PH108

Lecture 20:

Magnetic materials

(with a quick review of dielectrics)

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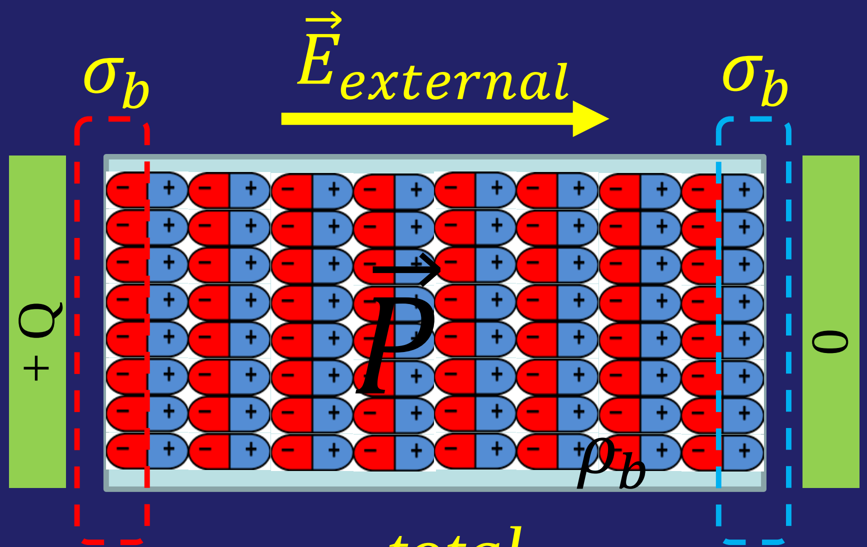
Recall the main points of dielectrics



$\vec{\nabla} \cdot \vec{P} = -\rho_b$

$\vec{P} \cdot \hat{n} = \sigma_b$

total

$$\epsilon_0 \vec{\nabla} \cdot \vec{\boldsymbol{E}} = \rho = \rho_f + \rho_b$$

total

$$\vec{\nabla} \cdot (\underline{\epsilon_0 \vec{E} + \vec{P}}) = \rho_f$$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

Gauss Law for dielectrics

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

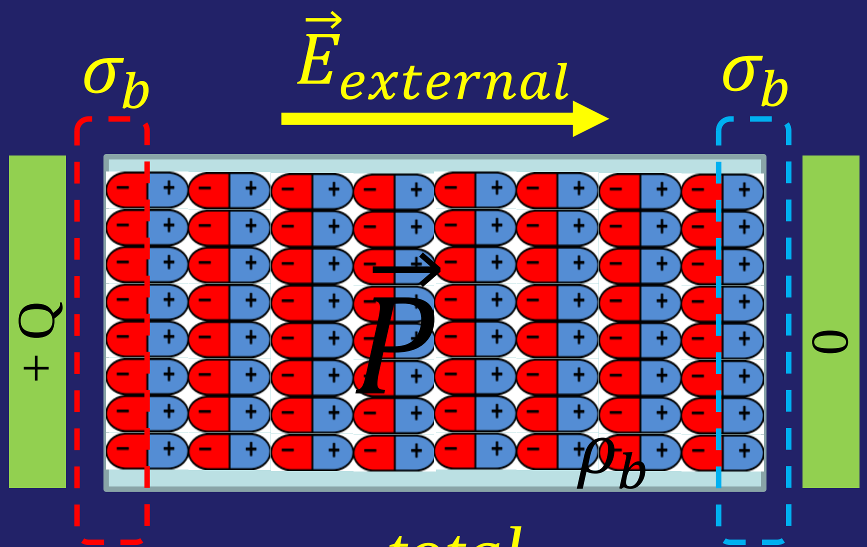
Integral form:

$$\oiint_{Gaussian\ surface} \vec{D} \cdot d\vec{S} = Q_{f|enclosed}$$

$\vec{D}$ is more useful than $\vec{P}$!

eg: $\vec{P}$ can be non-linear

What are the PHYSICAL quantities ?



$total$
$\vec{E}$

Can you measure $\vec{E}$? YES (force on test charge)

Can you measure $\vec{P}$? Calc. with σ_b, ρ_b

Can you measure $\vec{D}$? Calc. with ρ_f

Gauss Law for dielectrics

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

Integral form:

$$\oiint_{Gaussian\ surface} \vec{D} \cdot d\vec{S} = Q_{f|enclosed}$$

$\vec{D}$ is more useful than $\vec{P}$!

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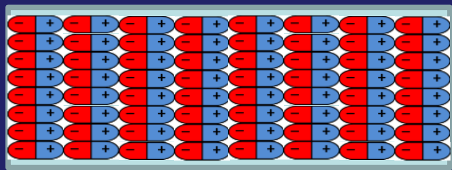
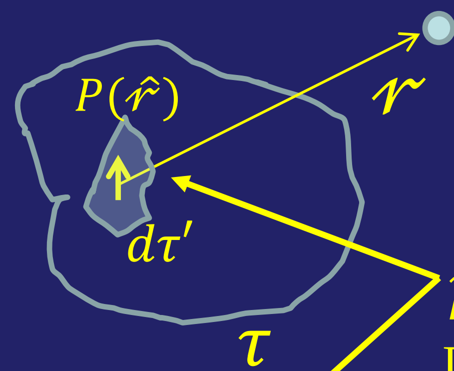
For magnetism, the math is similar

"Bound" charges	$\longrightarrow$	"Induced" currents
volume, surface	$\longrightarrow$	volume, surface
Dot products	$\longrightarrow$	Cross products
Divergence, V	$\longrightarrow$	Curl, $\vec{A}$
Polarization, $\vec{P}$, $\frac{1}{\epsilon_0}$	$\longrightarrow$	Magnetization, $\overrightarrow{M}$, μ_0
Displacement, $\overrightarrow{D}$	$\longrightarrow$	Magnetic field, $\overrightarrow{H}$

Polarized Dielectric creates potential



Ignore the external field that created these dipoles, what is the field created BY the dipoles?



$$\vec{p} = \vec{P}(\mathscr{r}')d\tau'$$

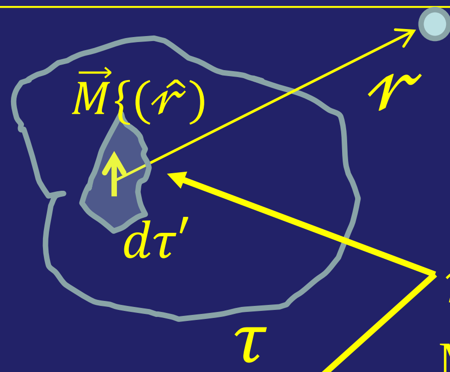
Dipole moment of this piece

$$V = \frac{1}{4\pi\epsilon_0}\int_\tau \frac{\hat{\mathscr{r}} \cdot \vec{P}(\mathscr{r}')d\tau'}{\mathscr{r}^2}$$

$$\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$V = \frac{1}{4\pi\epsilon_0}\int_\tau \vec{P} \cdot \vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

Magnetic material→Polarized by external $\vec{B}$→ Creates its own VECTOR potential



$$\vec{m} = \vec{M}(\mathscr{r}')d\tau'$$

Magnetic Dipole moment of $d\tau$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_\tau \frac{\vec{M}(\mathscr{r}') \times \hat{\mathscr{r}}\, d\tau'}{\mathscr{r}^2}$$

$$\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_\tau \vec{M} \times \vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

Dielectric potential has surface & vol terms

$$V = \frac{1}{4\pi\epsilon_0}\int_\tau \vec{P}\cdot\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)d\tau'$$

Integrate by parts

$$V = \frac{1}{4\pi\epsilon_0}\left\{\underbrace{\int_\tau \vec{\nabla}'\cdot\left(\frac{\vec{P}}{\mathscr{r}}\right)d\tau'}_{\text{Divergence theorem}} - \int_\tau \frac{1}{\mathscr{r}}\left(\vec{\nabla}'\cdot\vec{P}\right)d\tau'\right\}$$

$$V = \frac{1}{4\pi\epsilon_0}\left\{\oint_S \frac{\vec{P}\cdot\overrightarrow{ds'}}{\mathscr{r}} - \int_\tau \frac{1}{\mathscr{r}}\left(\vec{\nabla}'\cdot\vec{P}\right)d\tau'\right\}$$

SURFACE VOLUME

Magnetic potential has surface & vol terms

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{\tau} \vec{M} \times \vec{\nabla}'\left(\frac{1}{r}\right) d\tau'$$

Integrate by parts

$$\vec{A} = \frac{1}{4\pi\epsilon_0}\left\{\underbrace{\int_{\tau} \vec{\nabla}' \times \left(\frac{\vec{M}}{r}\right) d\tau'}_{\text{Divergence theorem}} - \int_{\tau} \frac{1}{r}\,(\vec{\nabla}' \times \vec{M}) d\tau'\right\}$$

$$V = \frac{1}{4\pi\epsilon_0}\left\{\oint_{S} \frac{\vec{M} \times \overrightarrow{ds}'}{r} - \int_{\tau} \frac{1}{r}\,(\vec{\nabla}' \times \vec{M}) d\tau'\right\}$$

SURFACE VOLUME

Currents are induced in the Surface & Vol

$$V = \frac{1}{4\pi\epsilon_0}\left\{\oint_S \frac{\vec{M} \times \overrightarrow{ds}'}{\mathcal{r}} - \int_\tau \frac{1}{\mathcal{r}}\left(\vec{\nabla}' \times \vec{M}\right)d\tau'\right\}$$

SURFACE VOLUME

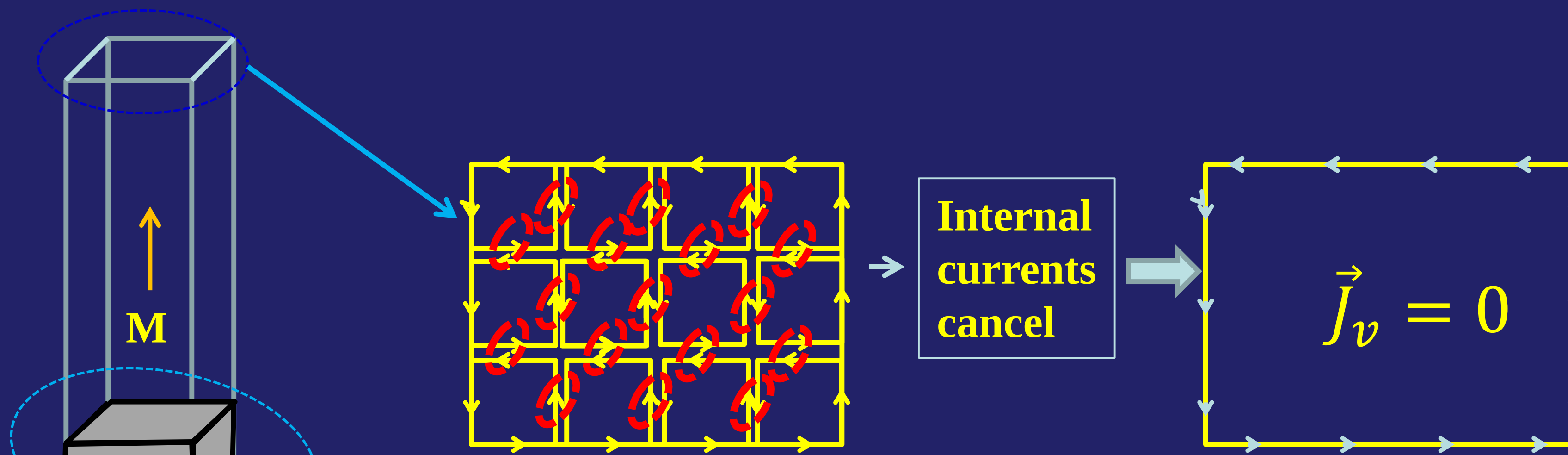
$$\vec{K}_s = \vec{M} \times d\vec{s} \qquad \vec{J}_b = \vec{\nabla} \times \vec{M}$$

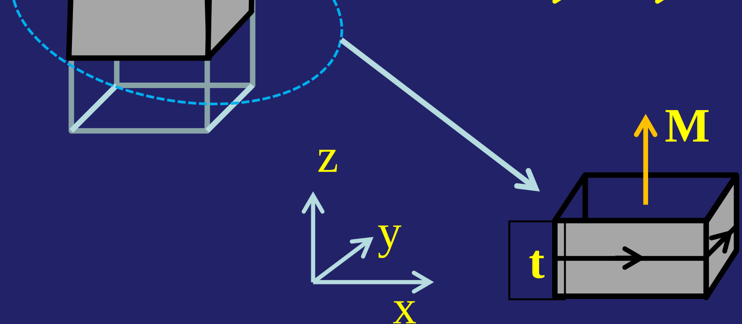
These are *bound* currents

i.e. they cannot leave the magnetized material

A physical picture of induced currents

Consider a rectangular block with Uniform magnetization $\vec{M} = M\,\hat{k}$





$$m = M * at = Ia \rightarrow M = \frac{I}{t}\,\hat{k}$$

$$\vec{K}_s = \vec{M} \times \hat{n}$$

Goes around the boundary
Zero at top and bottom face

Ampere's law in magnetic materials has to account for magnetic field created by the *induced current*

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

What is $\vec{B}$?

(1) $\vec{B}_{external}$ (2) $\vec{B}_{induced}$ (3) $\vec{B}_{total}$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_b + \vec{J}_f)$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$\vec{\nabla} \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{\equiv \vec{H}} = \vec{J}_f$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\oint \vec{H} \cdot \vec{dl} = I_{f|enclosed}$$

(some) problems on $\vec{H}$ in tutorials…