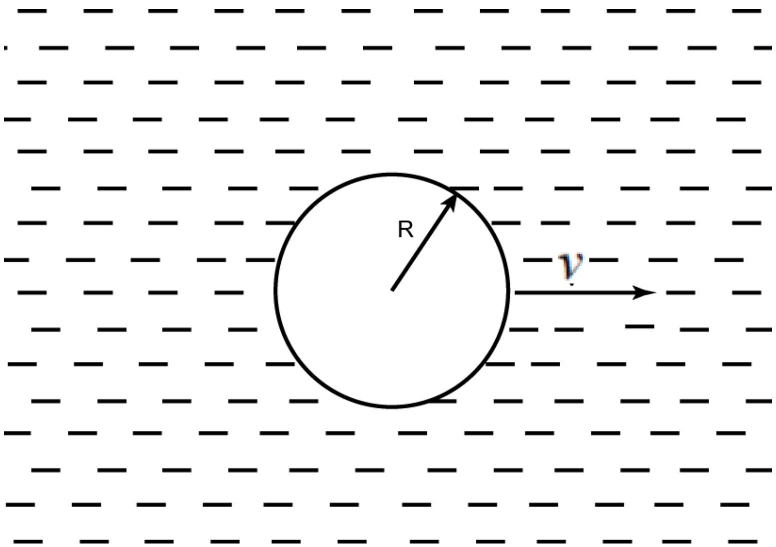
BB 101: MODULE II

PHYSICAL BIOLOGY

LECTURE 3

Review of Lecture 2: Reynolds number



A moving ball in water

Dimensional analysis

[η]:Pa s=kg m^{-1} s^{-1}

[R]:m

[ρ]: kg m^{-3}

[v]: m s^{-1}

Reynolds number

$$\boxed{\mathrm{Re}=\left[\frac{\rho R v}{\eta}\right]}$$

Dimensionless number

- Ratio of inertial to viscous forces
- Motion of microorganism is dominated by viscous forces

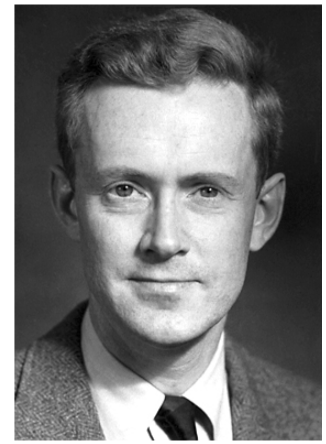
Life of Microorganisms at low Reynolds number

Purcell, E. M., Life at low Reynolds-number, Am. J. Phys. 45, 3–11 (1977)

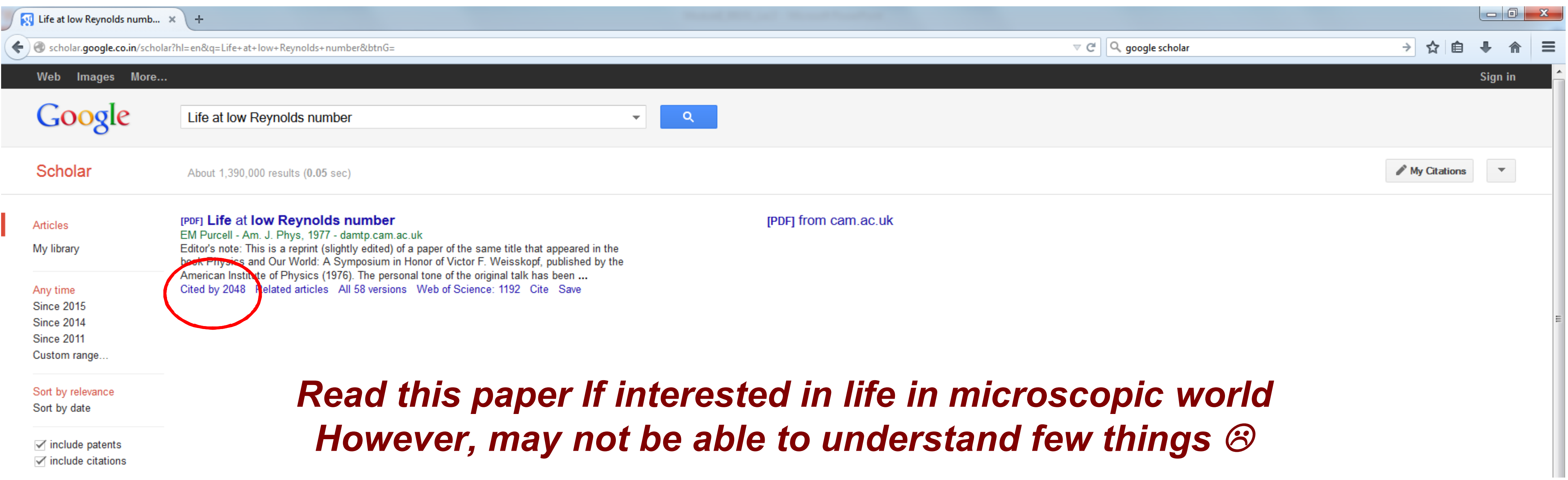
Life at low Reynolds number

E. M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 12 June 1976)

Editor's note: This is a reprint (slightly edited) of a paper of the same title that appeared in the book *Physics and Our World: A Symposium in Honor of Victor F. Weisskopf*, published by the American Institute of Physics (1976). The personal tone of the original talk has been preserved in the paper, which was itself a slightly edited transcript of a tape. The figures reproduce transparencies used in the talk. The demonstration involved a tall rectangular transparent vessel of corn syrup, projected by an overhead projector turned on its side. Some essential hand waving could not be reproduced.



Edward Mills Purcell
Noble Prize in 1952 for NMR



Read this paper If interested in life in microscopic world
However, may not be able to understand few things ☹

Life at low Reynolds number

- Most microorganisms live in fluid environments where they experience a viscous force that is many orders of magnitude stronger than inertial forces. Low Reynolds number (Re) regime

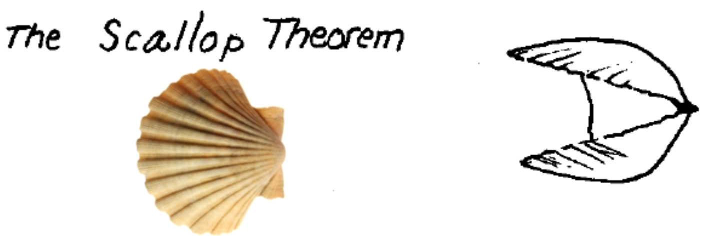
- A consequence of this is the '**scallop theorem**'

- If a low-Reynolds number swimmer executes ***geometrically reciprocal motion***, that is a sequence of shape changes that are identical when reversed, then the net displacement of the swimmer must be zero, if the fluid is incompressible and Newtonian

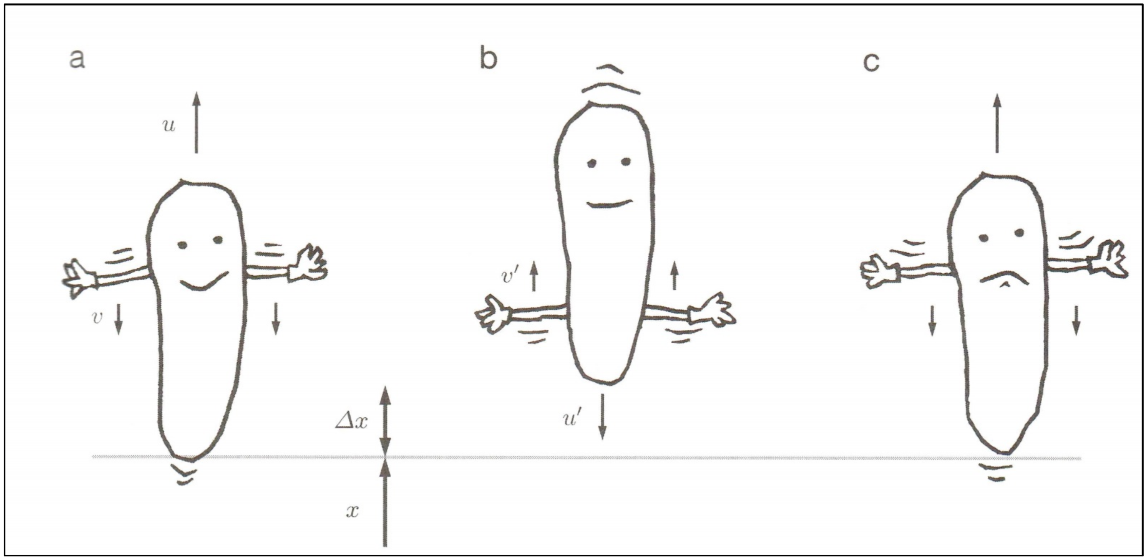
Proof of this theorem is beyond the scope of this course!!!

Life at low Reynolds number

In Purcell's own words, '***Fast, or slow, it exactly retraces its trajectory, and it's back where it started***'



A scallop opens its shell slowly and closes its shell fast, squirting out water. This reciprocal motion won't work at low Reynolds number

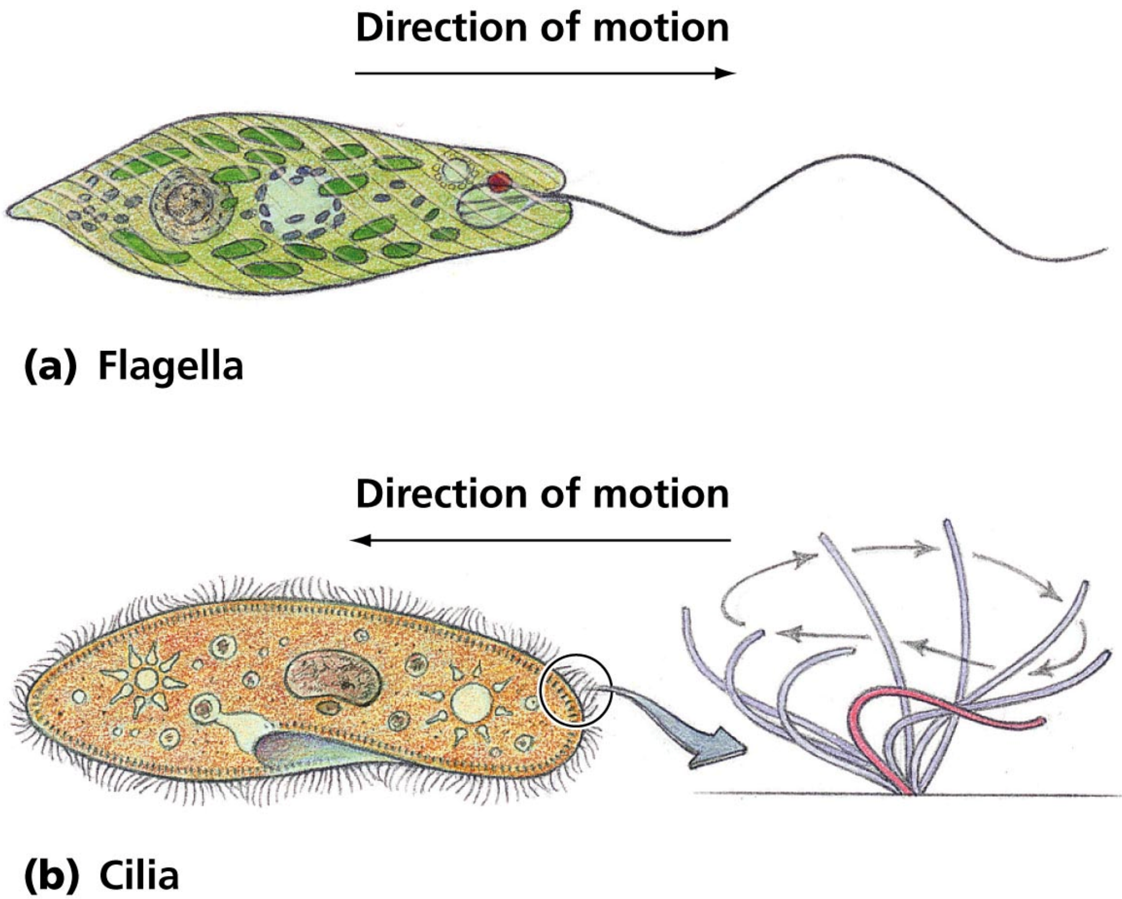


To be discussed in Tutorial 2

In a Newtonian incompressible fluid micro-scallop can't swim

Alternatives?

Alternative 1: Symmetry-Breaking (natural microorganisms)

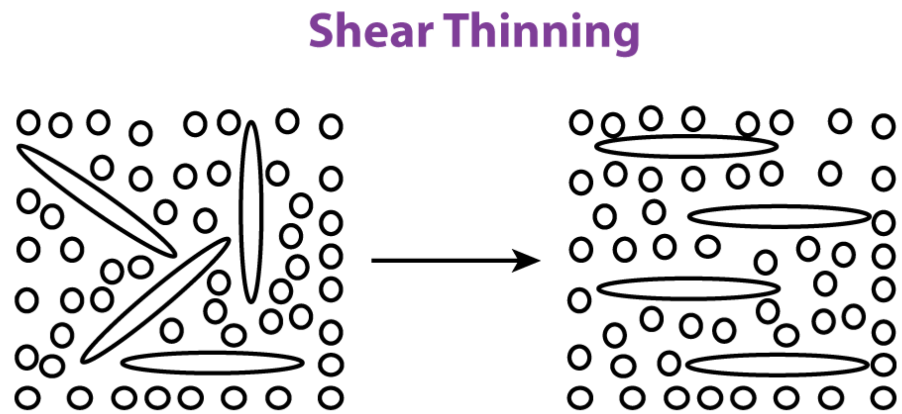


To be discussed in Tutorial 2

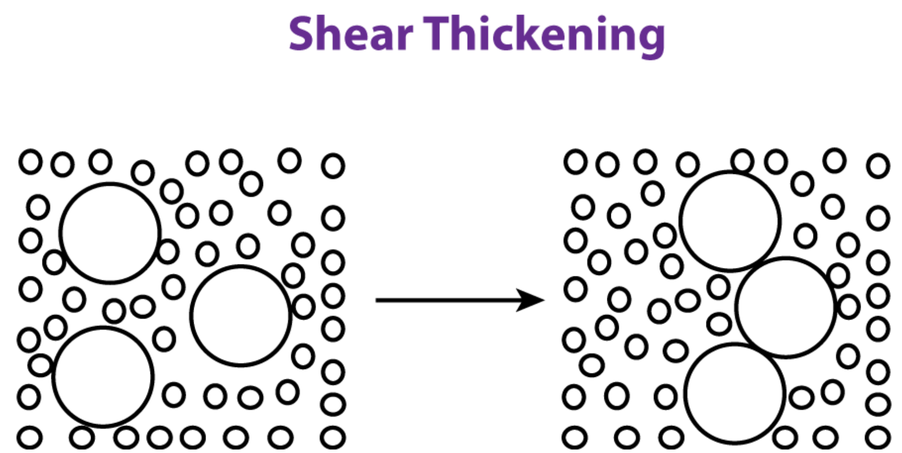
Figure Source: http://faculty.fmcc.suny.edu/mcdarby/majors101book/Chapter_04-Cell_Structure&Function/02-parts_of_advanced_cells.htm

Alternative 2: Use Non-Newtonian Fluids (artificial microorganism)

Shear Thinning:
-High molecular weight (polymers)
-Higher shear rate aligns molecules
-Results in decrease in viscosity



Shear Thickening:
-Large particles suspended in smaller particles
-Higher shear rate pushes out smaller molecules
-Results in viscosity increase



Watch "Non-newtonian fluid pool" video on following link

https://www.youtube.com/watch?v=D-wxnID2q4A

Micro-scallop in Non-Newtonian fluid

nature COMMUNICATIONS

ARTICLE

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OPEN

Swimming by reciprocal motion at low Reynolds number

Tian Qiu[1,2], Tung-Chun Lee[1], Andrew G. Mark[1], Konstantin I. Morozov[3], Raphael Münster[4], Otto Mierka[4], Stefan Turek[4], Alexander M. Leshansky[3,5] & Peer Fischer[1,6]

Watch the video “A swimming Micro-scallop”

http://vimeo.com/109797274

Summary so far.....

We looked at the **forces** at molecular and cellular scales

Cellular world is predominately governed by **viscous forces**

As a consequence, **inertial forces** can be safely ignored in most cases

What about energies?

Thermal Energy and Thermal Forces

- Proteins and cells are subjected to thermal forces, arises due from **collision** of water and other molecules in their surrounding fluid

- These collision forces are called **thermal forces** because their magnitude is proportional to temperature of the fluid molecules.

Thermal Motion and Thermal Energy

- The resulting movement of object is called **thermal motion**, and object is said to have **thermal energy**

- Since **thermal forces** are randomly directed, the resulting thermal motion is characterized by frequent changes in direction and is called **diffusion**

- Diffusion of a free particle or object is called **Brownian motion**

Brownian Motion

Figure Source:
http://www.nndb.com/people/050/000100747/

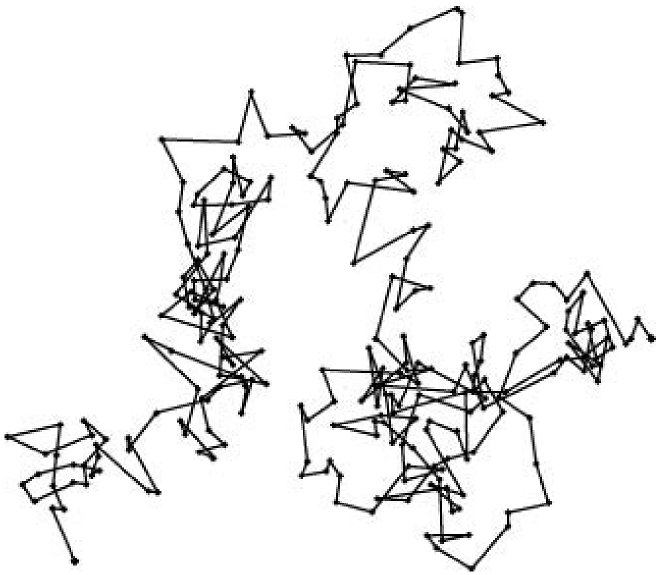


Figure Source: http://www.doc.ic.ac.uk/~nd/surprise_95/journal/vol4/ykl/report.html

In 1828, botanist Robert Brown noticed that pollen grains suspended in water dance in zig-zag manner

- Initially thought that it was signature of life
- Careful observer and proceeded to check his assumption
- Repeated observations with many lifeless particles, and all of them showed the same

Watch Video of Brownian Motion of pollens in water
https://www.youtube.com/watch?v=R5t-oA796to

Thermal Energy

- We saw that objects suspended in fluid can gain thermal energy and this thermal energy can make them to dance

- The thermal energy at temperature T is given by $k_B T$, where k_B is Boltzmann constant

- Thermal energy at room temperature

$$\mathbf{k_B T = 4.1 pN\ nm}$$

Is this thermal energy important in biology ?

Relative Importance of Thermal Energy

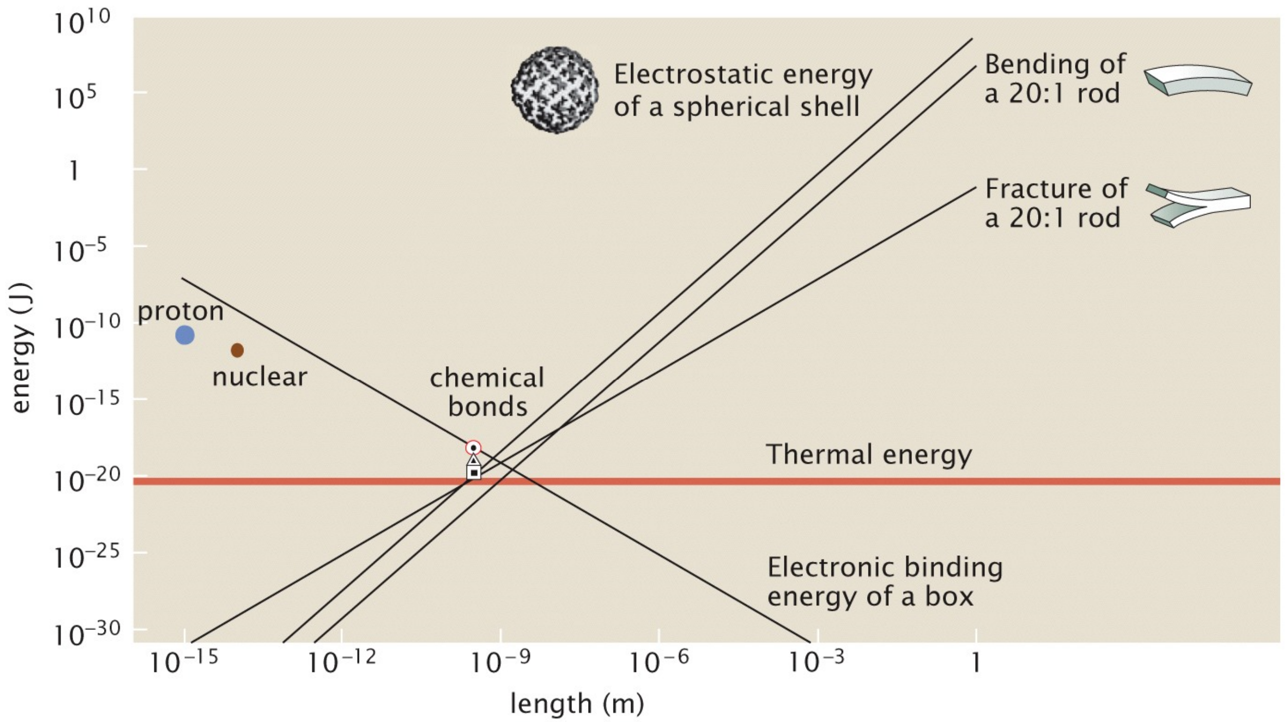


Figure 5.1 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

At the scale of macromolecule of the cells (nm) deterministic energies of bonding, charge rearrangement and molecular rearrangement are comparable

Thermal Energy

Therefore, it becomes important to consider thermal energy at macromolecular scales

Thermal energy can be safely ignored at macroscopic level.

It turn out that state of a biological system at molecular scale is decided by the competition between deterministic energy and thermal energy

Boltzmann’s Law

Fundamental physical law that describes how probability of a molecule having certain energy depends on the surrounding temperature

A particle or molecule always tends to remain in its lowest energy state

At non-zero temperature, due to molecular collisions, they can spend their time in higher energy states

Boltzmann's Law

Boltzmann's law says that if such a particle is in thermal equilibrium, then the probability pi of finding the particle in state i that has energy Ui is given by

$$\boxed{p_i = \frac{1}{Z} e^{-\frac{U_i}{k_B T}}}$$

Where $z = \sum_i e^{-\frac{U_i}{k_B T}}$ is called partition function

The exponential term is called Boltzmann factor

Boltzmann’s Law: Some comments

Boltzmann’s law is very general. The energy could correspond to particle’s potential energy (gravitational, elastic or electrical), its kinetic energy or the energy associated with its phase, or electronic or chemical state

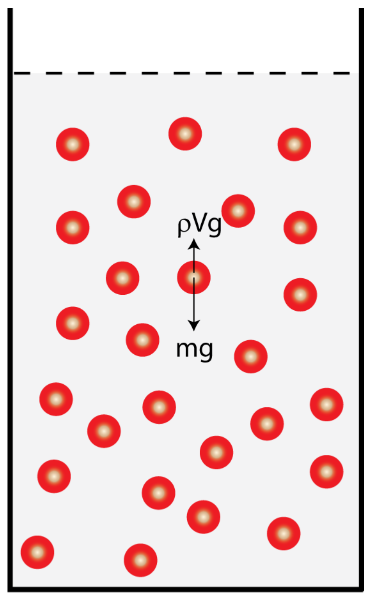
If there are just two states with energies U_1 and U_1 (and energy difference $\Delta U = U_2 - U_1$) then

$$\boxed{\frac{p_2}{p_1} = e^{-\frac{\Delta U}{k_B T}}}$$

Applications of Boltzmann's Law

Equilibrium Colloidal Suspension or Colloids

Macromolecules and many other soluble proteins form colloidal suspensions in water



colloidal suspension of globular proteins

Sedimentation Equilibrium in Gravity

Gravity: mg

Buoyant Force: $\rho V g$

Net force: $(m-\rho V)g \equiv m_{net}$

Profile of particle density is given by

$$\boxed{\frac{C_h}{C_0} = e^{-\frac{m_{net}gh}{k_B T}} = e^{-\frac{h}{h^*}}}$$

Competition between gravitational and thermal energy

For myoglobin protein

$$m_{net} = 4\frac{kg}{mol} \Rightarrow h_* = \frac{k_B T}{m_{net} g}$$

$$\approx 60m$$

In a 4 cm test tube $\frac{C_h}{C_0} = e^{-\frac{0.04}{60}} = 0.999 = 99.9\%$

Probability of finding particle at height h and the bottom are roughly same

Colloidal suspension will never sediment

Applications of Boltzmann's Law

Nernst Equation

Most spectacular application of biological electricity by cells is the action potential in the nerve cells

This is used to rapidly propagate information from the nerve cell body to the tip of axon

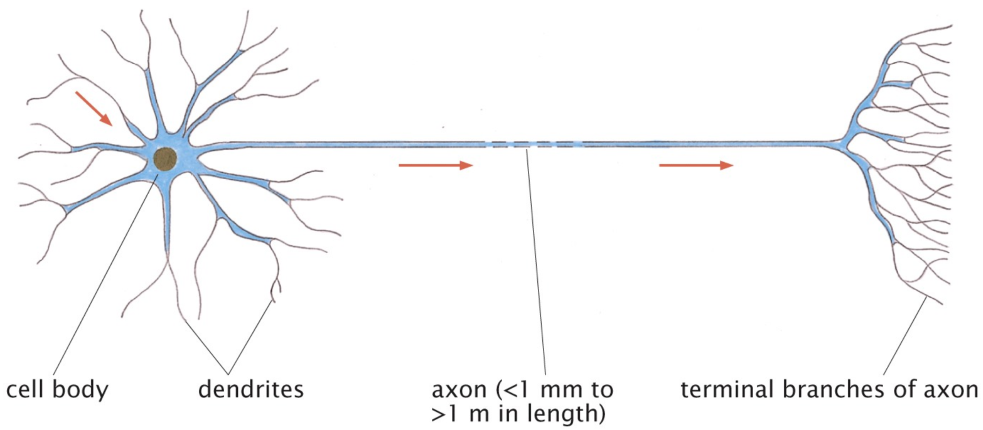


Figure 17.1 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

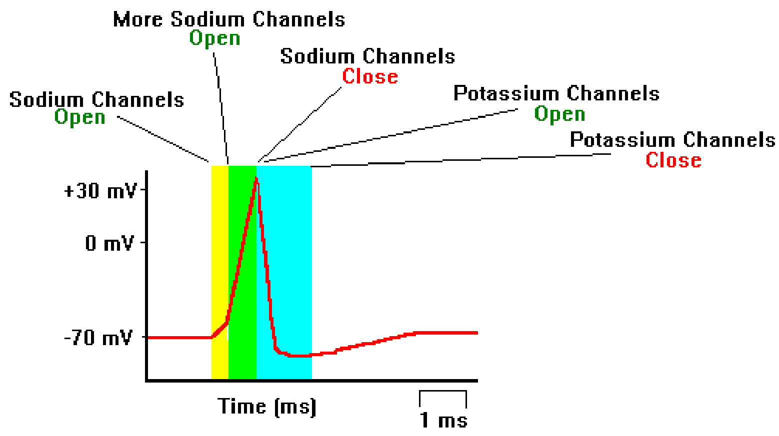
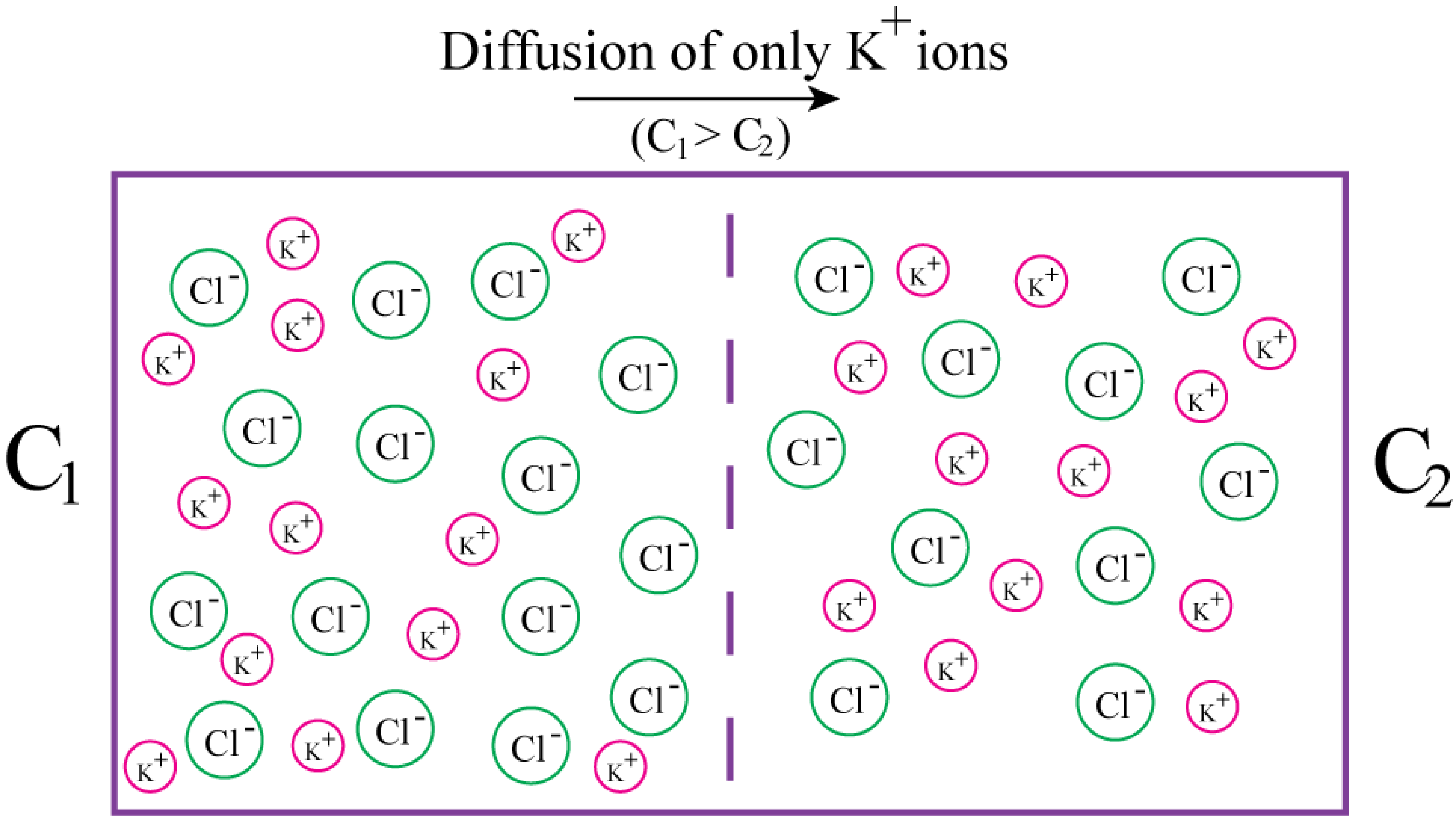


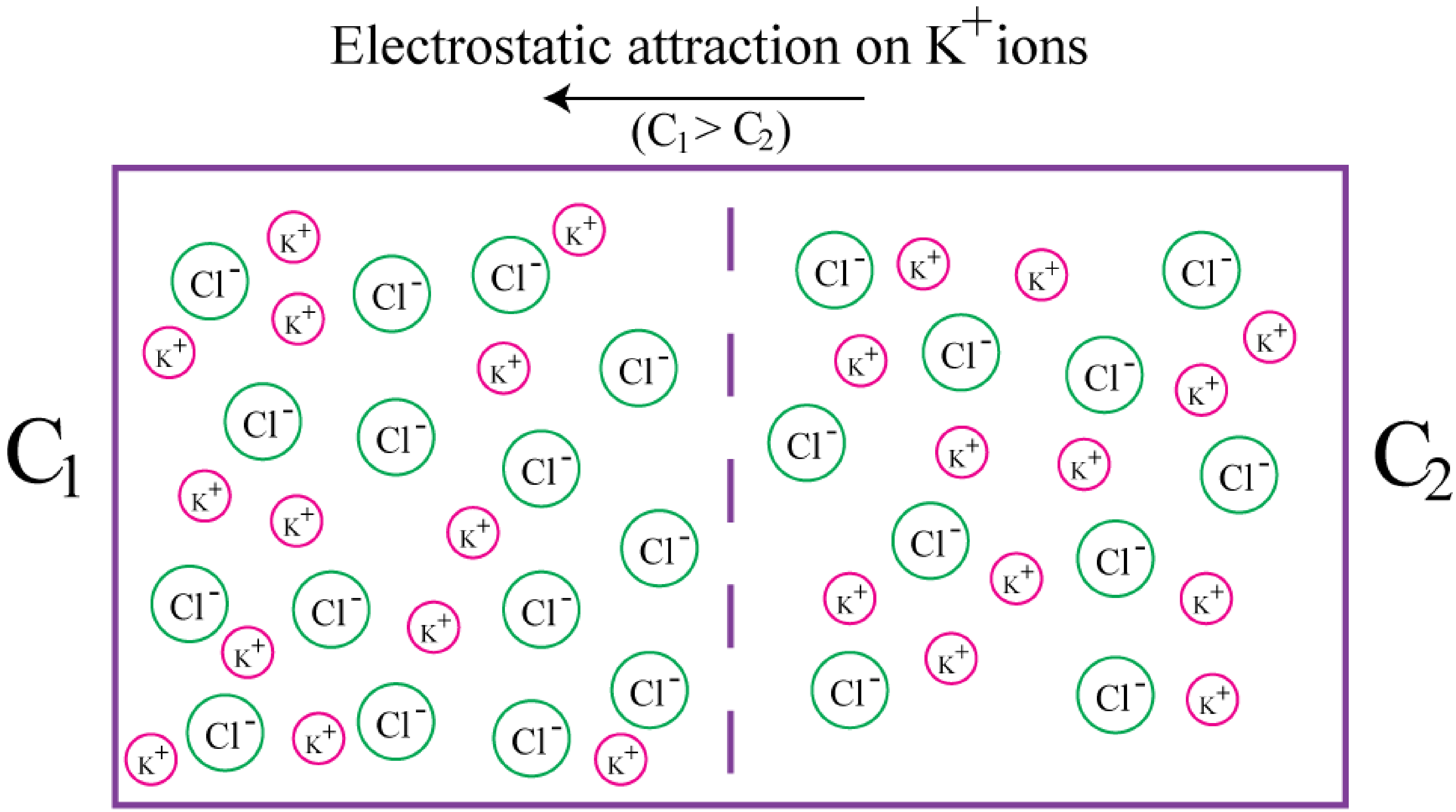
Figure Source: https://faculty.washington.edu/chudler/ap.html

Ion concentration difference across membrane lead to potential difference

Nernst Equation



Nernst Equation



Nernst Equation

$$\boxed{\frac{p_1}{p_2} = \frac{C_1}{C_2} = \frac{e^{-\frac{zeV_1}{k_BT}}}{e^{-\frac{zeV_2}{k_BT}}}}$$

$$\boxed{V_2 - V_1 = \frac{k_BT}{ze} ln \frac{C_1}{C_2}}$$

Ion species	Intracellular concentration (mM)	Extracellular concentration (mM)	Nernst potential (mV)
K^+	155	4	−98
Na^+	12	145	67
Ca^{2+}	10^{-4}	1.5	130
Cl^-	4	120	−90

Summary

- Life at low Reynolds number
- A low-Reynolds number microorganism can't swim by executing ***geometrically reciprocal motion***
- Thermal forces, Thermal energy and Brownian motion
- Thermal energy is comparable with other deterministic energy at molecular scales
- Boltzmann's law
- Applications of Boltzmann's law