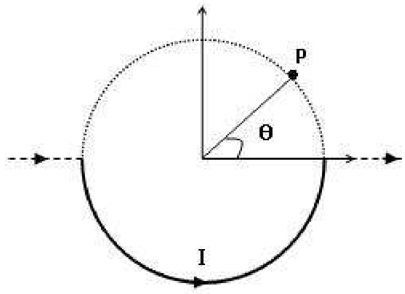
PH 103 : Electricity and Magnetism

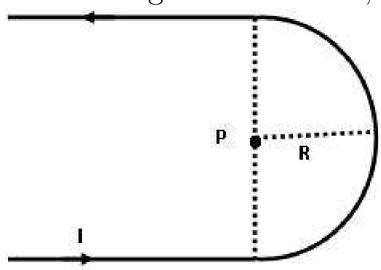
Tutorial Sheet 7 : **Magnetostatics**

1. A surface current density in the x-y plane is given by $\vec{K} = K_0(\hat{\imath} + \hat{\jmath})$. Calculate the rate of flow of charges across a straight line joining the points A(2,0) with B(2,2) in the plane.

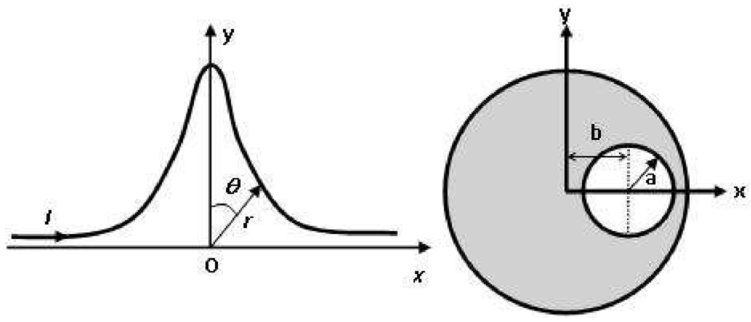
2. A semi-circular wire carries a steady current I as shown in the figure. Find the magnetic field at a point P on the other semi-circle due to the current I.



3. A long wire is bent into a hairpin-like shape as shown in the figure. Find an expression for the magnetic field at P, which lies at the centre of the semi-circular part of radius R.



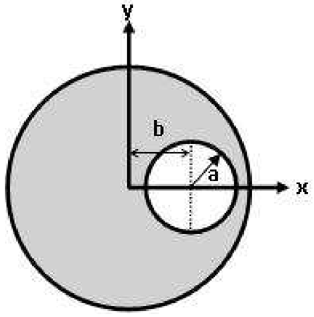
4. A current I flows through an infinitely long wire having the shape of a curve given by the equation, $r = a/\left\{\cos\theta\left(1 + \sin^2\theta\right)\right\}$, as shown in the figure. Calculate the magnetic field at the origin O. a is a constant where as r and θ are plane polar coordinates with θ defined with respect to the y-axis.



5. A spherical shell of radius R has uniform charge density σ on it. It is rotating with angular speed ω about an axis though its centre. Find the magnetic field at the centre.

6. A long cylindrical conductor of radius a has a current with current density given by $\vec{J} = \hat{k}(J_0 r/a)$ where r is the radial distance from the axis of the conductor. Calculate the magnetic field, both inside and outside the conductor.

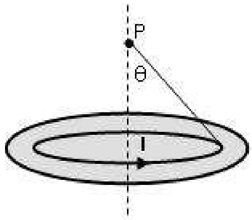
7. An infinitely long cylindrical conductor of radius R, carries a current I_0 along the z direction. An infinitely long cylindrical hole is drilled out of the conductor, parallel to the z axis, as shown in the figure. The center of the hole is on the x-axis at $x = b$, and the radius of the hole is a. Determine the magnetic field at a point inside the hole.



8. Fine insulated copper wire is wound in the form of a closed spiral, to form a circular disc of radius a, the ends of the spiral being at the centre and on the circumference. When a current I passes through the wire, show that the magnetic field at a point on the axis of the disc given by

$$B = \frac{\mu_0 n I}{2}\left\{\cosh^{-1}(\sec\theta - \sin\theta)\right\}$$

where n is the number of turns per unit length.



9. A circular loop of wire carries a current I_1. A long straight wire in the plane of the loop carries a current I_2. The loop subtends an angle 2θ at a point on the wire which is nearest to it. Show that the force between the wire and the loop has a magnitude $\mu_0 I_1 I_2 (\sec\theta - 1)$.

10. An infinite slab of thickness $2d$ is arranged in kept in the x-y plane such that $z = 0$ represents the bottom of the slab and $z = 2d$ represents the top of the slab. Find the magnetic field both inside and outside the slab.

11. Determine the vector potential for an infinite surface current sheet with a current density $\vec{K} = K_0\hat{\imath}$. Is your solution unique?

12. A long cylindrical conductor having radius R carries a current J. Find the vector potential of this current distribution at all points in space $r > R$ and $r < R$.

13. Show that $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}$ represents the vector potential of a uniform magnetic field $\vec{B}$. Also find the gauge of this potential.