PH 108 : Electricity and Magnetism

Tutorial Sheet 2 : **Coloumb's law, Gauss's law and Potential**

1. A semi-infinite slab of thickness t has a uniform charge density ρ distributed in its volume. Find the electric field intensity at a distance z from the median plane of the slab.

   [Ans. $E(z) = \rho z/\epsilon_0$ for $-t/2 < z < t/2$ and $E(z) = \rho t/2\epsilon_0$ for $-t/2 > z > t/2$]

2. A thin annular disc of inner radius a and outer radius b carries a uniform charge density σ. Determine the electric field intensity at a point on the z-axis (the axis of symmetry). Using this result determine the field due to an infinite sheet containing a charge density σ.

   [Ans. $\hat{k}\frac{\sigma z}{2\epsilon_0}\left\{\frac{1}{(a^2+z^2)^{1/2}} - \frac{1}{(b^2+z^2)^{1/2}}\right\}$]

3. A charge Q is uniformly distributed on a straight rod of length L. Find the potential at a distance d from the mid-point of the rod.

   [$(Q/4\epsilon_0 L)\ \ln\left\{\left(\sqrt{4d^2+L^2}+L\right)/\left(\sqrt{4d^2+L^2}-L\right)\right\}$]

4. Which one of the following is a possible expression for an electrostatic field? For the right expression, find a potential which determines this field with the origin as the reference.

   (a) $\vec{E} = A\left(xyz^2\,\hat{i} + 2xz\,\hat{j} - 3yz\,\hat{k}\right)$

   (b) $\vec{E} = A\left([3xz^2 + y^2]\,\hat{i} + 2xy\,\hat{j} + 3x^2z\,\hat{k}\right)$ (here A is a constant having appropriate dimensions).

   [Ans. (b) $V(x,y,z) = -(3x^2z^2/2) - xy^2 + c$]

5. A charge distribution produces an electric field

   $$\vec{E} = c\,(1 - \exp(-\alpha r)\,\frac{\hat{r}}{r^2}$$

   where c and α are constants. Find the net charge within a sphere of radius $r = 1/\alpha$.

   [Ans. $4\pi\epsilon_0 c\,\{1 - 1/\epsilon_0\}$]

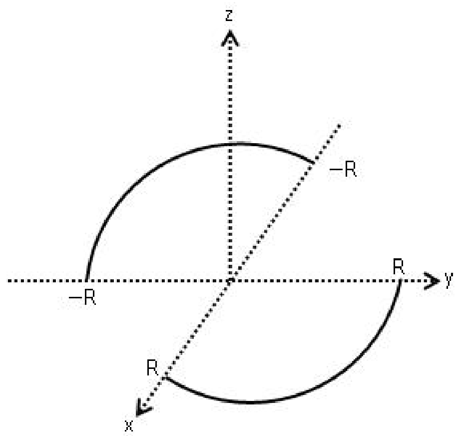
6. Show that the maximum value of the electric field $|E|$ for points on the axis of a uniform ring of radius R with total charge q occurs at $x = \pm R/\sqrt{2}$. If an electron is placed at the centre of the ring and then displaced by a small amount x ($x \ll R$) along the axis, show that it would execute simple harmonic oscillations. Determine the frequency of oscillations.

   [Ans. $\sqrt{eq/16mR^3\pi^3\epsilon_0}$]

7. A charged semicircular ring of radius R extending from $\theta = 0$ to $\theta = \pi$ lies in the x–y plane, centered at origin. If the charge distribution on the ring is $\lambda_0 \sin\theta$, compute the electric field intensity at P (0,0,z).

   [Ans. $\frac{\lambda_0 R}{8\pi\epsilon_0}\left\{\frac{1}{(R^2+z^2)^{3/2}}\left(-R\pi\,\hat{j} + z\,\hat{k}\right)\right\}$]

8. Two isolated surfaces in the shape of a quadrant of a circle of radius R lie in the x–y plane centered at the origin.The charge distribution on the surface in the first quadrant is $\sigma_0\cos\theta$ while that on the surface in the fourth quadrant is $-\sigma_0\cos\theta$. Obtain the field intensity at a point P along the z-axis (0,0,z).



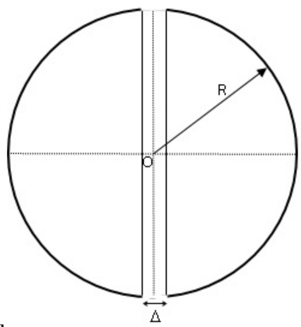
   [Ans. $\hat{\imath}\frac{\sigma_0}{8\epsilon_0}\left\{\frac{R}{z} - \ln\left[\sqrt{z^2+R^2}+R\right]/z\right\}$].

9. A hemisphere of radius R has $z = 0$ as its equatorial plane and lies entirely in the region $z \geq 0$. The hemisphere has a uniform charge density ρ. Determine the field at the centre.

   [Ans. $-\hat{k}\rho/4\epsilon_0$]

10. A sphere has a uniform volume charge density everywhere except inside an off-centre spherical cavity within. Show that the field inside the cavity is uniform.

11. A sphere of radius R has a uniform charge density ρ everywhere except in a very thin circular disk of thickness Δ, where $\Delta \ll R$, centered at the origin, which divides the sphere into two halves. Find the potential at the origin and at the point (0,R,0).



   [Ans. $V(0,0,0) = \rho R(R-t)/2\epsilon_0$; $V(0,R,0) = \rho R(R/3-t/\pi)/\epsilon_0$]

12. Two infinite sheets of planes intersect at right angles. The sheets carry charge densities $+\sigma$ and $-\sigma$. Find the magnitude and direction of electric field everywhere and sketch the electric field lines.

    [Ans. $|E| = \sigma/\sqrt{2}\epsilon_0$]

13. A electric dipole having moment $\vec{p} = p\,\hat{k}$ is placed at the origin of a coordinate system. Show that the electric field at a point P(r, θ) is given by

$$\vec{E}(r, \theta) = \frac{1}{4\pi\epsilon_0 r^3}\left\{2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right\}$$

which can be represented by the coordinate independent form by

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0 r^3}\left[3\left((\vec{p}\cdot\hat{r}\right)\hat{r} - \vec{p}\right]$$

Show that the magnitude of the field is given by

$$E = \frac{p}{4\pi\epsilon_0 r^3}\left(1 + 3\cos^2\theta\right)^{1/2}$$

14. A continuous charge distribution is spherically symmetric and has a volume charge density $\rho(r) = \rho_0\exp(-\alpha r)$. Find the potential $V(r)$ produced by this charge distribution.

    [Ans. $V(r) = \frac{\rho}{\epsilon_0}\frac{1}{r^3}\left\{-\alpha\exp(-\alpha r) - \frac{2}{r}\exp(-\alpha r) + \frac{2}{r}\right\}$]

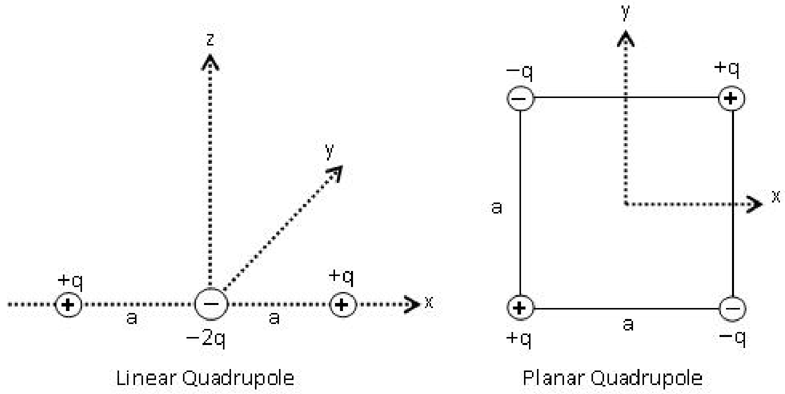
15. A spherical charge distribution has a volume charge density $\rho(r) = A/r$ for $0 \leq r \leq R$ and $\rho(r) = 0$ for $r > R$. Find the electric field $\vec{E}(r)$ and the potential $V(r)$ subject to $V(\infty) = 0$.

    [Ans. $E_{\text{out}} = \frac{AR^2}{2\epsilon_0 r^2}$, $E_{\text{in}} = \frac{A}{2\epsilon_0}$, $V_{\text{in}} = \frac{A}{2\epsilon_0}\{2R - r\}$; $V_{\text{out}} = \frac{A}{2\epsilon_0}\frac{R^2}{r}$]

16. Repeat the above problem for a charge distribution given by $\rho(r) = Ar$ for $0 \leq r \leq R$ and $\rho(r) = 0$ for $r > R$.

    [Ans. $V(r) = \frac{A}{6\epsilon_0}\{3R^2 - r^2\}$ $for\, r < R$; $V(r) = \frac{A}{3\epsilon_0}\frac{R^3}{r}$ $for\, r > R$]

17. A linear quadrupole is formed by placing a charge $+q$ each at ($\pm a$,0,0), and a charge $-2q$ at the origin. Find the potential and the electric field intensity at a point P(0,0,z), where $z \gg a$.

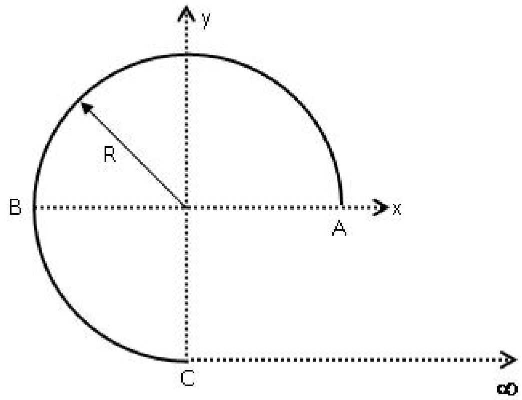


[Ans. $\vec{E} = \hat{k}\,\{3qa^2/4\pi\epsilon_0 z^4\}$]

18. A quadrupole can also be a configuration shown on right hand side of the above figure. Calculate the field and the potential due to such a quadrupole at a point P(0,0,z), where $z \gg a$.

19. A spherical surface of radius a has a uniform charge density σ on it. Calculate by direct integration the electric field at a distance $2a$ from its centre. [Ans. $\sigma/4\epsilon_0$]

20. Consider a line charge having the shape shown below. Portion ABC forms three-fourth of a circle of radius R while the straight portion CD is parallel to the x-axis and extends to infinity. Show that the electric field at the centre of the circular portion is zero.



21. One half of a spherical surface has a uniform charge density σ on it. Show that the magnitude of the field at the centre of the sphere is $\sigma/4\varepsilon_0$. What is its direction? [Ans. $-\hat{k}$

22. A point charge is located at the centre of a cylinder of length L and radius R. Show that the flux through the curved surface of the cylinder is

$$\frac{QL}{2\varepsilon_0}\frac{1}{\sqrt{R^2 + (\frac{L}{2})^2}}$$

23. A spherical distribution of charge consists of uniform charge density ρ_1 from $r = 0$ to $r = a/2$ and uniform charge density ρ_2 from $r = a/2$ to $r = a$. Using Gauss's law, calculate the electric field everywhere.

[Ans. $E(\frac{a}{2}) = \frac{\rho_1 a}{6\varepsilon_0}$, $E(r) = \frac{(\rho_1-\rho_2)a^3}{24\varepsilon_0 r^2} + \frac{\rho_2 a}{3\varepsilon_0}$ for $\frac{a}{2} < r < a$, $E(a) = \frac{(\rho_1+7\rho_2)a}{24\varepsilon_0}$]

24. A circle of radius a has a uniform chare density λ on its circumference. Determine the electric field and potential along its axis.

    [Ans. $E_{\max}$ at $z = \pm a/\sqrt{2}$; $V_{\max}$ at z = 0]

25. A circular sheet of radius a has a uniform charge density σ on it. Calculate the potential at a point on the circumference and at the centre.

    [Ans. circumference: $\sigma a/\pi\epsilon_0$; centre: $\sigma a/2\epsilon_0$]

26. Calculate the potential everywhere for the charge distribution in problem (25). What should be the relation between ρ_1 and ρ_2 so that the potentials at $r = a$ and $r = 0$ are equal.

    [Ans. $V(r = a) = a^2(\rho_1 + 7\rho_2)/24\epsilon_0$; $V(r = a/2) = a^2(2\rho_1 + 9\rho_2)/24\epsilon_0$; $V(r = 0) = a^2(\rho_1 + 3\rho_2)/8\epsilon_0$; $\rho_1 + \rho_2 = 0$]

27. A charge Q is uniformly distributed in a spherical volume of radius R. Find the potential inside the sphere.

    [Ans. $Q\left(3R^2 - r^2\right)/8\pi\epsilon_0 R^3$

28. An infinitely long cylinder of radius R with its axis along the z-axis has a volume charge density given by

    $$\rho(r, \theta, z) = \rho_0(R - r)$$

    for $r < R$ and

    $$\rho(r, \theta, z) = 0$$

    for $r > R$. Calculate (i) electric field for $r < a$ and $r > a$ and (ii) the potential difference between $r = a$ and $r = 0$, and between $r = 2a$ and $r = a$.

    [Ans. $\frac{\rho_0}{\varepsilon_0} r\left(\frac{R}{2} - \frac{r}{3}\right)$, $\frac{\rho_0}{\varepsilon_0}\frac{R^2}{6}$], [Ans. $\frac{\rho_0}{\varepsilon_0} r^2\left(\frac{R}{4} - \frac{r}{9}\right)$, $\frac{\rho_0}{\varepsilon_0}\frac{R^3}{6}\ln 2$]

29. A spherical volume of radius $4R$ centred at the origin (O) has constant volume charge density $+\rho$. Another spherical volume of radius $3R$ centred at O' ($5R$,0,0) has constant volume charge density $-\rho$. Calculate the electric field at any point in the overlap region.

    [Ans. $5\rho R/3\epsilon_0\,\vec{OO'}$]