PH108

Lecture 08:
Electrostatic energy and Conductors

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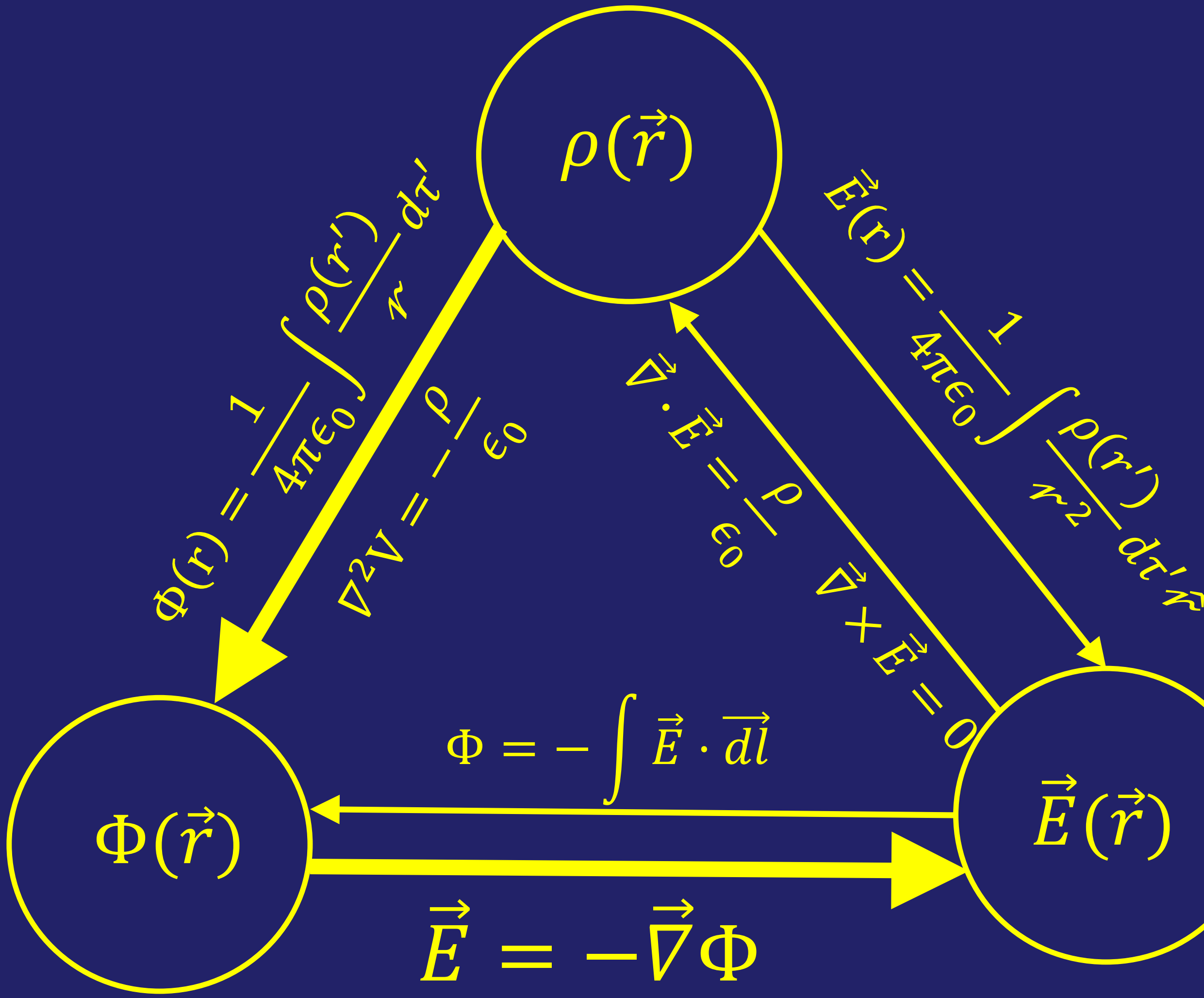
Reading: Griffiths 2.3.5 – 2.5.2

Our main focus so far: Have ρ, find V,E



Note: I use Φ to denote potential,
to avoid confusing with V $\equiv$ *voltage (an EE term)*

Electric potential *leads* to potential energy

Electric potential $\Phi(\vec{r})$ is a *scalar* field

$\vec{E} = -\vec{\nabla}\Phi$ is the electric *vector* field

$\vec{F} = q\vec{E}$ is the *vector* force due to $\vec{E}$ on q

Moving q against* a force requires *work*

Work requires *Energy*

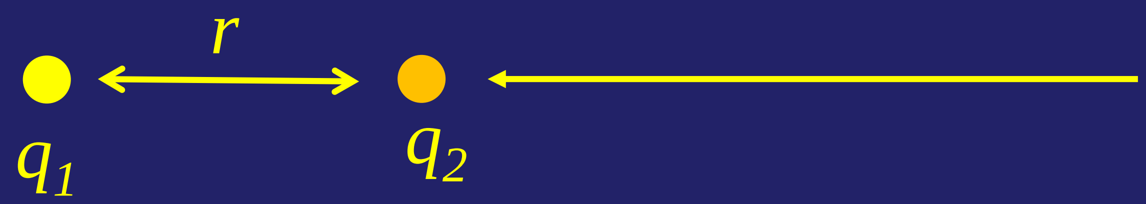
Where is the energy stored?

q_i or $\rho(r)$ charge distributions

For moving q_i around we spend energy or add energy

Where is the ‘bag’ in which we ‘store’ this energy?

Example: energy of a point charge



q_1 produces a scalar potential $\Phi(r)$, *vector* field $\vec{E}(r)$

Force on q_2 is $q_2\vec{E}(r)$. Suppose we want to move q_2

Work = Force * distance: $W = \int_{\infty}^{r} q_2\vec{E}\cdot\overrightarrow{dl} = q_2\Phi(r)$

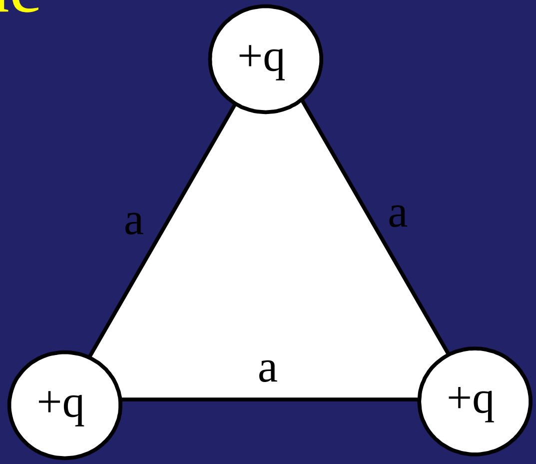
Energy $= \dfrac{Work}{q_2} = \Phi(r)$

in this case, move q_2 from ∞ to r

Question: Generalize to multiple point charges

Three identical charges $+q$ sit on an equilateral triangle with sides a.

What would be the final kinetic energy of the *top* charge if you released it (keeping the other two fixed)

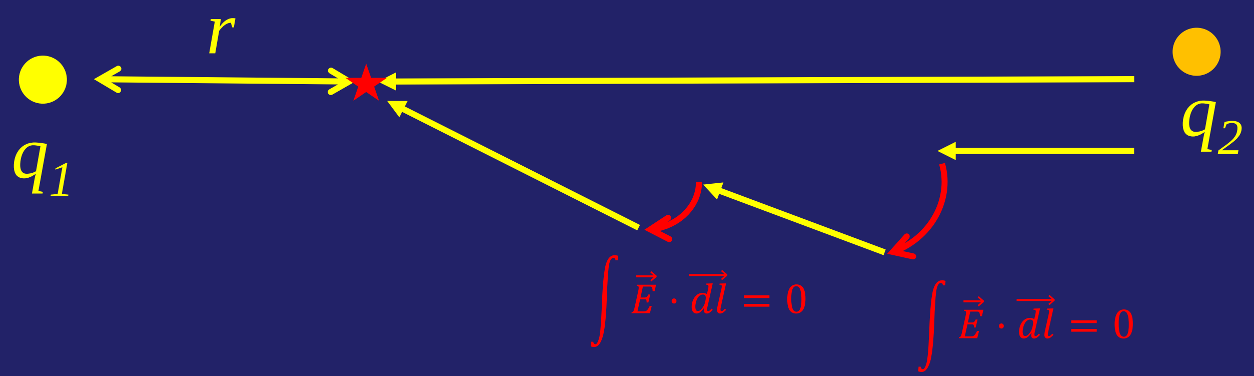


$A)\ \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}$ $\quad$ $B)\ \frac{1}{4\pi\epsilon_0}\frac{2q^2}{3a}$

$C)\ \frac{1}{4\pi\epsilon_0}\frac{2q^2}{a}$ $\quad$ $D)\ \frac{1}{4\pi\epsilon_0}\frac{3q^2}{a}$

$E)\ None\ of\ the\ above$

What does a ‘conservative’ field mean?



Can pick *any* path for $\int \vec{E} \cdot \overrightarrow{dl}$

break it into radial (∫≠0), and tangential(∫=0) pieces

$\Phi(r)$ is *independent* of the path from reference (∞) to *r*

More generally,

this is a result of $\vec{\nabla} \times \vec{E} = 0$ and Stokes theorem

What is the energy for arbitrary *ρ(r)* ?

Remember: Split the problem in two pieces

1) Calculate the field due to a set of charges, without worrying about *other* charges nearby

2) Calculate the effect of a field on a set of charges, without worrying about what charges produced the field

Can show using $\vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\epsilon}$

Energy is stored in $\vec{E}$

$$\boxed{\text{Energy} = \frac{\epsilon_0}{2} \int E^2 d\tau}$$

Limit of integral is tricky!

Where does $\rho(r)$ sit?

We have worked on problems like

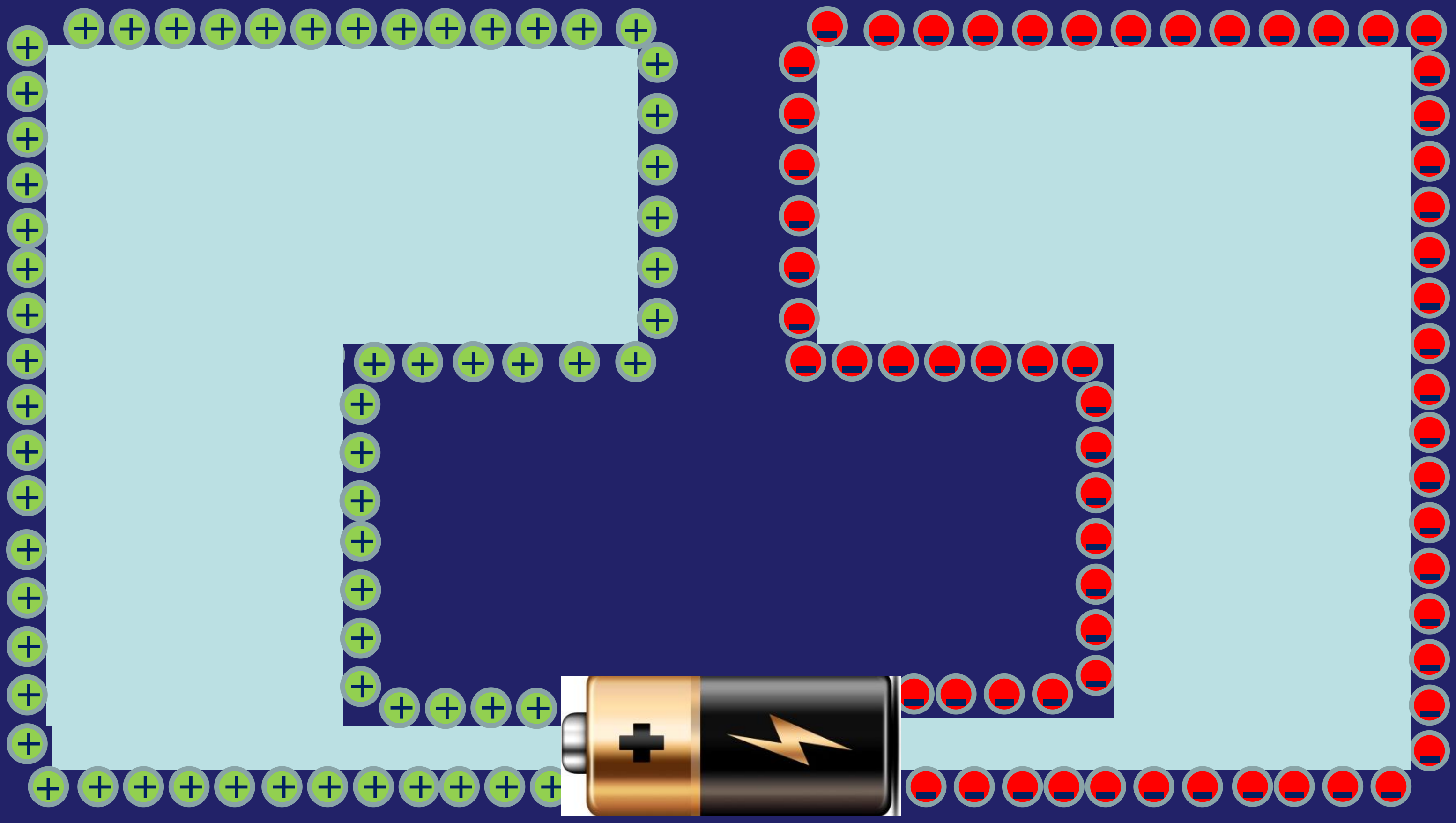
- charge distributed on the surface of a sphere
- charge on an infinite line or plane surface etc…

What are these volumes, surfaces, lines *made of*?

Material places constraints on how the charge is distributed!

Conductor

Recall the light bulb problem – how does charge travel in a conductor wire?



Question

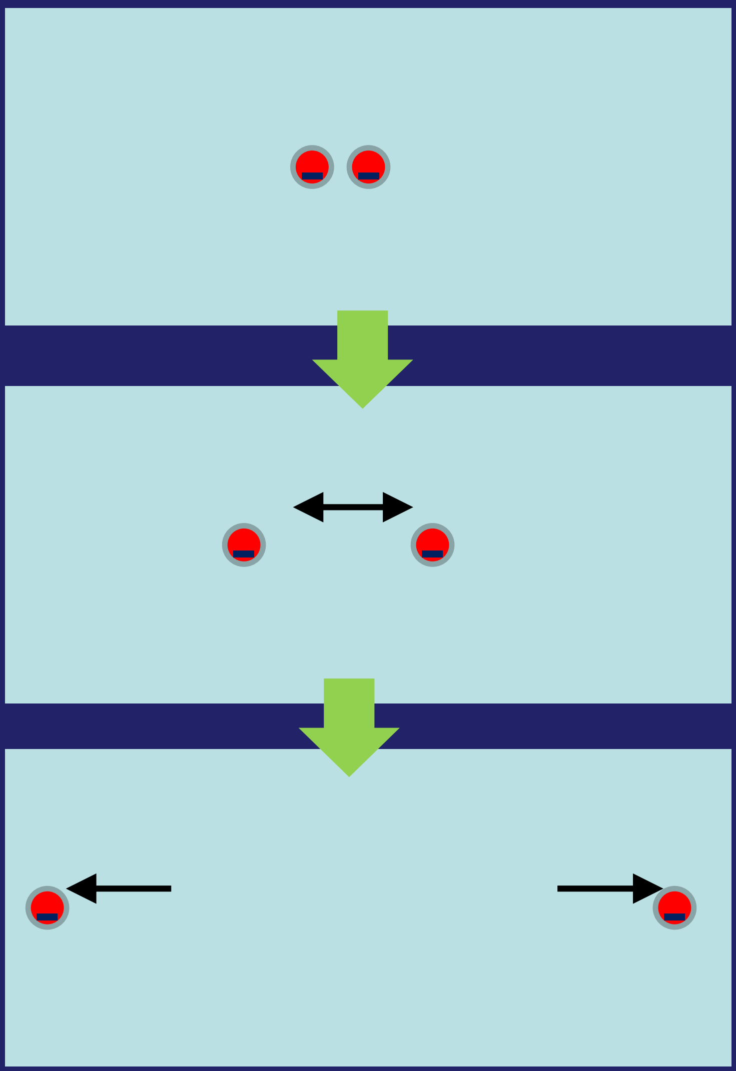
Two like sign charges repel and are free to move.
As they get farther, the total field energy $\int E^2 d\tau$:

A) Increases

B) Decreases

C) Stays the same

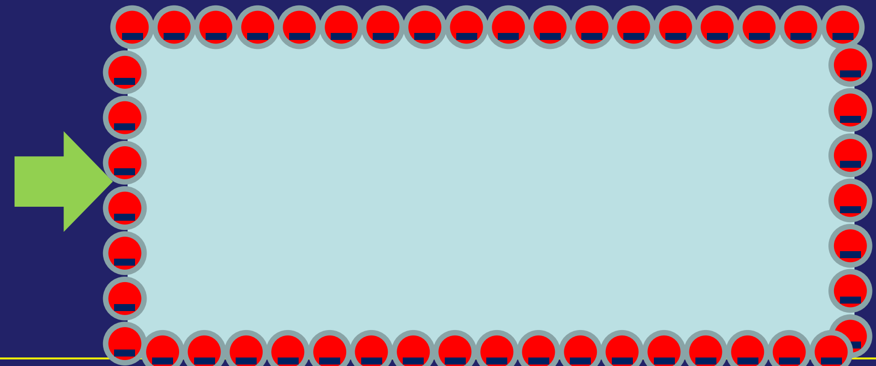
Charges in a conductor distribute themselves on the surface



All charge that is moveable (electrons) – repels each other

The farthest they can get is to the surface!

This is the static case – potential energy is minimized



What is the static configuration for a conductor?

$$\vec{\nabla}\cdot\vec{E} = \frac{\cancelto{0}{\rho}}{\epsilon_0}$$

$$\vec{E} = 0$$

$$-\vec{\nabla}\Phi = 0$$

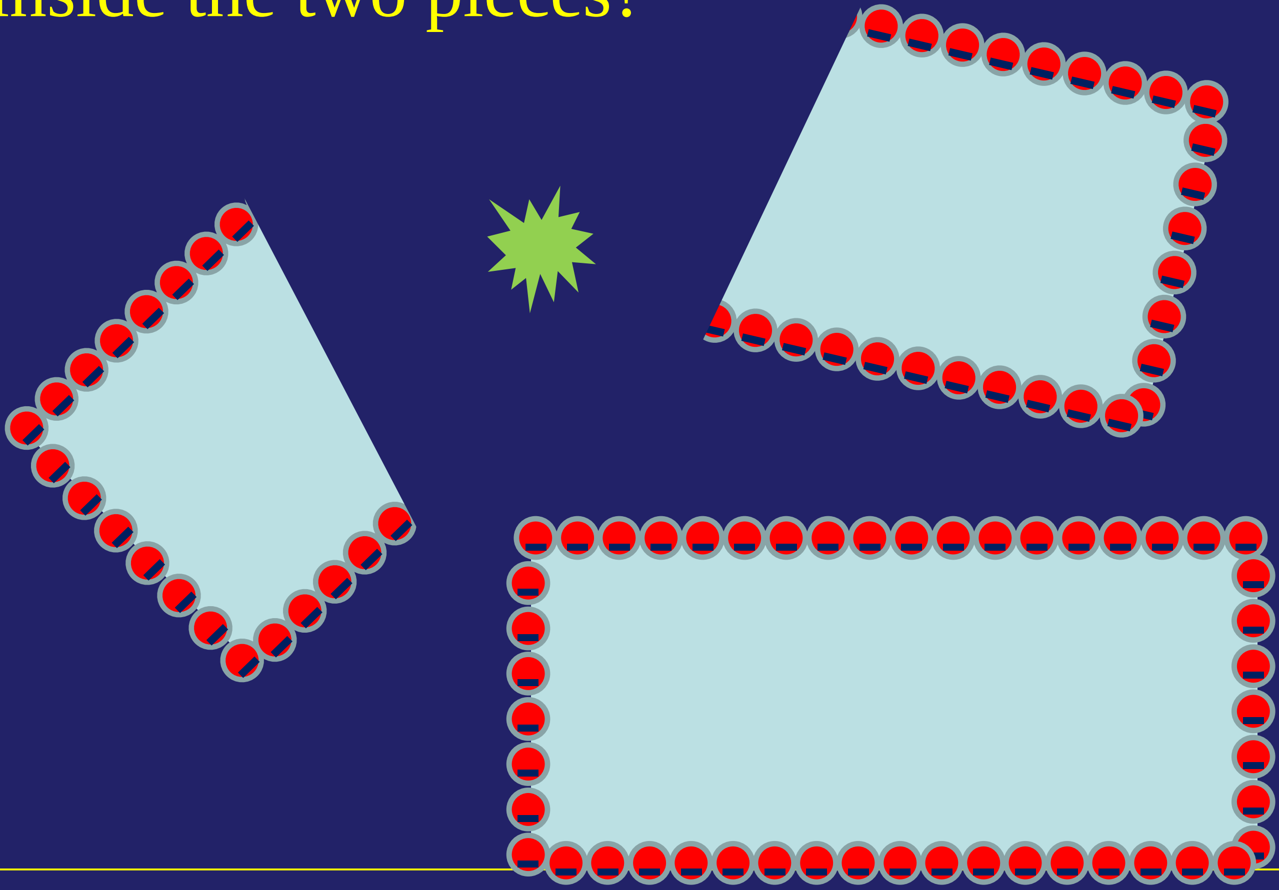
$$\Phi = constant$$

A conductor is an *equi*potential object

What happens if I break the conductor in two?

Where does the free charge go?

What is $\vec{E}$ inside the two pieces?

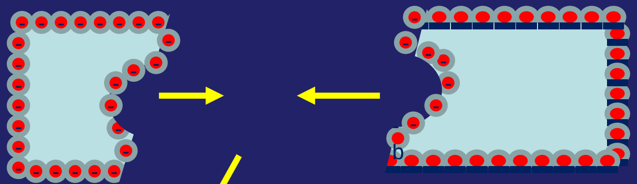


What happens if I make a hole inside the conductor?

$$\oint \vec{E}\cdot\overrightarrow{dl} = \int_{\text{b-c-a}} \vec{E}\cdot\overrightarrow{dl} + \int_{\text{a-b}} \vec{E}\cdot\overrightarrow{dl}$$

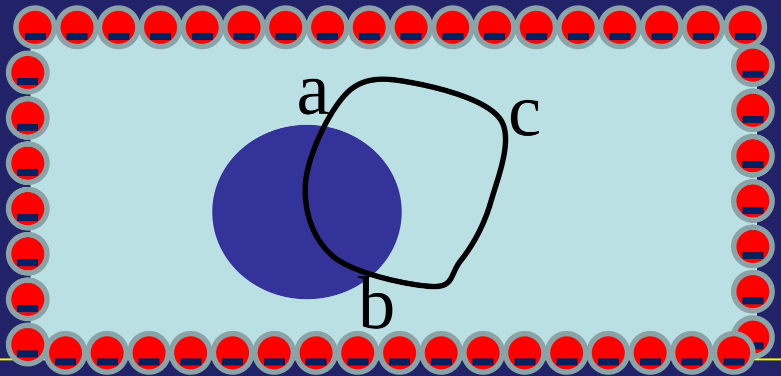
$\oint \vec{E}\cdot\overrightarrow{dl}$	$\int_{\text{b-c-a}}$	$\int_{\text{a-b}}$
0 (conservative)	0	?

Think practically! One way to make the hole is:
Scoop out a piece from each of the two pieces on previous slide



Join the two pieces back together

Where is the charge in static state after transients?



Logic thread of today's lecture

Any collection of charges has a potential energy

the charges generate an electric potential & field

work is needed to move a test charge in this field

Work costs energy – the energy is taken from or put into the electric field

Conductors

– minimize the potential energy for steady state by pushing free charge to the surface

– $\vec{E} = 0$ inside a conductor in steady state