CS207 (Discrete Structures) Practice problems on graphs, matchings

October 1, 2015

- 1. Show that in every simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.
- 2. A vertex is a cut-vertex if its deletion increases the number of (connected) components of the graph. Note that deleting the vertex also means deleting all edges incident on it.
 - (a) Let v denote a cut-vertex of simple graph G = (V, E), Let $\bar{G} = (V, E')$ denote its complementary graph (as we saw in class). That is, for all $u, v \in V$, $(uv) \in E'$ iff $(uv) \notin E$. Show that $\bar{G} \setminus \{v\}$ is connected.
 - (b) Show that a (simple) graph with at least two vertices has at least two vertices that are not cut vertices.

Solve the following questions from Douglas West's book, $Introduction\ to\ Graph\ Theory,\ 2nd\ Edition.$

- 3. Matchings and Maximum matchings: 3.1.2, 3.1.3, 3.1.11, 3.1.18,
- 4. Perfect matchings (next Monday's lecture): Exercise 3.1.19, 3.1.22, 3.1.23.