1989 experiment by Tonomura

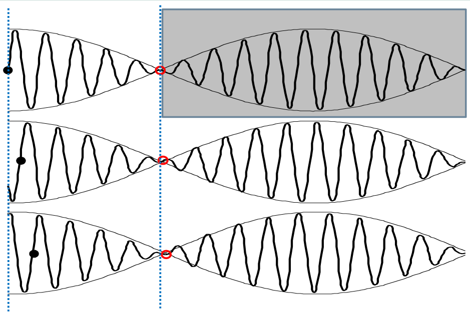
http://www.hitachi.com/rd/research/em/doubleslit.html

Order of Uncertainty Product

Wave Packet Approach

- Go back to old beats' problem.
- Assume that one modulation of the wave packet can be created by mixing waves of not just two wave lengths but all the wave lengths between k and $k+\Delta k$.

Beats



$$\Delta p = \Delta\left(\frac{h}{\lambda}\right) = \Delta\left(\frac{2\pi h}{2\pi\lambda}\right) = \hbar\Delta k$$

Position Uncertainty can be evaluated from the 'wave length' of 'modulation'.

$$\Delta x = \frac{1}{2}\left(\frac{2\times 2\pi}{\Delta k}\right) = \frac{2\pi}{\Delta k}$$

Estimate of Uncertainty Product

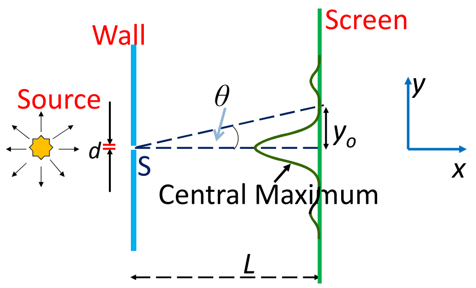
$$\boxed{\Delta x \Delta p \sim h}$$

Order of Uncertainty Product

Wave-Particle Duality Approach

- Consider a single slit diffraction of a beam of mono-energetic particles.
- Interpret this on the basis of particle model and find the order of the uncertainty product.

Single Slit Diffraction



Consequences

1. The smaller is the slit width, larger is the width of the central maximum.
2. The experiment can be understood well by the wave theory.
3. Now let us bring the particle nature assuming that the experiment is being performed with particles.

Particles Approaching

- Assume the source is infinite distance away. Therefore, electrons move in x-direction.

$$\Delta p_y = 0 \Rightarrow \Delta y = \infty$$

- Hence we are nor sure of the y-position of the particle at all.
- What happens when the particle passes through the slit?

Particles Passing through the Slit

- Δy is suddenly made finite. Hence Δp_y can also not be zero any more.
- Hence the particles would pick up a y-component of momentum in a totally unpredictable and uncontrolled manner.

- Let us use the wave formula to predict the order of uncertainty product.
- Let us assume that the electrons land themselves only in the central maximum of the diffraction pattern.

The time taken by the electron to reach the screen after passing through the slit is given by.

$$\frac{L}{v_x}$$

During this time the electron also gets displaced along y-direction due to the random velocity acquired after passing through the slit, due to uncertainty principle.

If v_y is the y-velocity component needed to take an electron to the end point of the central maximum distance y_o away from centre, then

$$y_o = \frac{Lv_y}{v_x} = \frac{Lp_y}{mv_x}$$

Using wave Diffraction Formula

$$d\sin\theta = \lambda$$

Assuming θ to be small

$$\theta d = \lambda \Rightarrow \frac{y_o}{L}d \approx \frac{h}{mv_x}$$

This gives

$$y_o = \frac{hL}{mv_x d}$$

Uncertainty Product

Equating the two values of y_o

$$\frac{Lp_y}{mv_x} = \frac{hL}{mv_x d} \Rightarrow p_y d = h$$

Take approximately d as the uncertainty in y and p_y as the uncertainty in momentum.

$$\Delta y \Delta p_y \sim h$$

Problem

In case one is able to ascertain in Young's double slit experiment that electron is able to pass only through one of the two slits, what should be the maximum allowed uncertainty in y co-ordinate? Show that this uncertainty will produce an uncertainty in the y component of momentum, that would be enough to destroy the interference pattern.

Uncertainty Defined

Uncertainty in any Physical Quantity G is defined as its standard deviation.

$$\Delta G = \sqrt{\left\langle \left(G - \langle G \rangle \right)^2 \right\rangle}$$

Minimum Uncertainty Product

The actual minimum uncertainty depend on the wave packet. But is can be shown that uncertainty product has always to obey the following relationship irrespective of any wave packet type.

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

Various Uncertainty Principles

The wave packet has to be localized in time also. An atom spends a very short time in the excited state before spontaneous emission. Hence the time spread of the emitted wave packet can not be larger than the life time of the state, causing an uncertainty of the energy.

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}; \Delta y \Delta p_y \geq \frac{\hbar}{2}; \Delta z \Delta p_z \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Resonant Absorption

Can a photon emitted as a result of a transition cause a similar reverse transition?