Tutorial-1, MA 106 (Linear Algebra)
Book - Linear Algebra and its Applications by Gilbert Strang

Problem Set 1.2 [2, 3, 5, 8, 12, 14]

1. Sketch these three lines and decide if the equations are solvable: 3×2 **system**

$$x + 2y = 2, \quad x - y = 2, \quad y = 1$$

2. For the equations $x + y = 4$, $2x - 2y = 4$, draw the row picture (two intersecting lines) and the column picture (combination of two columns equal to the column vector $(4, 4)$ on the right side).

3. Describe the intersection of the three planes in four dimensional space

$$u + v + w + z = 6, \quad u + w + z = 4, \quad u + w = 2$$

   Is it a line or a point or an empty set? What is the intersection if the fourth plane $u = -1$ is included? Find a fourth equation that leaves us with no solution.

4. Under what conditions on y_1, y_2, y_3 do the points $(0, y_1), (1, y_2), (2, y_3)$ lie on a straight line.

5. Starting with $x + 4y = 7$, find the equation for the parallel line through $x = 0, y = 0$. Find the equation of another line that meets the first at $x = 3, y = 1$.

6. For four linear equations in two unknowns x and y, the row picture shows four $- - - - - -$. The column picture is in $- - - - - -$ dimensional space. The equations have no solutions unless the vectors on the right-hand side is a combination of $- - - - - -$.

Problem Set 1.3 [3, 7, 9, 10, 13, 15, 19, 20, 22, 24, 26, 32]

7. Choose a coefficient b that makes this system singular. Then choose a right-hand side g that makes it solvable. Find two solutions in that singular case.

$$2x + by = 16, \quad 4x + 8y = g$$

8. What test on b_1, and b_2 decides whether these two equations allow a solution? How many solutions will they have? Draw the column picture.

$$3x - 2y = b_1, \quad 6x - 4y = b_2$$

9. For which three numbers k does elimination break down? Which is fixed by a row exchange? In each case, is the number of solutions 0 or 1 or ∞?

$$kx + 3y = 6, \quad 3x + ky = -6$$

10. Which number b leads later to a row exchange? Which b leads to a missing pivot? In that singular case find a non-zero solution x, y, z.

$$x + by = 0, \quad x - 2y - z = 0, \quad y + z = 0$$

11. Apply elimination (circle the pivots) and back-substitution to solve

$$2x - 3y = 3, \quad 4x - 5y + z = 7, \quad 2x - y - 3z = 5$$

12. It is impossible for a system of linear equations to have exactly two solutions. Explain why.

    (a) If (x, y, z) and (X, Y, Z) are two solutions, what is another one?

    (b) If 25 planes meet at two points, where else do they meet?

13. Construct a 3×3 example that 9 different coefficients on the left-hand side, but rows 2 and 3 become zero in elimination. How many solutions to your system with $b = (1, 10, 100)$ and how many with $b = (0, 0, 0)$?

14. Find the pivots and solutions for these four equations

$$2x + y = 0, \quad x + 2y + z = 0, \quad y + 2z + t = 0, \quad z + 2t = 5$$

15. Find three values of a for which elimination breaks down, temporarily or permanently, in

$$au + v = 1, \quad 4u + av = 2$$

    Breakdown at the first step can be fixed by exchanging rows, but not breakdown at the last step.

16. Consider a 3×3 system in variables u, v and w, with three (nonzero) pivots. State whether the following are true or false.

    (a) If the third equation starts with a zero coefficient (it begins with $0u$) then no multiple of equation 1 will be subtracted from equation 3.

    (b) If the third equation has zero as its second coefficient (it contains $0v$ then no multiple of equation 2 will be subtracted from equation 3.

    (c) If the third equation contains $0u$ and $0v$ then no multiple of equation 1 or no multiple of equation 2 will be subtracted from equation 3.

17. Use elimination to solve

$$u + v + w = 6, \quad u + 2v + 2w = 11, \quad 2u + 3v - 4w = 3 \quad \text{and}$$

$$u + v + w = 7, \quad u + 2v + 2w = 10, \quad 2u + 3v - 4w = 3$$

Extra Problems

For each of the following, set up and solve the system of linear equations which will solve the problem.

18. I have 10, 3, 6, 7, 2 and 2 currency notes respectively of the following denominations: Rs. 10, Rs. 20, Rs. 50, Rs. 100, Rs. 500 and Rs. 1,000. How many notes of each denomination will I need to pay for a purchase worth Rs. 1,760 from a shop that accepts only cash?

19. A cake shop offers three sizes of snack boxes containing chips packets, samosas, and cake slices. Each small box contains 1 chips packet, 3 samosas, and 3 cake slices. Each medium box contains 2 chips packets, 4 samosas, and 6 cake slices. Each large box contains 4 chips packets, 8 samosas, and 6 cake slices. You need 24 chips packets, 50 samosas, and 48 cake slices for your new year party, and want to know how many of the three type of snack boxes you should order.

20. It is known that sodium hydroxide and carbonic acid react to give sodium carbonate and water. The chemical equation can be written as:

$$wNaOH + xH_2CO_3 \to yH_2O + zNa_2CO_3.$$

Find the values of w, x, y and z to balance this equation.