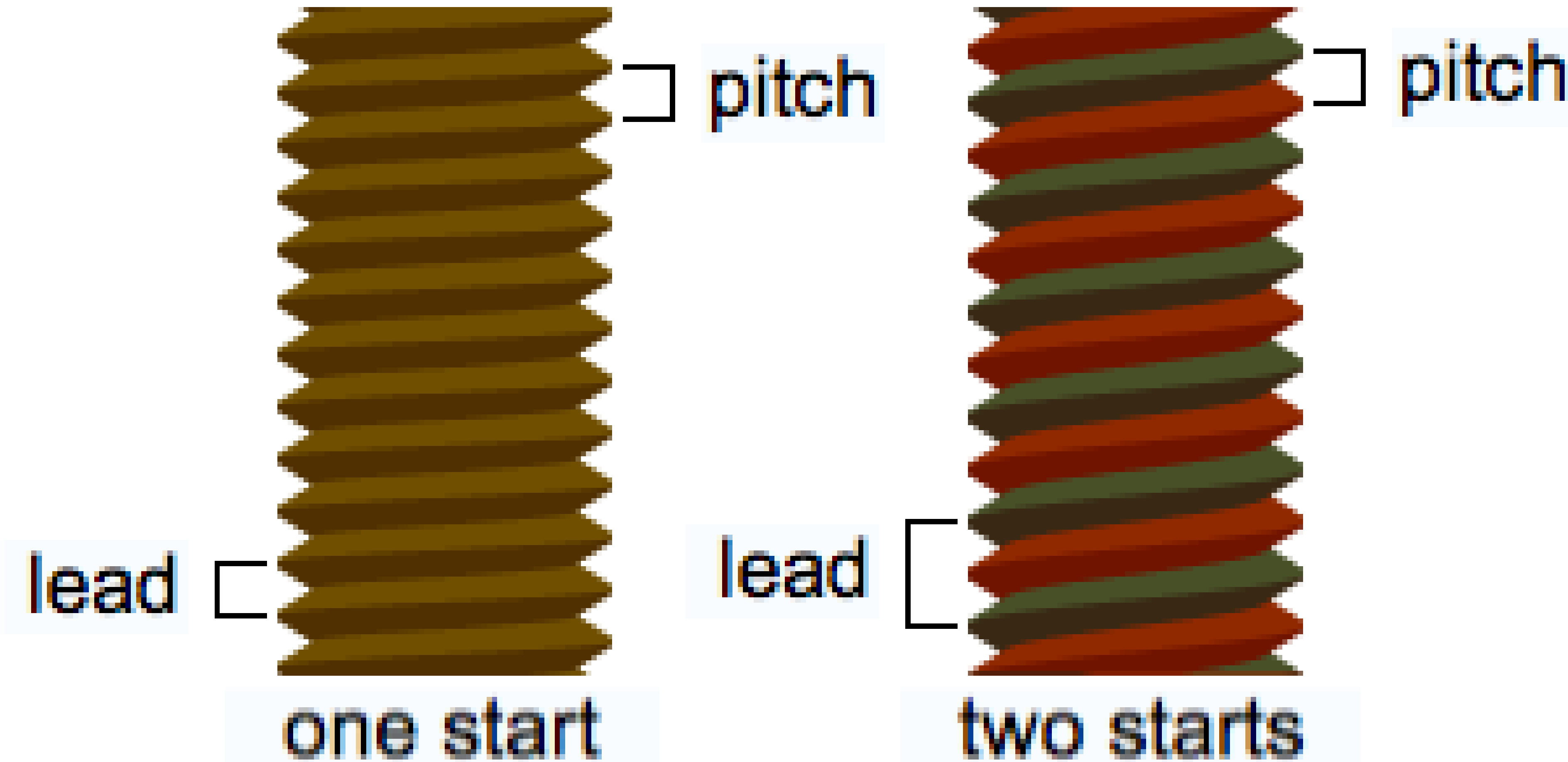
All the previous curves are planar; this is the first 3D curve we are studying. This is the extension of circle by adding its 3rd dimension.

The simplest helix is a curve generated by a point moving around the surface of a right circular cylinder in such a way that its axial advance is uniform with its movement around the surface of the cylinder. The axial advance per revolution is called lead. Note: Lead = pitch x number of starts.

The surface in a complex helix may be cone or any other regular shape and the relative of axial and peripheral velocities may be more complex.

Compare with spiral → spiral staircase is a misnomer. It should be helical staircase.

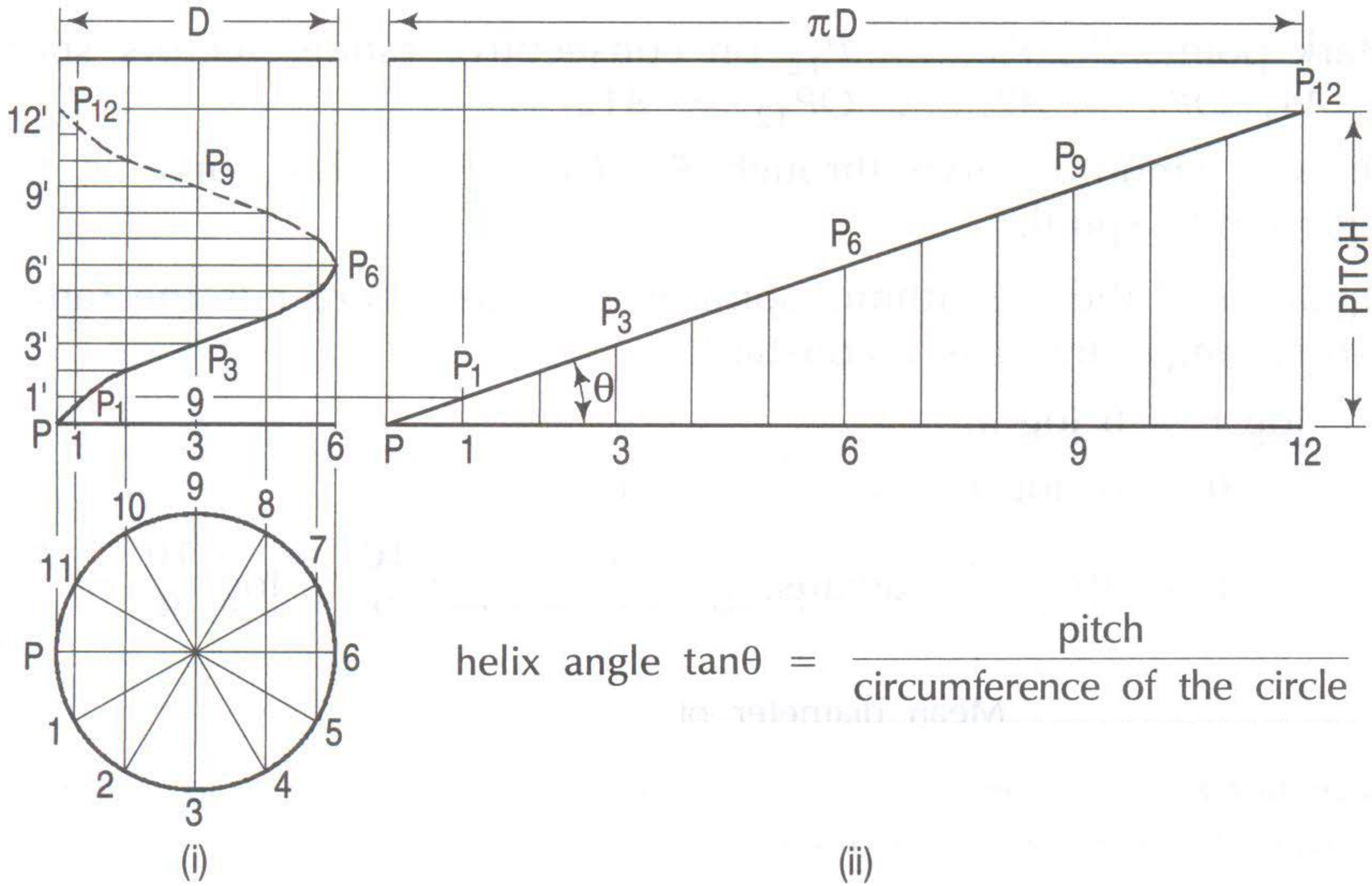
Helix



Helix

Ordinary helix

Problem 6-47: A helix has a pitch of 50mm and diameter of 75mm. Draw its front view.

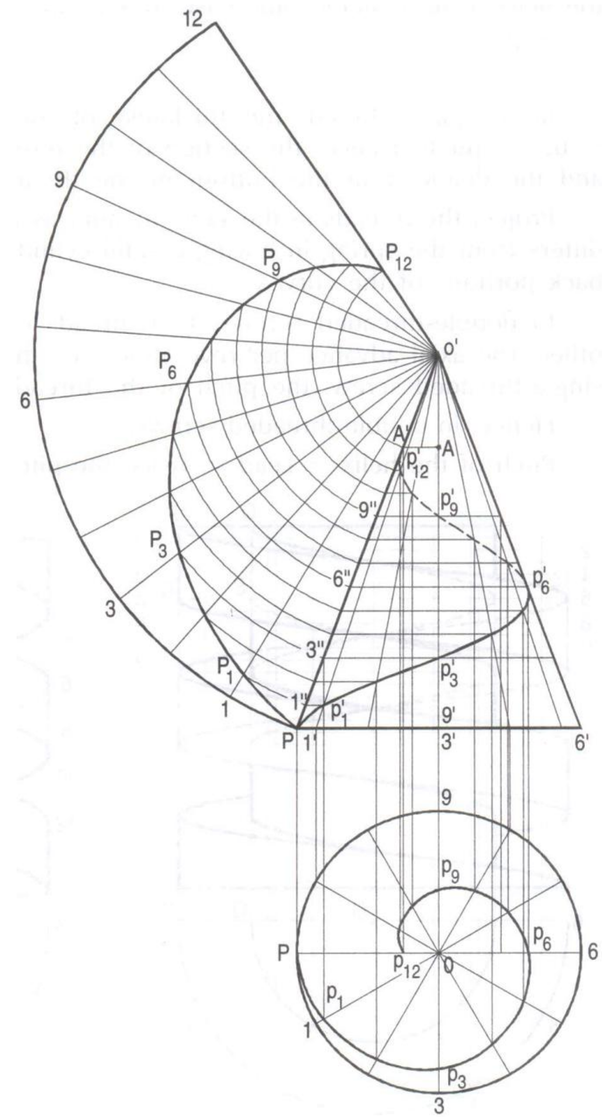


helix angle $\tan\theta = \dfrac{\text{pitch}}{\text{circumference of the circle}}$

Helix

Conical helix

Problem 6-54: The diameter of a cone is 75mm at the base and its height is 100mm. The pitch of a helix on it is 75mm. Draw its front view.



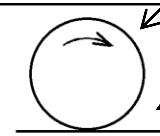
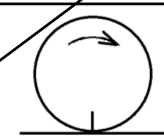
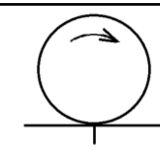
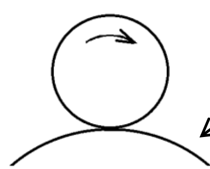
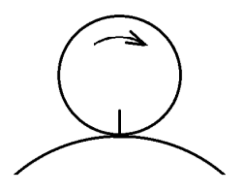
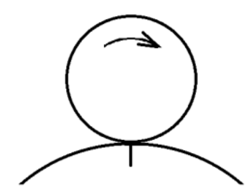
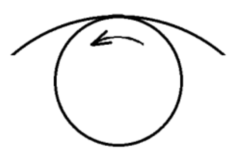
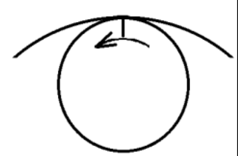
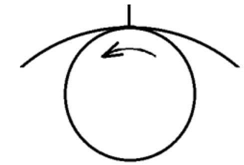
Cycloids and Trochoids

Cycloids (ON) and Trochoids (OFF)

Curve traced by a point on/inside/outside of a circular disc when it rolls on a line or outside/inside an arc.

Directrix line/circular
Circular directrix → Epi-/hypo
On → cycloid (3 types)
Off (inf/sup) → Trochoid

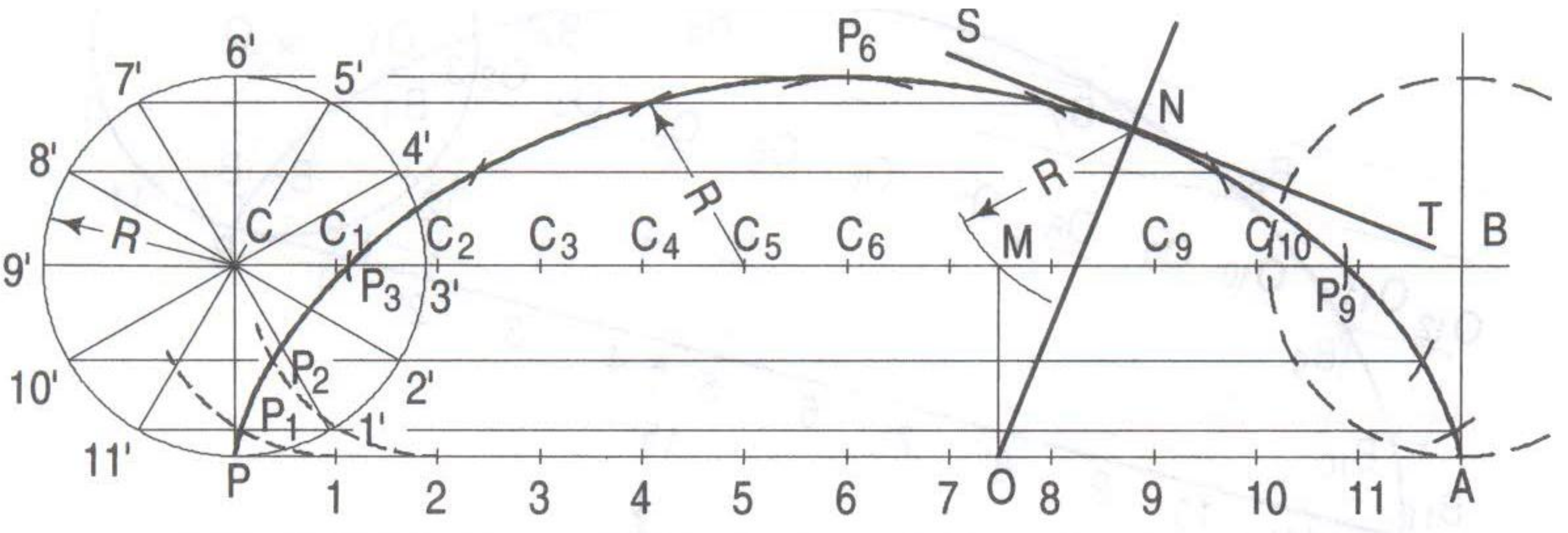
Generatrix   Directrix

Rolling Path (Directrix)	Position of Trace Point on the Circle (Generatrix)		
	On	Off (Inside)	(Off) Outside
Straight Line	Cycloid 	Inferior Trochoid 	Superior Trochoid 
Outside Circle	Epi-Cycloid 	Epi-Inferior-Trochoid 	Epi-Superior Trochoid 
Inside Circle	Hypo-Cycloid 	Hypo-Inferior-Trochoid 	Hypo-Superior Trochoid 

Cycloids and Trochoids

Construction of a cycloid

Problem 6-20: Construct a cycloid of the generating circle for 40mm diameter.



$PA = 2\pi R = CB$

Cycloids and Trochoids

Construction of the (inferior & superior) trocoids

Problem 6-23: Construct the inferior & superior trochoids of the generating circle for 40mm diameter.

Same method as cycloid.

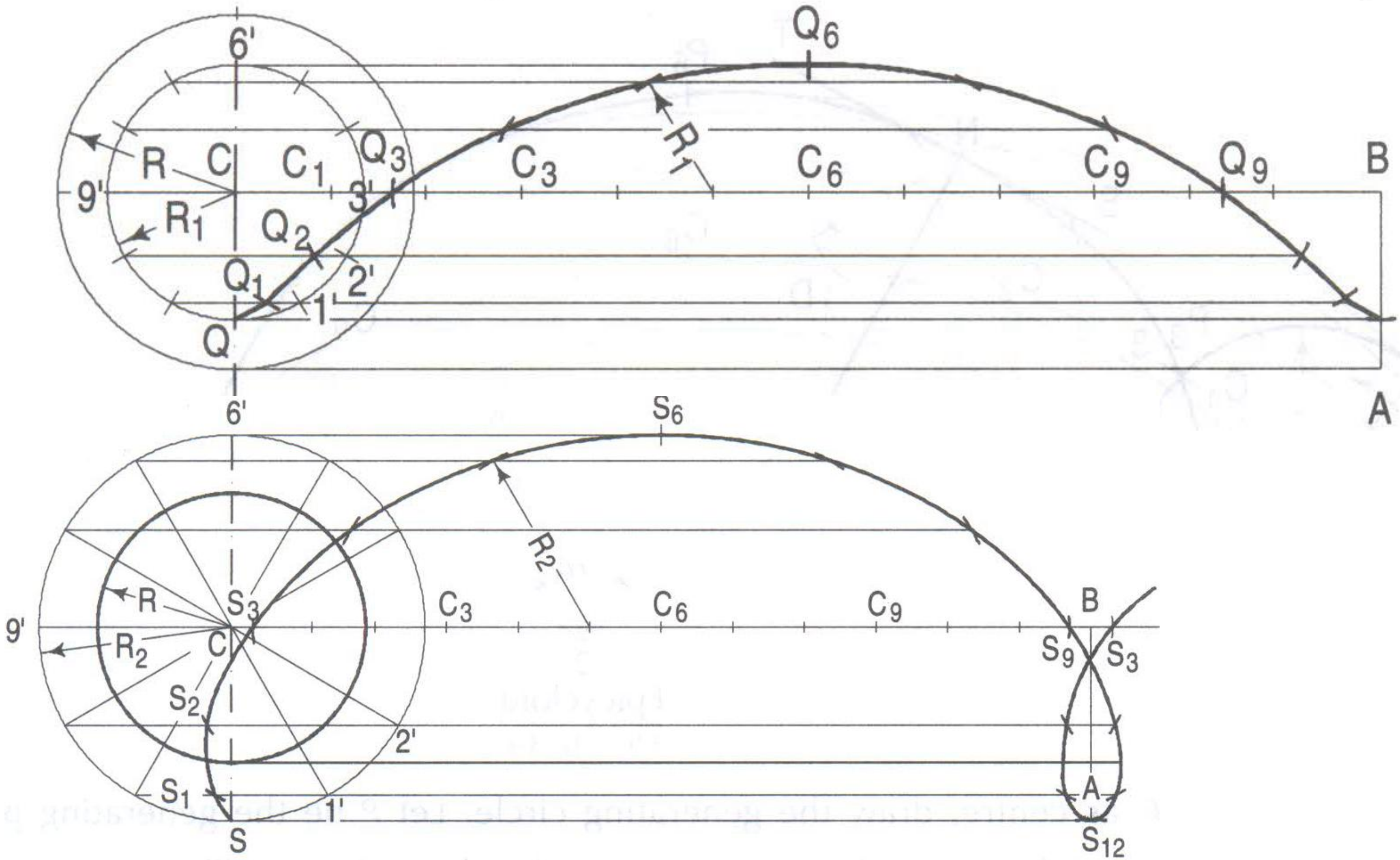
-Length of PA is $2\pi R$

-The circle to be divided is of radius R_1/R_2.

-The radius used for cutting is R_1/R_2.

Cycloids and Trochoids

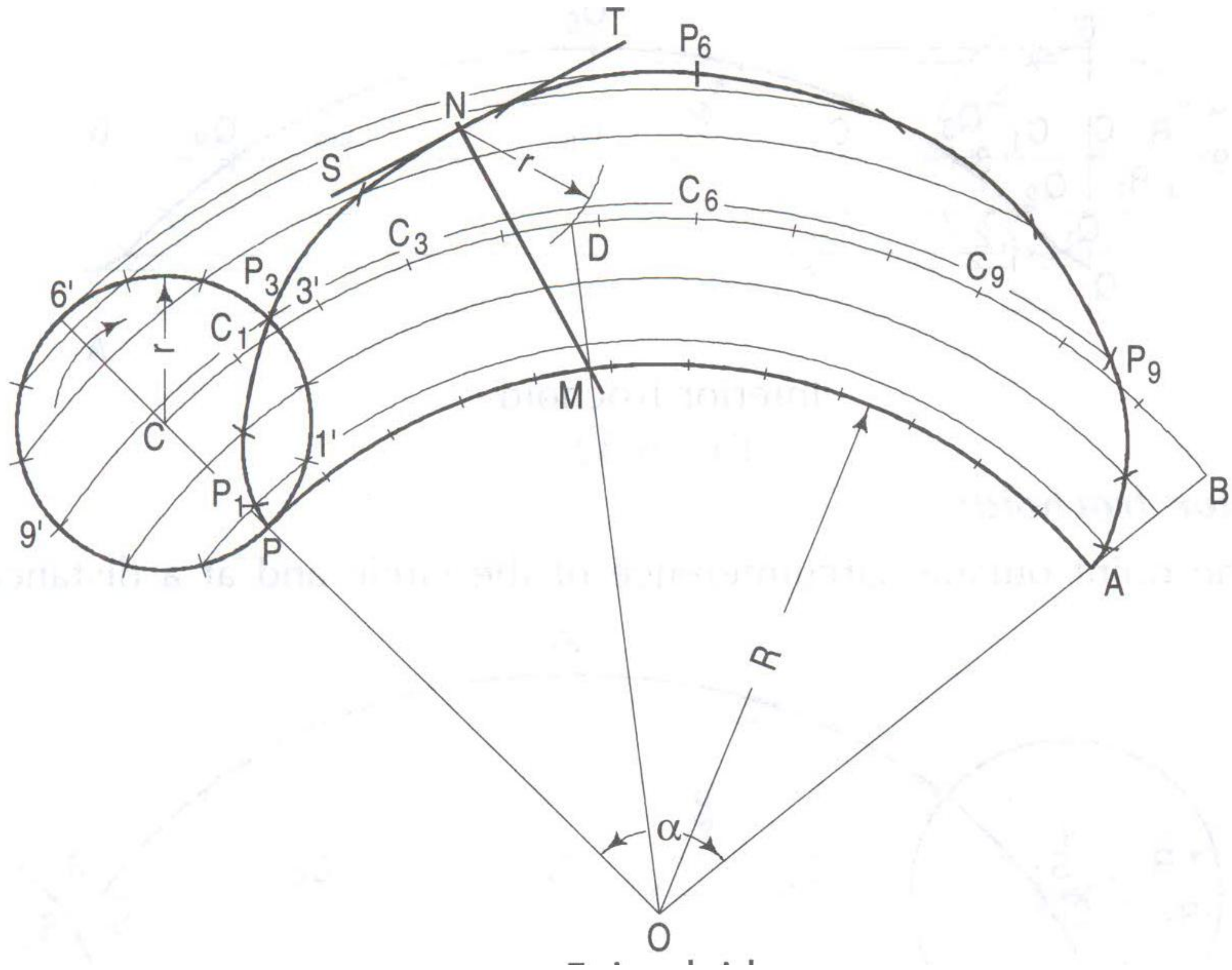
Construction of the (inferior & superior) trocoids …



Cycloids and Trochoids

Construction of an epi-cycloid

Problem 6-24a:
Construct an epi-cycloid from diameters of the generating and directing circles of 40mm and 100mm.



$$\alpha = \frac{arc}{R} = \frac{2\pi r}{R} \text{ radians}$$

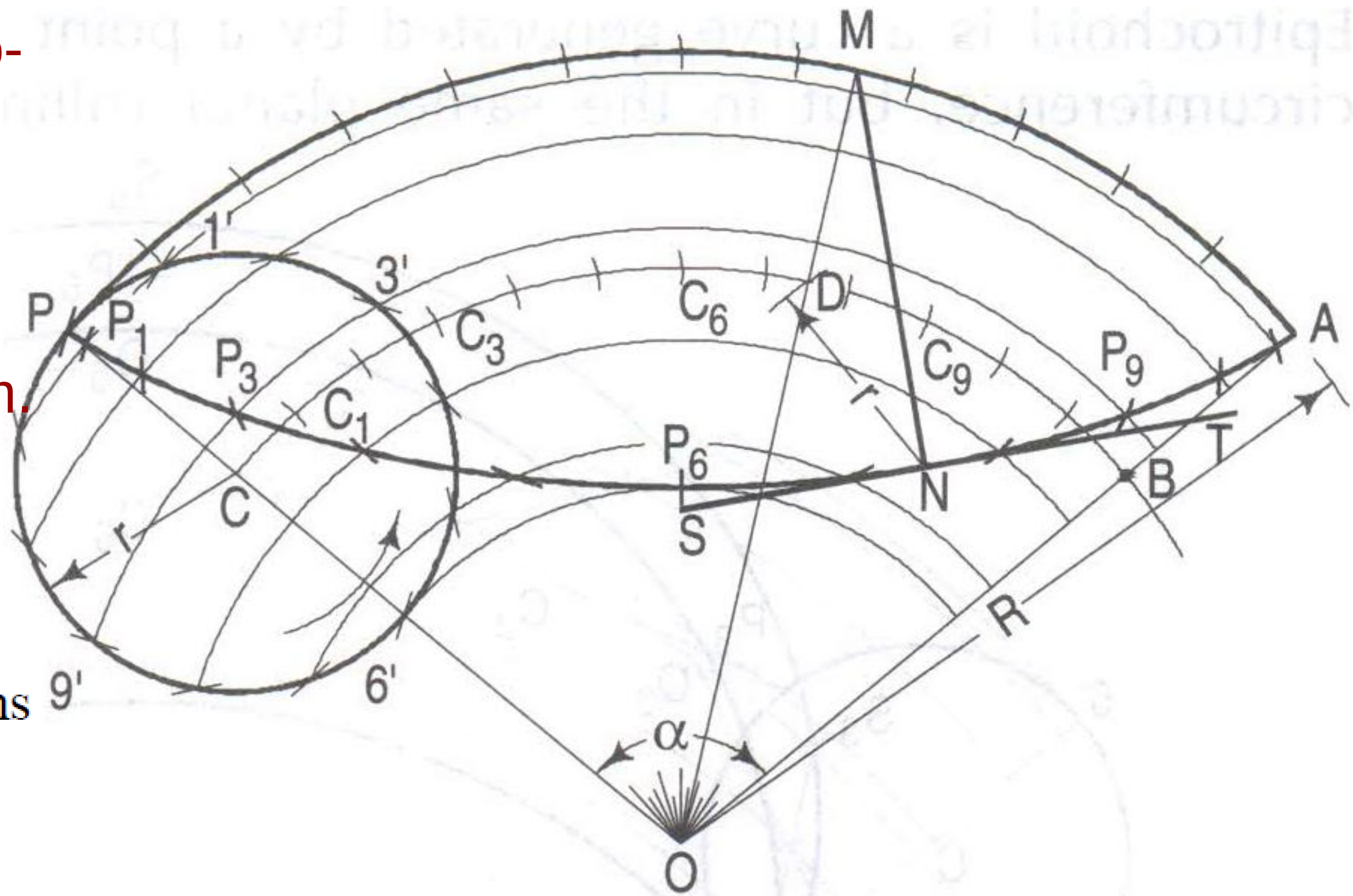
$$= 360 \times \frac{r}{R} \text{ degrees}$$

Cycloids and Trochoids

Construction of hypo-cycloid

Problem 6-24b:

Construct an hypo-cycloid from diameters of the generating and directing circles of 40mm and 100mm.



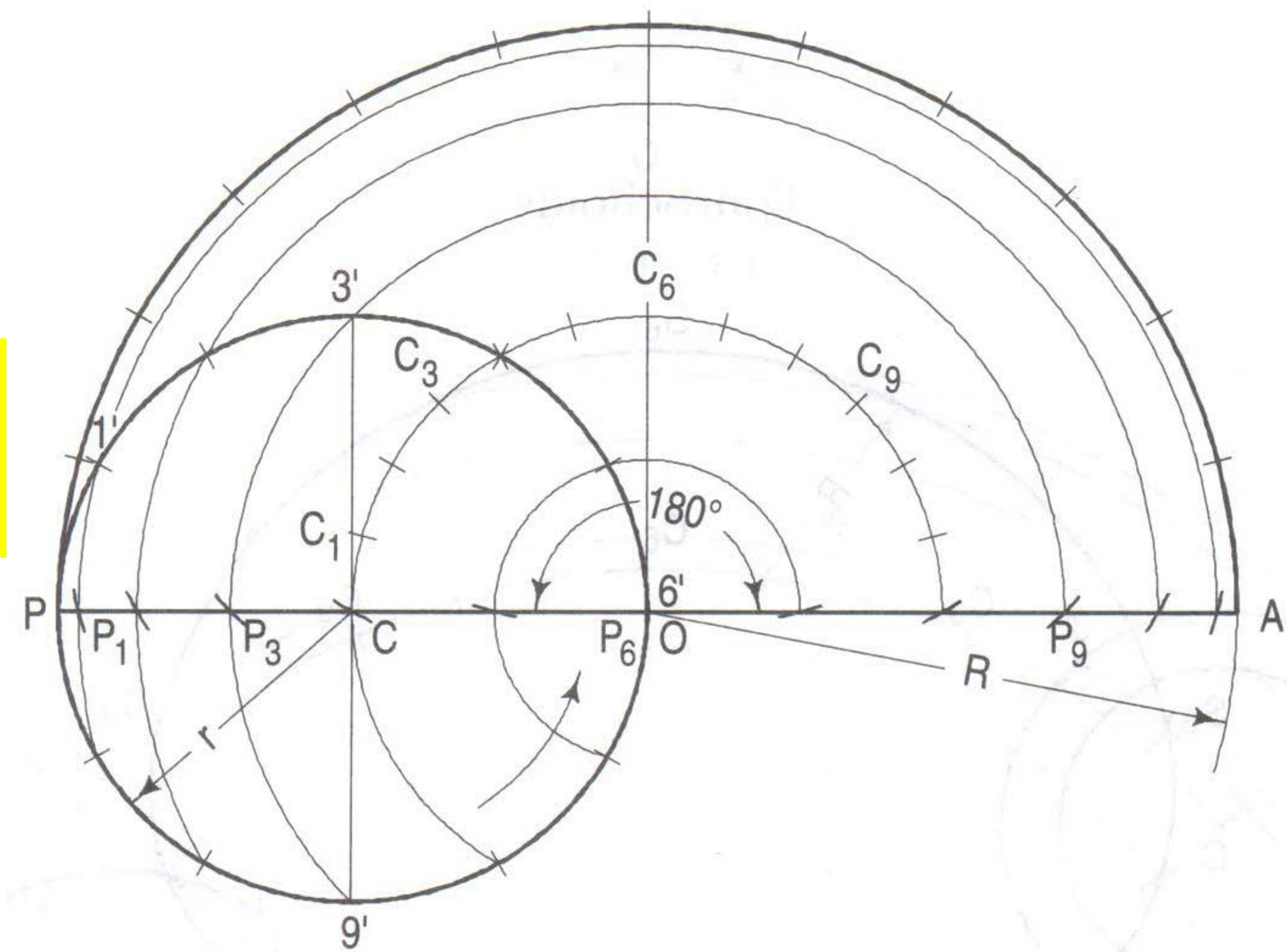
$$\alpha = \frac{arc}{R} = \frac{2\pi r}{R} \text{ radians}$$

$$= 360 \times \frac{r}{R} \text{ degrees}$$

Cycloids and Trochoids

Construction of hypo-cycloid …

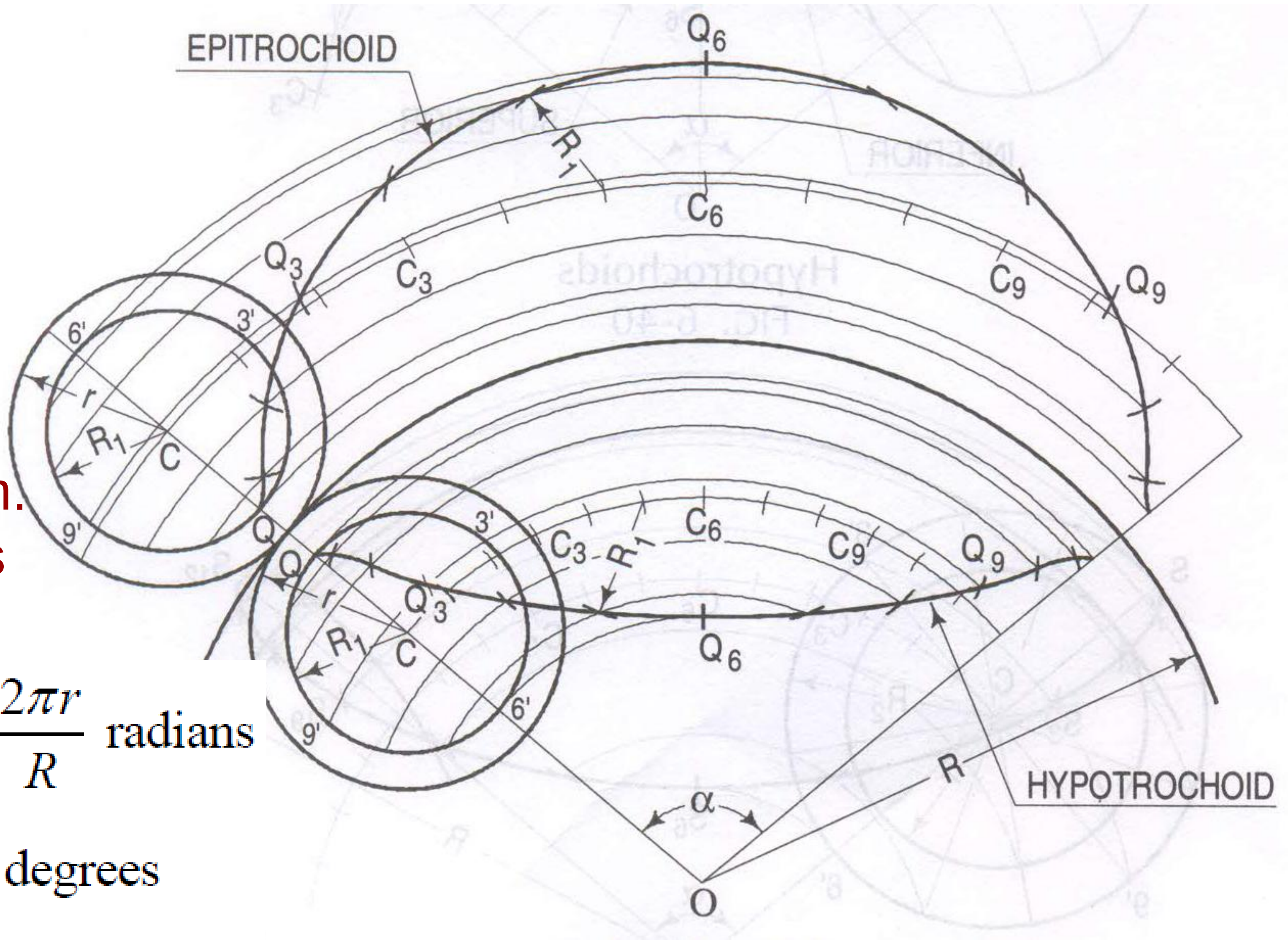
Note: When r = R/2, the hypo-cycloid will be a straight line.



Cycloids and Trochoids

Construction of an inferior epi-&hypo-trocoids

Problem 6-26a:
Construct an epi-trocoids (inferior and superior) from diameters of the generating and directing circles of 40mm and 100mm. The point traced is 5mm inside and outside.



$$\alpha = \frac{arc}{R} = \frac{2\pi r}{R} \text{ radians}$$

$$= 360 \times \frac{r}{R} \text{ degrees}$$

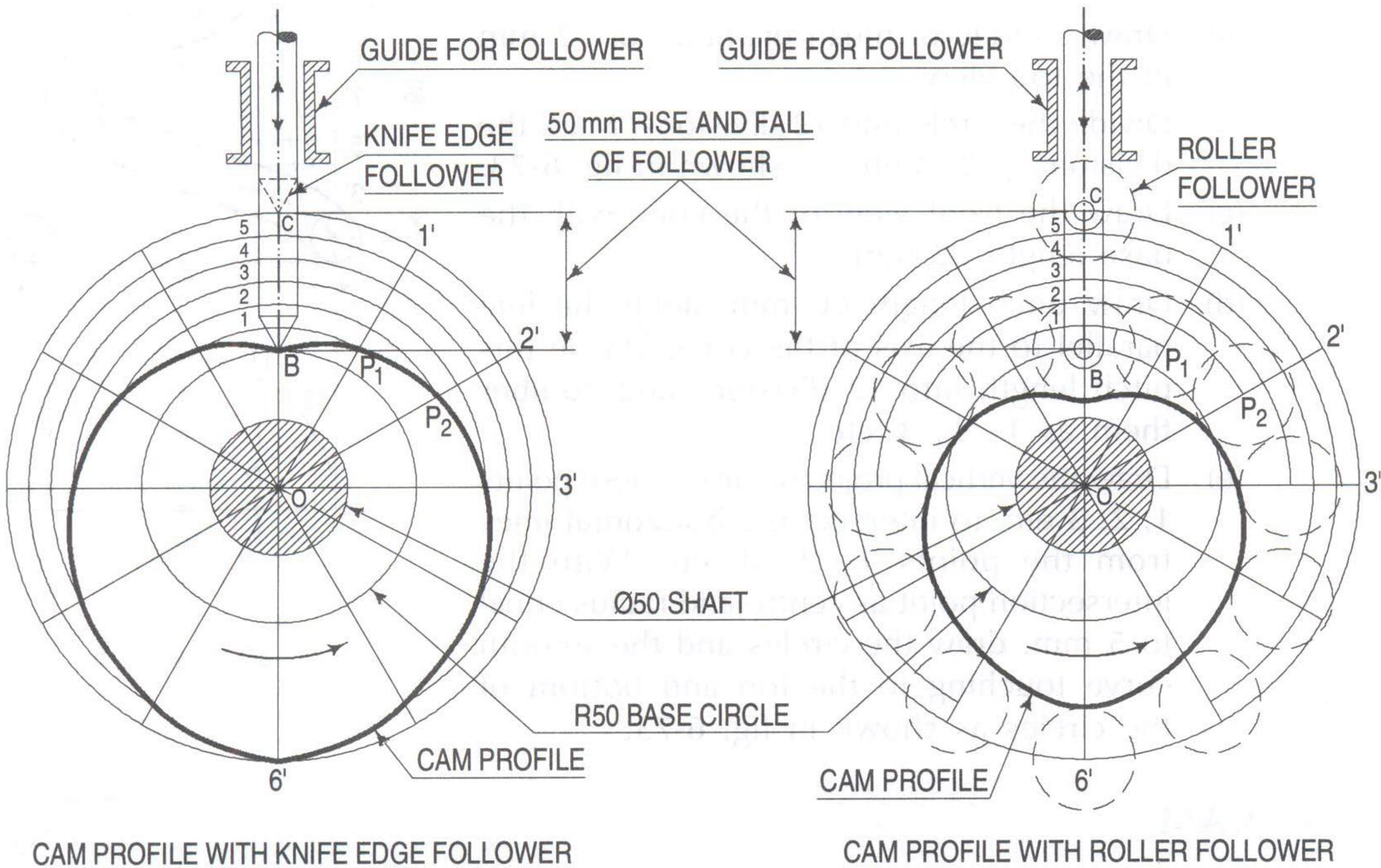
Cams
(not in the syllabus)

Cams

Disc cam is a profile whose radial distance from its centre of rotation varies to meet the required pattern. When a follower, which may be a sharp wedge or a roller ribs on it, moves (translates or oscillated) in the desired manner. 3D cam are also available which are used in various applications such as indexing.

Cams

Plate cam



Cam and follower

Cams

Plate cam - Exercise

The least radius of the cam is 50mm and its shaft diameter is 50mm. The diameter of its following roller is 25mm. Design this cam to give the following cyclically actions to the follower:

- A rise of 40mm over 90° of revolution
- Dwell in the upper position for the next 75° of revolution
- Fall by the same distance during the next 120° of revolution
- In lower most position for the remaining part of revolution

Locus of any Point in a Mechanism
(not in the syllabus)

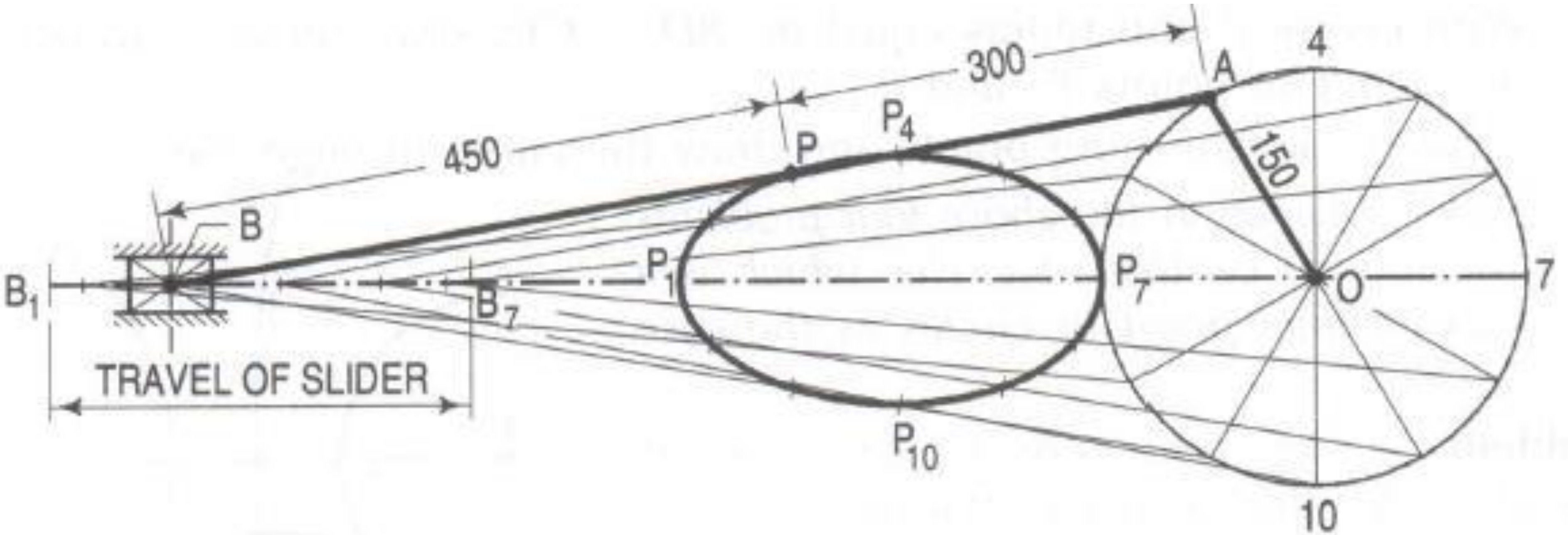
Locus of any Point in a Mechanism

Mechanisms are used to convert one motion into another. It is important to know the locus of any point of a mechanism which may be its hinge (joint) or a point on any link.

- Indexing motions
- Rotary to linear and vice versa
- Uniform rotation into non-uniform rotation
- etc.

Locus of any Point in a Mechanism

A simple and popular mechanism - “slider crank”



Connecting rod length = 750 mm
Crank = 150 mm
Draw the locus of a point P which is on the link AB 300 mm from point A.

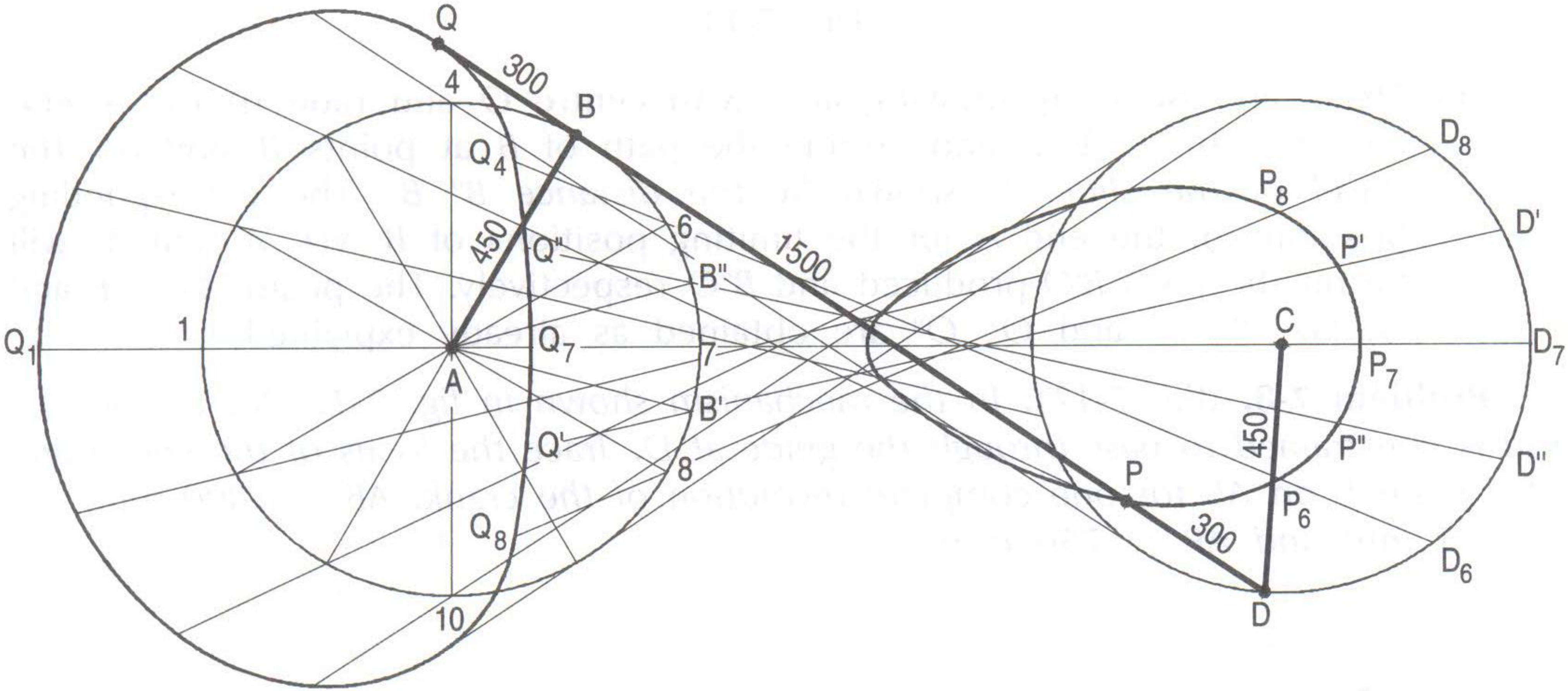
Locus of any Point in a Mechanism

Exercise

Two equal cranks AB and CD connected by the link BD as shown in Figure, rotate in opposite directions. Point P is on BD and point Q is along its extension beyond B. Draw the loci of P and Q for one revolution of AB. AB = CD = 450mm; AC = BD = 1500mm; PD = 300mm; BQ = 300mm.

Locus of any Point in a Mechanism

Exercise …



Conclusions

- Quickly skim through Chapters 1-5 of N.D. Bhat's book that cover the basic things such as
    - Use of drawing instruments
    - Sheet layout and Freehand sketching
    - Lines, Lettering and Dimensioning
    - Scales
    - Geometrical constructions
- Roughly work out all the problems given to you. Only if you come prepared, you will be able to complete all problems of the sheet in the drawing session.


Thank You!