PH108

Lecture 05

Electrostatic field and potential

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Supplementary reading:
Div, Grad, Curl pages 82-86 for polar and cylindrical coordinates

Tutorial 1 was… ?

A

B

C

Given $\rho(\vec{r})$, Determine $\vec{E}(\vec{r})$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

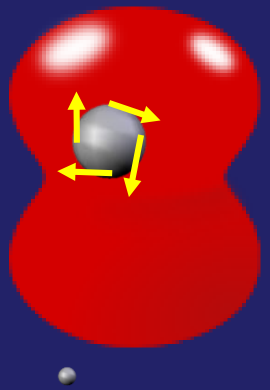
is ONE differential equation in THREE unknowns E_x E_y E_z

Need TWO more equations to solve for $\vec{E}$

Curl : $\vec{\nabla} \times \vec{E}$

Our ‘physical’, ‘visual’ definition:

$$\vec{\nabla} \times \vec{E} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint \vec{E} \cdot \overrightarrow{dl}$$



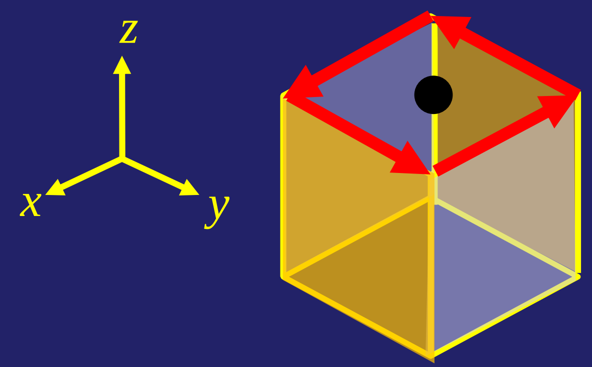
Circulation around a boundary,
as the surface enclosed by the boundary shrinks to zero

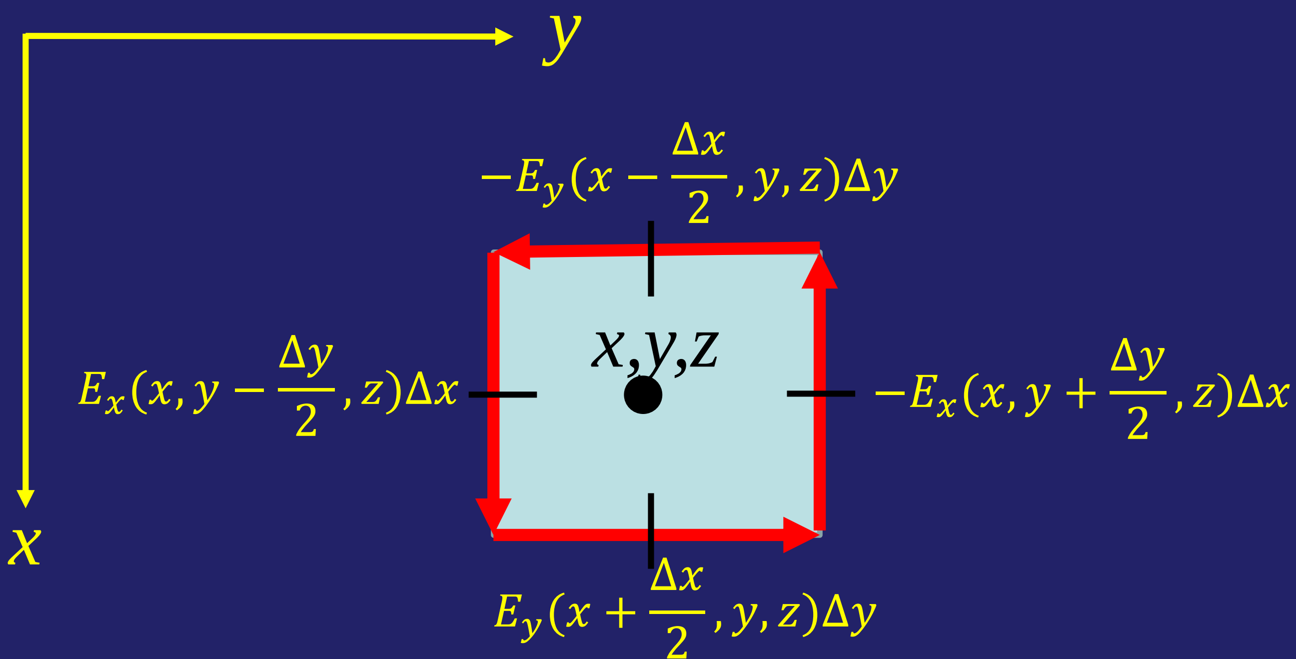
How does this turn into the operational expression: ?

$$\vec{\nabla} \times \vec{E} = \hat{\imath}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

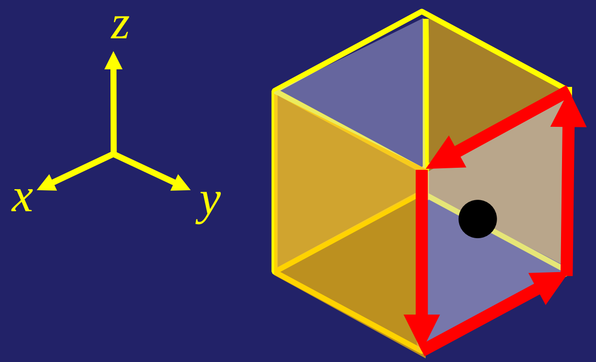
Calculate how $\vec{E}$ 'circulates' around a boundary

$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \ \hat{k}\ \Delta s_{xy}$$



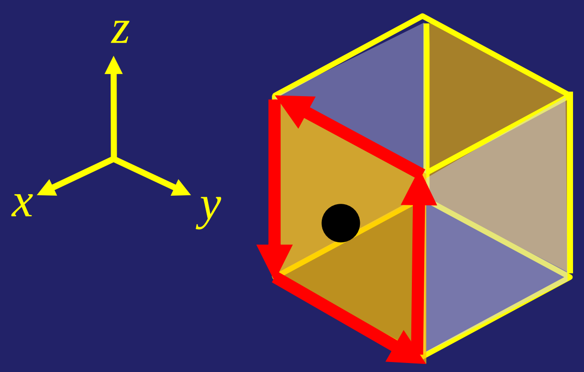


Calculate how $\vec{E}$ 'circulates' around a boundary



$$\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\ \hat{j}\ \Delta s_{xz}$$

Calculate how $\vec{E}$ ‘circulates’ around a boundary

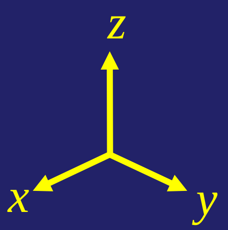


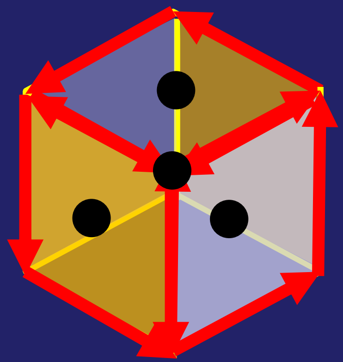
$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \ \hat{\imath}\ \Delta s_{yz}$$

Calculate how $\vec{E}$ 'circulates' around a boundary

Our definition of Curl :

$$\vec{\nabla} \times \vec{E} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint \vec{E} \cdot \overrightarrow{dl}$$



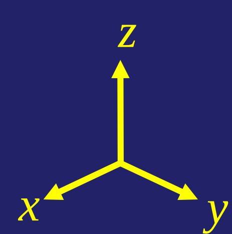


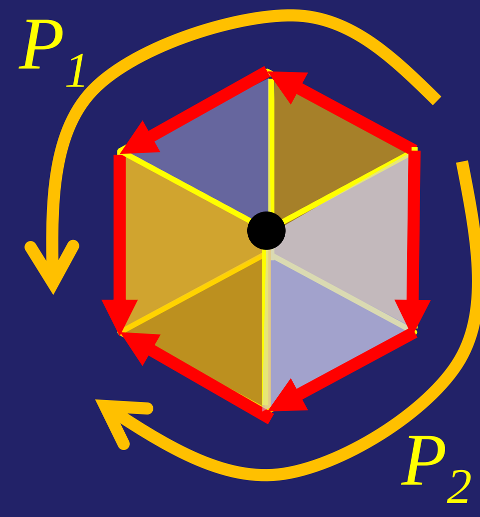
$$\vec{\nabla} \times \vec{E} = \hat{\imath}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

Shape of the surface does not matter,
Since we are taking $\Delta S \to 0$

Circulation of $\vec{E}$ is *independent* of path

Consider a constant $\vec{E} = C_x \hat{\imath}$





$$\int_{P_1} \vec{E} \cdot \overrightarrow{dl} = C_x$$

$$\int_{P_2} \vec{E} \cdot \overrightarrow{dl} = C_x$$

In general: $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

Evaluate this as the sum over many ΔP_i's
take E constant over each ΔP_i

$$\int_{r_a}^{r_b} \overbrace{\vec{E} \cdot \overrightarrow{dl}} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Circulation of $\vec{E}$ on a closed path is *ZERO*

$$\boxed{\oint \vec{E} \cdot \overrightarrow{dl} = 0}$$

From our *definition* of Curl:

$$\vec{\nabla} \times \vec{E} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint \vec{E} \cdot \overrightarrow{dl}$$

Stoke's theorem:

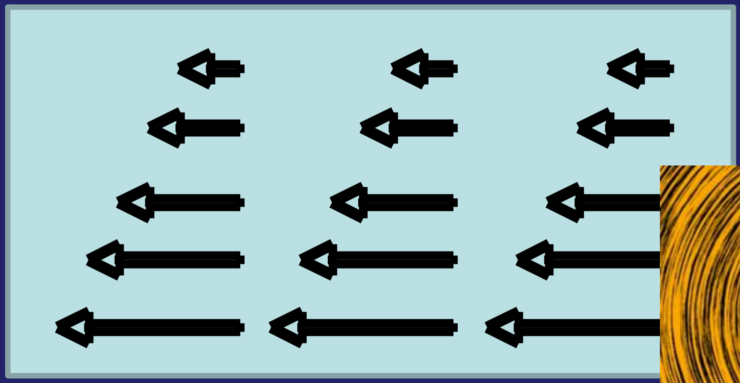
$$\oiint (\vec{\nabla} \times \vec{E}) \cdot \overrightarrow{d\sigma} = \oint \vec{E} \cdot \overrightarrow{dl}$$

$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$

Question

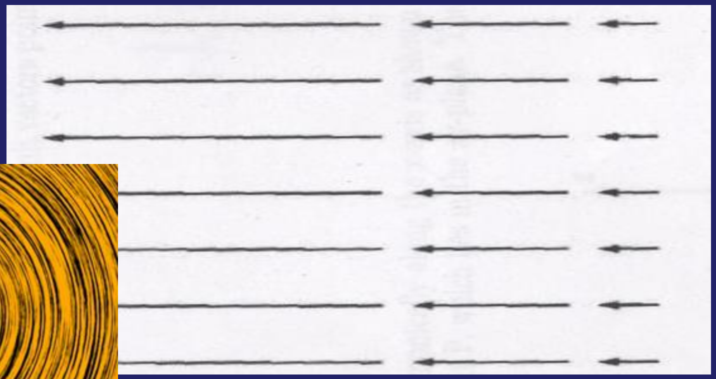
Which of the following *could* be a <u>static</u> E field in a small region?

I



?

II



A) Only I

B) Only II

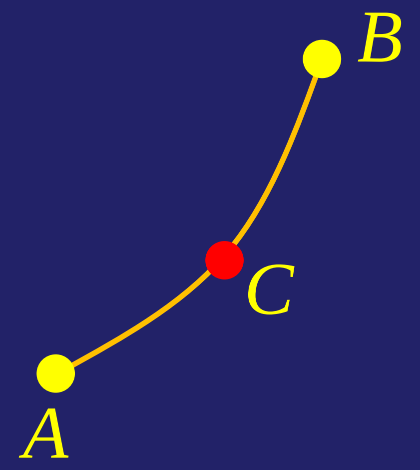
C) Both

D) Neither

E) Cannot answer without further info

Electric Potential (*not* potential energy!)

Path independence of $\int \vec{E} \cdot \overrightarrow{dl}$ implies:



$$\Phi(A, B) = \int_A^C \vec{E} \cdot \overrightarrow{dl} + \int_C^B \vec{E} \cdot \overrightarrow{dl}$$

$$\Phi(A, B) = \Phi(A, C) + \Phi(C, B)$$

$$\Phi(A, B) = \Phi(A) - \Phi(B)$$

<u>Define</u> electric potential at a point A: $\Phi(A) = -\int_{Ref}^{A} \vec{E} \cdot \overrightarrow{dl}$

Question

Why is $\oint \vec{E} \cdot \overrightarrow{dl} = 0$ in electrostatics?

A) Because $\vec{\nabla} \times \vec{E} = 0$

B) Because potential between two points is independent of the path between the points

C) Both the above

D) NONE of the above: its not true!

Summary so far

IMPORTANT: we are working with static charges

Given static $\rho(\vec{r})$, Determine $\vec{E}(\vec{r})$

Tools we have so far:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\oint_{any\ path} \vec{E} \cdot \overrightarrow{dl} = 0$$

Question

Could the following electrostatic field possibly exist in a finite region of space that contains no charges?

(A, and c are constants with appropriate units)

$$\vec{E} = A\left(\frac{z^2}{2}\hat{\imath} - cy\hat{\jmath} + xz\hat{k}\right)$$

A) Sure, why not?

B) No way

C) Not enough information to decide

Question

A cubical non-conducting *shell* has a ***uniform*** positive charge density on its surface. (There are no other charges around)

What is the field inside the box?

A: **E**=0 everywhere inside

B: **E**≠0 everywhere inside

C: **E**=0 only at the very center, but non-zero elsewhere inside.

D: Not enough info given