PH 103 : Electricity and Magnetism

Tutorial Sheet 4 : **Conductors and Electrostatic energy**

1. A charge of 3×10^{-10} C is placed on a conducting sphere of radius 1 cm. Another uncharged conducting sphere of radius 2 cm, which is far away from the charged sphere, is now connected by a fine conducting wire to the charged sphere. Calculate

   (a) the charge

   (b) the charge density

   (c) the potential of each sphere after they are connected together.

   [Ans: (a) $q_2 = 2q_1 = 0.2$ nC, (b) 80 & 40 nC/m^2, (c) 90 V]

2. A grounded conducting sphere of radius R is centred at the origin. A linear charge distribution $\lambda(x) = \lambda_0(x^2/a^2)$ is distributed along the x-axis from $x = 2a$ to $x = 3a$. Find the total image charge.

   [Ans.: $-(5/2)\lambda_0 a$]

3. A point charge $+q$ is placed at a distance d from the centre of a conducting sphere of radius R ($d > R$). Show that, if the sphere is grounded, the ratio of the charge on the part of the sphere visible from $+q$ to that on the rest is $\sqrt{(d+R)/(d-R)}$.

4. A point charge $+8$ nC is placed at a distance of 10 cm from the centre of an uncharged conducting sphere of radius 5 cm. Calculate the charge that will flow to earth if the sphere is now grounded. After the charge has flowed to earth, the connection to earth is broken and the 8 nC charge is moved to a distance of 20 cm from the centre of the sphere. What will be the potential of the sphere? [Ans.: 4 nC, -360 V]

5. Two infinite conducting planes (both grounded and perpendicular to the x-y plane) meet at an angle of 60°. A point charge $+q$ in the x-y plane has plane polar coordinates $(a, 20°)$. Find all the image charges and their positions in polar coordinates. [Ans.:$-q(a, -20°), -q(a, 100°), +q(a, -100°), -q(a, -140°), +q(a, 140°)$]

6. A circular ring of radius b has a uniform linear charge density λ and is placed on the x-y plane with the centre at the origin. Calculate the potential at point (0,0,z). A conducting sphere of radius a with $b = 2a$ is placed at the centre of the ring, with their centres coincident. Calculate the percentage change in the potential at the point (0,0,2a) when the conducting sphere is grounded. [Ans.: $(2\sqrt{2}/\sqrt{17} \times 100$]

7. A grounded conductor has a spherical cavity of radius R inside it. A point charge $+q$ is placed at a distance r ($r < R$) from the centre of the cavity. Find the electrostatic force experienced by $+q$ and show that for $r \ll R$, this force is proportional to r. Find expressions for the charge density on the surface of the cavity at the two points on the diameter passing through $+q$. [Ans.: for $r \ll R$, $F \cong -q^2 r/4\pi\varepsilon_0 R^3$]

8. A spherical soap bubble carries a charge Q. Its surface tension is T. Determine its equilibrium radius. [Ans. : $8T\epsilon_0/\sigma^2$]

9. If the radius and the surface tension of a spherical soap bubble are r and T, respectively, determine the charge that must be given to the bubble so that the equilibrium radius doubles. Assume the air inside the bubble to behave ideally and the process of expansion isothermal. [Ans. $8\pi(\epsilon_0 r)^{3/2}(7Pr + 12T)^{1/2}$]

10. Find the force of repulsion between the two hemispheres of a conducting sphere with charge Q. [Ans. : $Q^2/32\pi\epsilon_0 R^2$, R - radius]

11. A sphere of radius R carries a total charge Q uniformly distributed over its volume. What is the force of repulsion between the northern and the southern hemispheres? [Ans. : $3Q^2/64\pi\epsilon_0 R^2$]

12. By assembling charges from the first principles, show that the potential energy of a charge Q which is uniformly distributed in a sphere of radius R is $3Q^2/20\pi\epsilon_0 R$.

13. A sphere of radius R carries a charge density $\rho(r) = kr$, where k is a constant. Calculate the electrostatic energy using three different methods. [Ans. : $\pi k^2 R^7/7\epsilon_0$]