Introduction to (Solid) Modeling

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Outline

- Some Preambles
- Wire, Surface and Solid Representations
- Various Solid Representation Schemes
- STL Format
- Conclusions

Introduction

Parametric Definition

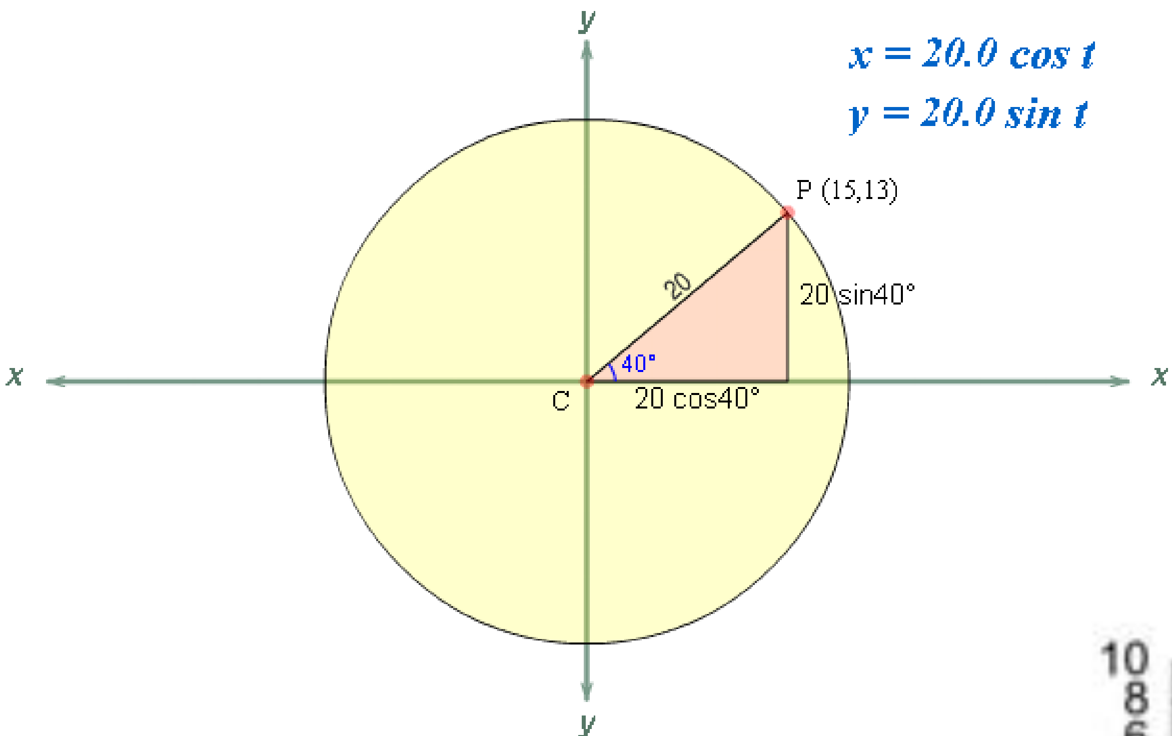
Entity	Number of Parameters	Generic Point of the Entity **p**
Point	0	$\mathbf{p} = [x, y, z]$ $\mathbf{p} = [20, -5, 30]$
Curve	1	$\mathbf{p}(u) = [x(u), y(u), z(u)]$ helix: $\mathbf{p}(u) = [r \cos(u), r \sin(u), lu/2\pi]$
Surface	2	$p(u , v) = [x(u, v), y(u , v), z(u, v)]$ Cylinder: $\mathbf{p}(u, v) = [r \cos(u), r \sin(u), v]$
Solid	3	$\mathbf{p}(u , v , w) = [x(u, v , w), y(u , v , w), z(u, v, w)]$ $x \leq r \cos(u), y \leq r \sin(u), v_1 \leq z \leq v_2$
Swept volume	4-1	$\mathbf{p}(u , v , w , t) = [x(u, v , w , t), y(u , v , w , t), z(u, v, w, t)]$

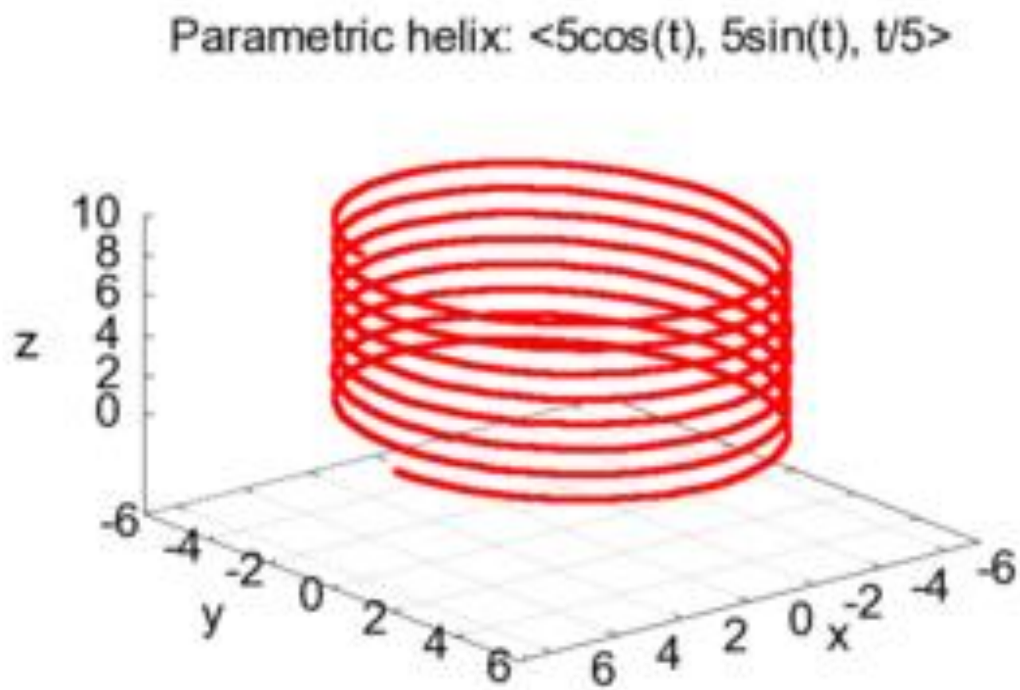
Introduction

Parametric Definition …

$x = 20.0 \cos t$

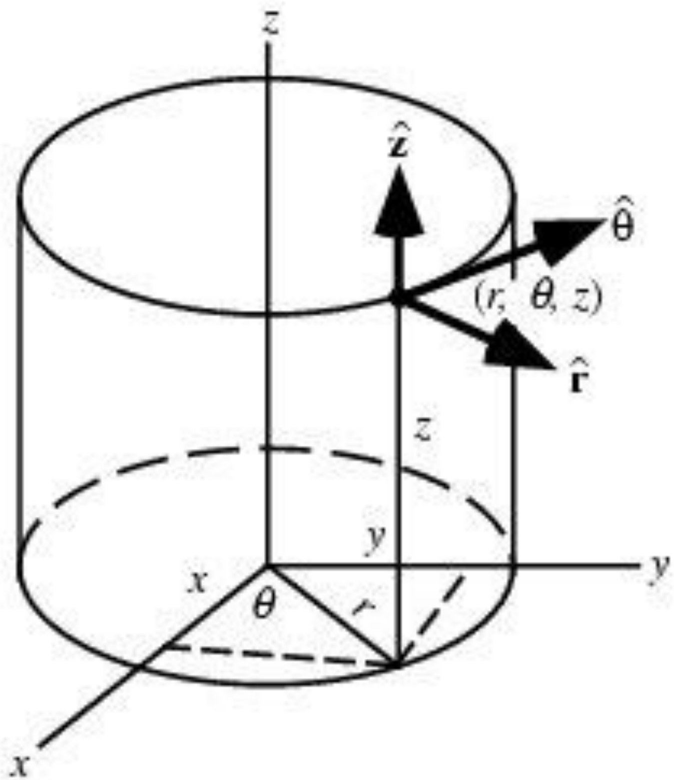
$y = 20.0 \sin t$

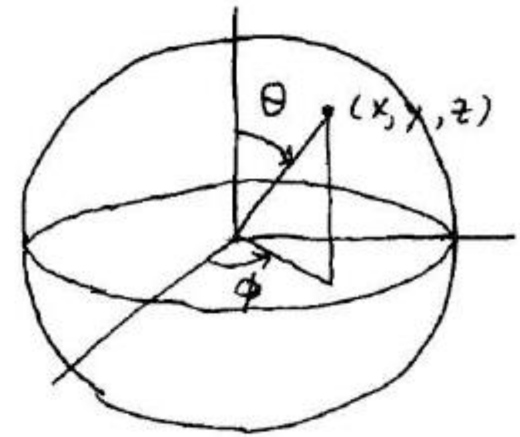




Introduction

Parametric Definition …



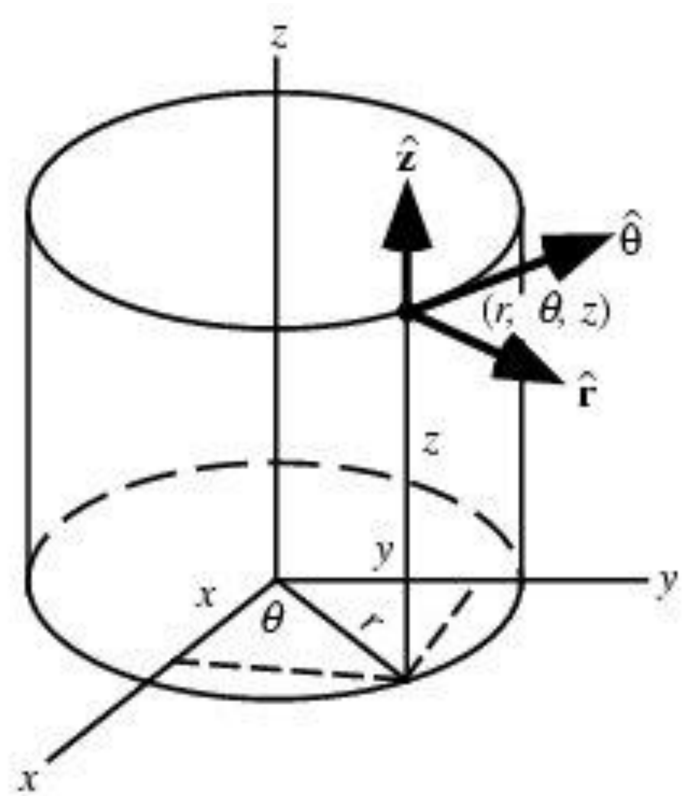


$$
\begin{aligned}
x &= R \sin\theta \cos\phi \\
y &= R \sin\theta \sin\phi \\
z &= R \cos\theta \,.
\end{aligned}
$$

p(u, v) = [r cos(u), r sin(u), v]

Introduction

Parametric Definition …



p(u, v, w) = [w cos(u), w sin(u), v]

Introduction

Geometry & Topology

Geometry: Describes the location and size of entities.

Examples: Coordinates of a point, position and radius of a circle etc..

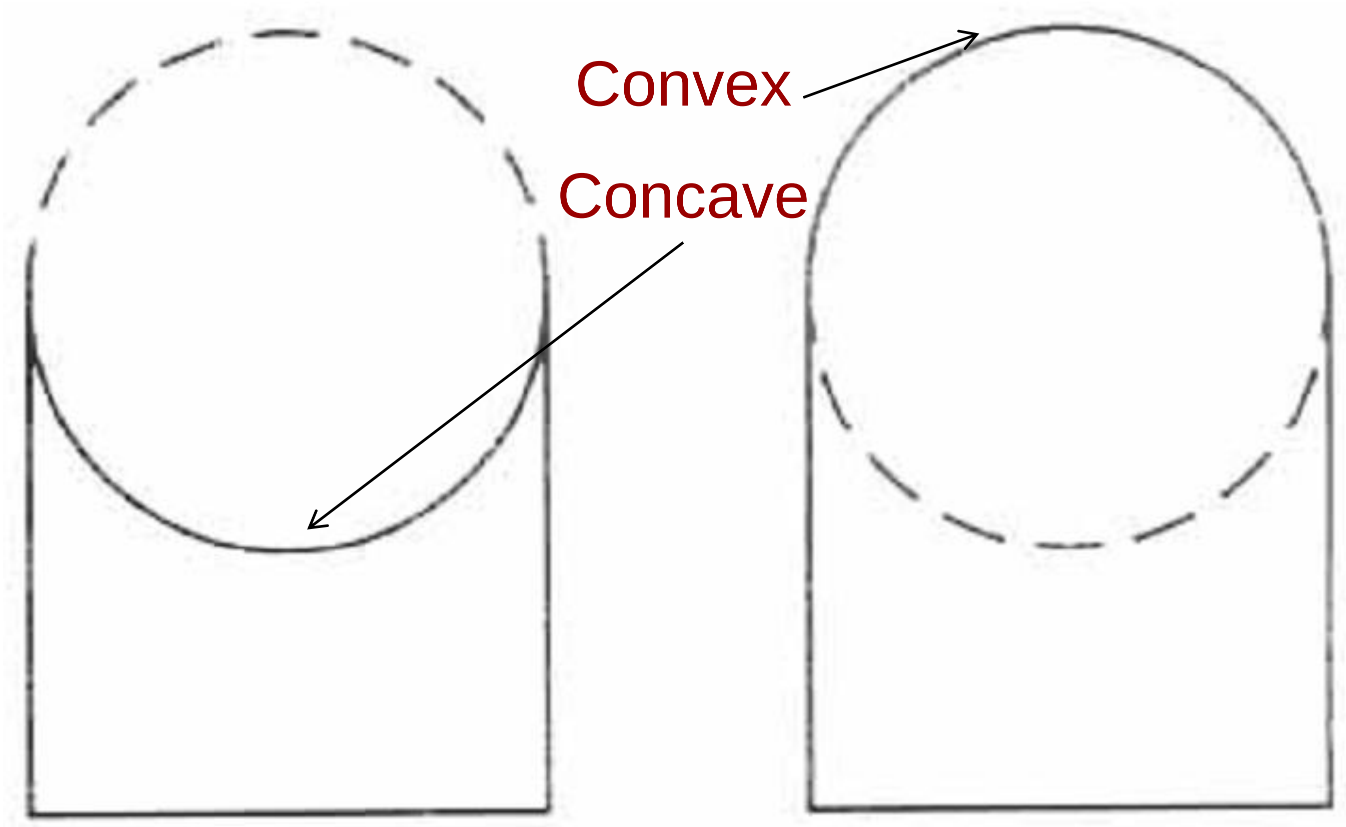
Topology: Tells how the entities are connected with one another. It describes how faces are bounded by edges, how edges are shared by faces, how vertices are shared by edges and so on.

Geometry is absolute while topology is relational.

Introduction

Geometry & Topology

The notion of concave and convex is topology.



Two objects having same geometry
but different topologies

Note:

There are convex bodies but nothing is a concave body. Bodies having concave features are called non-convex bodies.

Introduction

Geometry & Topology

Number of Parameters	Geometry	Topology
0	Point P = [20, -5, 30]	Vertex (end, mid, int, quad, cen etc.)
1	Curve line, arc, circle, helix, spiral, B-spline, NURB etc.	Edge
2	Surface Plane, cylinder, cone, B-spline, NURB etc.	Face
3	Solid	Body

Introduction

Three Basic Representations

We try to capture only the surface information. We assume the interior to be homogeneous.

The three basic representations used to capture the surface details:

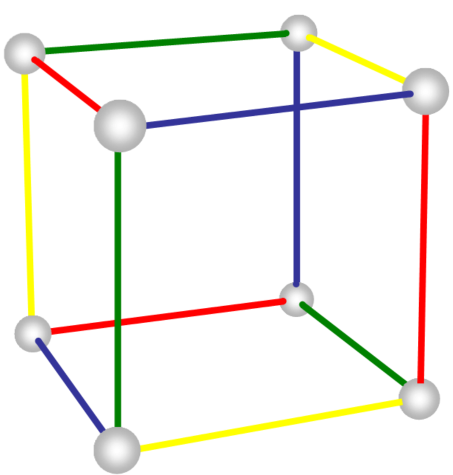
1. Wire-frame models
2. Surface models
3. Solid models

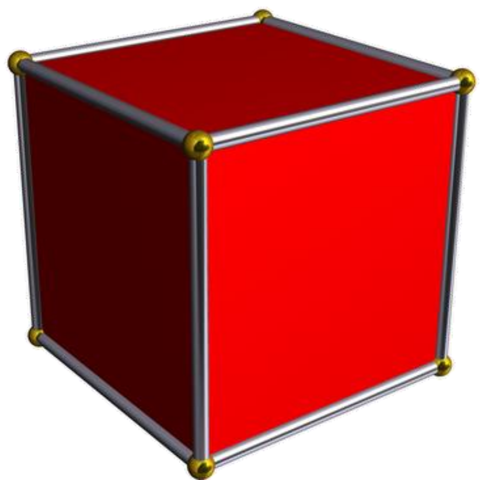
We shall distinguish among these using the notions of *geometry* and *topology*. Note that these are in the increasing order of topological information.

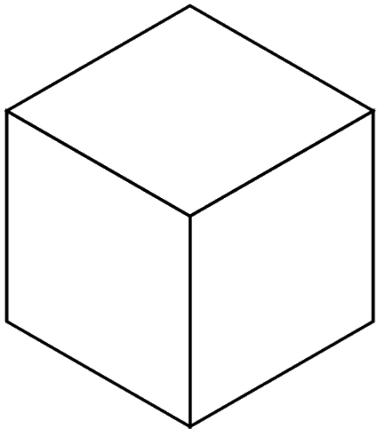
Introduction

Three Basic Representations

Model	Geometry	Topology
Wire-frame	Points, curves	Edges (Connectivity among vertices)
Surface	Points, curves, surfaces	Edges, Faces (Connectivity among edges)
Solid	Points, curves, surfaces	Edges, Faces, normals (Connectivity among faces)

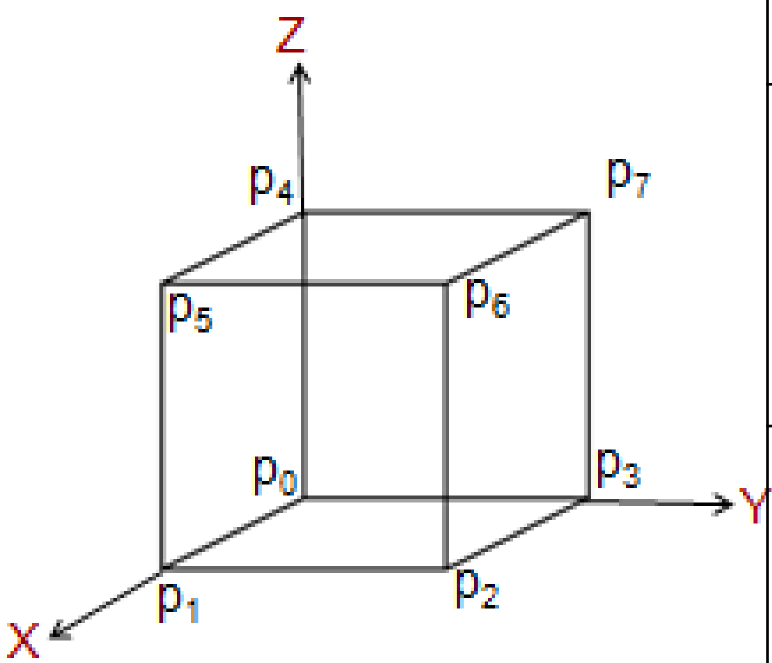






Introduction

Three Basic Representations

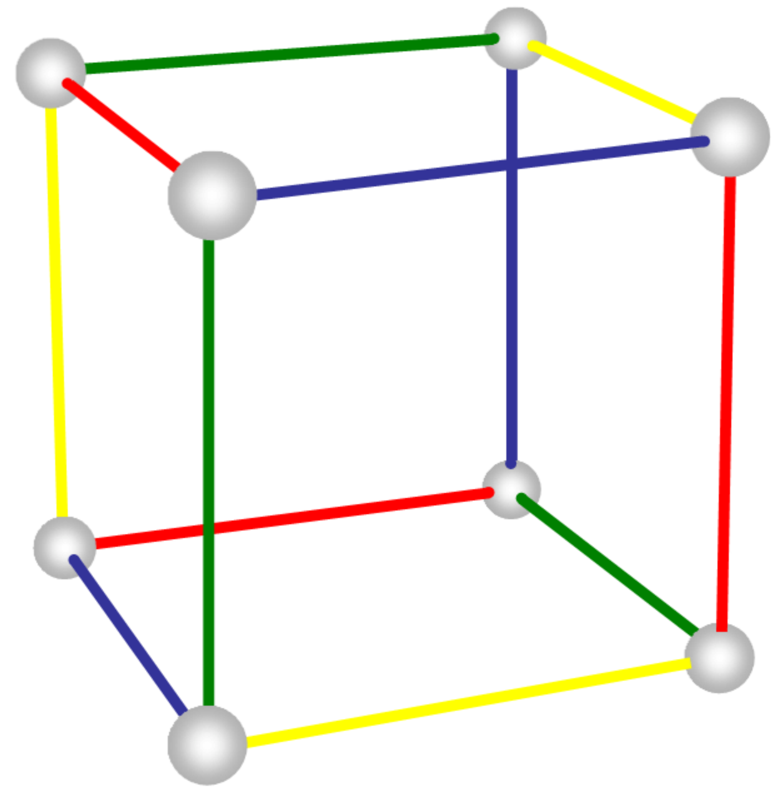


Model	Geometry	Topology
Wire-frame	8 points: p_0 to p_7	12 edges out of $12C_8$ (=495)
Surface		12 edges out of $12C_8$ (=495) 6 faces out of $12C_4$ (=990)
Solid		12 edges out of $12C_8$ (=495) 6 faces out of $12C_4$ (=990) 6 normals of the faces

Wire-Frame Model

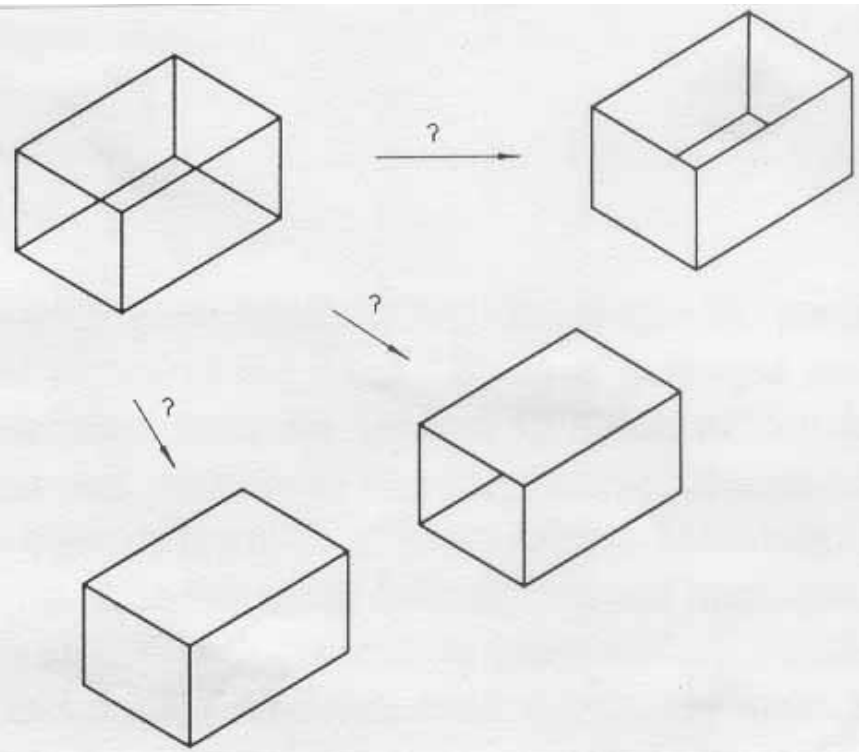
Definition, advantages & limitations

- It is analogous to obtaining the shape by welding the wires representing the edges.
- It is the simplest and the fastest 3D representation scheme.
- It is a subset of any other higher level modeling scheme.
- All modelers use it for quick displays.
- Limitations:
  - It is ambiguous leading to several interpretations of the same model.
  - Nonsense objects may be created.
  - Topological information available with a wire-frame model is inadequate. So, Hidden line removal and mass properties have no meaning.

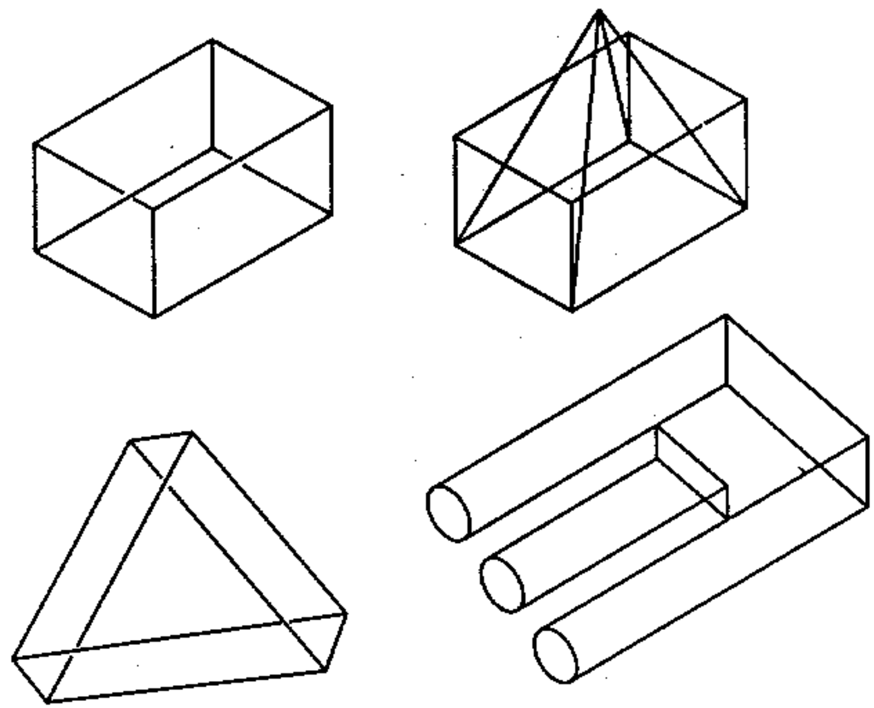


Wire-Frame Model

Definition, advantages & limitations …



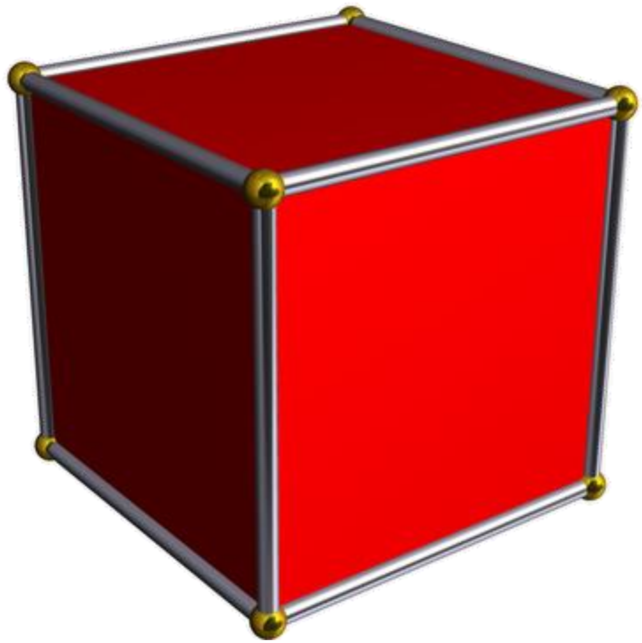
Ambiguity leading to multiple interpretations



Non-sense objects (physically impossible)

Surface Model
Definition, advantages & limitations

- It is analogous to obtaining the shape by joining the cardboard pieces representing the faces together.



- A surface model can be of two types (Faceted and Exact):
  i. Exact surface models (such as NURBS Based)
  ii. Faceted or tessellated or polyhedral models.
- Exact surface models are used for very precise applications such as free-form modeling for styling and NC machining. Faceted models are used in architecture, RP, FEA etc.
- It is a subset of any solid model.

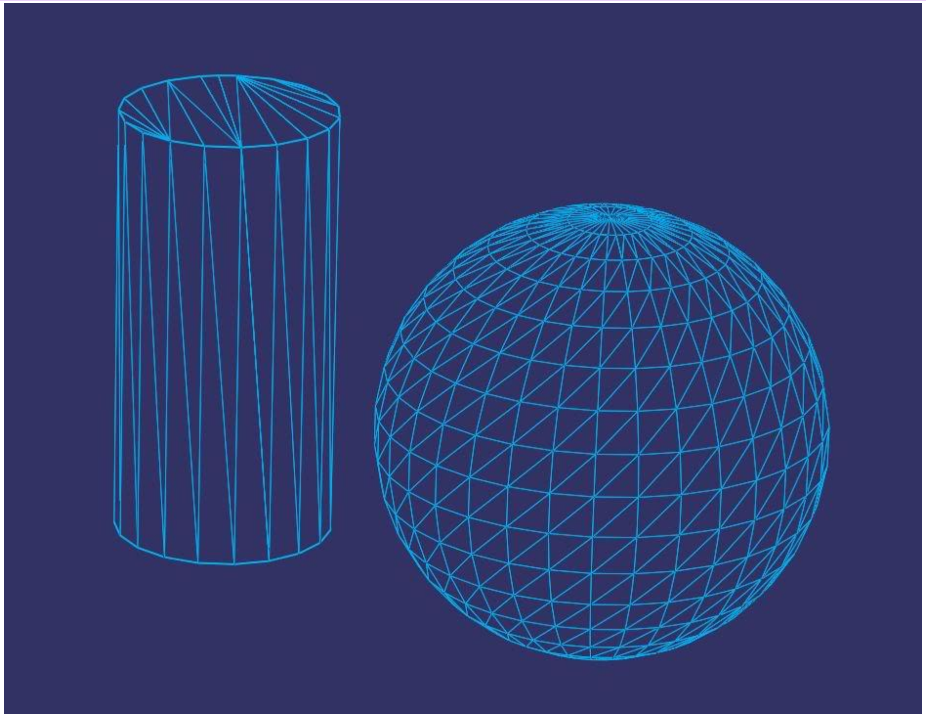
Surface Model

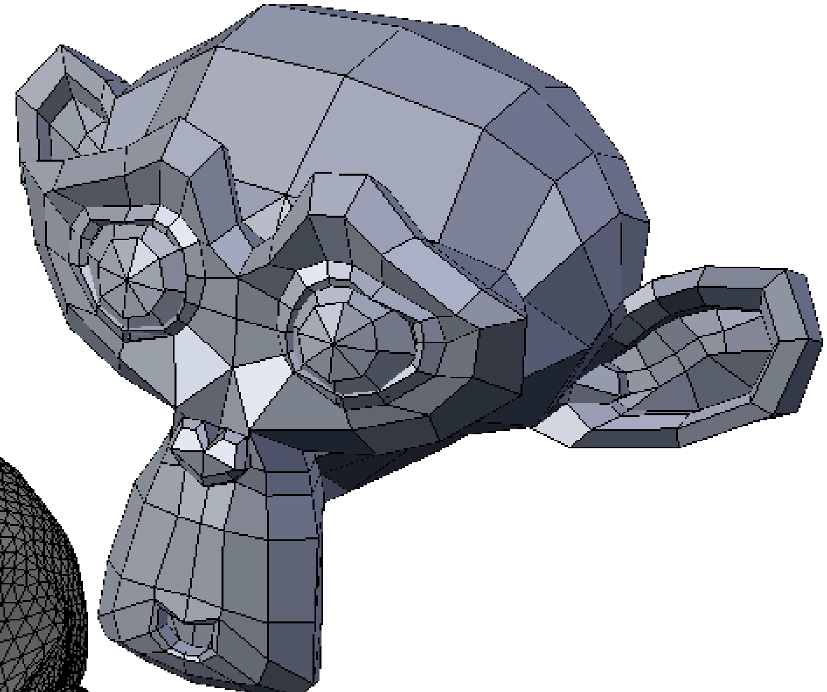
Definition, advantages & limitations

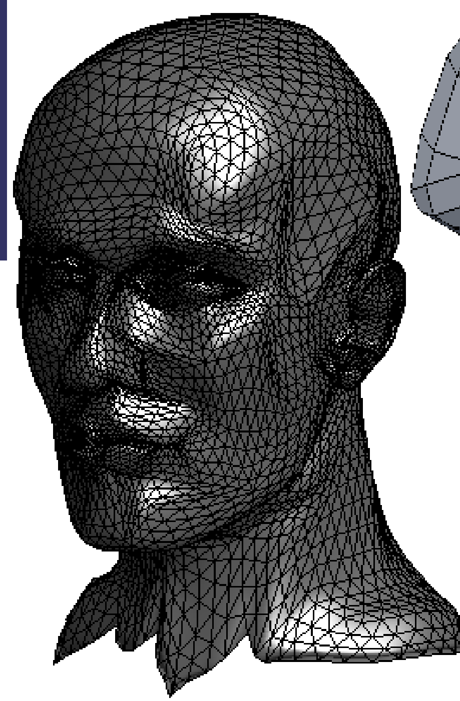
Smooth models

Surface Model

Definition, advantages & limitations







Faceted models

Surface Model

Definition, advantages & limitations …

- Faceted surface model is a simpler and faster way to display 3D objects with hidden lines removed.
- Limitations:
  - Faceted model is inaccurate. If accuracy is to be improved, the size of the model will become very high.
  - Nonsense/Invalid objects may be created (joints of the faces → gaps and overlaps). Stitching of edges – viz., obtaining water-tight models - exactly is difficult.
  - On a pure surface model, only area can be calculated since its inside is not defined.

Solid Model

Definition, advantages & limitations

- A solid model is an unambiguous and informatively complete representation of a physical object.
- A solid model can be created in several ways but the resulting object can be interpreted in only one way.
- Since inside the solid is also defined, all properties such as area, mass, c.g., moment of inertia etc. can be easily calculated.
- Limitations:
    - Costliest modeling scheme in terms of compute space (RAM and disc space) and time. However, this is no longer a limitation, thanks to the availability of more powerful computers at low cost.
    - Even a solid model does not capture the interior details.

Solid Model

Types

- Constructive Solid Geometry (CSG)
- Boundary Representation (B-Rep)
- Feature Based Modeling (FBM)
- Space Decomposition (SD)
  - Exhaustive
  - Hierarchical or adaptive (HSD)

There are two basic needs: (i) user-friendliness and (ii) system-friendliness. None meets both. Let us identify which is good in what.

Solid Model

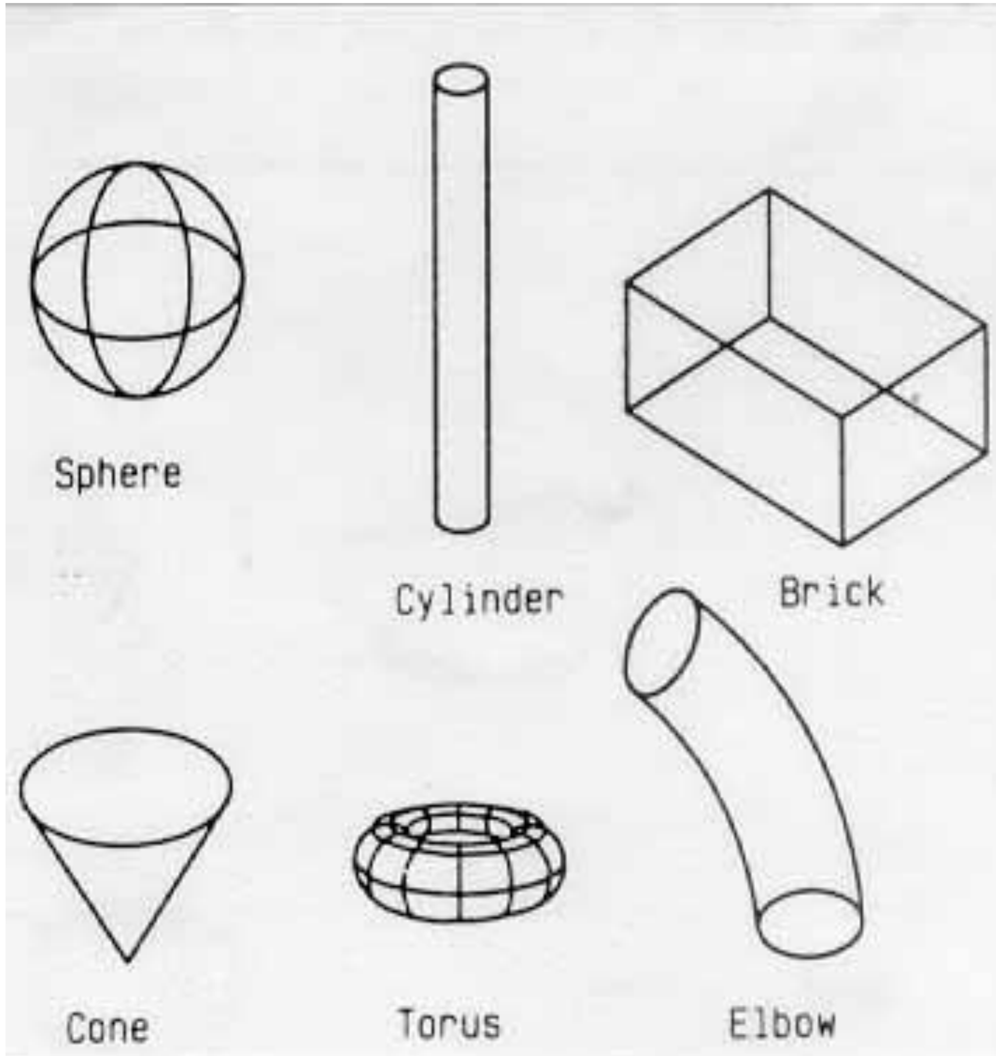
Constructive Solid Geometry (CSG)

- CSG is the first solid rep. scheme; developed at University of Rochester (PADL I & II).
- In CSG, a complex object is constructed from simple shapes such as box, cylinder etc. using Boolean operations.
- The three Booleans are:

  i. Union

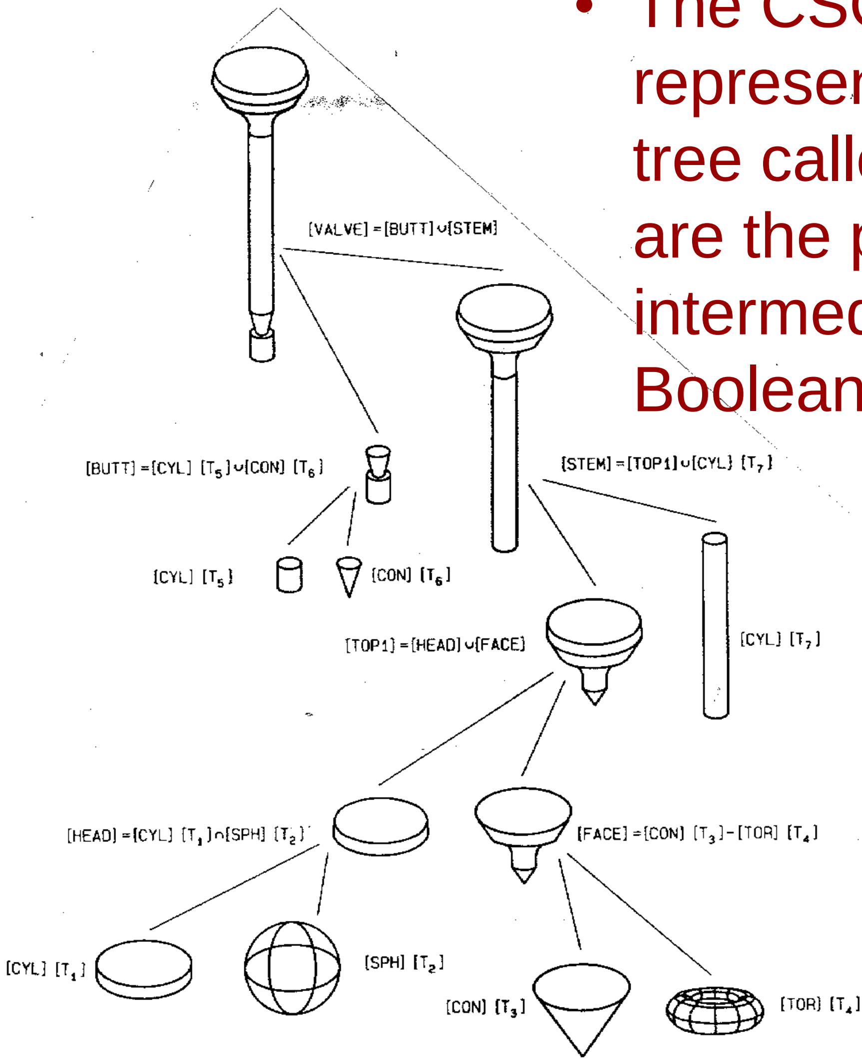
  ii. Subtraction

  iii. Intersection

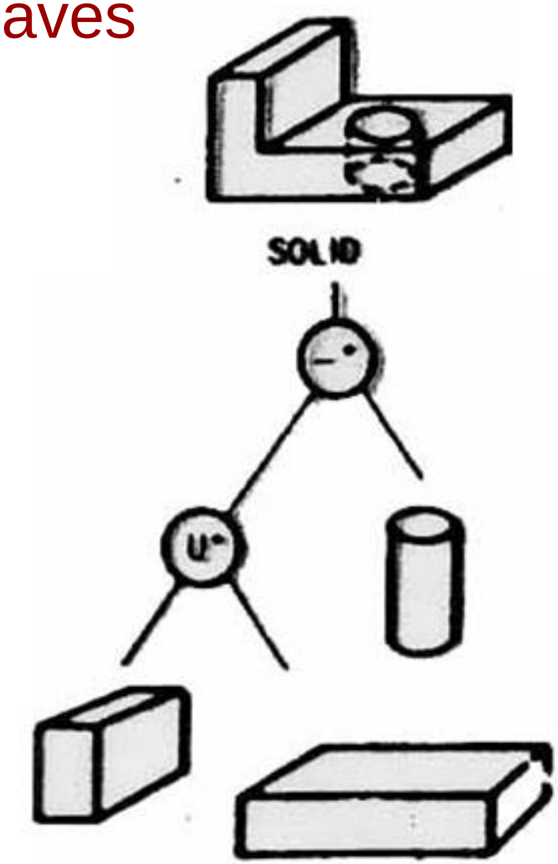


Solid Model

Constructive Solid Geometry (CSG)



- The CSG model of an object is represented in the form of a binary tree called CSG tree whose leaves are the primitives and the intermediate nodes are the Booleaned shapes.



Solid Model

Constructive Solid Geometry (CSG)

Advantages:

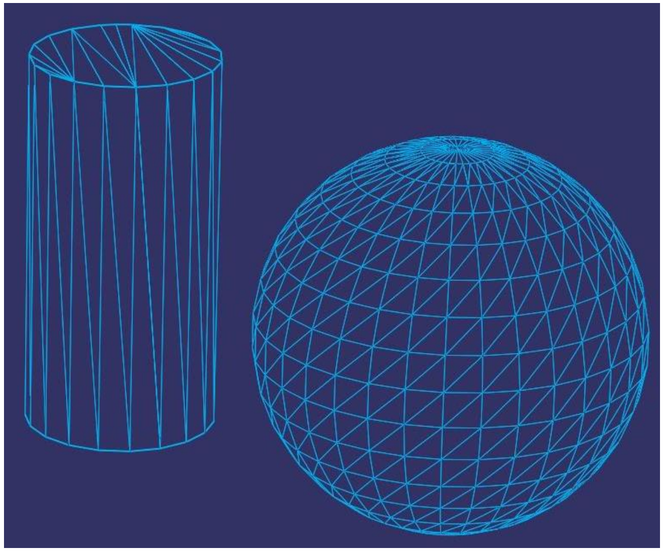
- Simple to understand and use.
- Low memory requirements

Limitations:

- CSG is an unevaluated or raw model. Therefore, for any operation such as display or property calculations, it takes more time.
- Data accession becomes more difficult as the complexity of the part increases.

Solid Model

Boundary Representation (B-Rep)

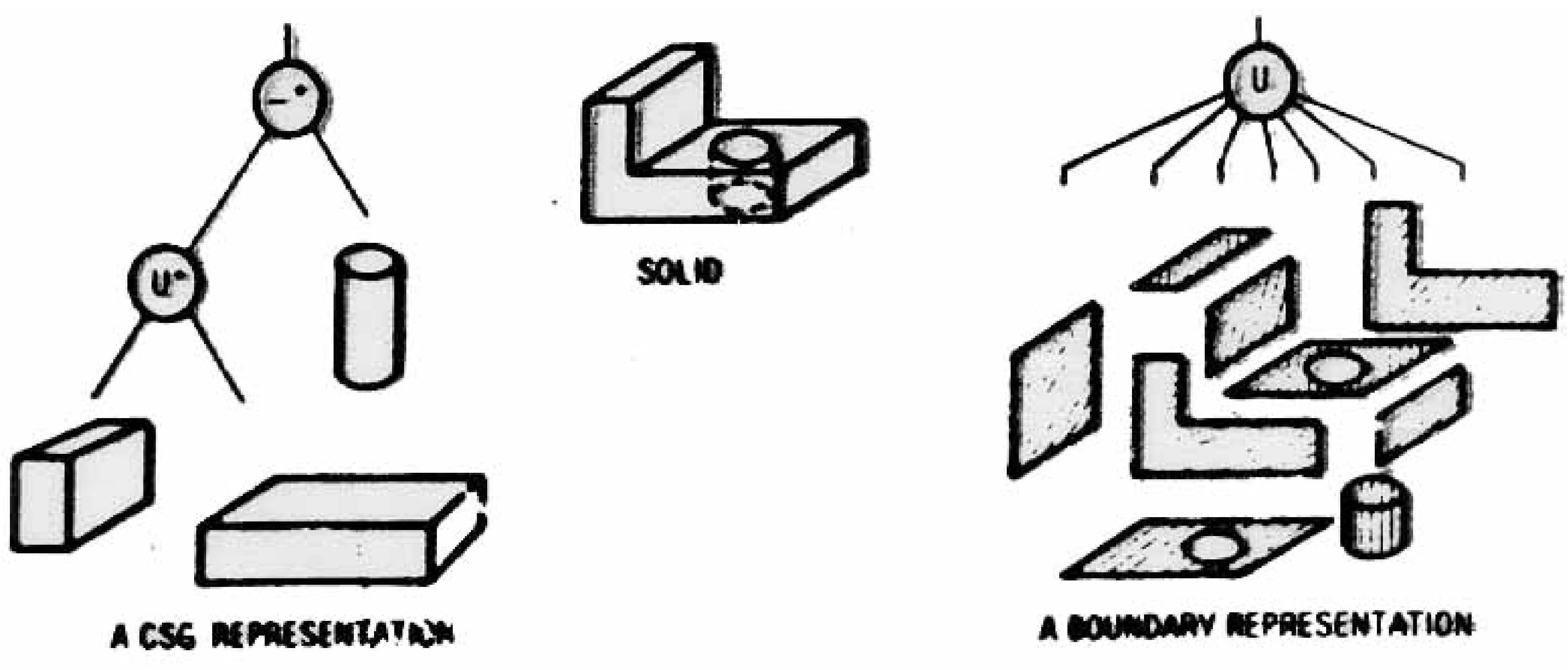


- B-Rep. is a very powerful solid representation scheme using the concept of half spaces.
- In B-Rep., a complex object is constructed out of its constituent surfaces.
- B-Rep. is nothing but a surface model with additional details and constraints:
  - Topology (outward normal)
  - Perfect water-tight stitching (no overlaps or gaps of surfaces)
  - Defining geometry in just one place and linking them extensively using topology. Thus no precision errors apart from memory saving.
  - Optimal data redundancy → Some data such as bounding box of the patches are stored. This little overhead on space complexity reduces time complexity.

Solid Model

Boundary Representation (B-Rep)

- The B-Rep. model of an object is generally represented in the form of winged data structure driven by the face that any edge is shared by two faces.



Solid Model

Boundary Representation (B-Rep)

Advantages:

- Algorithms work very fast due to its presence in evaluated condition and data redundancy.

Limitations:

- High memory requirements.
- Difficult for a user to create since the user has to calculate the intersections of various surfaces, i.e., poor user friendliness.
- Large amount of data redundancy (necessary evil like friction!).
- During the manipulative operations, the topology may be disturbed leading to nonsense object if proper check is not made.

Solid Model

Feature Based Modeling (FBM)

- Its full name is Constrained Parametric Feature-Based Modeling. The first two words are now taken for granted.
- The shape is constructed out of manufacturable features such as hole, slot, boss, protrusion etc. instead of a collection of primitives operated by the Booleans. In other words, Boolean operations are implicit in the features.
- Each feature has some constraints which it shall always adhere to.
- These constraints ensure adherence to topological conditions even when the dimensions are changed; in other words, the dimensions cannot be changed arbitrarily.
- FBM is design of a family of parts and not just a part.
- It is driven by constrained parameters.

Solid Model

Feature Based Modeling (FBM)

- Basically 2D sketches are involved.
- Procedure:
  - Choose a sketch plane.
  - Sketch a rough 2D sketch. This gives only topology.
  - Just constrain this sketch in three levels:

    i. Use the rules defined internally

    ii. Add more relational constraints

    iii. Add dimensional parameters for the rest.
  - Convert the 2D sketch into 3D features such as extrusion, revolution, sweep, cutout etc.
  - The first feature created in this manner is called base feature.
  - Repeat the above steps to create all other features.

Solid Model

Feature Based Modeling (FBM)

Advantages:

- By changing a few parameters, the object can be changed unambiguously since all the dimensions are related to each other by these parameters. this way, a family of parts can be designed with the same effort required to design a part in CSG.
- Changes propagate throughout the assembly automatically.
- Due to the presence of constraints, even if the dimensions change, the topological relations are preserved. For instance, a through hole remains ‘through’ even if the the thickness of the plate is increased.
- Creation of 2D and its conversion to 3D using familiar features makes this approach more elegant and natural.

Solid Model

Feature Based Modeling (FBM)

Advantages: …

- Unlike CSG, here, the Boolean operations are not explicit. They depend on the feature characteristics. This minimizes construction effort.
- Automatic and just dimensioning (but this is possible in any valid solid representation).

Solid Model

Feature Based Modeling (FBM)

Limitations:

- Solving constraints requires considerable geometric acumen.
- User should plan well in deciding the parameters to exploit the benefits of this philosophy.
- Not very amenable for freeform modeling (freeform and constraints do not go together!)

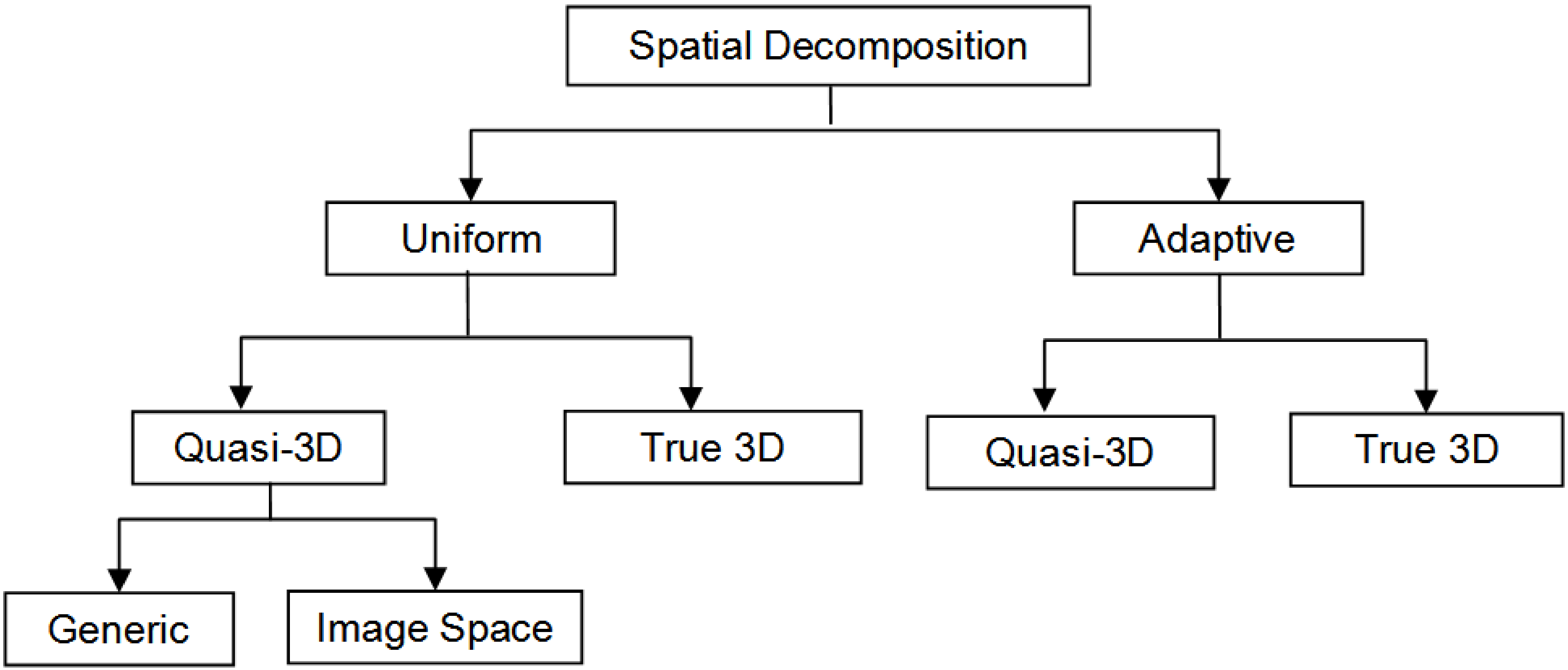
Solid Model

Spatial Decomposition (SD)

Two types:

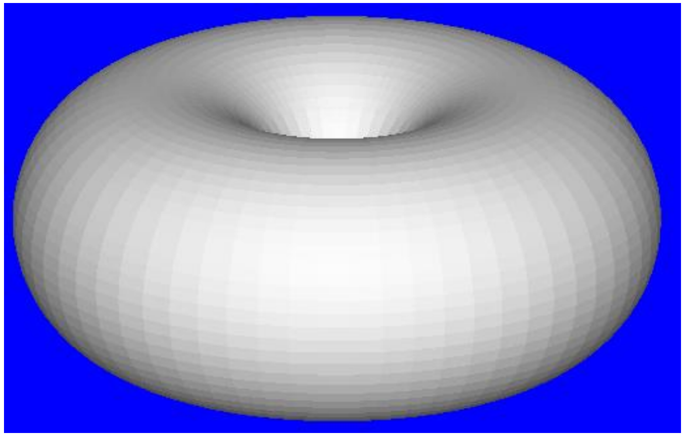
i. Uniform or exhaustive enumeration

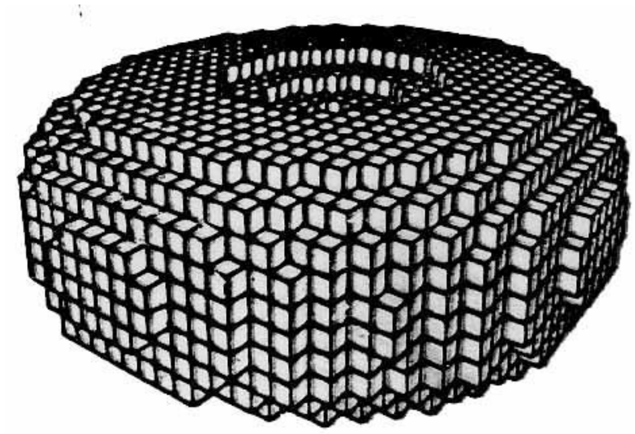
ii. Hierarchical or adaptive (HSD)



Solid Model

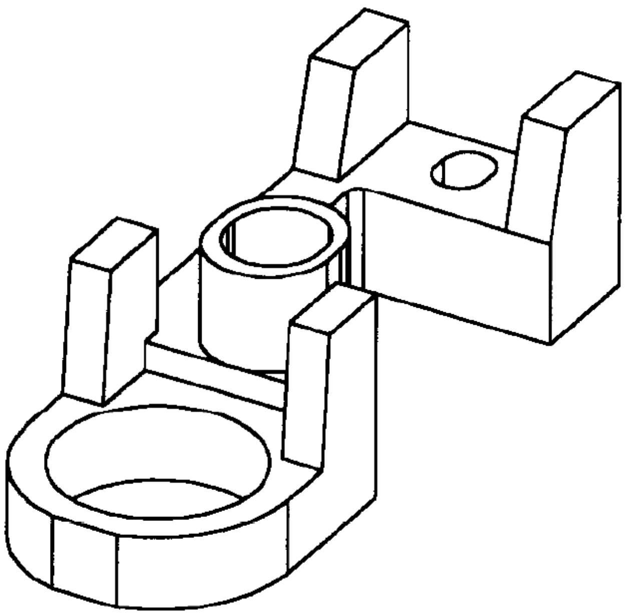
Spatial Decomposition (SD)

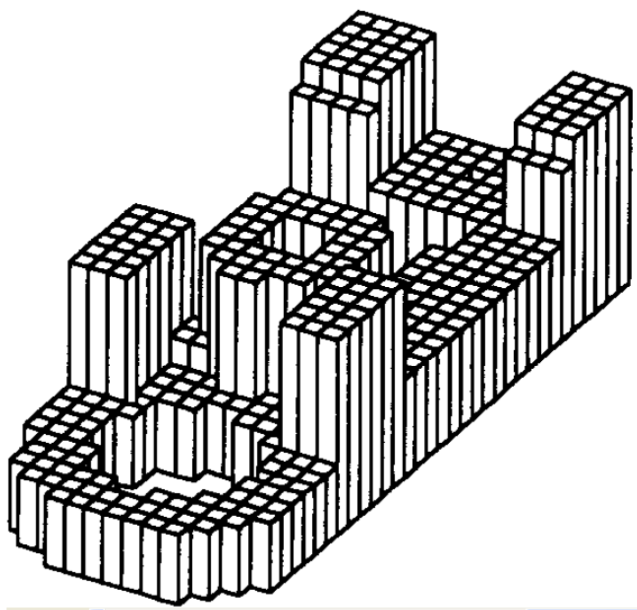




Uniform True-3D:

Voxel representation of a torus



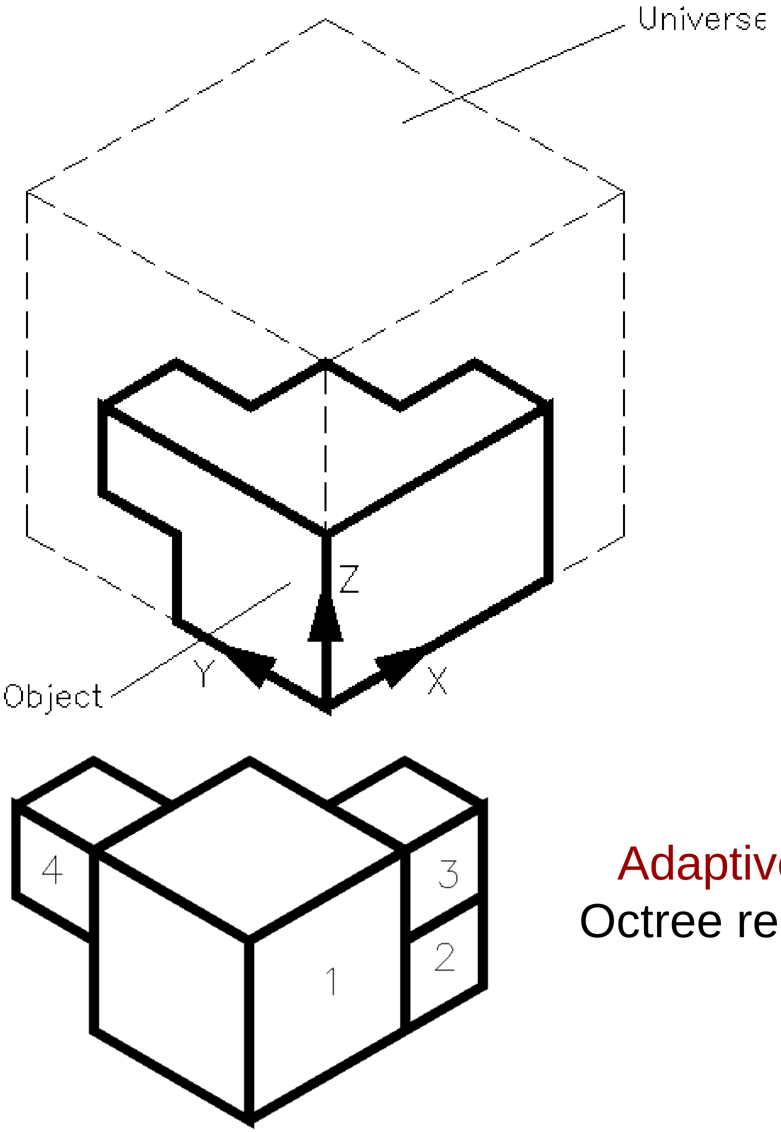


Uniform Quasi-3D:

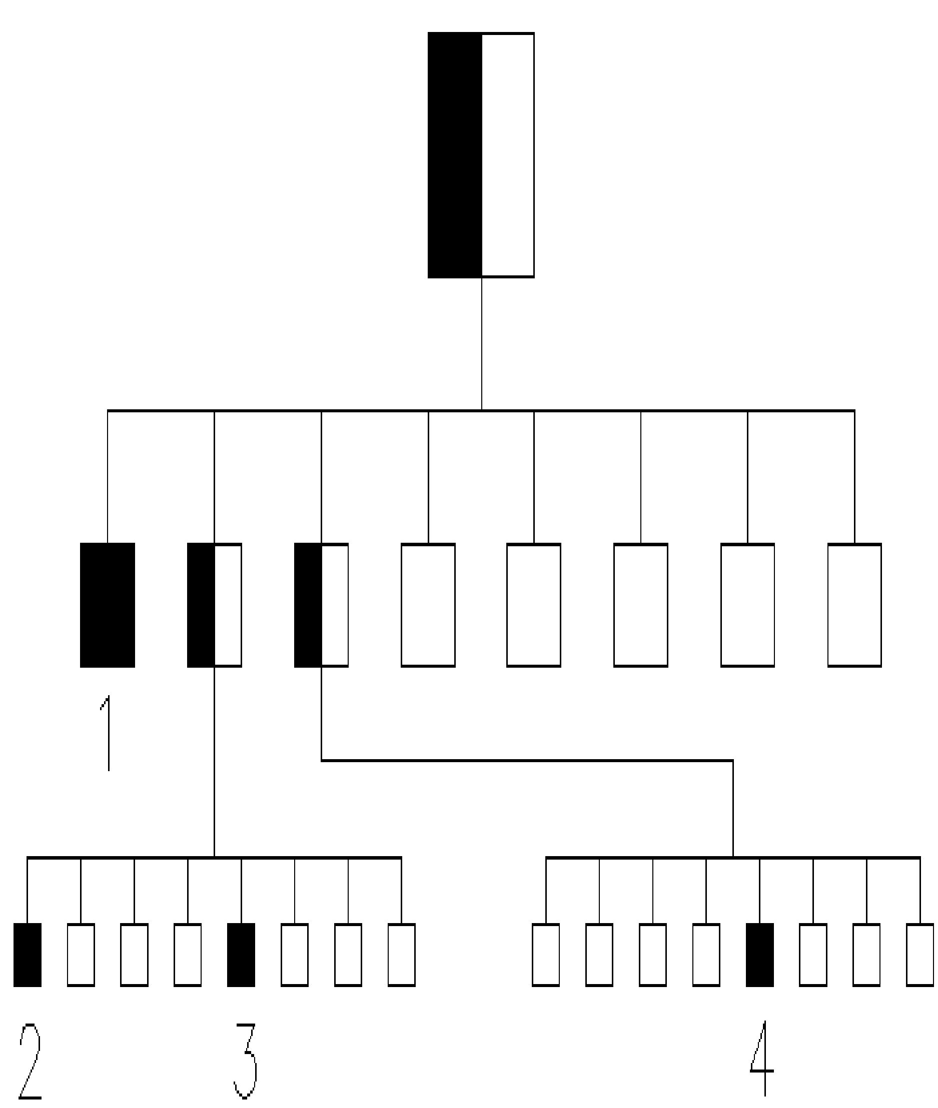
Stick representation of a Component

Solid Model

Spatial Decomposition (SD)

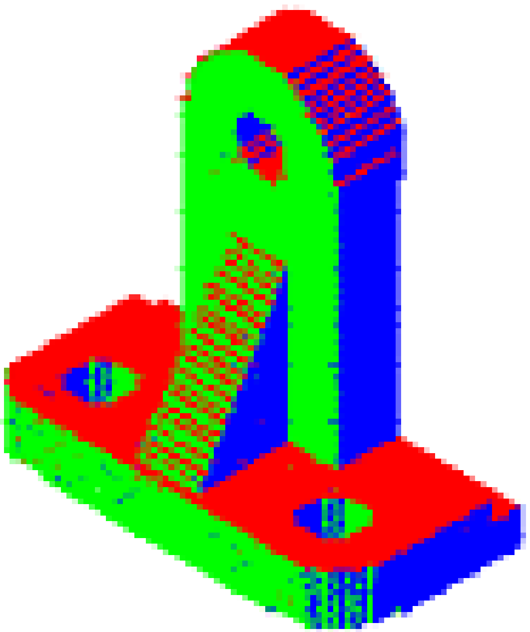


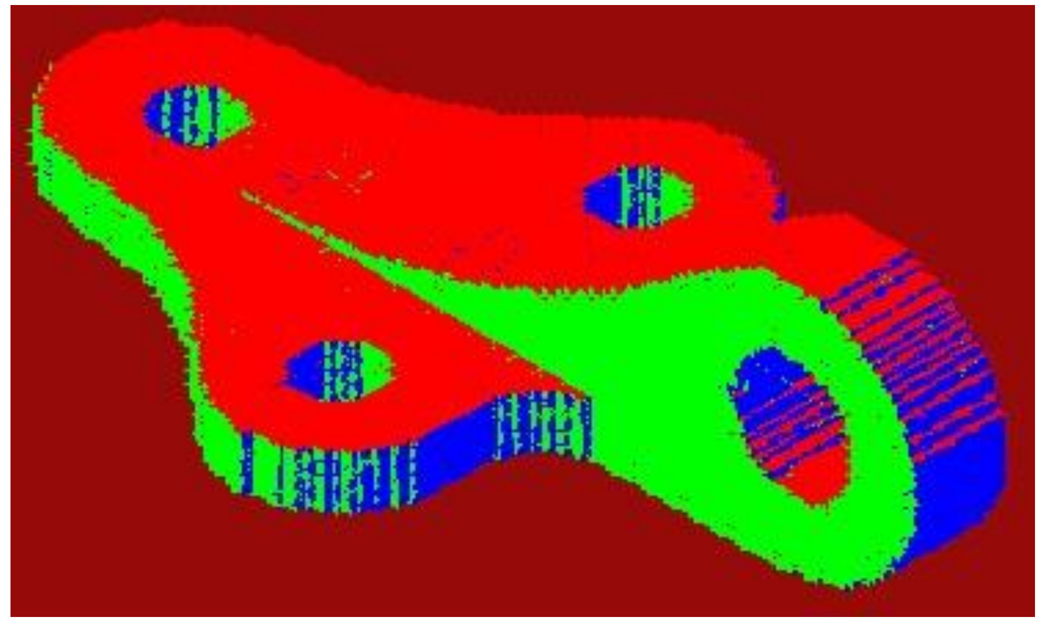
Adaptive True-3D:
Octree representation

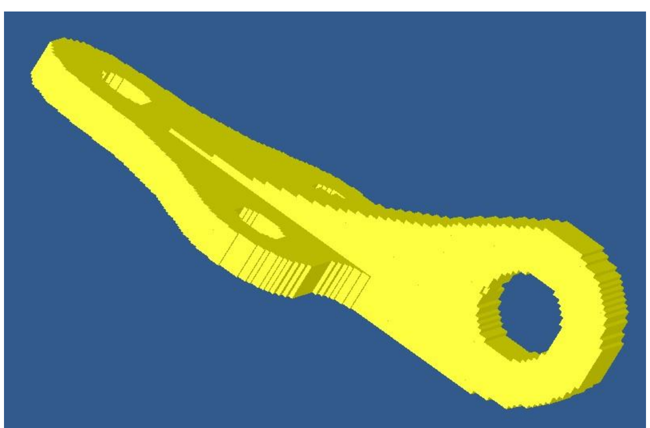


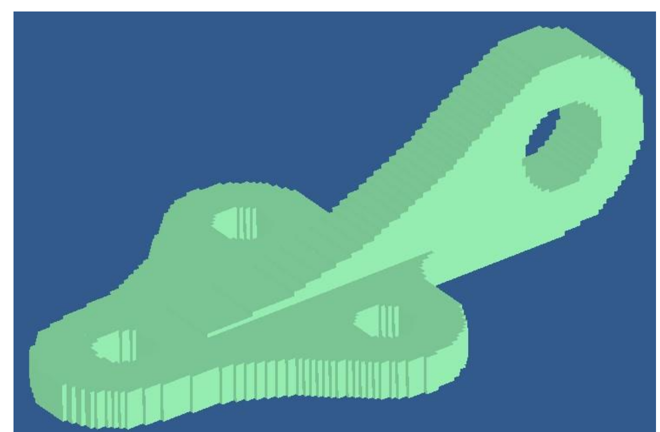
Solid Model

Spatial Decomposition (SD)









Solid Model

Spatial Decomposition (SD)

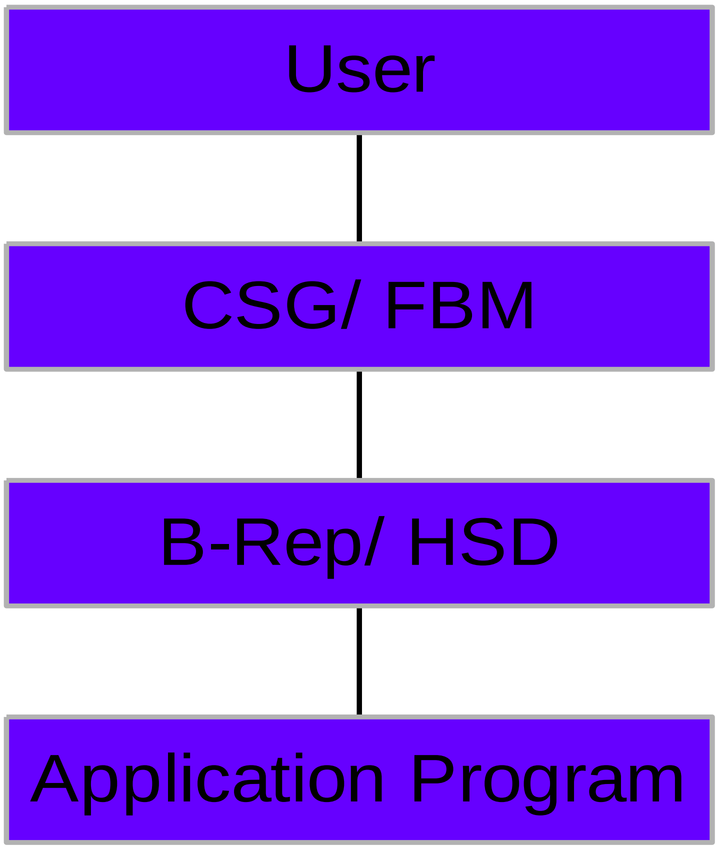
Advantages:

- Voxel is the simplest possible representation.
- Algorithms work very fast. Even the geometric Boolean operators happen as binary Boolean operations.
- Constant memory which is independent of the number of operations .

Limitations:

- Memory required increases exponentially with the increase in resolution.
- Inexact or approximate representation.
- Feature details are irretrievably lost.

A Practical Solid Modeling System



No single solid representation scheme is good enough in all aspects. Therefore, any practical solid modeling system invariably makes use of and maintains two or more representations - one being user-friendly and the other being system-friendly.

A Practical Solid Modeling System

Software/ Applications	User-friendly Representation	System-friendly Representation
AutoCAD, Unigraphics, Catia	CSG & FBM	B-Rep
Pro/E, SolidWorks	FBM	B-Rep
GIS, Medicine, NC Simulation …	Images, NC Program (DSG)	HSD (V-Rep or Octree)

Conclusions

- Wire-frame, Surface and Solid are the three possible representations of 3D objects.
- These representations differ in the amount of topological information they store.
- Solid Modeling is the most preferred model as of now.
- There are several solid representations in use, some more user-friendly and others more system-friendly – none have both.
- Any usable solid modeler has at least two representations simultaneously.

Conclusions

- An object has not only shape and size but many other properties such as tolerance, surface finish, color etc.
- No natural object is homogeneous. Due to human limitations, we define only boundary and assume interior to be homogeneous.




Conclusions

- We so far are representing the objects using our sense of vision. But we experience an object using our five senses. Therefore, its CAD representation is not complete without incorporating these other representations. We are still not fully successful in representing the object's geometry itself satisfactorily. This shows how much more to be achieved!
- It also may have other characteristics that go beyond our senses such as mind.
- Any simulation will require incorporation of gravity, friction, viscosity, forces, torques, deflections etc.

Thank You!

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