QM Tutorial Q.14

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Consider Compton Scattering. Show that if the angle of scattering θ increases beyond a certain value θ_0, the scattered photon will never have energy larger than $2m_oc^2$, irrespective of the energy of the incident photon. Find the value of θ_0.

Consider the equation relating the wavelengths of incident and scattered photon-
$\lambda_f - \lambda_i = \frac{h(1-cos\theta)}{m_0c}$
Using the relation $E_{photon} = \frac{hc}{\lambda}$, we replace the photon wavelengths by their energies and divide by hc to get

$\frac{1}{E_f} - \frac{1}{E_i} = \frac{(1-cos\theta)}{m_0c^2}$ Rearranging, we get

$E_f = \frac{E_i \times m_0c^2}{E_i \times (1-cos\theta) + m_0c^2}$

Dividing the numerator and denominator by E_i, we get

$E_f = \frac{m_0c^2}{(1-cos\theta) + \frac{m_0c^2}{E_i}}$

Clearly, the maximum value for a fixed θ this expression can have is $\frac{m_0c^2}{1-cos\theta}$, that is when $E_i \to \infty$ (since $E_i > 0$). Thus, no matter what the initial energy of the photon may be, there is a value of $\theta = \theta_0$ for which $E_f < 2m_0c^2$. To calculate θ_0, we put

$E_f = \frac{m_0c^2}{1-cos\theta} < 2m_0c^2 \Rightarrow \theta > 60^\circ$

Thus, $\underline{\theta_0 = 60^\circ}$