

PH-105 QM Sheet 1

Vipul Singh

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29. The band structure of a solid in the low wave vector limit is approximately given by the following equation.

$$\hbar\omega = Ak^2 - Bk^4$$

where A and B are constants.

- (a) Show that group and phase velocities are same when

$$\omega = 2A^2/9B\hbar$$

- (b) Show that if the second term in the dispersion relation is neglected, the group velocity of electrons would be twice the phase velocity.

Solution :

(a) Let v_p denote the phase velocity and v_g denote the group velocity.

Then, $\hbar v_p = \hbar\omega/k = Ak - Bk^3$ and $\hbar v_g = \hbar d\omega/dk = 2Ak - 4Bk^3$.

Equating the two, we get $k = \sqrt{\frac{2A}{3B}}$.

Using this value, we get $\hbar\omega = 2A^2/9B$.

(b) If we take $\hbar\omega = Ak^2$, then $\hbar v_p = \hbar\omega/k = Ak$ and $\hbar v_g = \hbar d\omega/dk = 2Ak$. So, we see that group velocity is double the phase velocity.