Quiz 1: CS 215

Name: ____________________ **Roll Number:** ____________________

Attempt all four questions. Each question carries 10 points for a total of 40.

Useful Information

1. Binomial theorem: $(x+y)^n = \sum_{k=0}^{n} C(n,k)x^k y^{n-k}$
2. The empirical mean of n independent and identically distributed random variables is approximately Gaussian distributed. The approximation accuracy is better when n is larger.
3. Defining $\Phi(x) = \int_{-\infty}^{x} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx$, we have the following table:

n	$\Phi(n) - \Phi(-n)$
1	68.2%
2	95.4%
2.6	99%
2.8	99.49%
3	99.73%

4. Integration by parts: $\int u dv = uv - \int v du$.
5. Gaussian pdf: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$
6. Poisson pmf: $P(X=i) = \frac{e^{-\lambda}\lambda^i}{i!}$

Additional space

1. Suppose I gather some n independent measurements of a quantity. Let us treat the measurements as independent random variables with mean μ and standard deviation σ. If I want to be 99% certain that the average of these measurements is accurate to within $\pm\frac{\sigma}{4}$ units, how many measurements must I take, i.e. what is the value of n? [10 points]

2. The joint density of random variables X and Y is given as $f_{XY}(x, y) = xe^{-(x+y)}$ when $x > 0, y > 0$ and 0 otherwise. Deduce whether X and Y are independent and whether they are uncorrelated. Show all steps clearly. [10 points]

3. Let $X_1, X_2, ..., X_n$ be n independent random variables each having a $[0,1]$ uniform random distribution. Define the random variable $Y = \max_{1 \leq i \leq n} X_i$. Write an expression for the CDF, PDF and expected value of Y. [4+3+3=10 points]

4. Show that the sum of two independent Poisson random variables with mean λ_1 and λ_2 respectively is another Poisson random variable. What is its mean? Show all steps clearly. [10 points]

Additional space