Solution to Relativity Tutorial Q 11

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11) Define Events as follows

A2 A1 B1 B2

E1: Ends A1 and B1 coincide

E2: Ends A2 and B2 coincide

$V_a = 0.6c$ $V_b = 0.8c$ $\gamma_a = 5/4$ $\gamma_b = 5/3$

In the ground frame the lengths would appear to be contracted as

$L_a = l_0 / \gamma_a = 4m$ $L_b = l_0 / \gamma_b = 3m$

a) Let time in S frame = t

$(0.6c)t + (0.8c)t = L_a + L_b = 7m$

$\Delta t = 1.67 \times 10^{-8}$ s

b) Let A1 and B1 coincide at t=0 in S frame

In $t = 1.67 \times 10^{-8}$ s , end A2 moves by
$(0.6c) * 1.67 \times 10^{-8}$ s = 3m in + X direction

And end B2 moves by
$(0.8c) * 1.67 \times 10^{-8}$ s = 4m in – X direction

Therefore $\Delta x = -1$ m

Apply Lorentz Transformation,
$\Delta t' = \gamma(\Delta t - \Delta x v/c^2)$
$\Delta t' = 1.25 * (1.67 \times 10^{-8} – (-1) * (0.6c)/ c^2)$

$\Delta t' = 2.33 \times 10^{-8}$ s

c) Similarly (as in part b)

$\Delta t'' = \gamma(\Delta t - \Delta x v/c^2)$

$\Delta t'' = 2.33 \times 10^{-8}$ s

d) Least possible time is the proper time

$\Delta\tau^2 = \Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)/c^2$

Substitute $\Delta t = 1.67 \times 10^{-8}$ s and $\Delta x = -1$ m

$\Delta\tau = 1.63 \times 10^{-8}$ s

e) The frame in which the time is the least would have
$\Delta x''' = \gamma(\Delta x - v \Delta t) = 0$
Therefore,
$v = \Delta x / \Delta t = (-1)/ 1.63 \times 10^{-8}$ s

v= -0.2c