QM Tutorial Sheet 3 Q.74

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Solution :

For $x < 0$, $V = 0 < E = 9V_0$. Hence wavefunction is given by

$$\phi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}, k_1^2 = \frac{2m(9V_0)}{\hbar^2}$$

For $0 < x < d$, $V = 5V_0 < E = 9V_0$. Hence wavefunction is given by

$$\phi_2(x) = Ce^{ik_2x} + De^{-ik_2x}, k_2^2 = \frac{2m(4V_0)}{\hbar^2}$$

Irrespective of the third region potential, the boundry conditions here hold. Thus at $x = 0$,

$$\phi_1(0) = \phi_2(0) \Rightarrow A + B = C + D$$

$$\phi_1'(0) = \phi_2'(0) \Rightarrow Ak_1 - Bk_1 = Ck_2 - Dk_2$$

$$\phi_3(x) = Ee^{ik_3x}, k_3^2 = \frac{2m(9-n)(V_0)}{\hbar^2}$$

(No component in -ve x direction) This holds for both $n > 9, n < 9n = 9$ Solving at $x = d, k_2d = \pi$, we have

$$\phi_2(d) = \phi_3(d) \Rightarrow -C - D = Ee^{ik_3d}$$

$$\phi_2'(d) = \phi_3'(d) \Rightarrow -C + D = E\frac{k_3}{k_2}e^{ik_3d}$$

$$\Rightarrow A = (-1 - (k_3/k_1)Ee^{ik_3d})/2$$

Back substituting all values, we have

$$D = A\frac{(k_2 - k_3)k_1}{(k_1 + k_3)k_2}$$

$$C = A\frac{(k_2 + k_3)k_1}{(k_1 + k_3)k_2}$$

$$B = A\frac{(k_1 - k_3)}{(k_1 + k_3)}$$

Transmission Coefficient is given by $\frac{|E|^2 k_3}{|A|^2 k_1} = 0.75$

$$\Rightarrow \frac{4k_3}{k_1(1 + k_3/k_1)^2} = 0.75$$

$$\Rightarrow \lambda = \frac{3}{16}(1 + \lambda)^2$$

$$\lambda = 3 \textbf{ or } 1/3$$

$$\frac{9 - n}{9} = 9 or \frac{|9 - n|}{9} = 1/9$$

$$n = 8 \textbf{ or } - 72$$

These coeffs can be subtituted to get all wavefuntions in terms of amplitude of incident wave.

Now,

$$B = A\frac{(k_1 - k_3)}{(k_1 + k_3)}$$

To get the phase change between incident and reflected beam, we consider $Im(B/A)$

$$= Im((\frac{k_1 - k_3}{k_1 + k_3})$$

$$= Im(\frac{3 - \sqrt{(9 - n)}}{3 + \sqrt{(9 - n)}})$$

For n¡9 it is 0. For n¿ 9, we have

$$= Im(\frac{3 - i\sqrt{(n - 9)}}{3 + i\sqrt{(n - 9)}}$$

$$= Im(\frac{(3 - i\sqrt{(n - 9)})^2}{n})$$

$$= \frac{-6\sqrt{(n - 9)}}{n}$$

for n=9, above expression gives B=A, hence no phase change