PH 103 : Electricity and Magnetism

Tutorial Sheet 3 : **Multipoles and conductors**

Multipole expansion

1. An electric dipole $\vec{p}$ is fixed at the origin, making an angle β with the z-axis. Calculate the electric flux through the hemispherical surface of radius R resting on the x-O-y plane and centred at the origin.

   [Ans.: $p\cos\beta/2\epsilon_0 R$]

2. A dipole $\vec{p} = p\,\hat{k}$ is fixed at the origin. Calculate the work done in moving a point charge +Q from the point A$(r, \theta = \pi/2)$ to the point B$(r, \theta = 0)$ [r and θ are polar coordinates] in two different ways: (a) from the potential at the two points, and (b) by calculating the line integral of E from A to B.

   [Ans.: $Qp/4\pi\epsilon_0 r^2$]

3. A circular area of radius a lies in the x-y plane and is centered at the origin. It has a charge density $\sigma(r,\theta) = \sigma_0 r\cos\theta$. Determine all the components of the dipole moment of this charge distribution and show that $\vec{p} = \hat{\imath}\,\sigma_0\pi a^4/4$.

4. Two pure dipoles $\vec{p}_1$ and $\vec{p}_2$ are oriented in space such that $\vec{p}_1$ has position vector $\vec{r}_1$ with respect to $\vec{p}_2$. The angle between $\vec{p}_1$ and $\vec{r}_1$ and $\vec{p}_2$ and $\vec{r}_2$ are θ' and θ, respectively, while the angle between the two dipoles is ϕ. Show that the mutual interaction energy between the two dipoles is $U = p_1 p_2\left(\cos\phi - 3\cos\theta\cos\theta'\right)/4\pi\epsilon_0 r^3$.

5. A sphere of radius a centered at the origin carries a volume charge density $\rho(r,\theta) = \rho_0 a\,(a-2r)\sin\theta/r^2$ where ρ_0 is a constant, and r, θ are the usual spherical polar coordinates. Find the approximate potential (up to dipole term) at large distances for points on the z-axis. [Ans. zero]

6. Four point charges $-2q$, $3q$, $-2q$ and q are placed at the points (0,a,0), (0,0,a), (0,$-a$,0) and (0,0,$-a$), respectively. Using spherical polar coordinates, calculate the approximate potential (up to dipole term), for large distances from the origin. [Ans. : zero]

7. A thin ring of radius a carrying a constant linear charge density λ is kept in the x-O-y plane with centre as origin. Using spherical coordinates, calculate the first three terms in the multipole expansion for the potential ($r \gg a$, $\theta = 0$) of the ring. Why is the term proportional to $1/r^2$ missing in the expansion. [Ans. : $V \simeq \frac{a\lambda}{2\epsilon_0 r}\left(1 - \frac{a^2}{2r^2}\right)$]

CONDUCTORS

1. Three identical parallel metallic plates are kept with separation a and b between the first and second and the second and third plates, respectively. A charge +Q is placed on the central plate while the outer two plates are uncharged and connected together by a very thin conducting wire. Determine the charges on all the six surfaces. Assume edge effects are negligible.

2. Consider a conducting sphere A which is initially uncharged. Another conducting sphere B is given a charge $+Q$, brought into contact with A and then moved far away. The charge on B is then increased to its original value $+Q$ and again brought into contact with A. Show that if this process is repeated many times, the charge on A will tend to the limit $Qq/(Q-q)$, where q is the charge acquired by A after its first contact with B.

3. Three parallel plates A, B and C each of area $10\,\text{cm}^2$, are maintained at potentials 50, 100 and 10 V, respectively The separation between A and B is 1 mm while that between B and C is 3 mm. Calculate the force required to prevent B from moving. [Ans. : 7.08×10^6 N]

4. A grounded conducting block of arbitrary shape has a spherical cavity of radius a within it. A point charge $+q$ is fixed at a distance x from the center C of the cavity where $x < a$. What is the potential at a point P on the surface of the cavity? What is the potential at C? [Ans: $V(P) = 0$; $V(C) = kq(\frac{1}{x} - \frac{1}{a})$].

5. Two thin co-axial conducting cylindrical shells of length L and radii $\sqrt{2}a$ and $2a$, respectively are kept on the x-y plane with the z-axis being the common axis. The inner cylinder is given a charge $+Q$ and the inner surface of the outer cylinder has a charge -Q. Calculate the electric flux through an infinite plane surface at $x = a$ (i.e. parallel to the y-z plane). The plane extends from $z = 0$ to $z = L$ and $y = -\infty$ to $+\infty$. Neglect edge effects. Assume zero charge on the outer surface of the outer cylinder. [Ans.: $Q/12\epsilon_0$]

6. Three conducting spherical shells (A, B and C) with a common centre have radii a, b and c ($a < b < c$), respectively. B has a charge $+Q$ on it while A and C are grounded. Calculate (a) the charges on shells A and C (b) the potential on B and (c) the electric field in the regions (i) $a < r < b$ and (ii) $b < r < c$.