PH108

Lecture 18:

Boundary conditions on $\vec{B}$
and
a Surprise!

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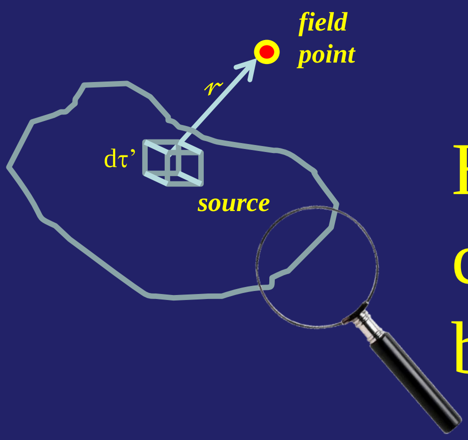
Recall: Ampere's Law lets us calculate $\vec{B}$

Biot-Savart

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\vec{\mathscr{r}}}{\mathscr{r}^3}\,d\tau'$$

Note: in unit vectors, this is $\dfrac{\hat{\mathscr{r}}}{\mathscr{r}^2}$



How does $\vec{B}$ change at the boundary?

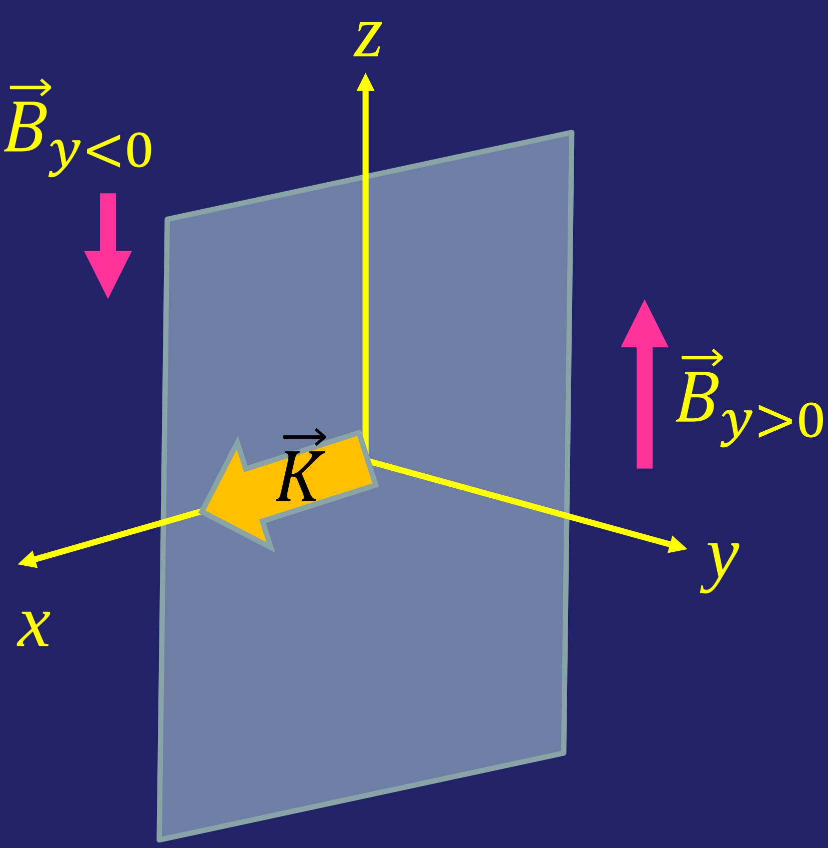
$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

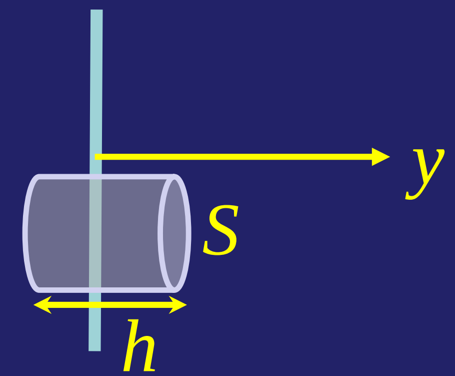
$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0 I$$

<u>Reminder:</u> We have assumed that $\vec{J}$ and I are independent of time – will continue to do so!

Normal component of $\vec{B}$ is continuous



An infinite *sheet* of current in the *x*-*z* plane carries surface current $\vec{K}$

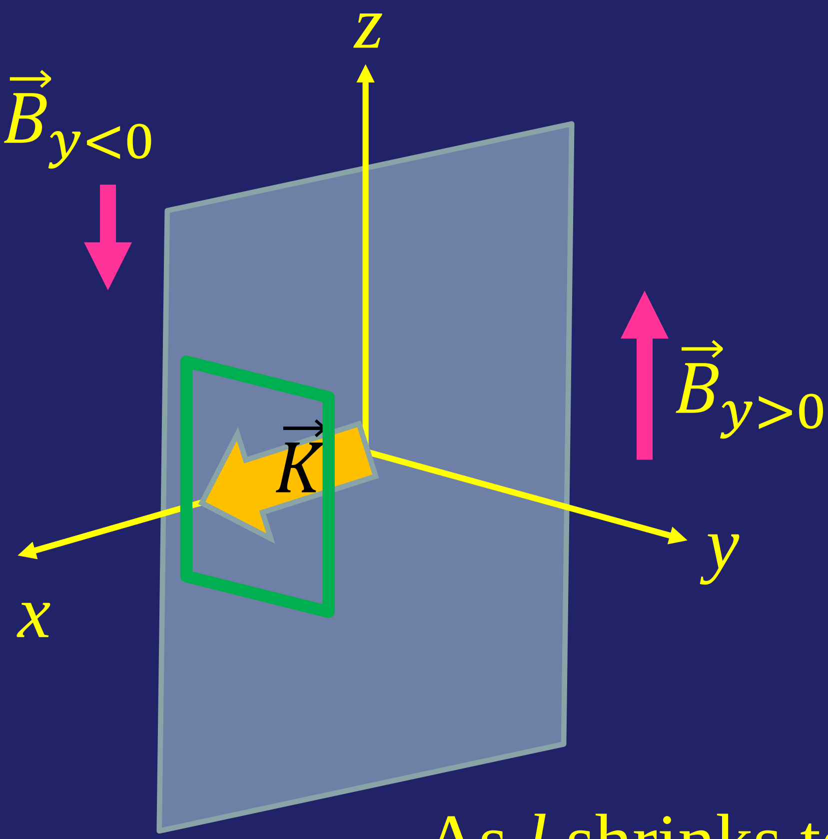


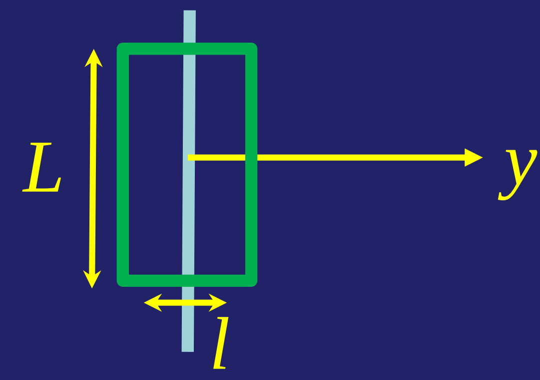
$$\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \oint \vec{B} \cdot \overrightarrow{dS} = 0$$

As h shrinks to 0, surface integral reduces to:

(independent of S) $\vec{B}_{\perp,y>0} - \vec{B}_{\perp,y<0} = 0$

Tangential component of $\vec{B}$ changes





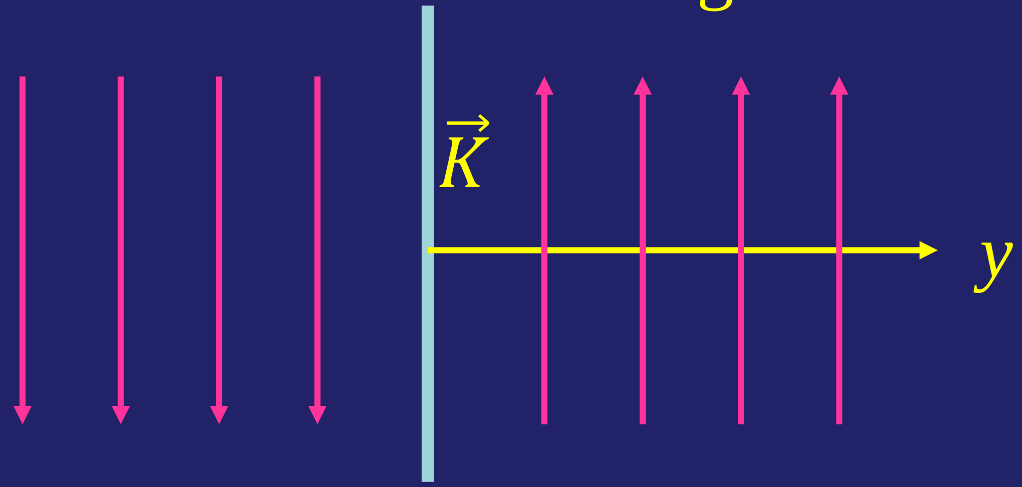
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \rightarrow \oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \vec{K}$$

As l shrinks to 0, line integral reduces to:

(independent of L) $\vec{B}_{||,y>0} - \vec{B}_{||,y<0} = \mu_0 \vec{K}$

Question 1

$\vec{K}$ on infinite sheet coming out of screen



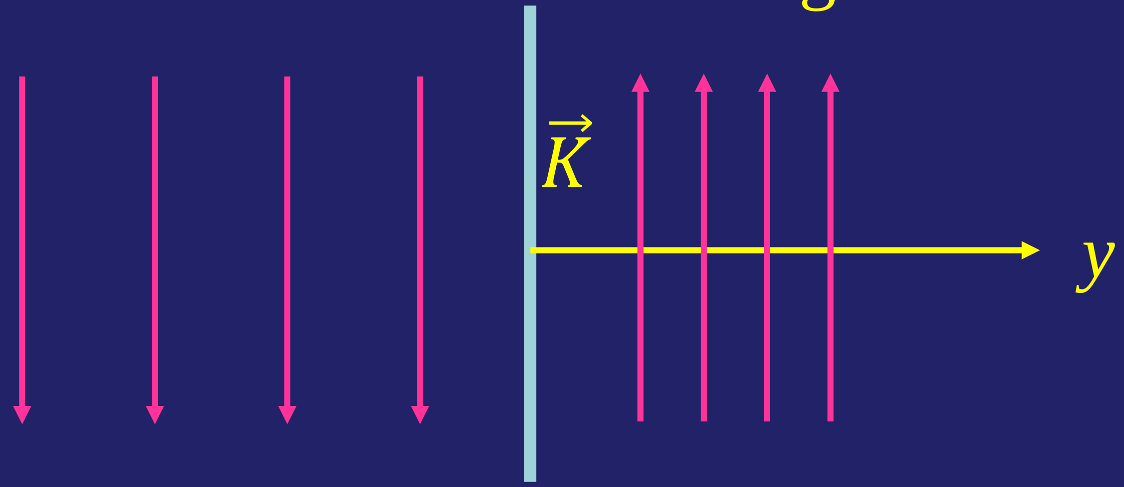
The pink arrows represent $\vec{B}$ field lines.

the surface current density is :

A) $\vec{K} = K\,\hat{x}$ B) $\vec{K} = K\,\hat{z}$

Question 2

$\vec{K}$ on infinite sheet coming out of screen



The pink arrows represent $\vec{B}$ field lines.

Is the above situation allowed by physics?

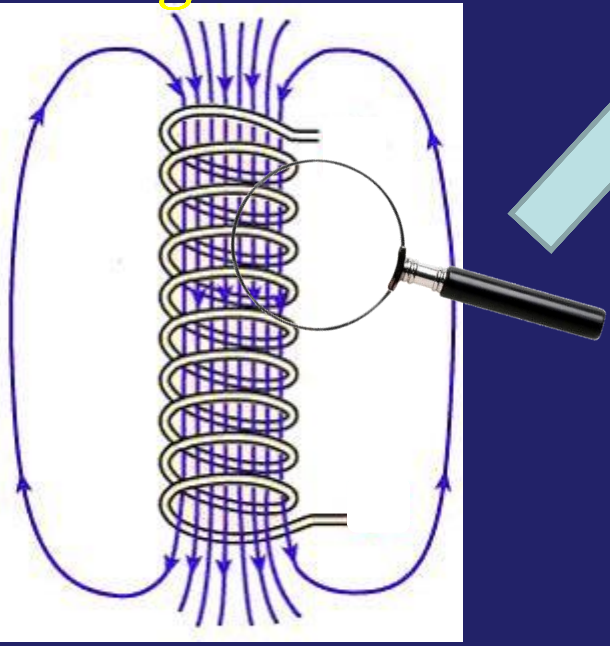
A) YES

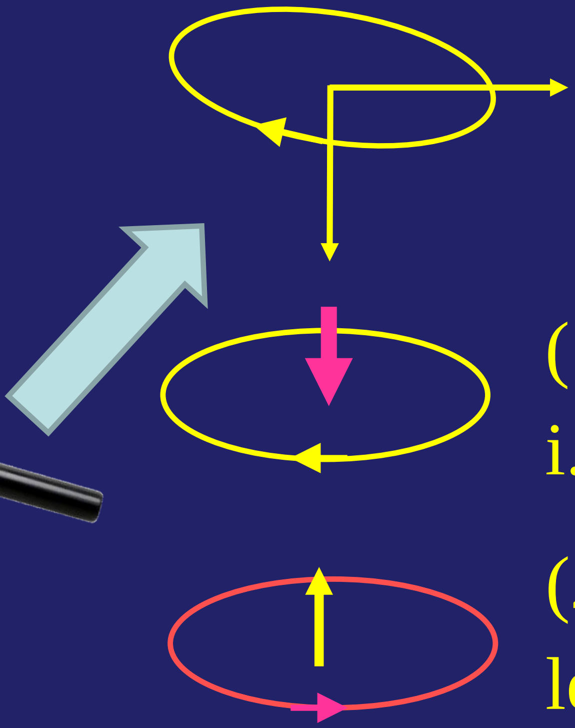
B) NO

$\vec{K} = K\hat{x}$ superimposed with fixed $B_0\hat{z}$!

Traditional method of defining a solenoid magnetic field has problems

"Traditional" textbook diagram





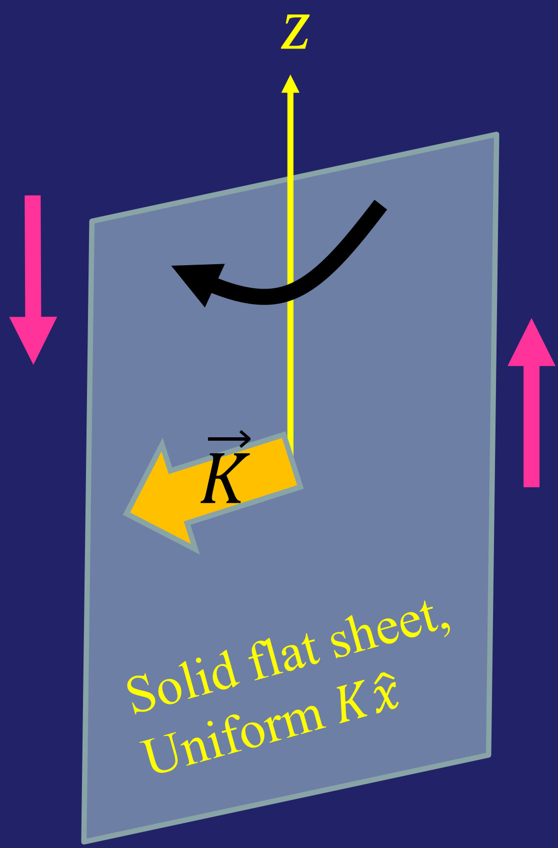
Each loop of this helical solenoid current has TWO components

(1) RADIAL current
i.e. ideal solenoid $\vec{B}$

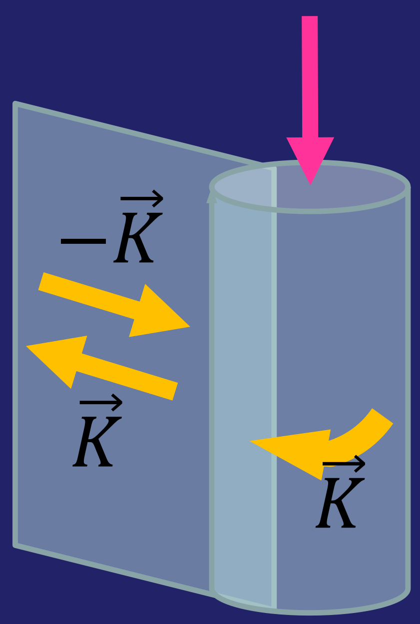
(2) AXIAL current
leads to a CIRCULAR $\vec{B}$

The net superimposed $\vec{B}$ is a distorted helical field.
This matters in high precision magnetic applications!

A better method of defining a solenoid magnetic field



Wrap into a ‘tube’



Hence high precision magnets are made from flat ‘ribbons’