LECTURE 6

BB 101: MODULE II

PHYSICAL BIOLOGY

Review of Lecture 5

- Thermal forces can bend the polymer

- Under what conditions thermal forces can bend filaments?

$$\boxed{E_b = \frac{k_b \pi}{R} = k_B T}$$

- Under what conditions we can treat polymers as freely jointed chain?

Polymers as random walk

- A polymers can be treated as consisting of a number of linear segments

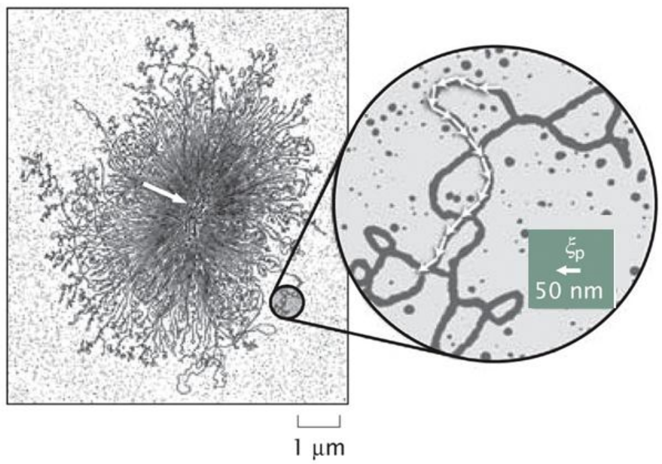


Figure 8.5 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

- Upto what characteristic length a filament would appear straight?

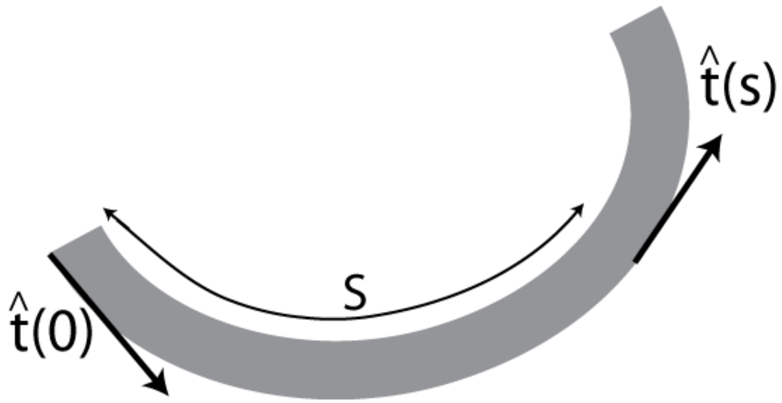
- What decides this characteristic length?

Persistence Length

- A filament would appear straight if successive tangent vectors points roughly in the same direction

- A filament would appear straight if successive tangent vectors are correlated

- Therefore, to find out the persistence length, we should calculate the correlation between tangent vectors which are separated by a distance of s



Persistence Length

$$g(s) = \langle \hat{t}(s) \quad \hat{t}(0) \rangle$$

- If tangent vectors at distance s are perfectly correlated then $g(s) = 1$
- On the other hand, if tangent vectors at distance s are completely independent then $g(s) \to 0$
- These properties can be easily captured by a decaying exponential function of the form

$$g(s) = e^{-\frac{s}{\xi_p}}$$

Persistence Length

- However, let's compute $g(s)$

$$g(s) = \langle \cos \theta(s) \rangle$$

- Bend can be approximated by an arc s of a circles of radius R such that angle subtended at center is θ

- Compute energy required to produce the bend

$$E_b = \frac{k_b}{2s}\theta^2$$

where $\theta = s/R$

Persistence Length

$$g(s) \approx \left\langle 1 - \frac{\theta^2(s)}{2} \right\rangle$$

- Let's calculate average of $\theta^2(s)$
- Recall definition of average and partition function

$$\langle \theta^2(s) \rangle = \frac{1}{Z} \int_0^{2\pi} d\varphi \int_0^{\pi} \theta^2 \sin\theta d\theta e^{-\frac{k_b \theta^2}{2k_B T s}}$$

where $$Z = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta e^{-\frac{k_b \theta^2}{2k_B T s}}$$

Persistence Length

$$\langle\theta^2(s)\rangle = \frac{1}{Z}\left(-2k_BTs\,\frac{\partial Z}{\partial k_b}\right) = -2k_BTs\,\frac{\partial lnZ}{\partial k_b}$$

where $\quad \mathrm{Z} = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta e^{-\frac{k_b\theta^2}{2k_BTs}}$

If θ is small $\quad \sin\theta \approx \theta$

Change of variable $\quad u = \frac{k_b\theta^2}{2k_BTs}$

$$\mathrm{Z} = \frac{2\pi k_BTs}{k_b}\int_0^{\infty} due^{-u} = \frac{2\pi k_BTs}{k_b}$$

Persistence Length

$$Z = \frac{2\pi k_B T s}{k_b}\int_0^{\infty} du e^{-u} = \frac{2\pi k_B T s}{k_b}$$

$$\frac{\partial lnZ}{\partial k_b} = -\frac{1}{k_b}$$

$$\langle \theta^2(s) \rangle = -2k_B T s \,\frac{\partial lnZ}{\partial k_b} = \frac{2k_B T s}{k_b}$$

Persistence Length

$$g(s) \approx \left\langle 1 - \frac{\theta^2(s)}{2} \right\rangle \approx \quad 1 - \frac{k_B T s}{k_b}$$

$$\approx \quad 1 - \frac{s}{(k_b/k_B T)}$$

$$\approx \quad 1 - \frac{s}{\xi_p}$$

$$\boxed{g(s) = e^{-\frac{s}{\xi_p}}}$$

Persistence Length

$$g(s) = e^{-\frac{s}{\xi_p}}$$

$$\boxed{\xi_p = \frac{k_b}{k_B T}}$$

Filament	Persistence Length
DNA	50 nm
Actin	15 μm
Microtubule	6 mm

Buckling of Filaments

- Microtubule and Actin filaments are tracks for active transportation in cells
- However, they also provide structural rigidity to cells
- **They should not buckle!**
- Can we estimates the buckling forces?

Buckling Force

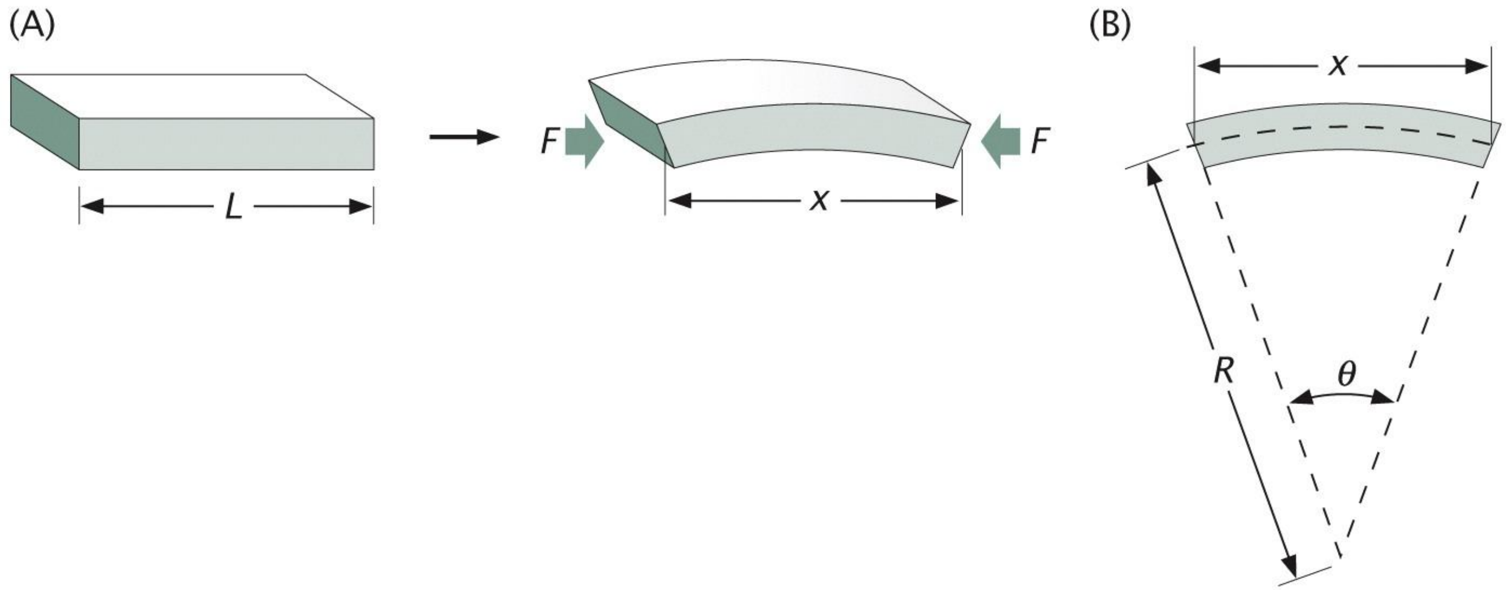


Figure 10.33 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

$$E_{total} = \frac{\xi_p k_B T}{2} \frac{L}{R^2} - F(L - x)$$

$$\text{where } x = 2R\sin\frac{\theta}{2}$$

Buckling Force

$$\frac{E_{total}}{k_B T} = \frac{\xi_p}{L}\frac{\theta^2}{2} - \frac{FL}{k_B T}\left(1 - \frac{2}{\theta}\sin\frac{\theta}{2}\right)$$

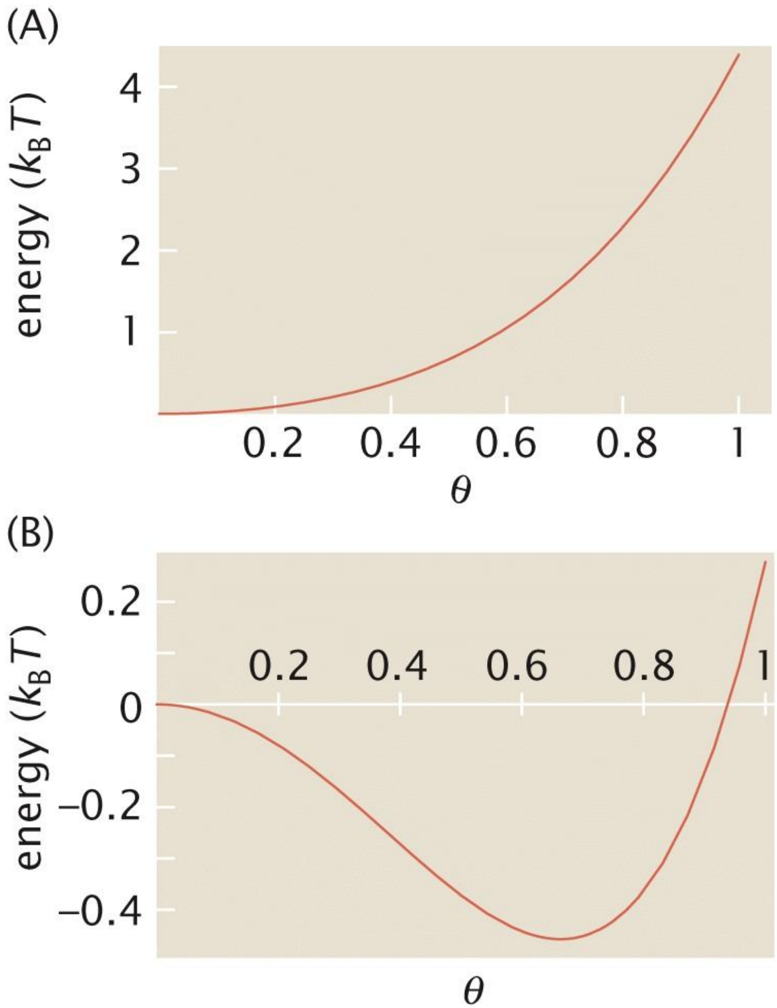


Figure 10.35 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

$$\frac{E_{total}}{k_B T} = \frac{\xi_p}{L}\frac{\theta^2}{2} - \frac{FL}{k_B T}\frac{\theta^2}{24}$$

$$\boxed{F_{critical} = 12\frac{k_B T \xi_p}{L^2}}$$

Microtubule and Actin Filaments are polymers

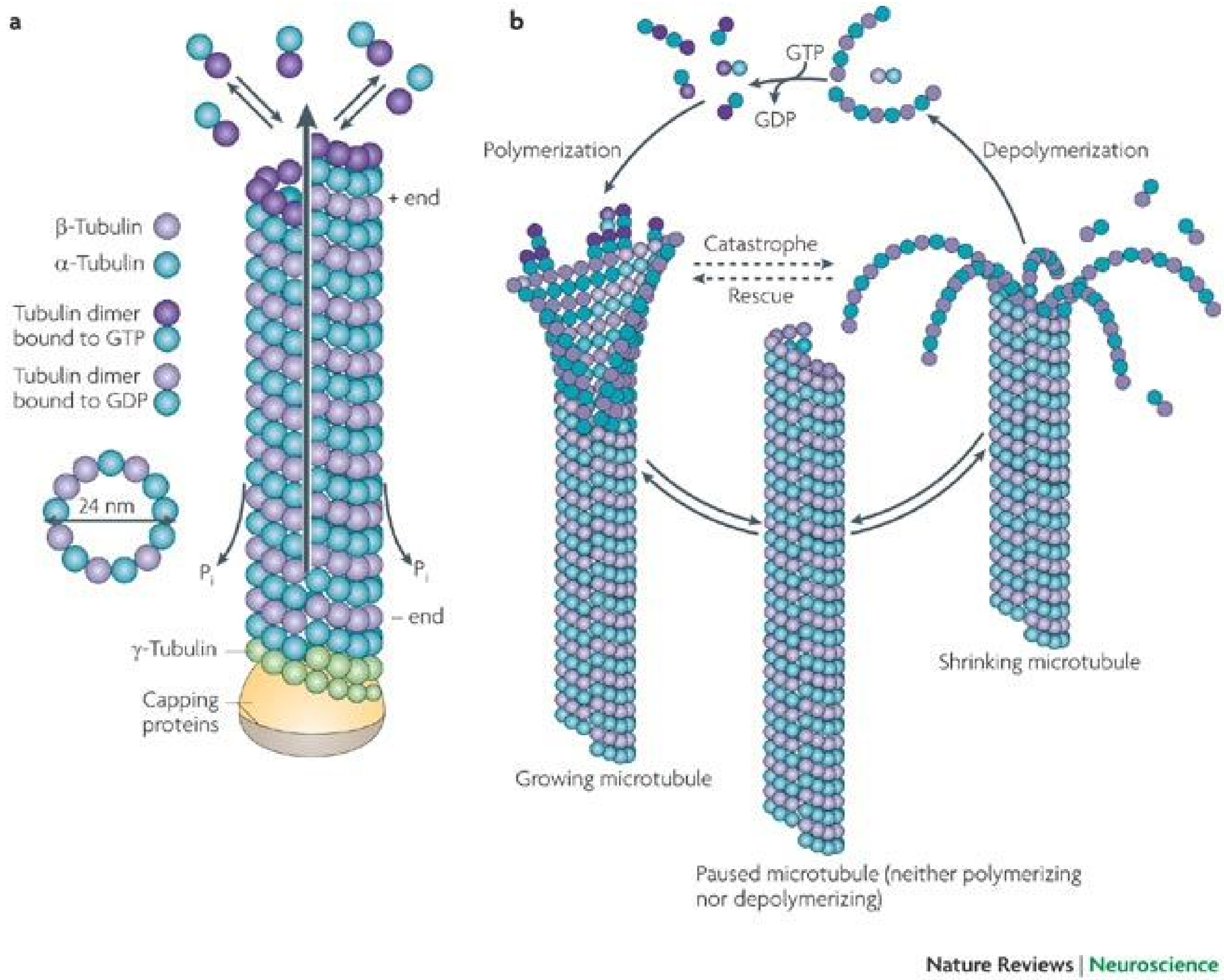


Figure Source: http://www.nature.com/scitable/topicpage/microtubules-and-filaments-14052932

Microtubule and Actin Filaments are polymers

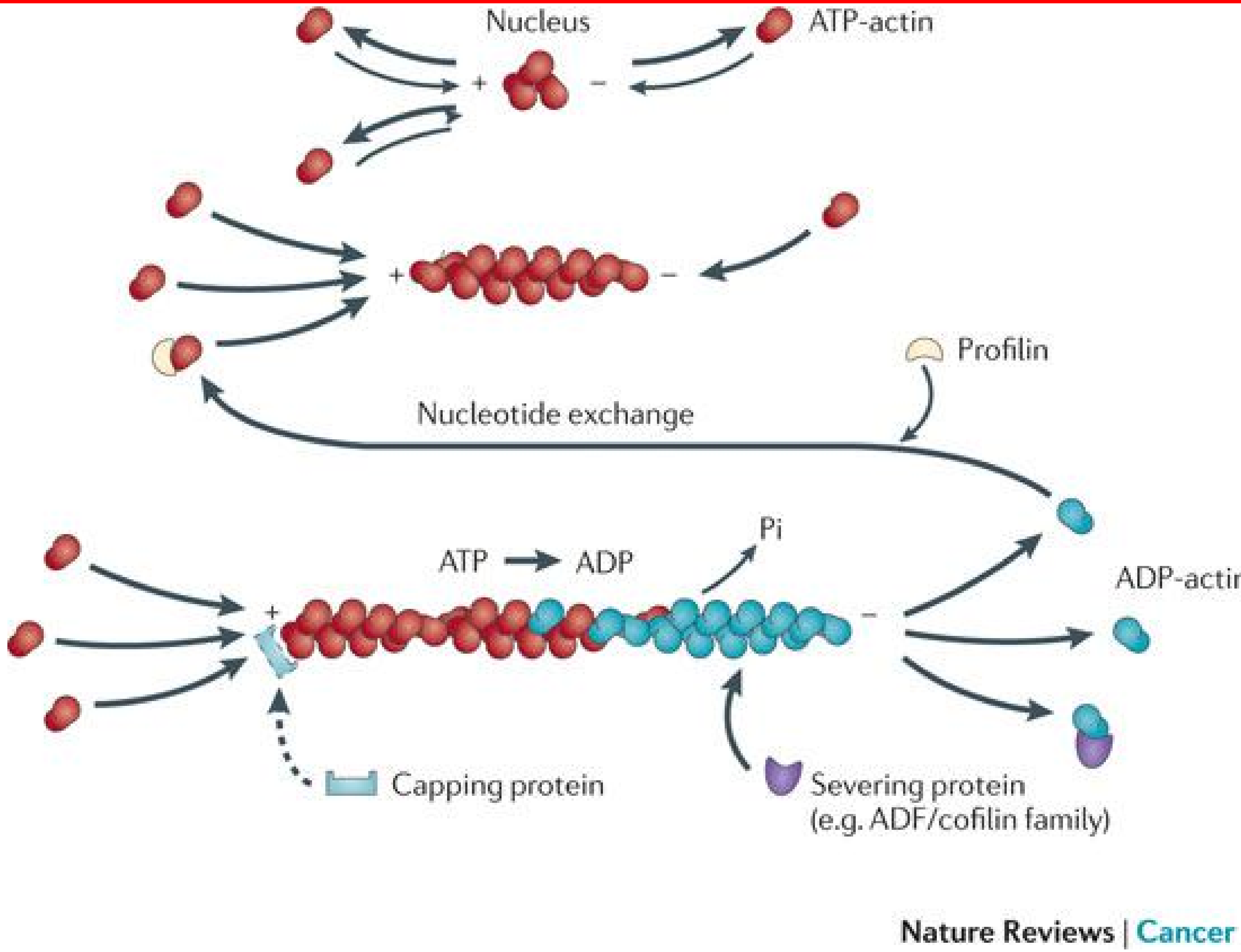


Figure Source: http://www.nature.com/nrc/journal/v11/n3/fig_tab/nrc3003_F1.html

Force generation Microtubule and Actin filaments

- Growing microtubule and actin filaments can exert forces against a barrier
- These forces can be measured using optical tweezers and Atomic Force Microscopy (AFM)
- This forces due to polymerization are useful in many cases

Finding the cell center using microtubules?

Forces generated by microtubule filaments can be used locate the center of the cell

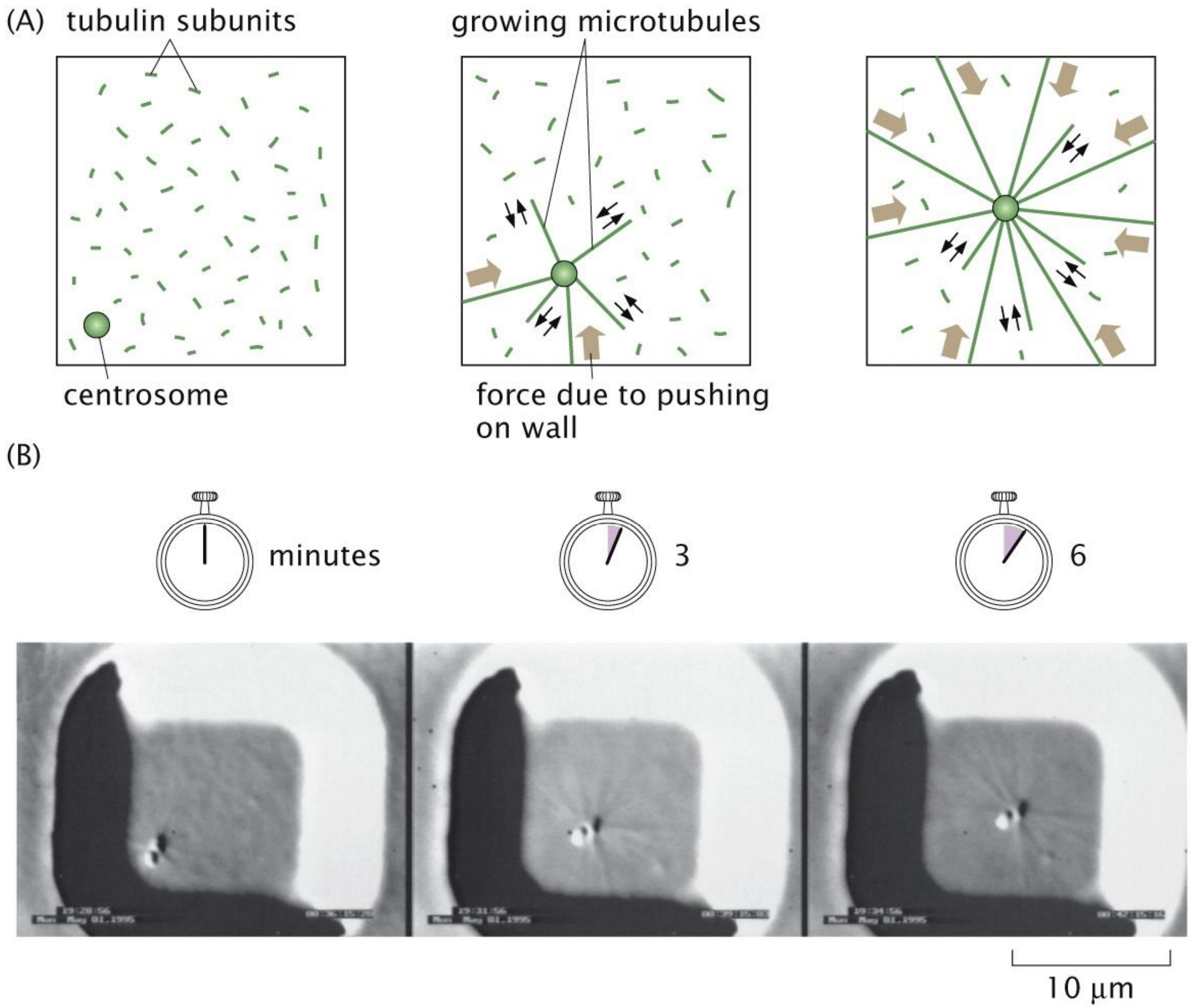


Figure 16.51 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Actin-based crawling of epithelial cells

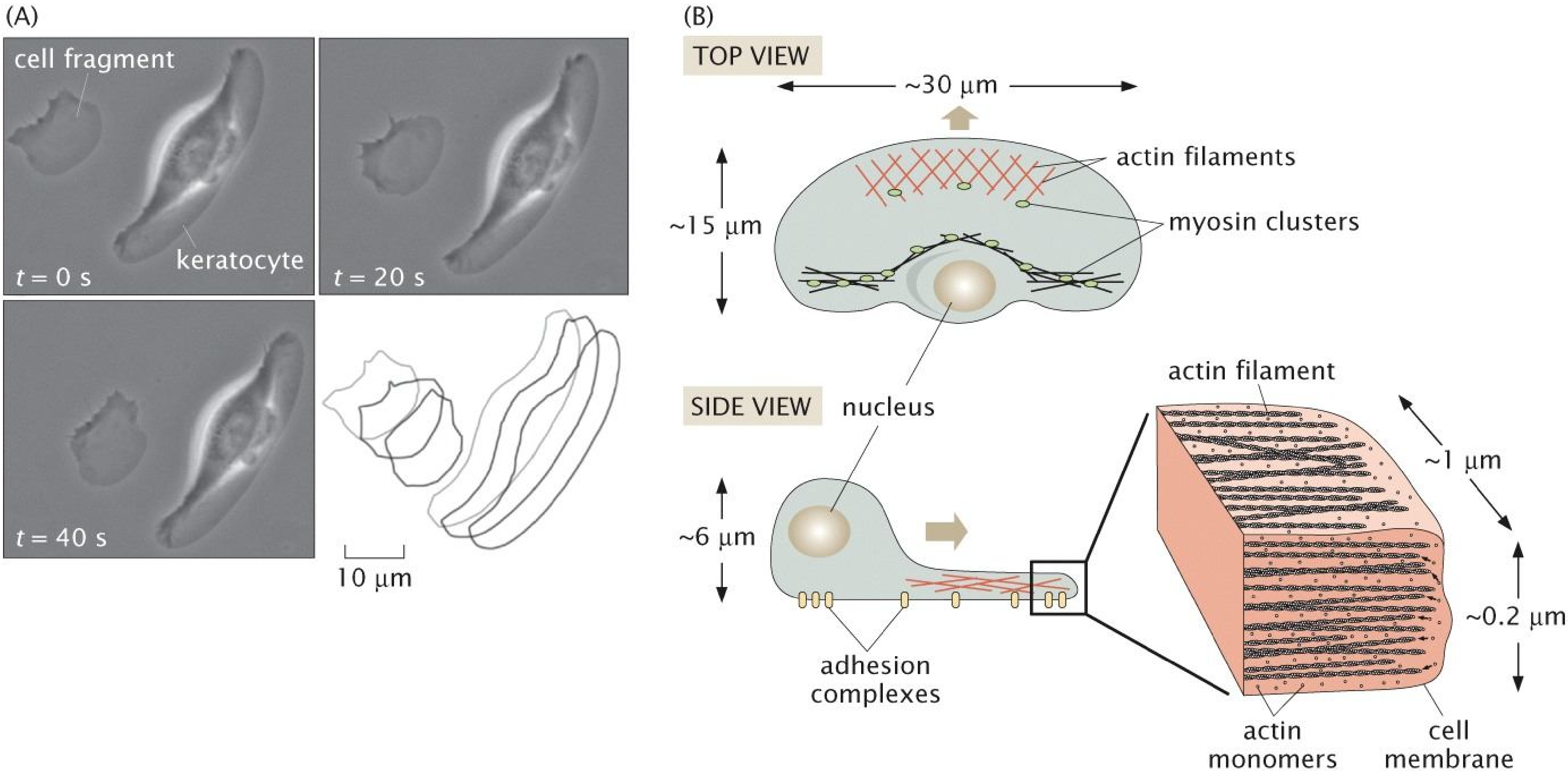


Figure 15.2 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Watch video of crawling fish keratocyte on following link:

https://www.youtube.com/watch?v=RTjYXBnMcgs

Actin polymerization driven motility of bacteria
Listeria monocytogenes

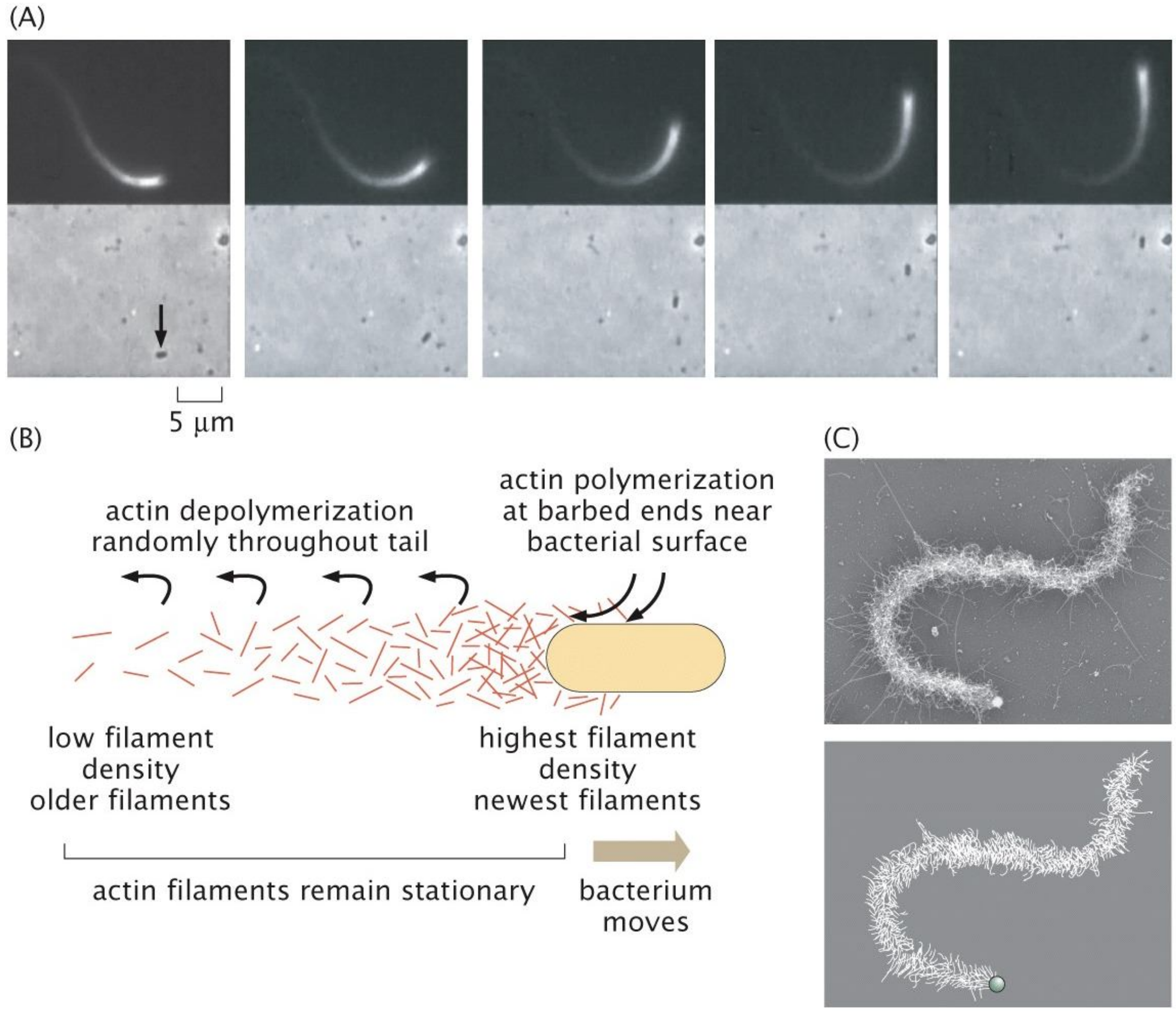


Figure 15.3 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Watch video of *Listeria monocytogenes* on following link:

https://www.youtube.com/watch?v=sF4BeU60yT8

Measuring force exerted by microtubule

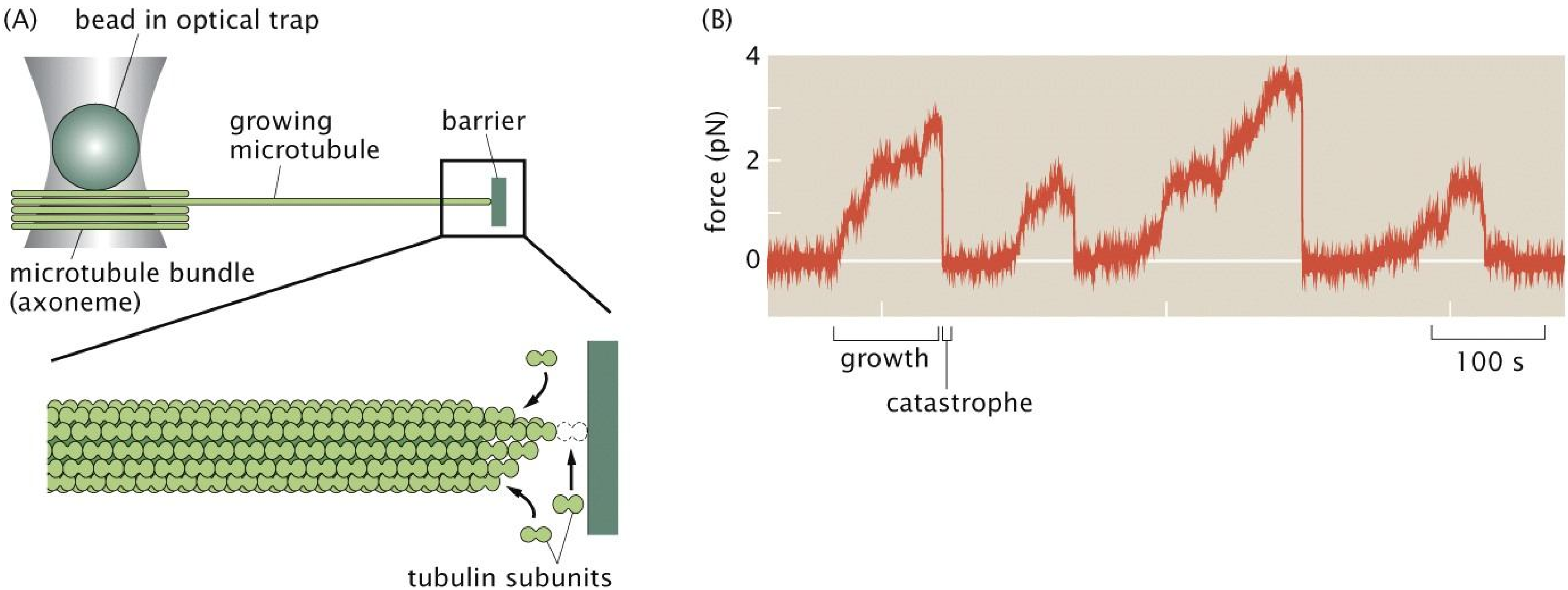


Figure 16.49 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Optical trap essentially behaves like a linear spring. If you know the displacement, you can calculate the force

Measuring force exerted by actin network

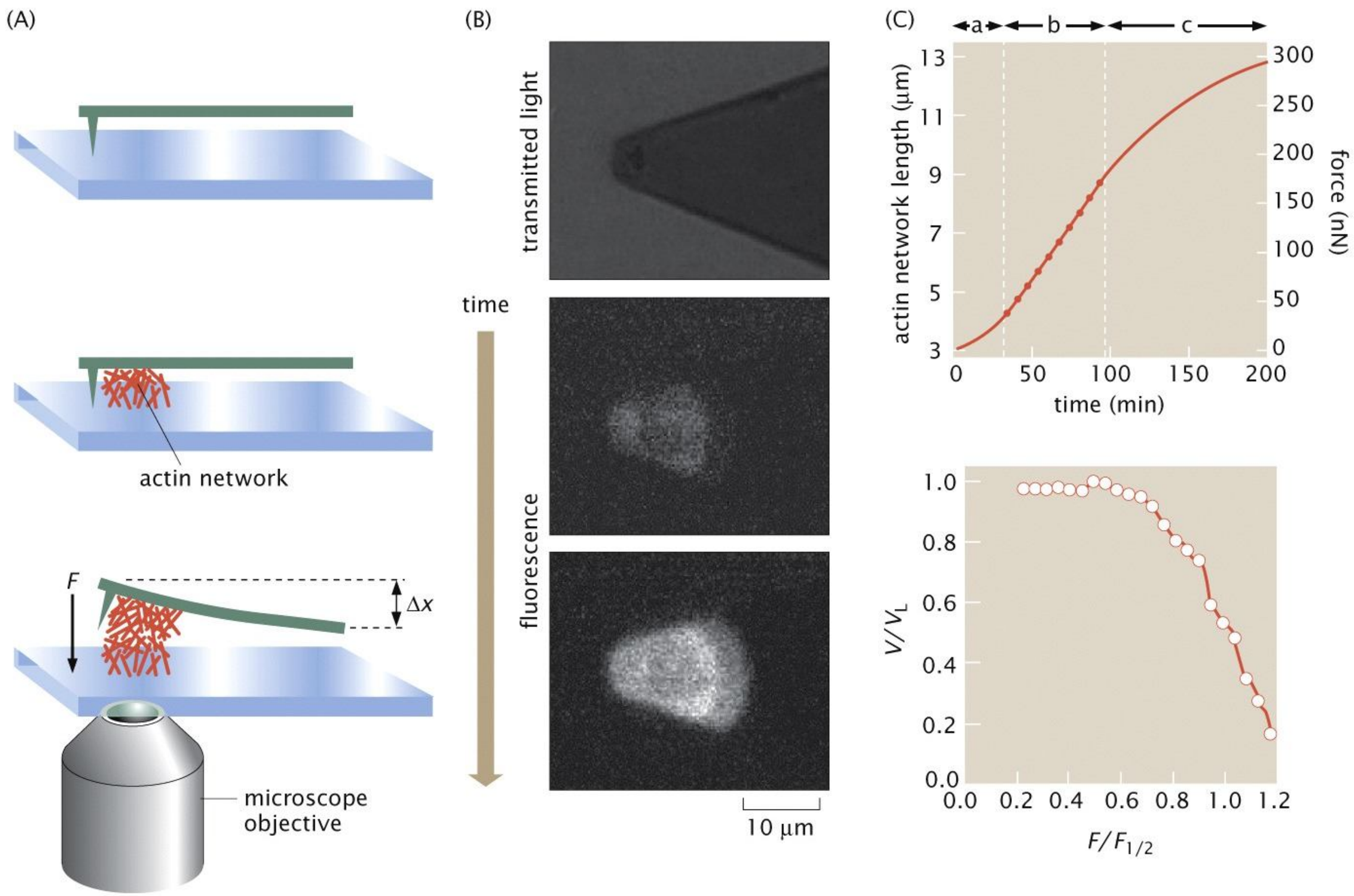


Figure 16.50 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Summary

- Under what conditions we can treat a polymer as freely jointed chain
- A filament would appears straight if their length is less than persistence length
- Externally applied forces can buckle filaments and critical buckling force
- Examples of force generation by microtubule and action filaments
- Measurement of forces exerted by microtubule and actin network