PH108

Lecture 07:

General validity of $\frac{1}{r^2}$ force laws

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Logistics of Quiz 1

Syllabus:

1. Coordinate systems
2. Vector calculus (necessary formulae for ∇ etc will be provided)
3. Coulomb's law
4. Gauss's law and its applications (flux etc)
5. Potential

PH108 marking scheme:

Quiz 1 = 10; MidSem = 20; Quiz 2 = 10; EndSem = 30

Tutorials = 10; D1 Assignments = 10

Discussion of D1: Assignment 1

A point charge $+Q$ is placed at the origin. 9 point charges each of value $-q$ each are placed at a radial distance R,2R,3R,4R...,9R from the origin. Assume $|q| \ll Q$, *eg* $q=0.01Q$ and $|q|$ is small enough that the interaction among the 9 charges is negligible.

1. If each of the 9 charges (-q) moves with a constant tangential speed along concentric circular orbits around the origin, calculate the tangential *linear* speeds of each of these charges (not the angular speed ω). The mass of each of the 9 charges is m.

   Make a plot of the tangential speed of charges q as a function of the radial distance.
1. In addition to the +Q charge at the origin, suppose there is a constant uniform volume density of charge ρ_0 (positive) throughout all space from $r=0$ to ∞. In this case, work out:
   a. Will the orbit of the 9 charges remain circular? Will the velocity continue to be purely tangential?
   b. What will be the tangential linear speeds of the 9 charges for this case of central charge +Q plus a uniform charge density ρ_0?

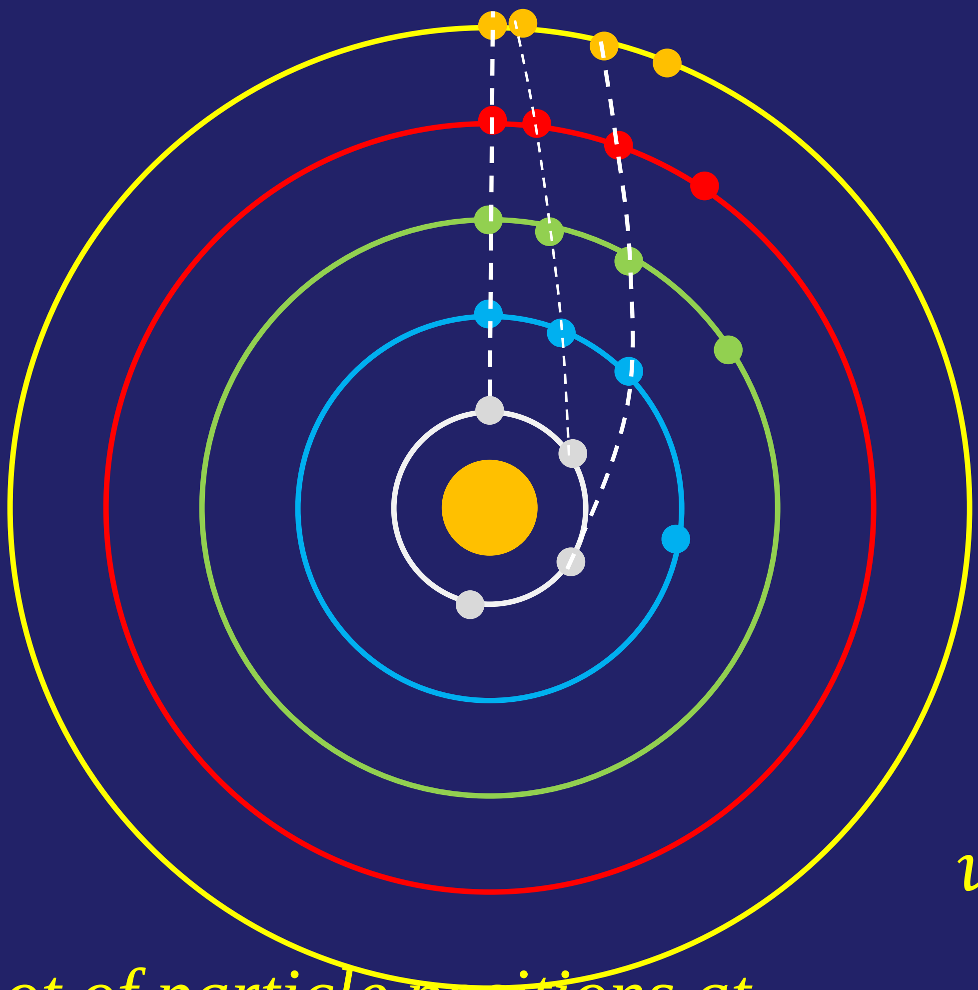
      Make another plot of the speed as a function of radial distance in this case of +Q plus a uniform charge density ρ_0

On a single polar (r,θ) plot in your solution sheet, mark the positions of all 9 charges for case 2 above at the following times:

$t=\pi/96$ *sec*, $\pi/80$ *sec*, $\pi/64$ *sec*, $\pi/48$ *sec*, $\pi/32$ *sec*, $\pi/16$ *sec*

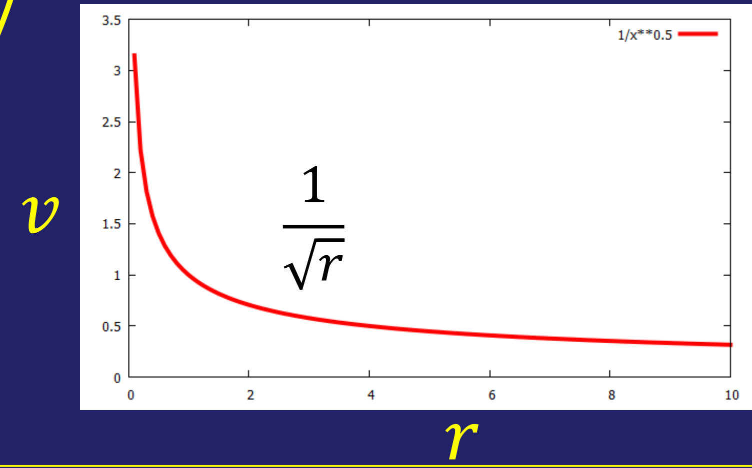
Assume that all 9 charges are placed at $\theta=0$ at $t=0$. Draw curves connecting the positions of the charges at equal times.

Trajectory of particles in $\frac{1}{r^2}$ field



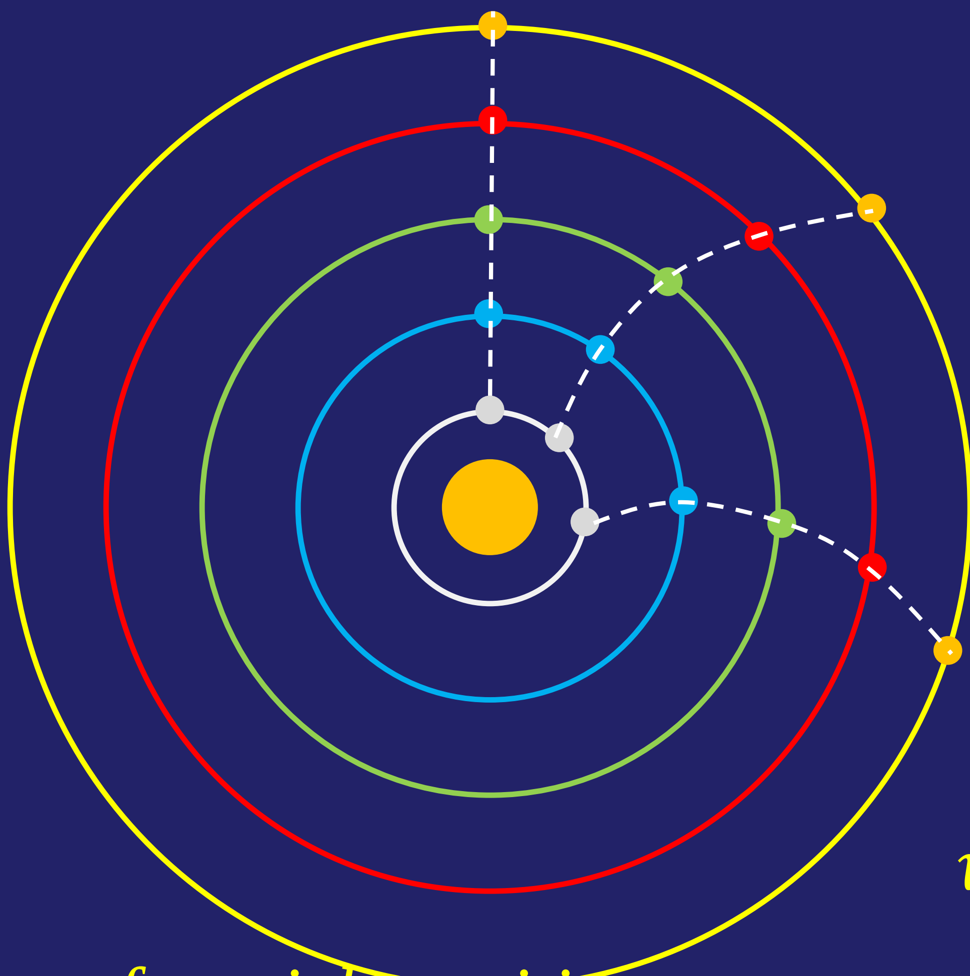
$$\frac{mv^2}{r} = \frac{Qq}{r^2}$$

$$v = \sqrt{\frac{Qq}{m} \cdot \frac{1}{r}}$$



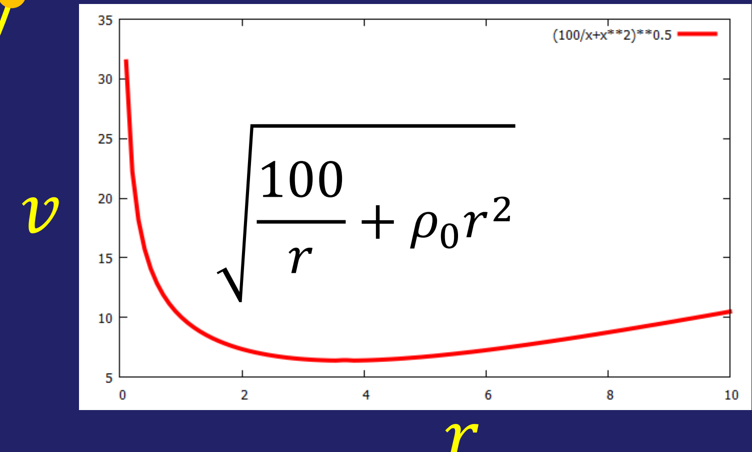
Snapshot of particle positions at any time is a right-handed spiral

In $(\frac{1}{r^2} + \rho_0 r)$ field, outer particles move faster



$$\frac{mv^2}{r} = \frac{Qq}{r^2} + \rho_0 r$$

$$v = \sqrt{\frac{Qq}{m} \cdot \frac{1}{r} + \rho_0 r^2}$$



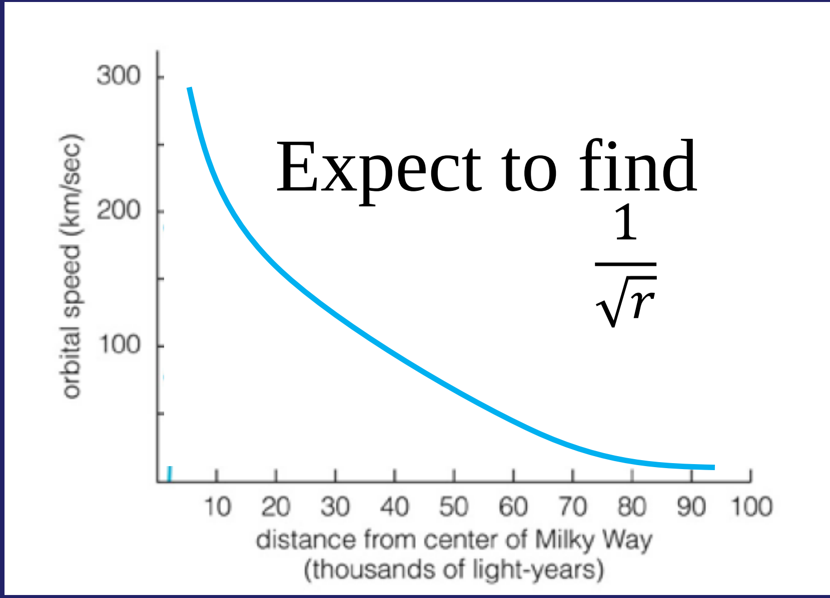
Snapshot of particle positions at any time is a LEFT-handed spiral

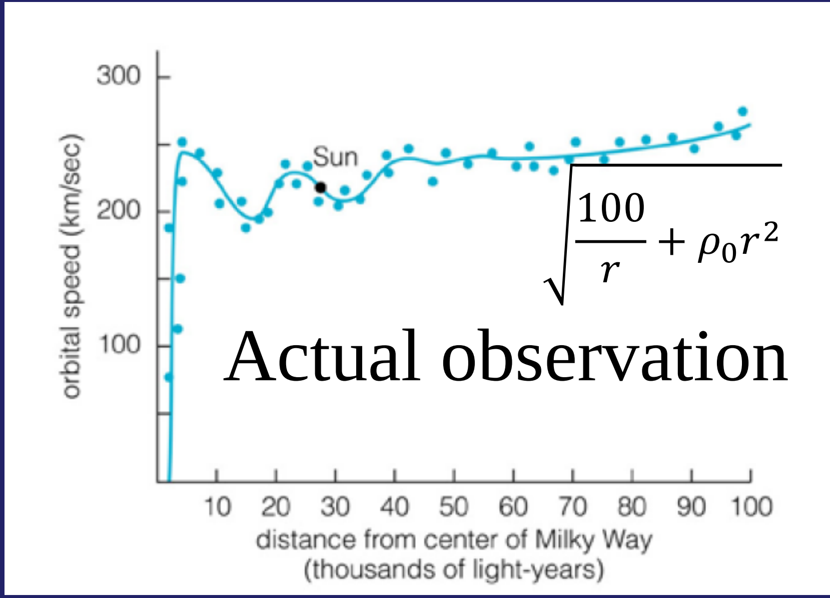
Is the $\frac{1}{r^2}$ law *universal* ?

Which other force in nature has $\frac{1}{r^2}$ behavior?

Gravitation $\quad F_G = \frac{GMm}{r^2} \quad$ (always attractive)

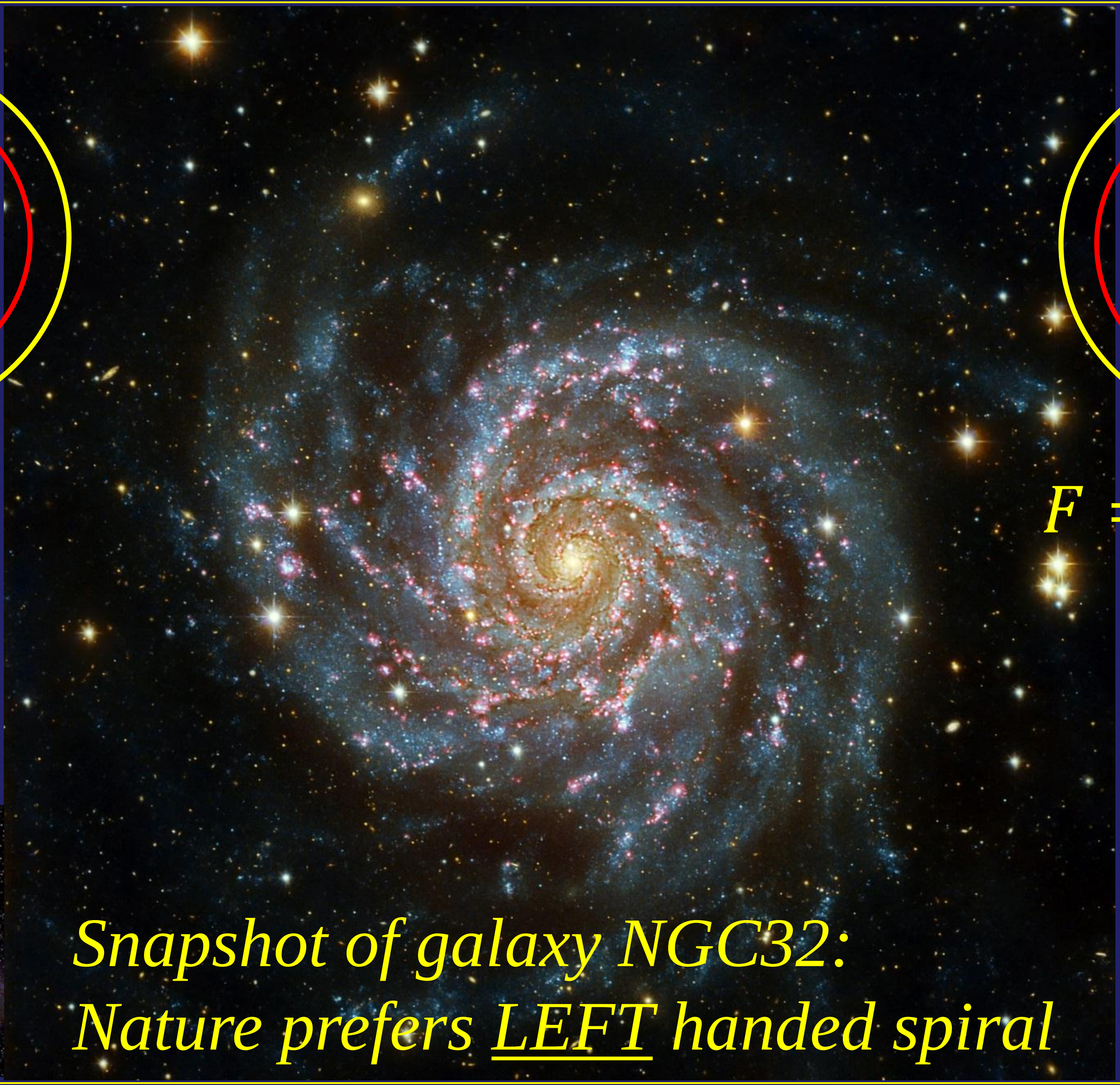
Astronomers observe orbital speed of stars in a galaxy

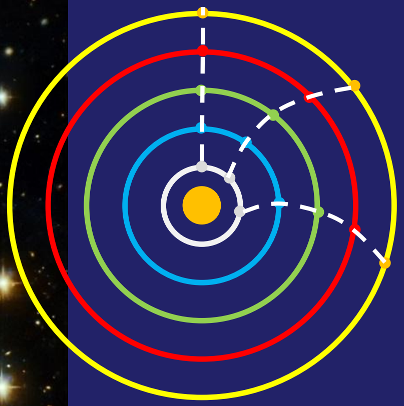




What do you *not* see in this picture?

$$F = \frac{1}{r^2}$$





$$F = \frac{1}{r^2} + \rho_0 r$$

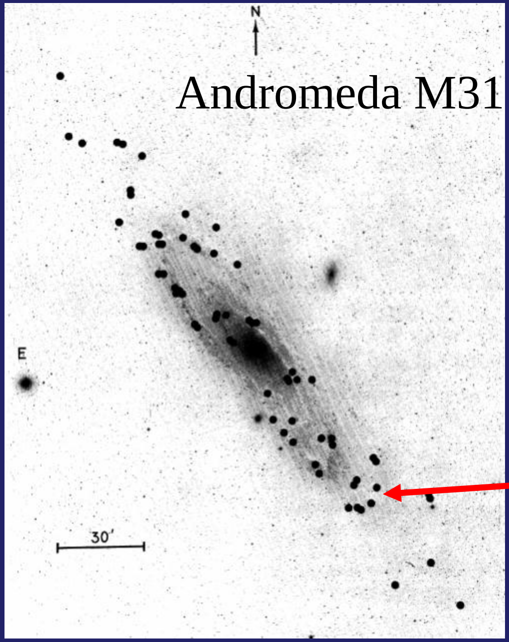
Snapshot of galaxy NGC32:
Nature prefers LEFT handed spiral

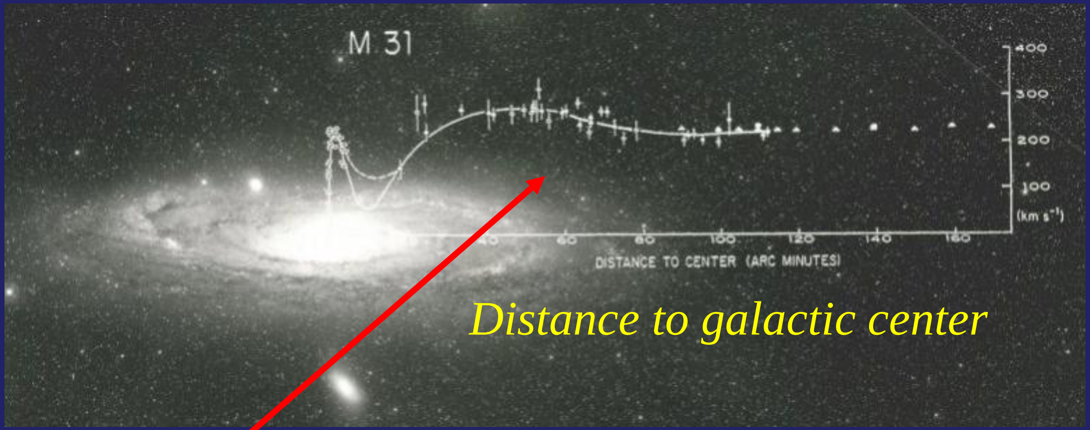
Hubble telescope © NASA

$\frac{1}{r^2}$ gravity force law works at galactic scale

… but there is matter we cannot 'see'

This was the first observation of dark matter





Measure spee
ds
of these stars
by Doppler shift

Expect orbital speed to decrease ~ $1/\sqrt{r}$
But it rises due to some $\rho_0(r)$

Plots from Rubin & Ford, Astrophysical Journal 159:379 (1970)

What is dark matter?

We don't know

All the visible matter in this picture does not account for the rising orbital velocity of stars at the periphery

Astronomers have studied many galaxies

Used the deviation from $\frac{1}{r^2}$

Found that ~ 24% of matter in the universe is dark

i.e. does not emit electromagnetic radiation – cannot be seen

Thread of logic so far…

Coulomb's force law between static charges was an *experimental* observation: Force ~ $\frac{1}{r^2}$

We try to understand *all* its implications

- Vector Electric field
- Maths of divergence and curl, Gauss's law
- Scalar potential

We find another force that also has $\frac{1}{r^2}$ behavior

Gravitation

We observe galaxy dynamics to check if $\frac{1}{r^2}$ works – it does

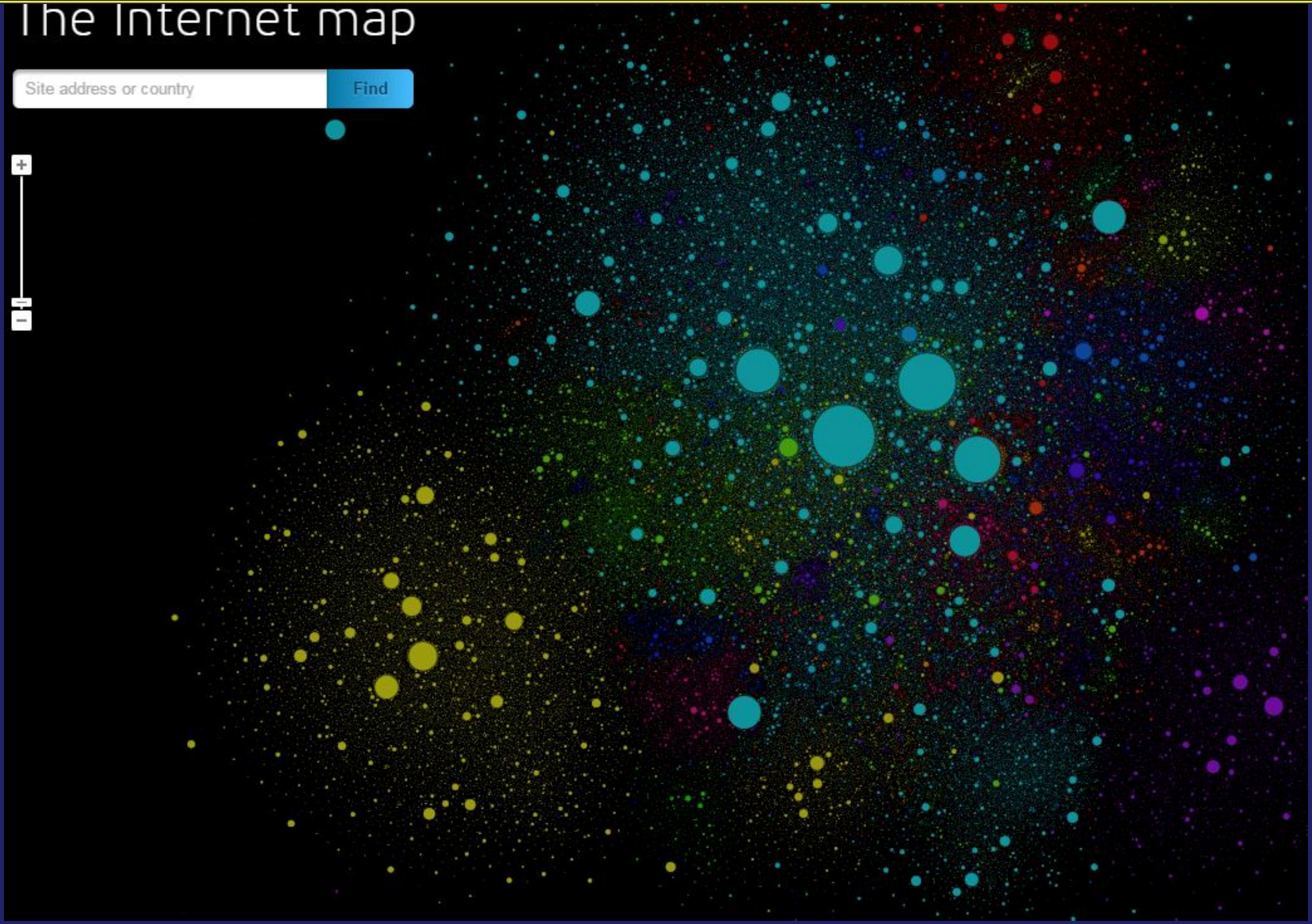
*Discover **new*** dark matter from our calculations

Statutory Warning

Material in the following slides is not strictly within the confines of electricity and magnetism

However, it may induce you to explore applications of $\frac{1}{r^2}$ connection between objects in other domains

Information ‘field’ on the www



Each circle on the map corresponds to a www address

Size of circle is $\propto$ number of other sites that link to it

Colors ~ geographic location

http://internet-map.net

Speculate on the scalar field of the internet

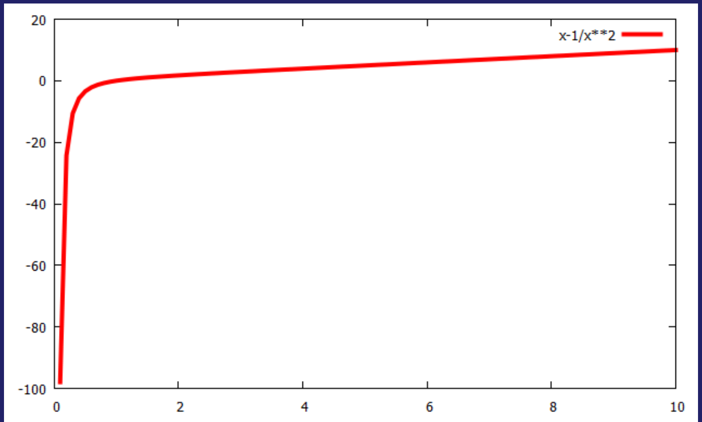
Let the area around each www $\rightarrow \rho$

Have to solve for Φ: $\nabla^2\Phi = -\frac{\rho}{c}$ $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\Phi) = -\frac{\rho}{c}$

$$\Phi = -\frac{\rho r^2}{6c} - \frac{b}{r} + a$$

Does this make sense?

$$E = -\nabla\Phi \sim \rho r - \frac{b}{r^2}$$



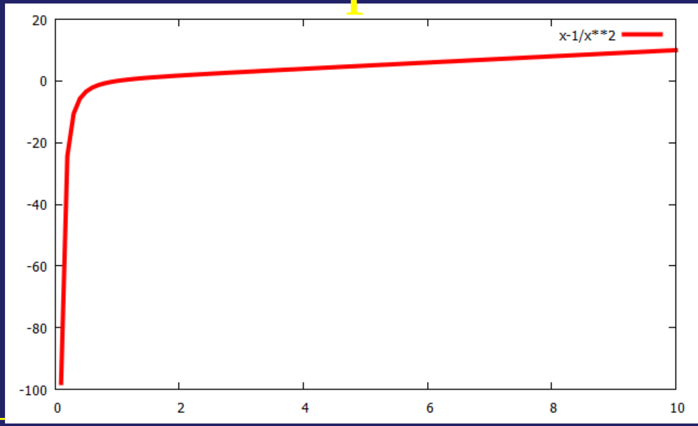
How do you search for information on the internet

Consider an information object (take for example, iPhone) which acts as the 'source' charge.

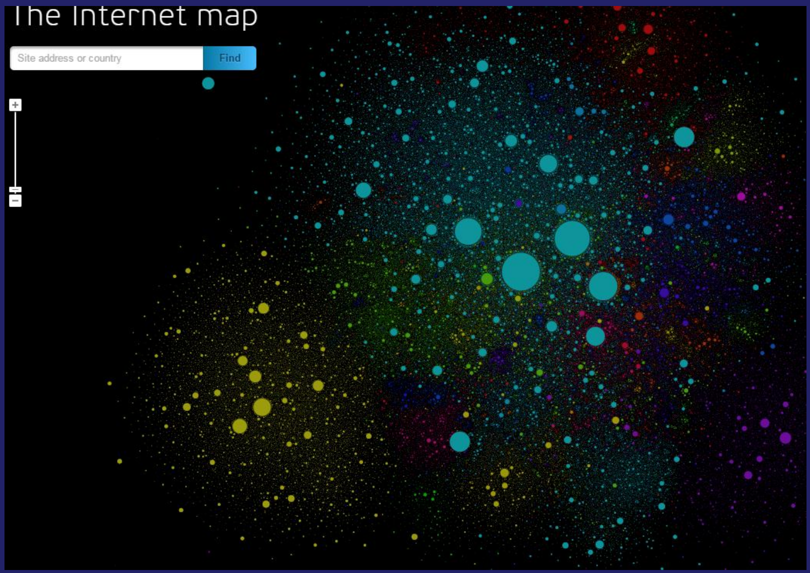
www.apple.com has maximum potential associated with this 'charge' iPhone

the information potential falls off rapidly (speculatively $\frac{1}{r^2}$?) as you move further away from the central concept

The information finding problem becomes an exercise in finding the local minima of this potential



The general solution is quite complex



… and unknown … and changing

There are many charge centers

They are spread in *(x,y)*
in this abstract representation

In general it is not possible to simply assume $\Phi = \Phi(r)$

But it is usually possible to *separate variables*

$$\Phi = \Phi_x(x)\Phi_y(y)$$

… and do a little bit of math to solve Poisson/Laplace equation