PH 103 : Electricity and Magnetism

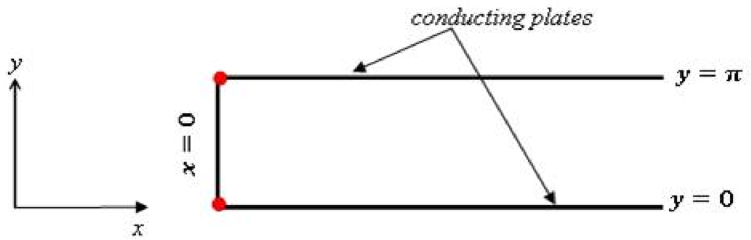
Tutorial Sheet 5 : **Solution to Laplace's equation**

1. A conducting sphere of radius R has a charge Q on its surface. Use Laplace's equation to find the potential everywhere in space and also determine the charge density on the surface of the sphere. [Ans. $V_{\text{out}} = \frac{V_0 R}{r}$, where$V_0 = Q/4\pi\epsilon_0 R$, $V_{\text{in}} = V_0$]

2. Sphere of radius a, kept at a constant potential V_0 and a concentric shell of radius b $(b > a)$ kept at a potential $V = 0$. Find the potential in the region $a < r < b$. [Ans. $V(a < r < b) = \frac{a}{b-a} V_0 \left(\frac{b}{r} - 1\right)$]

3. The potential on the surface of a sphere of radius R is given by $V(\theta) = V_0 \cos\theta$. Find the potential inside and outside the sphere as well as the surface charge density on the sphere. Assume that there are no charges inside or outside the sphere. [Ans. $V_{\text{out}} = \frac{V_0 R^2}{r^2} \cos\theta$, $V_{\text{in}} = \frac{V_0 r}{R} \cos\theta$]

4. Two semi-infinite grounded metal plates lie parallel to the xz plane, one at $y = 0$ and the other at $y = \pi$. The end at $x = 0$ is closed off with an infinite strip insulated from the two plates and maintained at a constatn potential V_0. Find the potential in the region between the plates.[Ans. $V(x, y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...} \frac{e^{-nx}\sin(ny)}{n}$]



5. Two infinitely long grounded metal plates at $y = 0$ and $y = a$ are connected at $x = \pm b$ by metal strips maintained at a constant potential V_0. A thin layer of insulation at each corner prevents the plates from shorting out. Find the potential inside the resulting rectangular pipe.

6. A spherical shell of radius R is kept at a constant potential $V_0 \sin^2\frac{\theta}{2}$. Find the potential (a) outside $(r > R)$ (b) inside $(r < R)$ the shell. [Ans. $V_{\text{out}} = \frac{V_0 R}{2r} - \frac{V_0 R^2}{2r^2}\cos\theta$, $V_{\text{in}} = \frac{V_0}{2} - \frac{V_0 R}{2r}\cos\theta$]

7. A sphere of radius R (earthed), is kept in a uniform electric field $\vec{E} = E_0\hat{k}$. Find the potential for $r > R$. [Ans. $V(r, \theta) = -E_0(r - \frac{R^3}{r^2})\cos\theta$]

8. A spherical shell of radius R has a charge density $\sigma(\theta) = \sigma_0 \cos\theta$. Find the resulting potential inside and outside the shell. [Ans. $V_{\text{in}}(r, \theta) = \frac{\sigma_0}{3\epsilon_0} r\cos\theta$, $V_{\text{out}}(r, \theta) = \frac{\sigma_0 R^3}{3\epsilon_0 r^2}\cos\theta$]

9. A thin (negligible thickness) spherical shell of radius R carries a surface charge density

$$\sigma(\theta) = \sigma_0 \left(\cos\theta + \cos^2\theta\right).$$

Using solutions of Laplace's equation, find the potential $V(r, \theta)$ everywhere; $r > R$ and $r < R$. [Ans. $V_{\text{out}}(r, \theta) = \frac{\sigma_0 R^2}{3\varepsilon_0 r} + \frac{\sigma_0 R^3}{3\varepsilon_0 r^2}\cos\theta + \frac{2\sigma_0 R^4}{15\varepsilon_0 r^3}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$]