MA-108 Ordinary Differential Equations

M.K. Keshari

Department of Mathematics
Indian Institute of Technology Bombay
Powai, Mumbai - 76

16th April, 2015
Extra Slides

Ex.

$$\begin{aligned} L^{-1}\left(\frac{1}{\sqrt{s^2+1}}\right) &= L^{-1}\left(\frac{1}{s}(1+\frac{1}{s^2})^{-1/2}\right) \\ &= L^{-1}\left(\frac{1}{s}\left(1-\frac{1}{2s^2}+\dots\right)\right) \\ &= 1-\frac{1}{2^2}t^2+\frac{1}{(2!)^2}\frac{t^4}{2^4}-\dots \end{aligned}$$

This is a well known function, known as Bessel function of first type and of order 0, and is denoted by $J_0(t)$.

Ex. $L^{-1}\left(\frac{s}{(s^2+1)^{3/2}}\right)$.

Either expand it into series and take L^{-1} of each term or try applying Convolution theorem, as we know

$L^{-1}\left(\frac{1}{(s^2+1)^{1/2}}\right)$.

If $F(s)$ contains $\dfrac{1}{1-e^{-Ts}}$, then it does Not automatically mean that it's inverse Laplace transform is periodic.

Ex.

$$\begin{aligned} L^{-1}\left(\frac{1}{s(1-e^{-s})}\right) &= \; = L^{-1}\left(\frac{1}{s}(1+e^{-s}+e^{-2s}+\dots)\right) \\ &= 1+u(t-1)+u(t-2)+\dots \\ &= n, \quad \text{if} \;\; t \in [n-1, n) \end{aligned}$$

In this case $f(t)$ is not periodic.

Ex.

$$\begin{aligned} L^{-1}\left(\frac{1}{s(1+e^{-s})}\right) &= \; = L^{-1}\left(\frac{1}{s}(1-e^{-s}+e^{-2s}-\dots)\right) \\ &= 1-u(t-1)+u(t-2)-\dots \end{aligned}$$

In this case $f(t)$ is periodic of period 2.