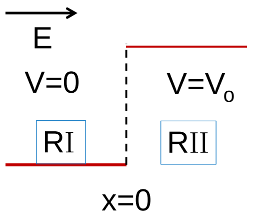
## Free State Problems

A particle approaching from left on a Step Potential




## General Solution

RI

$$\phi_I(x) = Ae^{ik_1x} + Be^{-ik_1x};\ k_1^2 = \frac{2mE}{\hbar^2}$$

RII

$$\phi_{II}(x) = Ce^{ik_2x} + De^{-ik_2x};\ k_2^2 = \frac{2m(E - V_o)}{\hbar^2}$$

Physical interpretation of different components yields

$$D = 0$$

Boundary Conditions yield

$$A + B = C$$
$$ik_1(A - B) = ik_2C$$

## Solving the equations

$$A + B = C$$
$$A - B = \frac{k_2}{k_1}C$$
$$A = \frac{C}{2}\left(1 + \frac{k_2}{k_1}\right)$$
$$B = \frac{C}{2}\left(1 - \frac{k_2}{k_1}\right)$$

$$\frac{C}{A} = \frac{2k_1}{k_1 + k_2}$$
$$\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}$$

## Transmission and Reflection coefficients

Have to look at the relative probability of finding particle in different parts of the beam.

Note: Probability is different from the number of particles crossing.

$$R = \left|\frac{B}{A}\right|^2 = \left[\frac{k_1 - k_2}{k_1 + k_2}\right]^2$$

$$T = \left|\frac{C}{A}\right|^2 \frac{v_2}{v_1} = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1} = \frac{4k_1k_2}{(k_1 + k_2)^2}$$

We can easily verify the following.

$$R + T = 1$$

## Momentum Eigen Values

$$\hat{p}_x(A\sin kx) = -i\hbar\frac{\partial}{\partial x}(A\sin kx)$$
$$= -i\hbar Ak\cos kx$$

Hence this is not an Eigen function of the momentum Operator.

$$\hat{p}_x(Ae^{ikx}) = -i\hbar\frac{\partial}{\partial x}(Ae^{ikx})$$
$$= \hbar k(Ae^{ikx})$$

Hence this is an Eigen function of the momentum Operator with Eigen value of $\hbar k$.

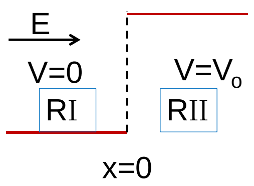
Is the following an Eigen function of the momentum Operator?

$$\phi(x) = Ae^{ikx} + Be^{-ikx}$$

What would a measurement of momentum would yield, if a particle is in the above state?

## Step Potential *(Case 2)*

A particle approaching from left on a Step Potential, with an energy E smaller than the step $V_o$.




## General Solution

RI $\phi_I(x) = Ae^{ikx} + Be^{-ikx}$; $k^2 = \dfrac{2mE}{\hbar^2}$

RII $\phi_{II}(x) = Ce^{\alpha x} + De^{-\alpha x}$; $\alpha^2 = \dfrac{2m(V_o - E)}{\hbar^2}$

Finiteness of wave function

$$C = 0$$

Boundary Conditions and solution yield

$$A + B = D$$

$$ik(A - B) = -\alpha D$$

$$A = \frac{D}{2}\left(1 + i\frac{\alpha}{k}\right)$$

$$B = \frac{D}{2}\left(1 - i\frac{\alpha}{k}\right)$$

## Reflection Coefficient

$$\frac{B}{A} = \frac{k - i\alpha}{k + i\alpha}$$

$$R = \left|\frac{B}{A}\right|^2 = \frac{k + i\alpha}{k - i\alpha} \times \frac{k - i\alpha}{k + i\alpha}$$

$$= 1$$