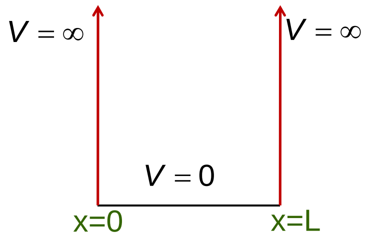
Example: *Particle in a Rigid Box*

$$V(x) = 0 \text{ for } 0 < x < L$$
$$= \infty \text{ for } x < 0 \text{ or } x > L$$



$\phi = 0$ For x<0 and x>L

$$\left.\begin{aligned} \frac{d^2\phi(x)}{dx^2} + \frac{2mE}{\hbar^2}\phi(x) = 0 \\ \frac{d^2\phi(x)}{dx^2} = -\frac{2mE}{\hbar^2}\phi(x) \end{aligned}\right\} \text{For } 0<x<L$$

The general solution of the latter equation is

$$\phi(x) = A\sin(kx) + B\cos(kx);\ k^2 = \frac{2mE}{\hbar^2}$$

Note: *k* is real

Boundary Conditions

The wave function must be continuous. This implies that

$$\phi(0) = \phi(L) = 0$$
$$\phi(0) = 0 \Rightarrow A\sin(0) + B\cos(0) = 0$$

This gives $B = 0$

$$\phi(L) = 0 \Rightarrow A\sin(kL) = 0$$

This is possible only when

$$kL = n\pi$$

$$\frac{\sqrt{2mE}}{\hbar} L = n\pi \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$n \geq 1$$

Notes

- We had two unknowns with three equation (including normalization condition). The additional equation quantized the energy levels.
- The values of *n* can only be positive and non-zero.
- The lowest energy is non-zero (zero-point energy).

The lowest energy corresponding to *n=1* for an electron in a box of 1Å comes out to be 37.6 eV. For a marble of 0.01 kg in 0.1m box it is 5.488×10^{-64}J. It would take 10^{20} years to move one mm.