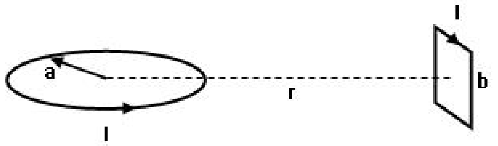
PH 103 : Electricity and Magnetism

Tutorial Sheet 8 : **Magnetic Materials**

1. Find the torque exerted on a square loop of side b due to a circular loop of radius a, at a distance r ($r \gg a, b$) as shown in the figure. The planes of the loop and the square are perpendicular to each other. [Ans. $\tau = -\frac{\mu_0}{4}\frac{(Iab)^2}{r^3}\,\hat{i}$]



2. A spherical shell of radius R and charge density σ is rotating with a constant angular velocity $\vec{\omega} = \omega_0\hat{k}$ about an axis though its centre. Find

   (a) the magnetic dipole moment

   (b) the field at the centre of the sphere and

   (c) the force of attraction between the northern and southern hemispheres of the shell.

   [Ans. (i). $\vec{m} = \frac{4\pi}{3}\sigma\omega_0 R^4\hat{k}$, (ii).$\vec{B} = \frac{2}{3}\mu_0\sigma\omega_0 R\hat{k}$, (iii). $\vec{F} = -\frac{\mu_0}{4}\pi\sigma^2\omega_0^2 R^4\hat{k}$]

3. A thin cylindrical glass rod of radius R, length L and surface charge density σ is set to rotate about its axis at an angular velocity ω. Find the magnetic field at a distance $r \gg R$ from the centre of the rod. [Ans. $B = \mu_0\omega\sigma R^3 L/\left\{4\left(\frac{L^2}{4} + R^2\right)^{3/2}\right\}$]

4. Find the magnetic field B at a point (r, θ) produced a magnetic dipole $\vec{m}$ kept at the origin along the $\hat{k}$ direction and show that it can be written in the coordinate independant form

$$\vec{B}(r,\theta) = \frac{\mu_0}{4\pi r^3}\left\{3\left(\vec{m}.\hat{r}\right)\hat{r} - \vec{m}\right\}$$

5. A cylindrical magnet of length $2L$ and radius R has a uniform magnetization $\vec{M} = M_0\hat{k}$.

   (a) Find the volume current density $\vec{J}_b$ and the surface current density $\vec{K}_b$. [Ans. $J_b = 0$, $\vec{K}_b = M_0\hat{\phi}$]

   (b) Find the magnetic field at a point P$(0, 0, z)$ where $|z| > L$. The origin of the coordinate system is fixed at the centre of the cylinder.

   [Ans. $\vec{B} = \frac{\mu_0 K_b}{2}\left(\cos\theta_2 - \cos\theta_1\right)\hat{k}$, $\cos\theta_1 = \frac{z-L}{[R^2+(z-L)^2]^{1/2}}$, $\cos\theta_2 = \frac{z+2L}{[R^2+(z+2L)^2]^{1/2}}$]

6. An infinitely long cylinder of radius R, carries a frozen in magnetization $\vec{M} = cr\hat{k}$, where r is the distance from the axis of the cylinder and c is a constant. Find the bound currents.

7. A sphere of radius R has its centre at the origin of the coordinates and carries magnetization $\vec{M} = M_0\hat{k}$. Calculate $\vec{J}_b$ and $\vec{K}_b$ in Cartesian and spherical co-ordinate systems. [Ans. $J_b = 0$, $\vec{K}_b = M_0\sin\theta\hat{\phi}$]

8. A region is occupied by an infinite slab of permeable material of constant relative permeability $\mu_r (= \mu/\mu_0) = 2.5$. Within the slab, the magnetic field (in $\mathrm{Wb/m^2}$) is given by

$$\vec{B} = 10y\,\hat{\imath} - 5x\,\hat{\jmath}$$

Determine $\vec{J}_f$, $\vec{J}_b$, $\vec{M}$ and $\vec{K}_b$.

[Ans. $\vec{J}_f = -\frac{6}{\mu_0}\hat{k}$, $\vec{J}_b = -\frac{9}{\mu_0}\hat{k}$, $\vec{M} = \frac{3}{5\mu_0}\left(10y\,\hat{\imath} - 5x\,\hat{\jmath}\right)$, $\vec{K}_b(z=0) = \frac{3}{5\mu_0}\left(10y\,\hat{\jmath} - +5x\,\hat{\imath}\right)$, $\vec{K}_b(z=2) = -\frac{3}{5\mu_0}\left(10y\,\hat{\imath} - +5x\,\hat{\jmath}\right)$]