BB 101: Module II

TUTORIAL 1: Solutions

1. Work done=Force × Distance

Given Force= 5 pN

Therefore, Distance =Work done/Force =$100 \times 10^{-21}/5 \times 10^{-12} = 20 \times 10^{-9}$ m = 20 nm

2. We know $F=\gamma v$

This implies, $v=F/\gamma$

Now $\gamma=6\pi\eta r$

Therefore, $v=F/6\pi\eta r$

Given F = 5 pN = 5×10^{-12} N

$\eta = 1000 \times 10^{-3}$ Pa.s

r= $3/\pi$ μm = $3/\pi \times 10^{-6}$ m

Therefore, $v=5 \times 10^{-12}$ N $/(6\pi\times 1000 \times 10^{-3}$ Pa.s$\times 3/\pi\times 10^{-6})$

$= (5/18) \times 10^{-6}$ m/s

= 5/18 μm/s

= 0.27 μm/s

3. Given t=15 min = 15 × 60 s

x=30μm = 30×10^{-6} m

Therefore, $v=x/t = 30 \times 10^{-6} / (15 \times 60)$

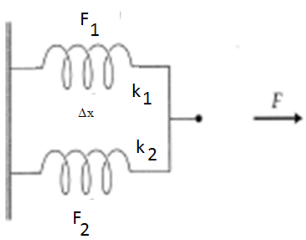
Now, $F=6\pi\eta rv= (6\pi\times 10000\times 10^{-3} \times 3/\pi\times 10^{-6} \times 30 \times 10^{-6})/ (15 \times 60) = 6 \times 10^{-12}$ N

$= 6$ pN

Since each motor can generate force of 1.0 pN

Therefore, total 6 motors would be required to generate force of 6 pN.

4. If springs are in parallel then when a force F is applied both of them will have same extension Δx as shown below



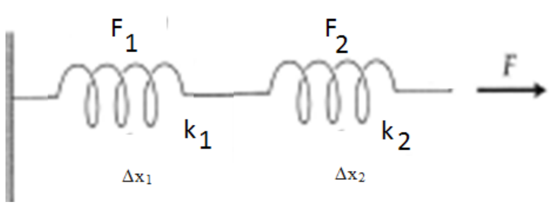
The force on each spring will add and total is F

i.e. $F=F_1+F_2$

or, $k_{eff}\,\Delta x = k_1\Delta x + k_2\Delta x$

or, $k_{eff} = k_1 + k_2$

If springs are placed in series as shown below then both springs feel the same force however their displacement would be different



$\Delta x = \Delta x_1 + \Delta x_2$

Or, $F/k_{eff} = F/k_1 + F/k_2$

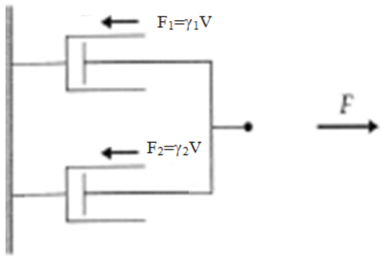
Or, $1/k_{eff} = 1/k_1 + 1/k_2$

When dashpots are in parallel as shown below then both of them would move with same velocity for any applied force and sum of the drag forces applied by each dashpot will be equal to F

Therefore, $F = F_1 + F_2$

Or, $\gamma_{eff}V=\gamma_1 V+=\gamma_2 V$

$\gamma_{eff}=\gamma_1+=\gamma_2$

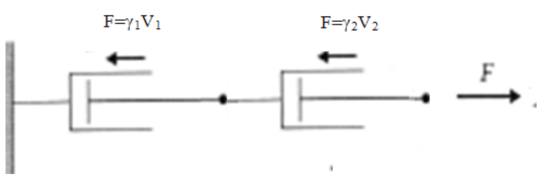


When dashpots are in series as show below then both of them would experience the same applied force. However, both of them would move with different velocities such that net velocity V is sum of the velocities of the individual dashpots

$V=V_1+V_2$

Or, $F/\gamma_{eff}= F/\gamma_1+ F/\gamma_2$

Or, $1/\gamma_{eff}= 1/\gamma_1+ 1/\gamma_2$



5. When an abrupt force F is applied at $t = 0$, let's assume that it leads to a net displacement of x_0. This net displacement would be the sum of the displacements x_s of spring and x_s of dashpot

i.e.$x_0 = x_s + x_d$

Further, both spring and dashpot would feel the same applied force and hence

$F = kx_s$ and $F = \gamma v = \gamma \frac{dx_d}{dt}$

Therefore, $kx_s = \gamma \frac{dx_d}{dt}$

Or, $k(x_0 - x_d) = \gamma \frac{dx_d}{dt}$

Or, $\int_0^{x_d} \frac{dx_d}{(x_0-x_d)} = \int_0^t \frac{k}{\gamma} dt$

Or, $\ln \frac{(x_0-x_d)}{x_0} = -\frac{k}{\gamma} t$

Or, $\frac{(x_0-x_d)}{x_0} = e^{-\frac{k}{\gamma}t}$

Or, $1 - \frac{x_d}{x_0} = e^{-\frac{k}{\gamma}t}$

Or, $x_d = x_0(1 - e^{-\frac{k}{\gamma}t})$

Therefore, $x_d = x_0(1 - e^{-\frac{k}{\gamma}t})$, and $x_s = x_0 e^{-\frac{k}{\gamma}t}$

Let's look at the limits $t = 0$ and $t = \infty$

At $t = 0$, $x_s = x_0$ and $x_d = 0$

At $t = \infty$, $x_s = 0$ and $x_d = x_0$

This mean when force is applied at $t = 0$, the spring instantaneously stretches to x_0. However, as time progress extension of the spring starts to decrease and applied load starts to elongate dashpot. In the limit $t = \infty$, extension of the spring would become equal to zero.

6. At $t = 0$, $x = x_s$

At $t = \infty$, $x = x_0$

Assume that at any arbitrary t the position is x

The force due to extension of the spring at any time t will be balance by the drag force

$$k(x - x_0) = -\gamma \frac{dx}{dt}$$

$$\frac{dx}{(x - x_0)} = -\frac{k}{\gamma} dt$$

$$\int_{x_s}^{x} \frac{dx}{(x - x_0)} = -\frac{k}{\gamma} \int_0^t dt$$

$$[ln(x - x_0)]_{x_s}^{x} = -\frac{k}{\gamma}[t]_0^t$$

$$ln\frac{(x - x_0)}{(x_s - x_0)} = -\frac{k}{\gamma}t$$

$$\frac{(x - x_0)}{(x_s - x_0)} = e^{-\frac{k}{\gamma}t}$$

$$(x - x_0) = (x_s - x_0)e^{-\frac{k}{\gamma}t}$$

$$x = x_0 + (x_s - x_0)e^{-\frac{k}{\gamma}t}$$