Special Theory of Relativity Portion-I (Tutorial-I)

1. Two observers A and B are close to a point where lightning strikes the earth. According to A, a second lightning strikes t_o seconds later at a distance *d* from him. B, on the other hand, finds the two events to be simultaneous, find his velocity with respect to A. Also find the distance between the two lightnings as seen by B. Assume earth to be inertial frame of reference.

[Ans.: $c^2 t_o / d$, $\sqrt{d^2 - c^2 t_o^2}$]

2. A meter stick is positioned so that it makes an angle 30° with the *x*-axis in its rest frame. Determine its length and its orientation as seen by an observer who is moving along the *x*-axis with a speed of 0.8c. [Ans.: 43°54']

3. An observer A sees two events at the same place and separated in time by 10^{-6} s. A second observer B sees them to be separated by 2×10^{-6} s. What is the separation in space of the two events according to B? What is the speed of B with respect to A?

4. An inertial frame S′ moves relative to another frame S with a velocity $v_1\hat{i} + v_2\hat{j}$ in such a way that the *x* and *x*′ axes, y and y′ axes and z and z′ axes are always parallel. Let the time $t = t'$ =0 when the origin of the two frames are coincident. Find the Lorentz transformation relating the coordinates and time of S′ to those in S.

[Ans.: $x' = x(\gamma\cos^2\theta + \sin^2\theta) + y(\gamma\cos\theta\sin\theta - \cos\theta\sin\theta) - \gamma vt\cos\theta$,

$y' = x(\gamma\sin\theta\cos\theta - \sin\theta\cos\theta) + y(\gamma\sin^2\theta + \cos^2\theta) - \gamma vt\sin\theta$,

$t' = \gamma\left(t - v\dfrac{x\cos\theta + y\sin\theta}{c^2}\right)$]

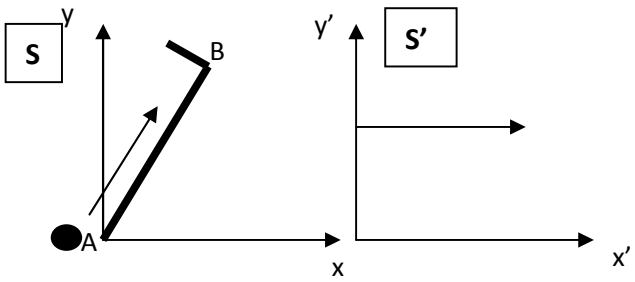
5. An observer is sitting in a train (call it frame S′ and assume that the observer is sitting at the origin). The train moves with a constant speed of 0.6c relative to ground (assume ground to be inertial frame S). Another observer is sitting at the origin of S. The two observers set the watches to zero time at the instance they cross each other. At time $t' = 2\times10^{-6}$ s, the observer in the train finds himself passing a building "A" on the ground. Also in the train frame at time $t' = 2\times10^{-6}$ s, lightning strikes the ground at a place where the origin of S is situated. Find the distance of train from origin when lightning struck, according to the observer on ground. Also find the distance of the building "A" from origin in S and the time the train would reach building "A", in S frame?

6. A rod AB of proper length 3 m is stationary in an inertial frame S making an angle of 60° with positive *x*-direction and 30° with positive *y*-direction as shown in the figure. In this frame a bullet is fired parallel to the rod from point A towards point B with a speed of 0.8c. This bullet hits a stopper at the end of the rod. Imagine another observer S′ is moving with a speed of 0.6 c along the x-axis of frame S. Find

(a) The time taken by the bullet to reach the stopper in frames S and S′.

(b) The *x*' and *y*' components of the velocity of the bullet in S'.

(c) According to the observer S', will the bullet hit the stopper or not? Substantiate your answer with appropriate calculations.



7. Two observers A and B, sitting at the origin of their respective inertial frames are moving with respect to each other with a relative speed of 0.6c. The observer A sends a light signal to observer B, 3×10^{-6} s after B passes A, as measured in A's frame. The observer B receives this light signal. He waits for 2×10^{-6} s after receiving the light signal as per his watch and then sends a light signal back to A.

(a) Find the time when the light signal is received by B as per the watches of A and B.

(b) Find the time when the light signal is received back by A as per the watches of A and B.

(c) What was the distance of B from A when B had sent the light signal according to A and B?

(d) What was the time taken by light to reach A after it was emitted from B, according to A and B?

8. An errant firecracker maniac cruising at 3×10^7 m/s in the positive x- direction sets off a firecracker at x= 20 km from the police headquarters (HQ) located at x=0. At the moment the HQ receives the resulting light signal, it instructs a patrol car just then passing in the +x direction under its window and cruising in the same direction at 6×10^7 m/s to catch the rogue. Assume that at this moment both HQ and patrol car clocks show time equal to zero. (i) What are the position and time of the firecracker burst in the frame of patrol car? (ii) Find the position of the boy in HQ frame at t=0 and in patrol car frame at t'=0. (iii) Find the time in HQ frame and the patrol car frames when the car reaches the maniac.

[Ans.:24.49 km, -81.65 μs, 22km, 21.995km,733.3 μs, 718.52 μs]

9. An observer O sees another observer A pass by him with a velocity v_1. At this instant, the watches of O and A read zero. After time t_1, O sees another observer B passing by him with velocity v_2. Sometimes later, B catches A. At this instant watch of O reads 245μs and watch of A reads 173 μs. According to B, the time difference between passing O and catching A is 100 μs. Assume that the observers O, A

and B are at the origins of their respective frames, calculate v_1, v_2 and t_1. Also calculate the relative velocity of B and A and the time in A's frame when B passes O.

[Ans.: 0.71c, 0.87c, 45 μs, 63.73 μs]

10. A light beam is propagating through a block of glass with index of refraction 1.2. If the glass is moving at a constant velocity 0.8c in the same direction as beam what is the velocity of light in the block as observed by an observer in the laboratory? [Ans.: 0.98c]

11. Two identical rods of proper length 5 m move in opposite direction to each other as seen in a frame S. While the speed of rod A is 0.6c in + *x* direction, the same of rod B is 0.8c in – *x* direction. Find

(a) The time taken by rods to cross each other in frame S.

(b) The time taken by rods to cross each other in A's frame.

(c) The time taken by rods to cross each other in B's frame.

(d) The least possible time in which the rods would cross each other in any frame.

(e) The speed and direction of a frame as seen in S for which the time taken by the rods to cross each other would be the least.

12. In a star war sequence three spaceships A, B and C are involved. In the frame of A the spaceship B is moving with a speed of 0.6c along +ve *x* direction and C with a speed of 0.75c along +ve *y* direction. At an instant, the distances of B and C from A are 360 and 450 km respectively, in A's frame. At the same instant, a war begins between B and C and both fire at each other simultaneously in A's frame. (a) Find the velocity of C in B's frame. (b) what is the time difference between B firing a bullet and C firing a bullet in B's frame and C's frame. (c) what is the proper time interval between B and C firing bullets at each other?

13. An observer in an initial frame S notices three events, the co-ordinates of which are given as follows.

E1: (x = 0, y = 0, z = 0, t = 0)

E2: (x = 9 km, y = 0, z = 0, t = 5×10^{-5}s)

E3: (x = 6 km, y = 3 km, z = 0, t = 1×10^{-5} s)

(a) Find the proper time interval between events E1 and E2. Also find the proper time interval between events E1 and E3.

(b) Is it possible to find a frame S′ moving along +*x*-direction of S, in which events E1 and E2 would occur at same place or same time? If yes find the speed of that frame relative to S. Please specify clearly, whether in frame S′ events are occurring at the same place or same time or both.

(c) Imagine another frame S″ moving along +*x*-direction of S. In the S″ frame, the difference $\Delta x''$ between the x'' coordinates of the events E1 and E3 is found to be $3\sqrt{3}$ km. Find $\Delta t''$, the time difference between events E1 and E3 as seen in the frame S″.

14. A particle of mass 'm' is moving with a constant velocity in the *x*-*y* plane of an inertial frame S. The observer in S finds that the particle travels a distance of 600 m in a time of 2.5×10^{-6}s. During this time both the *x* and *y* co-ordinates of the particle increase. The increase in the *x* co-ordinate is 300 m during the given duration.

(a) Find the proper time difference for the displacement described above.

(b) Find the components of the displacement four vector $\Delta\underset{\sim}{S}$ and the momentum four vector $\underset{\sim}{p}$.

(c) Find the components of the displacement four vector $\Delta\underset{\sim}{S}'$ and the momentum four vector $\underset{\sim}{p}'$ in a frame S′, which moves with a speed of 0.5c along +*x* direction of S.

15. A particle of rest mass m_o has a energy $4\ m_oc^2$. Find the momentum in the units of m_oc. What is the energy of the particle in a frame in which its momentum is $2\ m_oc$?

[Ans.: $\sqrt{15}\ m_o\ c$, $\sqrt{5}\ m_o\ c^2$]

16. A spaceship is moving away from an observer B on earth at a velocity 0.6c along +ve x-direction. The spaceship has a gun shooting the particles at a velocity 0.8c along the +ve x-direction relative to the spaceship. (a) Find out the velocity of the particles as seen by the observer B on earth. (b) Find out the kinetic energy measured by the observer B and an observer in the spaceship.

17. Find the minimum energy a neutrino must have to initiate the reaction $\nu + n \rightarrow \tau + p$ if the target neutron is at rest (rest energy of neutrino = 0 GeV, rest energy of neutron = 1 GeV, rest energy of proton = 1 GeV, rest energy of τ = 2 GeV).

18. In lab frame, a π particle collides with a proton at rest. A Δ particle is produced accordingly to the reaction $\pi + p \rightarrow \Delta$. The rest mass energies of these particles are $m_\pi c^2$ = 0.14 GeV, $m_p c^2$ = 0.94 GeVand $m_\Delta c^2$ = 1.24 GeV. Calculate the energy of the pions which produces the Δ particles. Calculate the speed of the Δ particles in the lab frame. Calculate the energy and momentum of the pions in the frame in which the Δ particle is at rest.

19. A proton of total energy 2 GeV (2×10^9 eV) moving along + *x* direction collides with another proton at rest. If this collision is viewed in the centre of mass frame of reference, it is found that one of the protons moves at an angle of 45 degrees from the initial direction of motion in *x*-*y* plane. Find the energy (in GeV) and the *x* and *y* components of the momentum of the two protons (in GeV/c) in the centre of mass frame and the original frame. What is the angle which the incoming proton makes after scattering in the original frame with the incident direction of motion? (Take rest mass of proton as 1 GeV)

20. A neutrino (ν), assumed to be of zero rest mass is incident on a neutron (n) at rest. After the reaction only two particles, a τ particle and a proton (p) are found. Take the rest mass energies of both neutron and proton to be 1 GeV and that of τ particle to be 2 GeV.

(a) Find the minimum energy that the neutrino must have, for this reaction to be possible.

(b) Assuming that the reaction takes place with the least possible energy of neutrino, find the speeds, energies and momenta of neutron and the neutrino in the centre of mass frame of reference.