LECTURE 2

BB 101: MODULE II

PHYSICAL BIOLOGY

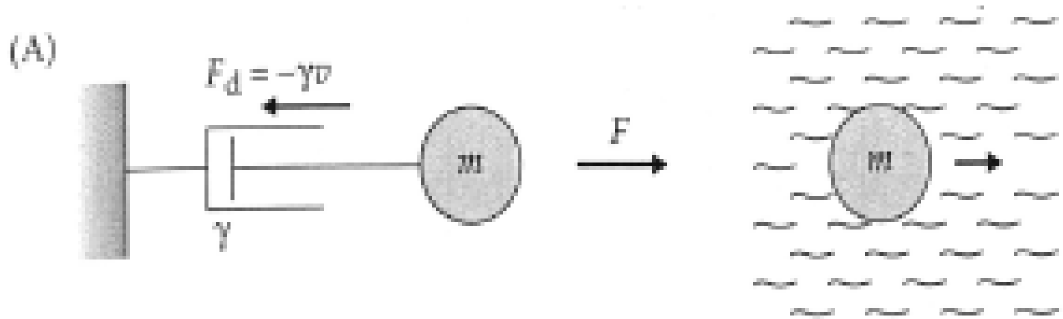
Review of Lecture 1

- *Proteins molecules and forces acting on a protein molecule*

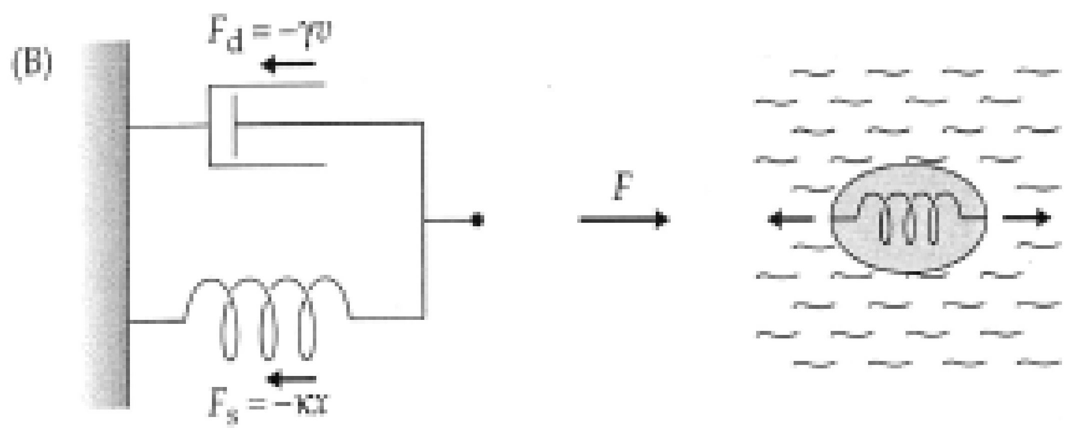
- *Inertial forces are negligible and effect of gravity can be ignored*

- *Many biological systems can be modeled as combination of three fundamental mechanical elements-mass, spring and dashpot*

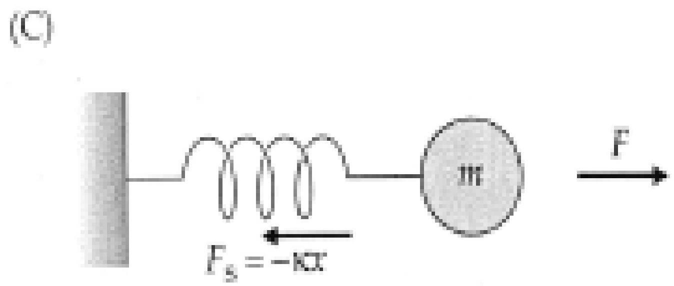
Motion of combination of mechanical elements



Object damped by viscous fluid



Low mass object deformed in viscous fluid



Undamped objects

Source: Mechanics of Motor Proteins and the Cytoskeleton, Jonathan Howard

Motion of combination of mechanical elements

(A) Mass & Dashpot

$$m\frac{dv}{dt} + \gamma v = F$$

$$v(t) = \frac{F}{\gamma}[1 - exp(-\frac{t}{\tau})]$$

(B) Spring & Dashpot

$$\gamma\frac{dx}{dt} + \kappa x = F$$

$$x(t) = \frac{F}{\kappa}[1 - exp(-\frac{t}{\tau})]$$

(c) Mass & Spring

$$m\frac{d^2x}{dt^2} + \kappa x = F$$

$$x(t) = \frac{F}{\kappa}[1 - cos(\omega t)]$$

Source: Mechanics of Motor Proteins and the Cytoskeleton, Jonathan Howard

Inertia of moving bacterium

Consider a bacterium swimming through water at a speed of 25 micron/s. How long bacterium will continue to move after its flagellar motors stop working? Assume bacteria to be a sphere of radius 1 micron with density that of water (1000 kg/m^3).

Time constant $\tau \sim 22.7 \times 10^{-6}$s $\sim 20\mu$s
Distance travelled $\sim 5 \times 10^{-10}$ m $\sim$ 5 Angstrom

Video Source: https://www.youtube.com/watch?v=6bEacq-ow50

Sedimentation of a 100 kDa globular protein

Consider the sedimentation of a globular protein of radius 3nm, initially right below the surface, in an Eppendorf tube filled with water upto 1cm height. How much time it would take for this protein to sediment under the effect of gravity?

Answer: ~1.2×10^9 seconds ~ 38.05 years

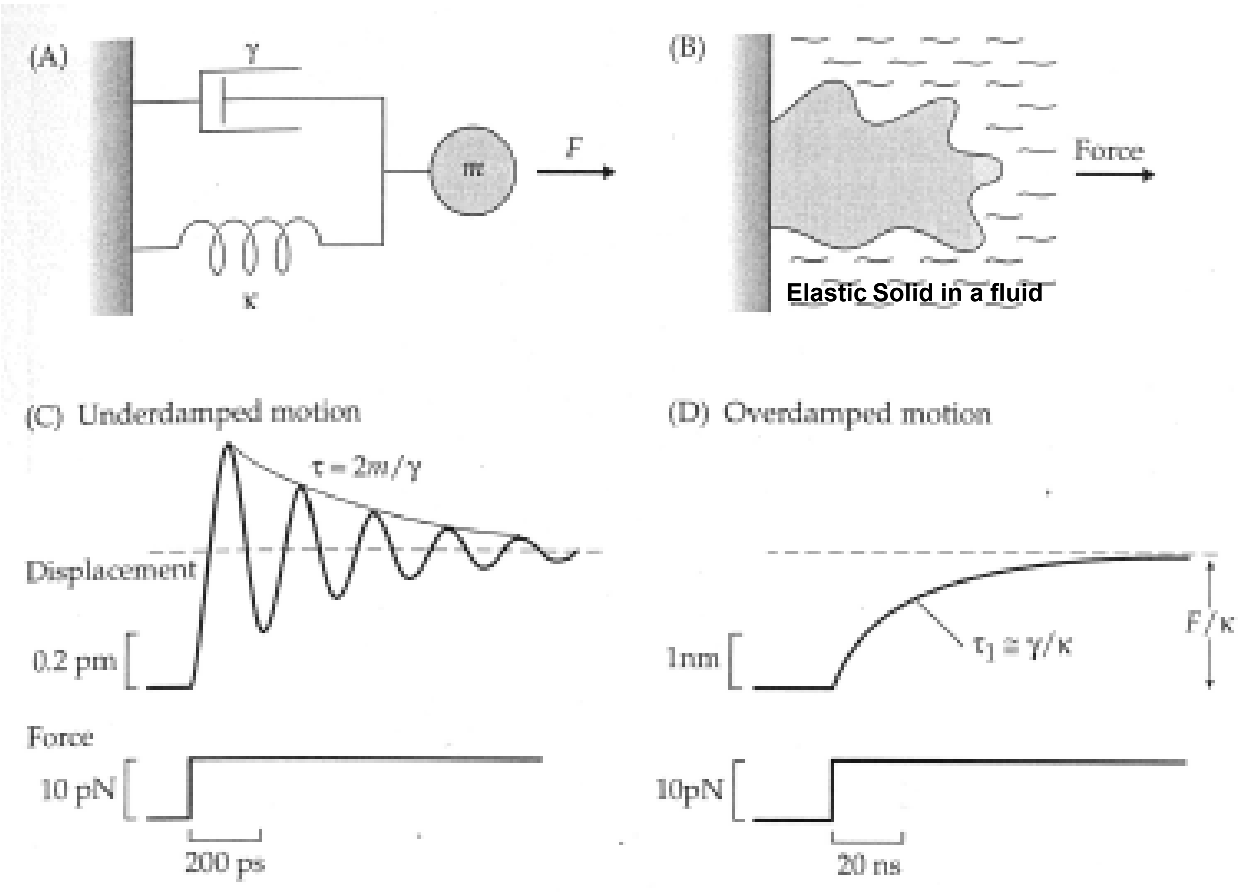
Figure Source: http://www.nature.com/scitable/content/eppendorf-tube-6488888

Why biologists need ultra-centrifuges

Ultracentrifuges are very expensive. Typical prices $150000

https://www.beckmancoulter.com/wsrportal/wsr/research-and-discovery/products-and-services/centrifugation/index.htm

Motion of combination of mechanical elements: Mass and Spring with Damping



Underdamped: $\gamma^2 < 4m\kappa$

Overdamped: $\gamma^2 > 4m\kappa$

Inertial forces
(tend to produce oscillations)

viscous forces
(tend to damp the oscillations out)

Source: Mechanics of Motor Proteins and the Cytoskeleton, Jonathan Howard

Homework

Derive expression for x(t) for overdamped, underdamped and critically damped motion

The equation of motion:

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \kappa x = F$$

Underdamped Motion ($\gamma^2 < 4m\kappa$)

$$x(t) = \frac{F}{\kappa}\left[1 - \exp\left(-\frac{t}{\tau}\right)\frac{\sin(\omega t + \phi)}{\sin\phi}\right]$$

where $\tau = \frac{2m}{\gamma}$, $\omega^2 = \omega_0^2 - \frac{1}{\tau^2}$, $\omega_0^2 = \frac{\kappa}{m}$, $\tan\phi = \omega\tau$

Overdamped Motion ($\gamma^2 > 4m\kappa$)

$$x(t) = \frac{F}{\kappa}\left[1 - \frac{\tau_1}{\tau_1 - \tau_2}\exp\left(-\frac{t}{\tau_1}\right) + \frac{\tau_2}{\tau_1 - \tau_2}\exp\left(-\frac{t}{\tau_2}\right)\right]$$

where $\tau_1 = \frac{\gamma + \sqrt{\gamma^2 - 4m\kappa}}{2\kappa}$ and $\tau_2 = \frac{\gamma - \sqrt{\gamma^2 - 4m\kappa}}{2\kappa}$

Critically Damped Motion ($\gamma^2 = 4m\kappa$)

$$x(t) = \frac{F}{\kappa}\left[1 - \left(1 + \frac{t}{\tau}\right)\exp\left(-\frac{t}{\tau}\right)\right]$$

where $\tau = \frac{2m}{\gamma} = \frac{\gamma}{2\kappa} = \sqrt{\frac{m}{\kappa}}$

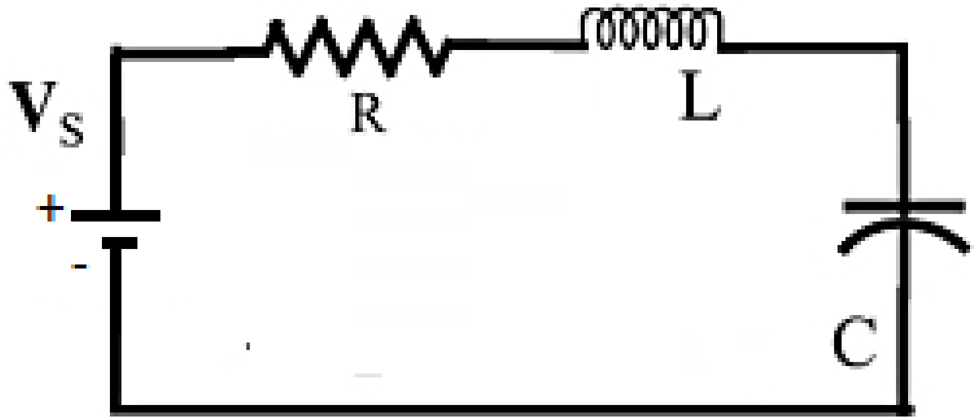
Source: Mechanics of Motor Proteins and the Cytoskeleton, Jonathan Howard

Hint

1. Write down equation of motion in terms of variable p (=kx-F), 2b(=γ/m) and $w_0^2 = \frac{k}{m}$

2. Assume a general solution either of the form $P = Ae^{-\frac{t}{\tau}}(easy)$ or $P = Ae^{\alpha t}(hard)$

Similarity with series LCR circuit



Viscosity and fluid flow

- *Stir the water slowly in a beaker*

- *Stir corn syrup slowly in a beaker*

Slowly Stirring of corn syrup causes an organized motion, in which successive layers of fluid simply slide over each other. Such a fluid motion is called **laminar flow**. Viscous forces are dominant in such scenario.

Newtonian fluid

viscosity doesn’t change with shear rate

- Dimensional analysis for Newtonian fluid
  [$\boldsymbol{\eta}$]: Pa s=kg m^{-1} s^{-1} **[$\boldsymbol{\rho}$]:** kg m^{-3}

- There is no intrinsic distinction between “thick” and “thin” fluids as there is no dimensionless measure of viscosity. Hence difficult say whether flow will be viscous or laminar

- We cannot construct a dimensionless quantity from η and ρ

- However, a situation-dependent characterization has been developed to distinguish between two flow regimes

Viscous Critical Force

$$\left[\frac{\eta^2}{\rho}\right]: \frac{\left(kg\, m^{-1} s^{2}\right)^2}{kg\, m^{-3}} = kg\ m\ s^{-2}$$

$$\boldsymbol{f_{crit}} = \eta^2/\rho$$

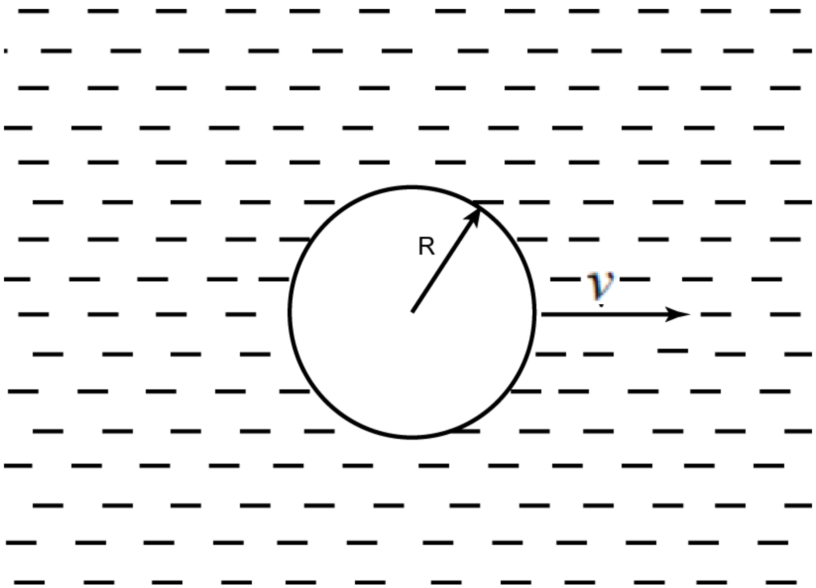
If applied force f is less than f_{crit} then fluid can be called "thick" and flow will be laminar. Friction will quickly damp out inertial effects. Flow is dominated by friction.

Fluid	ρ_m (kg m^{-3})	η (Pa · s)	f_{crit} (N)	
Air	1	$2 \cdot 10^{-5}$	$4 \cdot 10^{-10}$	
Water	1000	0.0009	$8 \cdot 10^{-10}$	~1nN
Olive oil	900	0.080	$4 \cdot 10^{-6}$	
Glycerine	1300	1	0.0008	
Corn syrup	1000	5	0.03	

Typical scale of forces inside cell is pN

Viscous forces rules the inner world of cells!!!

Reynolds number



A moving ball in water

Dimensional analysis

$[\eta]$:Pa s=kg m^{-1} s^{-1}

$[\rho]$: kg m^{-3}

$[R]$:m

$[v]$: m s^{-1}

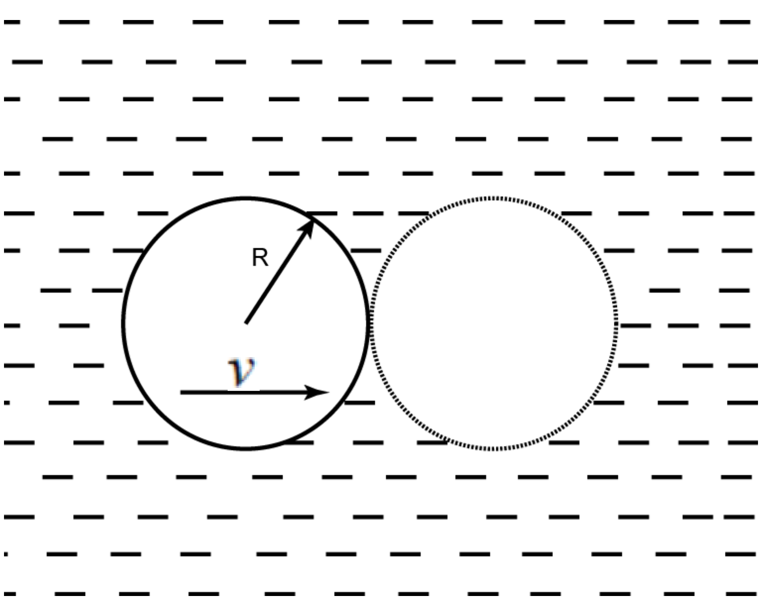
Reynolds number

$$\boxed{\mathrm{Re} = \left[\frac{\rho R v}{\eta}\right]}$$

Dimensionless number

A moving ball in water

Reynolds number



Time required for travelling distance 2R:

$t = \quad /v$

The same volume of water moves 2R in time t

Estimate for the mean acceleration of water

$\frac{1}{2}\mathrm{a}t^2 = \qquad \Rightarrow \mathrm{a} = \frac{\quad}{t^2} = \frac{v^2}{R}$

Inertial force=mass x acceleration $\sim\rho R^3(\frac{v^2}{R})$

Viscous force= $6\pi\eta R v \quad \sim \eta R v$

$$\text{Re}= \frac{\boldsymbol{Inertial\ force}}{\boldsymbol{Viscous\ force}} = \frac{\boldsymbol{\rho R v}}{\eta}$$

Inertial term can be safely ignored if Re <<1.

Reynolds number

Calculate Reynolds number in following cases:

(1) A 30 m whale, swimming in water at 10 m/s
(2) A 1 μm bacterium, swimming in water at 30 μm/s

Solutions:

(1) Re= $= \frac{\rho \boldsymbol{R v}}{\eta} = \frac{10^3 \times 30 \times 10}{10^{-3}} = 3 \times 10^8 \gg 1$

(2) Re= $= \frac{\rho \boldsymbol{R v}}{\eta} = \frac{10^3 \times 1 \times 10^{-6} \times 30 \times 10^{-6}}{10^{-3}} = 3 \times 10^{-5} \ll 1$

Microorganisms live in the world of low Reynolds number where viscous forces rule their motion

Summary

- Modeling biological systems with mass, springs and dashpots
- Similarly with LCR circuit
- Newtonian fluids
- Reynolds number
- We will consider life of microorganisms at low Reynolds numbers and its consequences