## Sommerfeld Model

- Used all the assumption of Drude Model but applied Quantum Mechanics and Quantum Statistics to the problem.
- Assumed the energy states of a free electron in metal are *similar* to that of a particle in 3-d box.
- Electrons obey F.D. statistics.

## Periodic Boundary Conditions

- We take a different boundary conditions so that solutions are travelling wave instead of the usual stationary waves.
- This helps in describing the motion of electrons, which has to be assigned a direction.

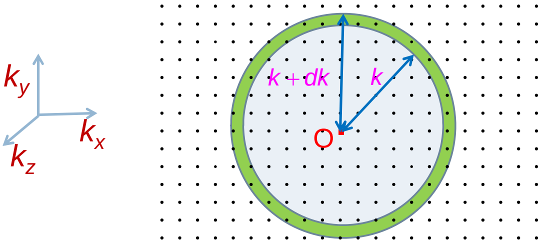
## Born and von Karman B.C.

$$\phi(x,y,z+L)=\phi(x,y,z)$$
$$\phi(x,y+L,z)=\phi(x,y,z)$$
$$\phi(x+L,y,z)=\phi(x,y,z)$$

$$A_x\sin\left[k_x(x+L)\right]+B_x\cos\left[k_x(x+L)\right]=$$
$$A_x\sin[k_x x]+B_x\cos[k_x x]$$
$$\Rightarrow k_x=\frac{2n_x\pi}{L}$$

Similarly

$$k_y=\frac{2n_y\pi}{L}$$
$$k_z=\frac{2n_z\pi}{L}$$




$$g(k)dk = 2\times\left(\frac{L^3}{2\pi}\right)\times 4\pi k^2 dk$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$g(E)dE = \left(\frac{V}{2\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} dE$$

## Number of particles *n(E)dE*

$$g(E)dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} dE$$

$$n(E)dE = \frac{g(E)dE}{e^{\frac{E-E_F}{kT}}+1}$$

## At T=0

$$N = \int_0^{E_F} g(E)dE$$

$$g(E)dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} dE$$

$$N = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \frac{2}{3} E_F^{3/2}$$

## Fermi Energy at T=0

$$N = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\frac{2}{3}E_F^{3/2}$$

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{\frac{2}{3}}$$

## Fermi Energy for Sodium

Density=0.971 g/cc
Atomic Weight=22.99

$$\frac{N}{V} = \frac{6.02\times10^{23}\times0.971}{22.99}\times10^6$$
$$= 2.543\times10^{28}$$

Substituting we get
$E_F$=3.16 eV

## Fermi Energy at Finite Temperature

$$N = \int_0^\infty n(E)dE = \int_0^\infty \frac{g(E)dE}{e^{\frac{E-E_F(T)}{kT}}+1}$$

$$g(E)dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E^{1/2}dE$$

$$E_F(T) \approx E_F(0)\left[1-\frac{\pi^2}{12}\left(\frac{kT}{E_F(0)}\right)^2\right]$$

To a good approximation be treated independent of temperature for metals at least at room temperature.

## Fermi Energies in eV

| Metal | $E_F$ |
|---|---|
| Li | 4.72 |
| K | 2.14 |
| Rb | 1.82 |
| Cs | 1.53 |
| Cu | 7.02 |
| Ag | 5.51 |

## Observations

- Fermi Energy is around two orders of magnitude higher than the value of $kT$ at room temperature.
- Even though the energy of the system at $T=0$ is much higher than the average energy of classical gas at room temperature, its increase with $T$ is quite small.

- The free electron contribution to specific heat is, therefore, quite small.
- It can be shown that the free electron contribution to specific heat is proportional to $T$ and hence can be seen at low temperatures.

$$C_v(FE) = \frac{\pi^2}{2}\left(\frac{kT}{\varepsilon_F(0)}\right)R$$

## Conduction

- Only electrons close to Fermi sphere take part in conduction.
- The Fermi velocity being quite large can explain the conduction, but mean free path also turns out to be larger than Drude model. In Sommerfeld model

$$\ell = v_F \times \tau$$