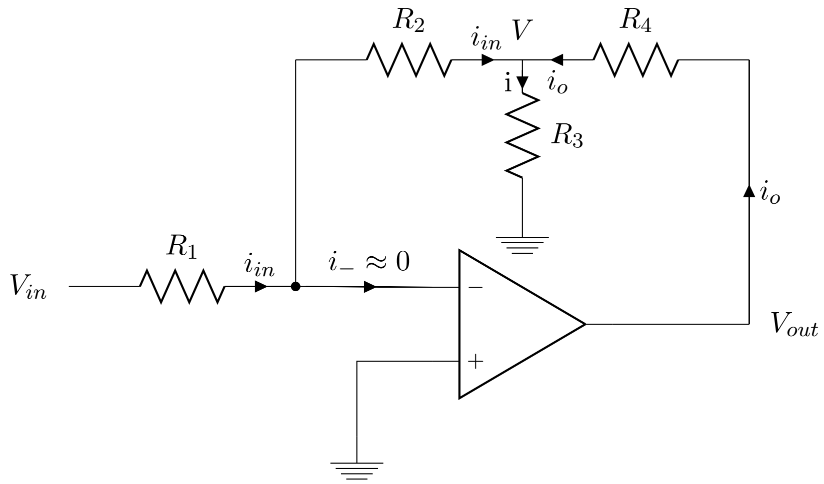
Tutorial 8-9 Solution

Question 4



Assuming ideal Op-Amp in negative feedback ensures $V_- = V_+$.
Here applying KCL at node V and V_+ yields,

$$\frac{V}{R_2} + \frac{V}{R_3} + \frac{V - V_{out}}{R4} = 0, \qquad \frac{V_{in}}{R_1} + \frac{V}{R_2} = 0$$

$$\therefore V = \frac{V_{out}(R_2 \parallel R_3 \parallel R_4)}{R_4}, \qquad V = \frac{R_2}{R_1} V_{in}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_2 R_4}{R_1(R_2 \parallel R_3 \parallel R_4)}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_2 + R_4}{R_1} + \frac{R_2 R_4}{R_1 R_3}$$

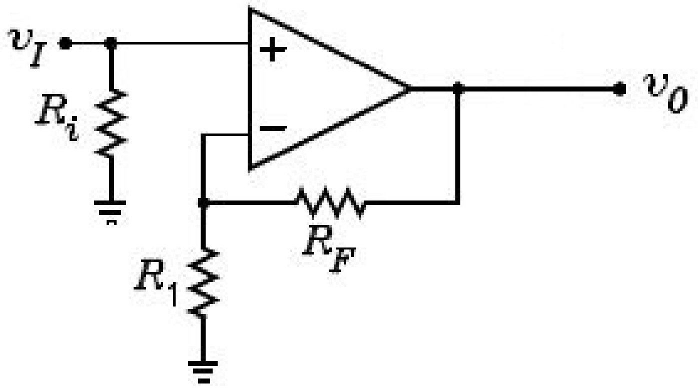
Now,

$$R_{in} = \frac{V_{in}}{i_{in}} = R_1$$

and

$$\begin{aligned} R_{out} &= \frac{V_{out}}{i_o} \\ &= \frac{R_4 V_{out}}{V_{out} - V} \qquad\qquad (\because i_o = \frac{V_{out} - V}{R_4}) \\ &= \frac{R_4^2}{R_4 - (R_2 \parallel R_3 \parallel R_4)} \\ \therefore R_{out} &= \frac{(R_2 R_3 + R_3 R_4 + R_2 R_4)}{R_2 + R_3} \end{aligned}$$

Question 5



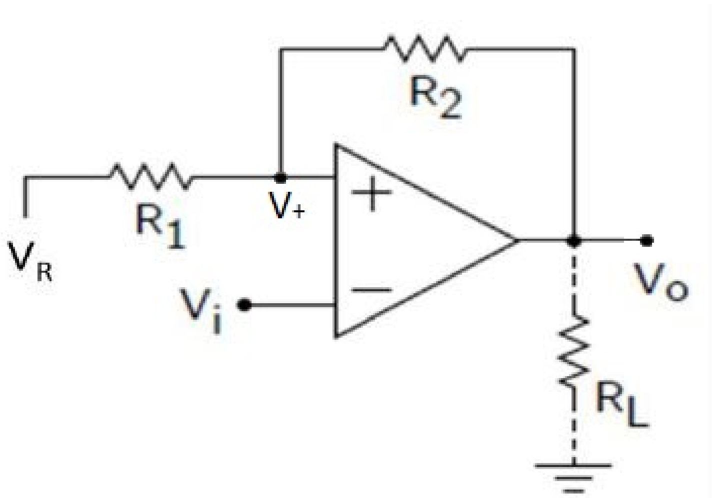
Here given specifications are:
input resistance: $10k\Omega$
gain: 26dB (≈ 20)
maximum feedback current: 0.1 mA for -10V $< v_o <$ 10V

Based on these specifications, we can write following relations for the circuit:

$$R_i = 10k\Omega$$
$$\left(1 + \frac{R_F}{R_1}\right) = 20$$
$$0.1\text{m} \geq \frac{10}{R_F + R_1}$$

Using equality condition for worst case, we obtain
$R_i = 10k\Omega$, $R_F = 95k\Omega$ and $R_1 = 5k\Omega$.

Question 6 (Q1-Tut9)



The op-amp circuit is in positive feedback and it implies $V_- \approx V_+$ is no longer a compulsory state. In this topology, output voltage can be written as following.

$$\begin{aligned} V_o &= +V_{sat} &\quad& \text{when } V_+ > V_- \\ &= -V_{sat} && \text{when } V_+ < V_- \end{aligned}$$

Case I: $V_o = +V_{sat}$

We can compute V_+ using voltage divider analogy to be

$V_{+,I} = \frac{R_2 V_R + R_1 V_{sat}}{R_1 + R_2}$

Now, V_o will remain at V_{sat} as long as $V_- < V_+$ is satisfied. When V_- crosses this value of V_+ the output will flip to $-V_{sat}$.

Case II: $V_o = -V_{sat}$

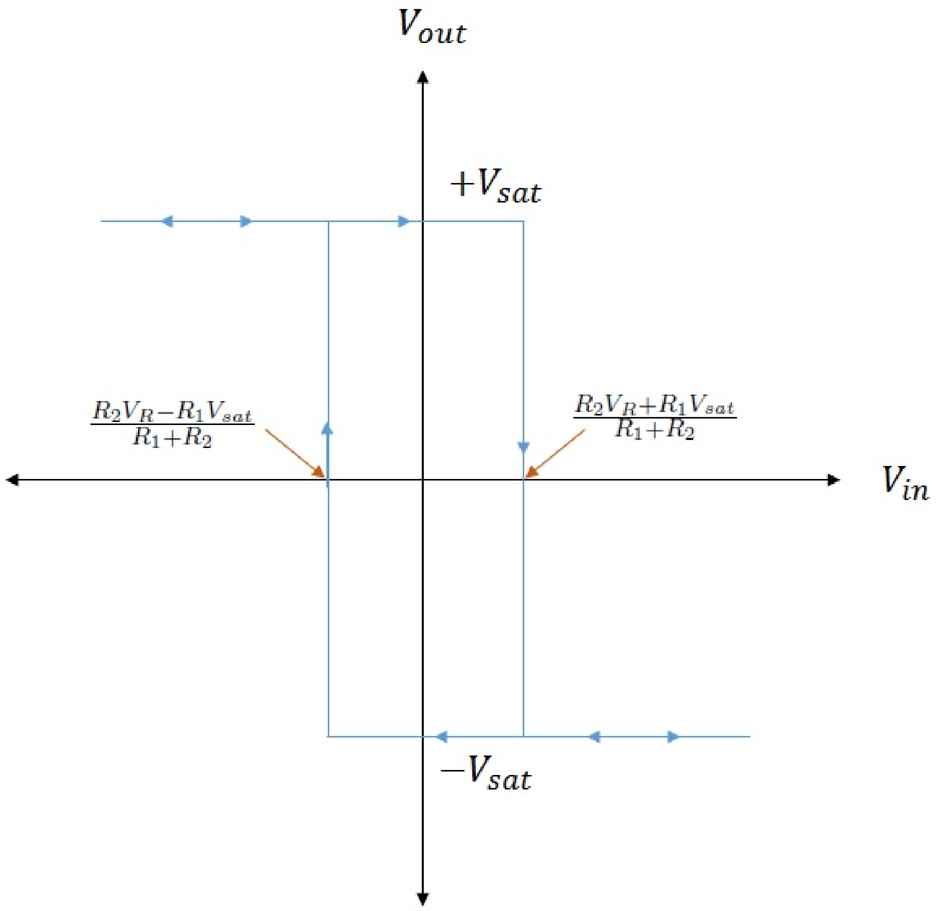
The same argument holds as case I with reversed polarities of input and output signals. Here V_+ will be

$V_{+,II} = \frac{R_2 V_R - R_1 V_{sat}}{R_1 + R_2}$

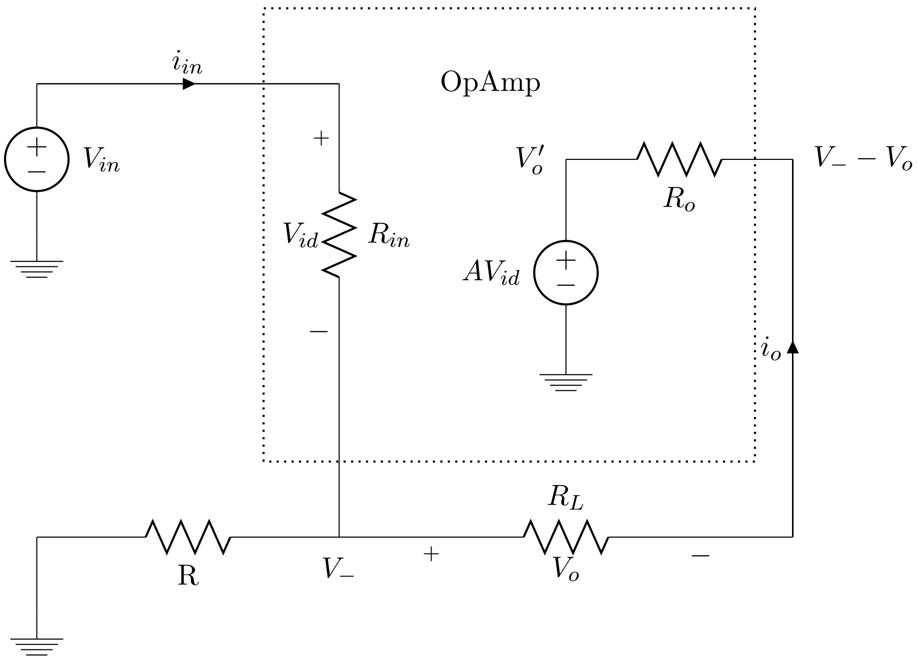
V_o will remain at $-V_{sat}$ as long as $V_- > V_+$ is satisfied. When V_- crosses this value of V_+ the output will flip to V_{sat}.

Transfer Characteristics

Transfer characteristics (Plot of $V_o \to V_i$) can be drawn as following.



Question 7 (Q2-Tut9)



Here, the equivalent circuit diagram for an op-amp is shown. Using KCL at V_- we can write

$$\frac{V_-}{R} + \frac{V_- - V_{in}}{R_{in}} + \frac{V_o}{R_L} = 0$$

and equating for i_o gives

$$\frac{V_o}{R_L} = \frac{V_- - V_o - AV_o}{R_o}$$

using $V_{id} = V_{in} - V_-$, we can write

$$V_o = \frac{(R - AR_{in})R_L}{R(AR_{in} + R_L + R_o)} V_-$$

and

$$V_{in} = \frac{R_{in}(R_L + R_o) + R((1 + A)Rin + R_L + R_o)}{R(AR_{in} + R_L + R_o)} V_-$$

7.a

$$\begin{aligned} A_F &= \frac{i_o}{i_{in}} \\ &= \left(\frac{V_o}{R_L}\right)\left(\frac{R_{in}}{V_{in} - V_-}\right) \\ &= \frac{R - AR_{in}}{R + R_L + R_o} \end{aligned}$$

Using $A \to \infty$, $R_{in} \to \infty$ and $R_o \to 0$;

$$A_F \to \infty$$

This can be interpreted as i_o being finite and $i_{in} \to 0$.

7.b

$$R_{if} = \frac{V_{in}}{i_{in}}$$
$$= R_{in} + \frac{R(AR_{in} + R_L + R_o)}{R + R_L + R_o}$$

Using finite A and R_{in} ($R_{in} >> R$) and $R_o \to 0$;

$$R_{if} = \left(1 + \frac{AR}{R + R_L}\right) R_{in}$$

7.c

Keeping V_{in}, we write equations as

$$i_{in} = \frac{V_-}{R} + i_o$$
$$V_- + i_{in}R_{in} = 0$$

with

$$V_o = V_- - A(-V_-) - i_oR_o$$

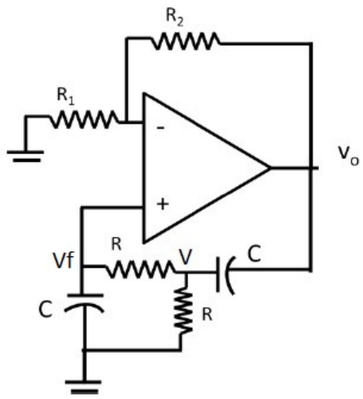
we get,

$$R_{of} = -\frac{V_o}{i_o}$$
$$= \frac{Rin((1 + A)R + R_o) + R_o}{R + R_{in}}$$

Using finite A, R_{in} ($R_{in} >> R$) and R_o;

$$R_{of} = (1 + A)R + R_o$$

Question 8 (Q3-Tut9)



Feedback factor for above circuit is defined as

$$\beta = \frac{V_f}{V_o}$$

Now, using voltage divider rule we can write

$$\frac{V_f}{V} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

and

$$\frac{V}{V_o} = \frac{R \parallel (R + \frac{1}{j\omega C})}{\frac{1}{j\omega C} + R \parallel (R + \frac{1}{j\omega C})} = \frac{j\omega RC(1 + j\omega RC)}{1 - \omega^2 R^2 C^2 + j3\omega RC}$$

Hence, feedback factor is

$$\beta = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + j3\omega RC}$$

Now for oscillator to be stable, loop gain should be 1 according to **Barkhausen stability criterion**[1]. Applying this, we get

$$A\beta = 1 \qquad \text{where} \qquad A = 1 + \frac{R_2}{R_1}$$

$$\therefore \left(1 + \frac{R_2}{R_1}\right) \left(\frac{3\omega^2 R^2 C^2 + j\omega RC(1 - \omega^2 R^2 C^2)}{(1 - \omega^2 R^2 C^2)^2 + 9\omega^2 R^2 C^2}\right) = 1$$

Equating imaginary part

$$\omega RC = 1 \qquad \text{or} \qquad \omega = \frac{1}{RC}$$

Using this result and equating the real part, we follow

$$(1 + \frac{R_2}{R_1})\frac{1}{3} = 1$$

$$\therefore R_2 = 2R_1$$

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[1] https://en.wikipedia.org/wiki/Barkhausen_stability_criterion