EE101: Circuit Analysis

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References: L. Bobrow

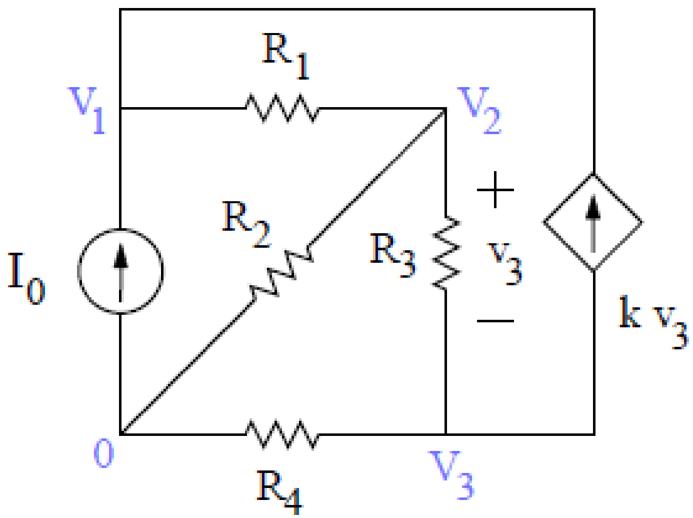
Outline

- Nodal analysis
- Mesh analysis
- Thevenin and Norton theorems
- Maximum power transfer
- Linearity and superposition

Nodal analysis

- Application of Kirchoff's current law (KCL)
- Variables are voltages
- Solve a set of simultaneous equations
- Can be used for any circuit
  - Non-planar also → element crossing over another

Nodal analysis: Example



- Define a reference node: "0"
- Define node voltages V_1, V_2 for other nodes
- Write KCL for the non-reference nodes in terms of node voltages

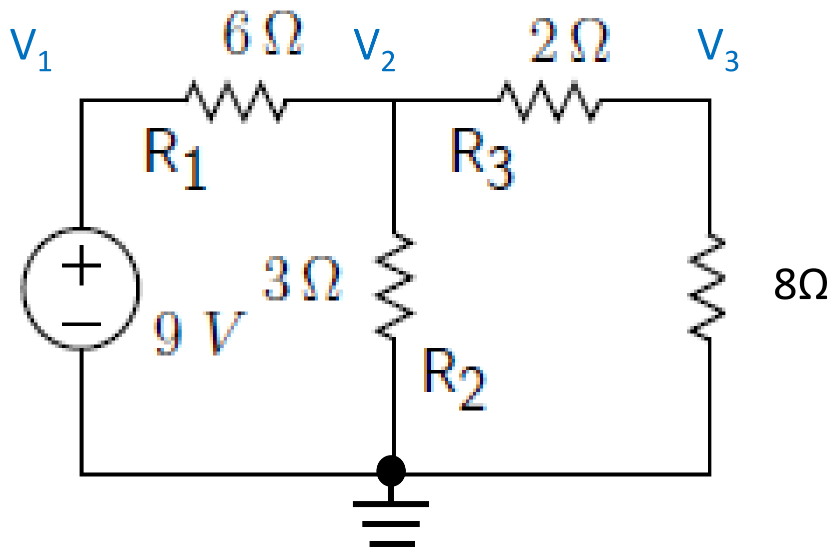
$$\frac{1}{R_1}(V_1 - V_2) - I_0 - k\,(V_2 - V_3) = 0\,,$$

$$\frac{1}{R_1}(V_2 - V_1) + \frac{1}{R_3}(V_2 - V_3) + \frac{1}{R_2}(V_2) = 0\,,$$

$$k\,(V_2 - V_3) + \frac{1}{R_3}(V_3 - V_2) + \frac{1}{R_4}(V_3) = 0\,.$$

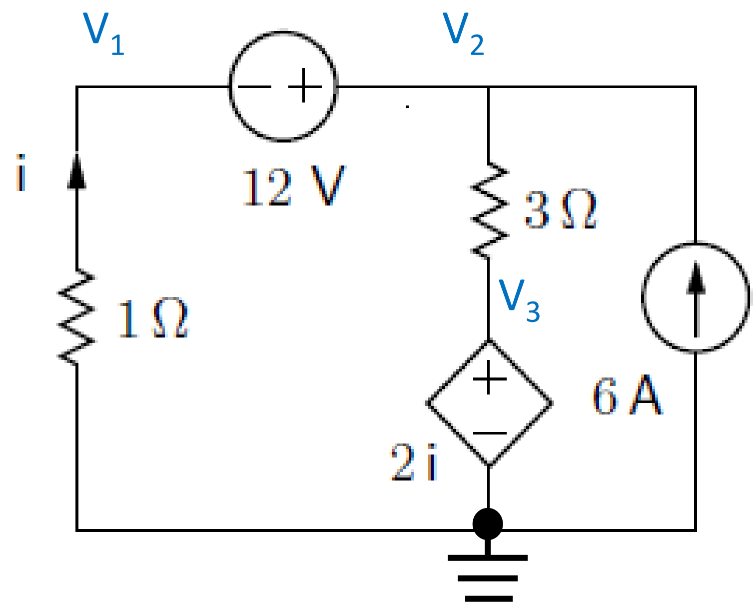
- Solve (n-1) simultaneous eqns for the node voltages → currents

Nodal analysis with voltage sources



- Voltage source where one terminal is connected to reference node
- By inspection, V_1=9 V
- Only two unknown variables remain
  - V_2 and V_3

Nodal analysis with voltage sources



V_2-V_1=12

-V_1+6=(V_2-V_3)/3 at node V_2

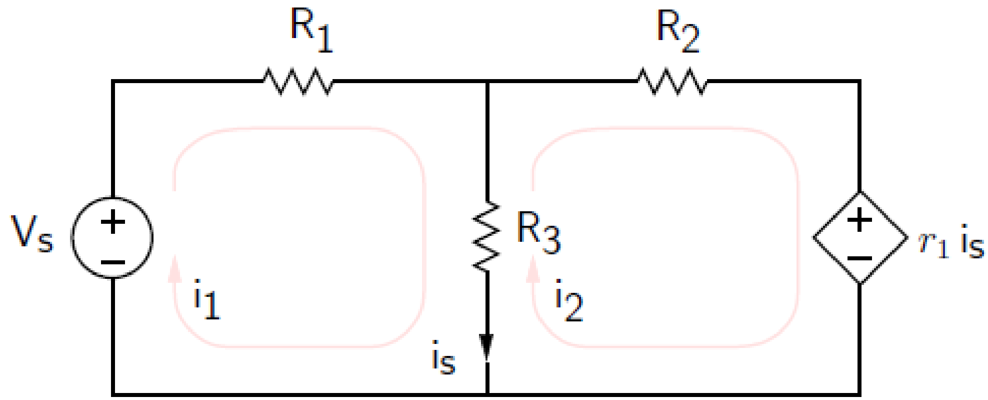
V_3=2i=-2V_1 at node V_3

- Voltage source between two non-reference nodes
- By inspection, V_2-V_1=12 V

Mesh analysis

- Application of Kirchoff's voltage law (KVL)
- Variables are currents
- Solve a set of simultaneous equations
- Can***not*** be used for any circuit
  - Planar circuits → element not crossing over another

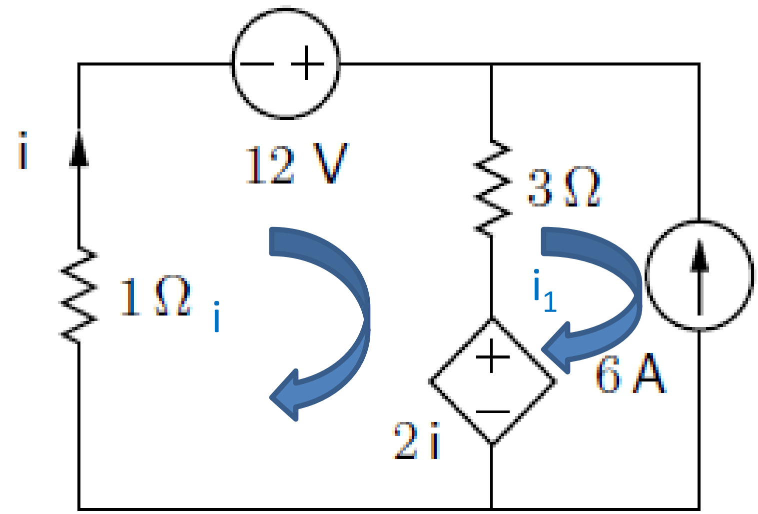
Mesh analysis: An example



- 2D space divided into finite and infinite windows (surrounding the ckt)→ meshes
- i_1 and i_2 are mesh currents (clock or anti-clockwise)
  - Current through R_3 is $i_s=i_1-i_2$
- Write KVL for the meshes and solve for i_1 and i_2 (m equations for m meshes)

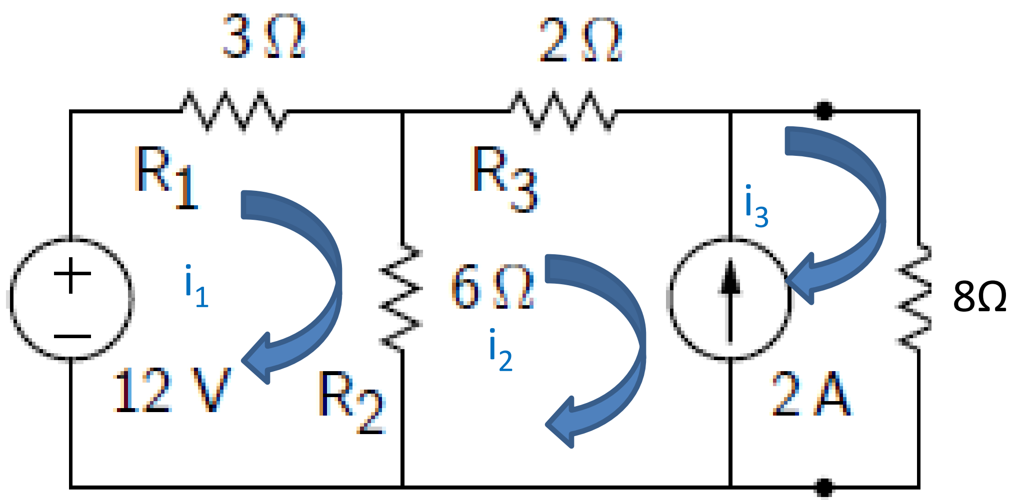
$$-V_s + i_1 R_1 + (i_1 - i_2) R_3 = 0\,,$$
$$R_2\, i_2 + r_1 (i_1 - i_2) + (i_2 - i_1) R_3 = 0\,.$$

Mesh analysis with current sources



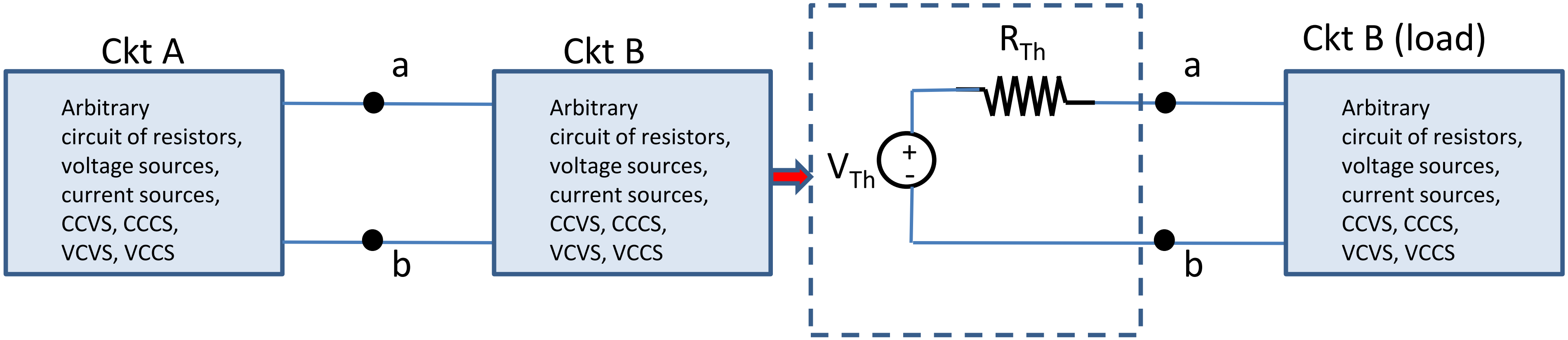
- Current source part of a single mesh
- By inspection, i_1=-6A
- Only one unknown quantity → i, one mesh

Mesh analysis with current sources



- Current source is part of two meshes
- By inspection,
  - i_3-i_2=2
- Remaining two equations can be obtained from meshes for i_1 and i_2

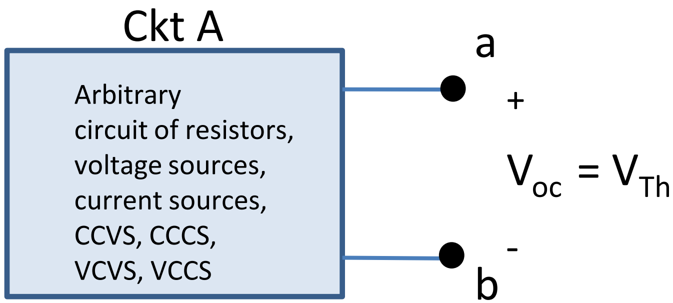
Thevenin's theorem

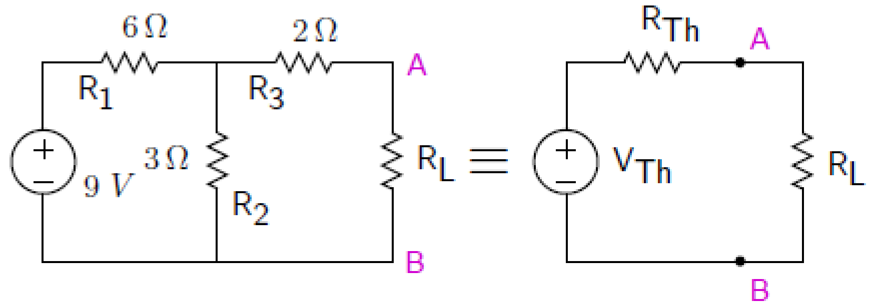


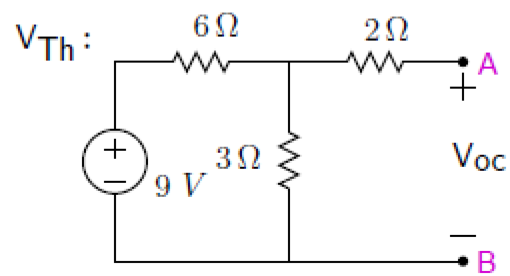
- Circuit A contains no dependent source that depends on a variable in Circuit B and vice versa

- Series combination of voltage source V_{Th} and R_{Th} is called the Thevenin equivalent of circuit A
  - V_{Th} → Open circuit voltage, also called V_{oc}
  - R_{Th} → Output resistance or Thevenin Resistance, also called R_o

How to find V_{Th}?





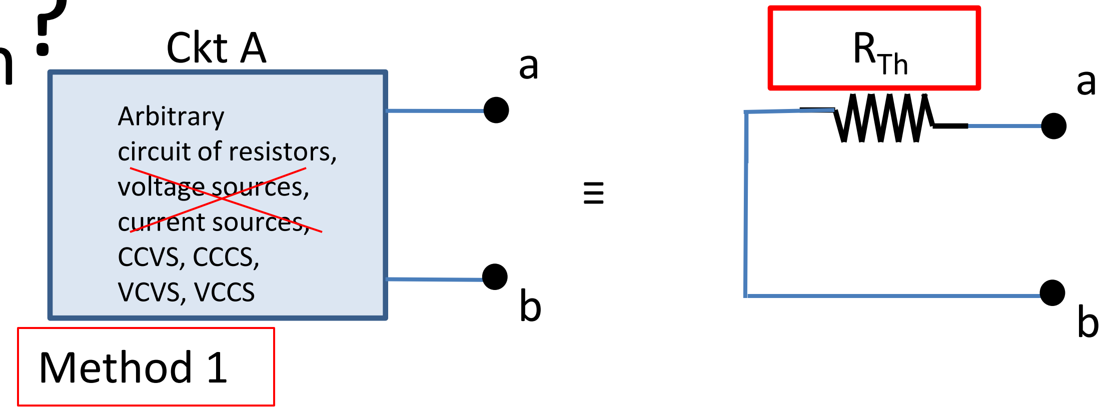


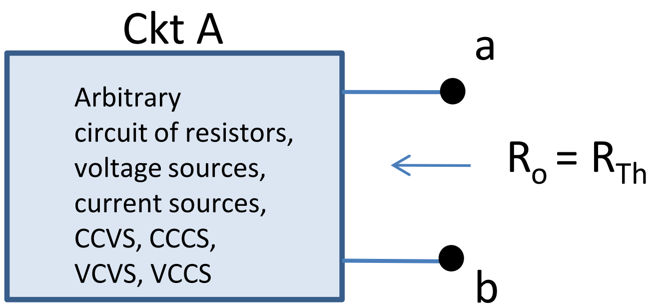
$$V_{oc} = 9\,\mathrm{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

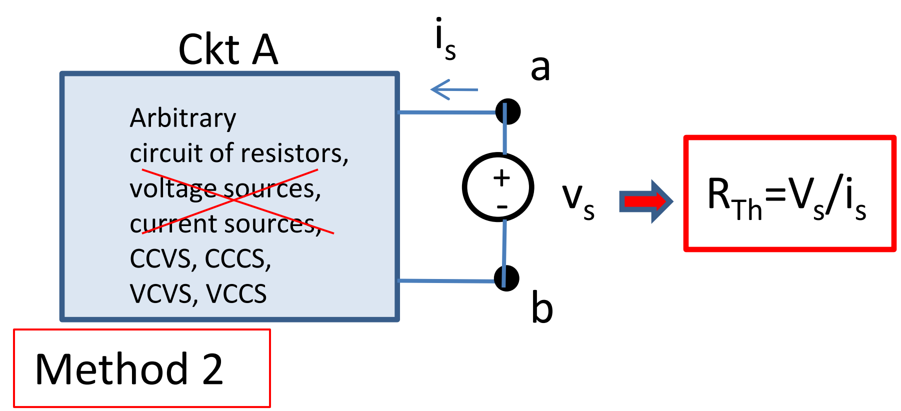
$$= 9\,\mathrm{V} \times \frac{1}{3} = 3\,\mathrm{V}$$

- Simply find the open circuit voltage between a and b.

How to find R_{Th}?

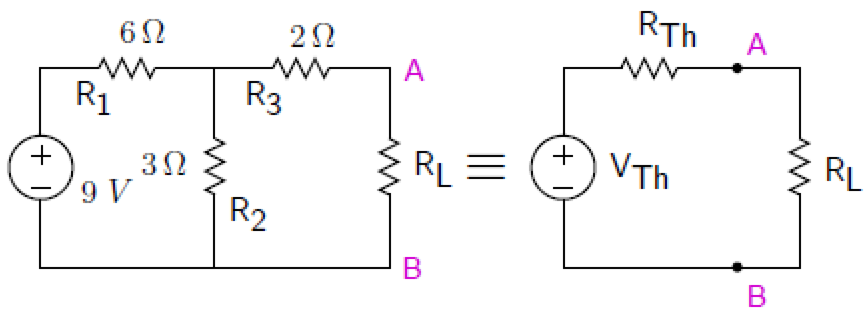


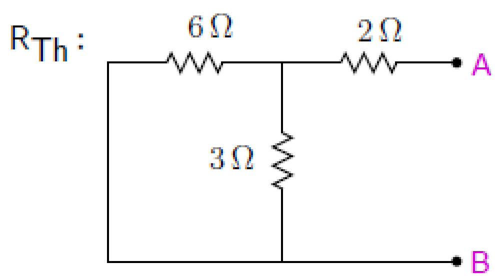




- R_o is the effective resistance of Ckt A as "seen" from a-b
- Method 1:
  - Deactivate all independent sources – short ckt for voltage sources and open ckt for current sources
  - R_{Th} can be found by inspection
- Method 2:
  - Apply an independent voltage test source and taking ratio of voltage to current

R_{Th} (R_o) Example

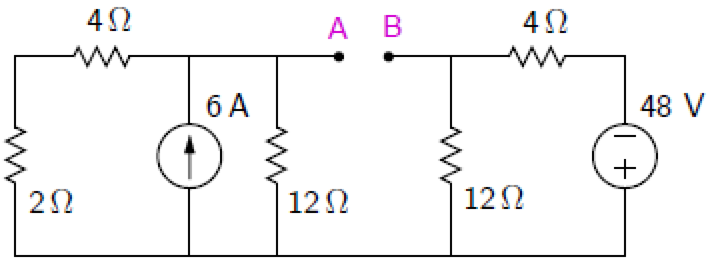




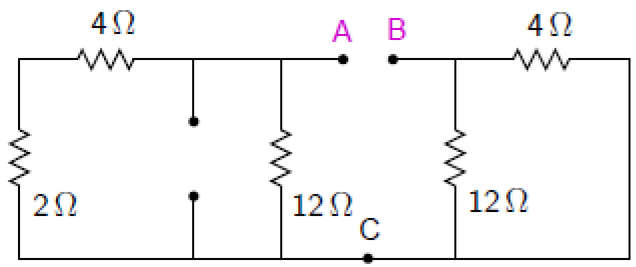
$$\boxed{\begin{aligned} R_{Th} &= (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2 \\ &= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega \end{aligned}}$$

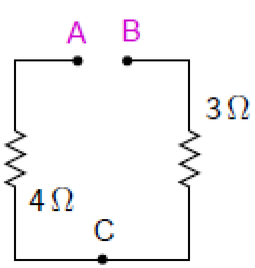
- Series and parallel combination of resistors

Example



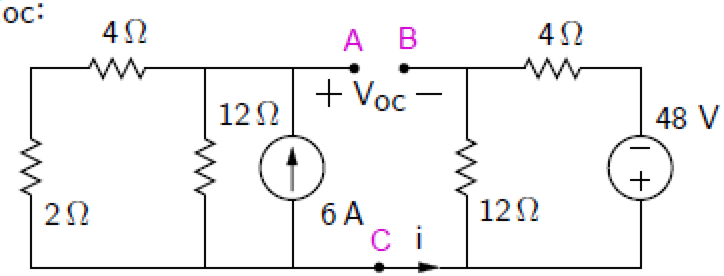
R_{Th}:





$\Rightarrow \quad R_{Th} = 7\,\Omega$

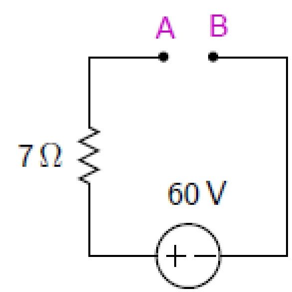
Voc:



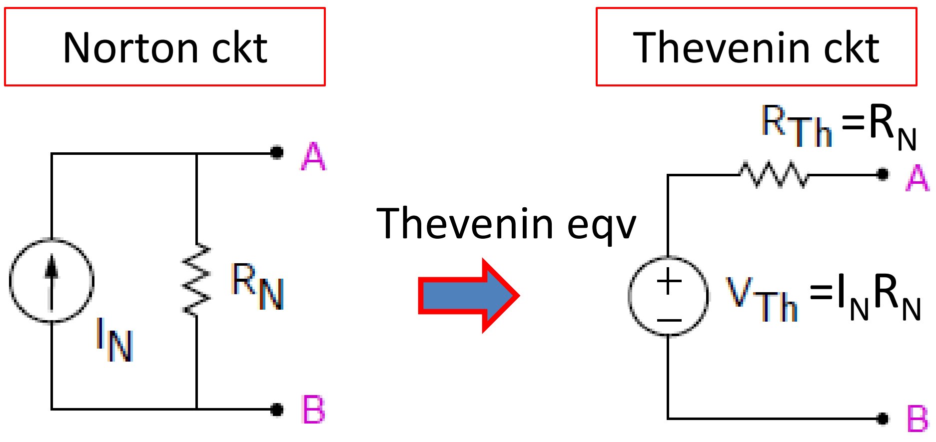
Note: $i = 0$ (since there is no return path).

$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= (V_A - V_C) + (V_C - V_B) \\ &= V_{AC} + V_{CB} \\ &= 24\,\text{V} + 36\,\text{V} = 60\,\text{V} \end{aligned}$$

$V_{Th} = 60\,\text{V}$
$R_{Th} = 7\,\Omega$



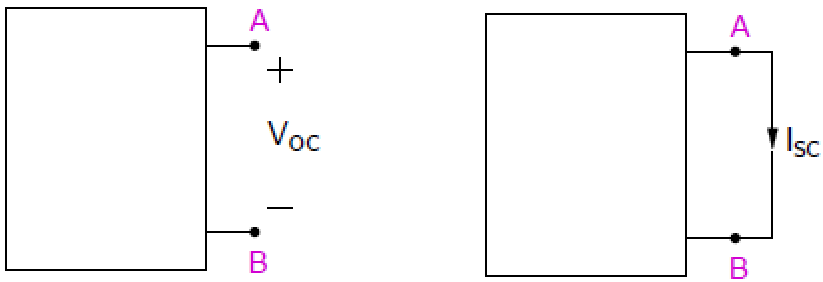
Norton's Theorem



- The two circuits above are equivalent
- I_N is also called "short circuit" current I_{sc}
- Note that $R_{Th}=V_{Th}/I_N \rightarrow R_{Th}=V_{oc}/I_{sc}$

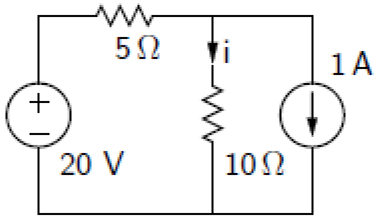
Method 3 for finding R_{TH}

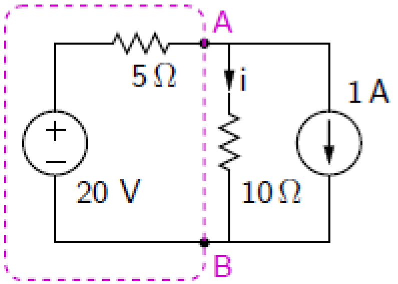
$$\boxed{R_{TH}=V_{TH}/I_N \rightarrow R_{TH}=V_{oc}/I_{sc}}$$



- Find V_{oc}
- Find I_{sc}
- $R_{TH}=V_{oc}/I_{sc}$
- Note that no sources are deactivated in this method

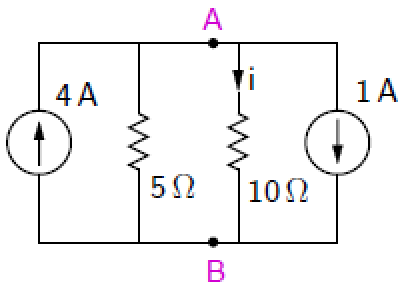
Example for Norton Equivalent Ckt

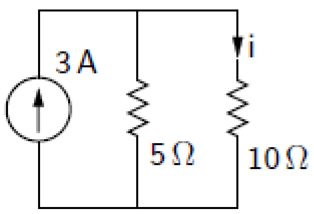




$$R_N = 5\,\Omega$$

$$I_N = \frac{20\text{ V}}{5\,\Omega} = 4\text{ A}$$

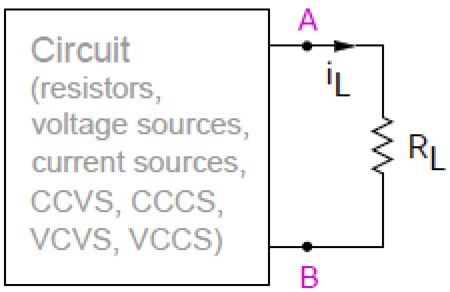


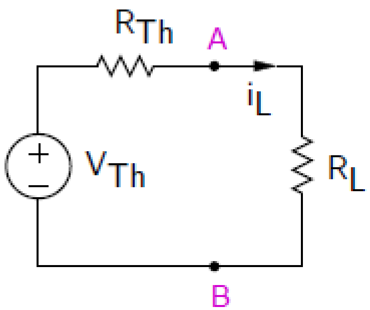


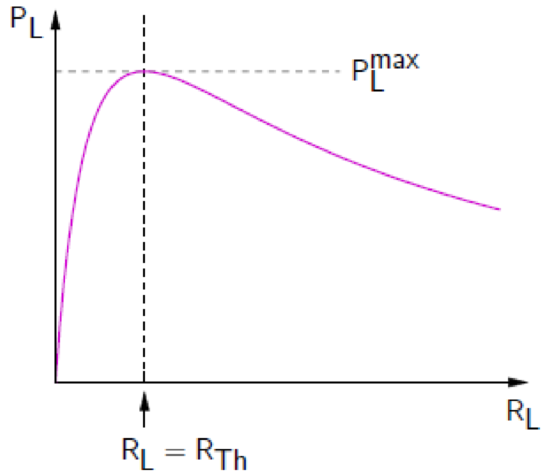
$$i = 3\text{ A} \times \frac{5}{5+10}$$

$$= 1\text{ A}$$

Maximum Power Transfer







$$i_L = \frac{V_{Th}}{R_{Th} + R_L},$$
$$P_L = V_{Th}^2 \times \frac{R_L}{(R_{Th} + R_L)^2}$$

For $\dfrac{dP_L}{dR_L} = 0$, we need

$$\frac{(R_{Th} + R_L)^2 - R_L \times 2\,(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0,$$

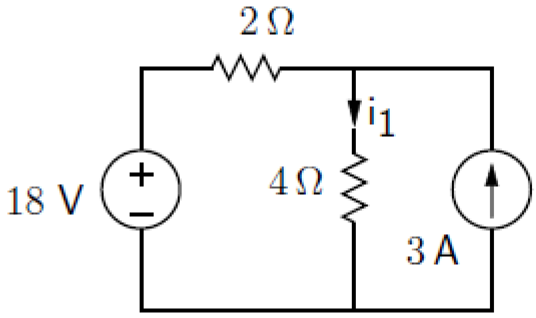
$$R_{Th} + R_L = 2\,R_L \Rightarrow R_L = R_{Th}.$$

- Power transferred to the load is $P_L = i^2_L R_L$
- For what value of R_L will the power transferred be maximum?
- Replace with Thevenin equivalent
- What is the maximum power transferred → P_l^{max}?

Linearity and Superposition

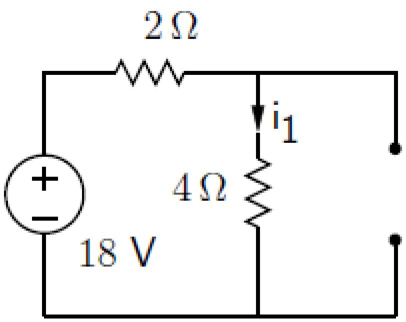
- Circuit containing independent sources, dependent sources, resistors is linear, i.e. system of equations describing the circuit is linear
- The dependent sources are assumed to be linear, i.e. $v=ai_c^2+b$ will make it non-linear
- For a linear system we can apply the principle of superposition
- For ckts this implies, computing the response for each independent source separately and add the individual contributions to get the final true response.
  - Cannot be applied to dependent sources
  - Superposition corresponds to superposition of response due to independent sources
  - One independent source at a time → deactivate all other independent sources
  - For a current source deactivation → $i_s=0$ → replace with an open circuit
  - For a voltage source deactivation → $v_s=0$ → replace with a short circuit

Example



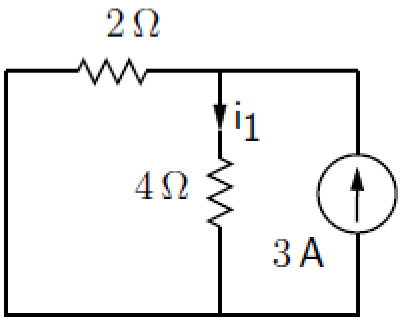
$$\boxed{i_1^{\text{net}} = i_1^{(1)} + i_1^{(2)} = 3 + 1 = 4\,\text{A}}$$

Case 1: Keep V_S, deactivate I_S.



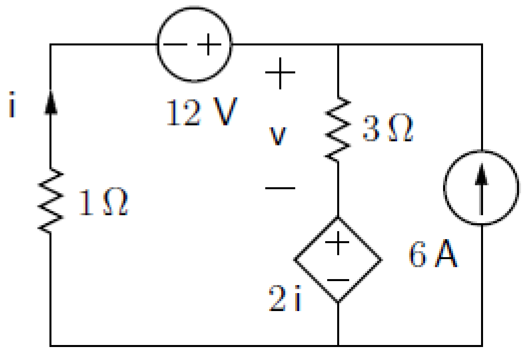
$$i_1^{(1)} = 3\,\text{A}$$

Case 2: Keep I_S, deactivate V_S.



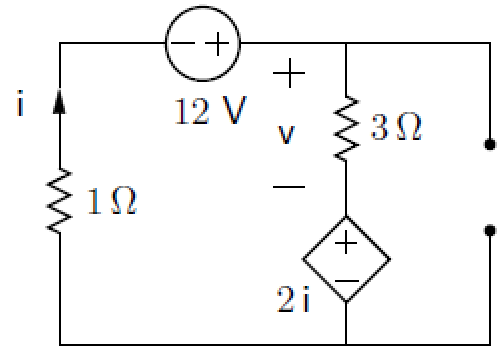
$$i_1^{(2)} = 3\,\text{A} \times \frac{2\,\Omega}{2\,\Omega + 4\,\Omega} = 1\,\text{A}$$

Example



$$v^{net} = v^{(1)} + v^{(2)} = 6 + 9 = 15\,\mathrm{V}$$

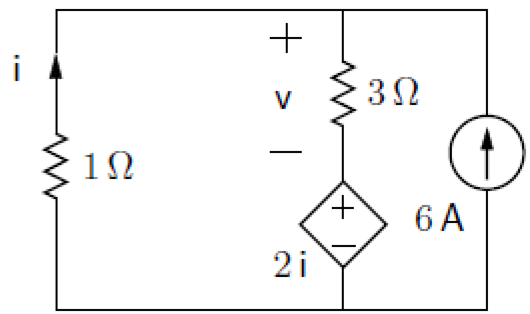
Case 1: Keep V_S, deactivate I_S.



KVL: $-12 + 3\,i + 2\,i + i = 0$

$\Rightarrow i = 2\,\mathrm{A}\,,\, v^{(1)} = 6\,\mathrm{V}\,.$

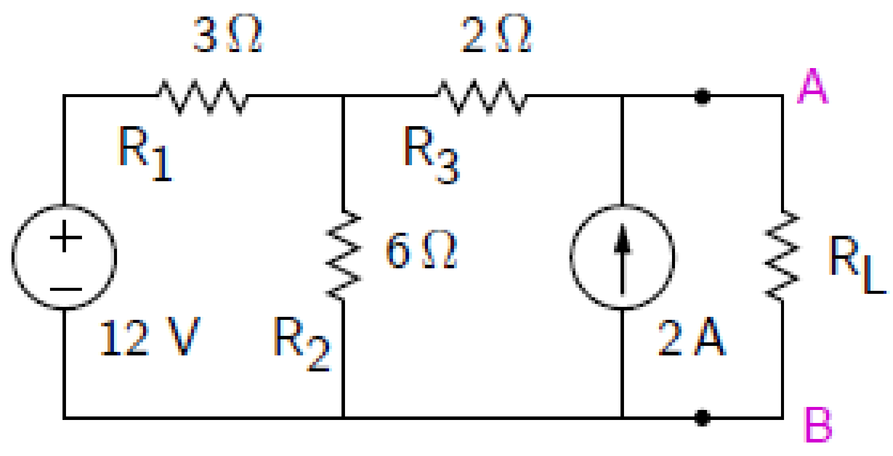
Case 2: Keep I_S, deactivate V_S.



KVL: $i + (6 + i)\,3 + 2\,i = 0$

$\Rightarrow i = -3\,\mathrm{A}\,,\, v^{(2)} = (-3 + 6) \times 3 = 9\,\mathrm{V}\,.$

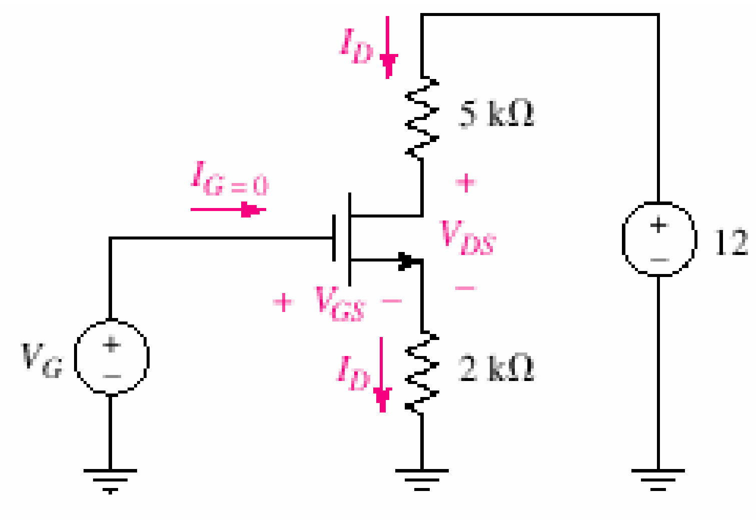
Try the following as HW



- Find R_L for which P_L is maximum

Backup

Problem 1



Solution

(*a*) By KVL, $-12 + 5000I_D + V_{DS} + 2000I_D = 0$

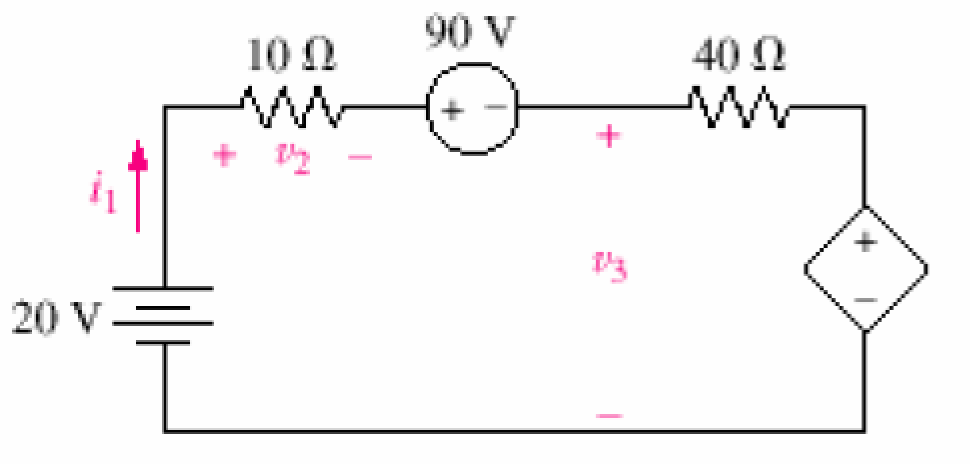
Therefore, $\boxed{V_{DS} = 12 - 7(1.5) = 1.5 \text{ V.}}$

(*b*) By KVL, $- V_G + V_{GS} + 2000I_D = 0$

Therefore, $\boxed{V_{GS} = V_G - 2(2) = -1 \text{ V.}}$

(a) If ID = 1.5 mA, compute VDS. (b) If ID = 2 mA and VG = 3 V, compute VGS.

Problem 2



Label the dependent source 1.8v3. Find v3 if (a) the 90-V source generates 180 W; (b) the 90-V source absorbs 180 W; (c) the dependent source generates 100 W; (d) the dependent source absorbs 100 W of power.

Applying KVL, we find that

$-20 + 10i1 + 90 + 40i1 + 1.8v3 = 0$ [1]

Also, KVL allows us to write

$v3 = 40i1 + 1.8v3$

$v3 = -50i1$

So that we may write Eq. [1] as

$50i1 – 1.8(50)i1 = -70$

or $i1 = -70/-40 = 1.75$ A.

Since $v3 = -50i1 = -87.5$ V, *no further information is required to determine its value.*

The 90-V source is absorbing (90)(*i1) = 157.5 W of power and the dependent source is*

absorbing (1.8*v3)(i1) = -275.6 W of power.*

Therefore, *none of the conditions specified in (a) to (d) can be met by this circuit.*