Introduction to Quantum Statistics

- We had initiated the course with problems involving statistics.
- Now with our quantum mechanical background, we should have a relook at this issue.

- Let us imagine that we have a very large number of particles (say around Avogadro's number).
- Imagine a box which is of macroscopic dimensions.

- Assume that the energies are given by particle in a three dimension box as given below.
- In some cases we use slightly different boundary conditions.

$$E = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x{}^2 + n_y{}^2 + n_z{}^2\right)$$

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$$= 3.76\times10^{-17}\left(n_x{}^2 + n_y{}^2 + n_z{}^2\right)\ \text{eV}$$

for an electron with L=0.1m

$$= 2.04\times10^{-20}\left(n_x{}^2 + n_y{}^2 + n_z{}^2\right)\ \text{eV}$$

for a proton with L=0.1m

- The energy levels thus are fairly closely spaced almost forming continuum .
- However, the number of levels in a given energy range are finite though quite large.

Assumption

- Energy levels do not get disturbed when more than one particle is present. Ignore particle-particle interaction.

Problem

- If we give particles some energy, say in the form of heat, how these particles would occupy these states.
- Finally assume that the system to be at a finite temperature and that is the form of energy being given to them.

- Find the number of particles having a particular energy.
- This number depends on the characteristics of the particles that are going to occupy the states.

Type of Particles

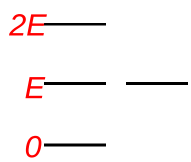
1. Classical Particles: Distinguishable and do not obey Pauli Exclusion Principle.

2. Quantum Particle 1 (Bosons): Indistinguishable, Do not obey Pauli Exclusion Principle. These are particle with integral spins.

3. Quantum Particle 2 (Fermions): Indistinguishable, Obey Pauli Exclusion Principle. These are particle with non-integral spins.

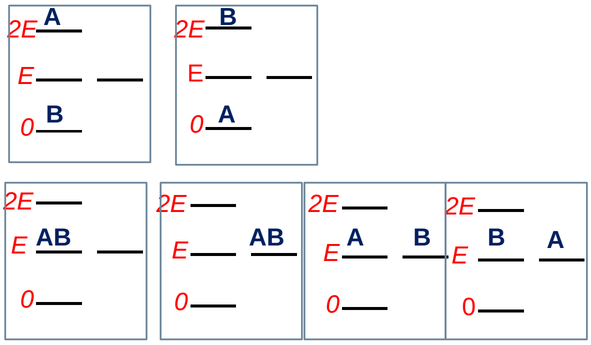
Example

- Two particles.
- Three energy values *0*, *E*, *2E*.
- Four levels, degeneracy *1*,*2*,*1*
- Total energy *2E*.



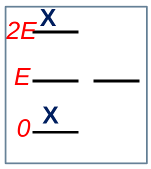
A distribution would mean the number of particles having a particular energy, irrespective of the state corresponding to that energy.

First type

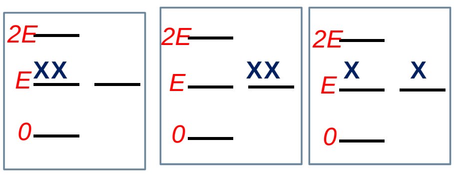


Distribution

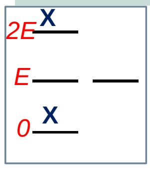
- Assume Equal a-priori probability.
- (0,2,0) is twice probable than (1,0,1).



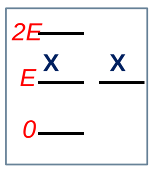
(0,2,0) is thrice probable than (1,0,1)



Third Type



(0,2,0) is equally probable to (1,0,1)



General Problem

- Given a fixed amount of energy and a fixed number of particles.
- Given a set of energy levels E_i, with degeneracies g_i.
- What is the most probable set of n_i?
- Finally assume that the energy is given by heat at a temperature T.