

# PH108

Lecture 18:

Boundary conditions on  $\vec{B}$   
and  
a Surprise!

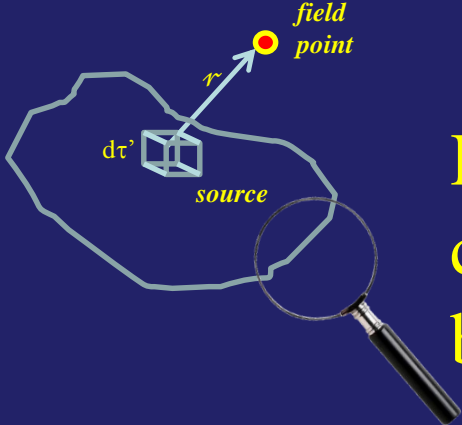
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# Recall: Ampere's Law lets us calculate $\vec{B}$

Biot-Savart

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \vec{r}}{r^3} d\tau'$$

Note: in unit vectors, this is  $\frac{\hat{r}}{r^2}$



The diagram shows an irregularly shaped region representing a current distribution. Inside this region, a small blue cube is labeled  $d\tau'$  and "source". A red dot outside the region is labeled "field point". A blue arrow labeled  $r$  points from the source cube to the field point. A magnifying glass is positioned over the boundary of the current distribution.

How does  $\vec{B}$  change at the boundary?

$$\vec{\nabla} \cdot \vec{B} = 0$$

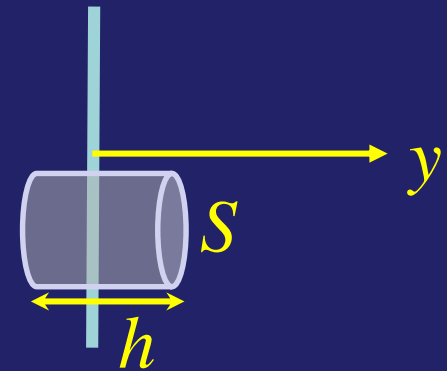
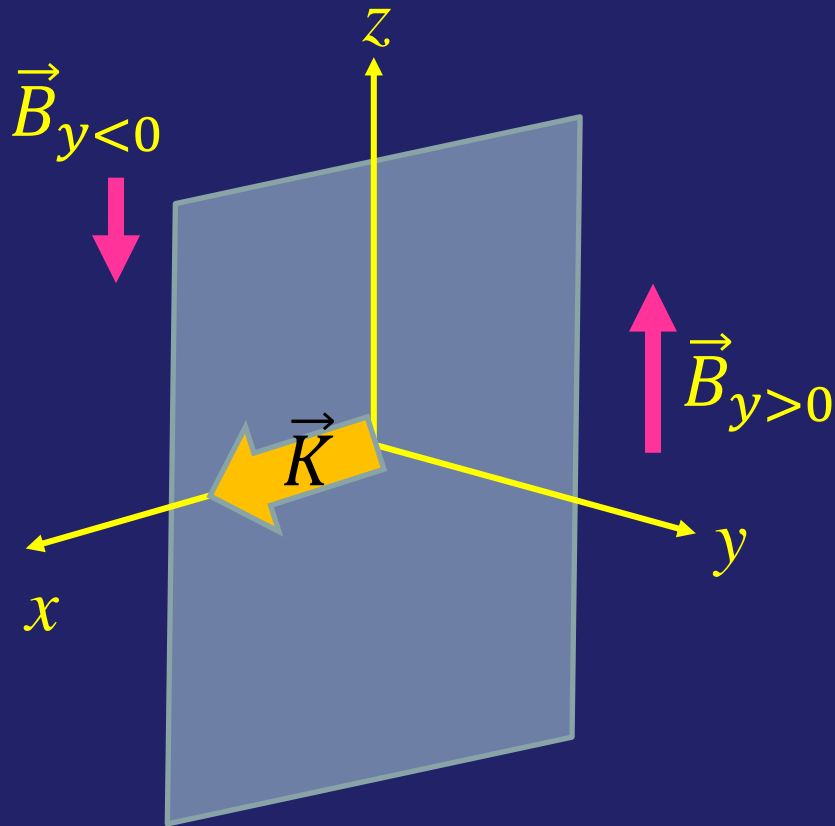
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Reminder: We have assumed that

$\vec{J}$  and  $I$  are independent of time – will continue to do so!

# Normal component of $\vec{B}$ is continuous

An infinite *sheet* of current in the  $x$ - $z$  plane carries surface current  $\vec{K}$

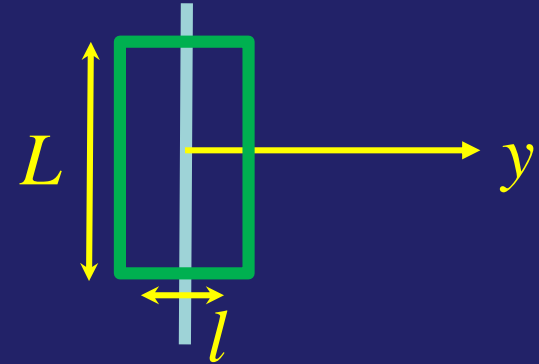
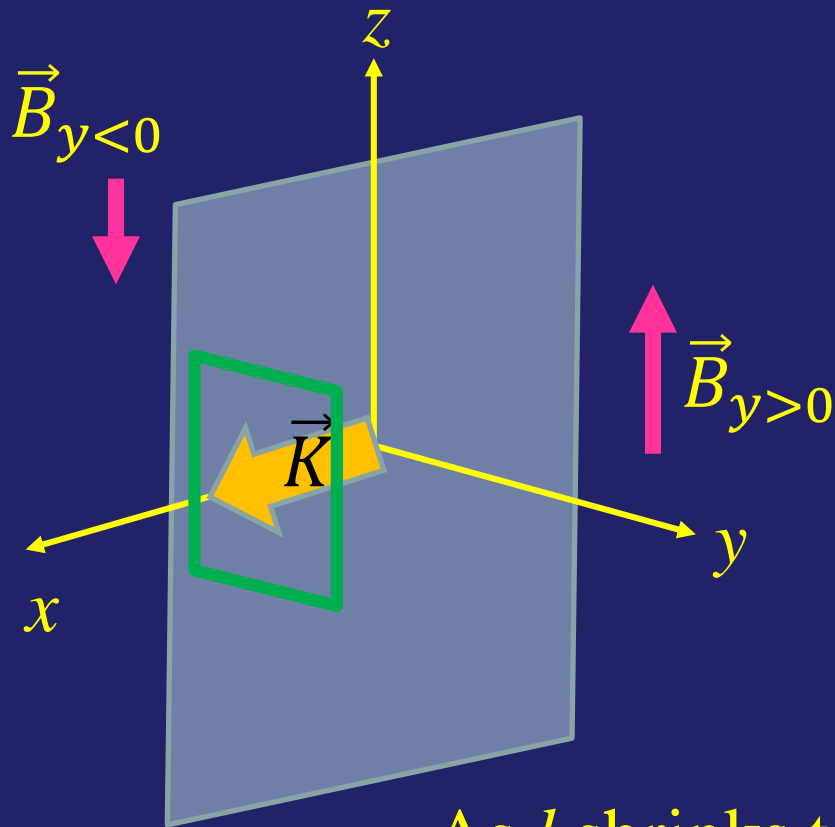


$$\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \oint \vec{B} \cdot d\vec{S} = 0$$

As  $h$  shrinks to 0, surface integral reduces to:

$$(\text{independent of } S) \quad \vec{B}_{\perp, y>0} - \vec{B}_{\perp, y<0} = 0$$

# Tangential component of $\vec{B}$ changes



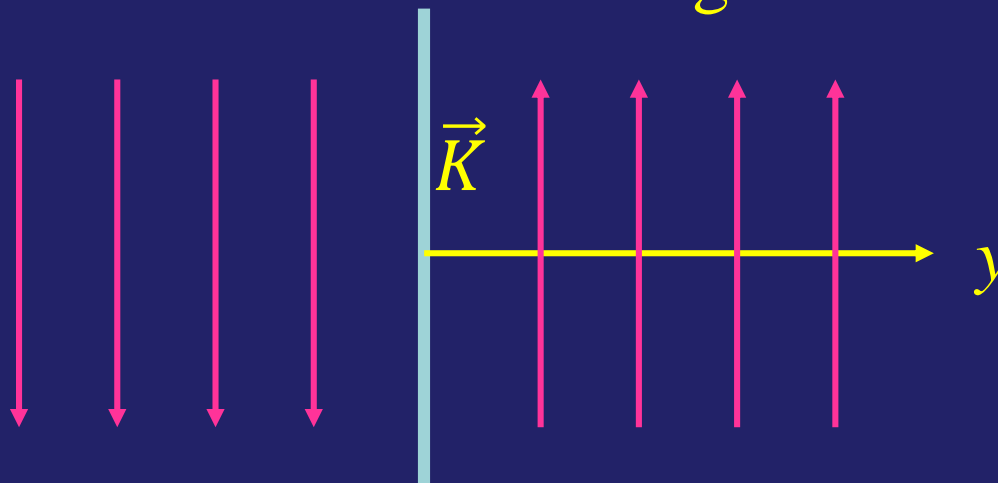
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \vec{K}$$

As  $l$  shrinks to 0, line integral reduces to:

$$(\text{independent of } L) \vec{B}_{||,y>0} - \vec{B}_{||,y<0} = \mu_0 \vec{K}$$

# Question 1

$\vec{K}$  on infinite sheet coming out of screen



The pink arrows represent  $\vec{B}$  field lines.

the surface current density is :

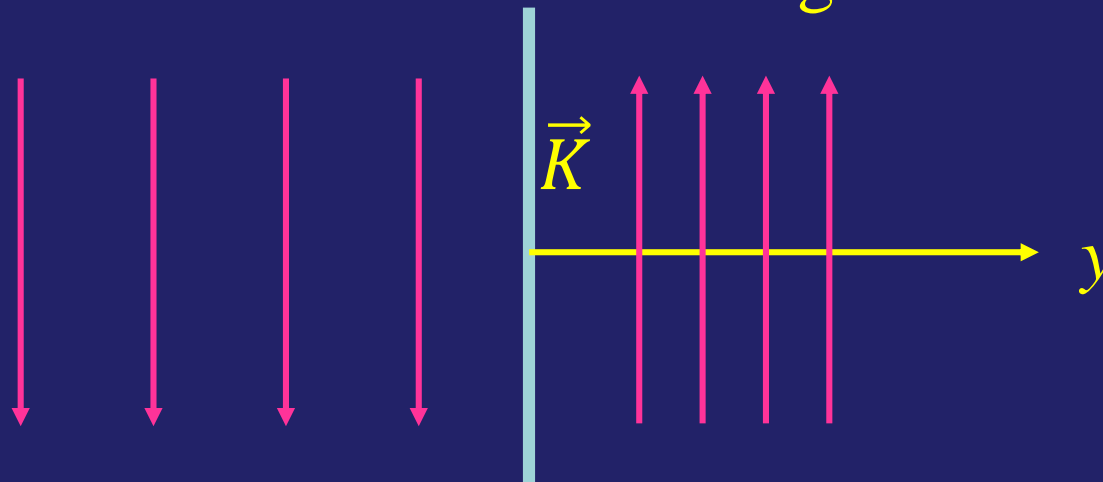
A)  $\vec{K} = K \hat{x}$

B)  $\vec{K} = K \hat{z}$



## Question 2

$\vec{K}$  on infinite sheet coming out of screen



The pink arrows represent  $\vec{B}$  field lines.

Is the above situation allowed by physics?

A) YES

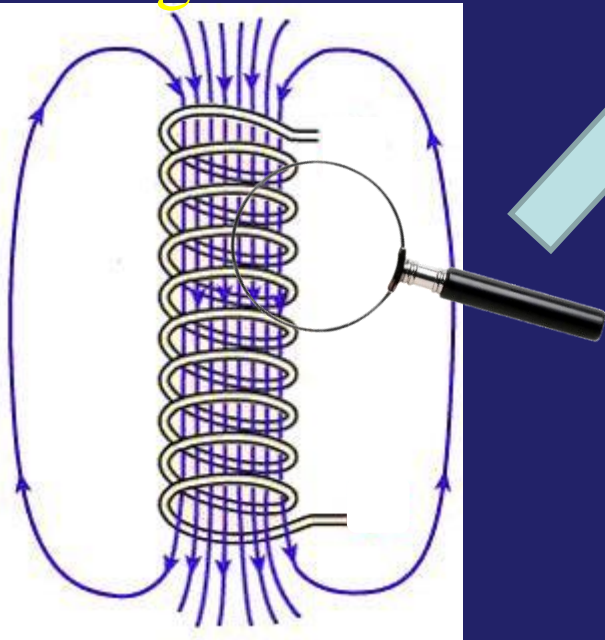
B) NO



$\vec{K} = K \hat{x}$  superimposed with fixed  $B_0 \hat{z}$  !

# Traditional method of defining a solenoid magnetic field has problems

“Traditional”  
textbook  
diagram



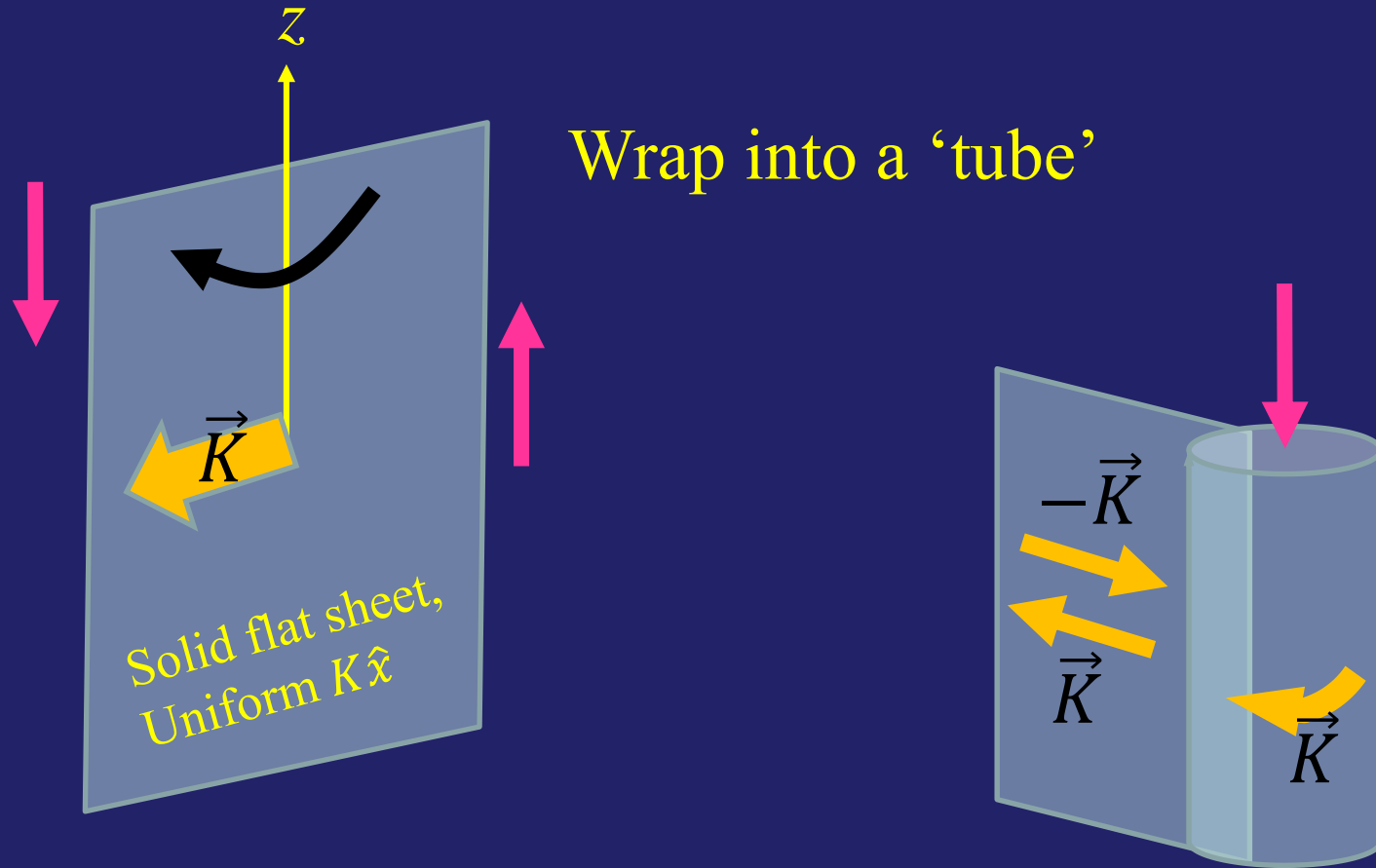
Each loop of this helical  
solenoid current has  
TWO components

(1) RADIAL current  
i.e. ideal solenoid  $\vec{B}$

(2) AXIAL current  
leads to a CIRCULAR  $\vec{B}$

The net superimposed  $\vec{B}$  is a distorted helical field.  
This matters in high precision magnetic applications!

# A better method of defining a solenoid magnetic field



Hence high precision magnets are made from flat 'ribbons'