A person in a flight suit is working on the tail rotor assembly of a helicopter. The person is kneeling on the ground, holding a long metal rod that extends upwards towards the tail rotor. The helicopter is white with blue and red accents. The background is a clear blue sky.

# **ME119: Engineering Drawing & Graphics**

## **10a. Isometric Projections**

**Department of Mechanical Engineering  
Indian Institute of Technology Bombay**

# Outline

- Isometric Projections
- Conclusions

# Isometric Projections

# Projections

- The media available for storing design is only surface.
- This surface is not necessarily plane always although we shall deal with planes (paper).
- The task here is to render 3D objects (3 parameters) onto this surface (2 parameters).
- Furthermore, we are constrained to use only points and curves and not shades (gray levels) and colors. In other words, the views we use are only silhouettes (which simply the borders of the object) and distinct edges.
- Due to these limitations, we may need more than one view to render an object completely.

# Projections

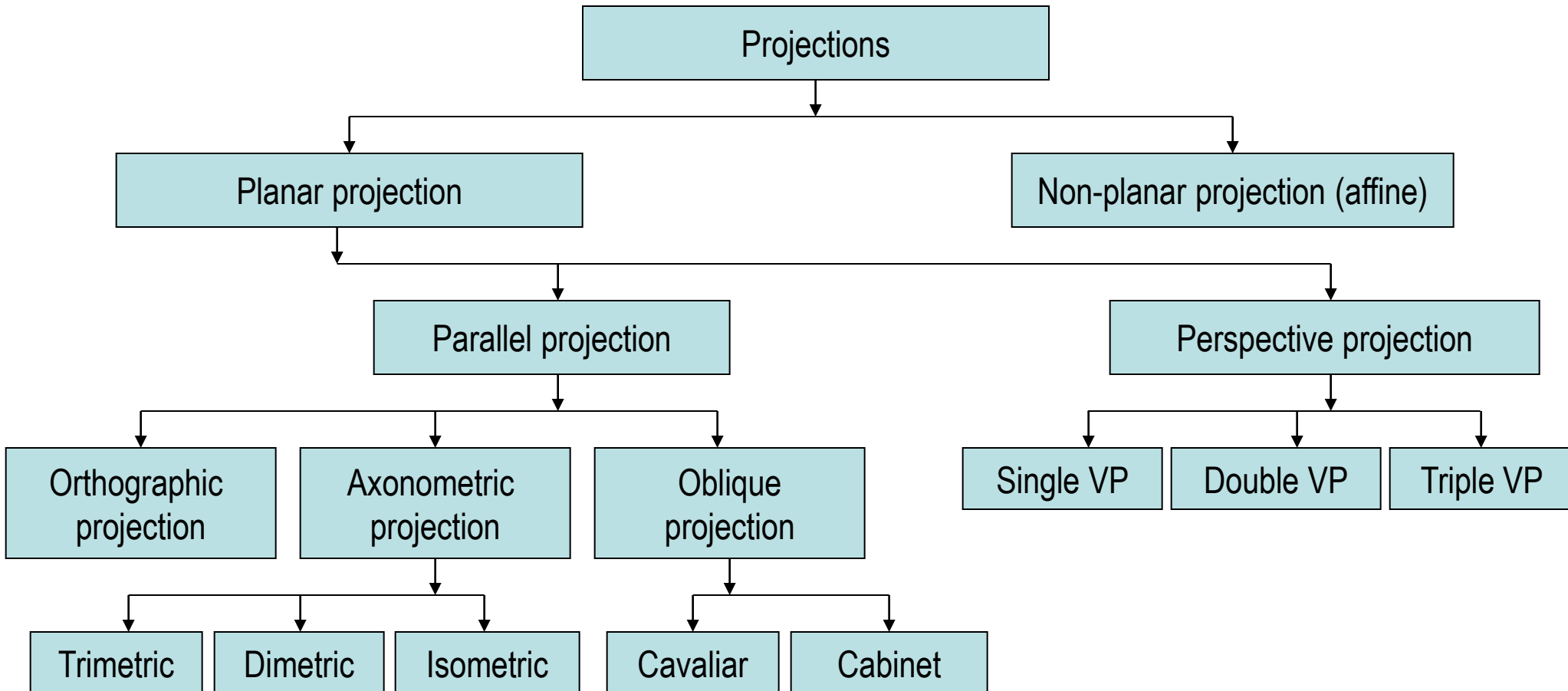
We deal with the following:

- Observer
- 3D object(s)
- Rays emanating from them called **projection lines** or **projectors**.  
The rays may emanate from a point (such as a bulb) and hence are divergent or may come from an infinite source and hence are parallel rays.
- The surface on which the view is captured called **projection surface**, in our case, it is a **projection plane**.
- View(s).

The **type of view** depends on the characteristics of the **projectors**, **projection surface** and their **relative orientation**.

# Projections

## Types



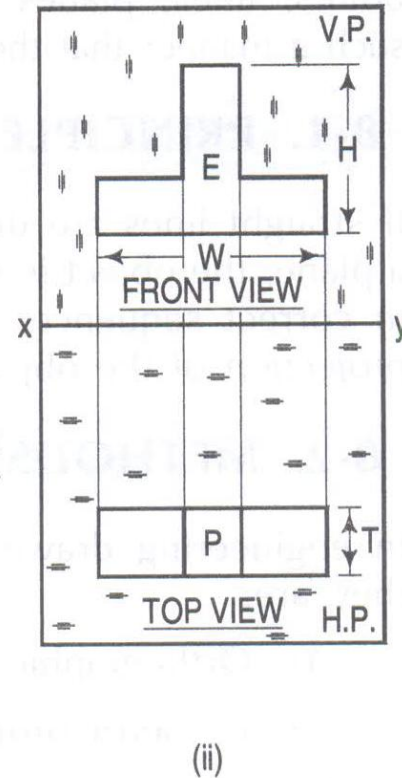
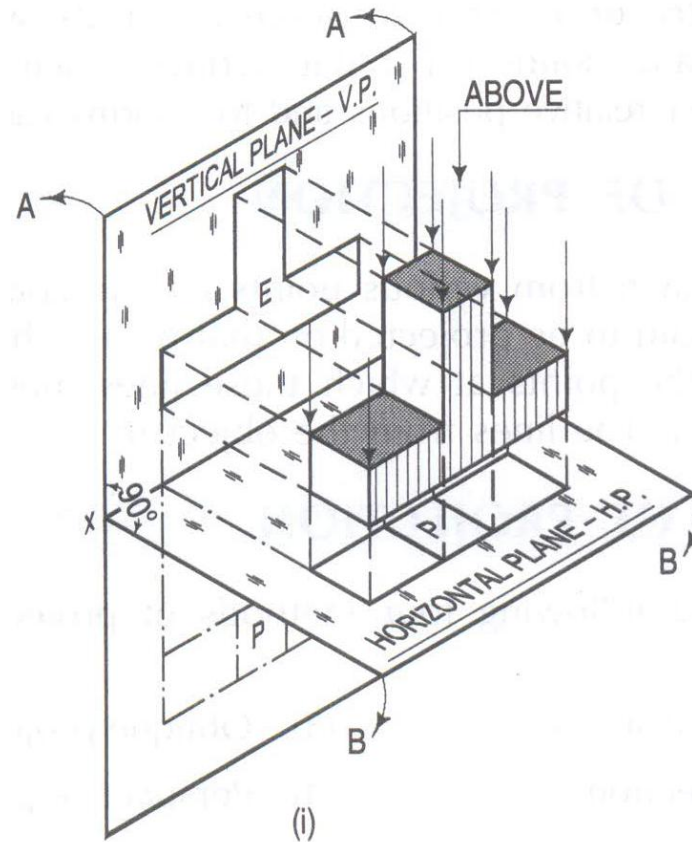
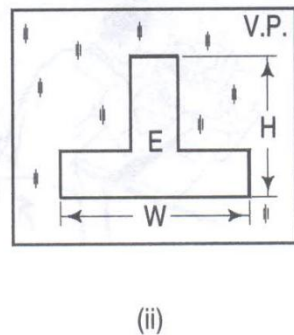
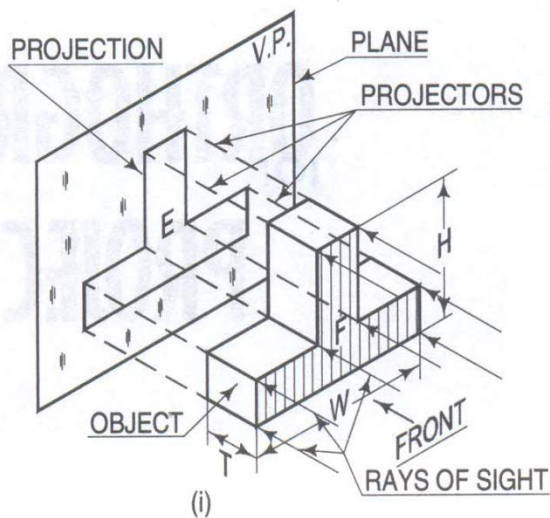
# Projections

## Types

Orthographic projections	Parallel projectors; projection on plane; both orthogonal. Hypothetical! Multiple views (At least 2) on mutually perpendicular planes.
Auxiliary projections	Parallel projectors; projection on plane; both orthogonal. Hypothetical! This too is orthographic. Views on any other plane in which the additional details are visible.
Sectional views	Parallel projectors; projection on plane; both orthogonal. Hypothetical! Cut-views to depict the interior features. This is mostly orthographic. However, the pictorial views also can use sectional views.
Isometric/ Dimetric/ Trimetric projections	Parallel projectors; projection on plane; both orthogonal. Hypothetical! Pictorial; so, single view in general. Isometric view is also an orthographic view after rotating the object appropriately in two directions!
Oblique projections	Parallel projectors; projection on plane; both not orthogonal. Hypothetical! Pictorial; so, single view in general.
Perspective projections	Non-parallel projectors; projection on plane; both orthogonal. Real (3VP alone). Pictorial; so, single view in general.
Affine projections	Projection on non-planar surface. Real.

# Orthographic Projections

Parallel projectors; projection on plane; both orthogonal.  
Hypothetical!



Single view does not capture all features. So, multiple views on mutually perpendicular planes are used. Even the simplest object of symmetry, viz., sphere, requires 2 views.



# Axonometric Projections

- Before projecting the object onto V.P./H.P., if it is rotated about X/Y/Z ax(e/i)s by some arbitrary angle(s), more details of the object become visible as 2 or 3 faces of its bounding cube becomes visible. Such an **orthographic view** preceded by the rotation(s) of the object is called **axonometric projection**. It is a **pictorial view** as it looks like a 3D view of the object. We have achieved such a view earlier too!
- One can think of a bounding sphere of the object. Each point of this sphere will define a possible direction of viewing an object. If the direction is orthogonal to V.P./H.P., the resulting view will be orthographic; otherwise axonometric.
- Instead of rotating the object, one can achieve the same by rotating the system of viewing planes V.P. and H.P.

# Axonometric Projections ...

- When the object is projected without rotation, only one of the 6 faces of its bounding cube is visible and all its four edges appear in true lengths (ignoring the hidden rear face and its edges).
- When the object is rotated about one axis, one more face and hence 3 additional edges become visible. Now we have 2 faces (out of 6) and 7 edges (out of 12) visible. When it is rotated about one more axis in addition, we shall have 3 faces (out of 6) and 9 edges (out of 12) visible.
- This additional visibility, however, comes with a price. The price is the loss of true length as the edges visible previously have to sacrifice some length to give way for new ones. The ratio of the current length with its original length is known as **foreshortening (f)**.

# Axonometric Projections ...

- The 12 edges of the bounding cube are, due to these two rotations, distributed equally along X, Y & Z; 4 edges parallel to each of [X, Y, Z]. The foreshortening of the edges along the same axis is same but this could be different among the axes. So, in principle, we shall have  $f_x$ ,  $f_y$  &  $f_z$ .

# Axonometric Projections ...

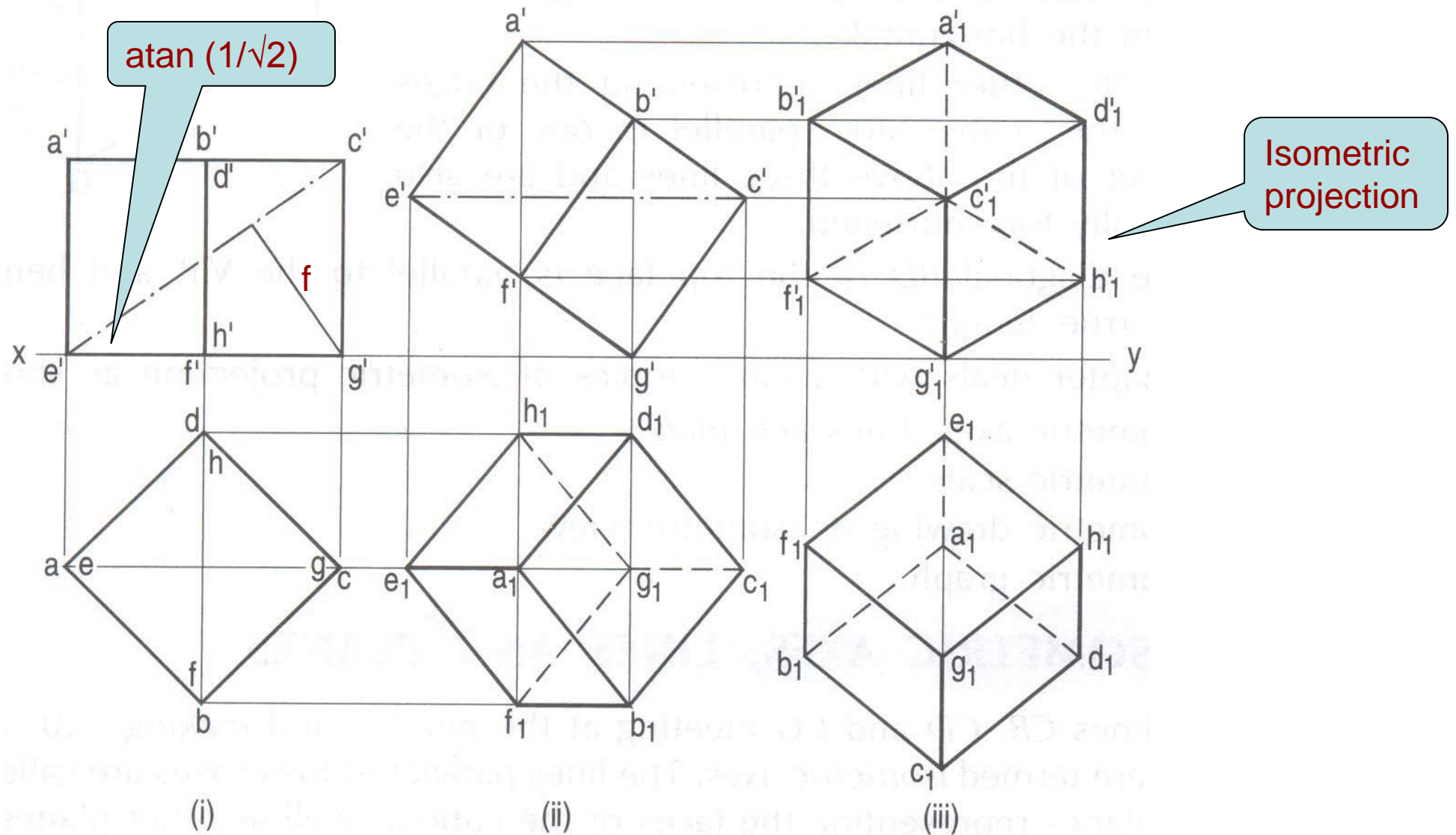
- Based on the value of the foreshortening  $f_x$ ,  $f_y$  &  $f_z$  along the three directions, we have three type of axonometric projections, viz., trimetric, dimetric and isometric projections.
- When all the foreshortening along three directions are different from each other, i.e.,  $f_x \neq f_y \neq f_z$ , it is trimetric projection.
- When any two of the foreshortenings are equal, i.e.,  $f_x = f_y$  or  $f_y = f_z$  or  $f_z = f_x$ , it is dimetric projection.
- When the foreshortening along all the three directions are equal, i.e.,  $f_x = f_y = f_z$ , it is Isometric projection.
- Trimetric is the most general. i.e.,  
Isometric  $\subset$  Dimetric  $\subset$  Trimetric

# Isometric Projections

- Isometric projection is obtained when the two angles are  $45^\circ$  and  $35.264^\circ$  ( $\text{atan}(1/\sqrt{2})$ ). At this condition,  $f_x = f_y = f_z = \sqrt{2} / \sqrt{3} = 0.815$ .
- Advantage of isometric: Apart from an appealing 3D-like look, one can measure the length and interpret it with the true length by dividing it by  $f=0.866$ .
- From the above discussions, it is clear that an isometric projection is nothing but a single orthographic view.
- An orthographic projection usually has multiple views. However, isometric projection is usually one view only. This is simply because (i) it is able to reveal more details of the object and (ii) it takes more time to draw an isometric view.
- Among the infinite viewing directions of a unit sphere, only 8 belong to Isometric views. These are the 8 corners of bounding cube.

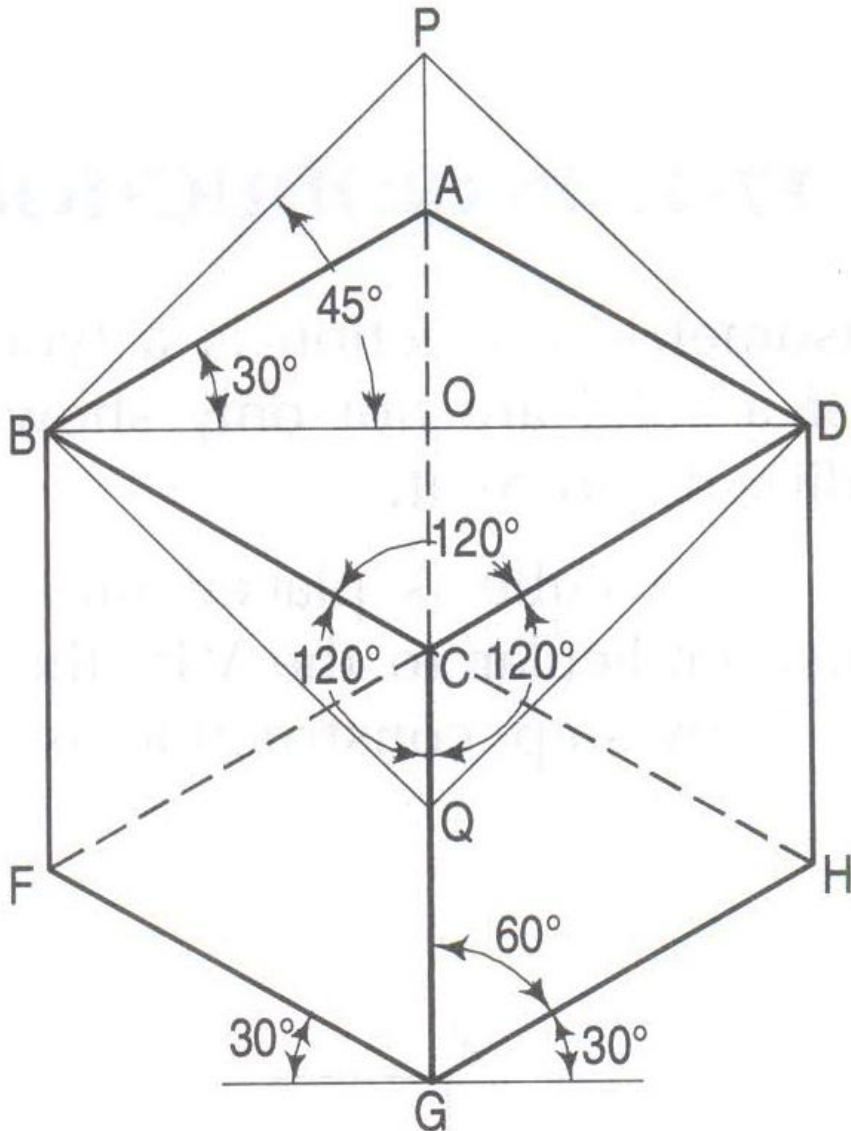
# Isometric Projections

## Projection of the bounding cube



# Isometric Projections

## Projection of the bounding cube ...

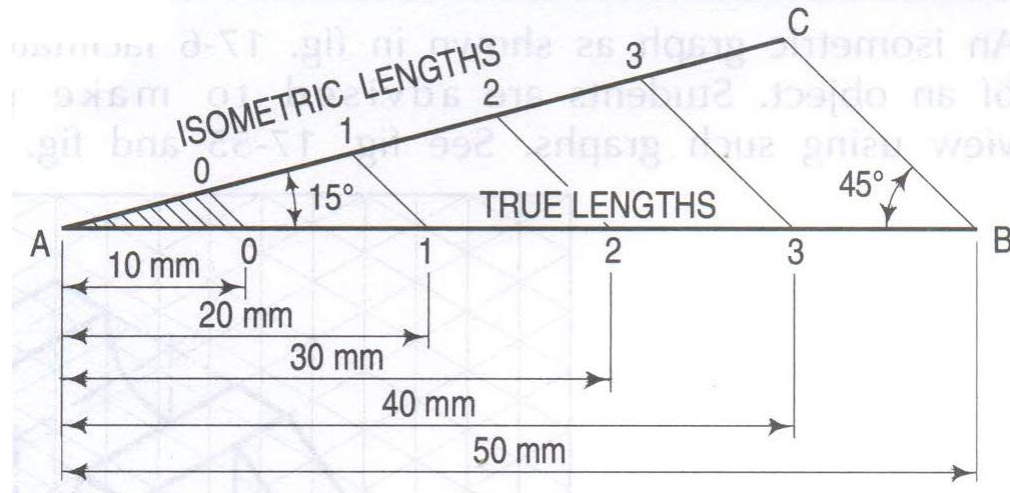
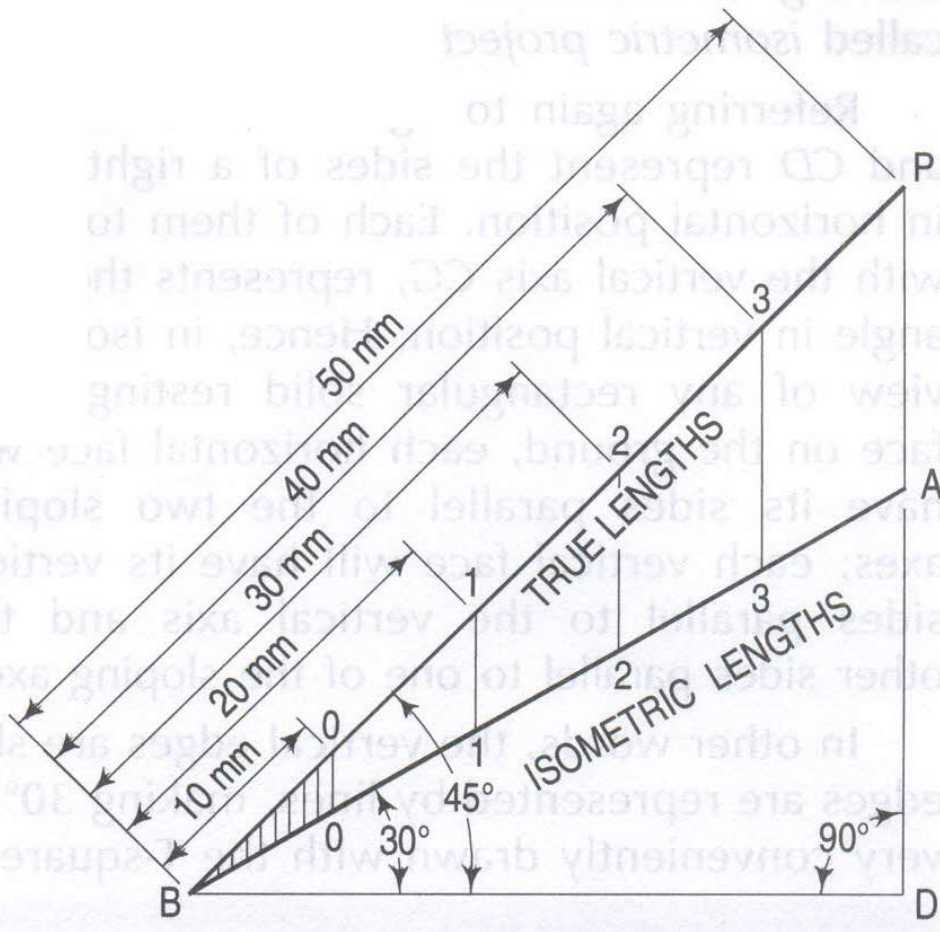


- Isometric axes: CB, CD, CG
- Isometric lines: Lines parallel to CB, CD, CG. These will have equal foreshortening.
- Isometric planes: Planes parallel to the faces of the cube.
- $f = BA/BP = (BA/BO) * (BO/BP)$   
 $= (\sec 30) * (\cos 45)$   
 $= \sqrt{2} / \sqrt{3} = 0.815.$



# Isometric Projections

## Isometric scales



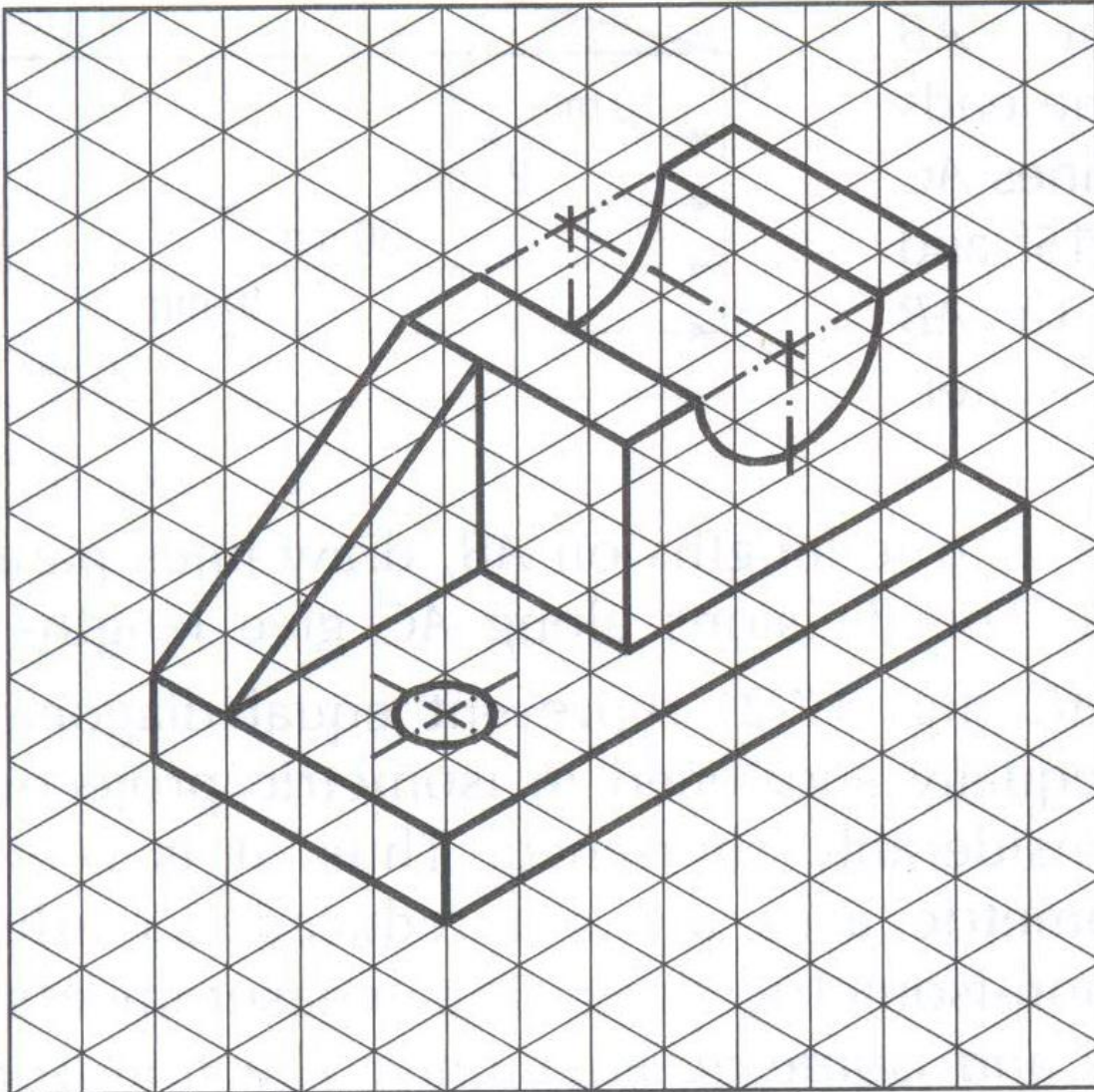
- One could draw the above figures and use for the scale conversion.
- Even isometric scales used to be available.

These are obsolete. Simply use your calculator instead!



# Isometric Projections

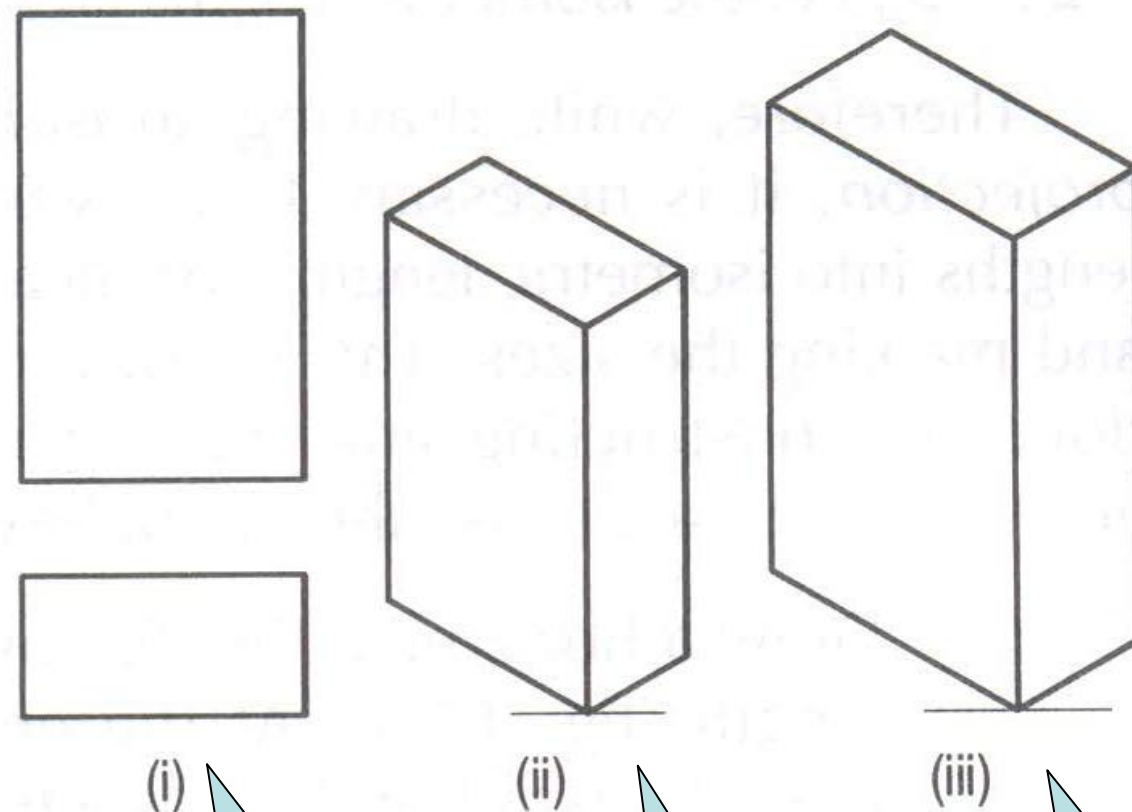
## Isometric scales ...



Graph sheets used to be available on which isometric views can be easily drawn. Even I have not used in my time. These are history now.

# Isometric Projections

## Isometric scales ...



Orthographic  
projection

Isometric  
projection

Isometric  
drawing/ view

One can simply forget the foreshortening and draw. The resulting view will be the same but larger. This will be called **Isometric drawing or isometric view** by convention to distinguish it from the actual **isometric projection** which will have the foreshortening.

# Isometric Projections

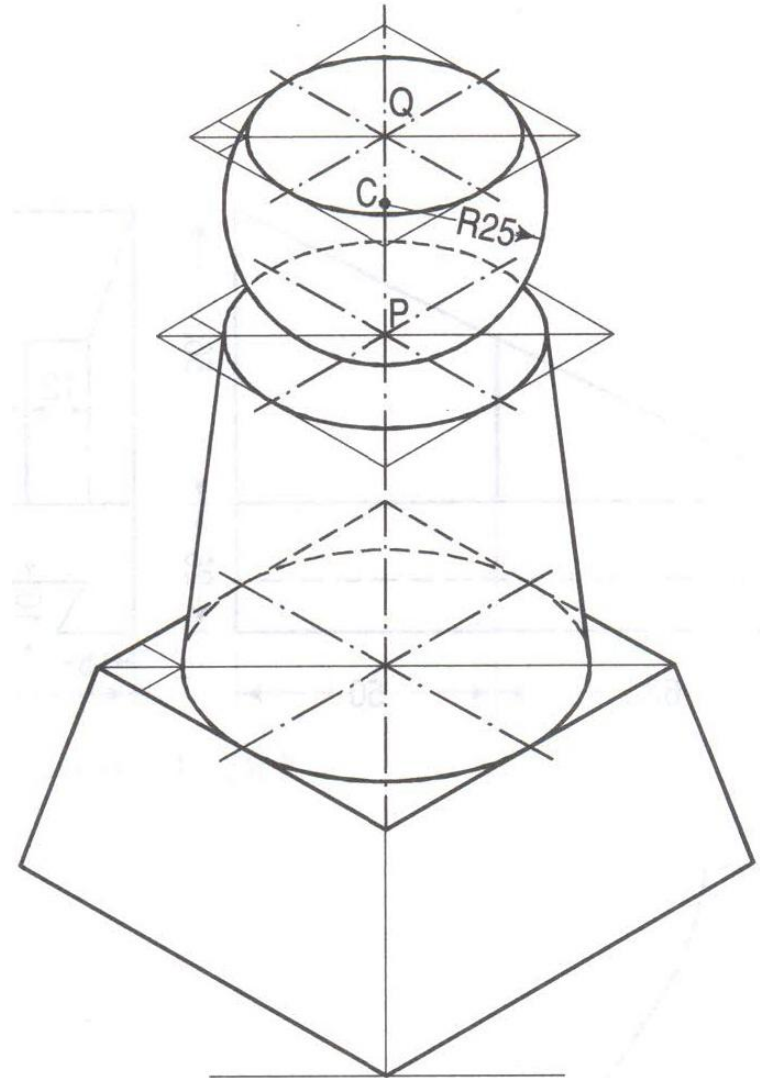
## Notes

- Visualize isometric view obtained through rotations about X & Y and projecting on to V.P. i.e., it is a front view.
- Any horizontal line along X and Y will be inclined respectively by  $30^\circ$  and  $150^\circ$ . A vertical line will be vertical in the view too.
- Let us draw to 0.815 scale and not to true scale. This is easy if you use a calculator. If you have to draw the orthographic views before isometric view, then, draw them in isometric scale itself so that whatever value you measure from the view will be directly usable in isometric view.

# Isometric Projections

## Notes

- Isometric scale shall be used where spherical shapes are involved. Because, sphere does not get foreshortened as in other shapes. However, the distance of its centre will undergo foreshortening.

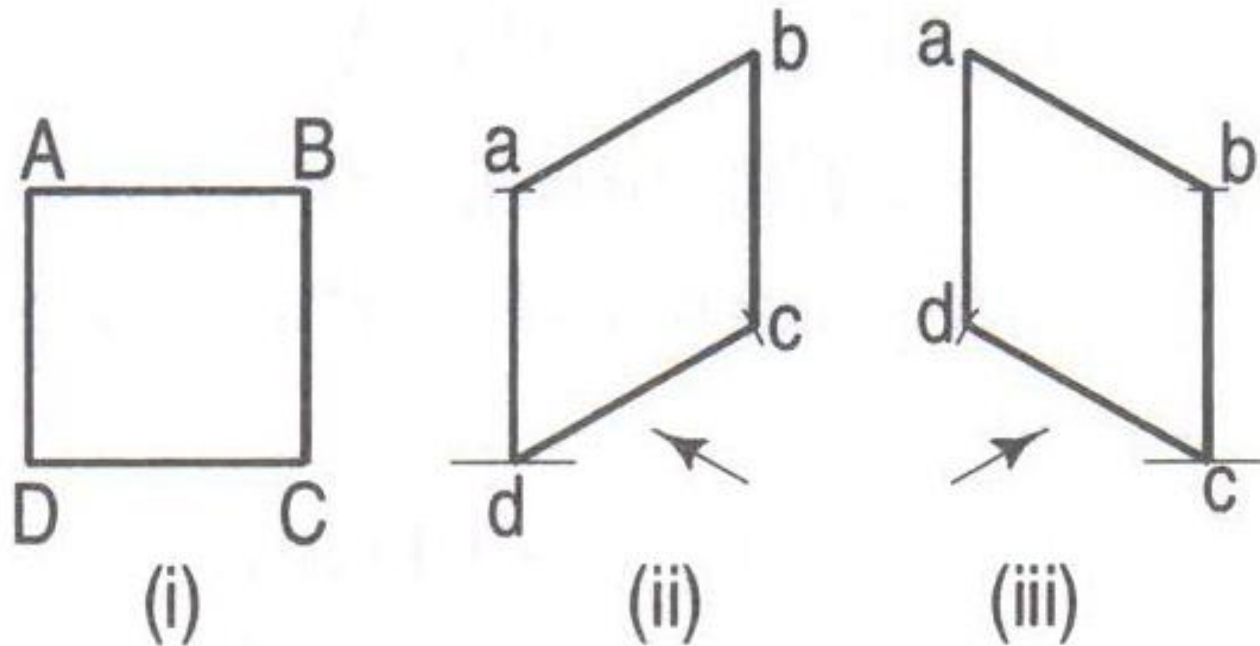


# Planar Features

# Isometric Projections

## Example-1 (Solved Pb. 17-1, pp. 412)

The front view of a square is given in (i). Draw its isometric projection.

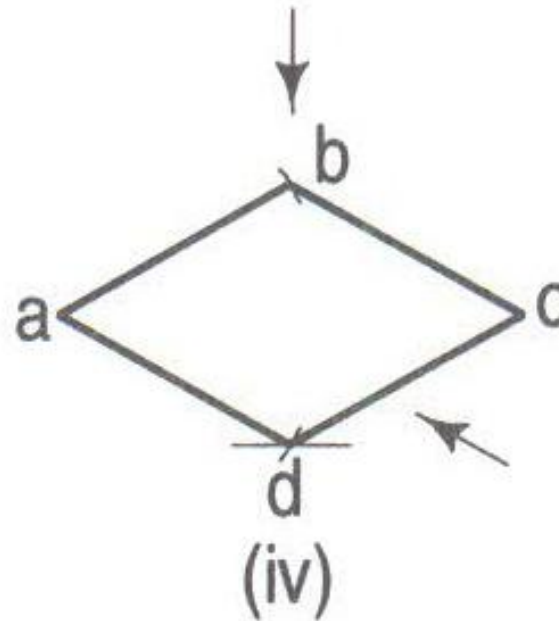
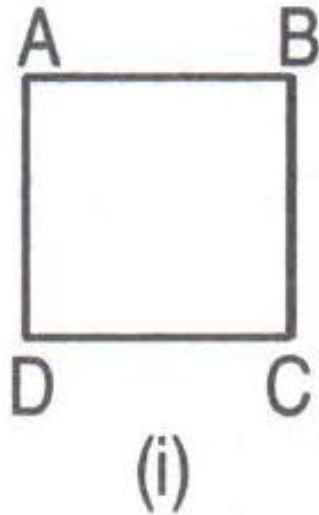


Note: Two possible answers.

# Isometric Projections

## Example-2 (Solved Pb. 17-2, pp. 421)

The top view of a square is given in (i). Draw its isometric projection.



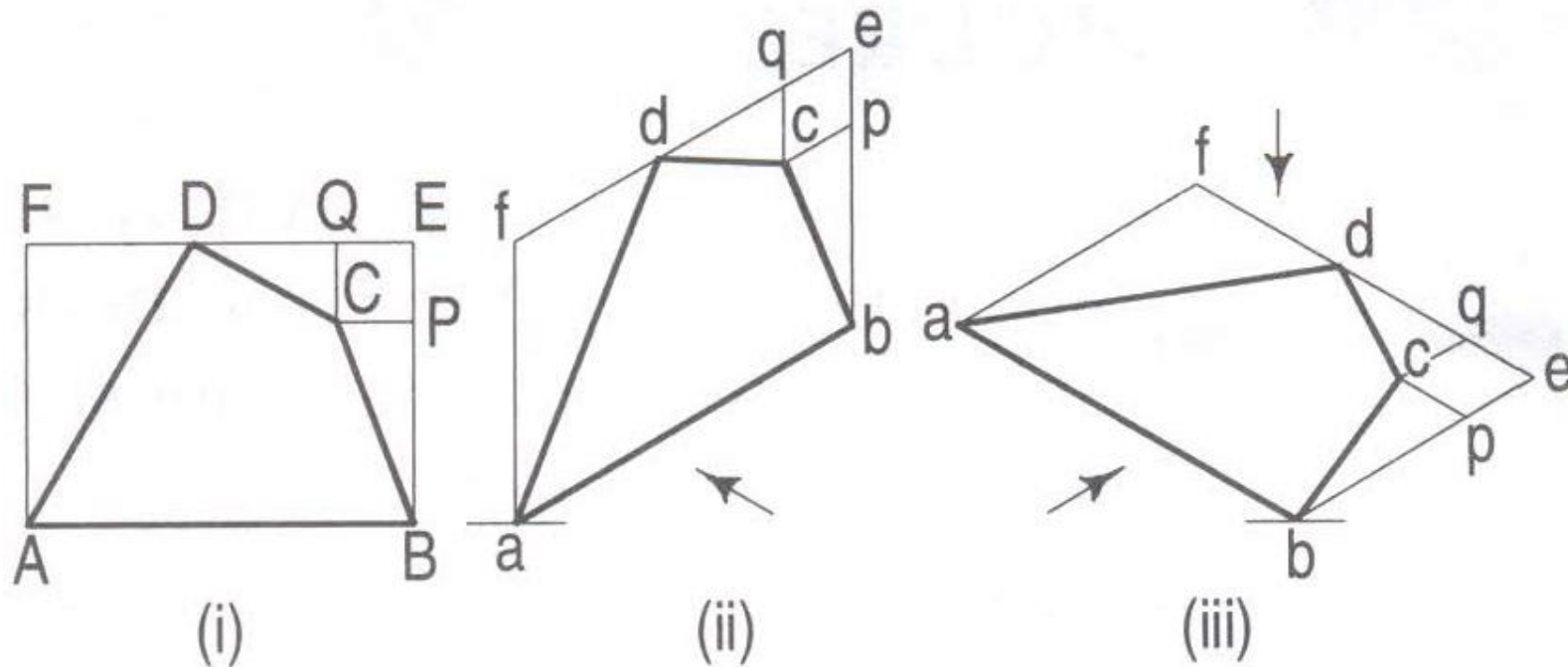
# Isometric Projections

## Example-5 (Solved Pb. 17-5, pp. 422)

A quadrilateral surface is shown in (i). Draw its isometric view when

(a) it is parallel to V.P.

(b) It is parallel to H.P.



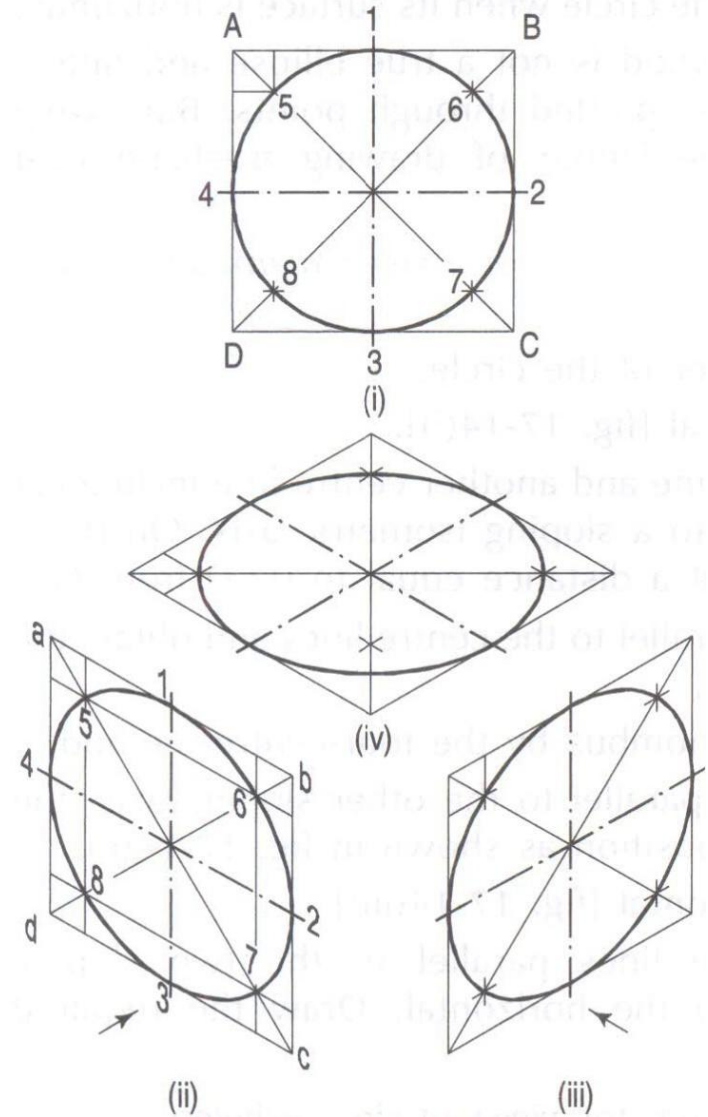


# Isometric Projections

## Example-7 (Solved Pb. 17-7, pp. 422)

Draw the isometric view of  
a circle when it is parallel to

(a) V.P. (b) H.P.

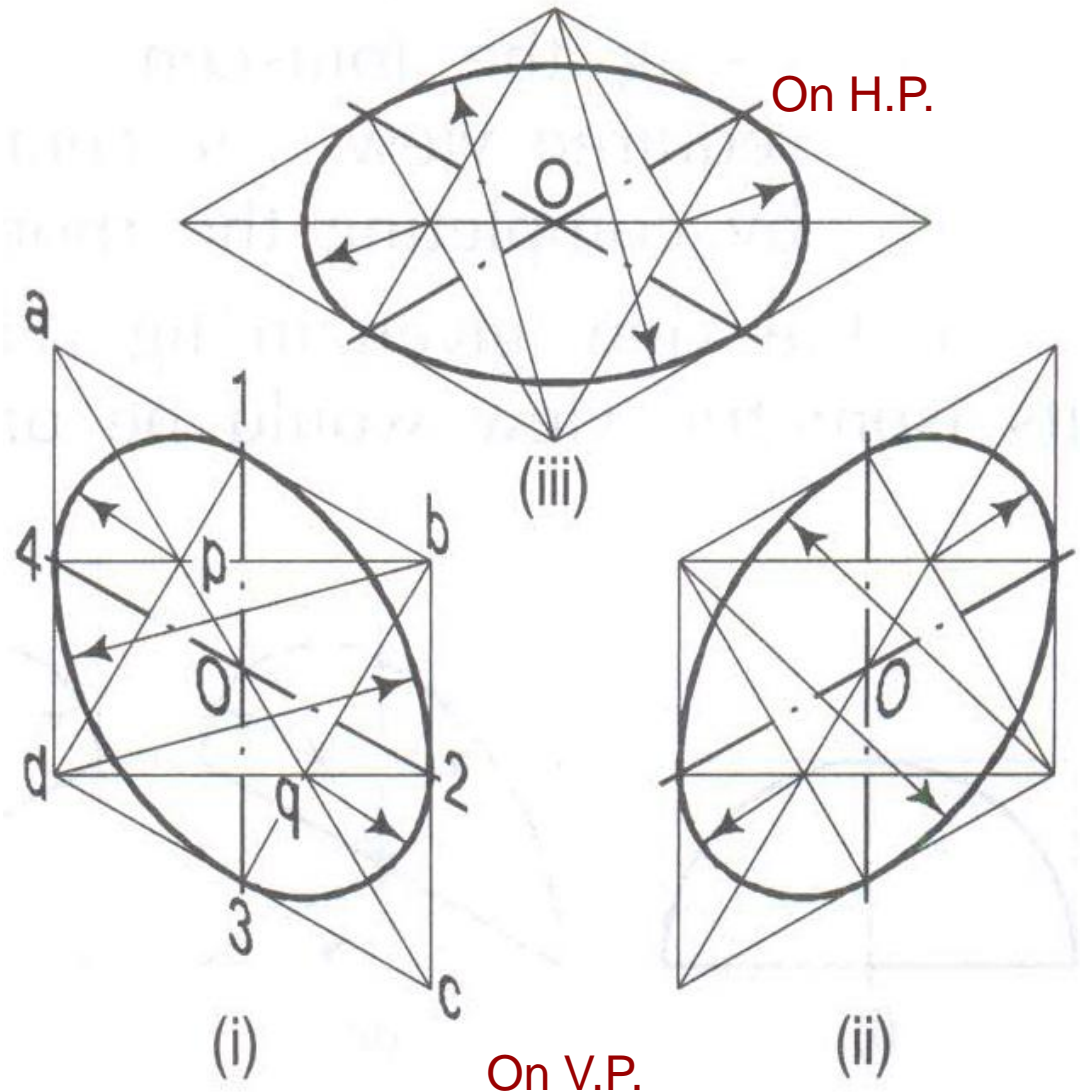


# Isometric Projections

## Approximating isometric ellipses into four arcs

Previous method is accurate but will take time. Since many isometric arcs and circles may be required in an isometric view, an approximate method is used as discussed in sheet 1.

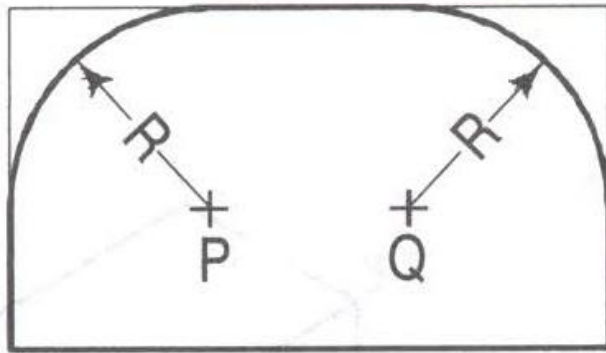
- It is an approximation of an ellipse by four arcs.
- The input is a bounding rhombus, in this case with included angles as  $60^\circ$  and  $120^\circ$ .



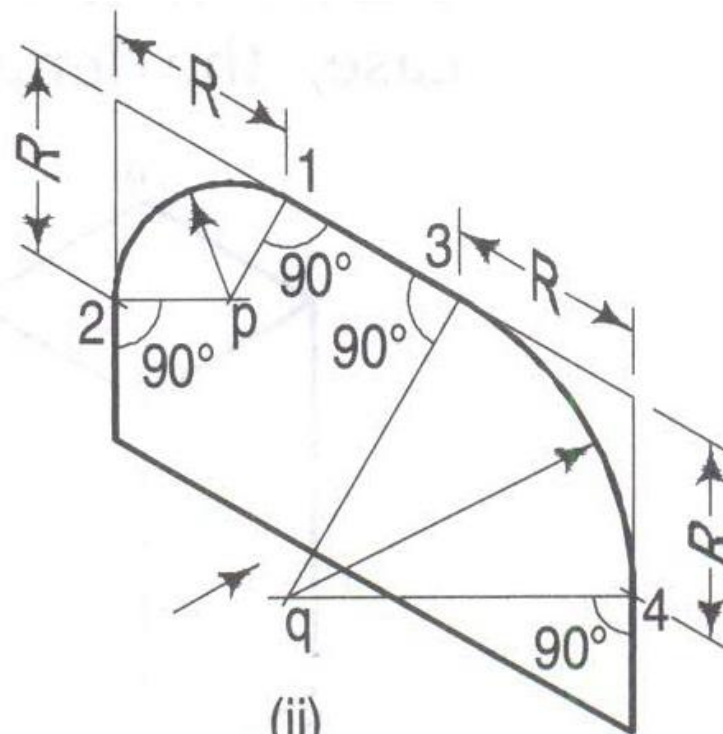
# Isometric Projections

## Example-11 (Solved Pb. 17-11, pp. 425)

The profiled surface is parallel to V.P. Draw its isometric view.



(i)



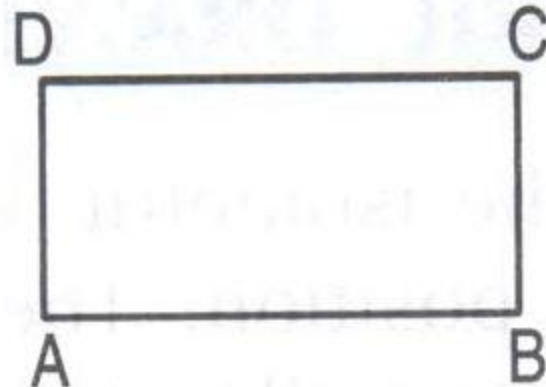
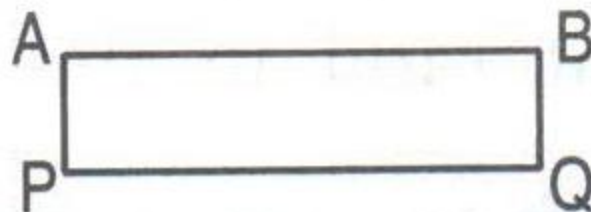
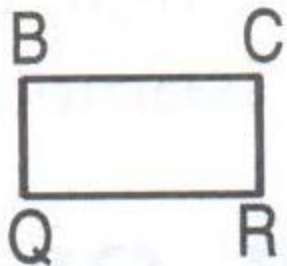
(ii)

# **Polyhedral Objects (Prisms & Pyramids)**

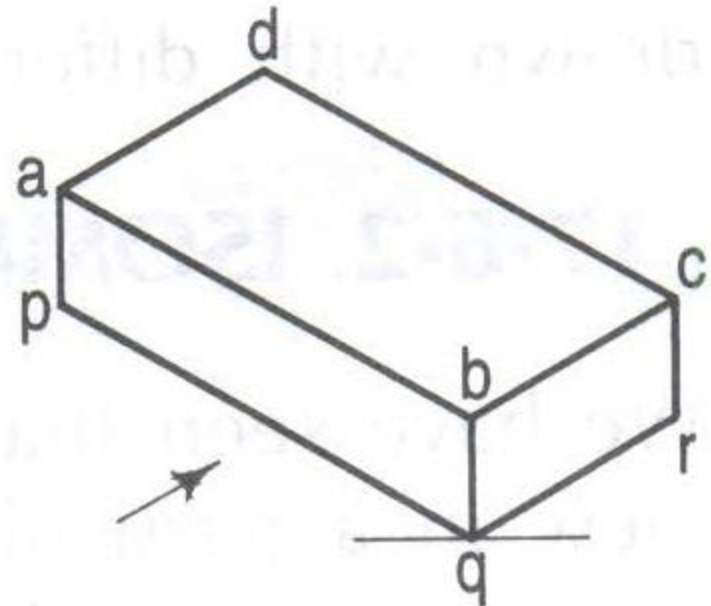
# Isometric Projections

## Example-13 (Solved Pb. 17-13, pp. 426)

Three views of a block are given in (i). Draw its isometric view.



(i)

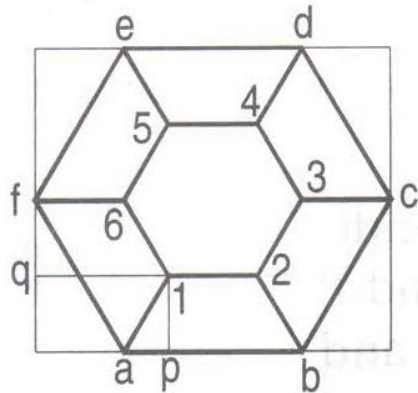
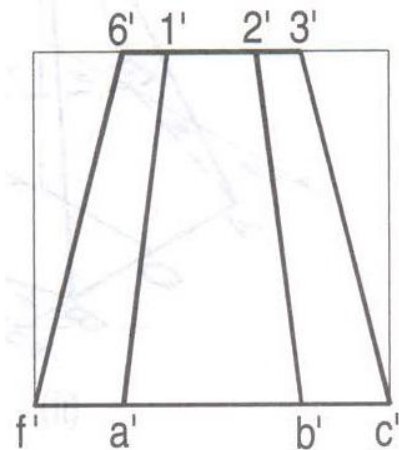


(ii)

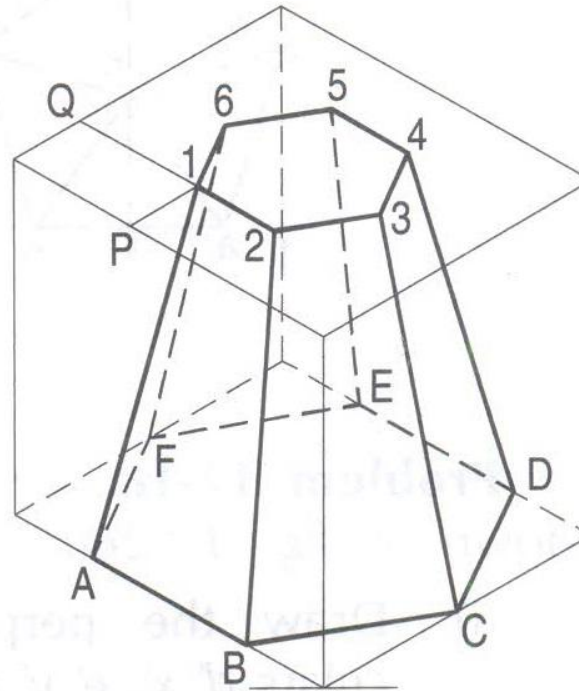
# Isometric Projections

## Example-16 (Solved Pb. 17-13, pp. 427)

Draw the isometric view of the frustum of the hexagonal pyramid shown in (i).



(i)

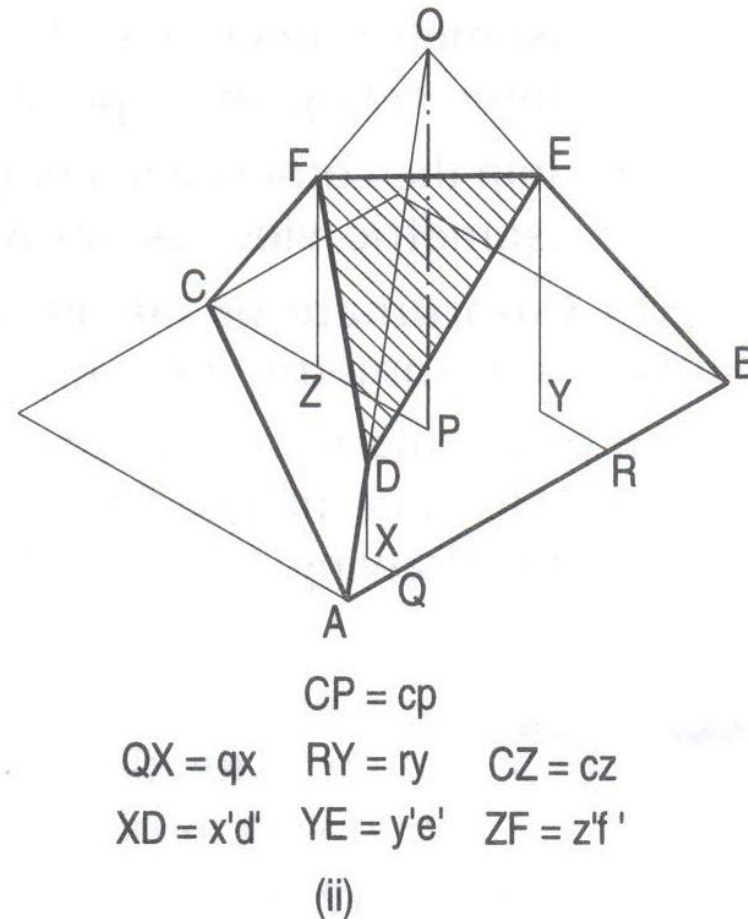
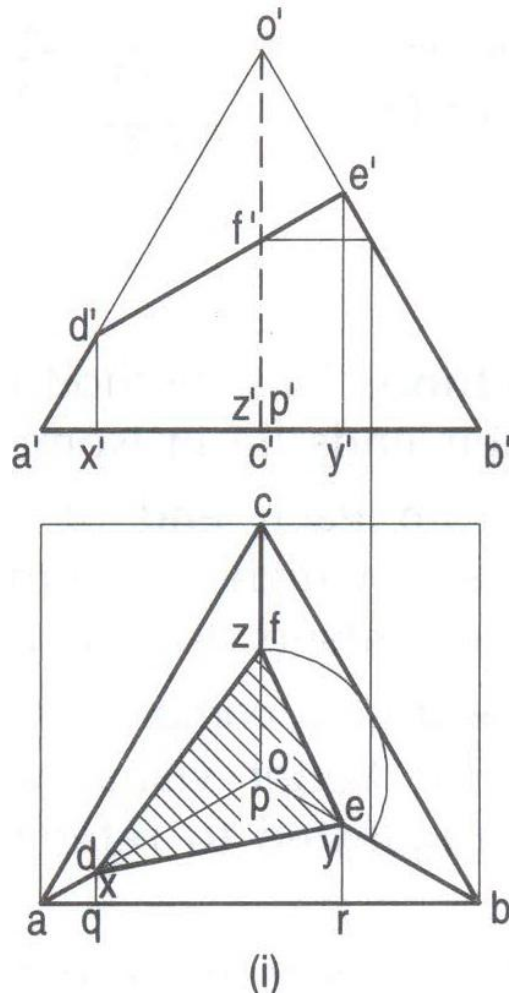


(ii)

# Isometric Projections

## Example-18 (Solved Pb. 17-18, pp. 428)

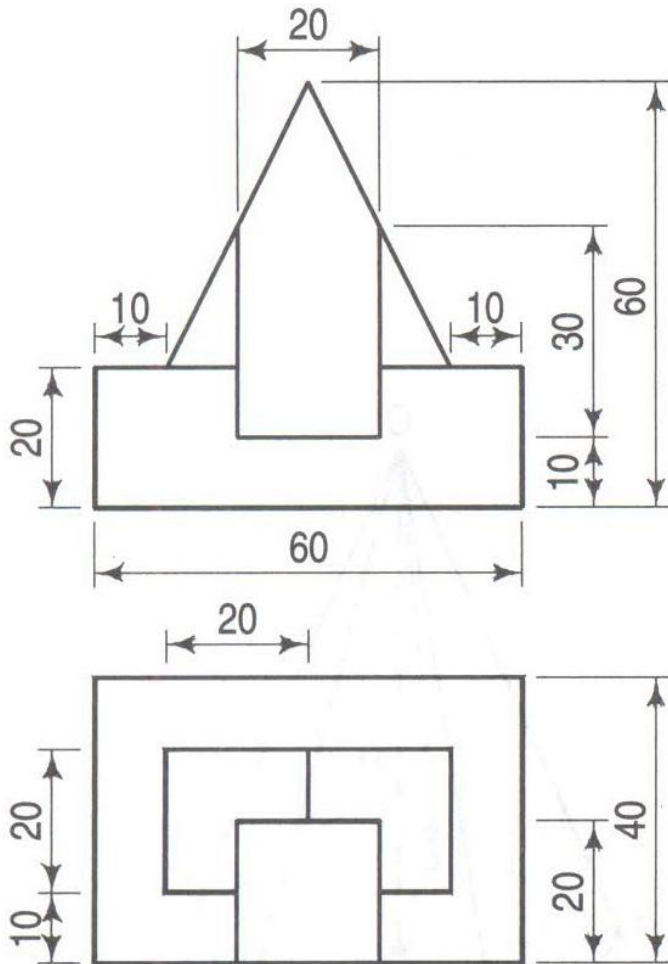
Draw the isometric view of the truncated triangular pyramid shown in (i).



# Isometric Projections

## Example-27 (Solved Pb. 17-27, pp. 434)

Draw the isometric view of the object shown.

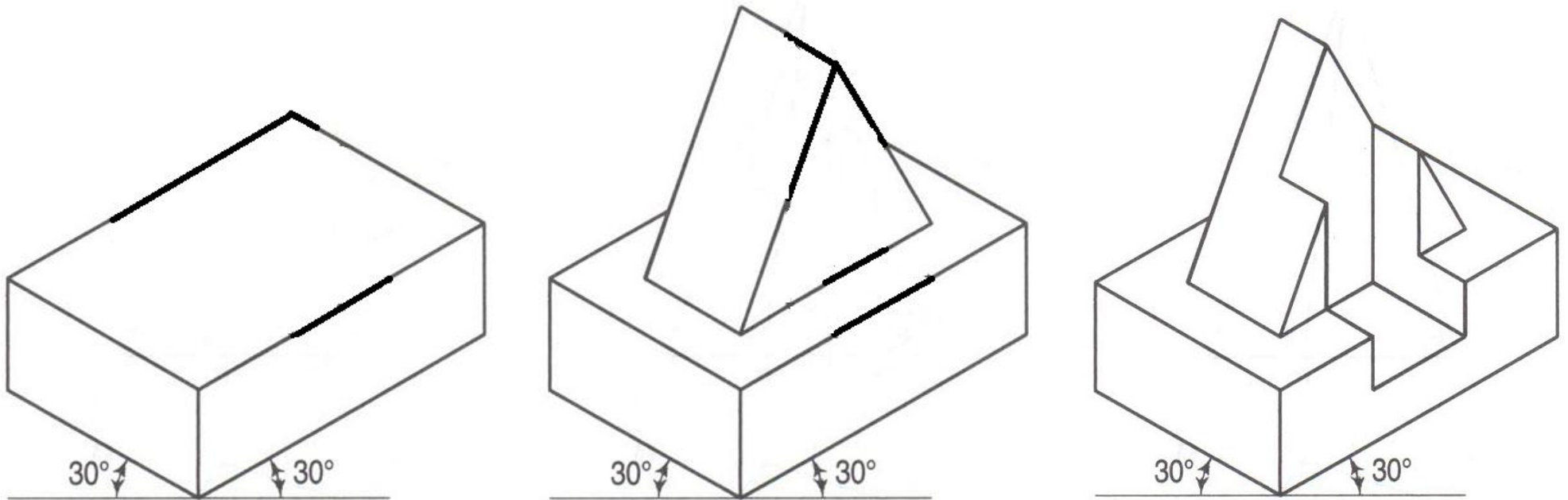


Visualize the object as a collection of features. Here, it has a block at the bottom and a prism at the top each of which have sub-features.



# Isometric Projections

Example-27 (Solved Pb. 17-27, pp. 434)

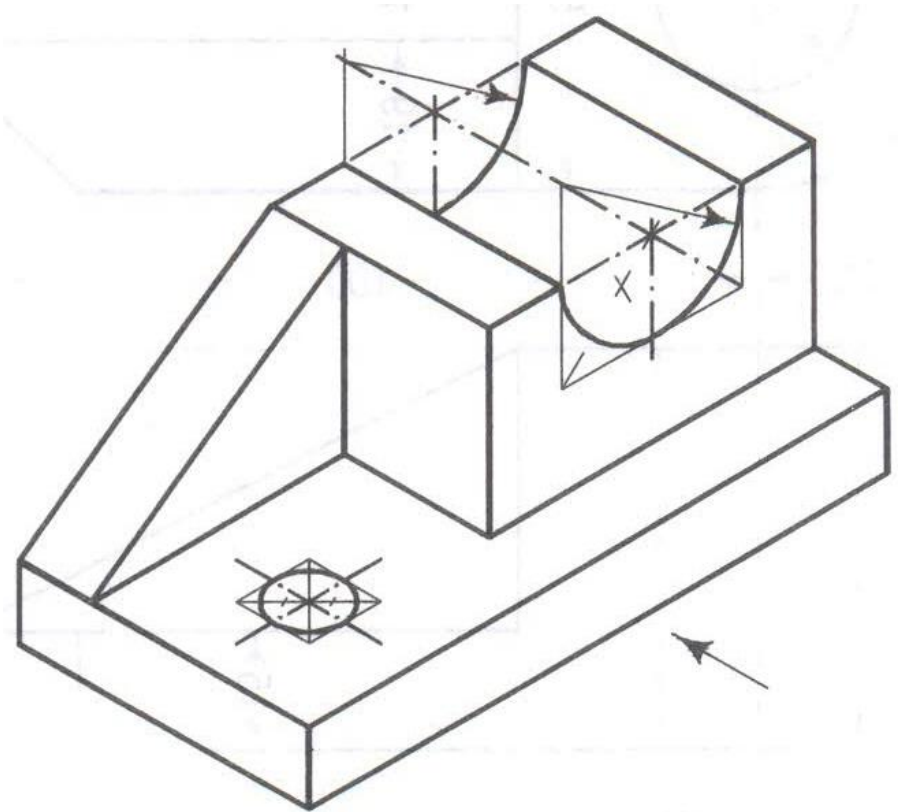
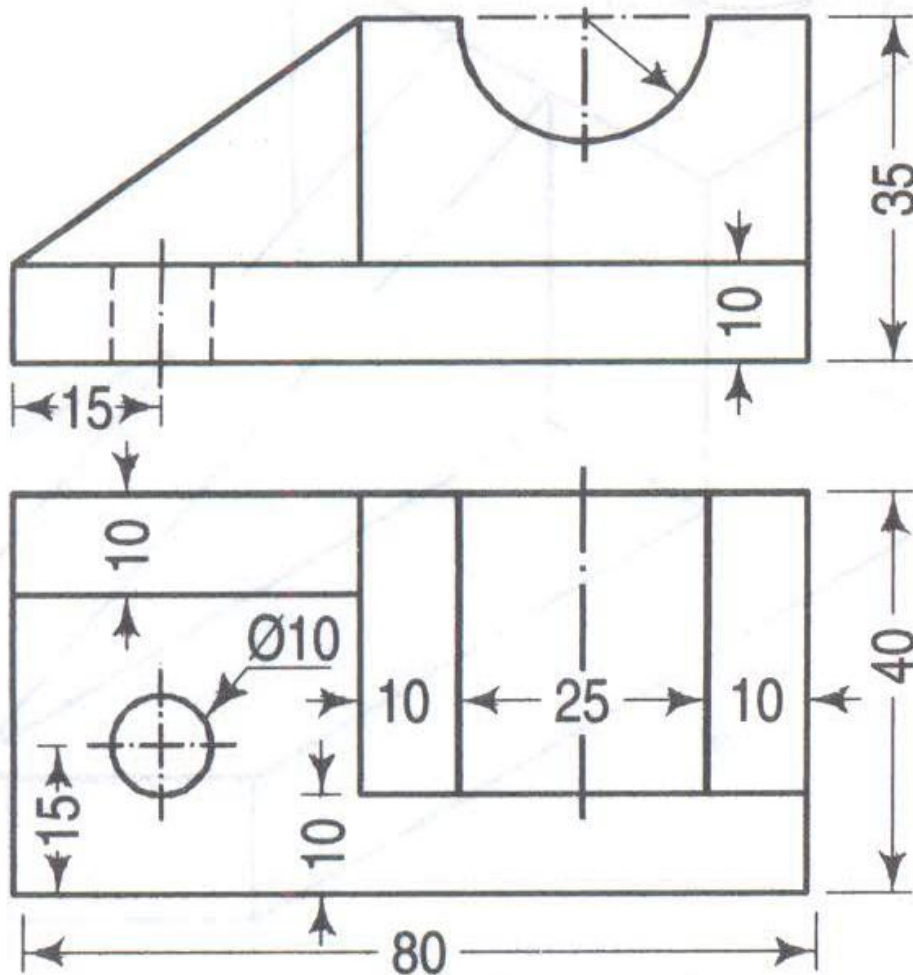


# **Objects with All Combinations**

# Isometric Projections

## Example-27 (Solved Pb. 17-27, pp. 434)

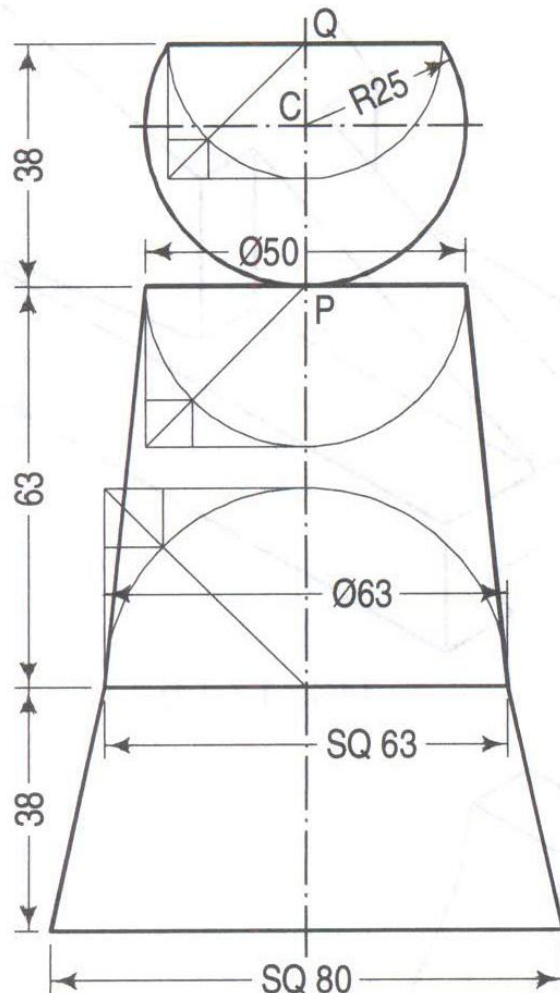
Draw the isometric view of the object shown in (i).



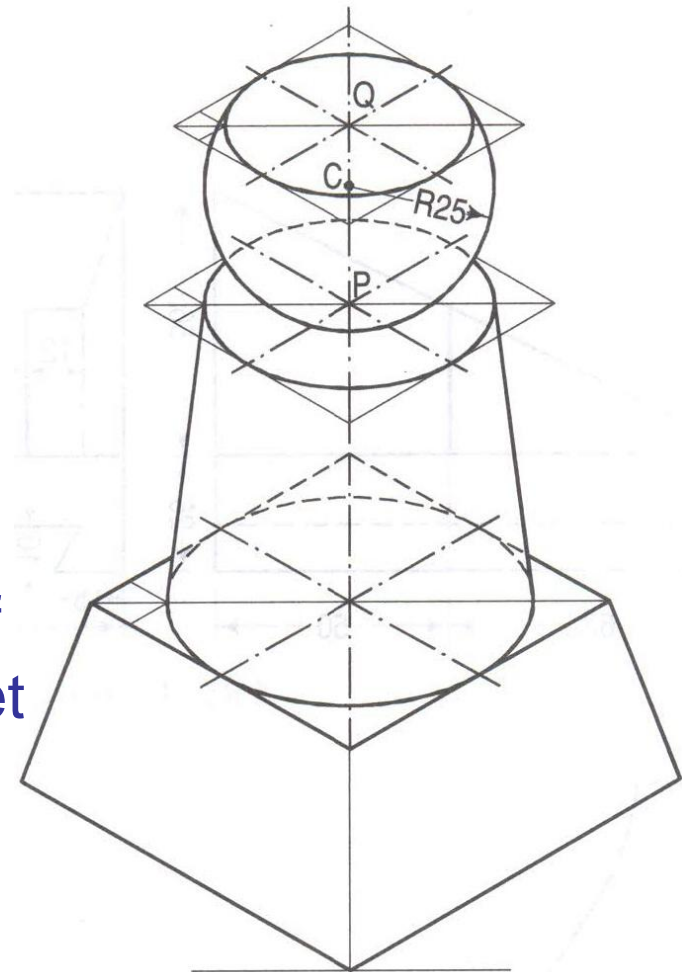
# Isometric Projections

## Example-40 (Solved Pb. 17-40, pp. 439)

Draw the isometric view of the object stack shown in (i).



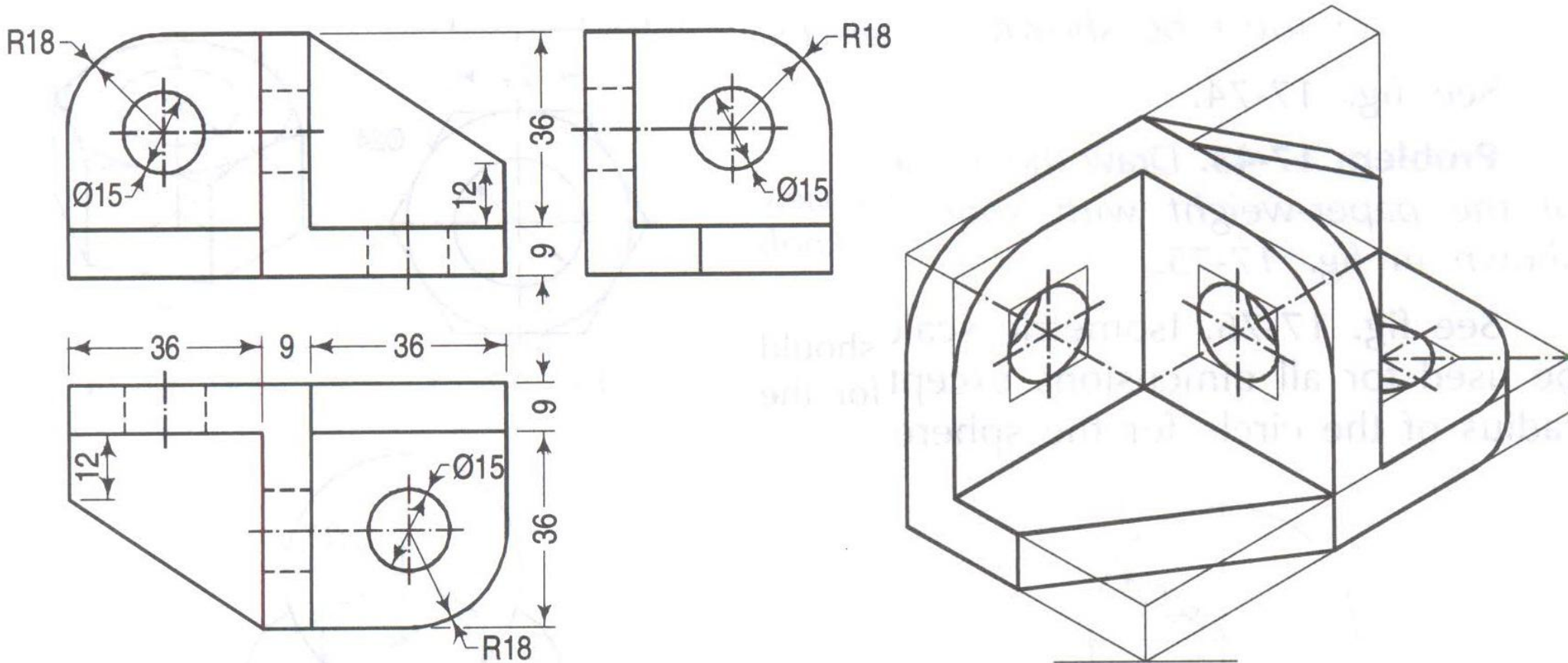
Note: The radius of sphere does not get foreshortened but the distance of centre does.



# Isometric Projections

## Example-45 (Solved Pb. 17-45, pp. 442)

Draw the isometric view of the object stack shown in (i).



# Conclusions

- Roughly work out all the problems given to you. Only if you come prepared, you will be able to complete all problems of the sheet in the drawing session.



**Thank You!**