BB 101: Module II

TUTORIAL 2: Solutions

1. Balancing the drag forces for the downward movement of paddle gives

$$u = \frac{\gamma_2 v}{\gamma_1 + \gamma_2} \qquad (1)$$

Similarly, drag forces for the upward movement of paddle gives

$$u' = \frac{\gamma_2 v'}{\gamma_1 + \gamma_2} \qquad (2)$$

Displacement Δx of the microorganism due to downward movement of paddle

$$\Delta x = tu$$

Since motion of paddle is geometrically reciprocal, therefore distance travelled by paddle during both upward and downward movement must be identical

$$tv = t'v' \qquad (3)$$

Now, displacement $\Delta x'$of the microorganism due to upward movement of paddle

$$\Delta x' = t'u'$$

$$= \left(\frac{tv}{v'}\right)\left(\frac{\gamma_2 v'}{\gamma_1 + \gamma_2}\right) \; using \; (2) \;\; and (3)$$

$$= \left(\frac{tv}{v'}\right)\left(\frac{\gamma_2 v'}{\gamma_1 + \gamma_2}\right)$$

$$= t(\frac{\gamma_2 v}{\gamma_1 + \gamma_2})$$

$$= tu = \Delta x \quad using \; (1)$$

Since $\Delta x' = \Delta x$, There will not be any net displacement

2. To understand how a net propulsive force is generated, when helix is cranked about its helix axis at a certain angular speed, let us consider a smaller segment of this axis which looks like a cylinder as show below

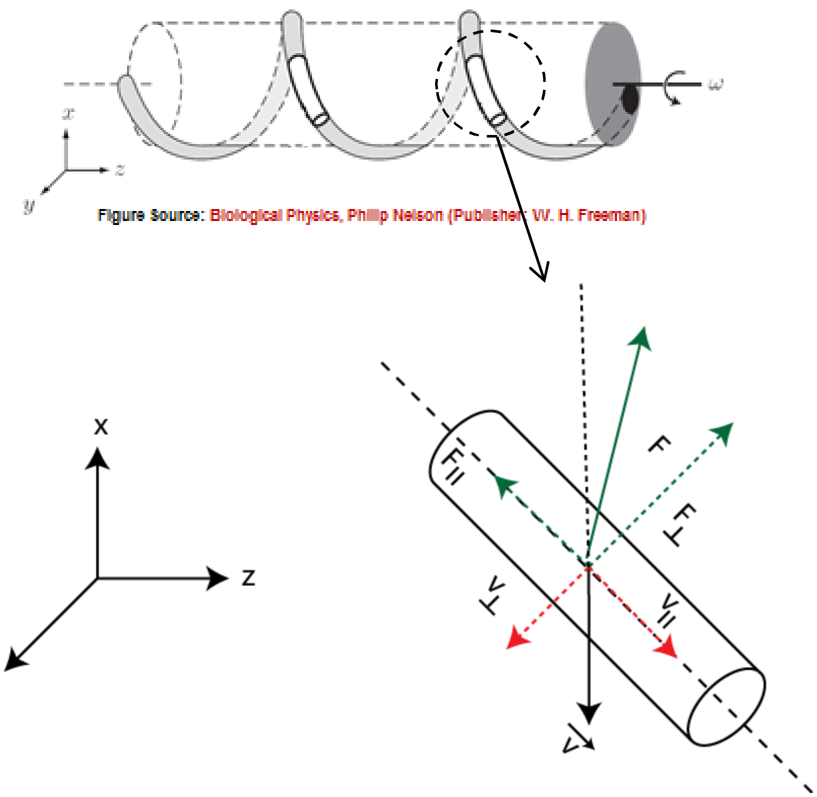


Figure Source: Biological Physics, Philip Nelson (Publisher: W. H. Freeman)

As shown in above figure, if the cylinder is moving with velocity v downward due to cranking of the helix then net drag force F on the cylinder will not be in a direction exactly opposite to v. The direction of the net drag force F will be tilted in the forward direction. This happens since drag force in perpendicular direction is higher drag force in parallel direction when cylinder moves.

One can think of a helix to be consisting of many such cylinders. Components of the drag force of these cylinders in x-y plane will be cancelled for the helix and only z-component survives. Thus, if a bacterial is placed to the right of the helix then it will be propelled due to these non-vanishing z-components of F.

3. If motors turn 10 times faster then swimming speed will be multiplied by a factor of 10 and hence swimming speed will be 200μm/s.

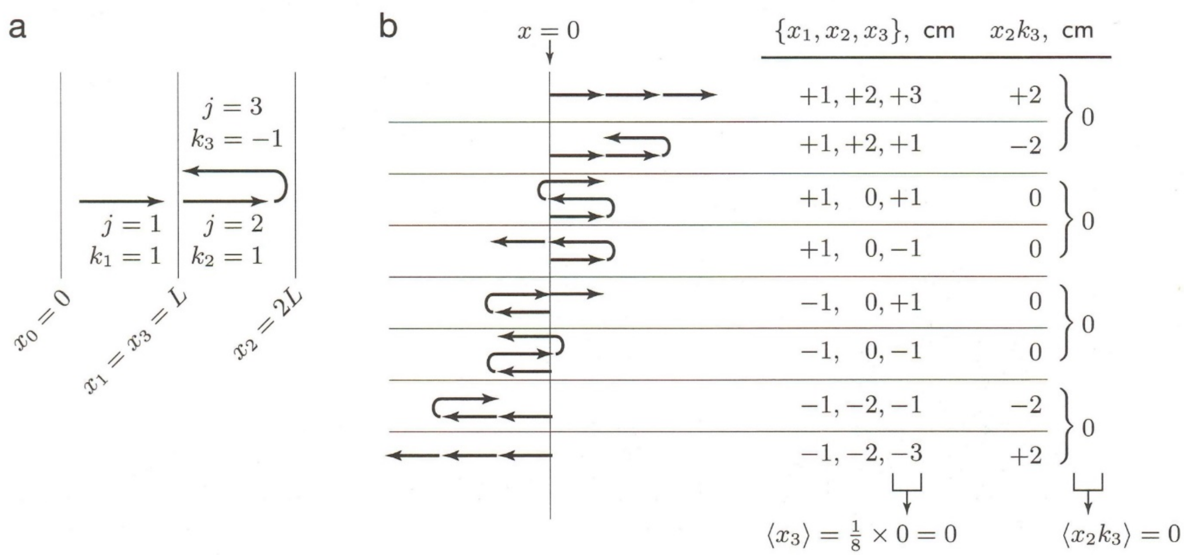
If fluid environment is made 10 time more viscous then velocity will become 10 times smaller i.e. swimming speed will be 2μm/s

In the first case the power output would increase by a factor of 10 whereas in second case it would decrease by a factor of 10.

4. Each step is of length L. Thus the displacement of step j is k_jL, where k_j is equally likely to be ±1. Call the position after j steps x_j ; the initial position is $x_0 = 0$ (see Figure)

Then$x_1 = k_1L$, and similarly the position after j steps is $x_j = x_{j-1} + k_jL$

We can't say anything about x_j because each walk is random. We can, however, make definite statements about the average of x_j over many different configurations: For example, Figure below shows that $< x_3 > = 0$. The diagram makes it clear why we got this result: In the average over all possible configurations, those with net displacement to the left will cancel the contributions of their equally likely analogs with net displacement to the right.



Please note that

$$\langle (x_N)^2 \rangle = \langle (x_{N-1} + k_NL)^2 \rangle = \langle (x_{N-1})^2 \rangle + 2\langle x_{N-1}k_N \rangle + L^2\langle (k_N)^2 \rangle$$

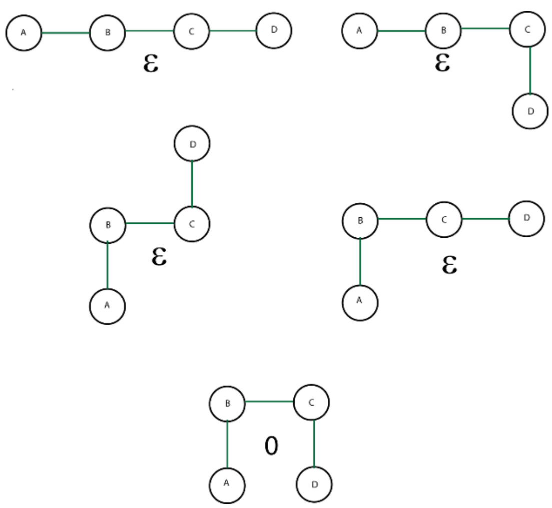
In the above expression, the final term just equals L^2, because $(\pm1)^2 = 1$. For the middle term, note that we can group all 2^N possible walks into pairs (see the last column of Figure above). Each pair consists of two equally probable walks with the same x_{N-1}, differing only in their last step, so each pair contributes zero to the average of $x_{N-1}k_N$.

Therefore, $\langle (x_N)^2 \rangle = \langle (x_{N-1})^2 \rangle + L^2$

Thus, above Equation says that a walk of N steps has mean-square displacement bigger by L^2 than a walk of N−1 steps, which in turn is L^2 bigger than a walk of N−2 steps, and so on. Carrying this logic to its end, we find

$$\langle (x_N)^2 \rangle = NL^2$$

5. Possible configurations with corresponding energies are shown in the figure below



Therefore, partition function

$$Z = e^{-\frac{0}{k_B T}} + e^{-\frac{\varepsilon}{k_B T}} + e^{-\frac{\varepsilon}{k_B T}} + e^{-\frac{\varepsilon}{k_B T}} + e^{-\frac{\varepsilon}{k_B T}}$$

$$Z = 1 + 4e^{-\frac{\varepsilon}{k_B T}}$$

At T=0,

$$Z = 1$$

This mean at T=0 it will have configuration which corresponds to zero energy.