PH108

Lecture 17: Magnetostatics – 1

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What is the difference between electricity and magnetism?

Magnetic charges are NOT isolated.

Every magnet has a north pole and a south pole

Electric charges can be isolated

Potential due to multiple charges of opposite sign placed close together can be expanded in polynomial powers of $\frac{1}{r^n}$: dipole, quadrupole, octupole …. multipole

Question

Magnetic charges are NOT isolated.

Every magnet has a north pole and a south pole

A rectangular bar magnet is broken in two pieces

The two broken pieces have poles that look like: (pick one)

(A)

(B)

(C)

(D) None of the above

What is the difference between electricity and magnetism?

From a practical point of view:

<u>Static</u> electric charges create an electrostatic field

<u>Moving</u> electric charge (current) creates a magnetic field

Recall the basics of electrostatics

Want to calculate the effect of one set of charges on another set of charges

Split problem in two pieces

1) Calculate the field due to a set of charges, without worrying about *other* charges nearby

2) Calculate the effect of a field on a set of charges, without worrying about what charges produced the field

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}^2}\hat{\mathscr{r}}$$

NOTE: *q* is the source charge
(generalized to λ σ ρ distributions)
Source charge is STATIC

Basics of Magnetism in are similar

Split problem in two pieces

1) Calculate the magnetic field due to a set of moving charges, without worrying about *other* charges nearby

2) Calculate the effect of a magnetic field on a set of charges, (Forget about the source)

Magnetic field of a moving *point* charge:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

1. $\vec{v}$ must be $\neq 0$ for $\vec{B} \neq 0$
2. $\vec{B}$ is always perpendicular to $\vec{v}$

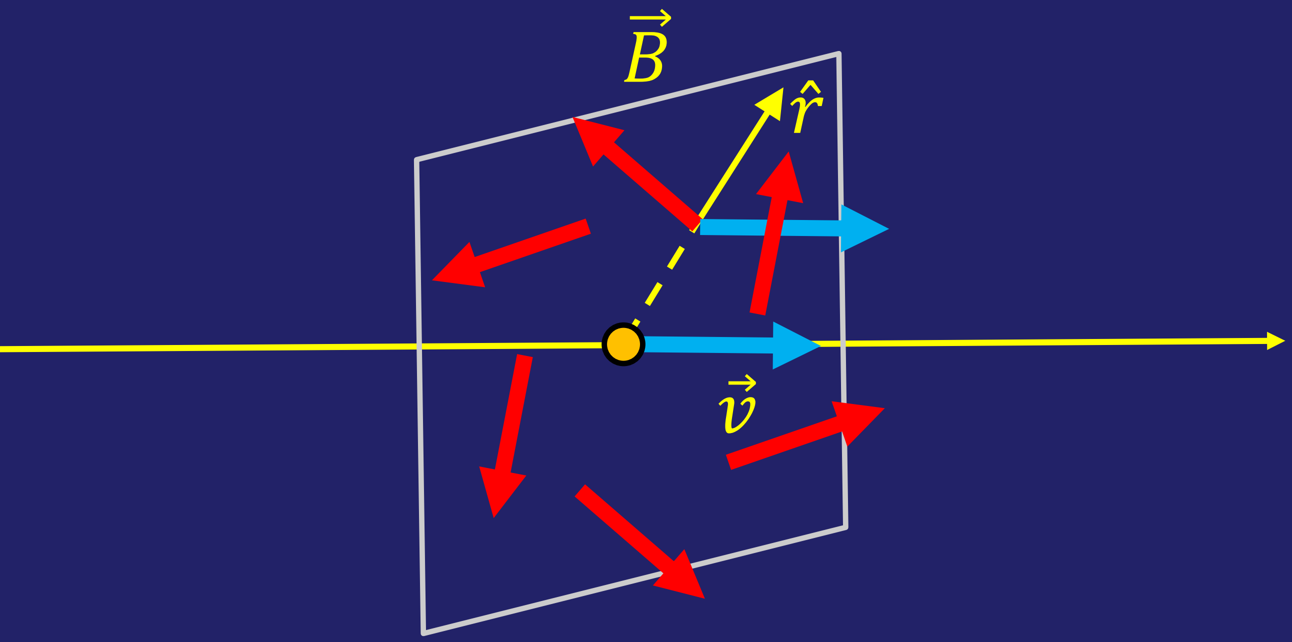
"Biot Savart Law for a point charge"

What does $\vec{B}$ of moving point charge look like?

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

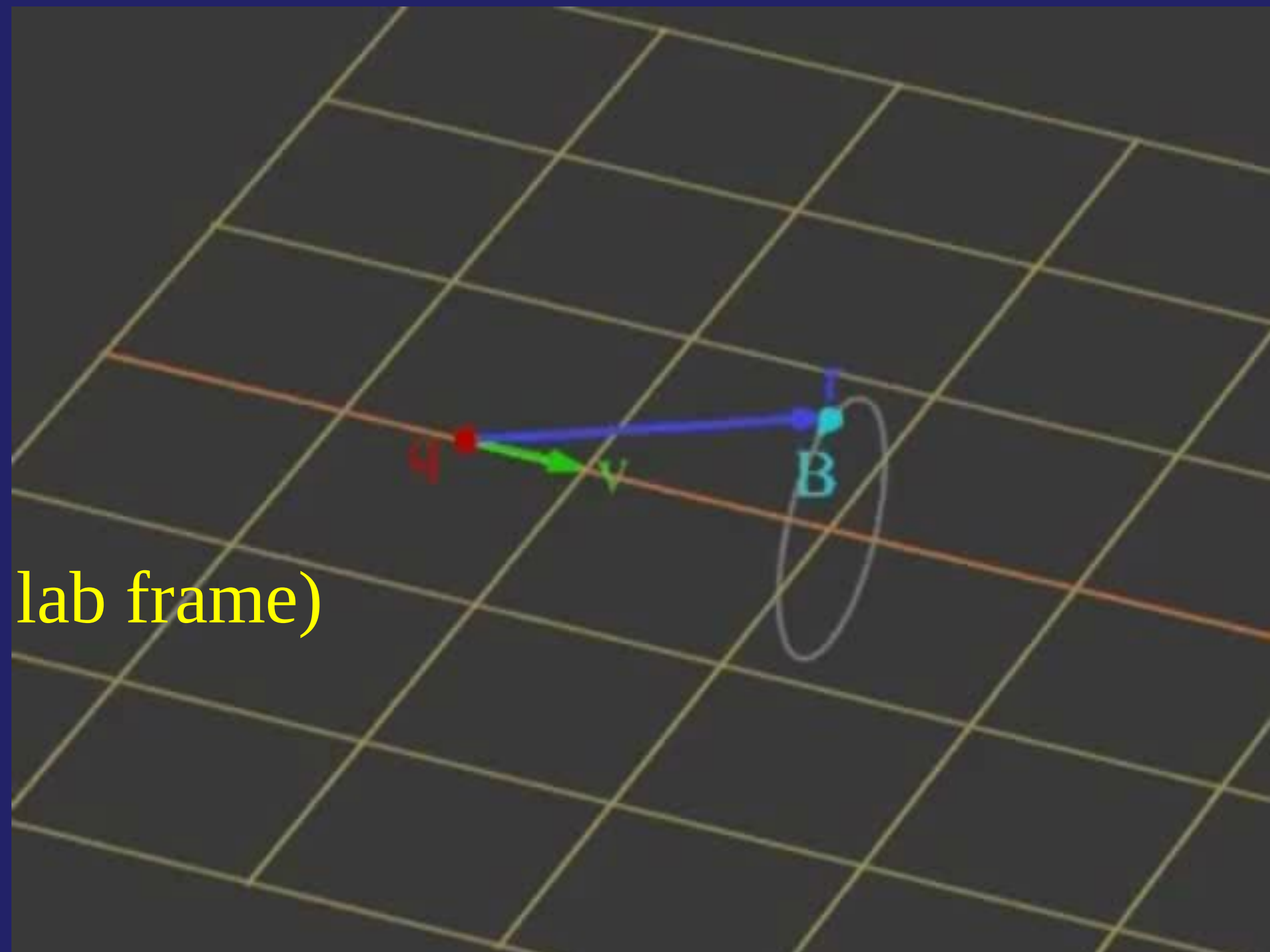
Note: $q = q_{source}$

$\vec{v} = \vec{v}_{source}$



Visualization of $\vec{B}$ created by a point charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\,\vec{v}\times\hat{r}}{r^2}$$



Things to note:

Our co-ordinate system is fixed (background grid = lab frame)

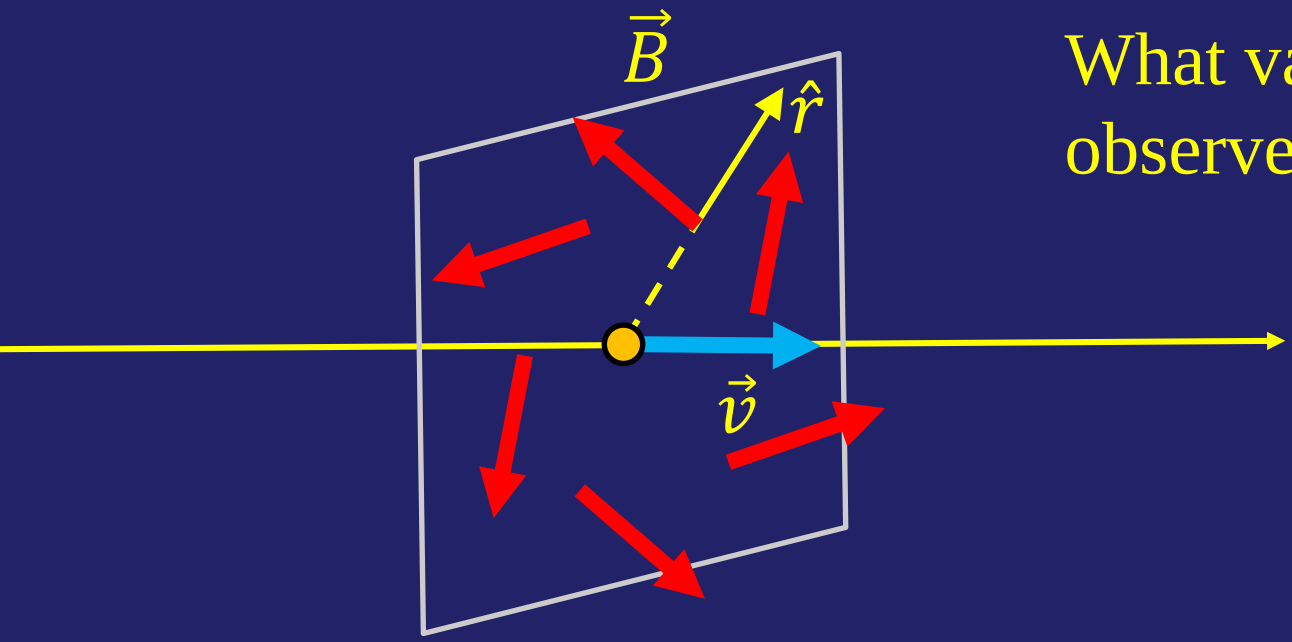
$\vec{B} = \vec{B}(\vec{r}, t)$

changes with time

www.youtube.com/watch?v=eWaA9RiLsno

$\vec{B}$ **depends** *on the observer frame of reference*

Consider an observer *moving* to the right at velocity $\vec{v}$

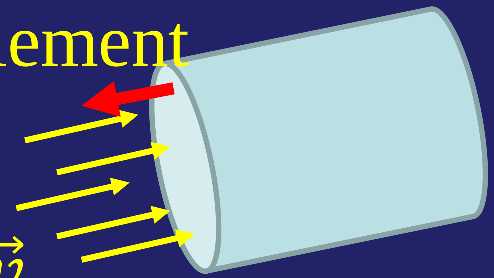


What value of $\vec{B}$ does the observer measure?

$$\vec{v} = 0 \;\therefore\; \vec{B} = 0$$

Current = collection of moving charge

$\overrightarrow{dS}$ surface element



Volume density of
charge ρ, velocity $\vec{v}$

Current density: [$Cm^{-2}s^{-1}$]

$$Current\ I = \oint (\rho \vec{v}) \cdot \overrightarrow{dS} = \oint \vec{J} \cdot \overrightarrow{dS} = -\frac{dQ}{dt} = -\frac{d}{dt}\int \rho dV$$

Divergence theorem

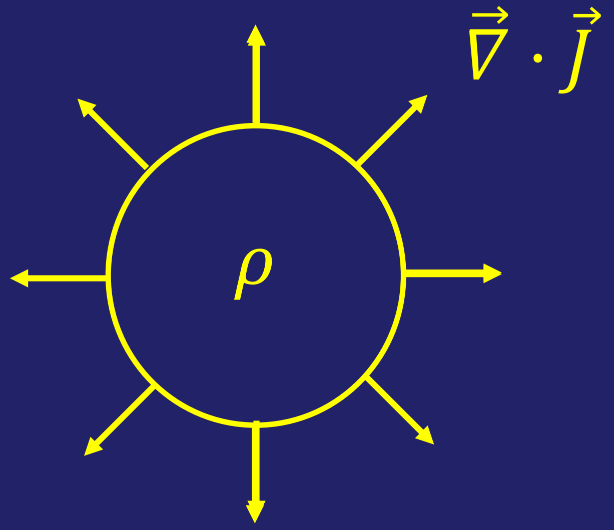
$$\int \frac{\partial \rho}{\partial t}\, dV = -\int \vec{\nabla} \cdot \vec{J}\, dV$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

Valid for any V, so…

Equation of continuity explicitly defines conservation of charge

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$



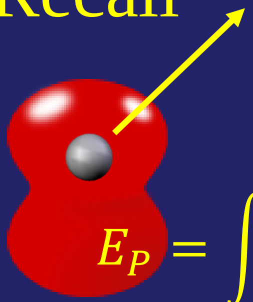
For the case $\frac{\partial \rho}{\partial t} = 0$

We get: $\vec{\nabla} \cdot \vec{J} = 0$ "Magnetostatics"

Note: this means $\vec{J} \neq 0$ Else no magnetic field!

Generalize from moving point charge to I

Recall



$$E_P = \int \frac{\rho dq}{\mathscr{r}^2}$$

Point charge moving $\vec{v}$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

Current line element $\overrightarrow{dl}$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,\overrightarrow{dl} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

We can calculate $\vec{B}$ as $\int I\, dl$ ...

Current surface element ds

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} ds$$

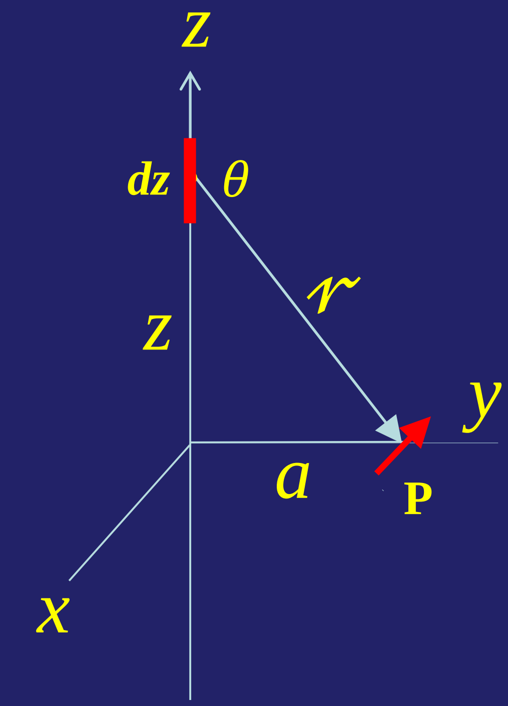
Current volume element $d\tau$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau$$

Example: $\vec{B}$ due to current line along $\hat{k}$

$$\hat{dl} = dz\,\hat{k}$$

$$\vec{\mathscr{r}} = a\,\hat{j} - z\,\hat{k} \qquad \hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{|\mathscr{r}|} = \frac{\mathrm{a}\,\hat{j} - z\,\hat{k}}{(a^2+z^2)^{\frac{1}{2}}}$$



$$\hat{dl} \times \hat{\mathscr{r}} = \frac{-adz}{(a^2+z^2)^{\frac{1}{2}}}\,\hat{\imath}$$

I

$$\mathscr{r}^2 = a^2 + z^2$$

$$\vec{B}_P = \frac{\mu_0}{4\pi}\, I \int_{-\infty}^{+\infty} \frac{\vec{dl} \times \hat{\mathscr{r}}}{\mathscr{r}^2} = -\frac{\mu_0 I}{2\pi a}\,\hat{\imath} \quad = \frac{\mu_0 I}{2\pi a}\,\hat{\phi}$$

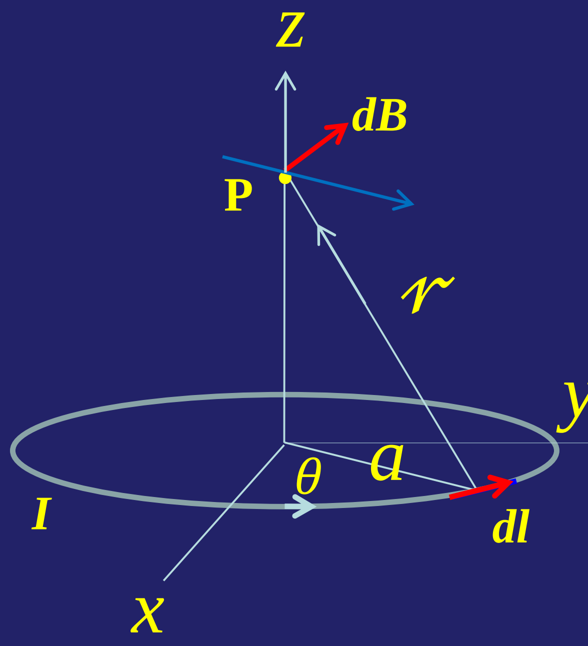
Right hand rule

Example: $\vec{B}$ due to a current loop along $\hat{k}$

$$\vec{\mathscr{r}} = z\hat{k} - (a\cos\theta\ \hat{\imath} + a\sin\theta\ \hat{\jmath}$$

$$\widehat{dl} = -a\sin\theta d\theta\ \hat{\imath} + a\cos\theta d\theta\ \hat{\jmath}$$

$$\mathscr{r}^2 = a^2 + z^2$$



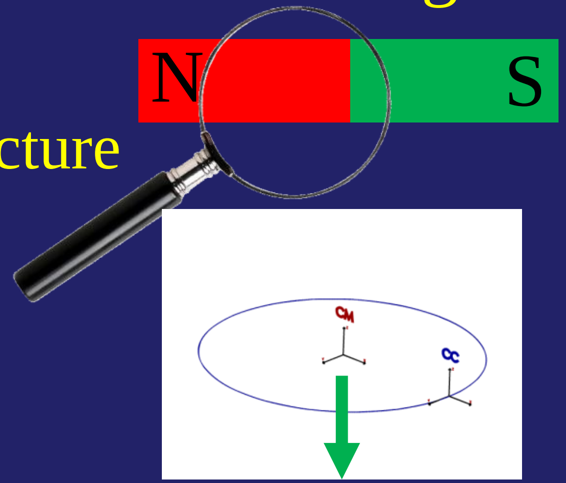
$$\widehat{dl} \times \vec{\mathscr{r}} = a^2\ \hat{k} + za\cos\theta d\theta\ \hat{\imath} + za\sin\theta d\theta\ \hat{\jmath}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\ I \int_0^{2\pi} \frac{\overrightarrow{dl} \times \hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0}{2} \frac{Ia^2}{(a^2 + z^2)^{\frac{3}{2}}}\ \hat{k}$$

How to resolve the bar magnet issue?

This picture of a bar magnet is incorrect!

Look at the atomic structure

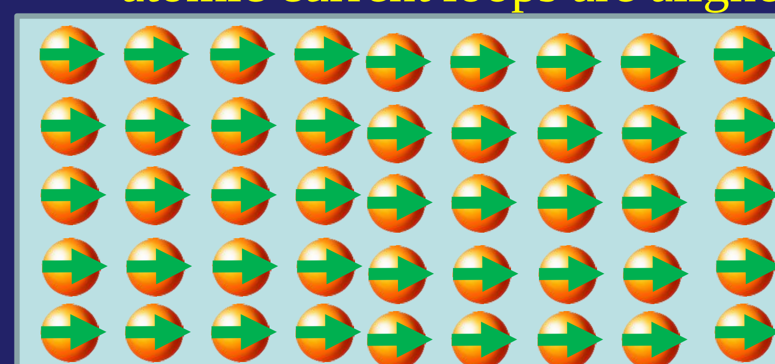


Atom: electron (current) in loop $\rightarrow \vec{B}_{atom}$

Non-magnetic: atomic current loops are randomly aligned

Ferromagnet:

atomic current loops are aligned



Note: Quantum Mechanics effect mainly driven by strong inter-atom electric field

$\vec{B}(r,t)$ is a vector field

Any vector field is defined by its divergence and curl

Given $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

What is $(\vec{\nabla} \cdot \vec{B})$ and what is $(\vec{\nabla} \times \vec{B})$?

Recall:

starting from $F_{electric} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}\,\hat{r}$ we got $\vec{\nabla} \cdot \vec{E}$ & $\vec{\nabla} \times \vec{E}$