PH108

Lecture 2:
Point charges, Coulomb's Law

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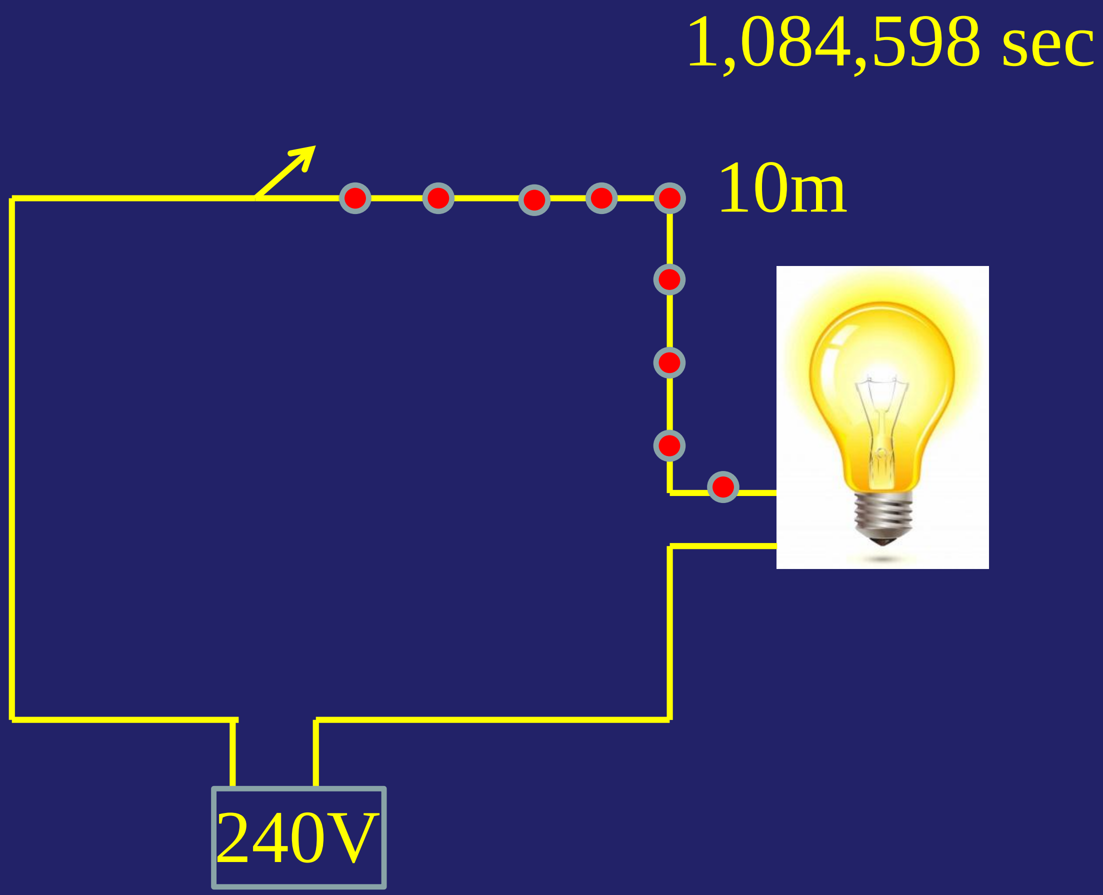
Reading: Griffiths, Chapter 2

Supplementary Reading: 'Div, Grad, Curl and all that' by H. M. Schey

Recall from lecture 1

Man walks into a room.
flicks the light switch.
How long for electrons
to get to the light bulb?



We need to re-check our understanding of electricity !

What is electricity?

Let's start with a **charge**

ideal, abstract 'point like' object

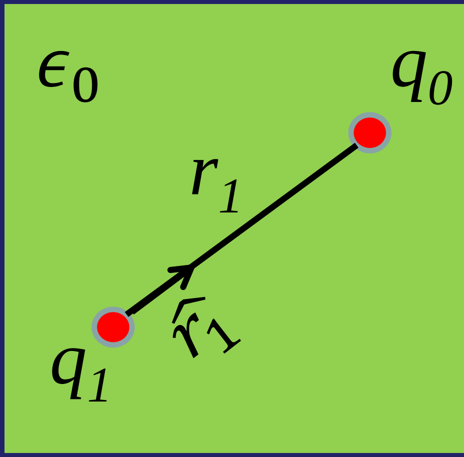
can have magnitude q

can have sign + –

For charge in Cu wire conductor, we took electrons

Electric Charge: + – q

What is the force BY q_1 ON q_0



Consider a stationery charge q_0

Stationery charge q_1 is located at distance r_1

Force **BY** q_1 **ON** q_0 is given by:

$$\overrightarrow{F_{10}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_0}{r_1^2}\hat{r}_1$$

Coulomb's Law

What is $\hat{r}_1$?
unit direction vector *from* q_1 to q_0

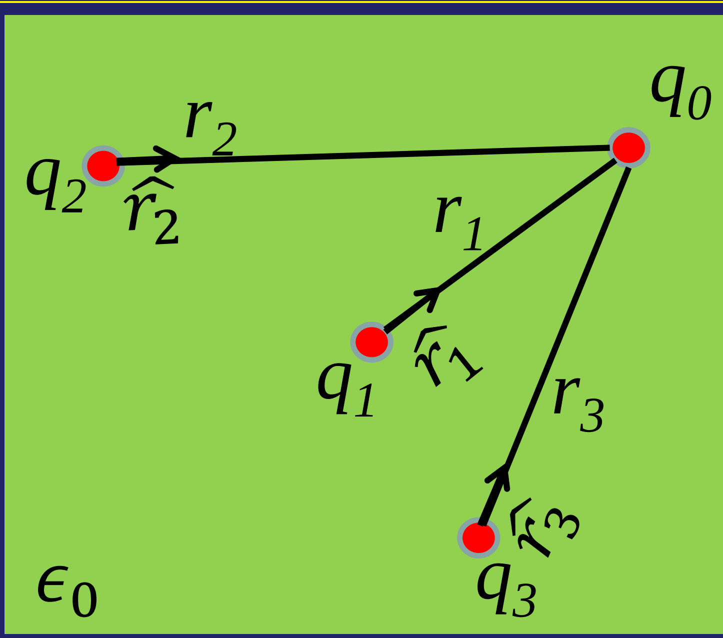
What is ϵ_0 ?
$\epsilon_0 \rightarrow$ 'permittivity' determines how q_1 and q_0 are 'permitted' to interact

What is the force from **many** charges?

$$\overrightarrow{F_{20}} = \frac{1}{4\pi\epsilon_0}\frac{q_2 q_0}{{r_2}^2}\widehat{r_2}$$

$$\overrightarrow{F_{30}} = \frac{1}{4\pi\epsilon_0}\frac{q_3 q_0}{{r_3}^2}\widehat{r_3}$$

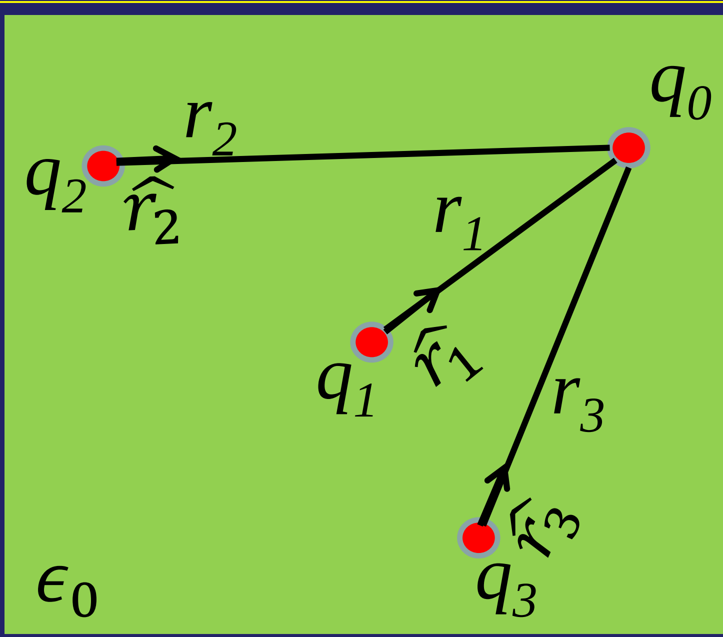
$$\overrightarrow{F_{tot}} = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i q_0}{r_i^2}\widehat{r_i}$$



note: *different* unit vectors,
all pointing to q_0

Superposition rule

Why is superposition important?



1) Will F_{20} affect F_{10} ?

NO.

2) Will F_{20} affect F_{21} , affect F_{23} ?

NO. All q_i are point charges.
We picked one q and labeled it q_0 – Can pick any q_i as q_0

$$\overrightarrow{F_{tot}} = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i q_0}{r_i^2}\hat{r}_i$$

Superposition rules OK

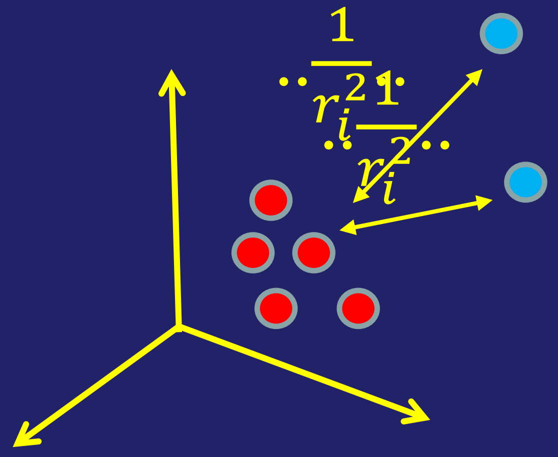
Superposition lets us define a vector FIELD

$$\overrightarrow{F_{tot}} = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i q_0}{r_i^2}\widehat{r_i} \rightarrow \vec{E} = \frac{\vec{F}}{q_0} = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i^2}\widehat{r_i}$$

How is $\vec{E}$ a Vector Field?

1) $\vec{E}$ is a vector : $\sum ..\widehat{r_i}$

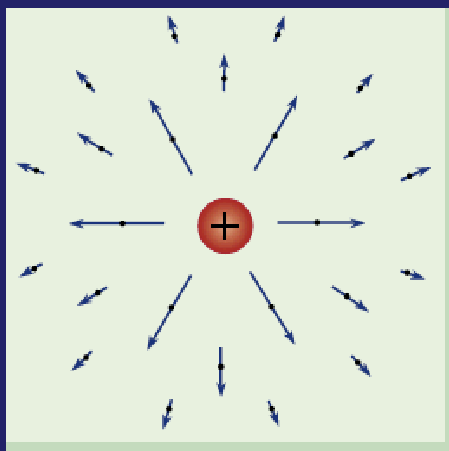
2) $|\vec{E}|$ changes as function of position

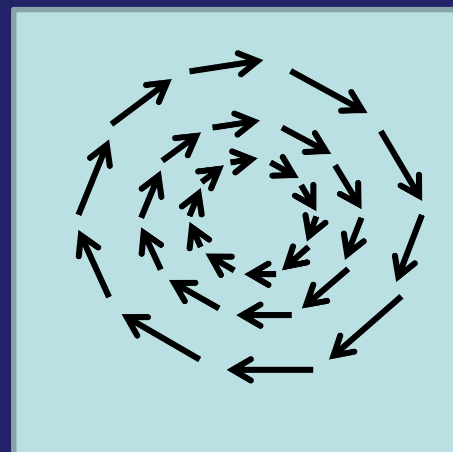


Here are some examples of Vector Field

We can use multiple representations

arrows





field lines

Why do you want the field ?

Objection: I was happy to calculate vector sum of forces

Why this vector field = $\frac{F}{q_0}$ with loopy diagrams?

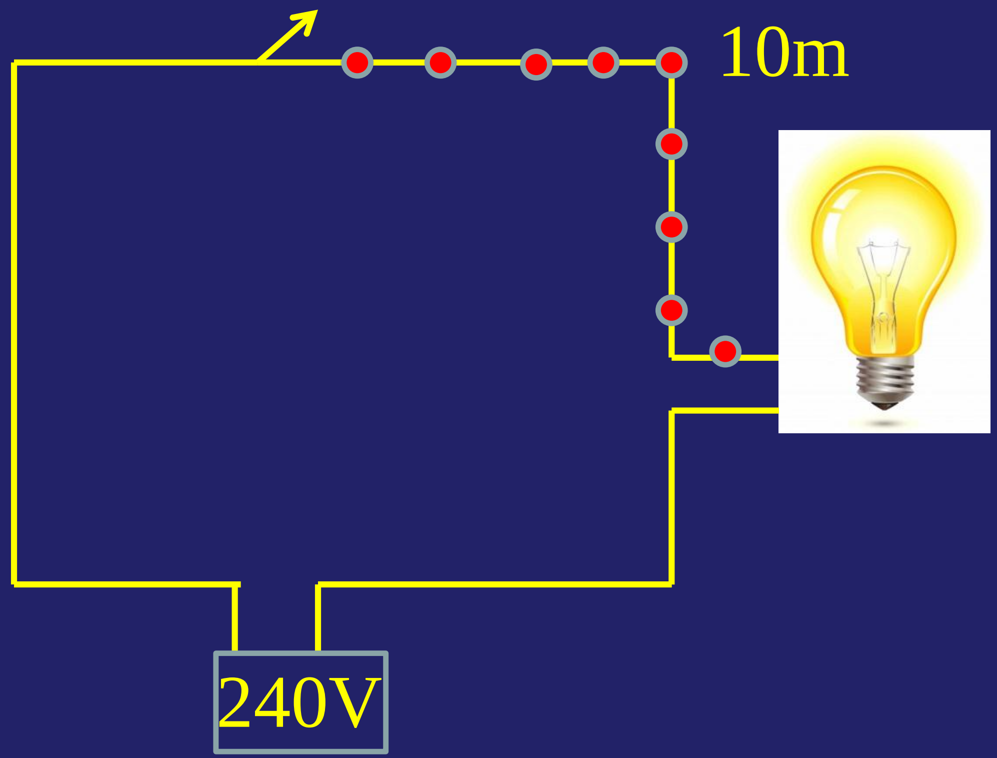
Resolution: We are interested in the effect one set of charges produces on another set (remember light bulb!)

Split this in two pieces

1) Calculate the field due to a set of charges, without worrying about *other* charges nearby

2) Calculate the effect of a field on a set of charges, without worrying about what charges produced the field

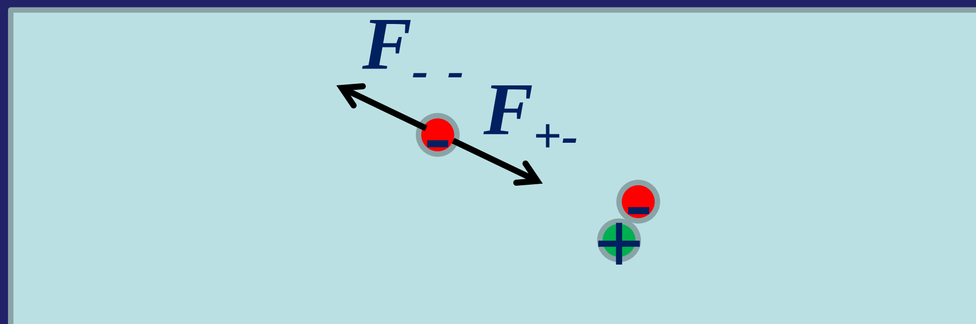
We will use a specific example to see how the field is useful



Where is all the charge in a conductor?

Each Cu atom contributes one electron $q = -e$

Therefore the Cu atom left behind has $q = +e$



$\vec{F}_{+-} + \vec{F}_{--} = 0$ $\rightarrow \vec{E} = 0$ inside Cu

+ charges (Cu atoms fixed to lattice)

– charges (electrons free to move)

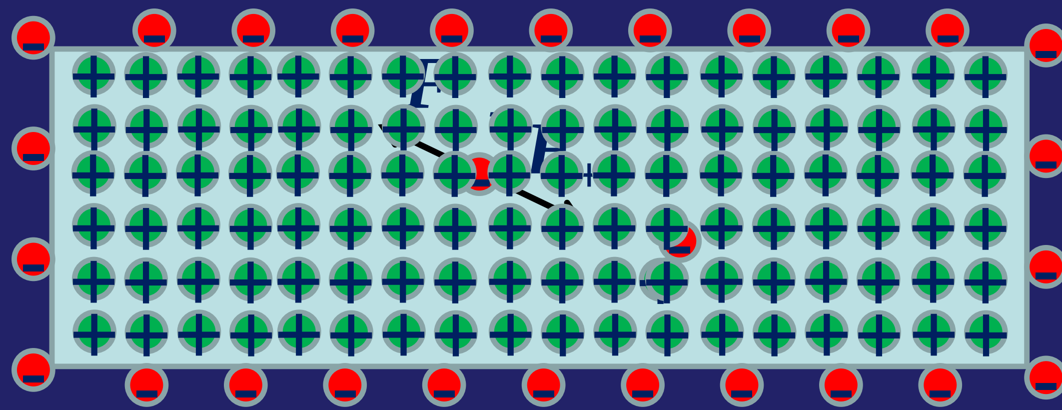
STATIC Equilibrium and STEADY STATE

We define two conditions we are interested in:

1) STATIC EQUILIBRIUM: no charges are moving

2) STEADY STATE: charges are moving with constant speed and there is no excess deposit of charge anywhere

All mobile charge in conductor is on the surface



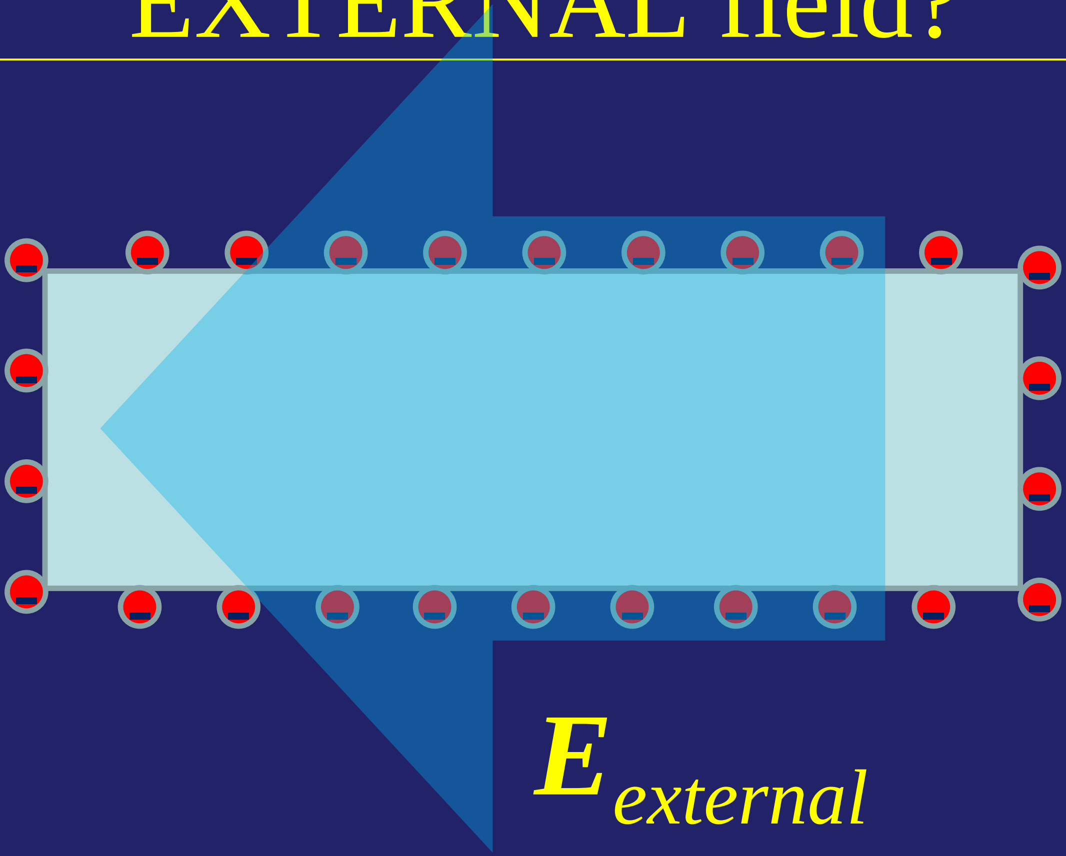
In STATIC EQUILIBRIUM

We will meet this later again in Gauss's Law

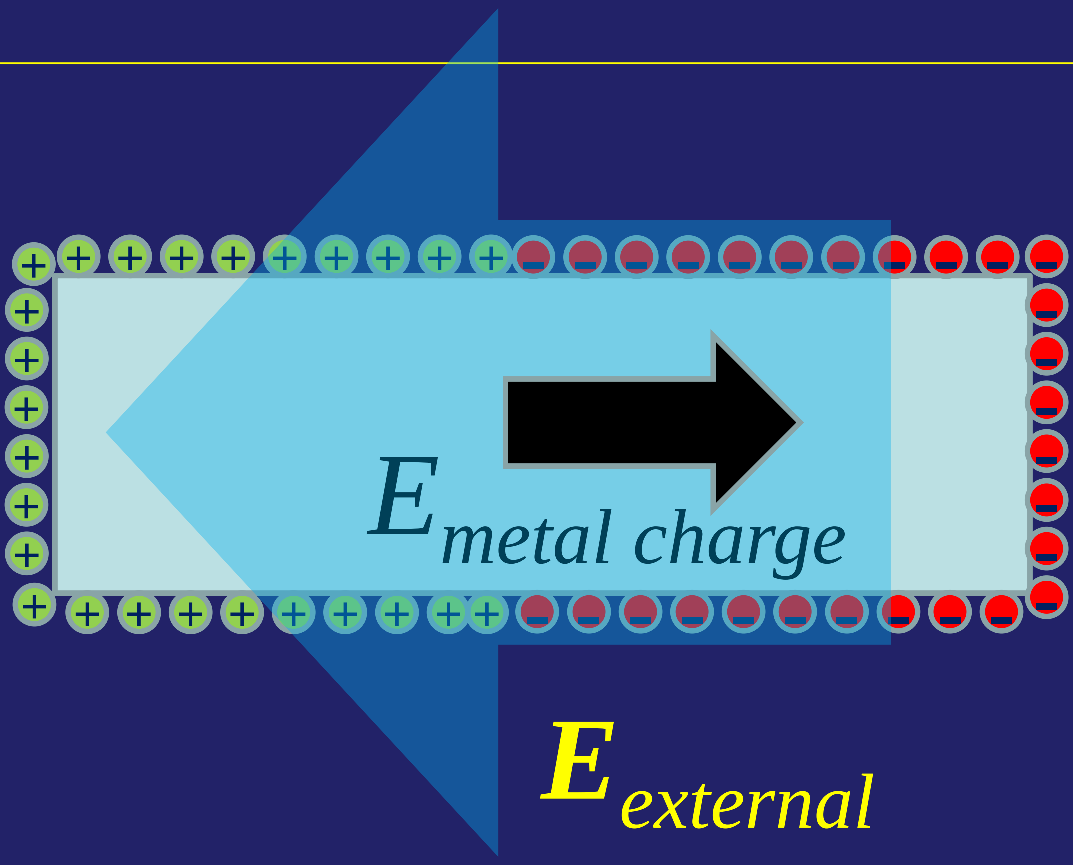
You can show that this works rigorously *because* $E \sim \frac{1}{r^2}$

It would not work if $E \sim \frac{1}{r^{2.5}}$ (our world would be very different!)

What happens when you apply an EXTERNAL field?

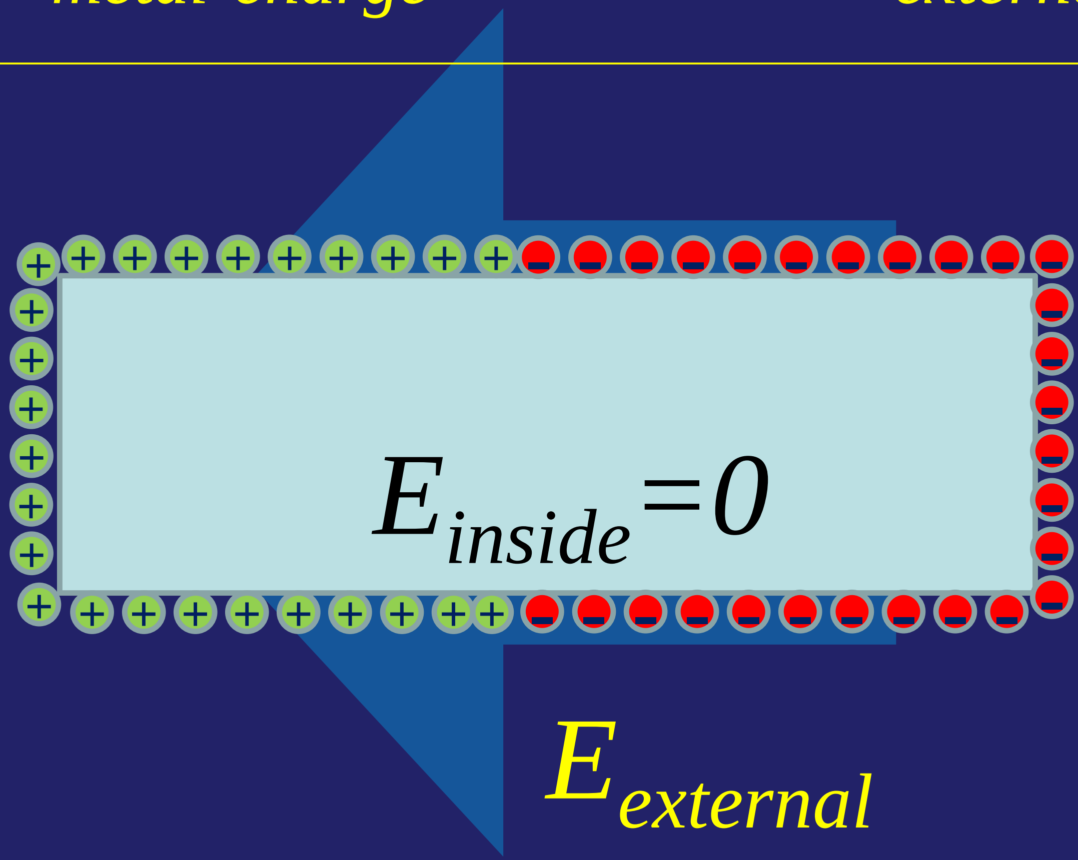


Free charges in the metal start rearranging



Field means force means acceleration.
Rearrangement of charges causes <u>transient</u> $\boldsymbol{E}_{metal\text{-}charge}$

$E_{metal\text{-}charge}$ opposes $E_{external}$

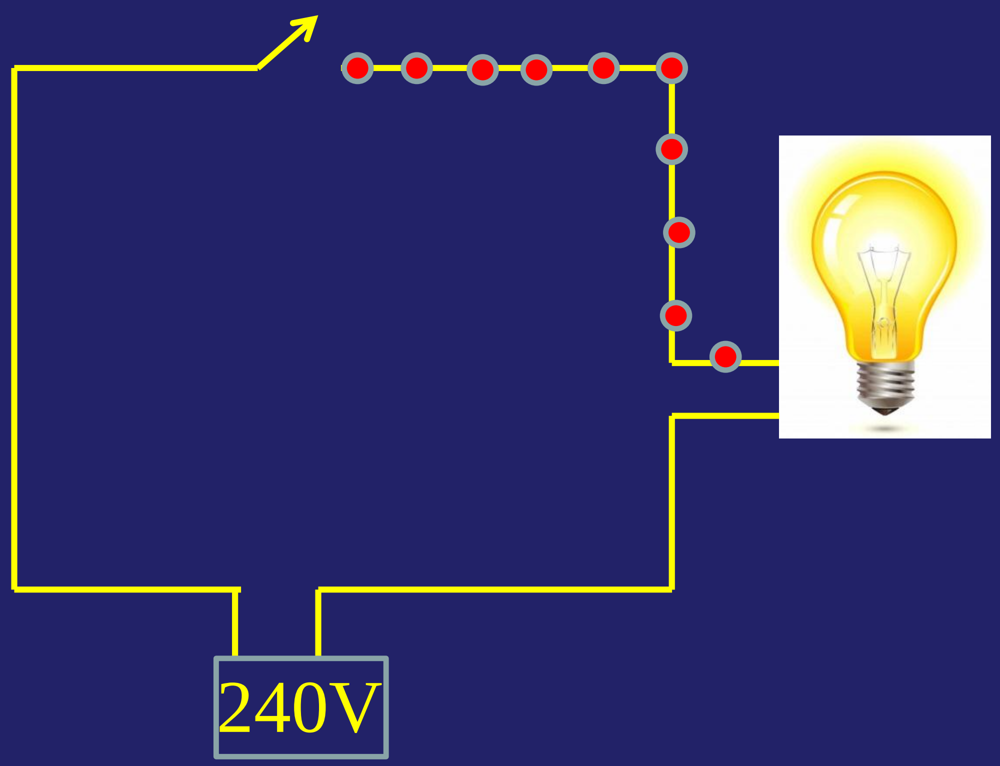


Why is $E_{inside} = 0$ in STEADY STATE?

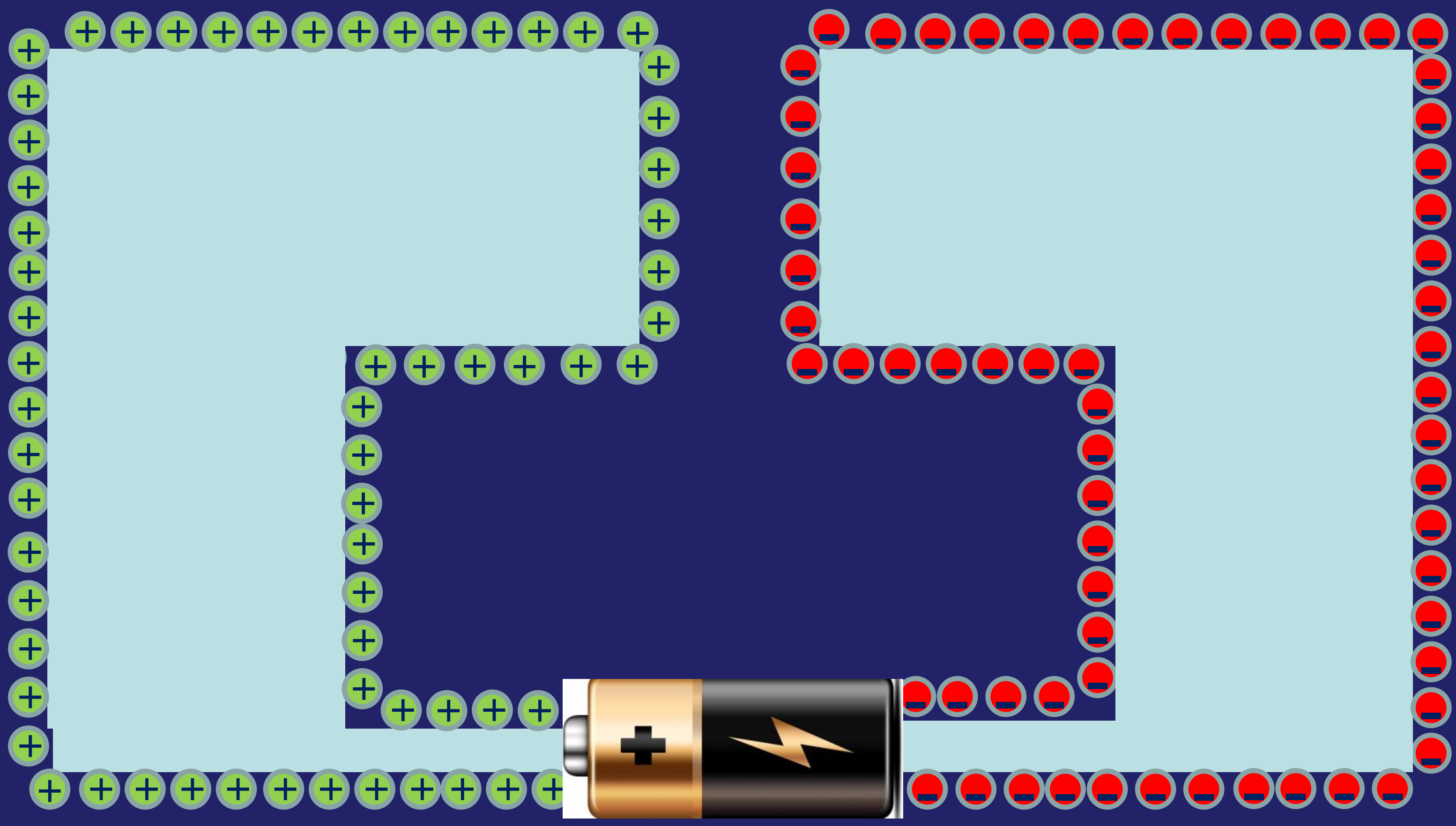
If $E_{inside} \neq 0$, charges would always be accelerating!

Use the field picture to re-look at the light bulb problem

Now that we know the distribution of charges in a conductor, we convert this circuit diagram to a physical diagram…..



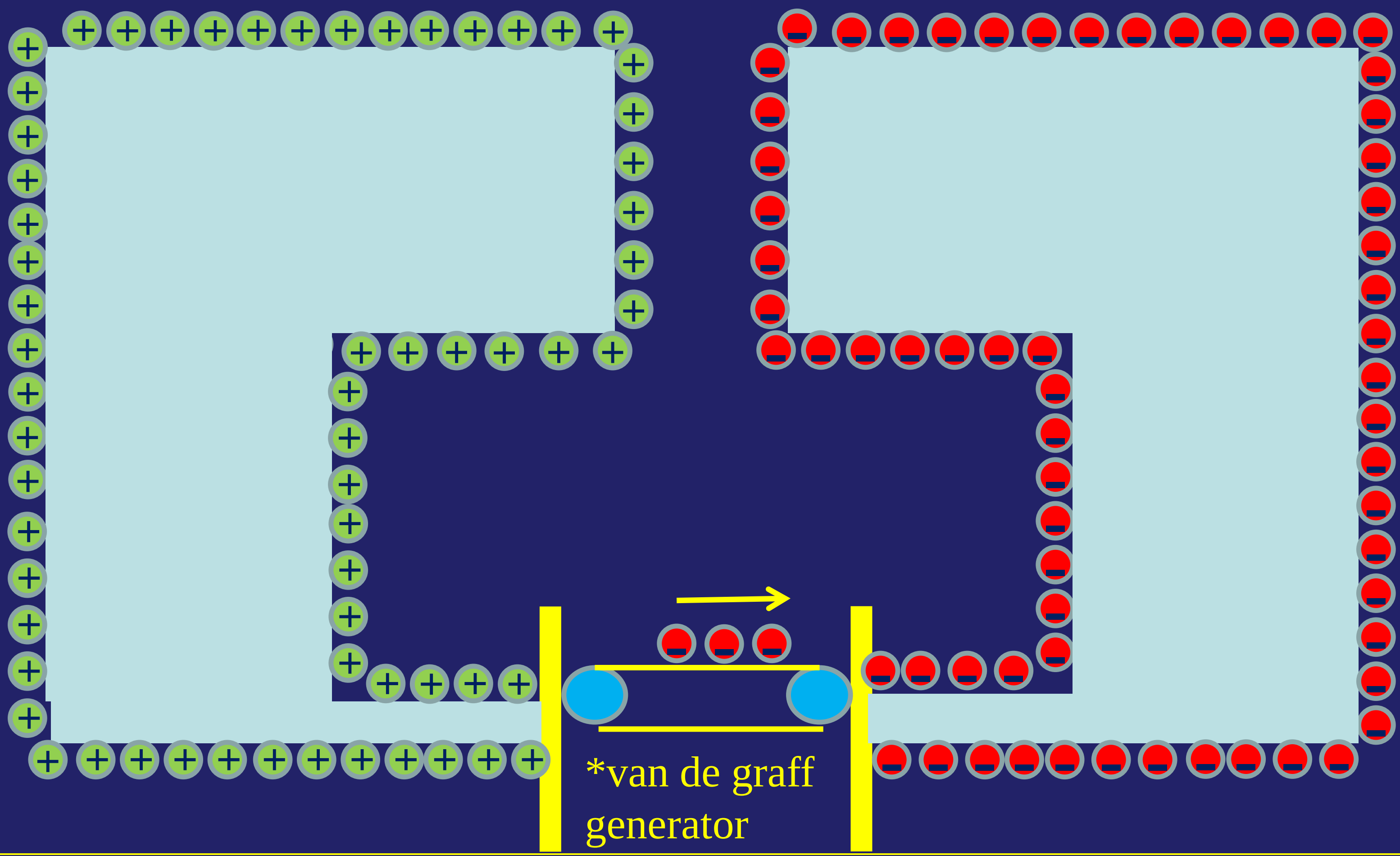
Use the field picture to (re)look at disturbances in the light bulb connection



Use a mechanical* battery to simplify

Two rollers cranked by a motor or hand crank

A belt takes ⊖ from left plate to right

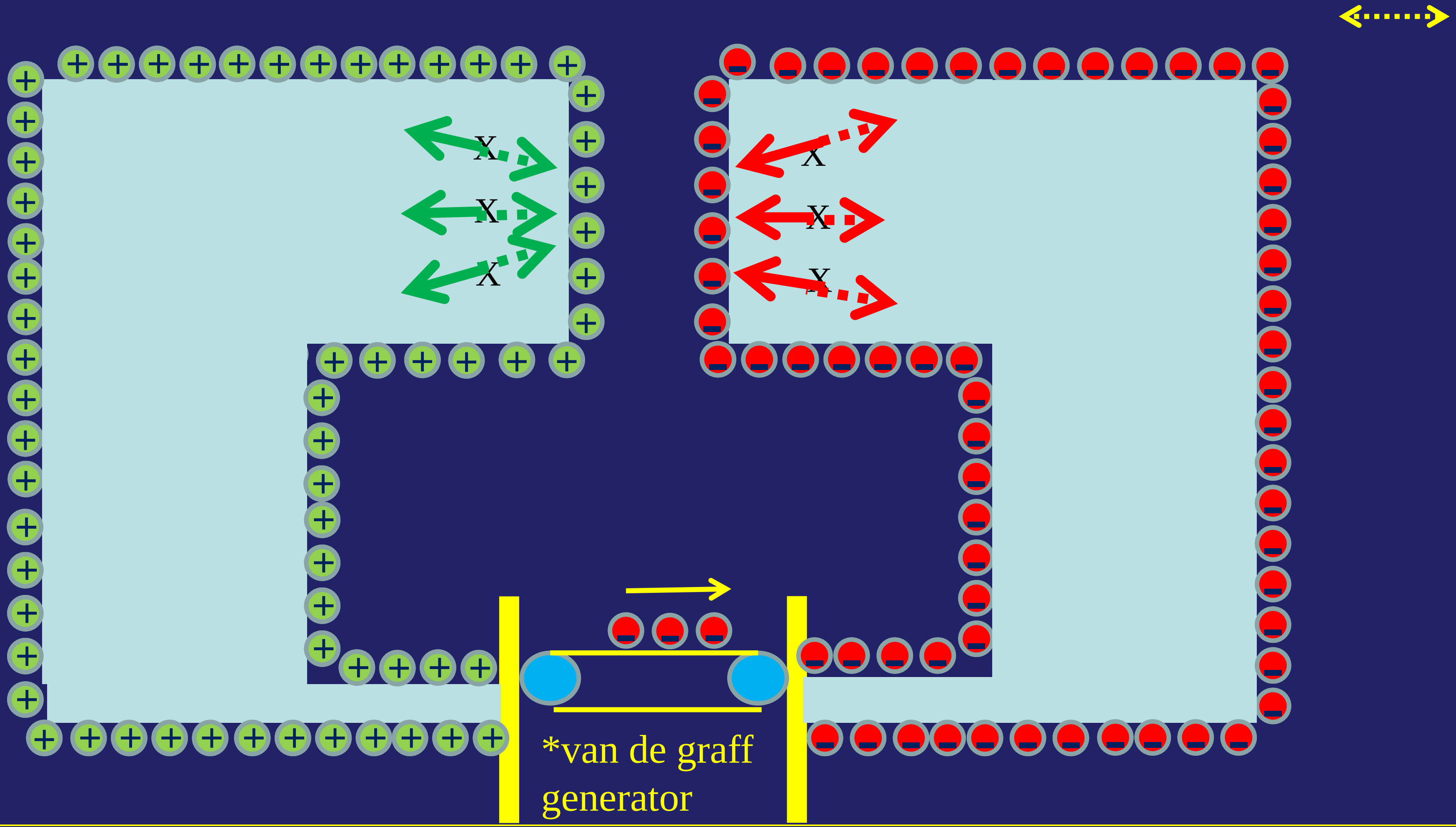


*van de graff generator

What is the field near the open switch?

1) What is the field at ‘x’ *due to charge at gap faces?*

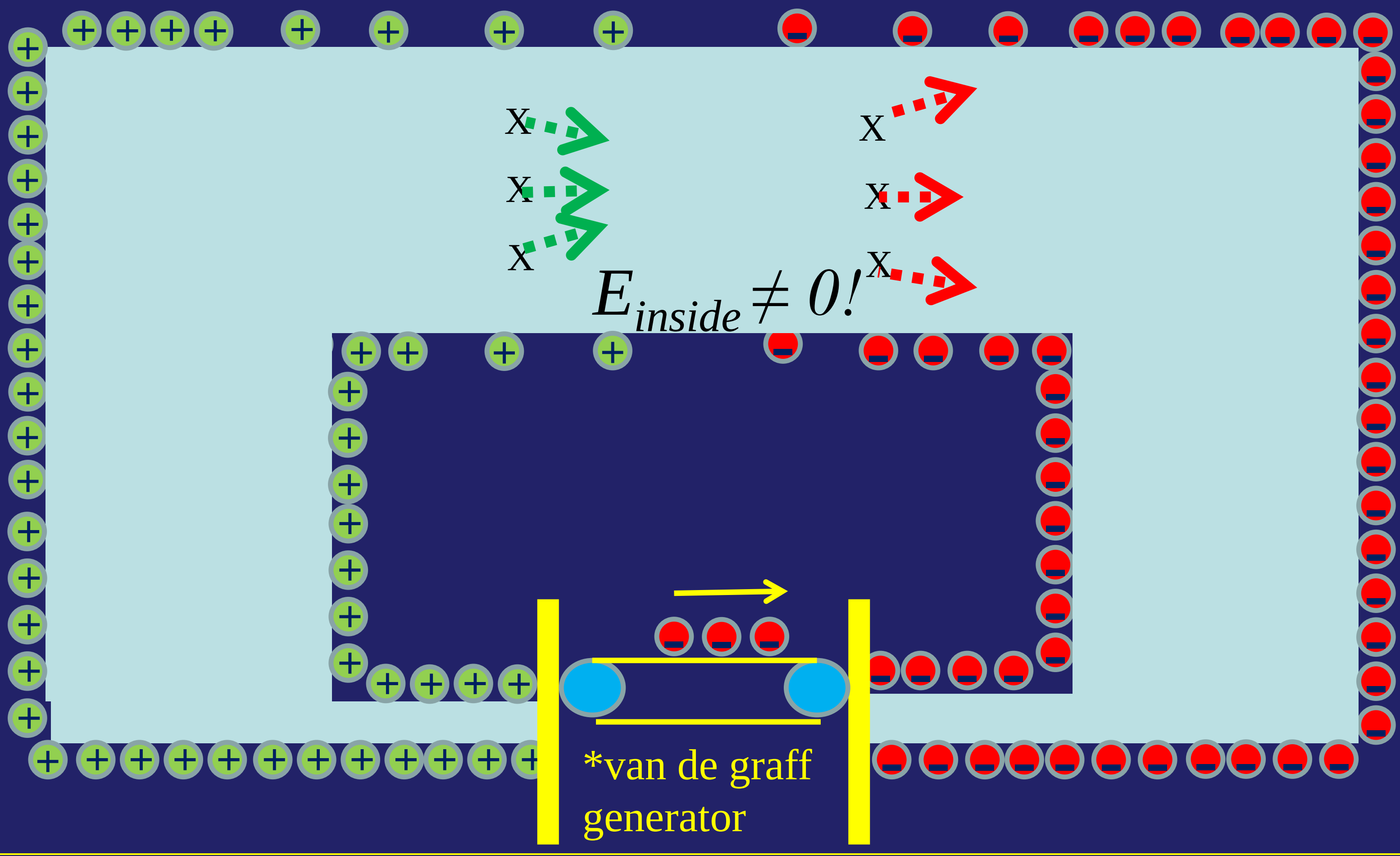
2) What is the field at ‘x’ *due to charge from the conductor?*



What happens when the switch is closed?

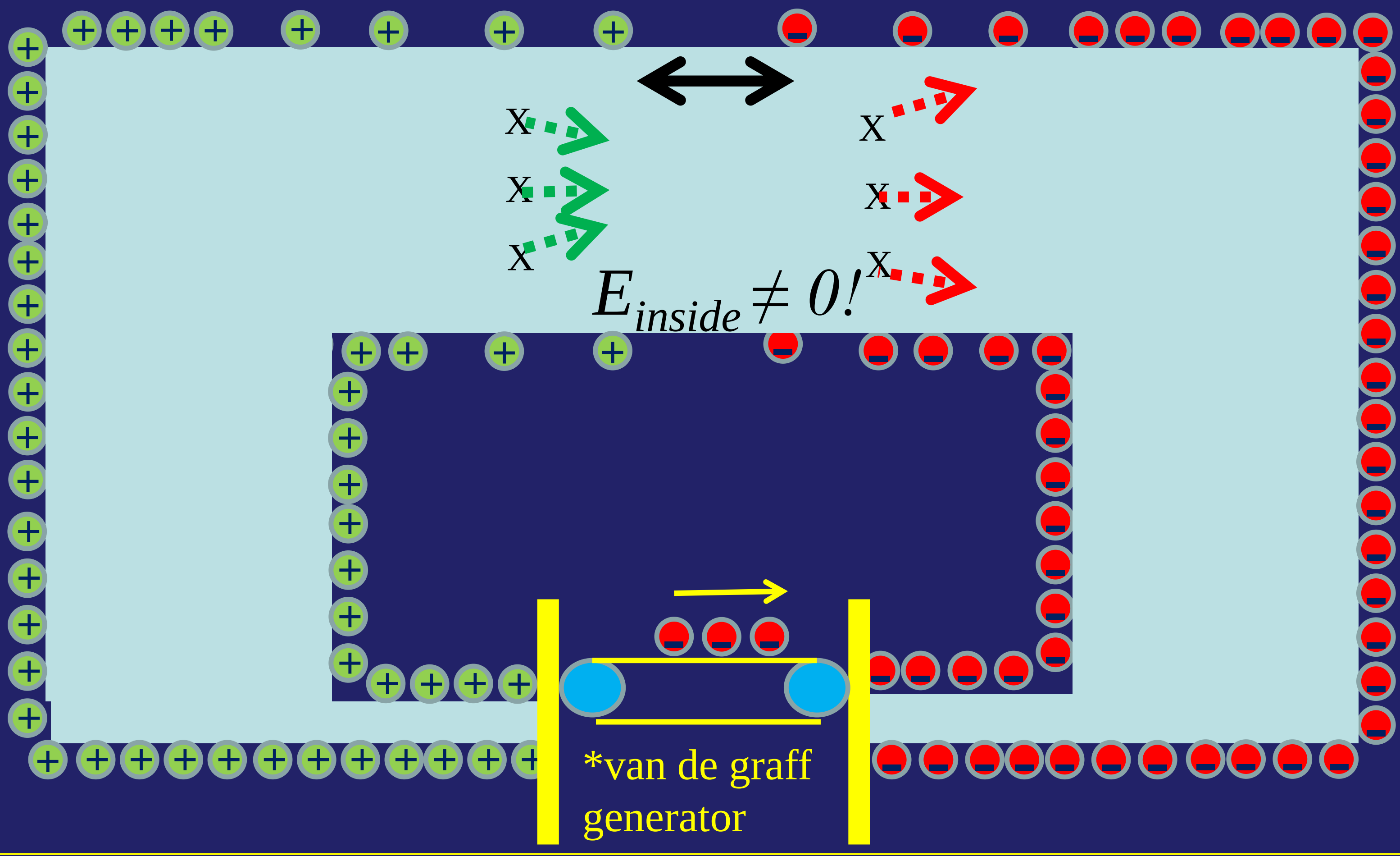
1) Are there any charges left on the gap faces?

2) *For a short time*, what are the fields at ‘x’ ?



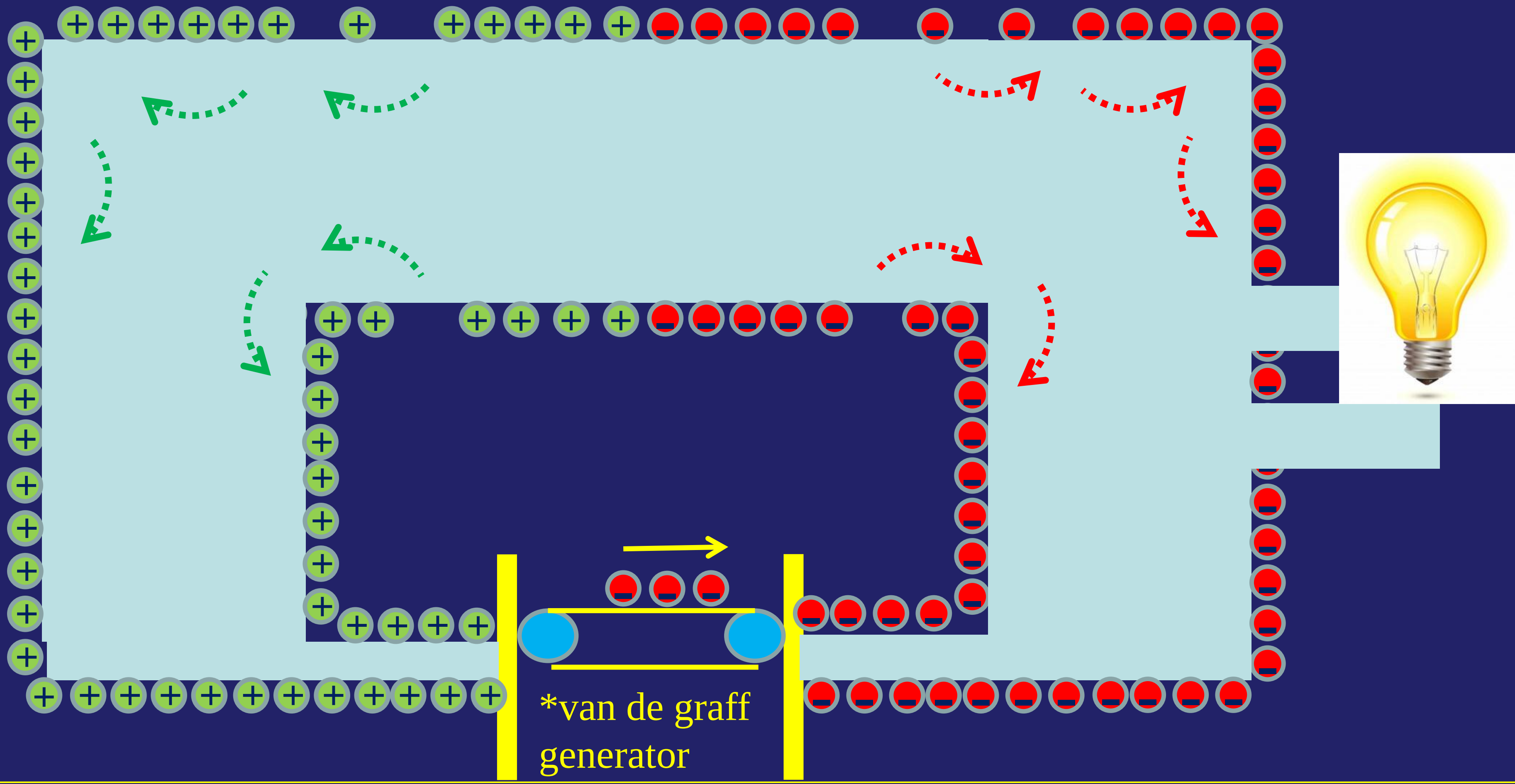
What is the result of this field disturbance?

Charge near the switch 'sees' the field and is nudged away



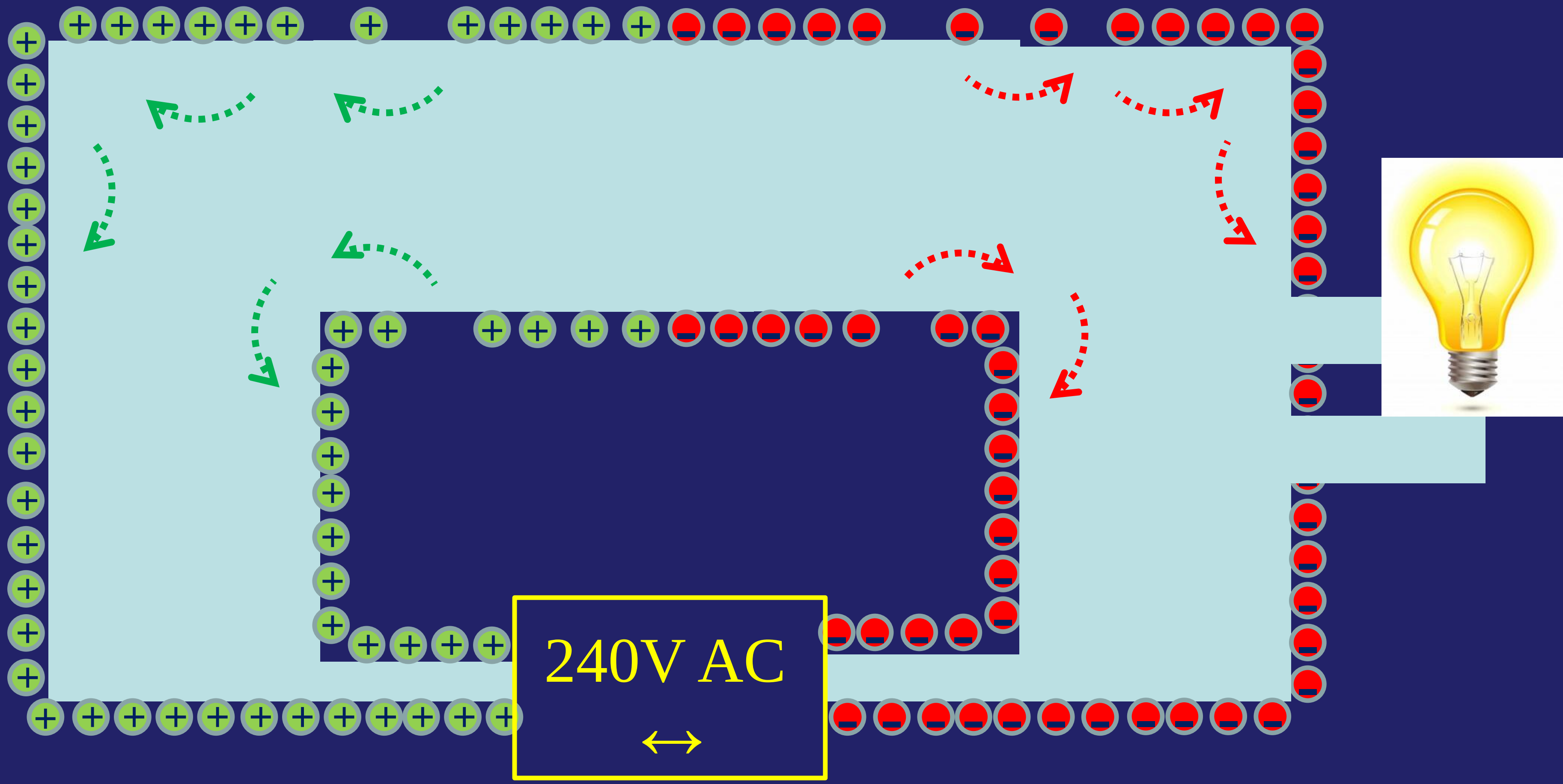
Any set of charges only needs to move a little bit

The disturbance is *conveyed by the field* at light speed.



The case for AC source is similar

Charges 'slosh' back and forth over short distance at 60 Hz



Logic thread of today's lecture

Light bulb goes on in ~ nanosec after switch is turned ON
How to explain?

Coulomb law of electric force

Force → Vector field

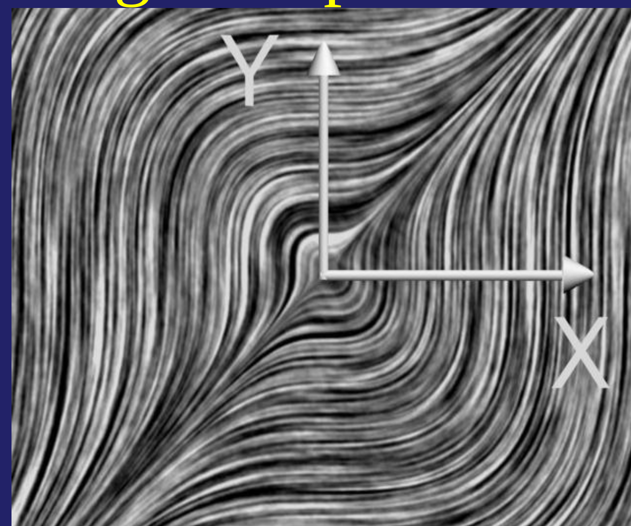
Use field to determine charge distribution in a conductor

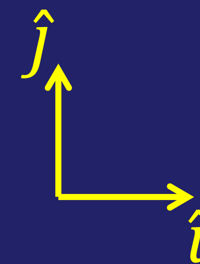
Determine how closing a switch creates a field disturbance, which nudges charges a short distance

Field disturbance travels close to speed of light to bulb

What are vector fields useful for, generally?

In *(x,y)* plane, this figure represents a vector field





The Vector field shown is….:

1. $\vec{E}(x,y) = x^2\hat{\imath} + y^2\hat{\jmath}$
2. $\vec{E}(x,y) = y^2\hat{\imath} + x^2\hat{\jmath}$
3. $\vec{E}(x,y) = \sin(x)\hat{\imath} + \cos(y)\hat{\jmath}$
4. $\vec{E}(x,y) = \cos(x)\hat{\imath} + \sin(y)\hat{\jmath}$
5. NOT SURE

Practice visualizing fields

Sketch the following vector function:

Use arrows of proper direction and length

$$\vec{E}(x,y) = \frac{1}{\sqrt{x^2 + y^2}}(-y\hat{\imath} + x\hat{\jmath})$$