PH 108 : Electricity and Magnetism

Tutorial Sheet 3 : **Conductorr, Electrostatic energy and method of images**

1. Three identical parallel metallic plates are kept with separation a and b between the first and second and the second and third plates, respectively. A charge +Q is placed on the central plate while the outer two plates are uncharged and connected together by a very thin conducting wire. Determine the charges on all the six surfaces. Assume edge effects are negligible.

2. Consider a conducting sphere A which is initially uncharged. Another conducting sphere B is given a charge $+Q$, brought into contact with A and then moved far away. The charge on B is then increased to its original value $+Q$ and again brought into contact with A. Show that if this process is repeated many times, the charge on A will tend to the limit $Qq/(Q-q)$, where q is the charge acquired by A after its first contact with B.

3. Three parallel plates A, B and C each of area $10\,\mathrm{cm}^2$, are maintained at potentials 50, 100 and 10 V, respectively The separation between A and B is 1 mm while that between B and C is 3 mm. Calculate the force required to prevent B from moving. [Ans. : 7.08×10^6 N]

4. A grounded conducting block of arbitrary shape has a spherical cavity of radius a within it. A point charge $+q$ is fixed at a distance x from the center C of the cavity where $x < a$. What is the potential at a point P on the surface of the cavity? What is the potential at C? [Ans: $V(P) = 0$; $V(C) = kq(\frac{1}{x} - \frac{1}{a})$].

5. Two thin co-axial conducting cylindrical shells of length L and radii $\sqrt{2}a$ and $2a$, respectively are kept on the x-y plane with the z-axis being the common axis. The inner cylinder is given a charge $+Q$ and the inner surface of the outer cylinder has a charge -Q. Calculate the electric flux through an infinite plane surface at $x = a$ (i.e. parallel to the y-z plane). The plane extends from $z = 0$ to $z = L$ and $y = -\infty$ to $+\infty$. Neglect edge effects. Assume zero charge on the outer surface of the outer cylinder. [Ans.: $Q/12\epsilon_0$]

6. Three conducting spherical shells (A, B and C) with a common centre have radii a, b and c ($a < b < c$), respectively. B has a charge $+Q$ on it while A and C are grounded. Calculate (a) the charges on shells A and C (b) the potential on B and (c) the electric field in the regions (i) $a < r < b$ and (ii) $b < r < c$.

7. A charge of 3×10^{-10} C is placed on a conducting sphere of radius 1 cm. Another uncharged conducting sphere of radius 2 cm, which is far away from the charged sphere, is now connected by a fine conducting wire to the charged sphere. Calculate

   (a) the charge

   (b) the charge density

   (c) the potential of each sphere after they are connected together.

   [Ans: (a) $q_2 = 2q_1 = 0.2\,\text{nC}$, (b) 80 & 40 nC/m^2, (c) 90 V]

8. A spherical soap bubble carries a charge Q. Its surface tension is T. Determine its equilibrium radius. [Ans. : $8T\epsilon_0/\sigma^2$]

9. If the radius and the surface tension of a spherical soap bubble are r and T, respectively, determine the charge that must be given to the bubble so that the equilibrium radius doubles. Assume the air inside the bubble to behave ideally and the process of expansion isothermal. [Ans. $8\pi(\epsilon_0 r)^{3/2}(7Pr + 12T)^{1/2}$]

10. Find the force of repulsion between the two hemispheres of a conducting sphere with charge Q. [Ans. : $Q^2/32\pi\epsilon_0 R^2$, R - radius]

11. A sphere of radius R carries a total charge Q uniformly distributed over its volume. What is the force of repulsion between the northern and the southern hemispheres? [Ans. : $3Q^2/64\pi\epsilon_0 R^2$]

12. By assembling charges from the first principles, show that the potential energy of a charge Q which is uniformly distributed in a sphere of radius R is $3Q^2/20\pi\epsilon_0 R$.

13. A sphere of radius R carries a charge density $\rho(r) = kr$, where k is a constant. Calculate the electrostatic energy using three different methods. [Ans. : $\pi k^2 R^7/7\epsilon_0$]

Method of Images:

1. A grounded conducting sphere of radius R is centred at the origin. A linear charge distribution $\lambda(x) = \lambda_0(x^2/a^2)$ is distributed along the x-axis from $x = 2a$ to $x = 3a$. Find the total image charge.

   [Ans.: $-(5/2)\lambda_0 a$]

2. A point charge $+q$ is placed at a distance d from the centre of a conducting sphere of radius R ($d > R$). Show that, if the sphere is grounded, the ratio of the charge on the part of the sphere visible from $+q$ to that on the rest is $\sqrt{(d+R)/(d-R)}$.

3. A point charge $+8\,\text{nC}$ is placed at a distance of $10\,\text{cm}$ from the centre of an uncharged conducting sphere of radius $5\,\text{cm}$. Calculate the charge that will flow to earth if the sphere is now grounded. After the charge has flowed to earth, the connection to earth is broken and the $8\,\text{nC}$ charge is moved to a distance of $20\,\text{cm}$ from the centre of the sphere. What will be the potential of the sphere? [Ans.: $4\,\text{nC}$, $-360\,\text{V}$]

4. Two infinite conducting planes (both grounded and perpendicular to the x-y plane) meet at an angle of $60°$. A point charge $+q$ in the x-y plane has plane polar coordinates $(a, 20°)$. Find all the image charges and their positions in polar coordinates. [Ans.:$-q(a, -20°), -q(a, 100°), +q(a, -100°), -q(a, -140°), +q(a, 140°)$]

5. A circular ring of radius b has a uniform linear charge density λ and is placed on the x-y plane with the centre at the origin. Calculate the potential at point (0,0,z). A conducting sphere of radius a with $b = 2a$ is placed at the centre of the ring, with their centres coincident. Calculate the percentage change in the potential at the point (0,0,$2a$) when the conducting sphere is grounded. [Ans.: $(2\sqrt{2}/\sqrt{17}\ \times 100$]

6. A grounded conductor has a spherical cavity of radius R inside it. A point charge $+q$ is placed at a distance r $(r < R)$ from the centre of the cavity. Find the electrostatic force experienced by $+q$ and show that for $r \ll R$, this force is proportional to r. Find expressions for the charge density on the surface of the cavity at the two points on the diameter passing through $+q$. [Ans.: for $r \ll R$, $F \cong -q^2 r/4\pi\varepsilon_0 R^3$]