Operational Amplifiers and Linear Op-Amp Circuits

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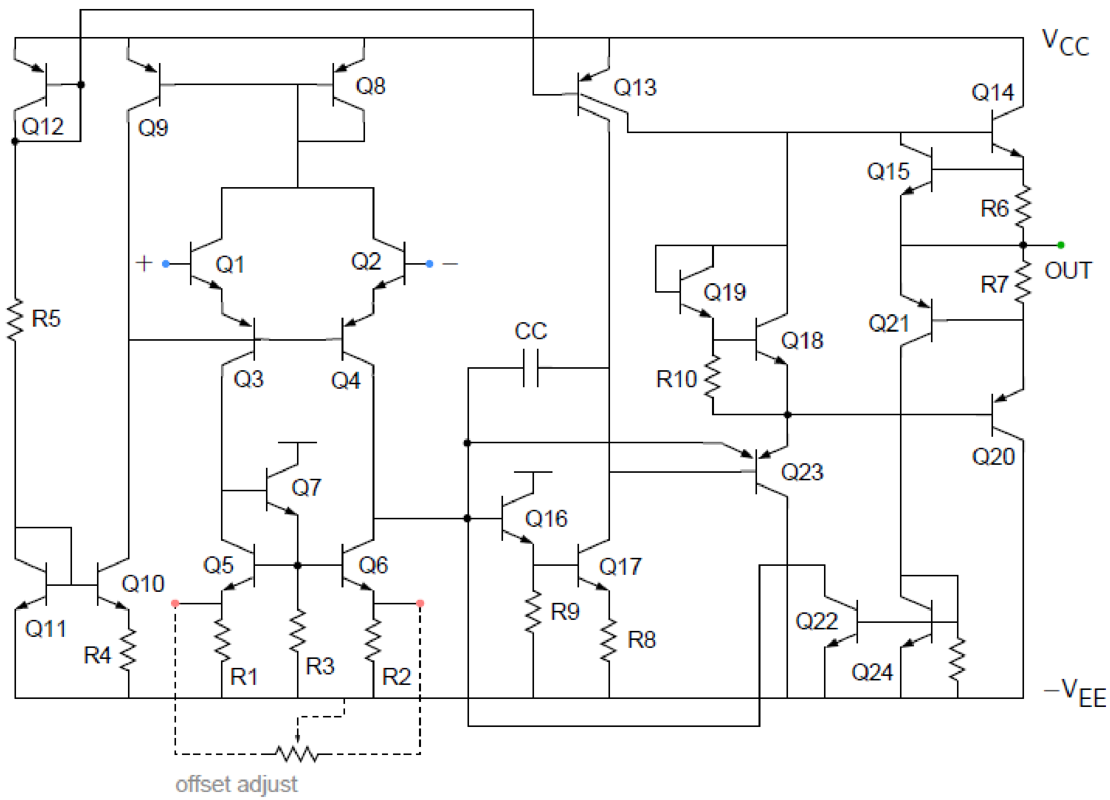
References: L. Bobrow's book and Prof. M. B. Patil's slides

Op-Amps

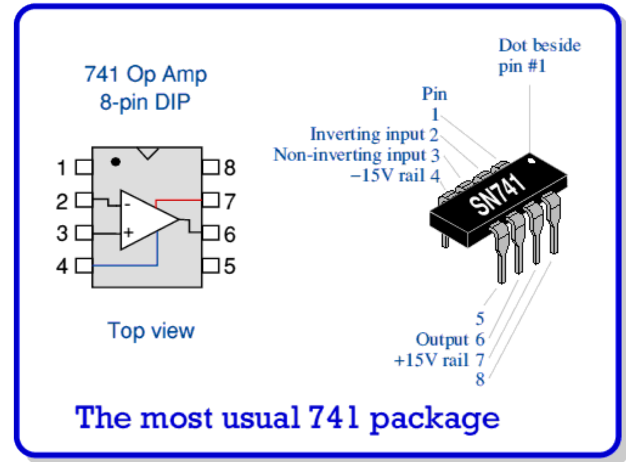
- Most basic Analog IC
  - Also called linear amplifier
- Why is it better than a BJT (ac) amplifier?
  - No need of coupling and bypass capacitors
    - Low frequency response can go to dc
    - Easy fabrication
      - Smaller in size and economical
- Versatile building block for large variety of electronic circuits
  - Oscillators, comparators, rectifiers, filters etc.
- Works with dc voltages also → applications in sensing, e.g. temperature, pressure etc.
- Nearly ideal characteristics → ckts work as per theoretical design

Op Amp 741

Internal Ckt

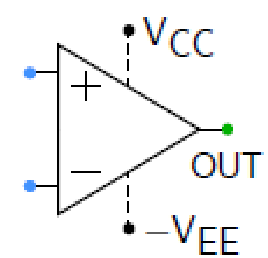


24 BJTs, 11 resistors and a capacitor



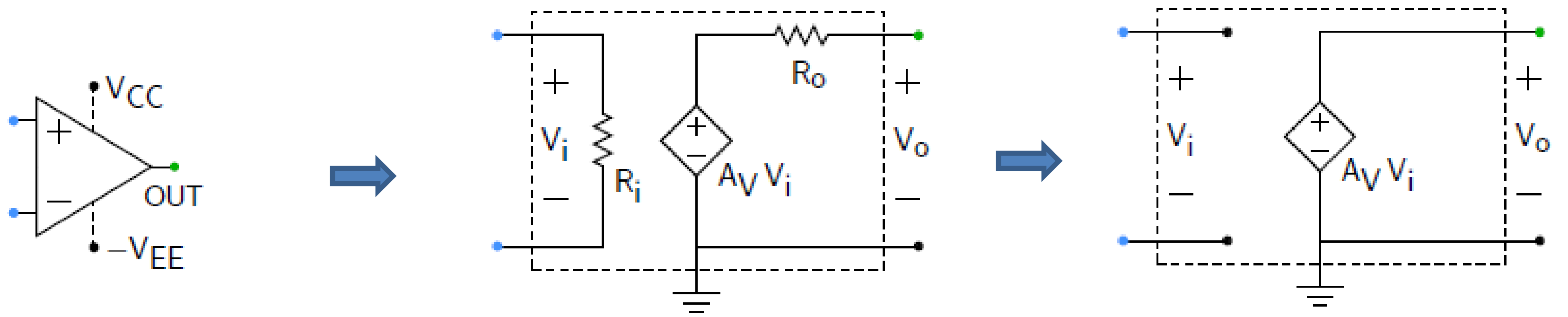
The most usual 741 package

Symbol



- The user need not worry about internal details
  - Easy to design circuits

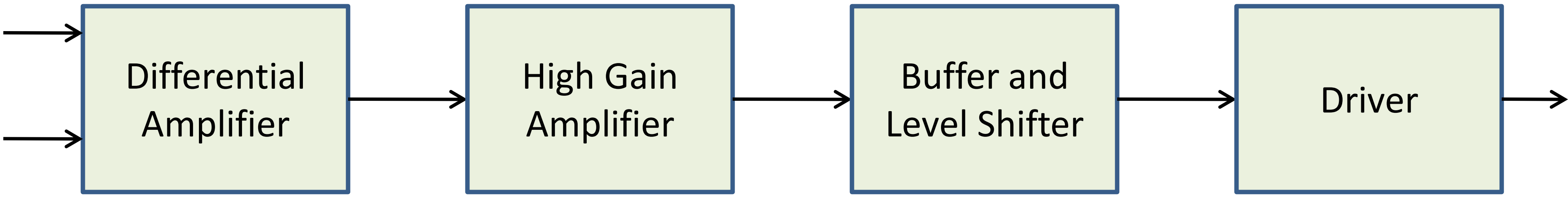
Equivalent Circuit



- Compared to external resistances, $R_i \rightarrow \infty$ and $R_o \rightarrow 0$
- V_{CC} and V_{EE} are supply voltages, typically +/-15 V
- Parameters

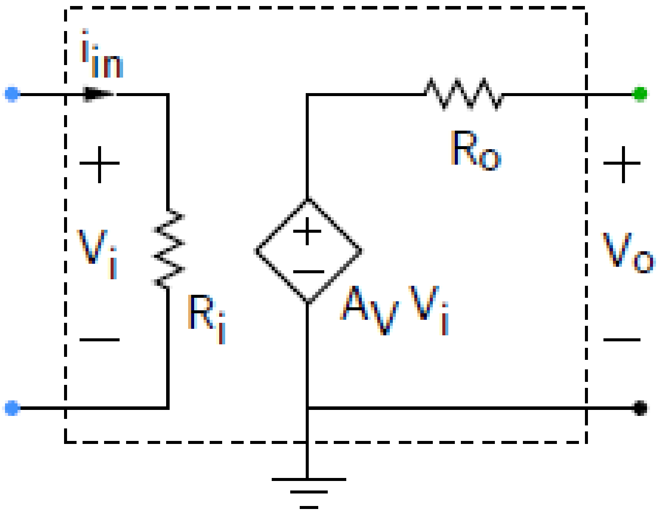
Parameter	Ideal Op-Amp	741
A_v	∞	10^5 (100 dB)
R_i	∞	2 MΩ
R_o	0	75 Ω

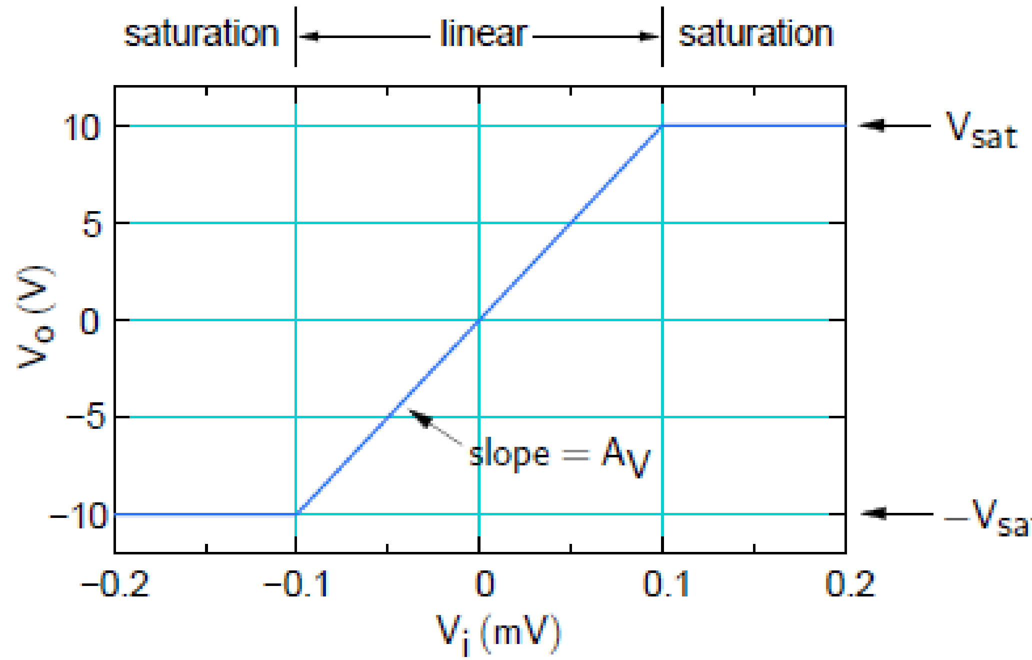
Op-Amp: Internal Blocks



- Differential amplifer provides high Rin
- High gain amplifier provides additional gain
- Buffer is typically an emitter follower
- Driver is a large signal (power) amplifier with low output resistance

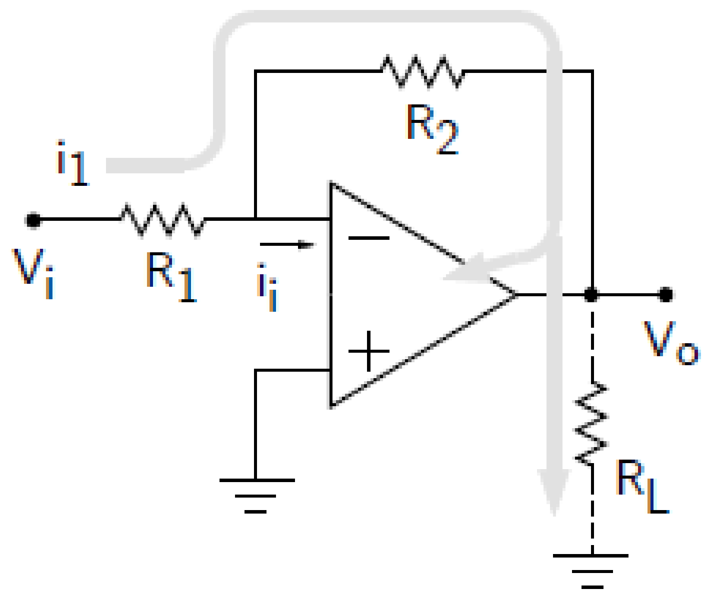
Modes of operation





- Two modes→ linear and saturation
  - Depends on magnitude of input voltage and feedback
- Output is limited to $V_{sat} \sim V_{CC}$-1.5 V
- In linear region
  - $V_i = V_+ - V_- = V_o/A_v \rightarrow 0$, hence **$V_+ \sim V_-$** ($V_+$ and V_- are virtually the same)
  - **$i_{in} \sim 0$**

Linear Op-Amp Circuits: Example 1



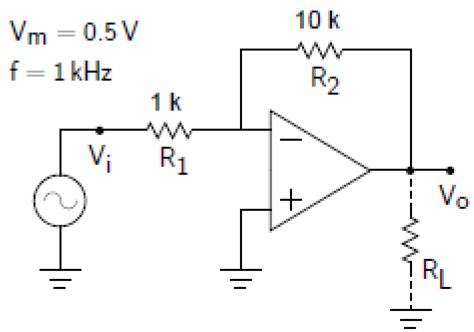
$$V_- \sim V_+ = 0V \Rightarrow i_1 = \frac{V_i}{R_1}$$

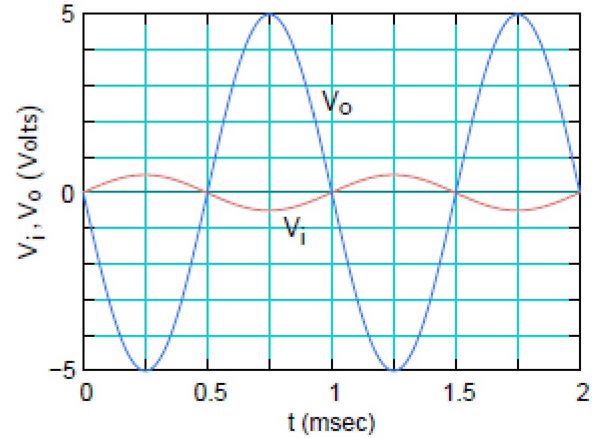
$$V_o = V_- - i_1 R_2 = 0 - \left(\frac{R_2}{R_1}\right) V_i = -\left(\frac{R_2}{R_1}\right) V_i$$

$$\boxed{V_o = -\left(\frac{R_2}{R_1}\right) V_i}$$

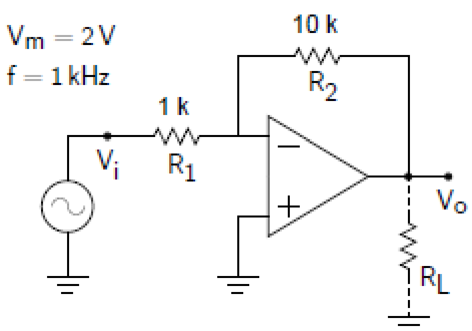
- Non inverting input is at real ground.
- Inverting input is at "virtual" ground.
- Inverting amplifier
- $-\left(\frac{R_2}{R_1}\right)$ is the "closed loop" gain. "Open loop" gain is the gain of the op amp (~10^5)
- Effective input resistance is $V_i/i_1 = R_1$

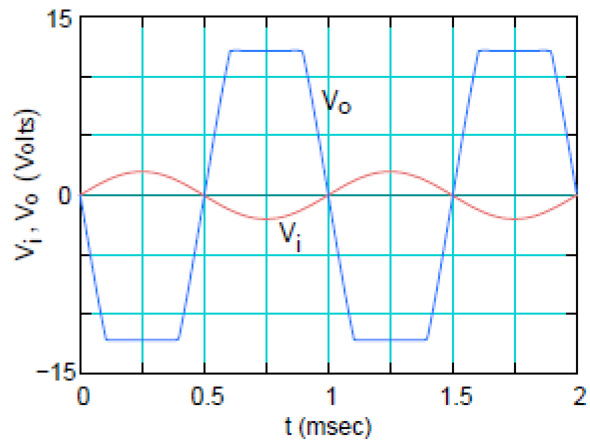
Simulation Examples



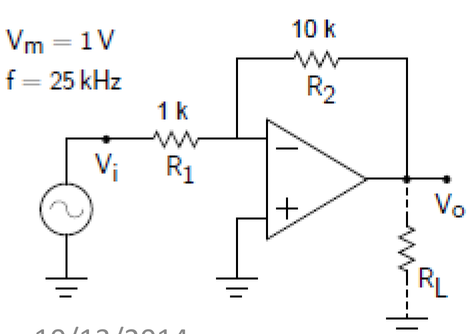


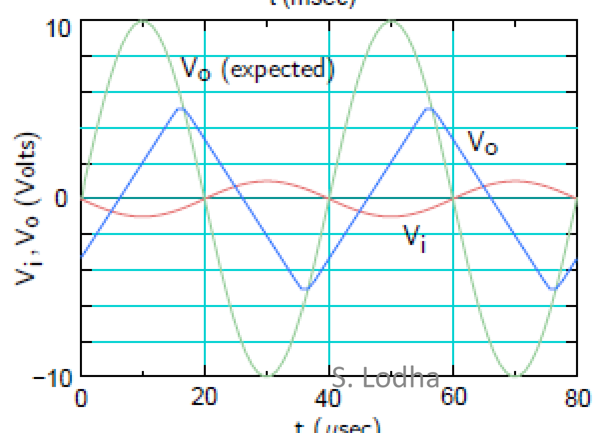
- Adjustable gain using R_2 and R_1
  - BJT → $-g_mR_T$ depends on biasing





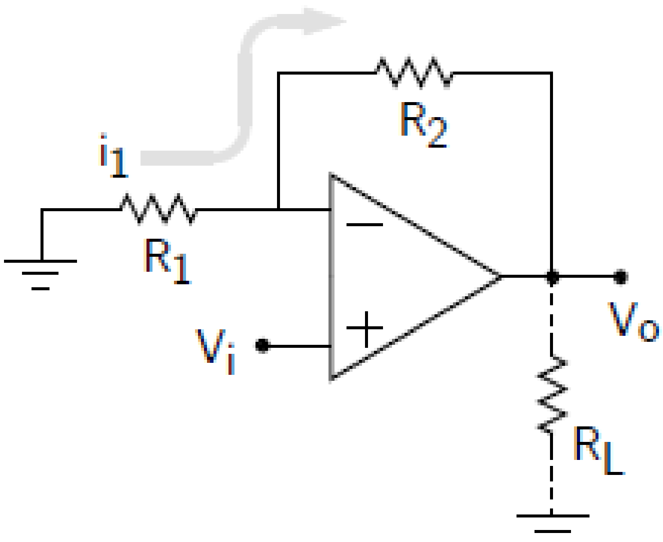
- Output voltage limited to V_{sat} ~ V_{CC}-1.5 V





- Output rise/fall not able to keep up with high frequency signals
  - Slew rate of 741 is 0.5 V/μsec

Linear Op-Amp circuits: Example 2



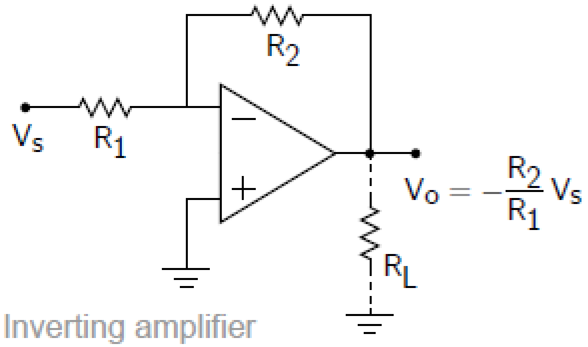
$$V_- \sim V_+ = V_i \Rightarrow i_1 = -\frac{V_i}{R_1}$$

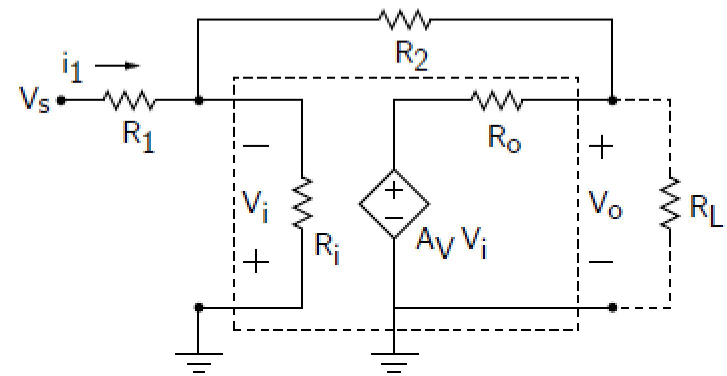
$$V_o = V_i - i_1 R_2 = V_i - \left(-\frac{V_i}{R_1}\right) V_i = \left(1 + \frac{R_2}{R_1}\right) V_i$$

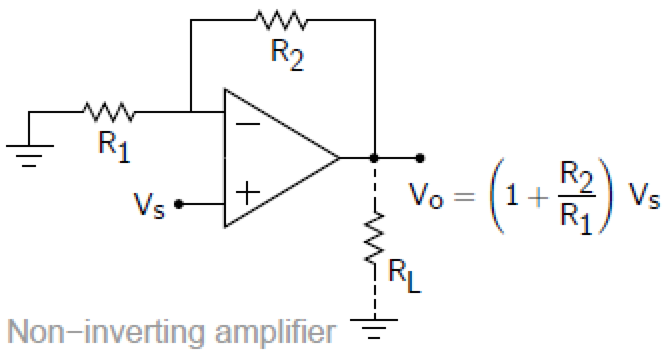
$$\boxed{V_o = \left(1 + \frac{R_2}{R_1}\right) V_i}$$

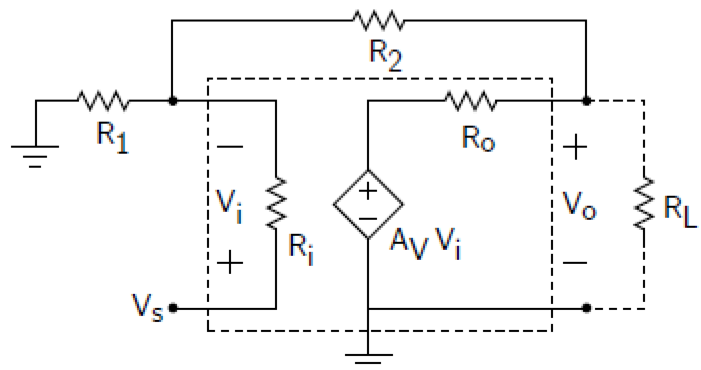
- Non-inverting amplifier
- Swapping terminals will change the entire circuit (negative → positive) feedback
- Effective input resistance is $R_i \rightarrow \infty$

Inverting or non-inverting?



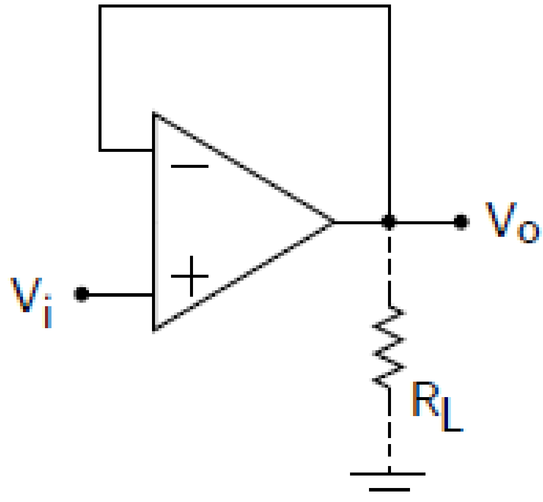






- For inverting amp, $R_{in}=V_s/i_1=R_1$
- For non-inverting amp, R_{in}~R_i, which is a few MΩ. Hence if large Rin is needed, non-inverting is preferred

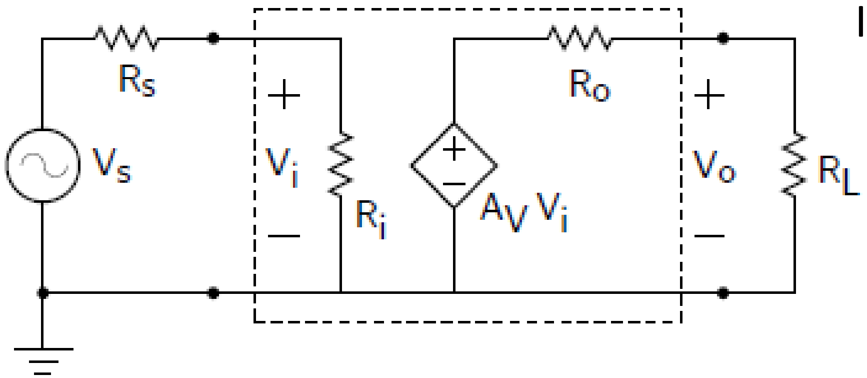
Linear Op-amp circuits: Example 3



$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_i = V_i$$

- Modified non-inverting amp, $R_1 = \infty$, $R_2 = 0$
- Unity-gain amplifier / voltage follower / buffer

Use of Buffer/voltage follower: Avoid Loading



Ideally $V_o = A_v V_s$

But $V_o = \dfrac{R_L}{R_o + R_L} A_v V_i = A_v \dfrac{R_L}{R_o + R_L} \dfrac{R_i}{R_i + R_s} V_s$

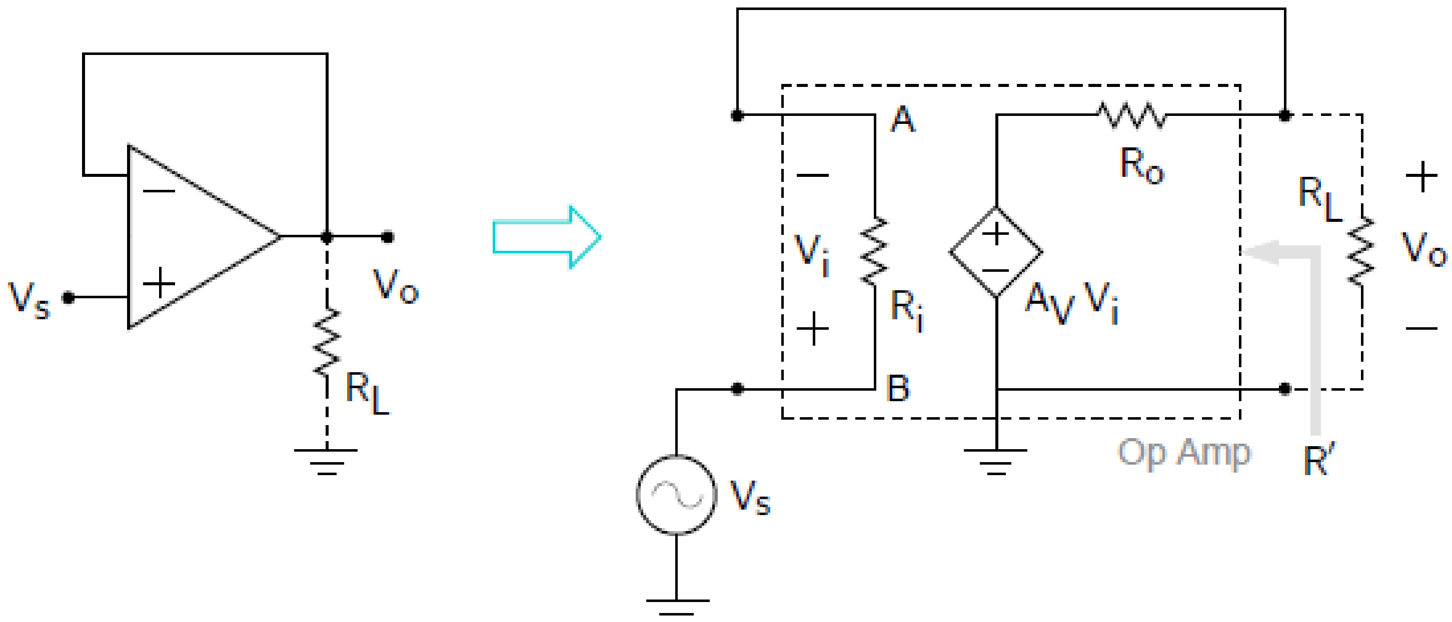
Requires $R_i \to \infty$

$R_o \to 0$

- Buffer/voltage follower provides both $R_i \to \infty$

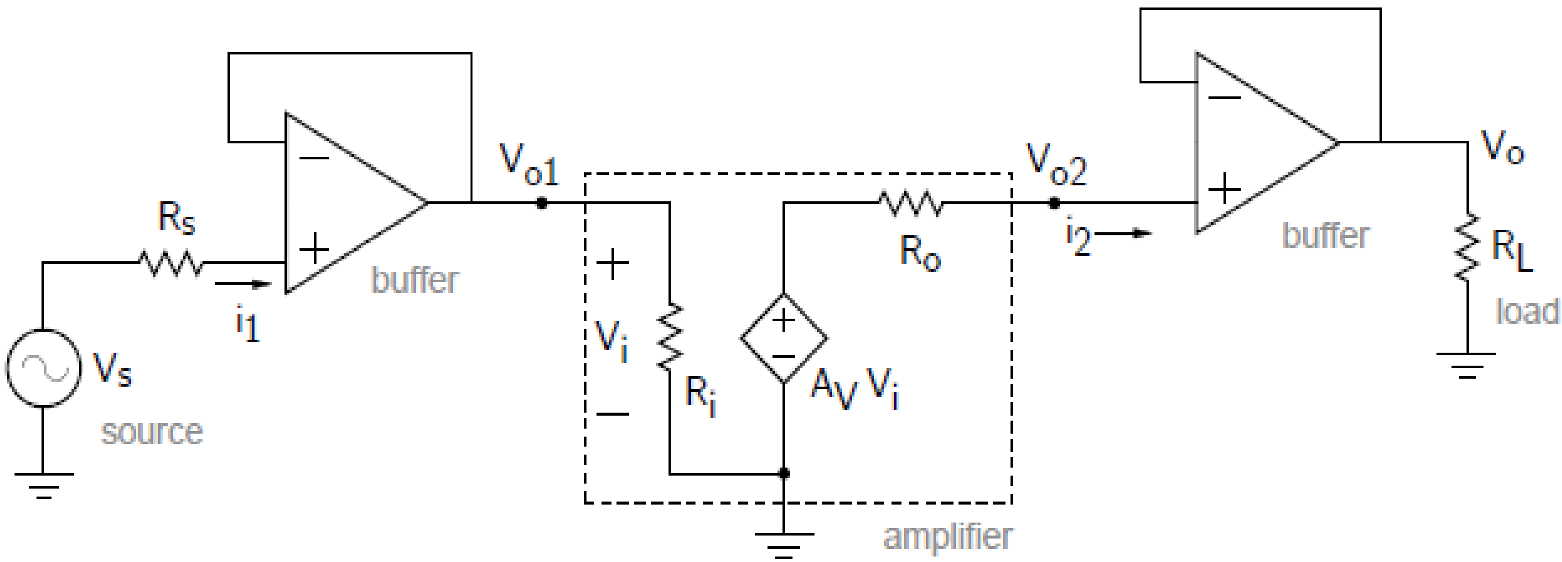
$R_o \to 0$

R_{in} and R_o of Buffer



- Current from V_s is small→ **High input resistance** due to high input resistance of the amplifier
- Output resistance is $R' = R_o$ → **low** because the output resistance of the amplifier is low

Use of buffer



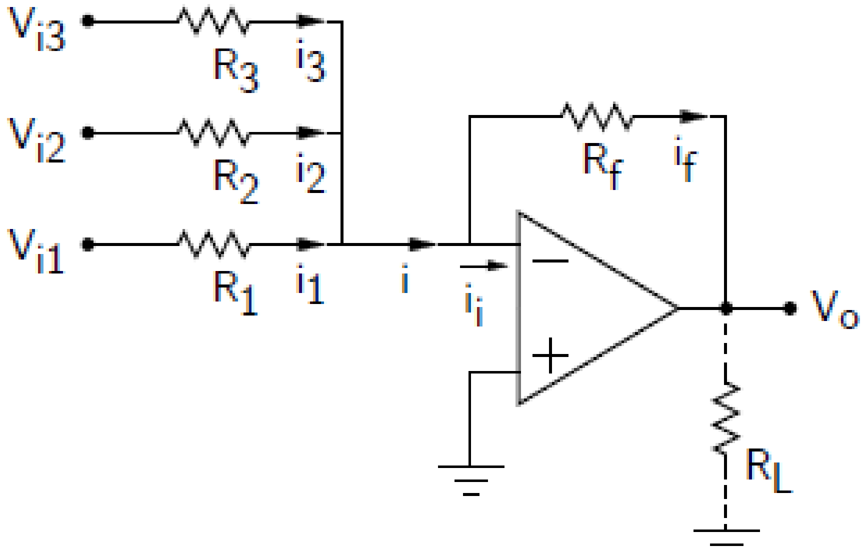
$$i_1 \approx 0, V_{o1} = V_s$$

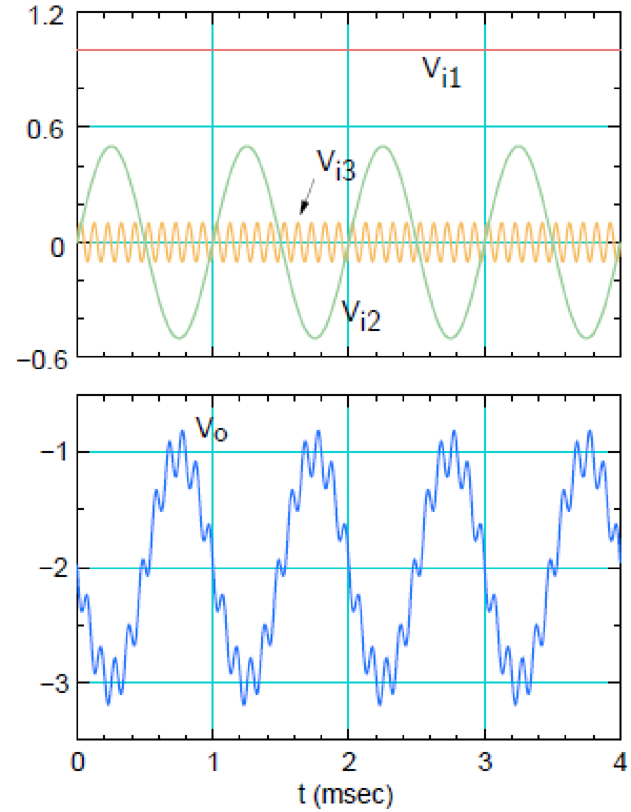
$$i_2 \approx 0, V_{o2} = A_v V_s$$

$$\boxed{V_o = V_{o2} = A_v V_s}$$

Irrespective of R_s and R_L

Linear Op-Amp circuits: Example 4



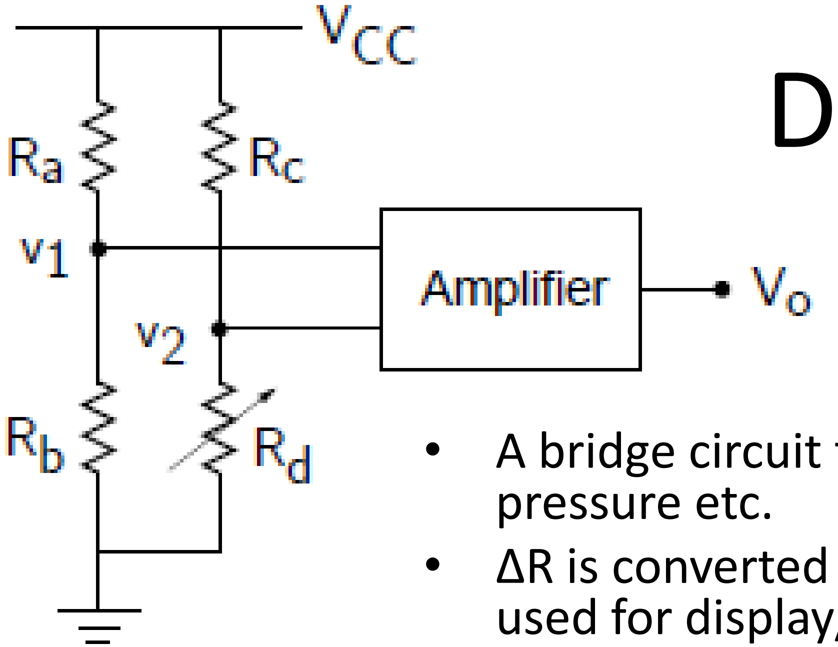


$$V_o = -\left(\frac{R_f}{R_1}V_{i1} + \frac{R_f}{R_2}V_{i2} + \frac{R_f}{R_3}V_{i3}\right) = -\frac{R_f}{R}(V_{i1} + V_{i2} + V_{i3})$$

$R_1 = R_2 = R_3 = R = 1k\Omega$

$R_f = 2k\ \Omega$

“Summer” if $R_1=R_2=R_3$



Difference Amplifier

- A bridge circuit for sensing changes in temperature, pressure etc.
- ΔR is converted to a signal voltage by bridge, amplified and used for display/control

$$R_a = R_b = R_c = R$$

$$R_d = R + \Delta R \longleftarrow$$ **Varies with quantity to be measured**

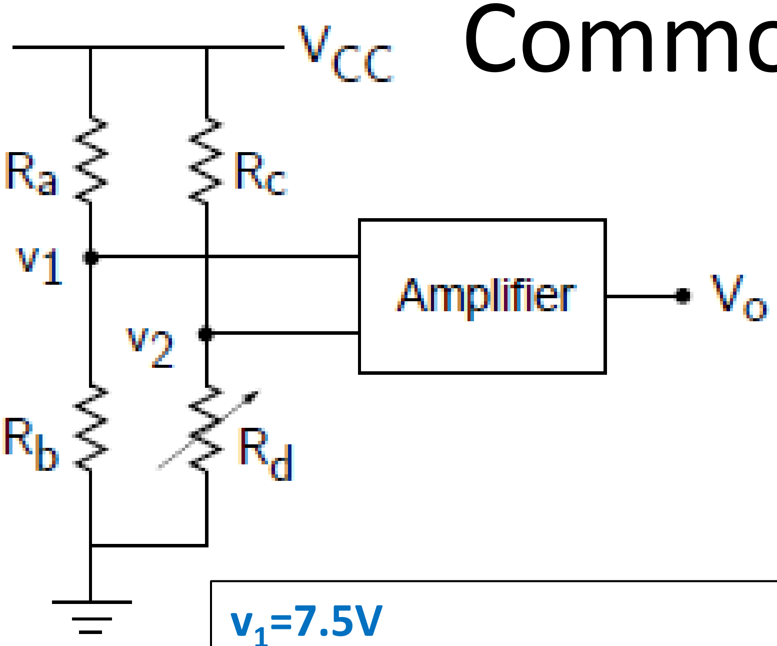
$$v_1 = \frac{R}{R+R} V_{CC} = \frac{1}{2} V_{CC}$$

$$v_2 = \frac{(R+\Delta R)}{R+(R+\Delta R)} V_{CC} = \frac{1}{2}\frac{1+x}{1+x/2} V_{CC} \approx \frac{1}{2}(1+x)(1-\frac{x}{2})V_{CC} = \frac{1}{2}(1+\frac{x}{2})V_{CC}$$

$$x = \frac{\Delta R}{R}$$

Example: V_{CC}=15V, R=1k, ΔR=0.01k
v_1=7.5V
v_2=7.5+0.0375 V

Common and differential mode



$$v_1 = \frac{1}{2} V_{CC}$$

$$v_2 = \frac{1}{2}(1 + \frac{x}{2}) V_{CC}$$

$$x = \frac{\Delta R}{R}$$

v_1=7.5V
v_2=7.5+0.0375 V
Amplifier should only amplify 0.0375V → signal from ΔR

Define:

$$v_c = \frac{1}{2}(v_1 + v_2)$$

Common-mode voltage

$$v_d = (v_2 - v_1)$$

Differential-mode voltage

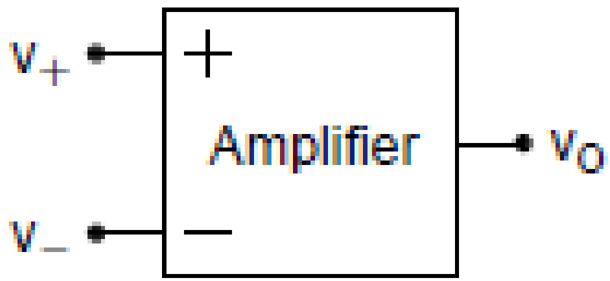
$$v_1 = v_c - \frac{v_d}{2}$$

$$v_2 = v_c + \frac{v_d}{2}$$

In example,

v_c~7.5 V, v_d~0.0375V

Common-mode rejection



$$v_+ = v_c + v_d/2$$

$$v_- = v_c - v_d/2$$

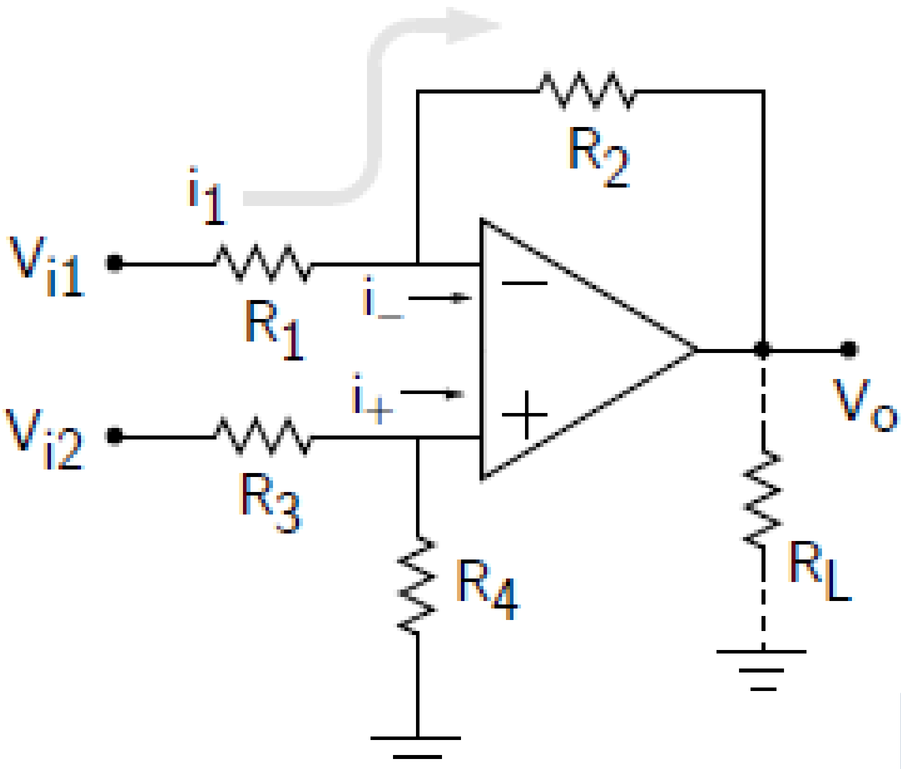
Ideally, $$v_o = A_d(v_+ - v_-) = A_d v_d$$ A_d = Differential gain

In practice $$v_o = A_d v_d + A_c v_c$$ A_c = Common-mode gain

Ability to reject common-mode signal $$CMRR = \frac{A_d}{A_c}$$ **C**ommon **M**ode **R**ejection **R**atio

For the 741 Op Amp, CMRR=90dB (~30,000)

Linear Op-Amps: Example 5 (Difference Amplifier)



$$i_+ = 0, V_+ = \frac{R_4}{R_3 + R_4} V_{i2}$$

$$i_1 = \frac{1}{R_1}(V_{i1} - V_+)$$

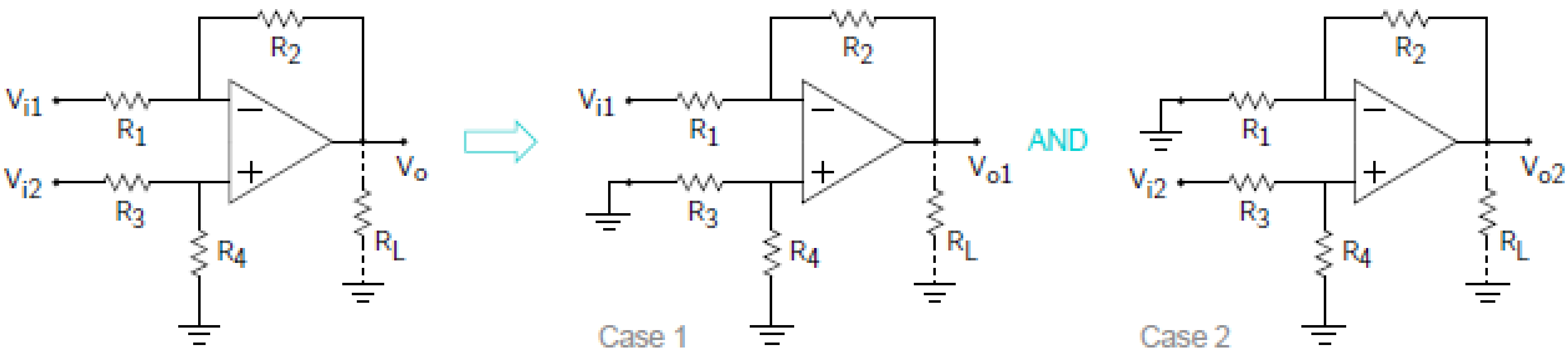
$$i_+ = 0 \Rightarrow V_o = V_+ - i_1 R_2 = V_+ - \frac{R_2}{R_1}(V_{i1} - V_+)$$

$$\frac{R_3}{R_4} = \frac{R_1}{R_2}$$

$$\boxed{V_o = \frac{R_2}{R_1}(V_{i2} - V_{i1})}$$

Difference Amplifier

Linear Op-Amps: Example 5 (Difference Amplifier)



Case 1 (Inverting Amp)

$$V_{o1} = -\frac{R_2}{R_1} V_{i1}$$

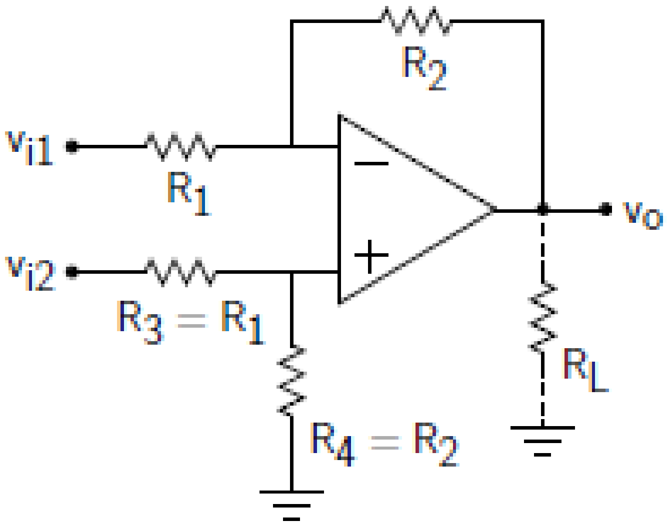
Case 2 (Non-inverting amp)

$$V_{o2} = (1 + \frac{R_2}{R_1}) V_+ = \frac{R_4}{R_3 + R_4} (1 + \frac{R_2}{R_1}) V_{i2}$$

$$\boxed{V_o = V_{o1} + V_{o2} = -\frac{R_2}{R_1} V_{i1} + \frac{R_4}{R_3 + R_4} (1 + \frac{R_2}{R_1}) V_{i2} = \frac{R_2}{R_1} (V_{i2} - V_{i1}) \text{ for } \frac{R_3}{R_4} = \frac{R_1}{R_2}}$$

Difference Amplifier

Difference Amplifier: Problem 1



$v_{i1} = v_c - v_d/2$

$v_{i2} = v_c + v_d/2$

$$\boxed{v_o = \frac{R_2}{R_1}(v_{i2} - v_{i1}) = A_d v_d}$$

Common-mode gain A_c=0

If $R_3 = R_1 + \Delta R$

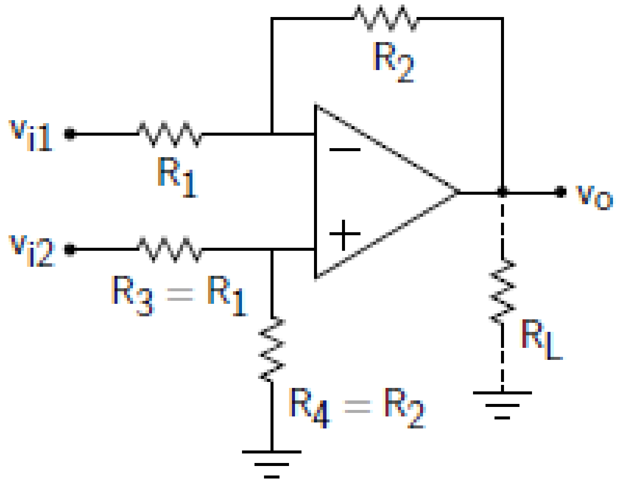
$$v_o = -\frac{R_2}{R_1} v_{i1} + \frac{R_2}{R_1 + \Delta R + R_2}(1 + \frac{R_2}{R_1}) v_{i2}$$

$$\boxed{v_o \cong \frac{R_2}{R_1}(v_d - x v_c) \rightarrow x = \frac{\Delta R}{R_1 + R_2}}$$

$$|A_c| = x\frac{R_2}{R_1} << |A_d| = \frac{R_2}{R_1}$$

Even though A_c is small w.r.t A_d (x~0.01), v_c>>v_d and hence common-mode amplification cannot be ignored

Difference Amplifier: Problem 1



$v_{i1} = v_c - v_d/2$

$v_{i2} = v_c + v_d/2$

If $R_3 = R_1 + \Delta R$

$$\boxed{v_o \cong \frac{R_2}{R_1}(v_d - x v_c)}$$

$$|A_c| = x\frac{R_2}{R_1}, |A_d| = \frac{R_2}{R_1}, x = \frac{\Delta R}{R_1 + R_2}$$

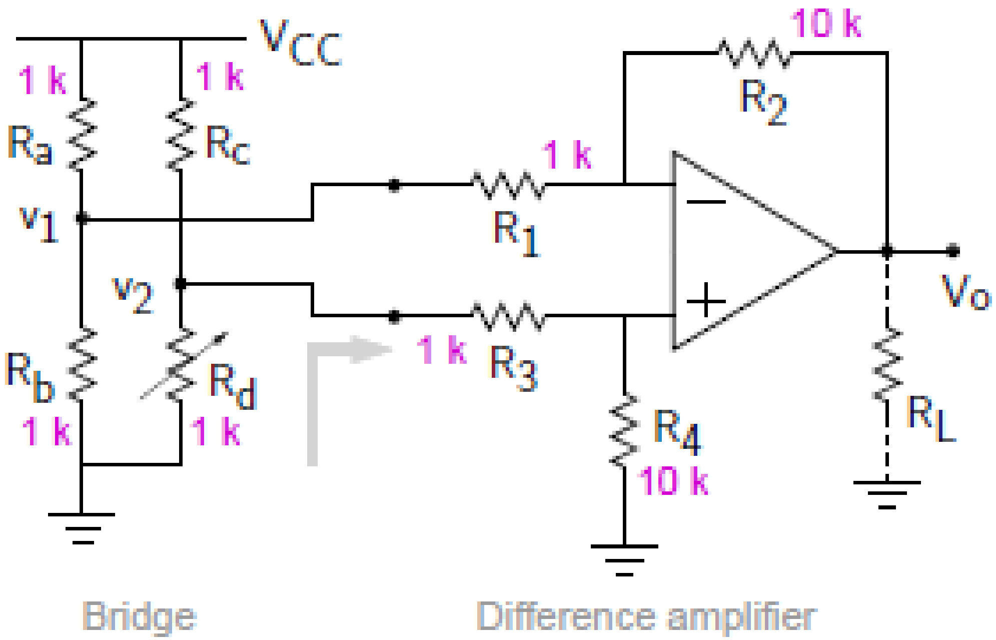
$$v_d = 0.0375V, v_c = 7.5V$$

$$R_1 = 1k, R_2 = 10k, x = 0.01$$

$$\boxed{\begin{array}{l} |A_c v_c| = x\frac{R_2}{R_1} v_c = 0.75V \\ \\ |A_d v_d| = \frac{R_2}{R_1} v_d = 0.375V \end{array}}$$

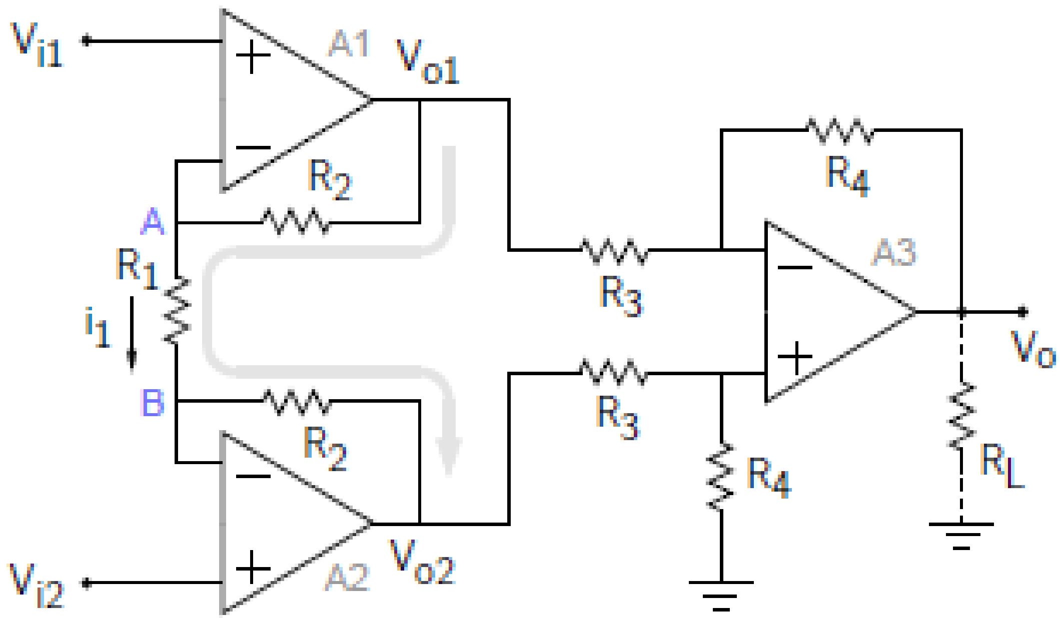
Substantial common-mode contribution at the output

Difference Amplifier: Problem 2



- Resistance seen from v_2 is R_3+R_4 → small enough to cause v_2 to change
- Need higher input resistance

Improved Difference Amplifier: Instrumentation Amplifier

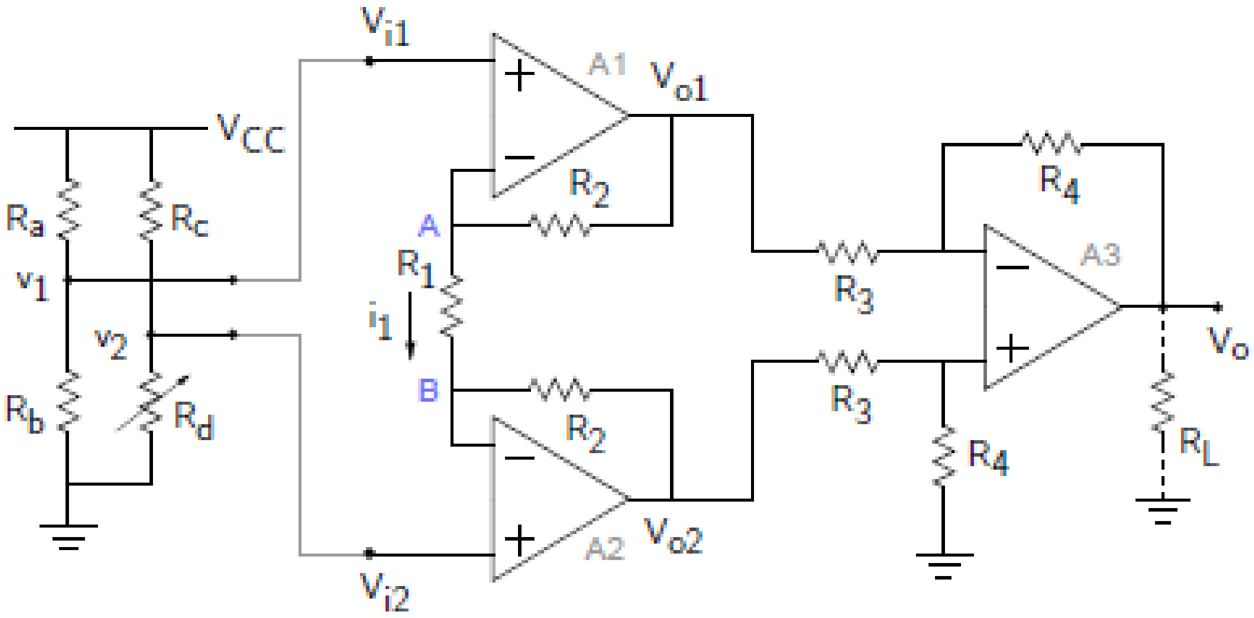


$$V_A = V_{i1,} V_B = V_{i2} \Rightarrow i_1 = \frac{V_{i1} - V_{i2}}{R_1}$$

$$V_{o1} - V_{o2} = i_1 (R_1 + 2R_2) = \frac{V_{i1} - V_{i2}}{R_1}(R_1 + 2R_2) = (V_{i1} - V_{i2})(1 + \frac{2R_2}{R_1})$$

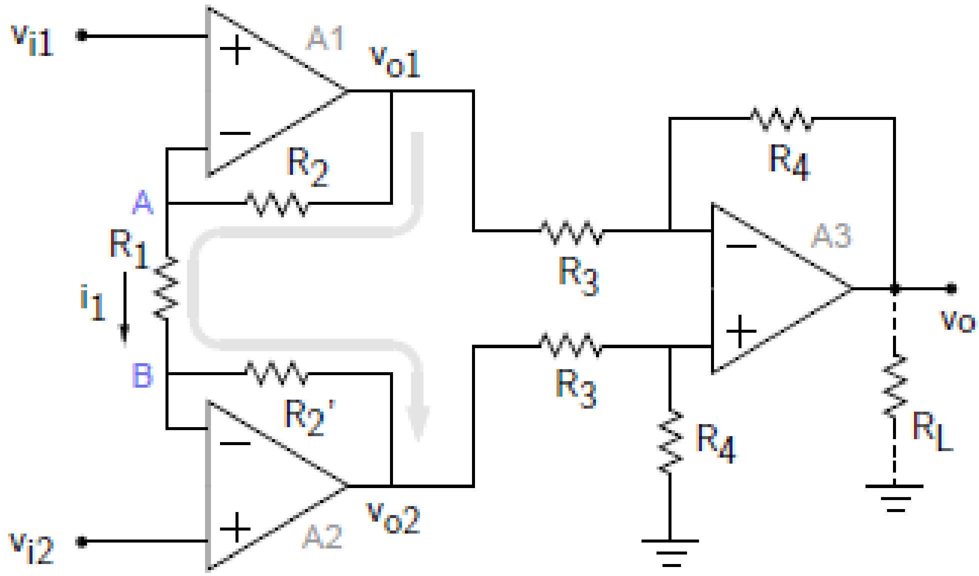
$$\boxed{V_o = \frac{R_4}{R_3}(V_{o2} - V_{o1}) = \frac{R_4}{R_3}(1 + \frac{2R_2}{R_1})(V_{i2} - V_{i1})}$$

Input Resistance?



- Input resistance seen from V_{i1} and V_{i2} is large
- Does not "load" the bridge circuit
  - v_2 does not change

Common mode contribution?



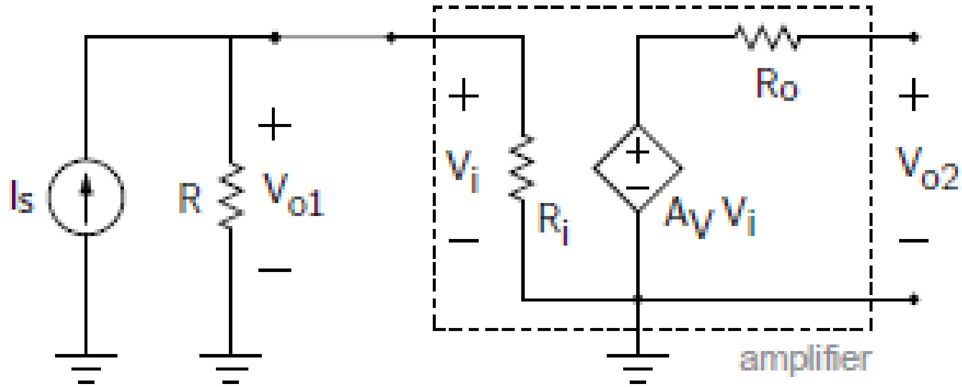
$v_{i1} = v_c - v_d/2$

$v_{i2} = v_c + v_d/2$

$$v_A = v_c - \frac{v_d}{2}, v_B = v_c + \frac{v_d}{2} \Rightarrow i_1 = \frac{v_A - v_B}{R_1} = -\frac{v_d}{R_1}$$

- What happens to the large common-mode component v_c?
- Gets cancelled (even if R_2 and R_2' are not matched)

Current→Voltage Conversion



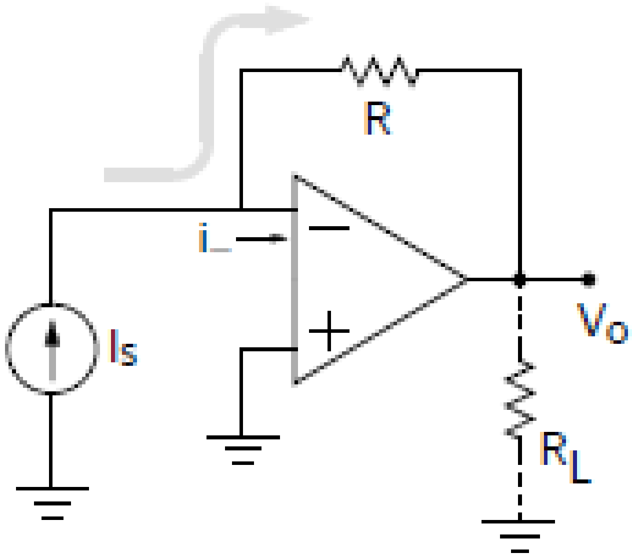
$$V_{o1} = I_s R$$ Ideally

$$V_{o1} = I_s (R_i \parallel R)$$ In reality, R_i affects V_{o1}

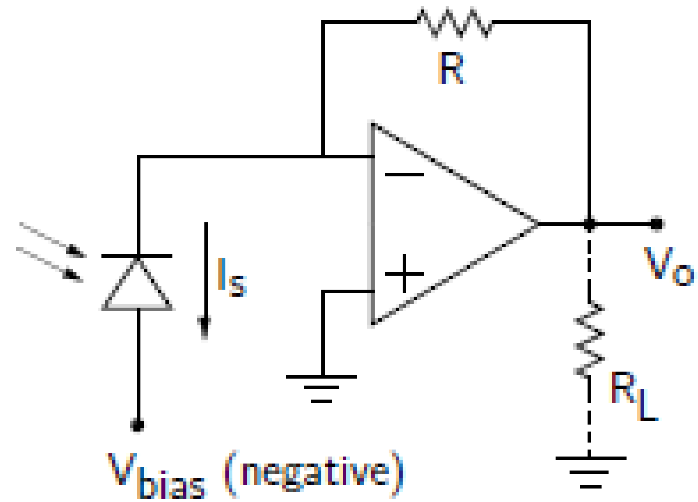
Some circuits produce current output that needs to be converted to voltage to simplify further processing

If next stage is an amplifier, it can modify V_{o1}→ not desirable.

Linear Op-Amps: Example 6 Current → Voltage Conversion



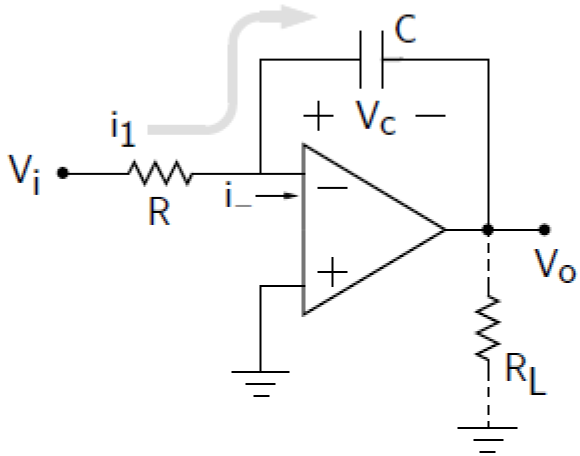
$$V_o = -I_s R$$



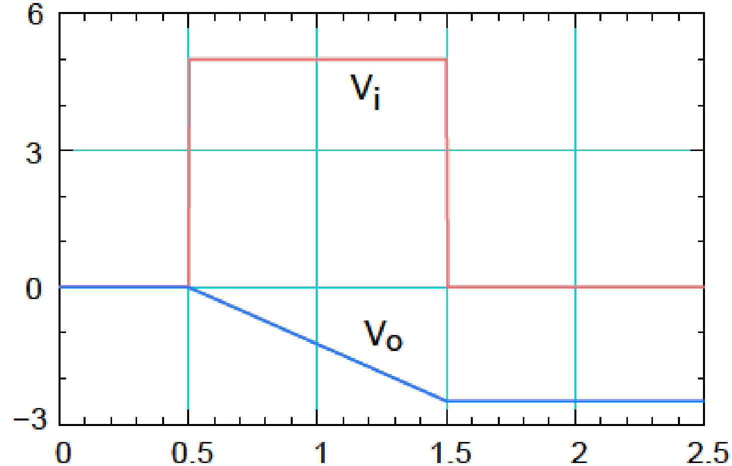
$$V_o = I_s R$$

- Output voltage is $-I_sR$ irrespective of R_L, i.e. irrespective of the next stage
- Practical Example → Photocurrent detector ($V_o=I_sR$)

Linear Op-Amps: Example 7 Integrator

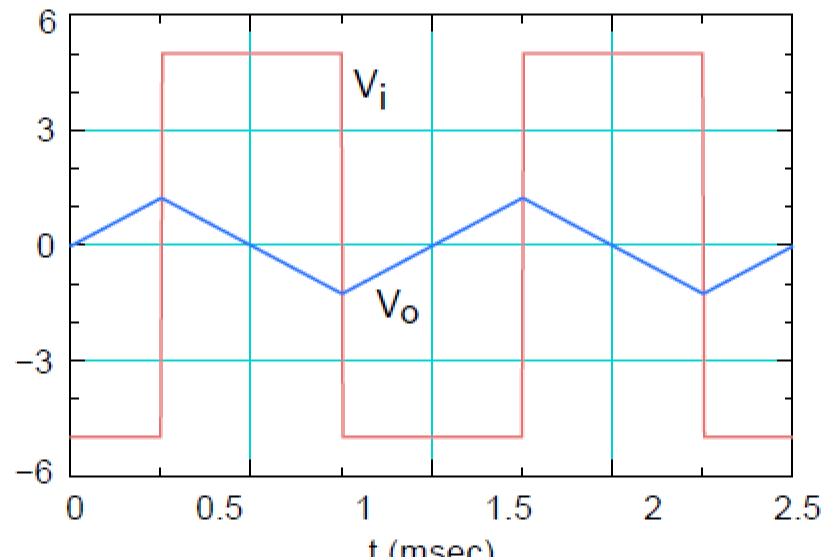


If R=1 kΩ and C=0.2 μF

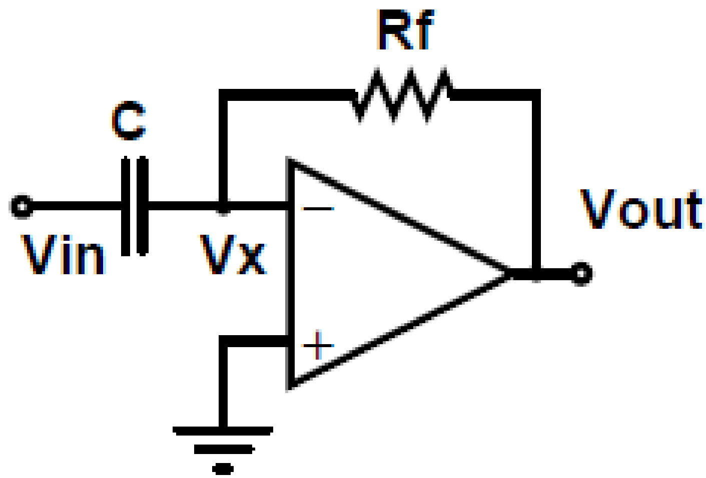


$$i_1 = \frac{V_i}{R} = C\frac{dV_C}{dt} = C\frac{d(0-V_o)}{dt} = -C\frac{dV_o}{dt}$$

$$\boxed{\Rightarrow V_o = -\frac{1}{RC}\int V_i dt}$$



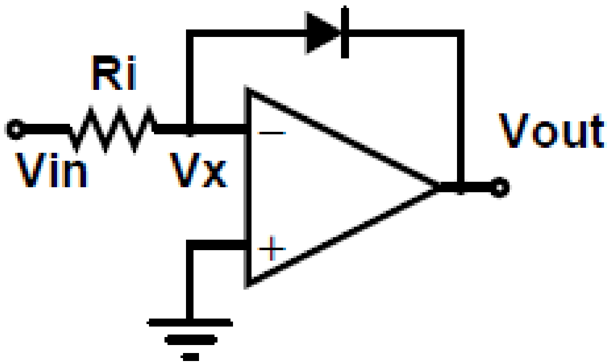
Linear Op-Amps: Example 8
Differentiator



$$-\frac{V_{out}}{R_f} = C\frac{dV_{in}}{dt}$$

$$\boxed{\Rightarrow V_{out} = -R_f C\frac{dV_{in}}{dt}}$$

Linear Op-Amps: Example 9 Logarithmic Amplifer



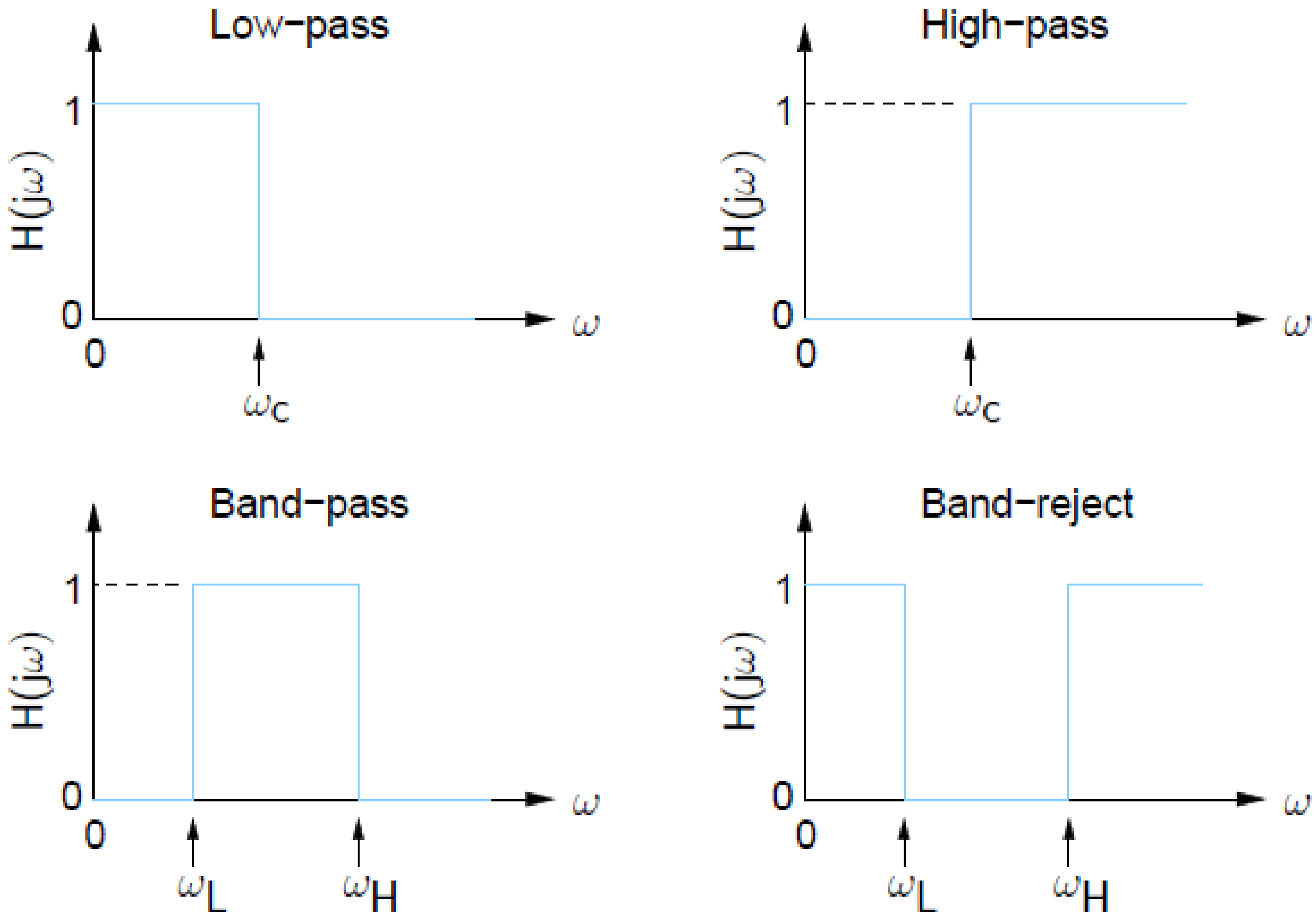
Assume V_{in} is always positive ➔ V_{out} will be negative and diode will be forward biased

$$-\frac{V_{in}}{R} = I_o e^{-\frac{qV_{out}}{kT}}$$

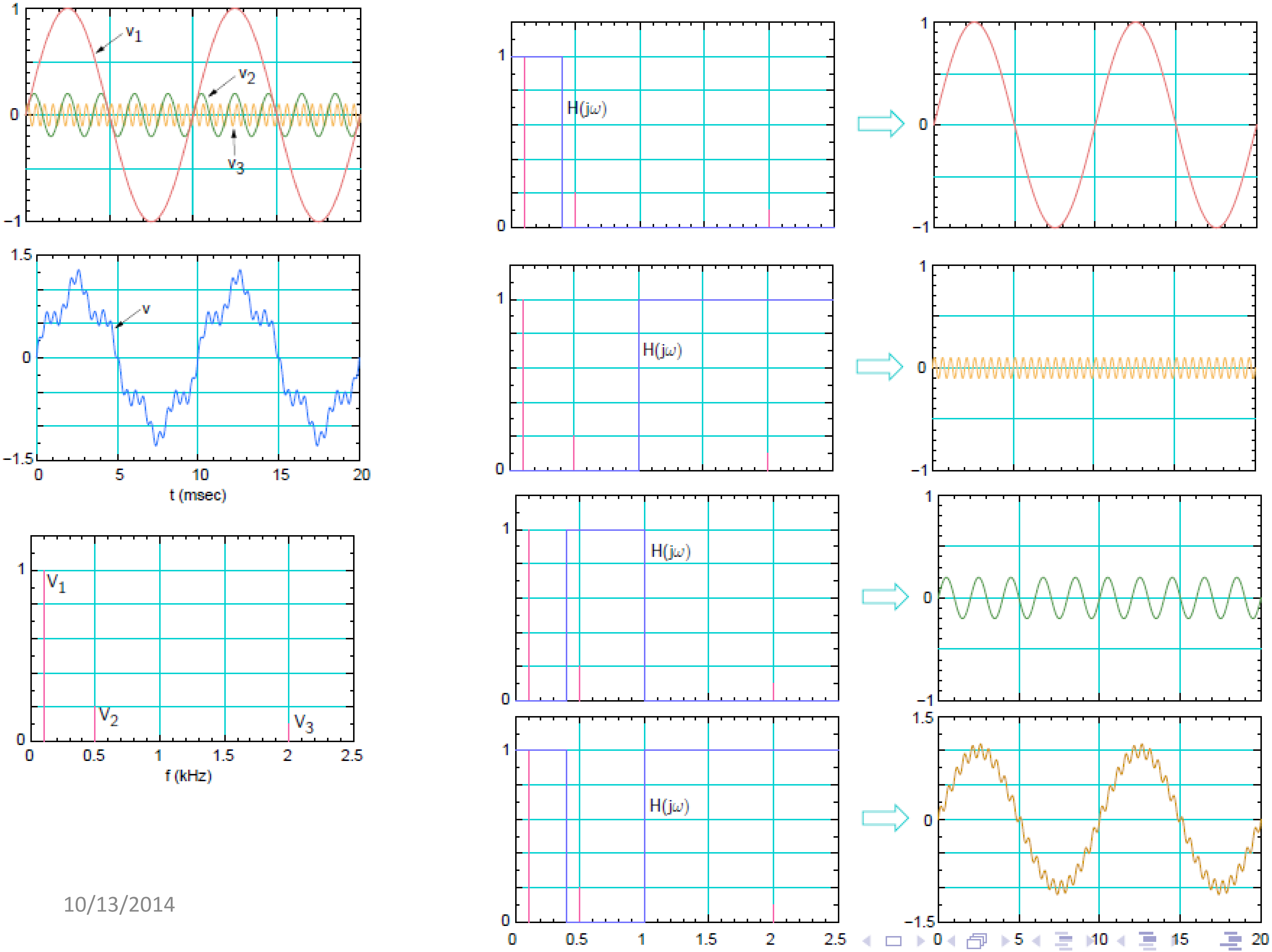
$$\boxed{\Rightarrow V_{out} = -\frac{kT}{q}\ln\left(\frac{V_{in}}{RI_o}\right)}$$

Filters using Op-Amps

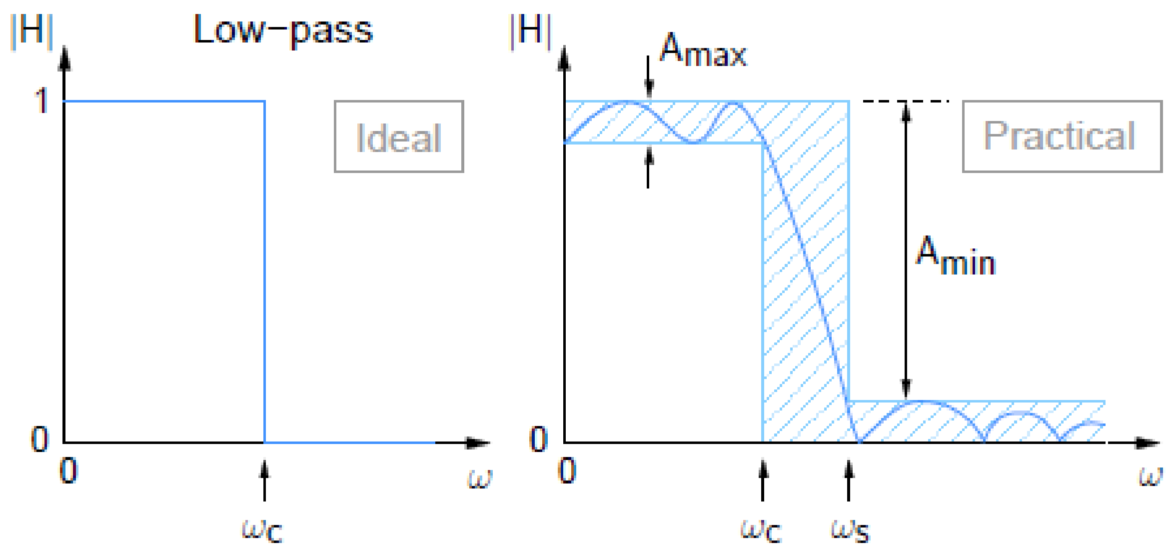
Ideal Filters



Ideal Filters



Practical filters



- A_{max} is called the maximum pass band ripple, e.g. A_{max}=1dB
- A_{min} is the minimum attenuation provided by the filter, e.g. A_{min}=60dB
- ω_s → edge of stop band
- ω_s/ω_C → selectivity/sharpness of the filter
- $\omega_C < \omega < \omega_s$ → transition band of the filter

Practical filters

- A low pass filter $H(s) = \dfrac{1}{\sum_{i=0}^{n} a_i (s/\omega_C)^i}$
- E.g. 5th order low pass filter $H(s) = \dfrac{1}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$
- Commonly used approximations (polynomials) are:
  - Butterworth, Chebyshev, Bessel and elliptic functions
- Butterworth filters: $|H(j\omega)| = \dfrac{1}{\sqrt{1+\varepsilon^2 (\omega/\omega_C)^{2n}}}$
- Chebyshev filters:

$$|H(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2 (\omega/\omega_C)^{2n}}}$$

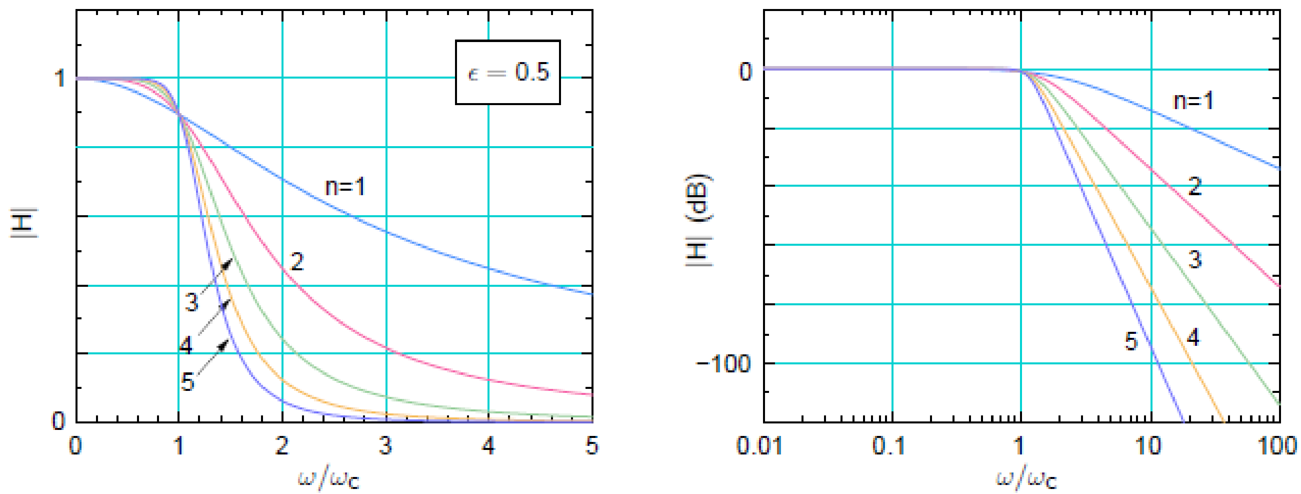
$$C_n = \cos\left[n \cos^{-1}(x)\right], x \le 1$$

$$C_n = \cos\left[n \cosh^{-1}(x)\right], x \ge 1$$

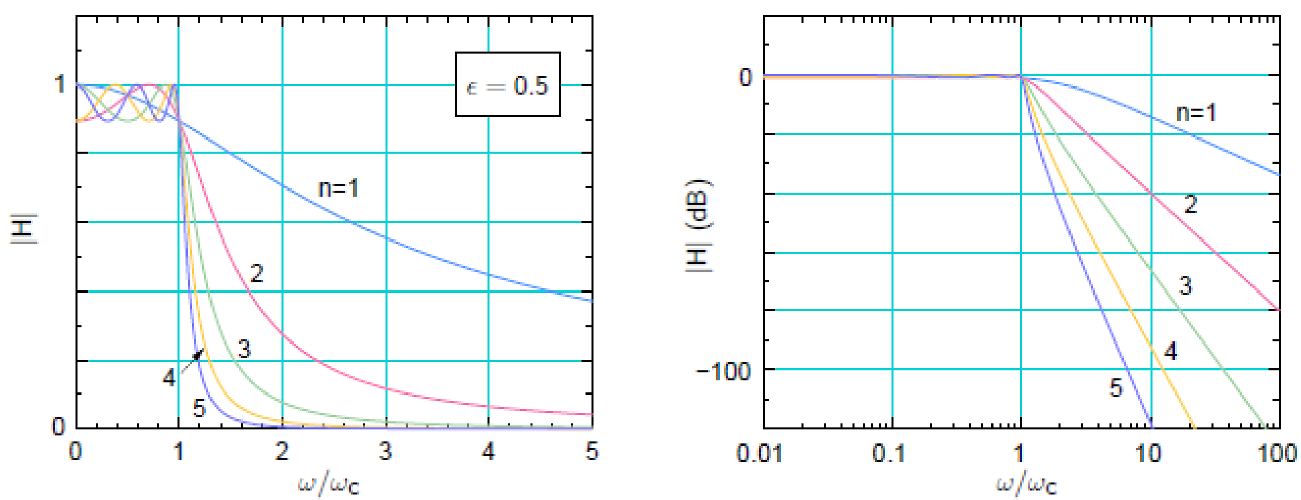
- High pass filter can be obtained from low pass filter by replacing s/ω_c with ω_c/s

Practical low-pass filters

Butterworth filters:

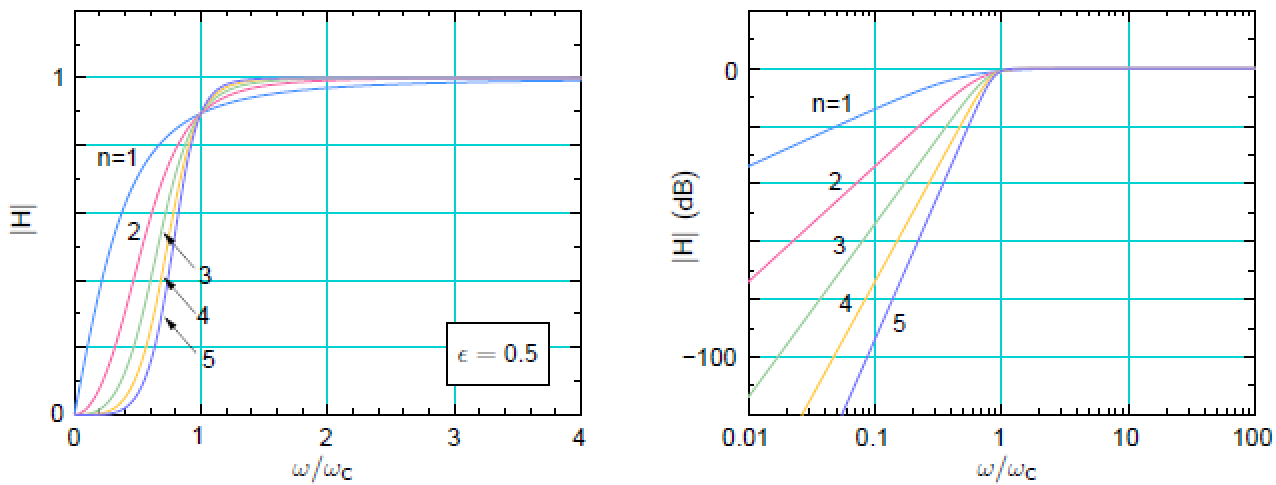


Chebyshev filters:

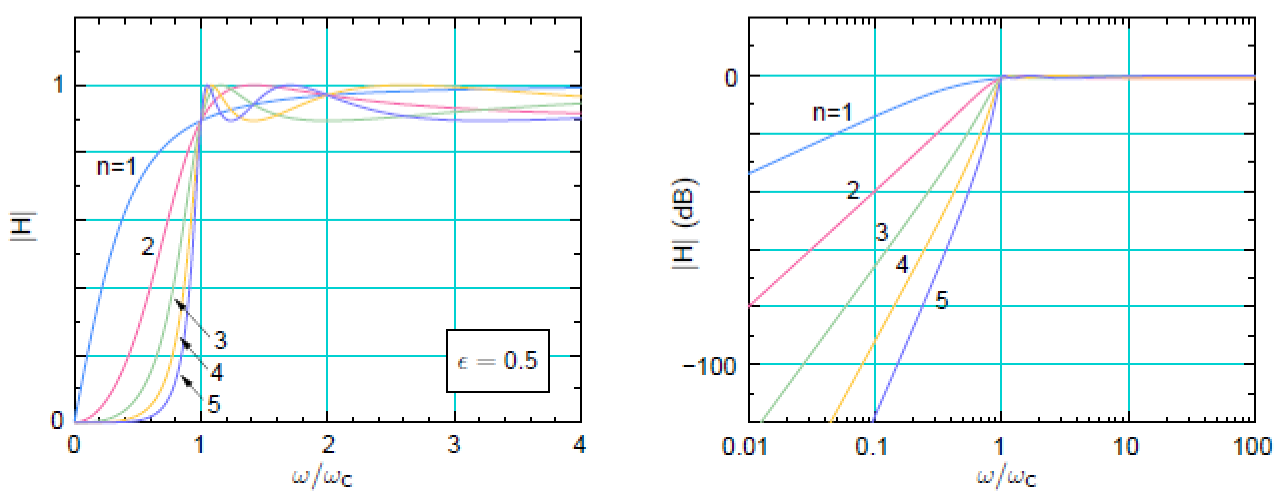


Practical high-pass filters

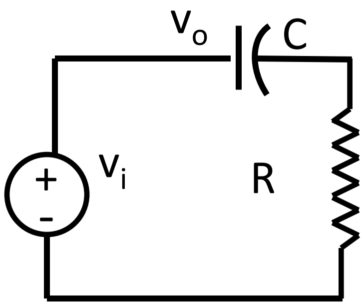
Butterworth filters:



Chebyshev filters:



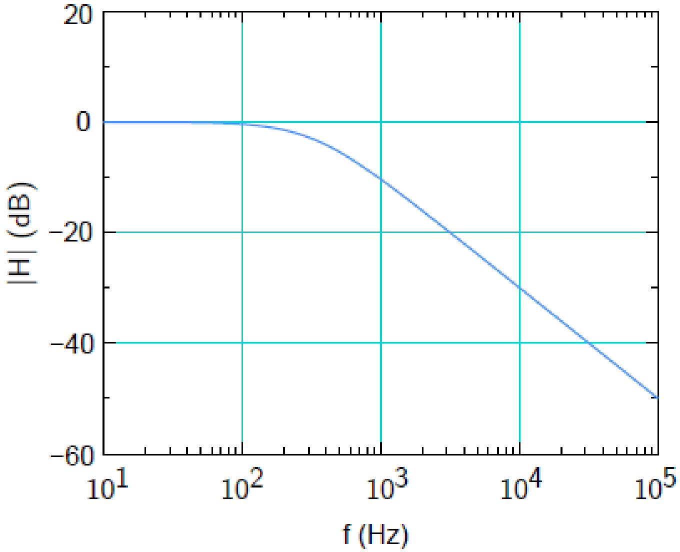
Passive low pass filter



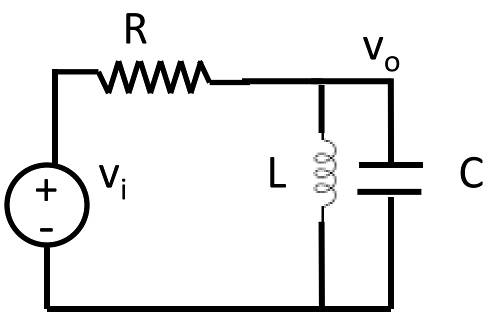
$$H(s) = \frac{1}{1 + s/RC} = \frac{1}{1 + s/\omega_o}$$

$$\omega_o = RC$$

R=100 Ω, C=5 μF



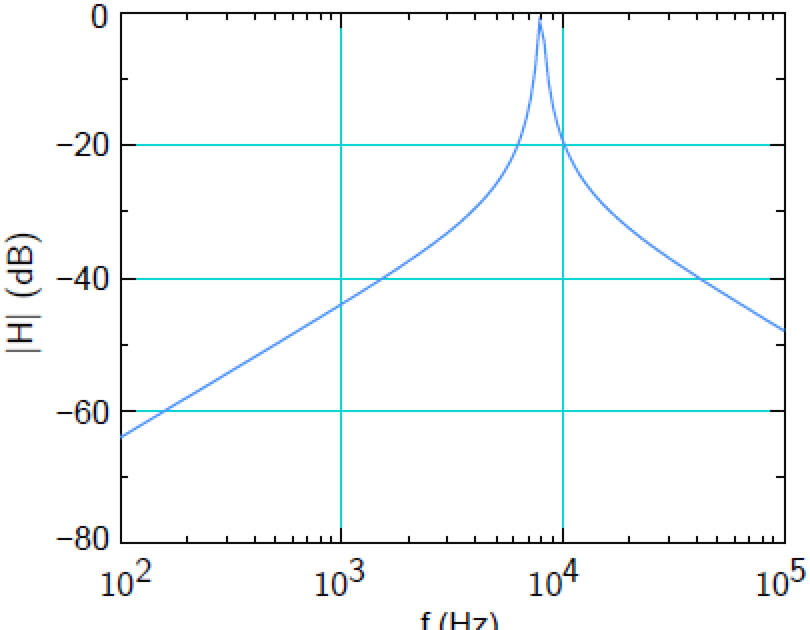
Passive band pass filter



$$H(s) = \frac{s(L/R)}{1 + s(L/R) + s^2 LC}$$

$$\omega_o = 1/\sqrt{LC}$$

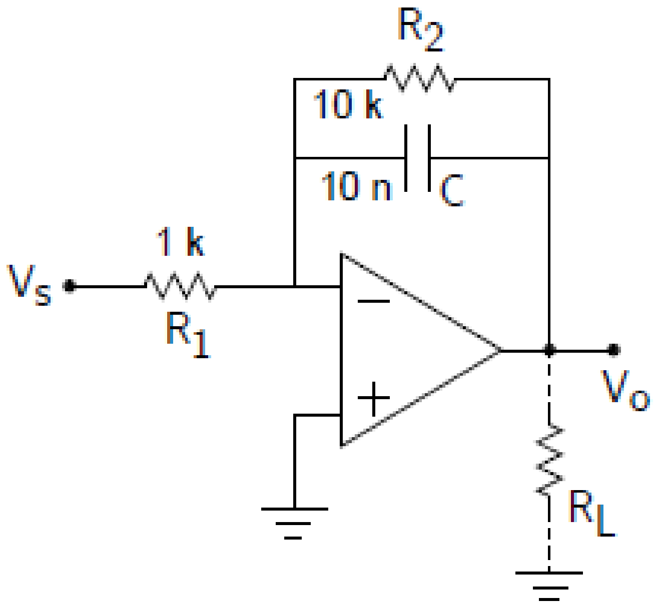
R=100 Ω, C=4 μF, L=0.1 mH

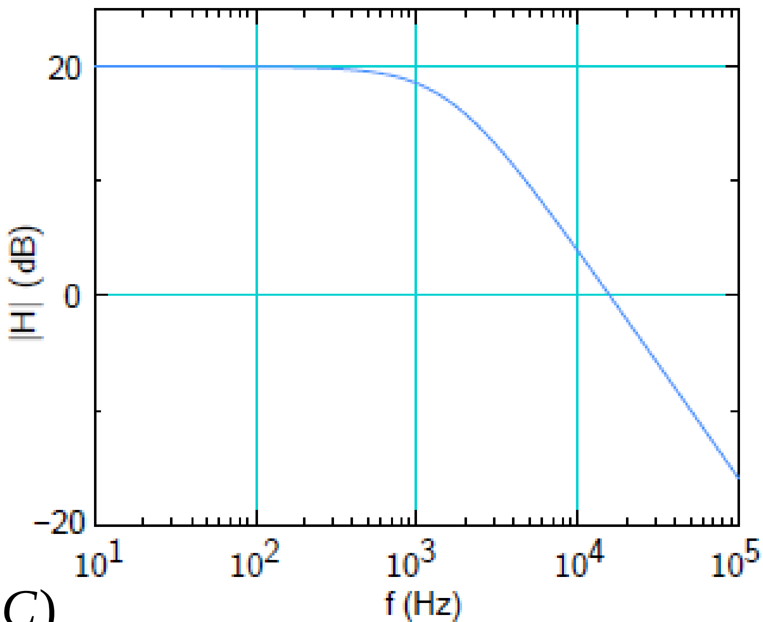


Op-Amp “Active” Filters

- Advantages
  - No inductors (bulky, expensive, non-linear)
  - “Gain” in pass-band
  - Easy to integrate in ICs
  - High input impedance and low output impedance
- Limitations
  - No gain at High frequencies (MHz)
  - Not good for high power requirement

Op-Amp Active Low Pass Filter



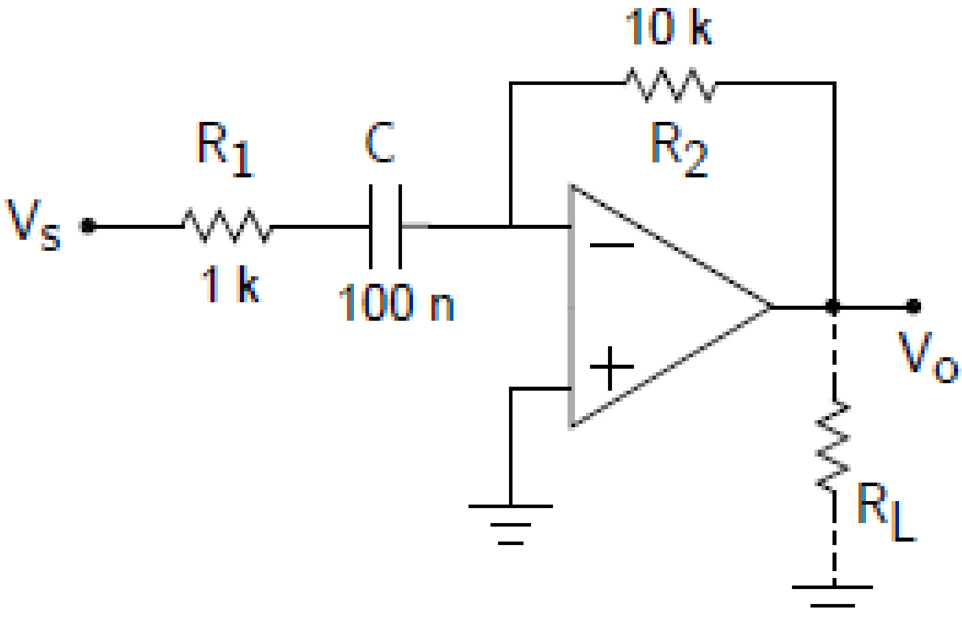


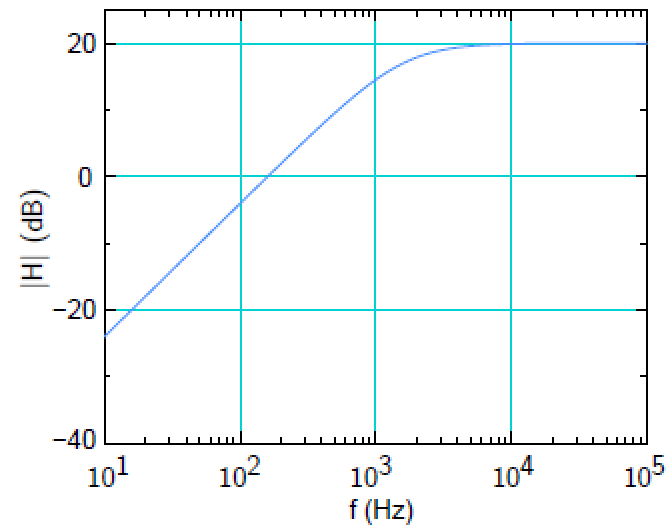
$$V_o = -\frac{R_2 \parallel (1/sC)}{R_1} V_s$$

$$\boxed{H(s) = -\frac{R_2}{R_1}\frac{1}{1+sR_2C}}$$

$$\omega_o = 1/R_2C$$

Op-Amp Active High Pass Filter

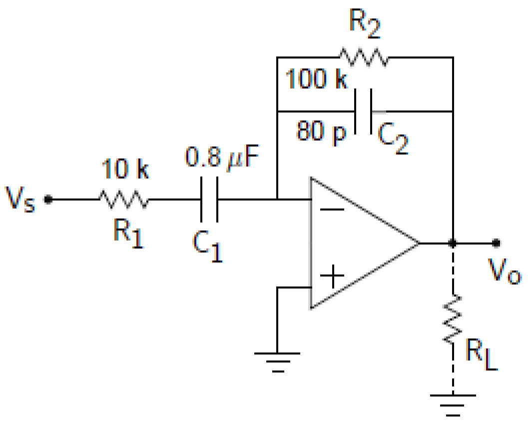


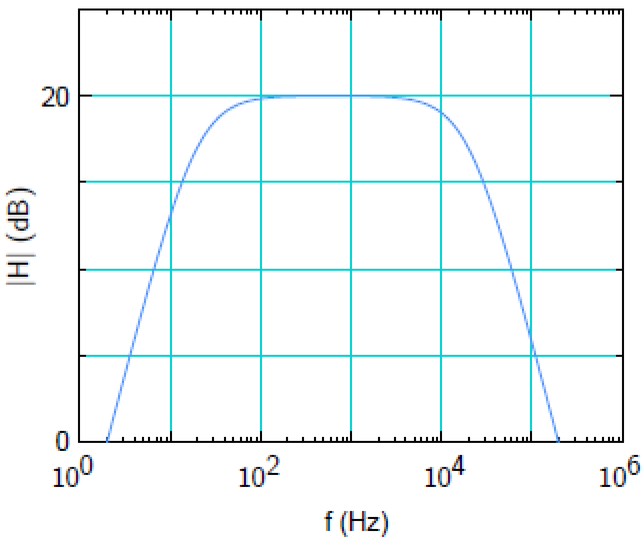


$$\boxed{H(s) = -\frac{sR_2C}{1+sR_1C}}$$

$$\omega_o = 1/R_1C$$

Op-Amp Active Band Pass Filter

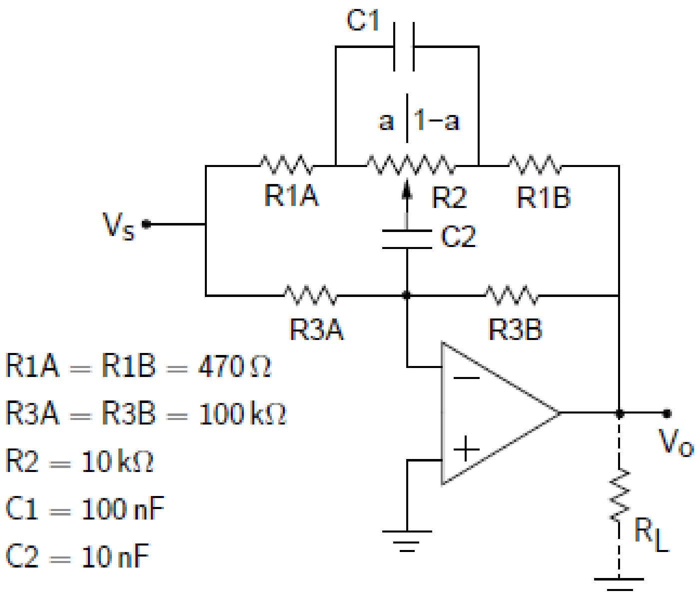


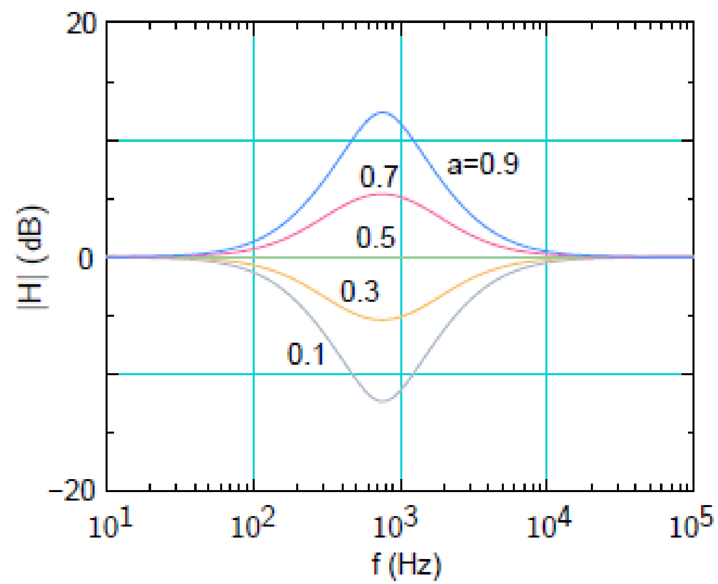


$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = \boxed{-\frac{R_2}{R_1}\frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)}}$$

$$\omega_L = 1/R_1C_1, \omega_H = 1/R_2C_2$$

Example of a Graphic Equalizer

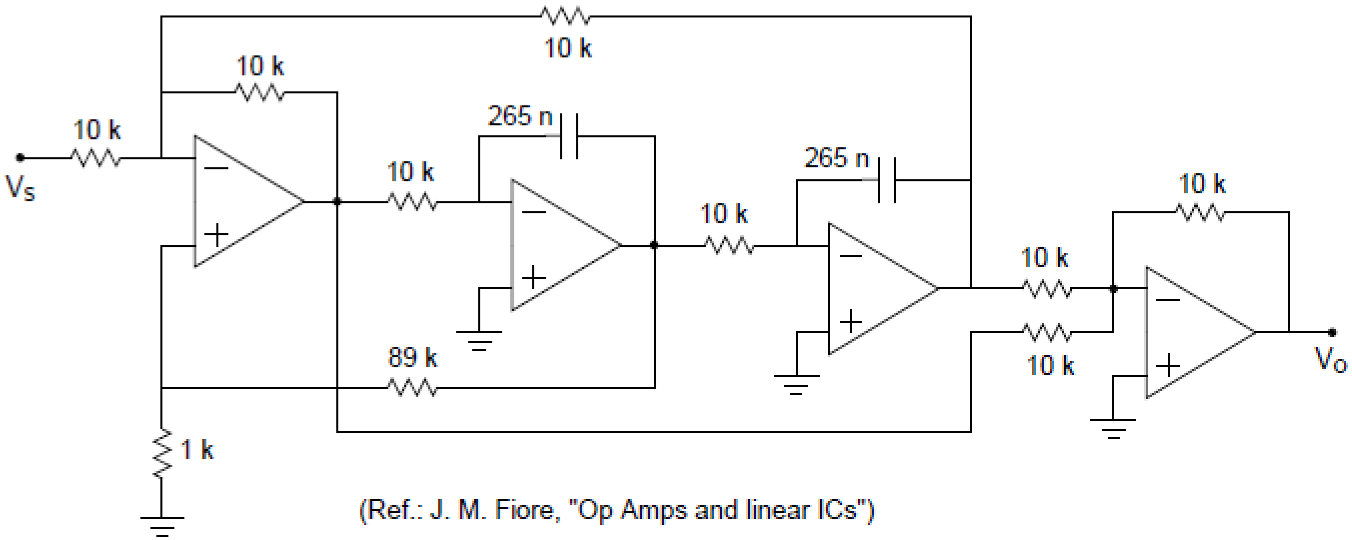




(Ref.: S. Franco, "Design with Op Amps and analog ICs")

- Adjustable gain around a center frequency
- Array of such narrow-band filters

Example of a Notch Filter



(Ref.: J. M. Fiore, "Op Amps and linear ICs")