PH-107 (2014)
Tutorial Sheet 5
(Quantum Statistics and Free Electron Theory)

O: Quantum Statistics:

P93: In how many ways three electrons can occupy ten states (the states include spin degeneracy)? Is the number same as the way in which three persons can occupy ten chairs in a room? State the reason. In case the number is different, find the other number also.

P94: A system has three energy states with energies E_1,E_2, and E_3,with respective degeneracies of 1, 4 and 8. In how many ways a distribution of six indistinguishable particles obeying Pauli exclusion principle, can be obtained, where 1 particle has energy E_1, 2 particles have energy E_2 and 3 particles have energy E_3 (call it 1,2,3). With an equal a priory probability, is this distribution more probable than (1,3,2). What are the ratios of the probability?

P95*: Three electrons are to be arranged in three different energy levels of energy 0, E and 2E, with respective degeneracies (including spin degeneracy) 2, 10 and 20. The total energy available is 3E. What are the possible distributions and what are their probabilities?

P96: (a) Using the Fermi Dirac Statistics, find the probability that a state is occupied if its energy is higher than E_F by *0.1 kT, 1.0 kT, 2.0 kT* and *10.0 kT*, where ε_F is the Fermi Energy. How good is the approximation of neglecting *1* in the denominator for an energy equal to *10 kT*.(b) In the Fermi Dirac distribution substitute $E = E_F+\delta$. Compute δ for the probability of occupancy equal to *0.25* and *0.75*. (c) Show that for a distribution system governed by F.D. distribution, the probability of occupation of a state with energy higher than E_F by an amount ΔE is equal to the probability that a state with energy lower than E_F by ΔE is unoccupied.

P97*: Assume that in the limit of large energies, i.e. $\varepsilon \gg \hbar\,\omega_o$, the energy levels of a two dimensional harmonic oscillators are given by the following expression:

$$\varepsilon = \left(n_x + n_y\right)\hbar\omega_o$$

Here n_x and n_y are zero or positive integers. Find an expression for the number of allowed energy states in the energy range ε and $\varepsilon + d\varepsilon$ in the large energy limit.

P98: The energy for a particle in 3-d cubical box is given as follows:

$$\varepsilon = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^{\;2} + n_y^{\;2} + n_z^{\;2}\right)$$

If these levels are going to be occupied by electrons, write the energy values corresponding to the five lowest levels, taking into account the spin degeneracy. If three electrons occupy these states, find out the possible distributions which would yield a total energy of $\frac{18\pi^2\hbar^2}{2mL^2}$. Also find out the probability of each of the distributions.

P99: Find the value of $n\lambda^3$ for bosons at the critical temperature for Bose Einstein Condensation. Here *n i*s number of particles per unit volume and λ is their de Broglie wavelength at critical temperature (assuming equipartition law). Using the above value, express the condition of B E condensation in the form of a relationship between the de Broglie wavelength and the average distance between particles

P100: Find the value of critical temperature at which Bose Einstein Condensation occurs for liquid He^4 (Density 140 kg/m^3). Also find the temperature at which (a) 10% (b) 50% of the He atoms will condense to ground state.

P101*: Use Bose-Einstein Statistics and the density of state expression, with suitable modifications, if any, to derive Planck's formula of black body radiation.

P: Free Electron Theory:

P102*: A copper wire of cross-sectional area $3.3\text{x}\ 10^{-6}\ m^2$ is carrying a current of *25 A*. One meter length of this wire has a resistance of $5.8\text{x}\ 10^{-3}$ *ohms* at room temperature. Calculate the conductivity, average drift velocity of the electrons, electron mobility and mean free path of the electrons in Drude and Sommerfeld models at room temperature. *[Take atomic weight of Cu as 63.5 and density as 8.94 g/cc]*

P103*: For Sodium the conductivity at *300 K* is $2.17\text{x}10^7\ ohm^{-1}m^{-1}$ and the effective mass of the electron is *1.2* times the mass of the free electron. Calculate the relaxation time and the mean free path. Calculate the drift velocity of the electrons in an electric field of *100V/m*. *[Density of Na = 970* kg/m^3*, Atomic weight 23]*

P104*: Show that the kinetic energy of a three dimensional gas of *N* free electrons at *0 K* is $(3/5)N\varepsilon_F$.

P105: Using the data given and any other constants, evaluate the Fermi energy of the alkali metals.

	Li	Na	K	Rb	Cs
Density (g/cc)	0.534	0.971	0.860	1.530	1.870
Atomic weight	6.939	22.99	39.102	85.47	132.905

P106: The Fermi energy of Cu is 7.04 eV. Calculate the velocity and de Broglie wavelength of electrons at the Fermi energy of Cu. Can these electrons be diffracted by a crystal?

P107*: Assuming that Silver is a monovalent metal obeying Sommerfeld model, calculate the following quantities.(a) Fermi energy and Fermi temperature. (b) Radius of Fermi sphere.(c) Fermi velocity. (d) the average energy of free electrons at 0 K. (e) the temperature at which the average molecular energy in the ideal gas will have the same value as the average energy of free electrons at 0 K.(f) the speed of electron with this energy.(g) Mean free path of electrons at room temperature and near absolute zero.(h) the ratio of the Fermi velocity to drift velocity at room temperature in a field of 1 V/cm. [*Given density of Ag= 10.5 g cc ; Atomic wt. of Ag= 107.87; Resistivity of Ag at 295 K= 1.61 x 10^{-6} ohm cm and at 20K= 3.8 x 10^{-9} ohm cm*.]

P108*: Consider a gas of electrons (mass m) confined to a dimensional square box of size a. Find an expression of density of state using periodic boundary conditions and that of Fermi energy at 0^oK.

P109: Find an expression of density of state for free electrons confined within a distance L in one dimension.

P110*: Show that the fraction of electrons within kT of the Fermi energy is $1.5kT/\varepsilon_F$, under the assumption that the temperature is so low that the probability of occupancy of levels is not altered from the one at 0^oK. Calculate numerically the value of this fraction for copper ($\varepsilon_F = 7.04$ eV) at 300^oK and 1360^oK (approximate melting point of Cu). This fraction is of interest because it is a rough measure of the percentage of electrons excited to higher energy states at a temperature T. Find roughly the electronic contribution to specific heat of Cu using this expression.

P111*: In a Hall experiment on Silver, a current of *25 A* passes through a long foil which is *0.1 mm* thick (in the direction of **B**) and *3 cm* wide. What is the Hall voltage for $B = 1.4T$, if the Hall coefficient is $-0.84x10^{-10}\ m^3\ /C$. Given the conductivity of Ag = $6.8x10^7$ *mho/m*, estimate the Hall angle and the mobility of electrons. What would be the values of the Hall voltage, Hall angle and the mobility if the width of the foil is doubled?

P112: For Cu, the Hall coefficient is $R_H = -0.55x10^{-10}\ m^3\ /C$. If the relaxation time $\tau = 2.1x10^{-14}\ s$ and the conductivity $\sigma = 6.0x10^7$ *mho/m*, calculate the mobility of the electron and the ratio of the average effective mass of the electron to the electron mass.

P113*: Consider a metal that follows the Sommerfeld model exactly and has a Fermi Energy as 5.0 eV. A Hall Effect experiment is planned on this metal. For this purpose a strip of metal is made, the length of which is 8 cm, the width is 2 cm and thickness is 0.1mm. Initially a current is passed lengthwise through this metal strip in the absence of the magnetic field. The potential drop across the length is found to be 10 mV.

Now a magnetic field of 2 T is applied along the thickness of the foil. A potential drop of 32 micro volts is measured across the width of the strip. Find the relaxation time, the conductivity and the Hall coefficient of the metal. Also find the value of current in the foil.