PH108

Lecture 06:

Electrostatics … solved!

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Summarizing Lectures 1 – 5

Given static $\rho(\vec{r})$, Determine $\vec{E}(\vec{r})$

Tools we have so far: Coulomb's Law of electric force: $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ Gauss's Law; 1 equation; 3 unknowns

$\vec{\nabla} \times \vec{E} = 0$ Consistency check for *static* field

$\oint_{any\ path} \vec{E} \cdot \overrightarrow{dl} = 0$ → Idea of Electric potential

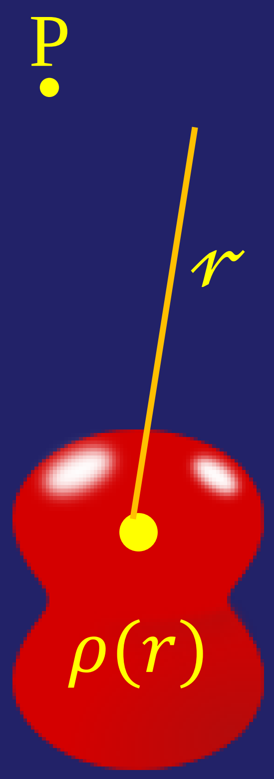
How to calculate the potential Φ(P)

IF we know $\vec{E}$, $\Phi(\mathrm{P}) = \int_{ref}^{P} \vec{E} \cdot \overrightarrow{dl}$

We *DON'T* know $\vec{E}$ for a general $\rho(r)$

But we DO know $\vec{E}$ for a point charge:

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}^2}\hat{\mathscr{r}}$$



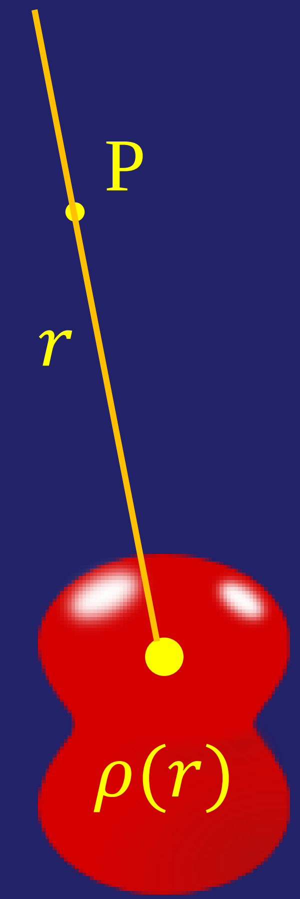
How to calculate the potential Φ(r)

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$\xRightarrow[\substack{\textit{any path is OK,}\\ \textit{so choose } \hat{r}}]{} \quad \Phi(P) = -\int E\, dr$$

$$\Phi(\mathrm{P}) = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q}{r'^2}\, dr$$

OK, because $E \rightarrow 0$ at ∞



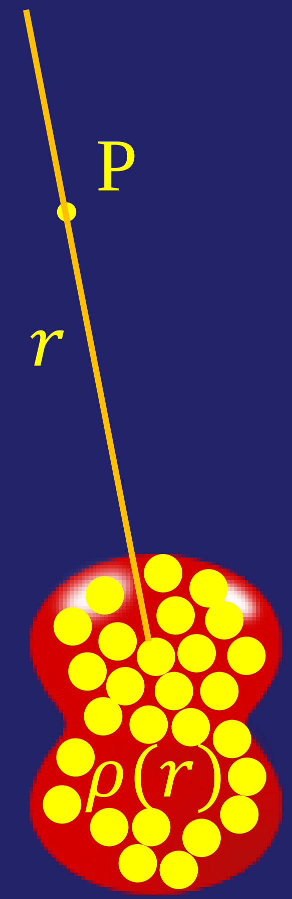
$$\boxed{\Phi(\mathrm{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

The potential $\Phi(\mathrm{r})$ due to $\rho(r)$

$\Phi(\mathrm{P}) = \int_{ref}^{P} \vec{E} \cdot \overrightarrow{dl}$: $\vec{E}$ superposition OK,

Any path OK

So $\Phi(P)$ superposition OK



$$\Phi(\mathrm{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$\Phi(\mathrm{r}) = \frac{1}{4\pi\epsilon_0}\sum \frac{q_i}{\mathscr{r}_i}$$

$$\boxed{\Phi(\mathrm{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{\mathscr{r}} d\tau'}$$

$\vec{E}(r)$ and $\Phi(\mathrm{r})$ are close cousins

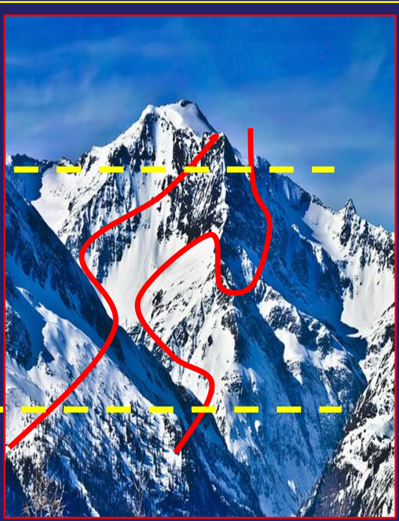
Fundamental gradient theorem:

$$\int_A^B \vec{\nabla}\Phi \cdot \overrightarrow{dl} = \Phi(B) - \Phi(A)$$

any path

$\Phi(B)$

$\Phi(A)$

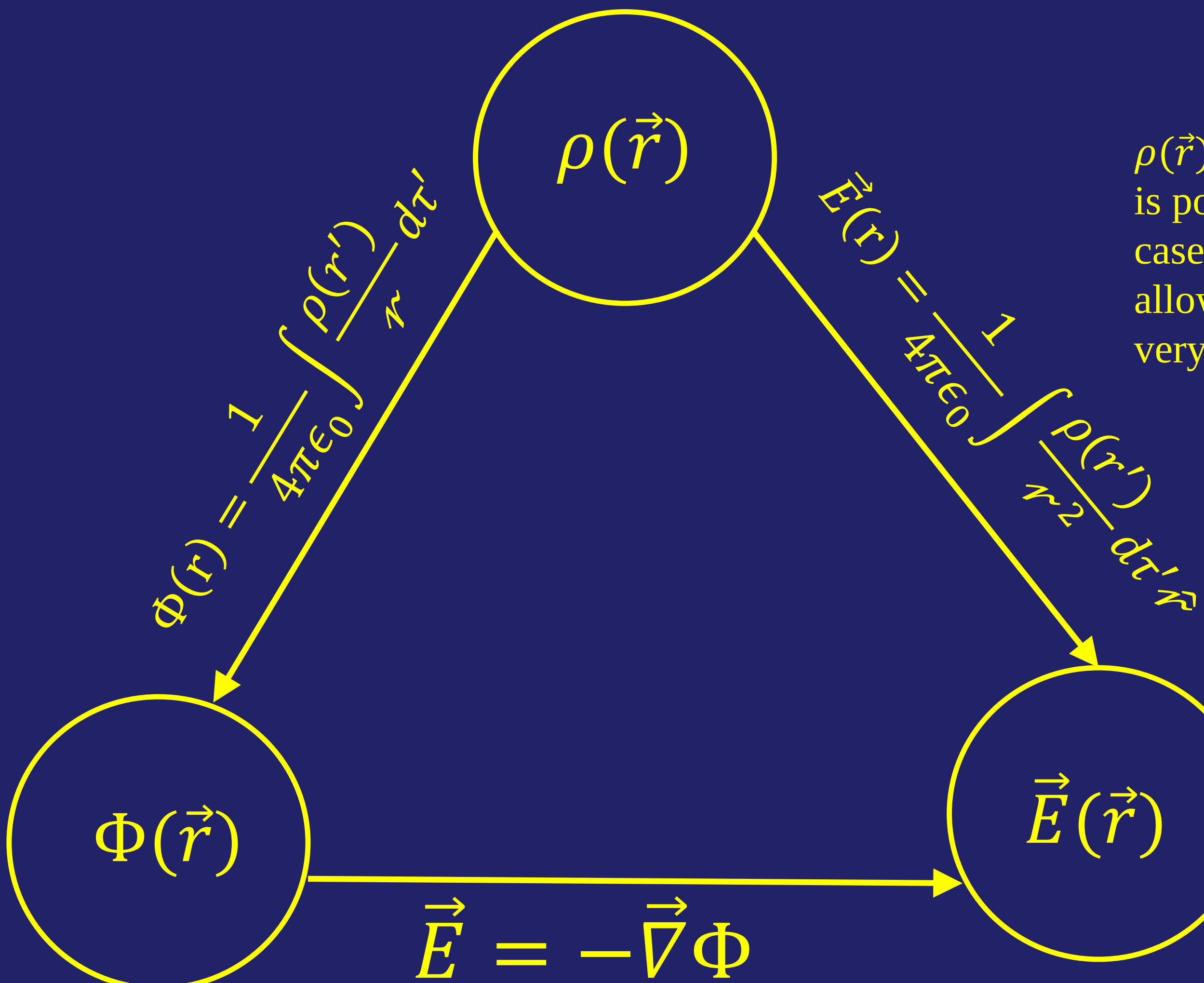


$$\Phi(A) - \Phi(B) = \int_A^B \vec{E} \cdot \overrightarrow{dl}$$

any path

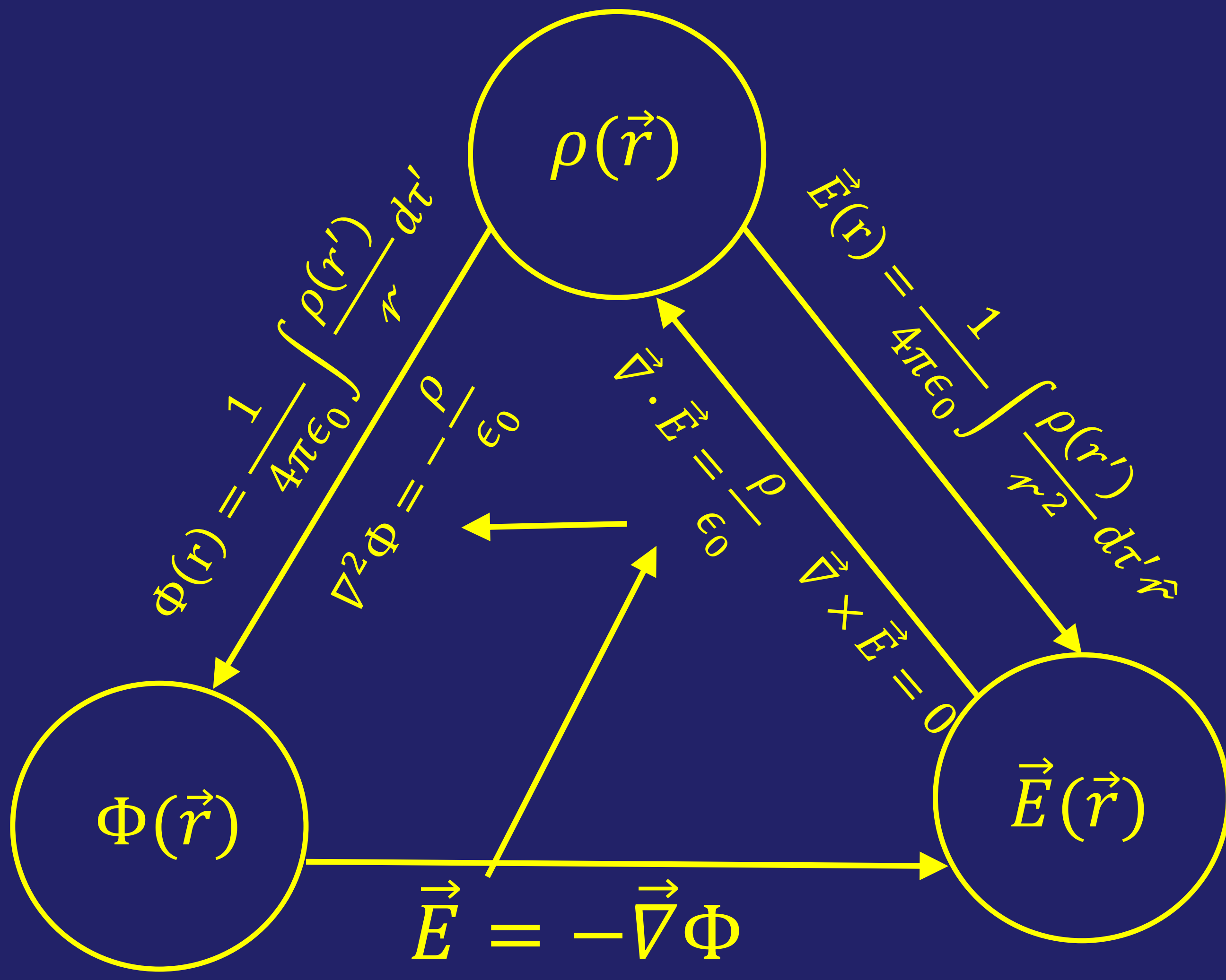
$$\boxed{\vec{E} = -\vec{\nabla}\Phi}$$

… and we have solved electro***statics***



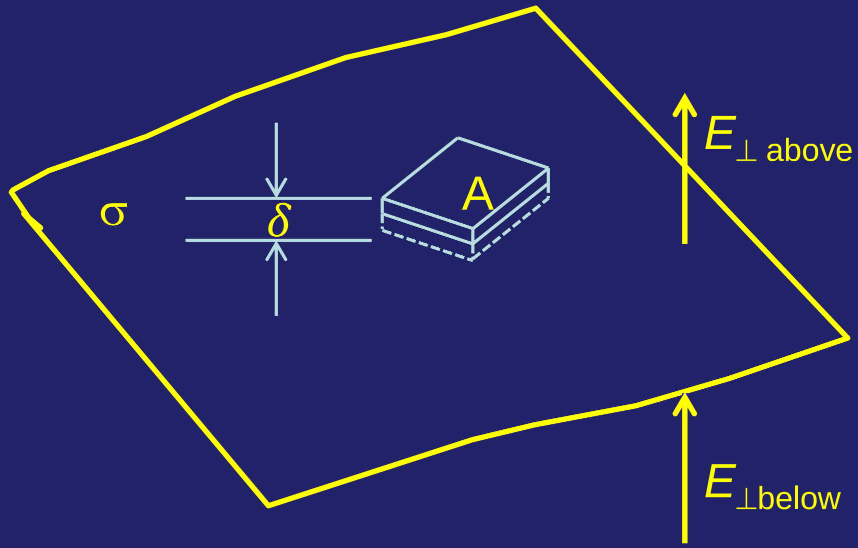
$\rho(\vec{r}) \rightarrow \vec{E}(r)$ calculation is possible only in special cases where symmetry allows. Else $\hat{r}$ makes it very difficult

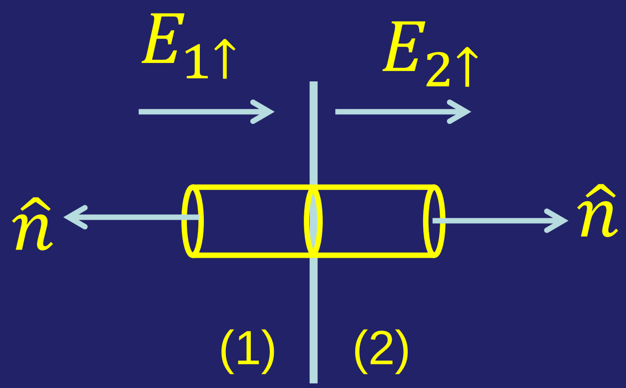
… and gone beyond *statics*



Flux is used to set boundary conditions

Normal component of $\vec{E}$ discontinuos cross a surface charge



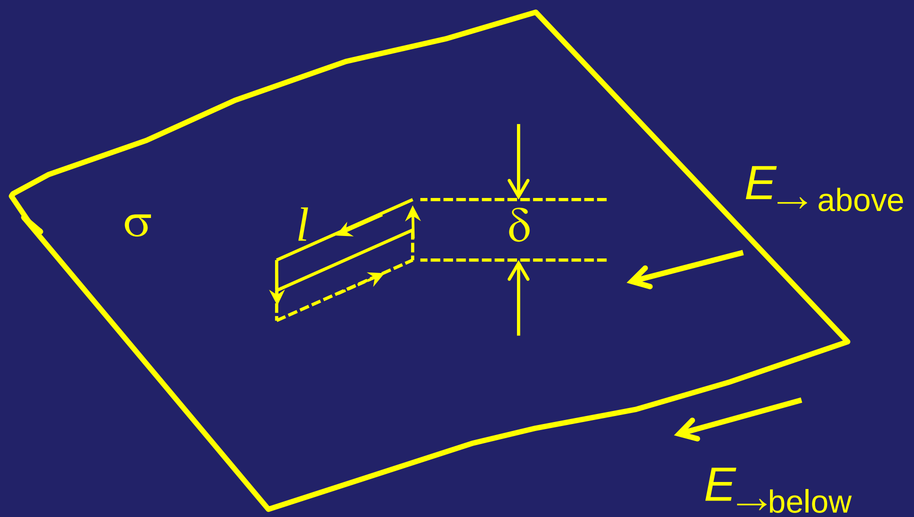


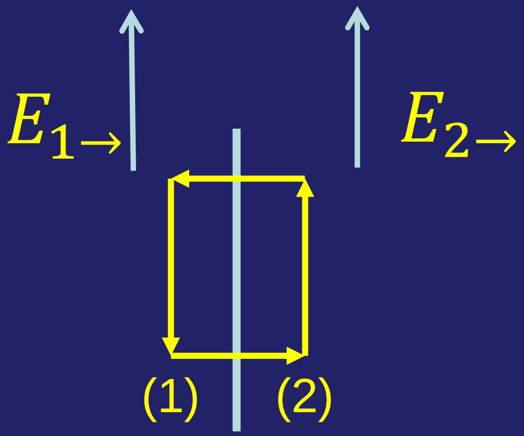
$$\oiint \vec{E} \cdot \overrightarrow{d\sigma} = (E_{2\uparrow} - E_{1\uparrow})A = \frac{Q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{\Delta E_{\uparrow} = \frac{\sigma}{\epsilon_0}}$$

Flux is used to set boundary conditions

Parallel component of $\vec{E}$ is continuous





$$\oint \vec{E} \cdot \overrightarrow{dl} = (E_{2\rightarrow} - E_{1\rightarrow}) = 0$$

$$\Delta E_{\rightarrow} = 0$$