# Tutorial-3, MA 108 (ODE)
# Spring 2015, IIT Bombay

1. Use Euler method and Improved Euler's method with step size $h = 1$ to find approximate values of the solution of the IVP $y' = \dfrac{y^2 + xy - x^2}{x^2}$, $y(1) = 2$ at $x = 2, 3$. Compare these approximate values with the values of the exact solution $y = \dfrac{x(1 + x^2/3)}{1 - x^2/3}$.

2. An implicit solution of the IVP $y' = -\dfrac{4x^3y^3 + 2xy^5 + 2y}{3x^4y^2 + 5x^2y^4 + 2x}$, $y(1) = 1$ is given by $x^4y^3 + x^2y^5 + 2xy = 4$. Use Euler method and Improved Euler method to find the approxinmate values of the solution at $x = 1, 2$.

3. Use Euler method and Improved Euler's method with step size $h = 1$ to find approximate values of the solution of the IVP $y' + \dfrac{(y+1)(y+2)(y+3)}{x+1} = 0$, $y(1) = 0$ at $x = 2, 3$.

4. A radioactive substance with decay constant $k$ is produced at a constant rate of $a$ units of mass per unit time. Assuming $Q(0) = Q_0$, find the mass $Q(t)$ of the substance present at time $t$.

5. Newton's law of cooling states that if an object with temperature $T(t)$ at time $t$ is in a medium with temperature $T_m(t)$, then rate of change of $T$ $T' = -k(T - T_m)$, where $k > 0$ is temperature decay constant of the medium.

   Aceramic insulator is baked at $400°C$ and cooled in a room in which the temperature is $25°C$. After 4 minutes the temperature of the insulator is $200°C$. What is the temperature after 8 minutes?

6. Consider 2nd order **autonomous** ODE $y'' = F(y, y')$. Convert it to first order ODE in $v$ and $y$, where $v = y'$.

   (a) Solve $y'' + p(y) = 0$. (b) Solve $y'' + y(y-1) = 0$.

7. Find the general solution of $y'' - 2y' + 2y = 0$. Solve it with initial conditions (a) $y(0) = 3, y'(0) = -2$. (b) $y(0) = k_0, y'(0) = k_1$.

8. (a) Verify that $y_1 = 1/(x-1)$ and $y_2 = 1/(x+1)$ are solutions of $(x^2-1)y'' + 4xy' + 2y = 0$ on $\mathbb{R} - \{\pm 1\}$. Find the general solution. (b) Find the solution with initial conditions $y(0) = -5, y'(0) = 1$. (c) What is the interval of validity of this solution?

9. Compute the Wronskians of the given set of functions.

   (a) $\{e^x, e^x \sin x\}$, (b) $\{x^{1/2}, x^{-1/3}\}$, (c) $\{x \ln|x|, x^2 \ln|x|\}$.

10. Find the Wronskian of a given set of solutions of $y'' + 3(x^2 + 1)y' - 2y = 0$, given that $W(\pi) = 0$.

11. Find the Wronskian of a given set of solutions of $(1 - x^2)y'' - 2xy' + a(a + 1)y = 0$, given that $W(0) = 1$.

12. Find the Wronskian of a given set of solutions of $x^2y'' + xy' + (x^2 - \nu^2)y = 0$, given that $W(1) = 1$.

13. Given one solution $y_1$, find other solution $y_2$ s.t. $\{y_1, y_2\}$ is linearly independent set.

    (a) $y'' - 6y' + 9y = 0$; $y_1 = e^{3x}$, (b) $x^2y'' - xy' + y = 0$; $y_1 = x$.

    (c) $(x-1)y'' - xy' + (3 - 16x^2)y = 0$; $y_1 = e^x$, (d) $(x^2 - 4)y'' + 4xy' + 2y = 0$; $y_1 = 1/(x-2)$.

14. Suppose $p_1, p_2, q_1, q_2$ are continuous on $(a, b)$ and the equations $y'' + p_1(x)y' + q_1(x)y = 0$ and $y'' + p_2(x)y' + q_2(x)y = 0$ have the same solutions on $(a, b)$. Show that $p_1 = p_2$ and $q_1 = q_2$ on $(a, b)$. [Hint. Use Abel's formula.]

15. Find a linear homogeneous ODE for which the given functions form a fundamental set of solutions on some interval.

    (a) $e^x \cos 2x,\ e^x \sin 2x$; (b) $x,\ e^{2x}$ (c) $\cos(\ln x),\ \sin(\ln x)$.

16. Solve IVPs. (a) $y'' + 14y' + 50y = 0,\ y(0) = 2, y'(0) = -17$. (b) $6y'' - y' - y = 0,\ y(0) = 10, y'(0) = 0$.

    (c) $4y'' - 4y' - 3y = 0,\ y(0) = 13/12, y'(0) = 23/24$; (d) $4y'' - 12y' + 9y = 0,\ y(0) = 3, y'(0) = 5/2$.

17. Find a particular solution of $x^2y'' + xy' - 4y = 2x^4$.

18. (Principle of Superposition) Assume $y_1$ is a solution of $a(x)y'' + b(x)y' + c(x)y = f_1(x)$ and $y_2$ is a solution of $a(x)y'' + b(x)y' + c(x)y = f_2(x)$. Show that $y_1 + y_2$ is a solution of $a(x)y'' + b(x)y' + c(x)y = f_1(x) + f_2(x)$.

19. Find the general solution of (a) $x^2y'' - 3xy' + 3y = x$; (b) $y'' - 3y' + 2y = 1/(1 + e^{-x})$; (c) $x^2y'' + xy' - 4y = -6x - 4$;

    (d) $(1 - 2x)y'' + 2y' + (2x - 3)y = (1 - 4x + 4x^2)e^x$, one solution is $y_1 = e^x$.

20. Find a particular solution of (a) $x^2y'' - 2xy' + 2y = x^{9/2}$; (b) $y'' - 2y' + y = 14x^{3/2}e^x$; (c) $y'' + 4y = \sin 2x \sec^2 2x$; (d) $y'' + 4xy' + (4x^2 + 2)y = 4e^{-x(x+2)}$, given that $y_1 = e^{-x^2}$, $y_2 = xe^{-x^2}$ are solutions of homogeneous part.