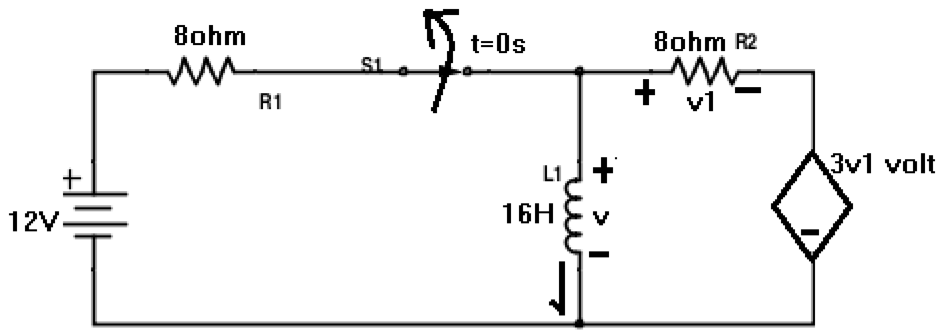
1)) **For the circuit shown in Figure below the switch opens at time *t* =0 s. Write a differential equation *in i(t)* for *t>=* 0 s. Find *i(t)* and *V(t)* for all time .**

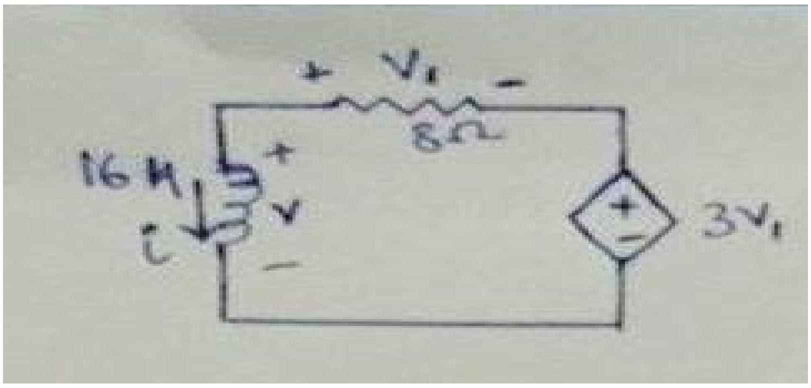


Solution

At t=0 or just before switch is open: Inductor acts as short circuit so no current flows through the 8 ohms resistor branch and $3V_1$ branch.

i(0)=12/8=1.5 amps

After t>=0 : $V_s = 0$, so circuit looks like:



By KVL in this loop :

$$-3V_1 + \mathrm{V} - V_1 = 0$$

$\mathrm{V} = 4V_1$

$V_1 = 8i$

So, V=-32i

$L\frac{d}{dt} = -32\mathrm{i}$

$\int \frac{di}{i} = \int -2dt + \ln C$

i=Ce^{-2t}

At t=0, i(0)=1.5 amps

So , i=1.5e^{-2t} amp

Hence,

When t<0   i=1.5 amp

t>=0   i=1.5e^{-2t} amp

for voltage

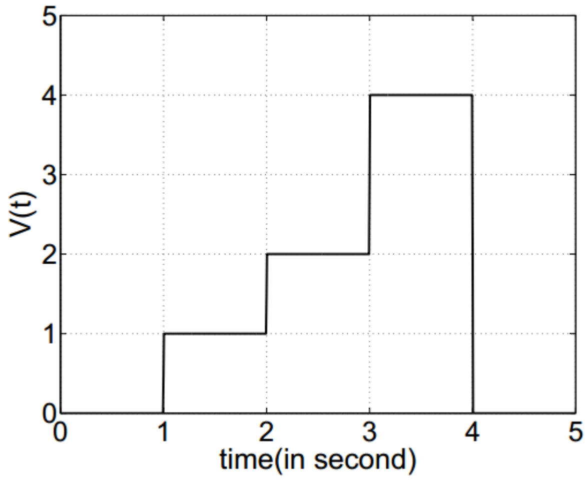
when t<0   V=0 volts

t>=0   V=16$\frac{d}{dt}$[1.5e^{-2t}]

V= (-)*16*1.5*2e^{-2t}

V=-48e^{-2t} $volts$

2) **The staircase input voltage wave form shown in Fig. (a) is applied across a series LR circuit consisting of R=1Ω and L=1H. Find the current through the circuit.**



Sol:- The given waveform is the summation of four unit step function

V(t)= u(t-1) + u(t-2) + 2u(t-3) - 4u(t-4)

= V_a + V_b + V_c + V_d

For RL ckt. Current i_L for u(t) is given by

$$i(t) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)u(t) \qquad where \quad \tau = \frac{L}{R} = 1$$

$$I_0 = \frac{V}{R}$$

for V_a I_0=1 A $\quad i_{La}(t) = \left(1 - e^{-(t-1)}\right)u(t-1)$

for V_b I_0=1 A $\quad i_{Lb}(t) = \left(1 - e^{-(t-2)}\right)u(t-2)$

for V_c I_0=2 A $\quad i_{Lb}(t) = 2\left(1 - e^{-(t-3)}\right)u(t-3)$

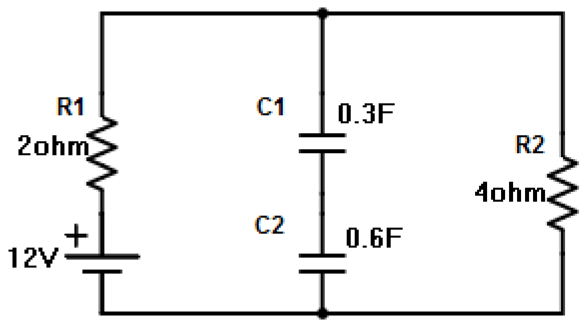
for V_d I_0=-4 A $\quad i_{Lb}(t) = -4\left(1 - e^{-(t-4)}\right)u(t-4)$

So, the total current due to this complete wave form would be

$$i_L(t)=i_{La}(t)+i_{Lb}(t)+i_{Lc}(t)+i_{Ld}(t)$$

TUTORIAL 2

3) [redacted]) **Given $V_{C1}(0^-) = 6$ and $V_{C2}(0^-) = 24$, Find V_o:**



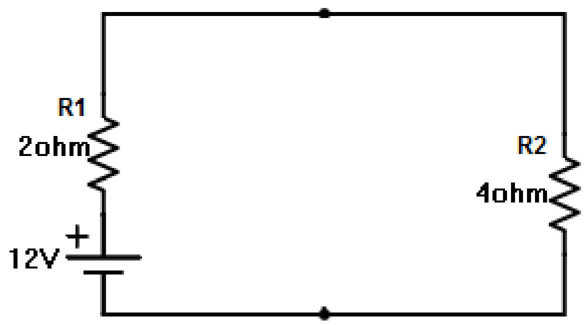
Solution:

At t>=0

$$V_o(0^-) = V_{C1}(0^-) + V_{C2}(0^-)$$

$$V_o(0^-) = 24 + 6 = 30 \text{ volts}$$

$$V_o(0^-) = V_o(0^+) = 30 \text{ volts}$$



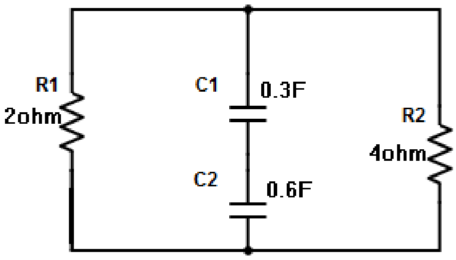
V_o= **(Initial value-Final value)**$e^{\frac{-t}{\tau}}$**+ (Final value)**

$$=[V_o(0^+) - V_o(0^-)]e^{\frac{-t}{\tau}} + [V_o(+\infty)] \qquad \textbf{(1)}$$

Final value at t=$+\infty$

$$V_o(+\infty_o) = \frac{12 * 4}{4 + 2} = 8\, volts$$

Time constant (τ)



$$C_{eq} = \frac{0.6 * 0.3}{0.3 + 0.6} = 0.2F$$

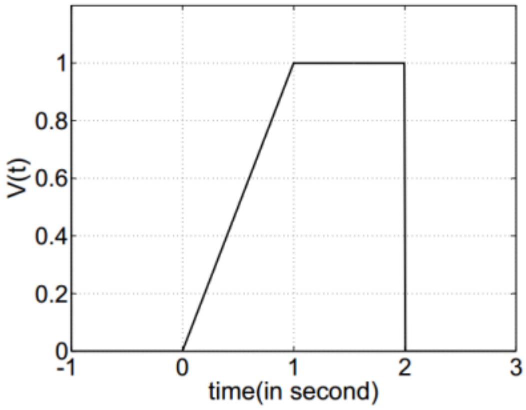
$$R_{eq} = \frac{2 * 4}{2 + 4} = \frac{4}{3}\, ohms$$

$$\tau = R_{eq}C_{eq} = 0.266\, seconds$$

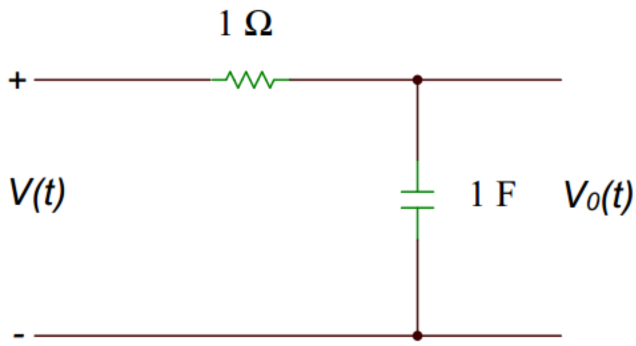
Hence, by (1)

V_o= (22) $e^{\frac{-t}{0.266}} + 8$

4)The input voltage wave form shown in Fig. (a) is applied across a RC circuit shown in Fig. (b). Find the output voltage across the capacitor C. Plot $i_c(t)$, $v_c(t)$.



(a)



(b)

Sol:- V(t) = t for 0<t≤1 Region 1

= 1 for 1<t≤2 Region 2

= 0 otherwise Region 3

As t<0 V(t)=0, so for t<0 ckt. was in steady state so $V_c(0_-) = 0 = V_c(0_+)$

For Region 1 V(t)=t.

using KVL $V_R + V_C = V(t)$

or, $Ri + \frac{1}{C}\int i\,dt = V(t)$

$$\frac{di}{dt} + i = \frac{dV}{dt} \qquad \text{as } R=1\Omega \text{ and } C=1F$$

$$\frac{di}{dt} + i = \frac{dV}{dt} \qquad \text{as } V(t)=t \text{ so } \frac{dV}{dt} = 1$$

So, the sol. $i = 1 + K_1 e^{-t}$ $K_1 = const.$

at t=0 i(0)=0 so, $0 = 1 + K_1$ so, $K_1 = -1$

so, $i(t) = 1 - e^{-t}$ ……….(1)

as, $V_c = \frac{1}{C}\int i\,dt$ $so,$ $V_C = t + e^{-t} + K_2$

as, Vc(0)=0 so, K_2= -1. So, $V_C(t) = t - 1 + e^{-t}$ ………(2) Reg. 1

For Region 2 using KVL $V_R + V_L = V(t)$

$$Ri + \frac{1}{C}\int i\,dt = V(t)$$

$$\frac{di}{dt} + i = \frac{dV}{dt}$$

$$\frac{di}{dt} + i = 0 \quad \text{as V(t)=1} \quad \text{so} \quad \frac{dV}{dt} = 0$$

So, $i = K_3 e^{-t}$

For this region at t=1 $i(1) = (1 - e^{-1}) = K_3 e^{-1}$ or, $K_3 = (e - 1)$

So, $i = (e - 1)e^{-t}$ ....(3)

as, $V_c = \frac{1}{C}\int i\, dt$ $so,$ $V_C = -(e - 1)e^{-t} + K_4$

at t=1 $V_C(1) = e^{-1}$

so, $e^{-1} = -(e - 1)e^{-1} + K_4$ $or,$ $e^{-1} = -1 + e^{-1} + K_4$

so, K_4=1. $V_C(t) = 1 + e^{-t} - e^{-(t-1)}$ ....(4) for Reg. 2

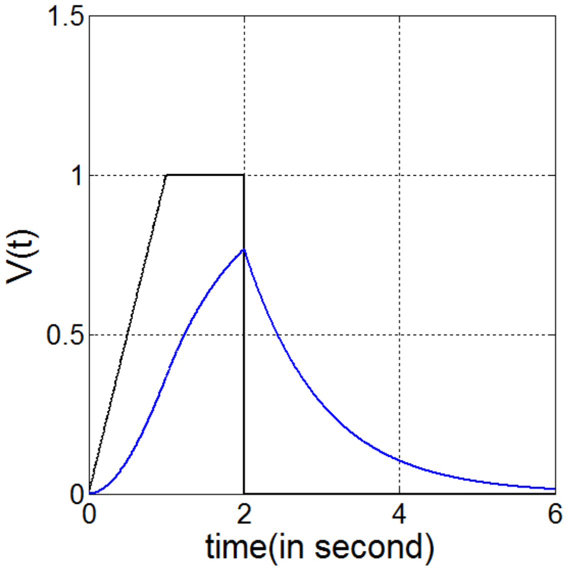
<u>For Region 3</u> as V(t)=0 for this region so $V_C(t) = V_0 e^{-t}$

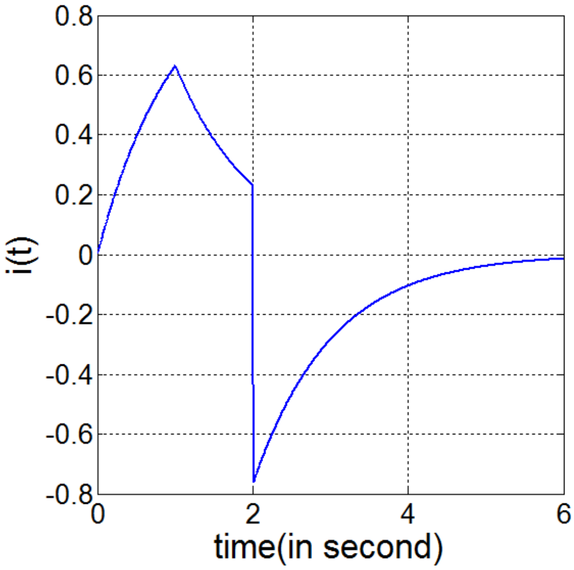
so $V_C(2-) = V_C(2+) = 1 + e^{-2} - e^{-1}$

so, at t=2 $(1 + e^{-2} - e^{-1}) = V_0 e^{-2}$ $so,$ $V_0 = e^2 - e^1 + 1$

$$V_C(t) = (e^2 + 1 - e)e^{-t}$$

or, $$V_C(t) = (e^{-t} - e^{-(t-1)} + e^{-(t-2)})$$





4) The given signal can be written as

$$V(t) = t\,u(t) - (t-1)u(t-1) - u(t-2)$$
$$= \quad V_a + \qquad V_b + \qquad\qquad V_c$$

For V_a for CR ckt we know

$$\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V}{RC}$$

As here R=1Ω and C=1 F so RC=1.

So eq. becomes $\frac{dV_C}{dt} + V_C = V$

As we know the sol. of $\frac{dx}{dt} + ax(t) = f(t)$

is $x(t) = e^{-at}\int e^{at} f(t)dt + Ae^{-at}$

so here $V_{ca}(t)$ will be $V_{ca}(t) = (t-1) + Ae^{-t}$

at t=0 Vc(0)=0 so A=1 So $\boldsymbol{V_{ca}(t) = t - 1 + e^{-t}}$,

For V_b $V_b = -(t-1)u(t-1) = -t'u(t')$ $\quad where \quad t' = t-1$

we know the response for t u(t)

so, the response for $-t'u(t') = -[t' - 1 + e^{-t'}]$

now, put $t' = t-1;$ $\quad so, \quad V_{bc} = -\left[(t-1) - 1 + e^{-(t-1)}\right]$

$$= -t + 2 - e^{-(t-1)}$$

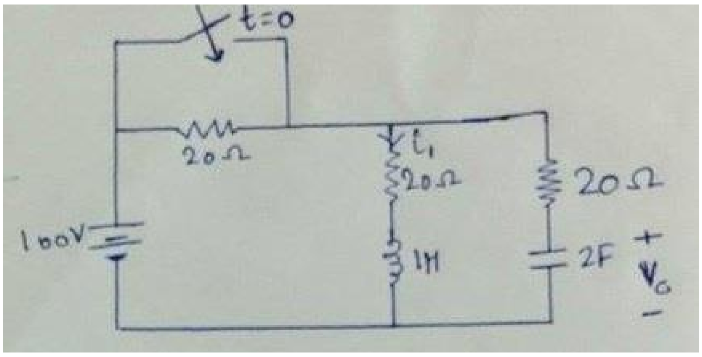
For V_c for V u(t) we know $V_C(t) = V\left(1 - e^{-\frac{t}{RC}}\right)$

so here for –u(t-2) $\quad V_{Cc}(t) = -\left(1 - e^{-(t-2)}\right)$

So, the final solution for Vc(t) will be

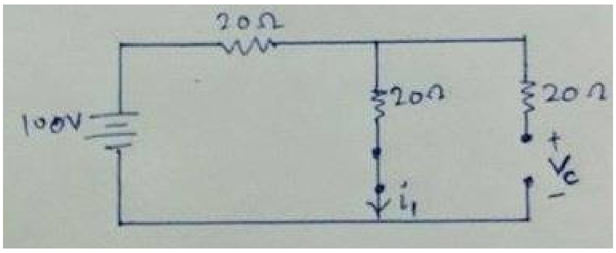
$$V_c = V_{ca}(t)u(t) + V_{cb}(t)u(t-1) + V_{cc}(t)u(t-2)$$

5) [redacted]) **FIND** $\frac{di_1}{dt}(0^+), \frac{dv_c}{dt}(0^+), i_1(0^+), v_c(\infty)$ **in the below figure**



Solutions:

At t=0^-: Inductor is short circuited and capacitor is open circuited

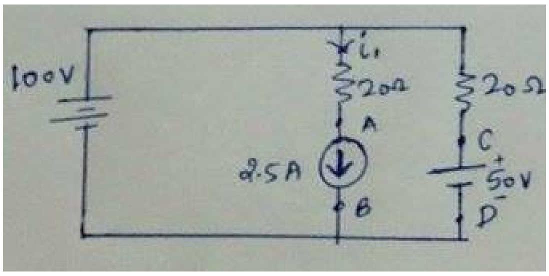


$$i_1(0^-) = \frac{100}{40} = 2.5\,A = i_1(0^+)$$

$$V_c(0^-) = 100 - 50 = 50\,Volts = V_c(0^+)$$

At t=0^+

Replace inductor by a current source of value $i_1(0^+)$=2.5 A and capacitor by voltage source of value $V_c(0^+)$



$$V_{20\ ohm} = 50 volts$$

$$i_{20\ ohm} = \frac{50}{20} = 2.5A$$

50=$V_L = \ L\frac{di}{dt} = 1\frac{di}{dt}$

50 = $\frac{di}{dt}(0^{+})$ A/sec

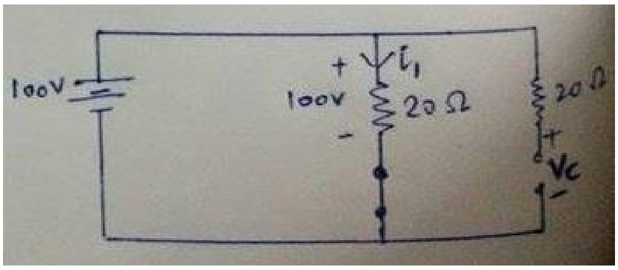
Also,

$$i_c = C\frac{dv_c}{dt}$$

2.5=$i_c = 2 * \frac{dv_c}{dt}$

$$\frac{dv_c}{dt}(0^{+}) = 1.25\ volts/sec$$

At t=+∞

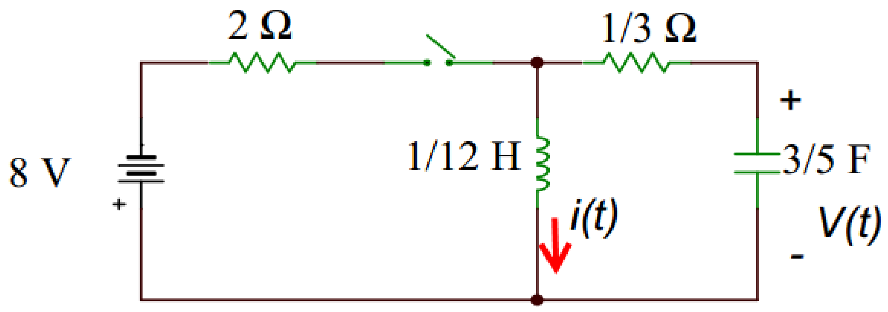


$$i_1 = \frac{100}{20} = 5A$$

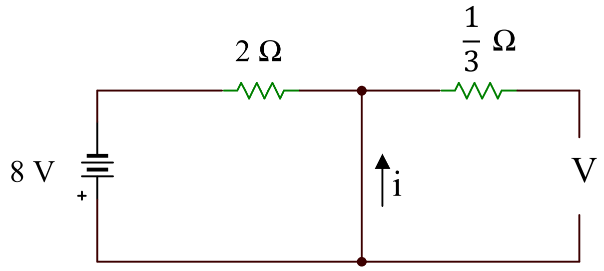
$$i_1(+\infty) = 5A$$

$$v_c(+\infty) = 100\ volts$$

6)For the circuit shown in the Fig.6 , the switch opens at time t = 0. Find V(t) and i(t) for all time.

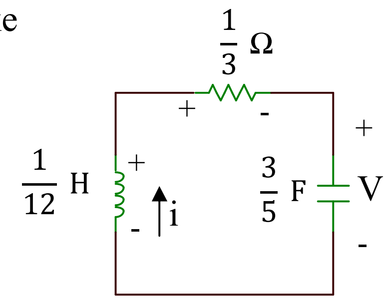


Sol:- for t<0 ckt. was in steady state so inductor was short and capacitor was open and the ckt. looks like



so i=4A and V=0 V

for t>0 the ckt. looks like



so, $L\frac{di}{dt}+Ri+V=0 \quad or \quad \frac{1}{12}\frac{di}{dt}+\frac{1}{3}i+V=0$ ……….(1)

as, $i=C\frac{dV_C}{dt} \quad or\ i=\frac{3}{5}\frac{dV_C}{dt}$ so eq. (1) becomes

$$\frac{1}{20}\frac{d^2V}{dt^2}+\frac{1}{5}\frac{dV}{dt}+V=0 \quad or \quad \frac{d^2V}{dt^2}+4\frac{dV}{dt}+20V=0$$

so, $\alpha=2 \ and\ \omega_n^2=20$ so, roots are $s=-\alpha\pm\sqrt{\alpha^2-\omega_n^2}=-2\pm 4j$

$$V_C(t)=e^{-2t}(A_1 cos4t+A_2 sin4t)$$

at t=0 $V_C(0) = 0;$ $so,\ A_1 = 0.$ $V_C(t) = A_2 e^{-2t} sin4t$

again $i_C = C\frac{dV_C}{dt} = \frac{3}{5} A_2[-2e^{-2t} sin4t + 4e^{-2t} cos4t]$

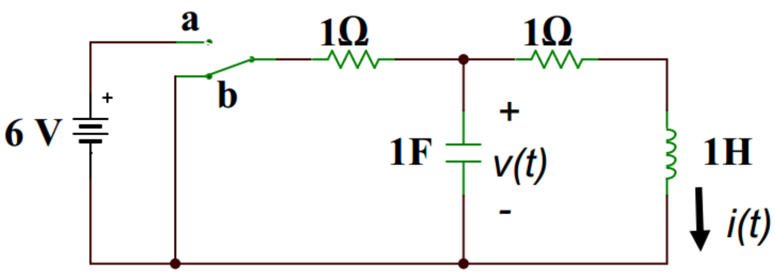
at t= 0 $i_C(0) = i_L(0+) = i_L(0-) = 4$ $so\ \frac{3}{5} A_2 4 = 4$ or $A_2 = \frac{5}{3}$

so, $V_C(t) = \frac{5}{3}\ e^{-2t} sin4t$

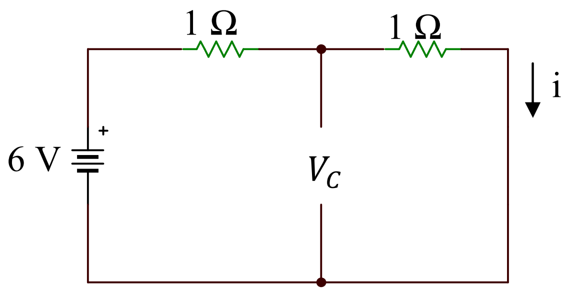
then $i_C(t) = C\frac{dV_C}{dt} = \frac{3}{5}\frac{5}{3}[-2e^{-2t} sin4t + 4e^{-2t} cos4t]$

$= e^{-2t}[4cos4t - 2sin4t]$

7) For the circuit shown in fig. switch was moved from position *a* to position *b* at time t=0; Find *i(t)* and *v(t)* for t>0.

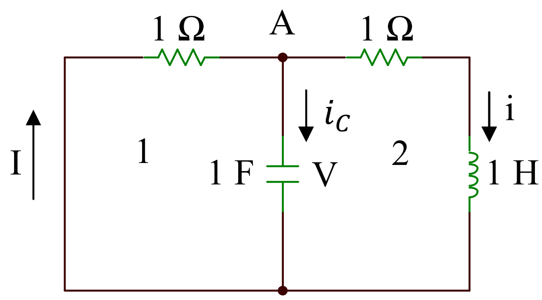


Sol:- for t<0 ckt. was in steady state and it looks like



$$so \quad V_C = 3V = V_C(0-) \;\; and \;\; i = 3A = i_L(0-)$$

for t>0 ckt. looks like



KVL in loop 2 gives $\quad V = Ri + L\frac{di}{dt} \quad or \;\; V = i + \frac{di}{dt}$ ……….(1)

applying KCL at node A we get $\quad I = i_C + i$

or, $\frac{0-V}{1} = C\frac{dV}{dt} + i \quad or \;\; -V = \frac{dV}{dt} + i$

putting V from eq. (1) $\quad -i - \frac{di}{dt} = \frac{di}{dt} + \frac{d^2i}{dt^2} + i \quad or \;\; \frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i = 0$

$$so \;\; \alpha = 1 \;\; and \; \omega_n^2 = 2 \qquad so, roots\ are \qquad s = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$$
$$= -1 \pm j$$

so, sol. of current i(t) is $i(t) = e^{-t}(A_1 cost + A_2 sint)$

at t=0, i(0)=3, so $A_1 = 3$ then $i(t) = e^{-t}(3cost + A_2 sint)$

from eq. (1) we $V(t) = i + \frac{di}{dt}$

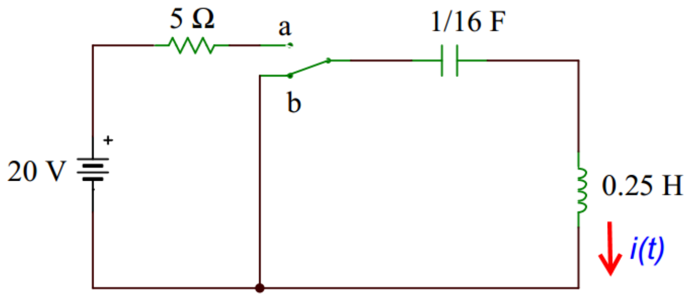
$$= e^{-t}(3cost + A_2 sint) - e^{-t}(3cost + A_2 sint)$$
$$+ e^{-t}(A_2 cost - 3sint)$$
$$= e^{-t}(A_2 cost - 3sint)$$

at t=0 V(0)=3 so A_2=3

so, *current* $i(t) = 3e^{-t}(cost + sint)$

voltage $v(t) = 3e^{-t}(cost - sint)$

8) For the circuit shown in fig. switch was moved from position *a* to position *b* at time t=0; Find *i(t)* for t>0.



Sol:- for t<0 ckt. was in steady state

so, $i(0-) = 0\ A \quad and \quad V_C(0-) = 20\ V$

for t>0 ckt. looks like as shown in the problem figure

so, $V_C + V_L = 0 \quad or, \quad CV_C + CV_L = 0$

$$C\frac{dV_C}{dt} + C\frac{dV_L}{dt} = 0 \quad or, \quad i + C\frac{d}{dt}\left(L\frac{di}{dt}\right) = 0$$

$$or, \quad \frac{d^2i}{dt^2} = -\frac{1}{LC}i = -64i \qquad so, i(t) = A_1cos8t + A_2\sin 8t$$

$i(0+) = i_L(0-) = 0 \qquad so, A_1 = 0 \quad so, i(t) = A_2sin8t$ ....(1)

$at\ t = 0+ \quad so, \quad V_C(0+) + V_L(0+) = 0; \qquad so, V_L(0+) = -V_C(0+) = -20$

$$so, \quad V_L(0+) = L\left.\frac{di}{dt}\right|_{t=0+} = -20; \quad so, \quad \left.\frac{di}{dt}\right|_{t=0+} = -80$$

$$so, from\ eq.(1)\ \frac{di}{dt} = 8A_2cos8t$$

at t=0+ $\quad -80=8A_2 \qquad A_2=-10 \qquad$ so, current $i(t)=-10\sin8t$