PH 103 : Electricity and Magnetism

Tutorial Sheet 1 : **Coordinate system**

1. For a circle given by the equation, $r = 2R\cos\theta$, where R is the radius of the circle and (r, θ) are the polar coordinates, calculate the following:
   (a) area of the circle
   (b) centroid of the semi-circular area in the first quadrant
   (c) length and centroid of the semi-circular arc in the first quadrant
   (d) area common to the given circle and the circle given by $r = R$

2. The area bounded by the curve $r = 2R\cos\theta$ has surface charge density given by $\sigma(r, \theta) = \sigma_0(\frac{r}{R})\sin^4\theta$. Show that the total charge on the curve is $\frac{32}{105}\sigma_0 R^2$.

3. Consider a sphere of *unit* radius with its centre at the origin of the coordinate system. Show that the area of the surface enclosed between $\theta = 0$ and $\theta = \alpha$ is $2\pi(1 - \cos\alpha)$.

4. Consider the frustum of a cone given by the equation $z^2 = x^2 + y^2$ between the planes $z = 1$ and $z = 2$. Determine the volume of the frustum using
   (a) spherical polar coordinates,
   (b) cylindrical coordinates.
   (c) If the above frustum has a volume charge density given by
   $$\rho = \rho_0 \left(x^2 + y^2 + z^2\right)/a^2$$
   where ρ_0 is a constant, what is the total charge?

5. Compute the divergence of the function $\vec{v} = r\cos\theta\,\hat{r} + r\sin\theta\,\hat{\theta} + r\sin\theta\cos\phi\,\hat{\phi}$. Check the divergence theorem for this function using the volume of an inverted hemisphere of radius R, resting on the xy-plane and centered at the origin.

6. Test the Stokes theorem for the vector $\vec{v} = xy\,\hat{i} + 2yz\,\hat{j} + 3zk\,\hat{k}$ using a triangular area with vertices at (000), (020) and (002).

7. A vector field is given by $\vec{v} = ay\,\hat{i} + bx\,\hat{j}$, where a and b are constants.
   (a) Find the line integral of this field over a circular path of radius R, lying in the xy-plane and centered at the origin using (i) the plane polar coordinate system (ii) the Cartesian system.
   (b) Imagine a right circular cylinder of length L with its axis parallel to the z-axis standing on this circle. Use cylindrical coordinate system to show that the Stokes' theorem is valid over its surface.