PH-105 Assignment Sheet - 1

Ashwin P. Paranjape

4. An inertial frame S' moves relative to another frame S with a velocity $v_1\hat{i} + v_2\hat{j}$ in such a way that x and x' axes and y and y' axes and z and z' axes are always parallel. Let the time $t = t' = 0$ when the origin of the frames are coincident. Find the lorentz transformation relaing the coordinates and time of S' to those in S.

Solution :
As we know the Lorentz transformation when relative velocity is along x axis, we need to convert the coordinates of the two given frames S and S' to S_R and S'_R respectively. Here the frames S_R and S'_R are rotated frames such that the relative velocity between these frames is along the x_R and x'_R axes. Let $\theta = tan^{-}1(v_2/v_1)$ and v be the magnitude of velocity vector. Thus angle between S_R and S is θ, S'_R and S' is θ.

To get S_R from S, we take take components of x and y axes along x_R and y_R. Note that time coordinate does not change due to rotation.

$$x_R = xcos\theta + ysin\theta$$

$$y_R = ycos\theta - xsin\theta$$

$$t_R = t$$

Now to get x'_R and y'_R we use lorentz transformation.

$$x'_R = \gamma(x_R - vt_R) = \gamma(xcos\theta + ysin\theta - vt)$$

$$y'_R = y_R = ycos\theta - xsin\theta$$

$$t'_R = \gamma(t_R - \frac{vx_R}{c^2}) = t - \frac{v(xcos\theta + ysin\theta)}{c^2}$$

To get x' and y' we use take components of x'_R and y'_R on x' and y'

$$\begin{aligned}
x' &= x'_R cos\theta - y'_R sin\theta \\
&= \gamma(xcos\theta + ysin\theta - vt)cos\theta - (ycos\theta - xsin\theta)sin\theta \\
&= x(\gamma cos^2\theta + sin^2\theta) + y(\gamma sin\theta cos\theta - cos\theta sin\theta) - \gamma vtcos\theta \\
y' &= y'_R cos\theta + x'_R sin\theta \\
&= (ycos\theta - xsin\theta)cos\theta + \gamma(xcos\theta + ysin\theta - vt)sin\theta \\
&= (ycos^2\theta - xsin\theta cos\theta) + \gamma(xcos\theta sin\theta + ysin^2\theta - vtsin\theta) \\
&= y(cos^2\theta + \gamma sin^2\theta) + x(\gamma cos\theta sin\theta - cos\theta sin\theta) - \gamma vtsin\theta \\
t' &= t'_R \\
&= \gamma(t - v\frac{xcos\theta + ysin\theta}{c^2})
\end{aligned}$$