

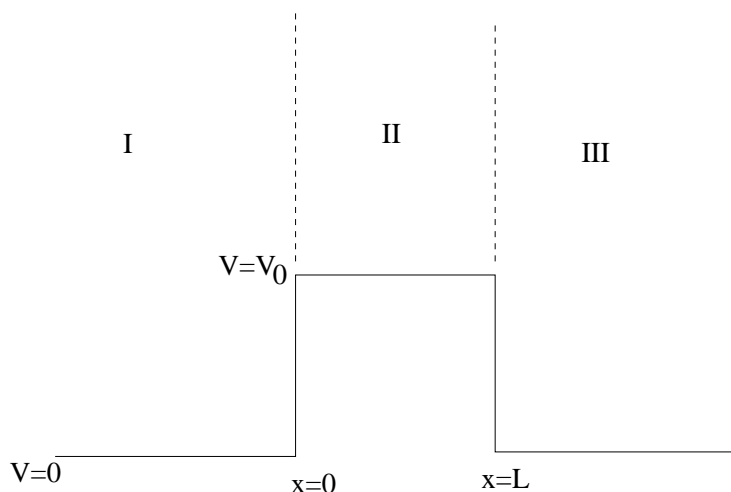
PH-105 QM Sheet 2

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71. A beam of particles of energy E and de-Broglie wavelength λ , travelling along the positive x -axis in potential free region, encounters a one-dimensional potential barrier of height $V_0=E$ and width L .
- (a) Obtain an expression for the transmission coefficient.
- (b) For what value of L (in terms of λ), will the reflection coefficient be half?

Solution :



Let $k^2 = \frac{2mV_0}{\hbar^2}$. Then, the wave-functions in the three regions are given by:

$$\phi_I(x) = Ae^{ikx} + Be^{-ikx}$$

$$\phi_{II}(x) = Cx + D$$

$$\phi_{III}(x) = Fe^{ikx}$$

At $x=0$, we have $A + B = D$ and $ik(A - B) = C$. At $x=L$, we have $CL + D = Fe^{ikL}$ and $C = ikFe^{ikL}$.

On solving, we get $\frac{F}{A} = \frac{2e^{-ikL}}{2 - ikL}$

Transmission coefficient, $T = \left|\frac{F}{A}\right|^2 = \frac{4}{4 + (kL)^2}$.

For $T = 0.5$, we get $kL = 2$ and hence $L = \frac{2}{k} = \frac{\lambda}{\pi}$.