PH108

Lecture 21:

Faraday’s Law – Magnetic induction

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Our understanding of electrostatics and magnetostatics so far is summarized in four differential equations

STATIC CHARGE | CONSTANT CURRENT

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \leftrightarrow \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = 0 \quad \leftrightarrow \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

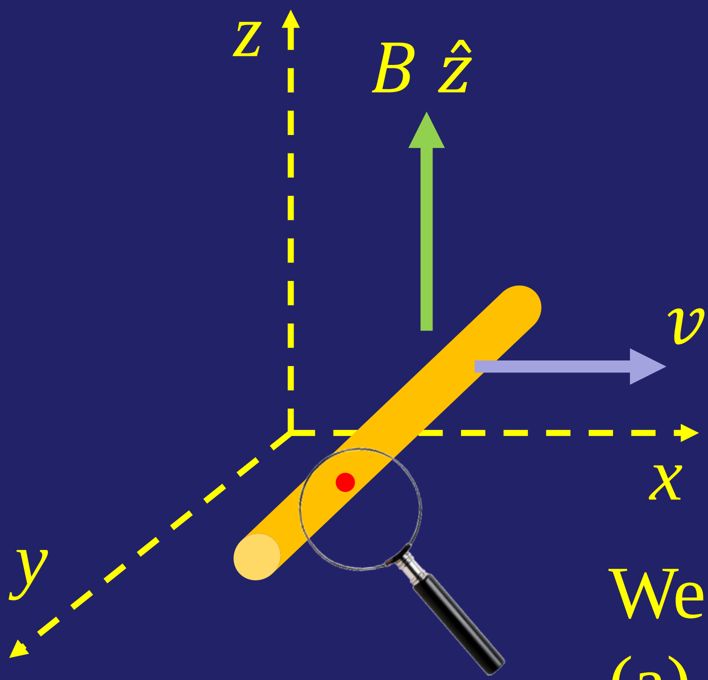
The equations for $\vec{B}$ are inconsistent!

Divergence of Curl $\equiv 0$

But $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{B}\right) = \mu_0 \vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

$\neq 0$

A conductor rod moving through $\vec{B}$ "generates" $\vec{E}$



Uniform magnetic field $B\hat{z}$

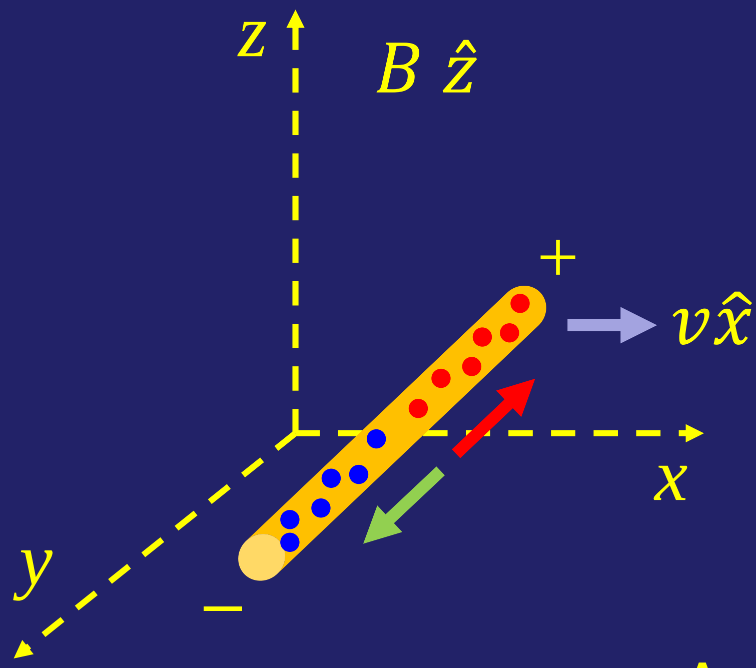
Uncharged conductor rod oriented along y *starts moving* with velocity $v\hat{x}$

We want to see in detail what happens (a) initially (transient) & (b) steady state

Charge q in the conductor feels a force:

$$\vec{f} = q(v\hat{x} \times B\hat{z}) = -qvB\hat{y}$$

Free charge in the rod is pushed to the ends by the force $\vec{f}$



Charge q in the conductor feels a force:

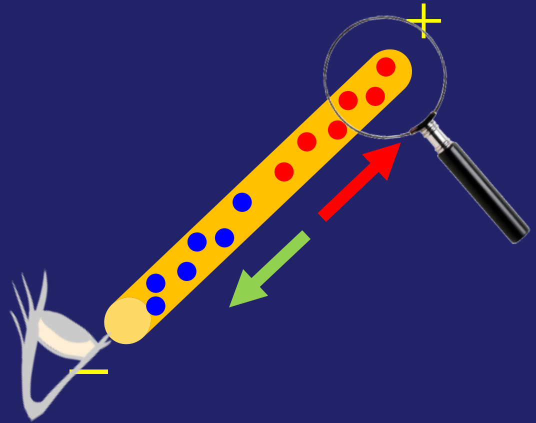
$$\vec{f} = q(v\hat{x} \times B\hat{z}) = -qvB\hat{y}$$

Free charges in the conductor are pushed towards ends of the rod

At equilibrium, an electric field is established that balances the force:

$$q\vec{E} = -\vec{f}$$

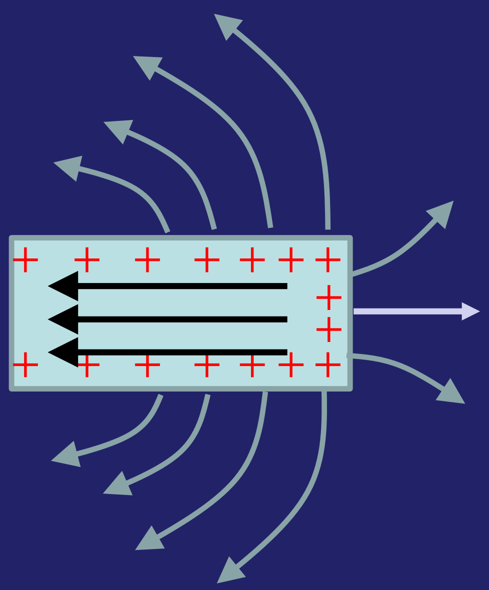
Question



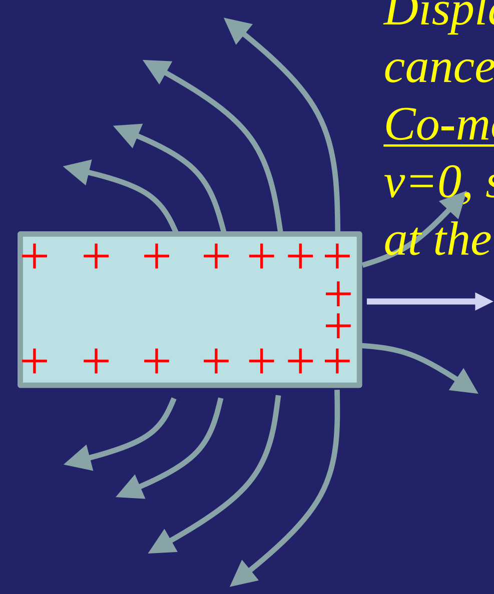
At equilibrium, the <u>electric field</u> near one end of of the rod (inside and outside) looks like:
(pick one answer)

Depends on frame of reference!
Both observers see E=0 inside the conductor, but for different reasons:
<u>Stationary observer:</u>
Displacement field qE=-f cancels the Lorentz force
<u>Co-moving observer:</u>
v=0, so no force, charge just sits at the surface of the conductor

(A)



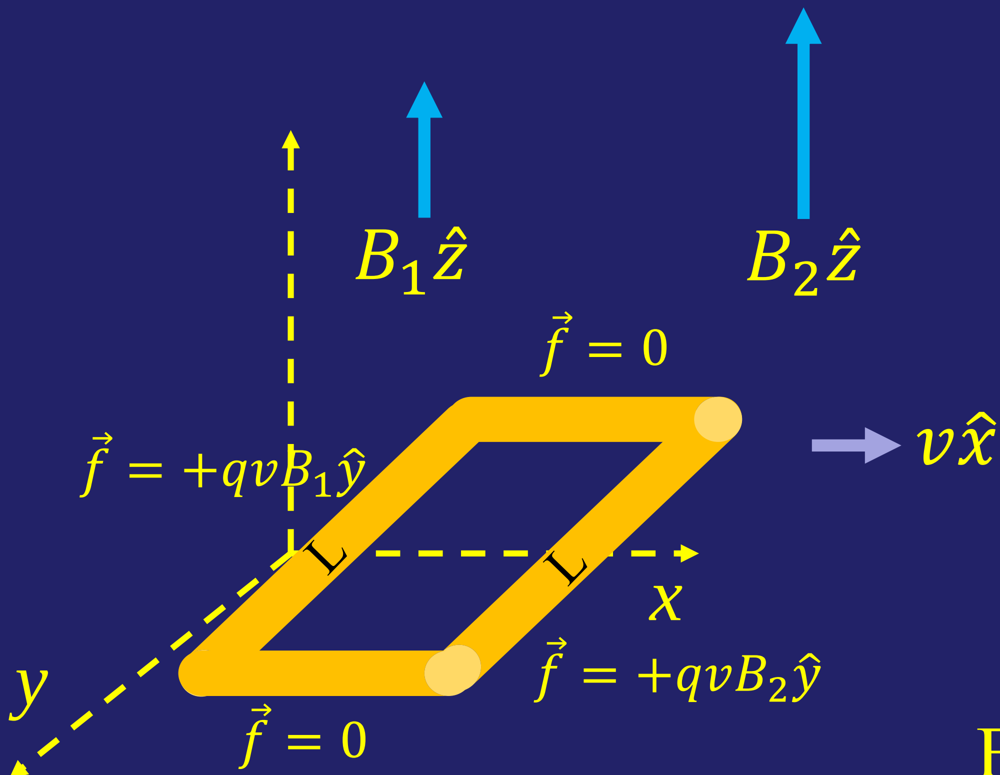
(B)



(C)

Invalid Question

What is the force on a LOOP of wire in $\vec{B}$?



Total force around the loop:

$$\oint \vec{f} \cdot d\vec{s} = qv(B_1 - B_2)L$$

Electromotive force ≡ force per unit charge

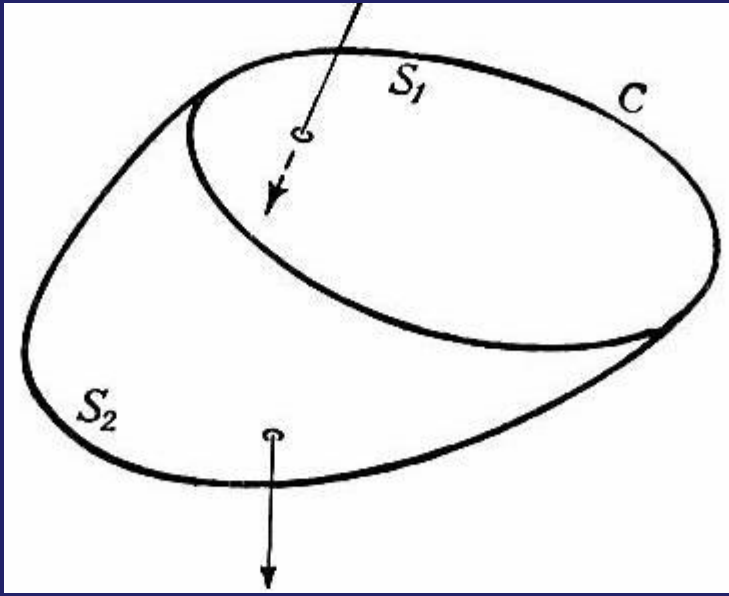
$$\mathcal{E} = v\underbrace{L(B_1 - B_2)}_{\text{"Flux of } \vec{B}}$$

Flux = Magnetic Field * Area

$$\Phi_{S_1} \equiv \int_{S_1} \vec{B} \cdot d\vec{a}$$

Φ_{S_1} is the flux through a surface S_1 that bounds a loop C

Is Φ_{S_1} unique? What if we take a difference surface S_2 ?



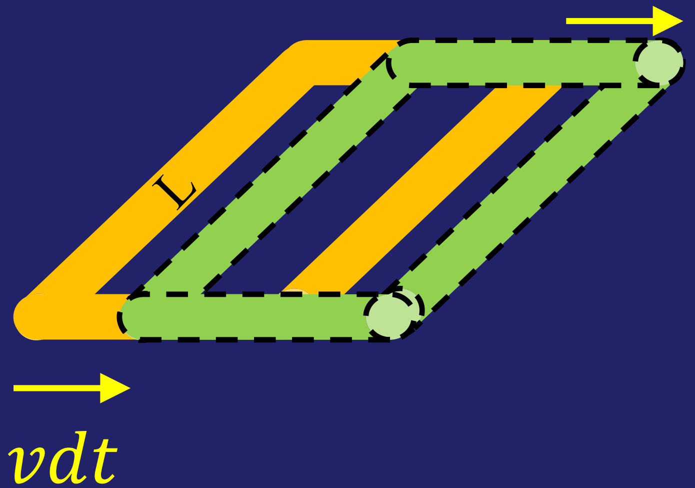
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$So \oiiint \vec{\nabla} \cdot \vec{B}\, d\tau = \oiint \vec{B} \cdot d\vec{a} = 0$$

$$\Phi_{S_1} = \Phi_{S_2}$$

Electromotive force = Rate of flux change

$Flux$ gained $= B_2 Lvdt$



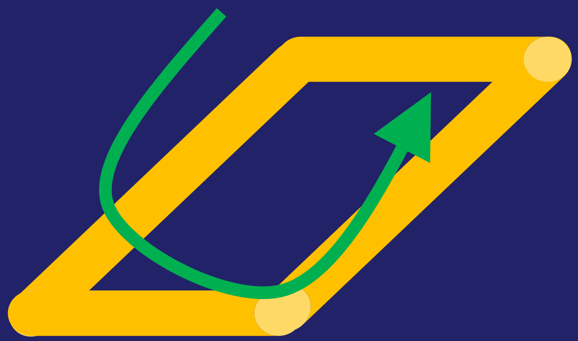
$Flux$ lost $= B_1 Lvdt$

Change in Flux:

$$d\Phi = -\underbrace{(B_1 - B_2)Lv}_{\mathcal{E}\ \text{emf}}\,dt$$

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

$\mathcal{E}$ forces us to re-examine $\vec{\nabla} \times \vec{E} = 0$



There is now an electromotive force *going around* the loop.
If there is a force, there must be a field!

$$\int \vec{E} \cdot d\vec{l} = \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{a}$$

valid for *any* loop bounding surface of flux

So: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$