Frequency response

S. Lodha

References:

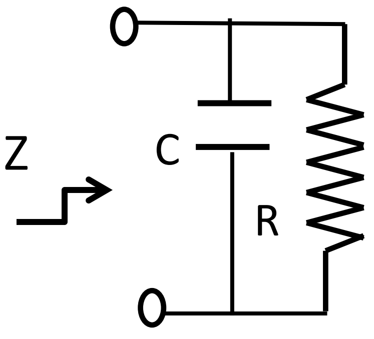
1. L. Bobrow's textbook
2. Slides from Prof. M. B. Patil

Frequency response

- Variation of magnitude and phase of an impedance or network function vs frequency

- In certain circuits, the network function peaks to a maximum when impedance becomes purely real
  - Resonance
  - Frequency selectivity
    - Quality Factor
    - Bandwidth

Frequency response: Impedance



$$|\mathbf{Z}| = \frac{R}{\sqrt{1+(\omega RC)^2}}$$

At ω=0, |**Z**|=R

At ω=1/RC, |**Z**|=R/1.414

At ω→inf, |**Z**|→0

$$\mathbf{Z} = \frac{R}{1+j\omega RC}$$

$$\mathbf{Z} = \frac{R}{\sqrt{1+(\omega RC)^2}} \angle -\tan^{-1}(\omega RC)$$

$$\mathbf{Z} = |\mathbf{Z}|\angle\theta = mag(\mathbf{Z})ang(\mathbf{Z})$$

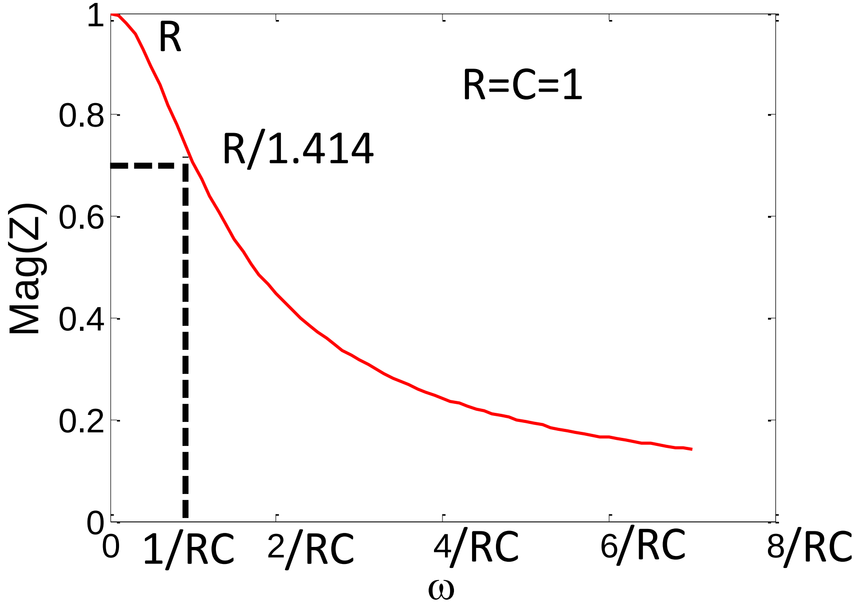
$$ang(\mathbf{Z}) = -\tan^{-1}(\omega RC)$$

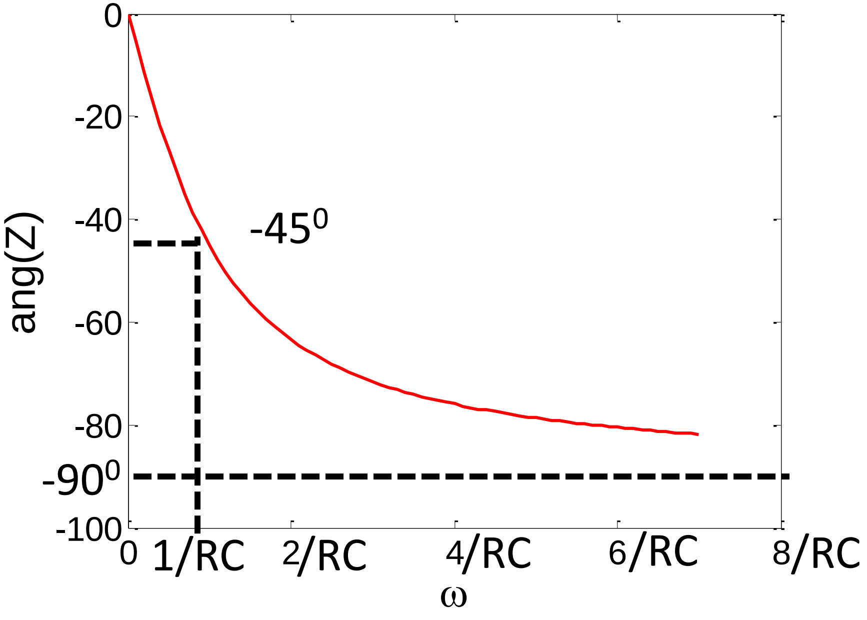
At ω=0, ang(**Z**)=0

At ω=1/RC, ang(**Z**)=-45^0

At ω→inf, ang(**Z**)→-π/2

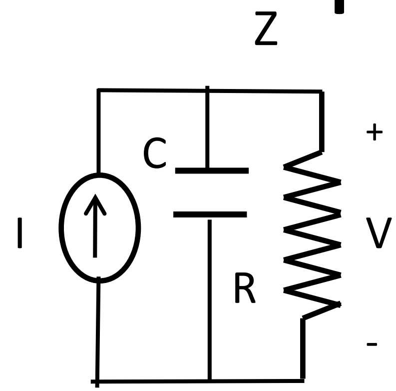
Frequency response: Impedance

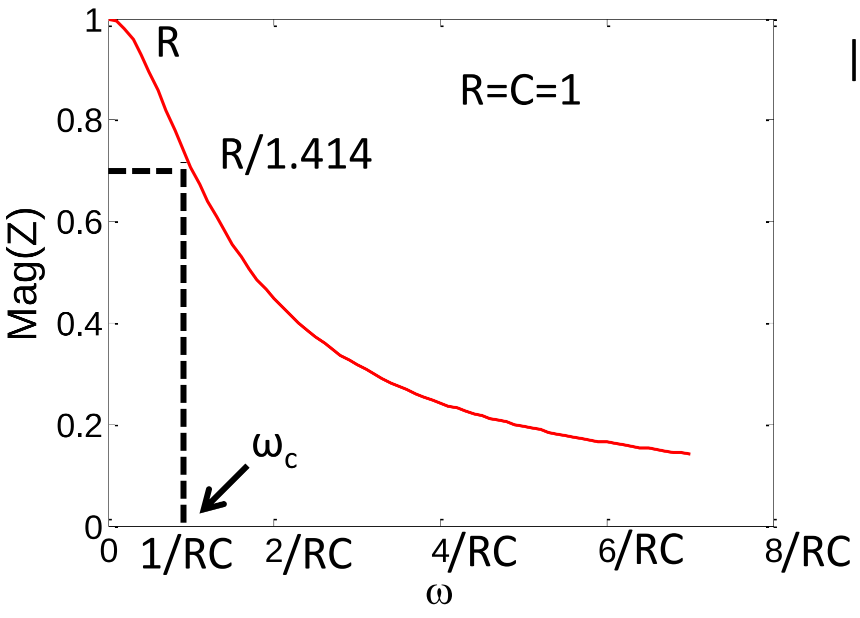




- Amplitude and Phase response of Z
- In most cases amplitude response is called frequency response

Half power/cut-off frequency ω_c





$$|\mathbf{Z}| = \frac{R}{\sqrt{1+(\omega RC)^2}}$$

$$|\mathbf{V}| = |\mathbf{ZI}| = |\mathbf{Z}||\mathbf{I}|$$

Given a current **I**, |**V**| is max when |**Z**| is max

|**Z**| is max when ω=0 rad/s, |**Z**|=R

$$|\mathbf{V}| = R|\mathbf{I}|$$

$$P_o = \frac{1}{2}\frac{|\mathbf{V}|^2}{R} = \frac{1}{2}R|\mathbf{I}|^2$$

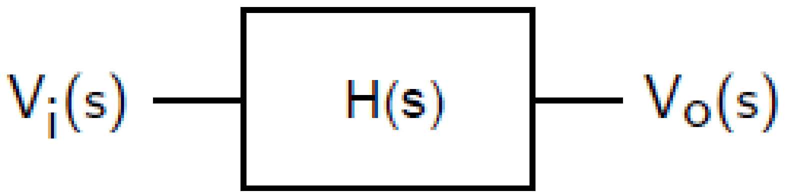
When ω=1/RC rad/s, |**Z**|=R/1.414

$$|\mathbf{V}| = \frac{R}{\sqrt{2}}|\mathbf{I}|$$

$$P_1 = \frac{1}{2}\frac{|\mathbf{V}|^2}{R} = \frac{1}{2}\frac{R|\mathbf{I}|^2}{2} = \frac{1}{2}P_0$$

Hence ω_c is called the half-power/cut-off frequency
In general, frequency at which amplitude response falls to 1/1.414 of maximum

Transfer function

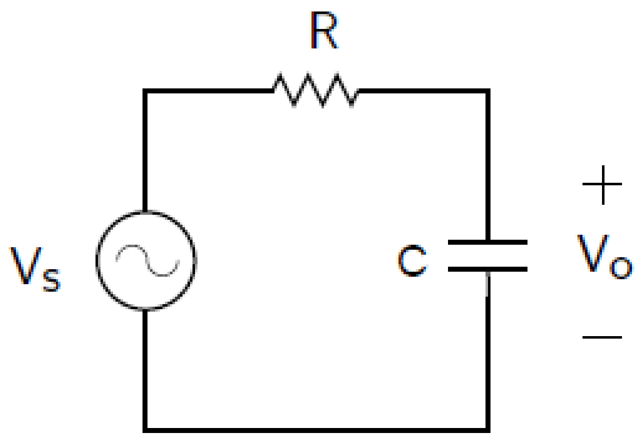


$$H(s) = V_o(s)/V_i(s), \quad s = j\omega.$$

$$\text{e.g., } H(s) = \frac{K}{1 + s\tau} = \frac{K}{1 + j\omega\tau}$$

- The transfer function of a circuit such as an amplifier or a filter is given by H(jω) → output voltage/input voltage
- H(jω) has an amplitude and phase response

Example



$$V_o = \frac{(1/sC)}{R + (1/sC)} V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (j\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC}\,.$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right).$$

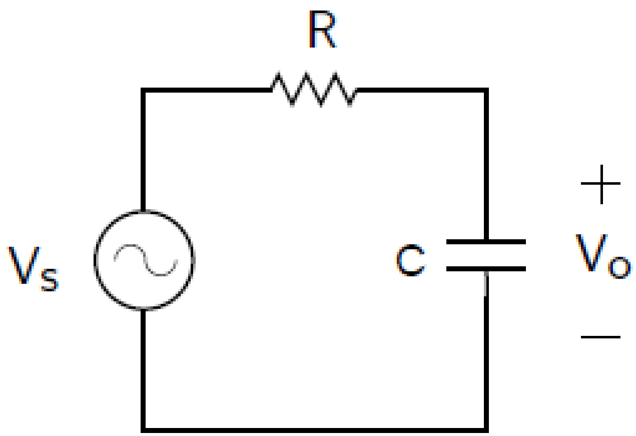
For $\omega \ll \omega_0$, $|H(j\omega)| \rightarrow 1$.

For $\omega \gg \omega_0$, $|H(j\omega)| \propto 1/\omega$.

The phase ($\angle H$) varies from 0 (for $\omega \ll \omega_0$) to $-\pi/2$ (for $\omega \gg \omega_0$).

- In general we are interested in both, |H| and <H vs ω for a wide range of ω

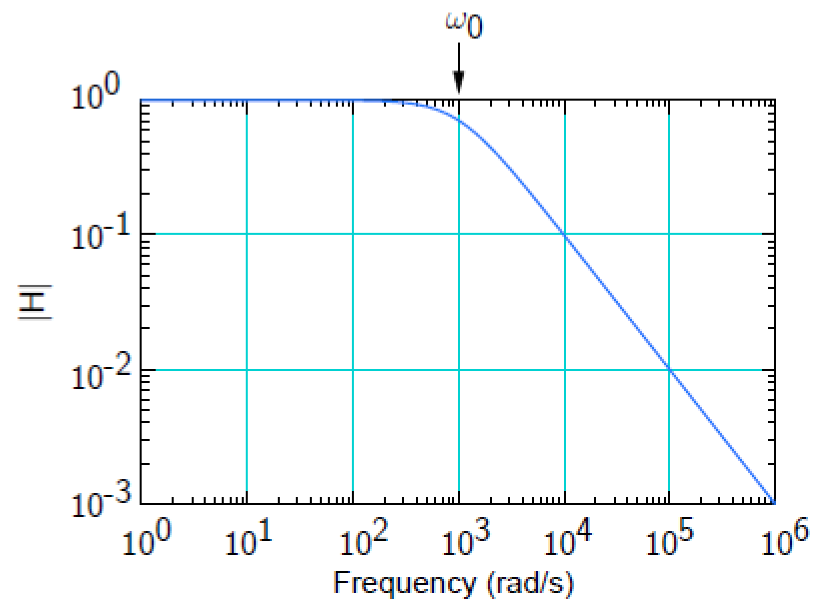
Example (contd.)



$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right).$$

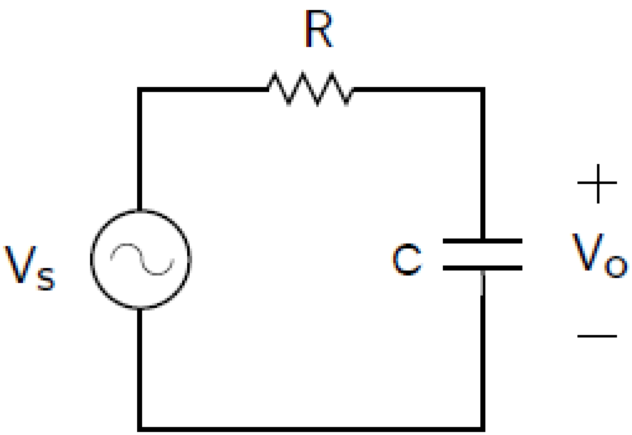
For $\omega \ll \omega_0$, $|H(j\omega)| \to 1$.

For $\omega \gg \omega_0$, $|H(j\omega)| \propto 1/\omega$.

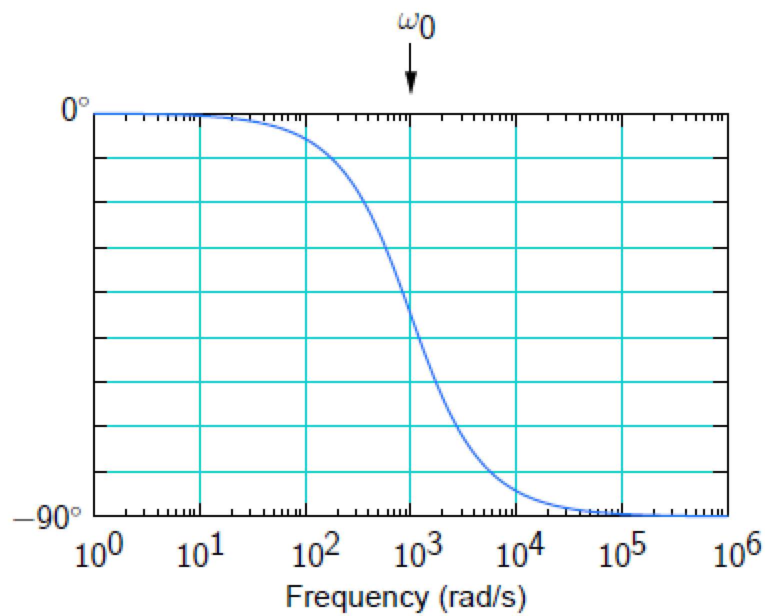


- Plot on a log-log scale
- Low-pass filter, suppresses frequencies higher than ~ω_0
- Note that $|H|=0.707$ at $\omega=\omega_0$ which is the half-power/cutoff frequency

Example (contd.)



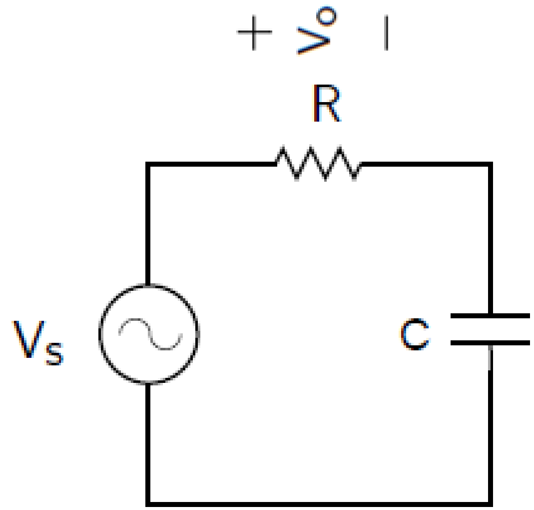
$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right).$$



The phase ($\angle H$) varies from 0 (for $\omega \ll \omega_0$) to $-\pi/2$ (for $\omega \gg \omega_0$).

- Plot on a log-linear scale

H. W.



- Find |H(ω)| and ang(ω)
- Plot these for a wide range of frequency
- What kind of a filter is this?
- What is the cut-off frequency?

Decibel (dB)

- The unit dB is used to represent quantities on a logarithmic scale
- Because of the log scale, dB is convenient for representing numbers that vary in a wide range
- log scaling roughly corresponds to human perception of sound and light
- log scale allows × and ÷ to be replaced by + and − → simpler!
- The unit "Bel" was introduced in the 1920s by Bell Labs engineers to quantify attenuation of an audio signal over one mile of cable
- Bel turned out to be too large in practice → deciBel (i.e., one tenth of a Bel)

Decibel (dB)

- dB is a unit that describes a quantity, on a log scale, with respect to a reference quantity
  - X (in dB) = 10 $\log_{10}(X/X_{ref})$.
  - For example, if P_1 = 20W and P_{ref} = 1W,
  - P_1 = 10 log (20W/1W) = 10 log (20) = 13 dB

- For voltages or currents, the ratio of squares is taken (since $P \alpha V^2$ or $P \alpha I^2$ for a resistor)

- For example, if V_1 = 1.2 V, V_{ref}=1 mV, then
  - $V_1 = 10 \log (1.2\ V/1mV)^2 = 20 \log(1.2/10^{-3})^2$= 61.6 dBm

- The voltage gain of an amplifier is A_V in dB = 20 log (V_o/V_i), with V_i serving as the reference voltage.

Example

$$A_V = 20\log\frac{V_o}{V_i}$$

$$36.3 = 20\log\frac{V_o}{2.5mV} \Rightarrow V_o = 162.5mV$$

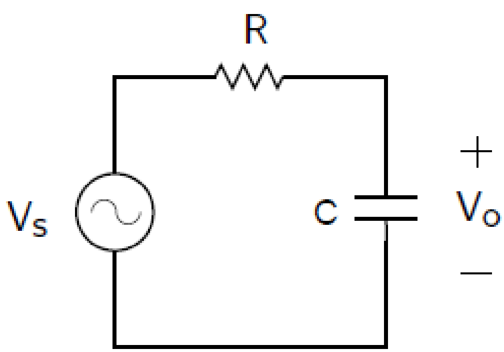
$$V_o(dBm) = 20\log\frac{V_o}{1mV} \Rightarrow V_o = 44.22dBm$$

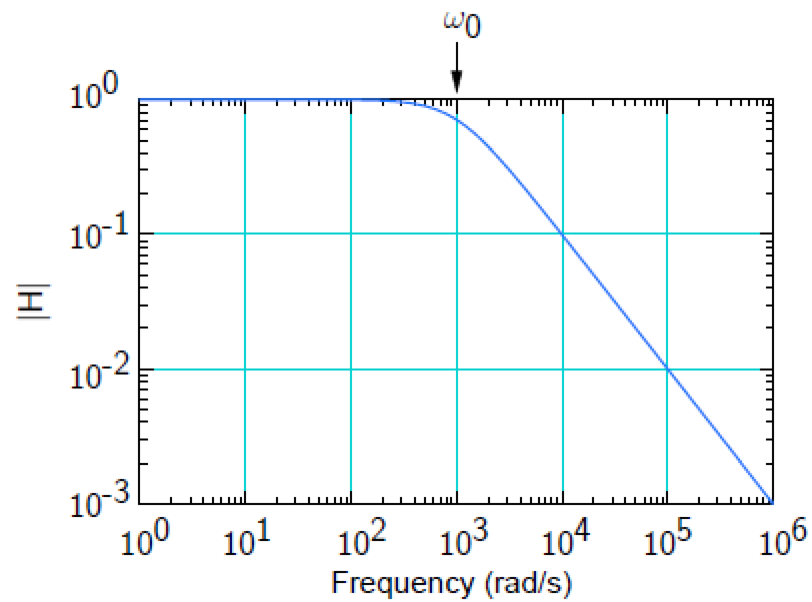
- Given V_i = 2.5mV and A_V = 36.3 dB, compute V_o in dBm and in mV. (V_i and V_o are peak input and peak output voltages, respectively).

dB in sound measurements

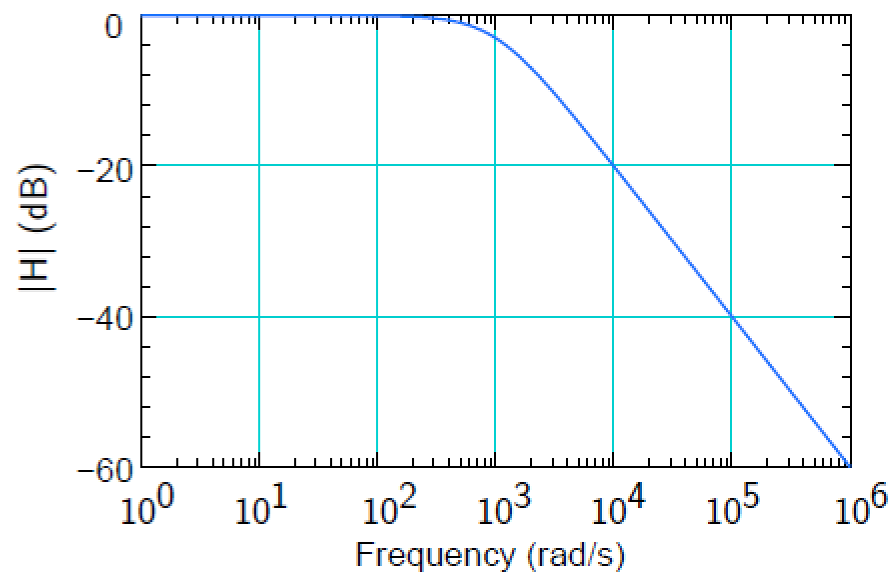
- When sound intensity is specified in dB, the reference pressure is P_{ref} = 20 μPa (our hearing threshold)
- If the pressure corresponding to the sound being measured is P, we say that it is 20 log (P/P_{ref}) dB
- Some interesting numbers:
  - mosquito 3m away 0 dB
  - whisper 20 dB
  - normal conversation 60 to 70 dB
  - noisy factory 90 to 100 dB
  - loud thunder 110 dB
  - loudest sound human ear can tolerate 120 dB
  - windows break 163 dB
- Also check → https://en.wikipedia.org/wiki/DBm

Example (revisited)





$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$



- |H| (dB) = 20 log |H| is simply a scaled version of log |H|.

Bode Plots

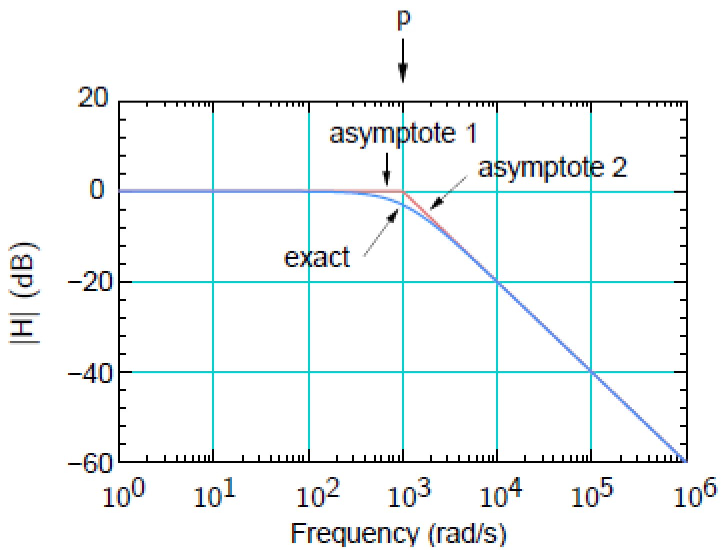
$$H(s) = \frac{K\,(1+s/z_1)(1+s/z_2)\cdots(1+s/z_M)}{(1+s/p_1)(1+s/p_2)\cdots(1+s/p_N)}$$

- $-z_1$, $-z_2$, .. are called the “zeroes” of H(s)
- $-p_1$, $-p_2$, .. are called the “poles” of H(s)
- In addition there could be terms like s, s^2, · · · in the numerator
- Assume that the zeroes (and poles) are real and distinct
- Construction of Bode plots involves

  (a) computing approximate contribution of each pole/zero as a function of ω.

  (b) combining the various contributions to obtain |H| and <H versus ω.

Contribution of a pole: Magnitude



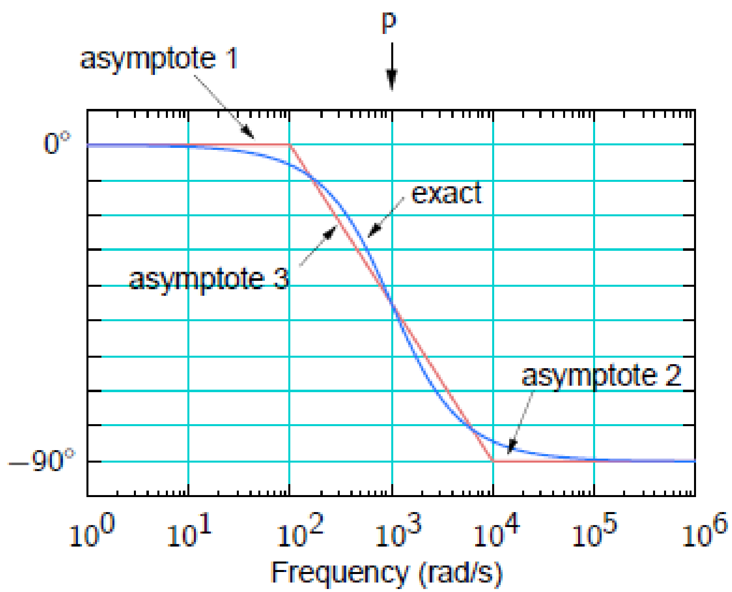
$$H(s) = \frac{1}{1+s/p} \rightarrow H(j\omega) = \frac{1}{1+j\,(\omega/p)}\ , |H(j\omega)| = \frac{1}{\sqrt{1+(\omega/p)^2}}\ .$$

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1,\ 20\log|H| = 0\,\text{dB}.$

Asymptote 2: $\omega \gg p$: $|H| \rightarrow \dfrac{1}{\omega/p} = \dfrac{p}{\omega} \rightarrow |H| = 20\ \log\ p - 20\ \log\omega\ (\text{dB})$

- Note that |H| vs ω has the slope of -20 dB
- At ω=p, actual value of |H|=-3 dB (|H|=1/1.414)

Contribution of a pole: Phase



$$H(s) = \frac{1}{1 + s/p} = \frac{1}{1 + j\,(\omega/p)} \rightarrow \angle H = -\tan^{-1}\left(\frac{\omega}{p}\right)$$

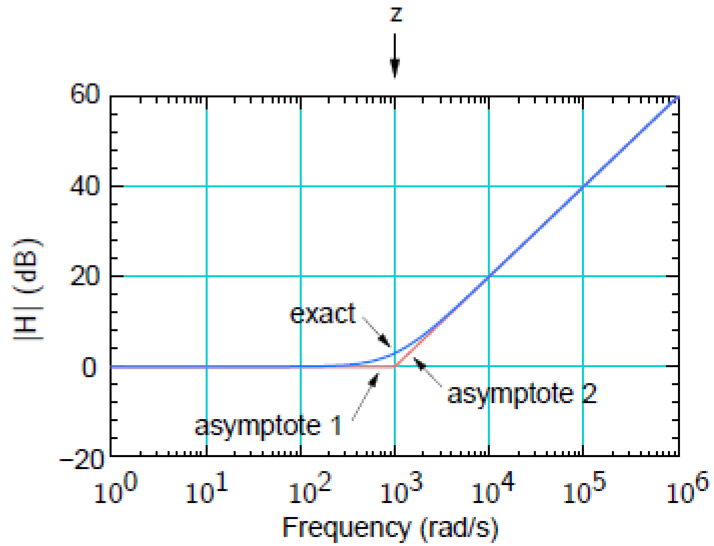
Asymptote 1: $\omega \ll p$ (say, $\omega < p/10$): $\angle H = 0$.

Asymptote 2: $\omega \gg p$ (say, $\omega > 10\,p$): $\angle H = -\pi/2$.

Asymptote 3: For $p/10 < \omega < 10\,p$, $\angle H$ is assumed to vary linearly with $\log \omega$

$\rightarrow$ at $\omega = p$, $\angle H = -\pi/4$ (which is also the actual value of $\angle H$).

Contribution of a zero: Magnitude



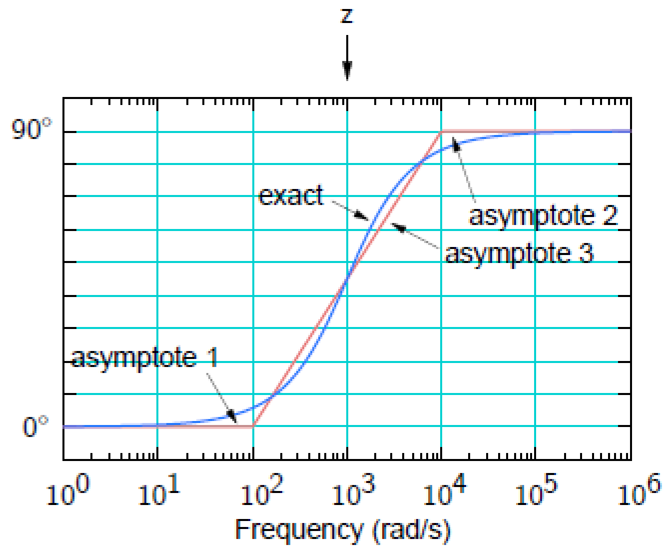
$$H(s) = 1 + s/z \longrightarrow H(j\omega) = 1 + j\,(\omega/z)\,,\, |H(j\omega)| = \sqrt{1 + (\omega/z)^2}\,.$$

Asymptote 1: $\omega \ll p$: $|H| \to 1,\ 20\log|H| = 0\,\mathrm{dB}.$

Asymptote 2: $\omega \gg p$: $|H| \to \dfrac{\omega}{z} \to |H| = 20\ \log\omega - 20\ \log\ z\ (\mathrm{dB})$

- Note that |H| vs ω has the slope of +20 dB
- At ω=z, actual value of|H|=+3 dB (|H|=1.414)

Contribution of a zero: Phase



$$H(s) = 1 + s/z = 1 + j\,(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\frac{\omega}{z}\right)$$

Asymptote 1: $\omega \ll z$ (say, $\omega < z/10$): $\angle H = 0$.

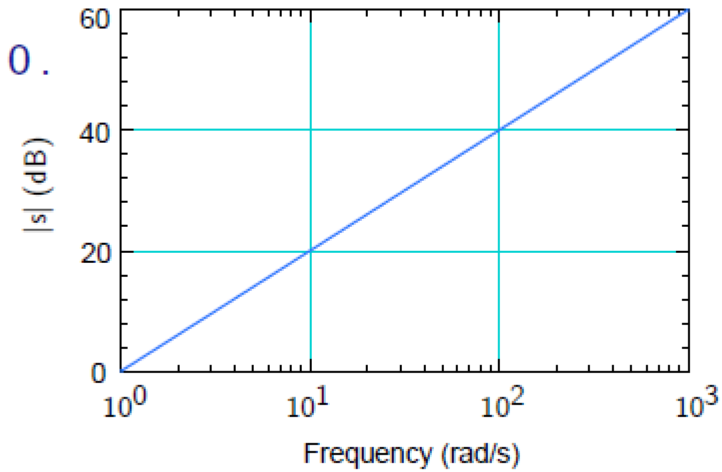
Asymptote 2: $\omega \gg z$ (say, $\omega > 10\,z$): $\angle H = \pi/2$.

Asymptote 3: For $z/10 < \omega < 10\,z$, $\angle H$ is assumed to vary linearly with $\log \omega$

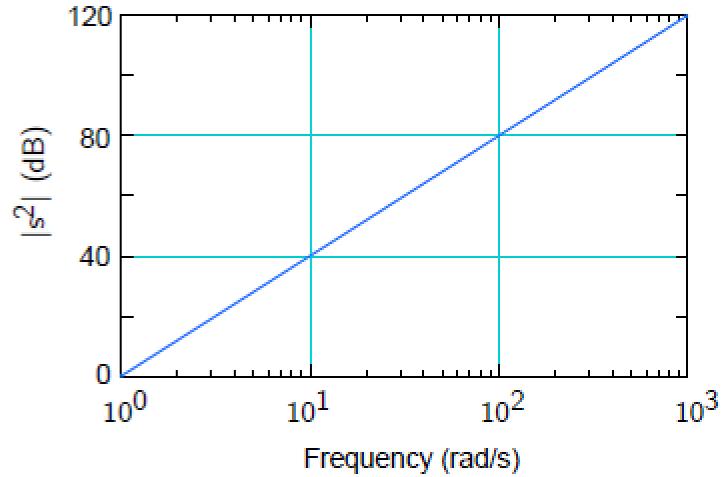
$\rightarrow$ at $\omega = z$, $\angle H = \pi/4$ (which is also the actual value of $\angle H$).

Contribution of K (constant), s, s^2

$H(s) = K$, $20 \log |H| = 20 \log K$ (a constant), and $\angle H = 0$.



For $H(s) = s$, i.e., $H(j\omega) = j\omega$, $|H| = \omega$.
$\rightarrow 20 \log |H| = 20 \log \omega$,
i.e., a straight line in the $|H|$ (dB)-$\log \omega$ plane with a slope of 20 dB/decade, passing through $(1, 0)$.
$\angle H = \pi/2$ (irrespective of ω).



For $H(s) = s^2$, i.e., $H(j\omega) = -\omega^2$, $|H| = \omega^2$.
$\rightarrow 20 \log |H| = 40 \log \omega$,
i.e., a straight line in the $|H|$ (dB)-$\log \omega$ plane with a slope of 40 dB/decade, passing through $(1, 0)$. $\angle H = \pi$ (irrespective of ω)

Combining different terms

$H(s) = H_1(s) \times H_2(s)$

Magnitude

$|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|$.

$20 \log |H| = 20 \log |H_1| + 20 \log |H_2|$.

In the Bode magnitude plot, the contributions due to H_1 and H_2 simply get added

Phase

$H_1(j\omega)$ and $H_2(j\omega)$ are complex numbers.

At a given ω, let $H_1 = K_1 \angle \alpha = K_1\, e^{j\alpha}$, and $H_2 = K_2 \angle \beta = K_2\, e^{j\beta}$.

Then, $H_1 H_2 = K_1\, K_2\, e^{j(\alpha+\beta)} = K_1 K_2 \angle (\alpha + \beta)$.

i.e., $\angle H = \angle H_1 + \angle H_2$.

In the Bode phase plot, the contributions due to H_1 and H_2 also get added

Example

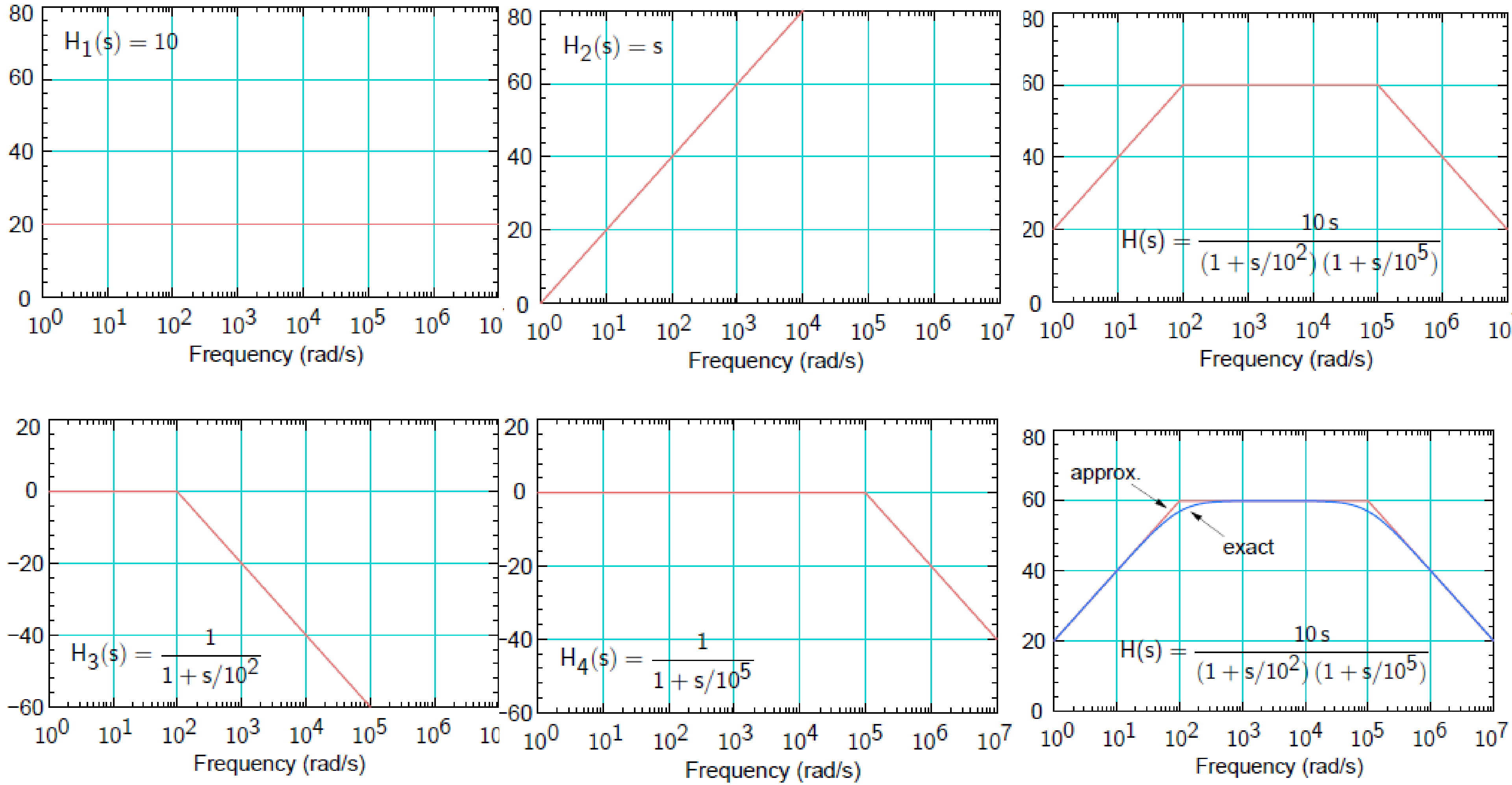
$$H(s) = \frac{10\,s}{(1 + s/10^2)\,(1 + s/10^5)}$$

Let $H(s) = H_1(s)\,H_2(s)\,H_3(s)\,H_4(s)$ where

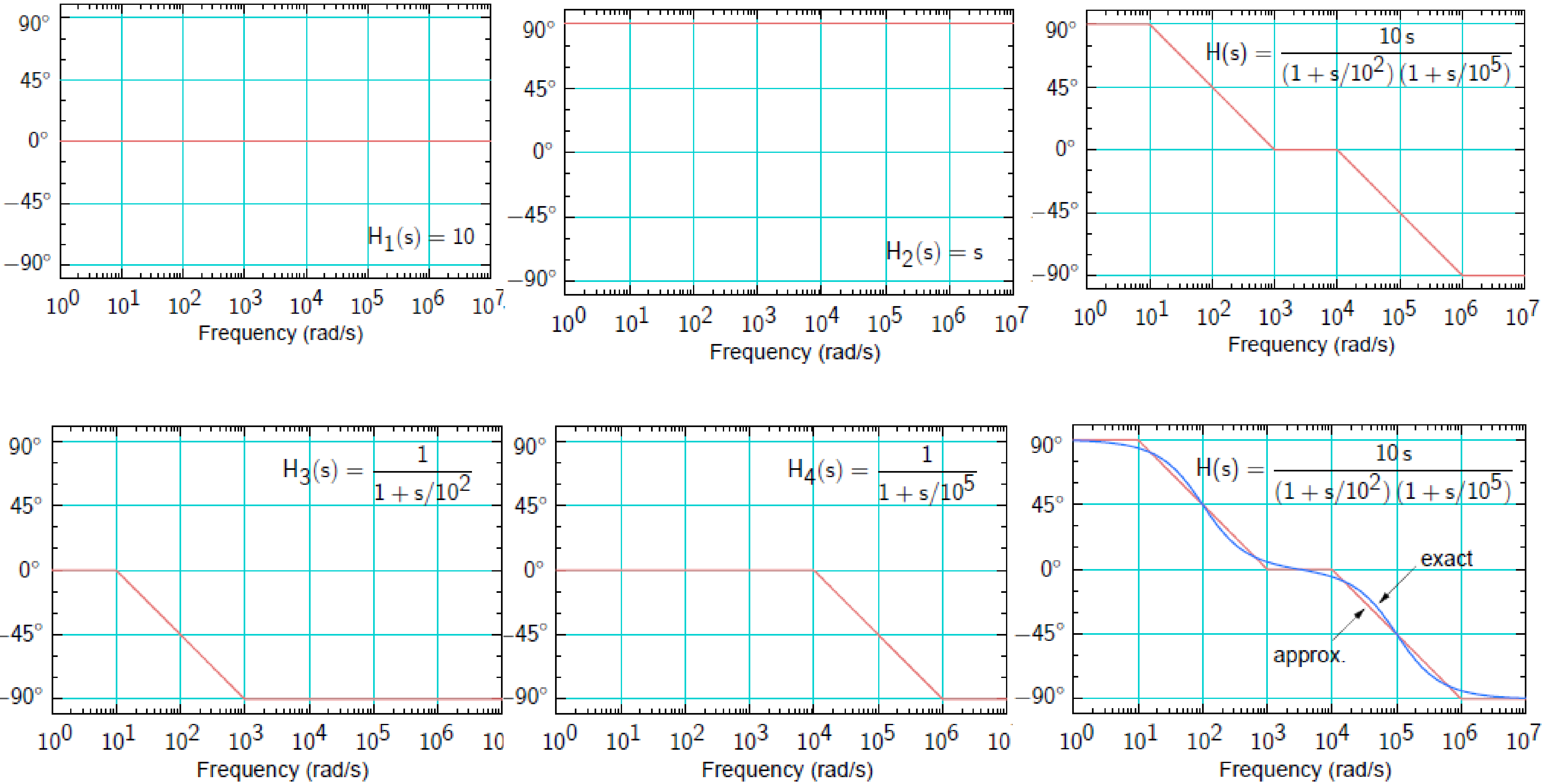
$$\begin{aligned}
H_1(s) &= 10\,, \\
H_2(s) &= s\,, \\
H_3(s) &= \frac{1}{1 + s/p_1}\,,\ p_1 = 10^2\ \mathrm{rad/s}, \\
H_4(s) &= \frac{1}{1 + s/p_2}\,,\ p_2 = 10^5\ \mathrm{rad/s}.
\end{aligned}$$

- Plot the magnitude and phase of H_1, H_2, H_3, H_4 individually versus ω and then simply add them to obtain |H| and <H.

Example (contd.)



Phase Plot of H(s)



How good are the approximations?

- The contribution of a pole to the magnitude and phase plots is well represented by the asymptotes when $\omega << p$ or $\omega >> p$ (similarly for a zero)

- Near $\omega = p$ (or $\omega = z$), there is some error

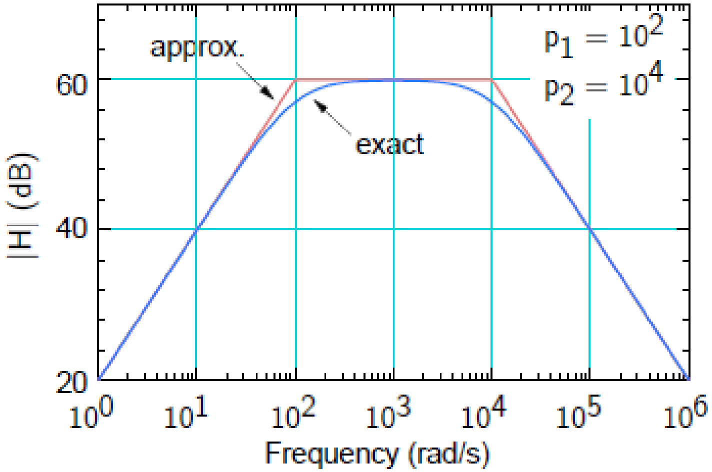
- If two poles p_1 and p_2 are close to each other (say, separated by less than a decade in ω), the error becomes larger (next slide)

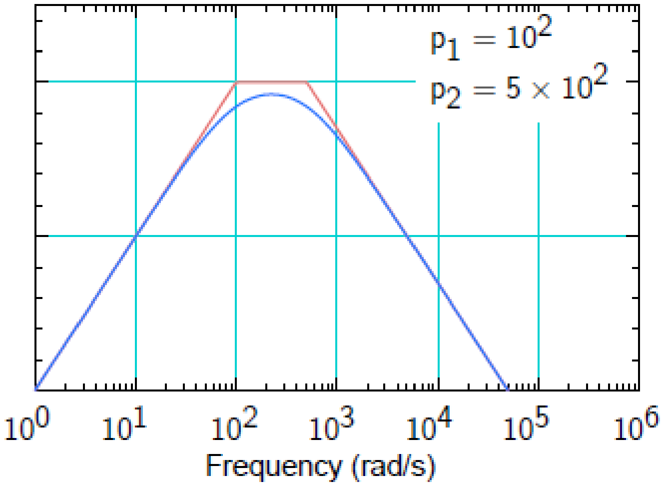
- When the poles and zeros are not sufficiently separated, the Bode approximation should be used only for a rough estimate, followed by a numerical calculation

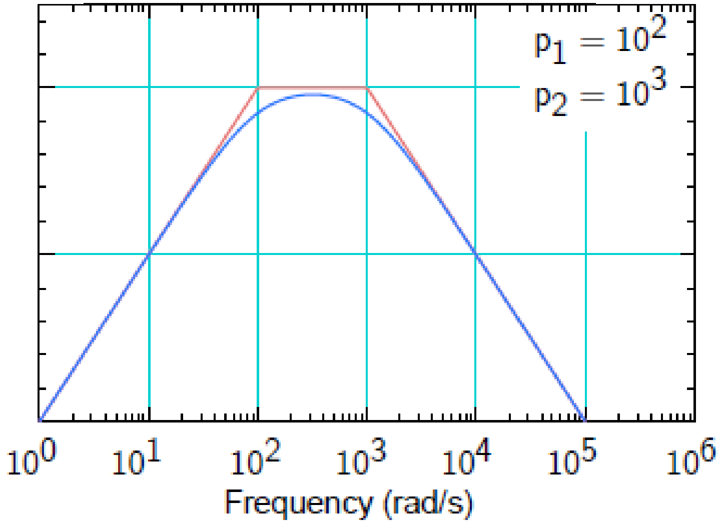
- However, even in such cases, it does give a good idea of the asymptotic magnitude and phase plots, which is valuable in amplifier design

Example

$$H(s) = \frac{10\,s}{(1 + s/p_1)\,(1 + s/p_2)}$$

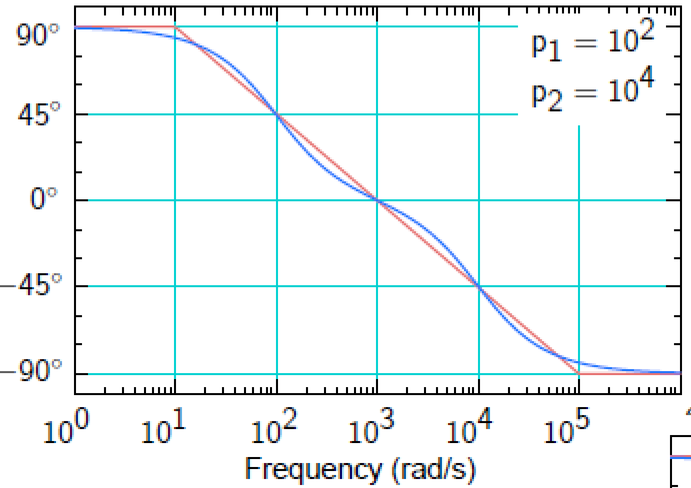


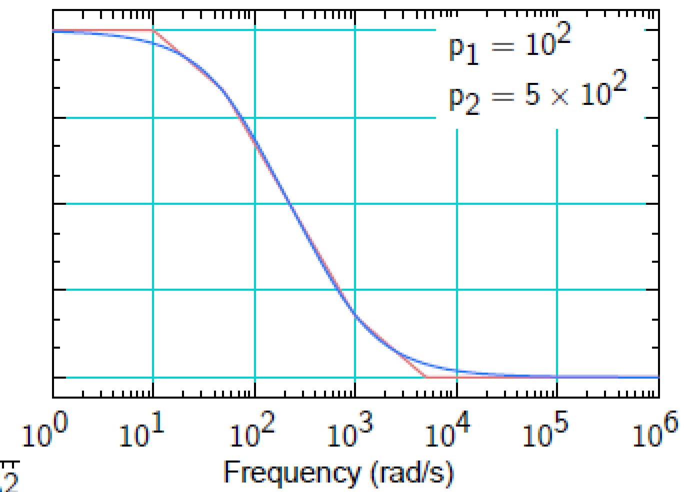


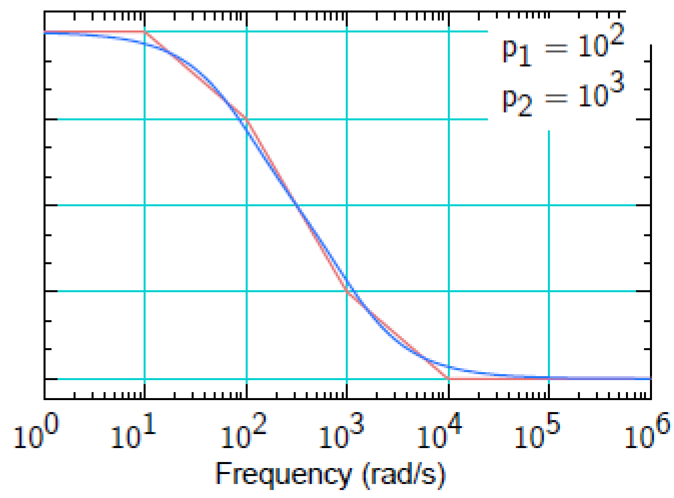


Example

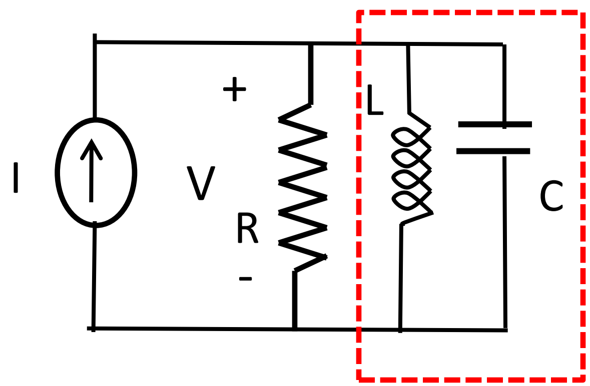
$$H(s) = \frac{10\, s}{(1 + s/p_1)\,(1 + s/p_2)}$$







Resonance in parallel RLC circuits



Tank ckt, open ckt at resonance

$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

$$\omega_r C - \frac{1}{\omega_r L} = 0$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

At $\omega = \omega_r$

$$|\mathbf{Y}| = \frac{1}{R}$$ is minimum

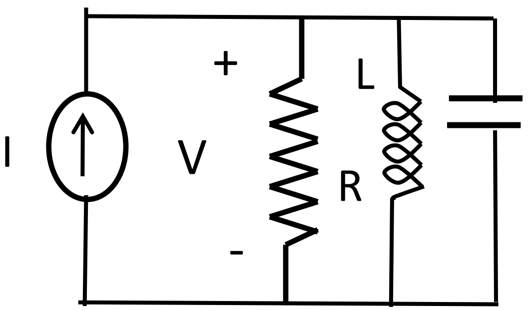
$$|\mathbf{V}| = \frac{|\mathbf{I}|}{|\mathbf{Y}|} = R|\mathbf{I}|$$ is maximum

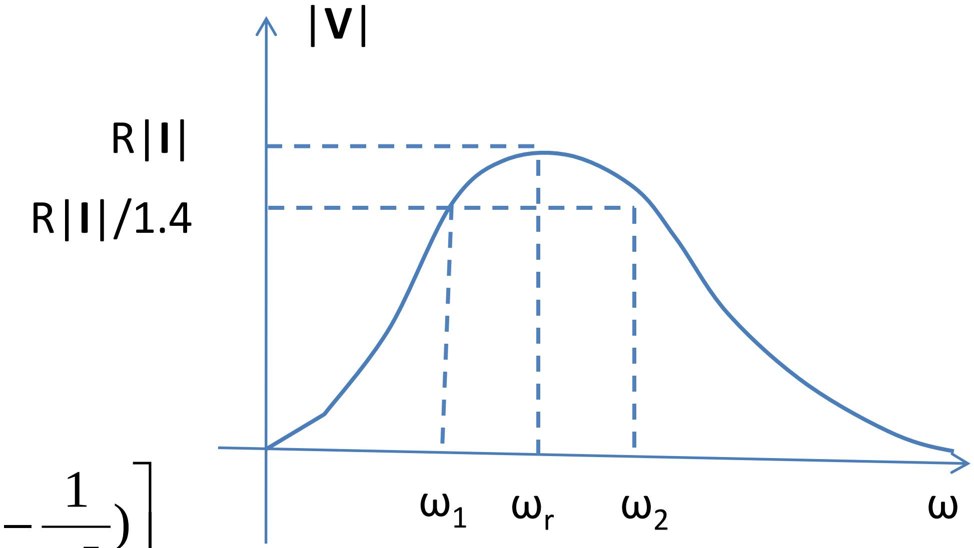
$$\omega \to 0,\ j\omega L \to 0, |\mathbf{V}| \to 0$$

$$\omega \to \infty, \frac{1}{j\omega C} \to 0, |\mathbf{V}| \to 0$$

- Circuit with at least one capacitor and one inductor is in resonance when imaginary part of the admittance (or impedance) = 0
- ω_r is the resonance frequency
- At resonance, parallel RLC acts simply as R, the LC ckt (tank ckt) acts as an open ckt

Parallel RC circuit





$$\mathbf{V} = \frac{|\mathbf{I}|}{\sqrt{\frac{1}{R^2} + (\omega C - \frac{1}{\omega L})^2}} \angle - \tan^{-1}\left[R(\omega C - \frac{1}{\omega L})\right]$$

- Bandwidth BW=ω_2-ω_1
- Smaller the bandwidth, sharper is the amplitude response

Quality Factor: Sharpness of the amplitude response

$$Q = 2\pi\left(\frac{\text{max energy stored}}{\text{total energy lost in a period}}\right)$$

$$Q = \frac{2\pi[\mathrm{W_C}(t) + \mathrm{W_L}(t)]_{\max}}{P_R T}$$

$$i(t) = I\cos\omega_r t,\ v(t) = RI\cos\omega_r t\ (\text{since } \mathbf{Y} = 1/R)$$

$$\mathrm{W_C}(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}CR^2I^2\cos^2\omega_r t$$

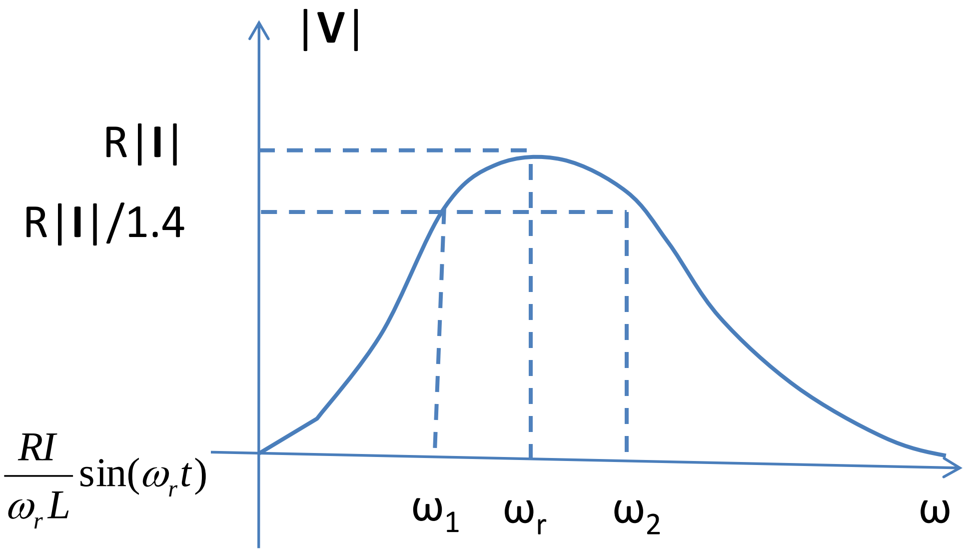
$$\mathbf{I_L} = \frac{\mathbf{V}}{j\omega_r L} = \frac{RI\angle 0^0}{\omega_r L\angle 90^0} \Rightarrow i_L(t) = \frac{RI}{\omega_r L}\cos(\omega_r t - 90^0) = \frac{RI}{\omega_r L}\sin(\omega_r t)$$

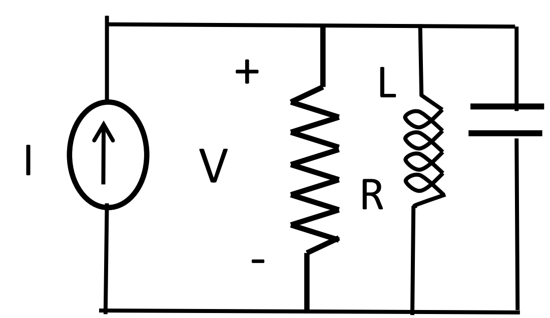
$$\mathrm{W_L}(t) = \frac{1}{2}Li_L^2(t) = \frac{1}{2}\left(\frac{RI}{\omega_r L}\right)^2\sin^2(\omega_r t) = \frac{1}{2}CR^2I^2\sin^2\omega_r t$$

$$\boxed{\mathrm{W_C}(t) + \mathrm{W_L}(t) = \frac{1}{2}CR^2I^2\cos^2\omega_r t + \frac{1}{2}CR^2I^2\sin^2\omega_r t = \frac{1}{2}CR^2I^2}$$

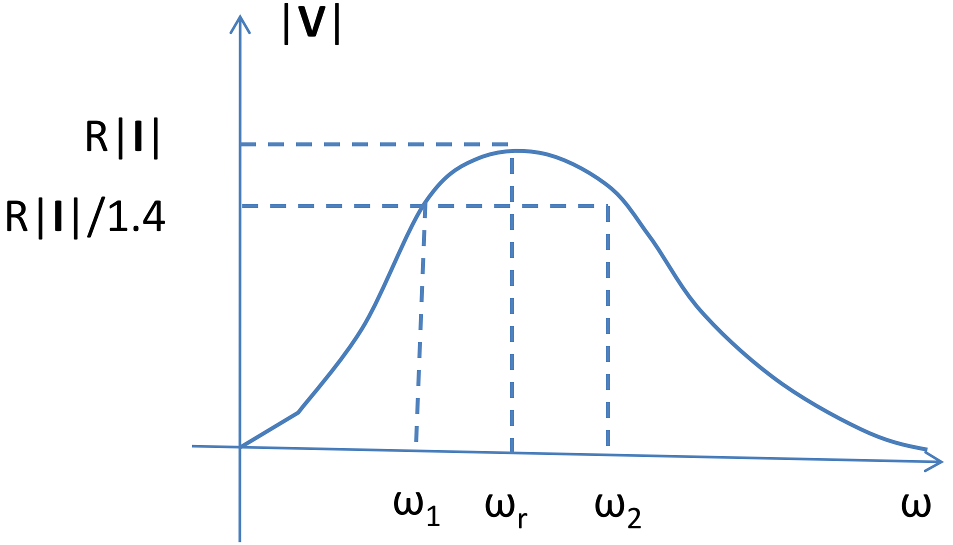
$$\boxed{P_R T = \frac{1}{2}RI^2T = \frac{1}{2}RI^2\left(\frac{2\pi}{\omega_r}\right) = \frac{\pi RI^2}{\omega_r}}$$

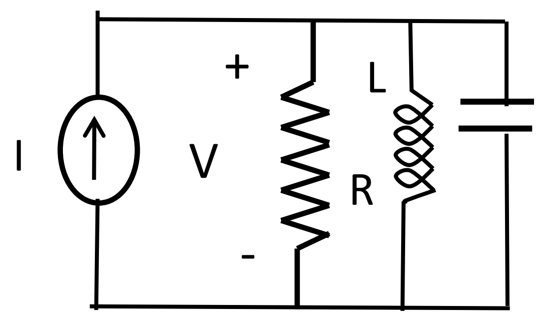
$$\boxed{Q = \frac{2\pi\left(\frac{1}{2}CR^2I^2\right)}{\frac{\pi RI^2}{\omega_r}} = \omega_r RC = \frac{R}{\omega_r L} = R\sqrt{\frac{C}{L}}}$$





Bandwidth





- Higher Q → smaller BW
- Higher BW → smaller Q

$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

$$\boxed{Y = \frac{1}{R}\left[1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)\right]}$$

At half power frequencies ω_1 and ω_2

$$|\mathbf{V}| = \frac{R|\mathbf{I}|}{\sqrt{2}} = \frac{|\mathbf{I}|}{|\mathbf{Y}|} \Rightarrow |\mathbf{Y}| = \frac{\sqrt{2}}{R}$$

$$Q\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right) = 1,\ Q\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right) = -1$$

Solve the quadratic equation in ω

and keep only the positive roots,

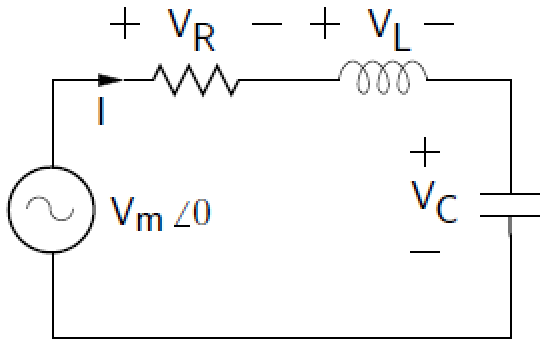
$$\omega_1 = -\frac{\omega_r}{2Q} + \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$$

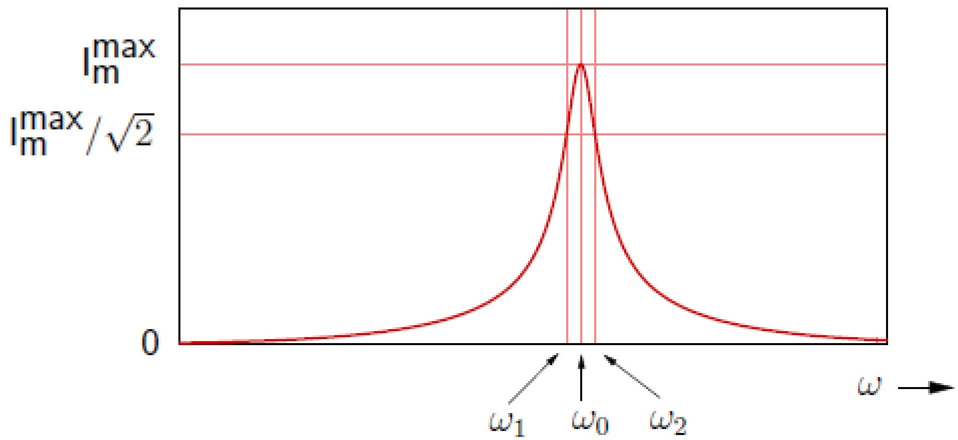
$$\omega_2 = \frac{\omega_r}{2Q} + \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$$

$$Bandwidth = \omega_2 - \omega_1 = \frac{\omega_r}{Q} = \frac{1}{RC}$$

$$Selectivity = \frac{\omega_r}{BW}$$

Resonance in series RLC ckt





$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R$

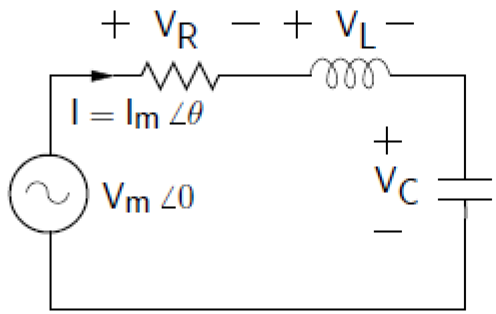
For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}$

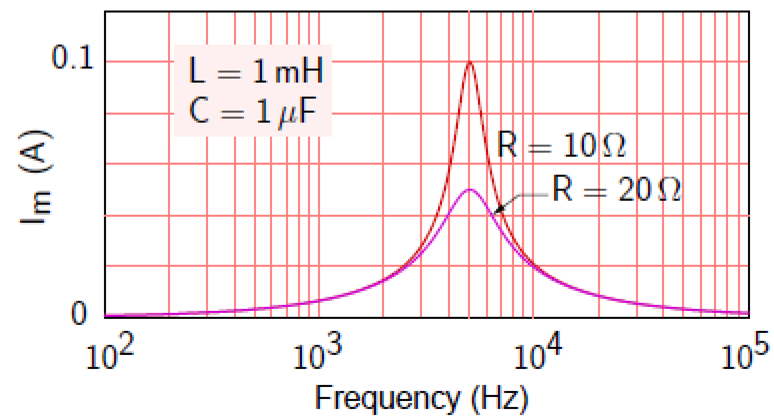
$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

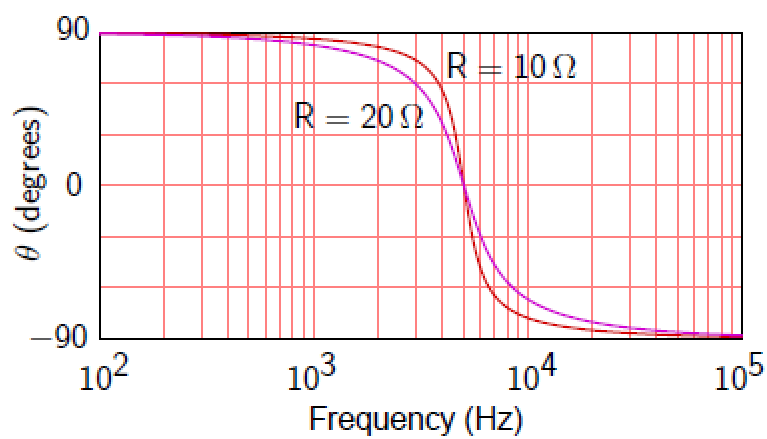
$$\boxed{\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

- Bandwidth BW= $\omega_2 - \omega_1$= R/L
- Quality Q =ω_0/BW=ω_0L/R
- Show sqrt($\omega_1\omega_2$)=ω_0
- Show that at resonance,
  - $|\mathbf{V_L}| = |\mathbf{V}_C| = Q\ V_m$

Resonance in series RLC ckt



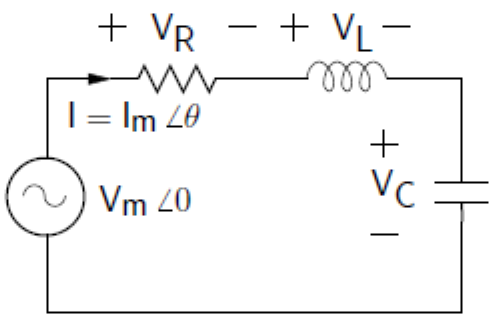




As R is increased,

- The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader
- The maximum current (at $\omega=\omega_0$) decreases (since $I_m^{max} = V_m/R$).
- The resonance frequency ($\omega_0=1/LC$) is not affected
- Bandwidth=ω_0/Q increases

Resonance in series RLC ckt



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} \equiv I_m \angle \theta$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$$

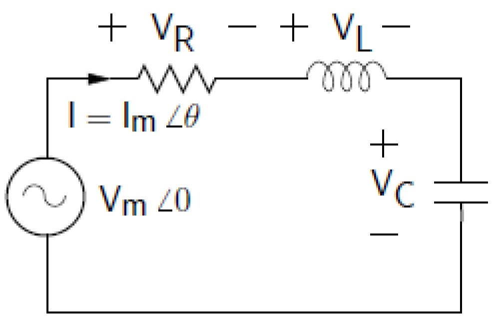
For

$$\omega < \omega_0, \omega L < \frac{1}{\omega C} \Rightarrow \text{net impedance is capacitive, I leads V}$$

$$\omega = \omega_0, \omega L = \frac{1}{\omega C} \Rightarrow \text{net impedance is resistive, I is in phase with V}$$

$$\omega > \omega_0, \omega L > \frac{1}{\omega C} \Rightarrow \text{net impedance is inductive, I lags V}$$

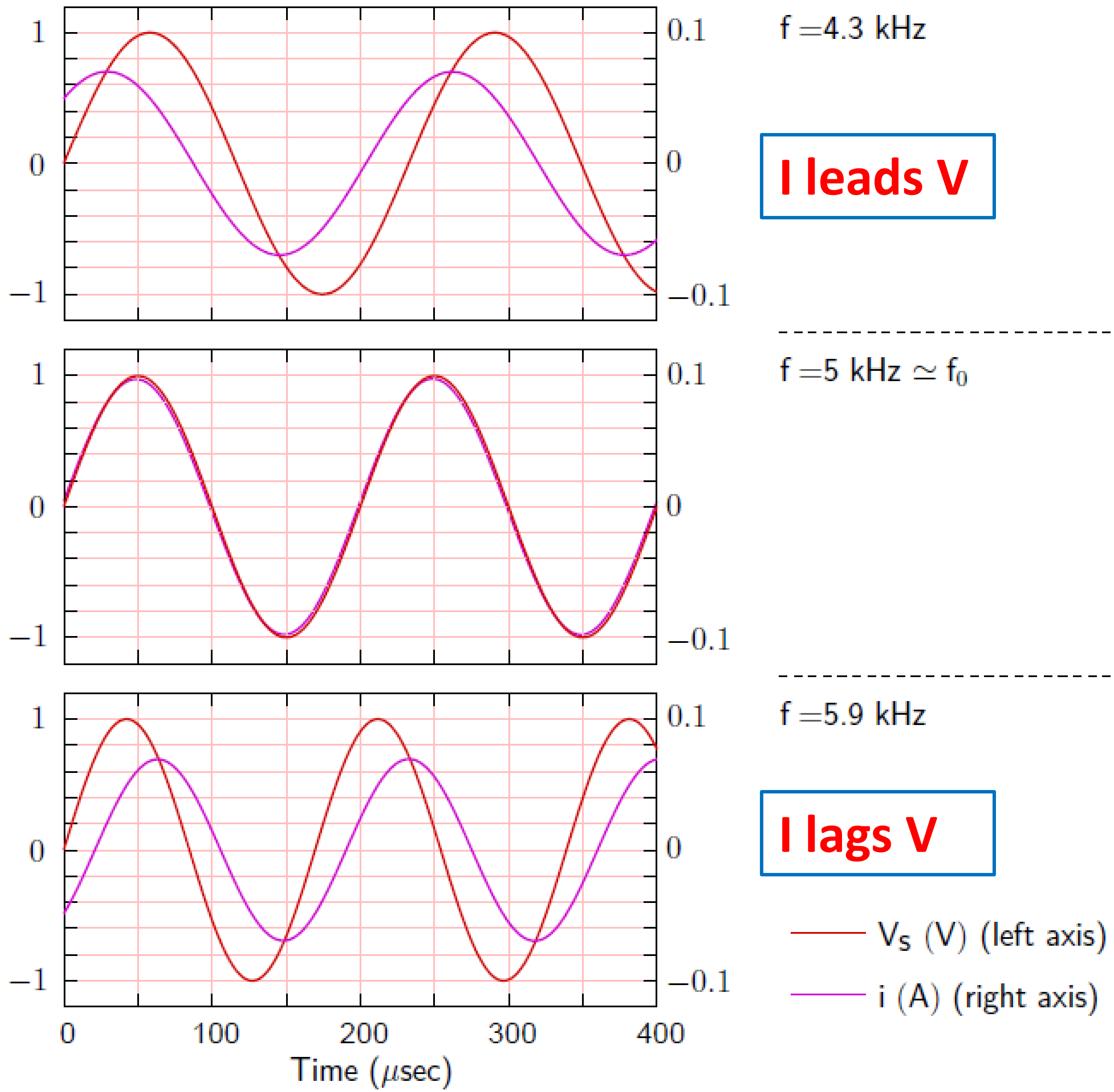
Example



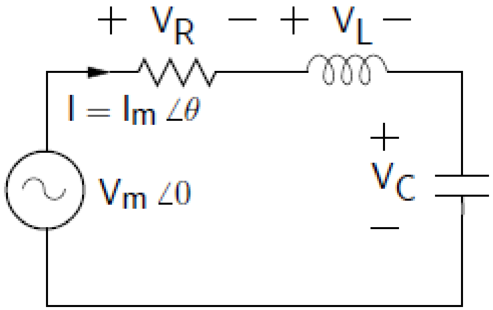
$R = 10\,\Omega$

$L = 1$ mH

$C = 1\ \mu$F



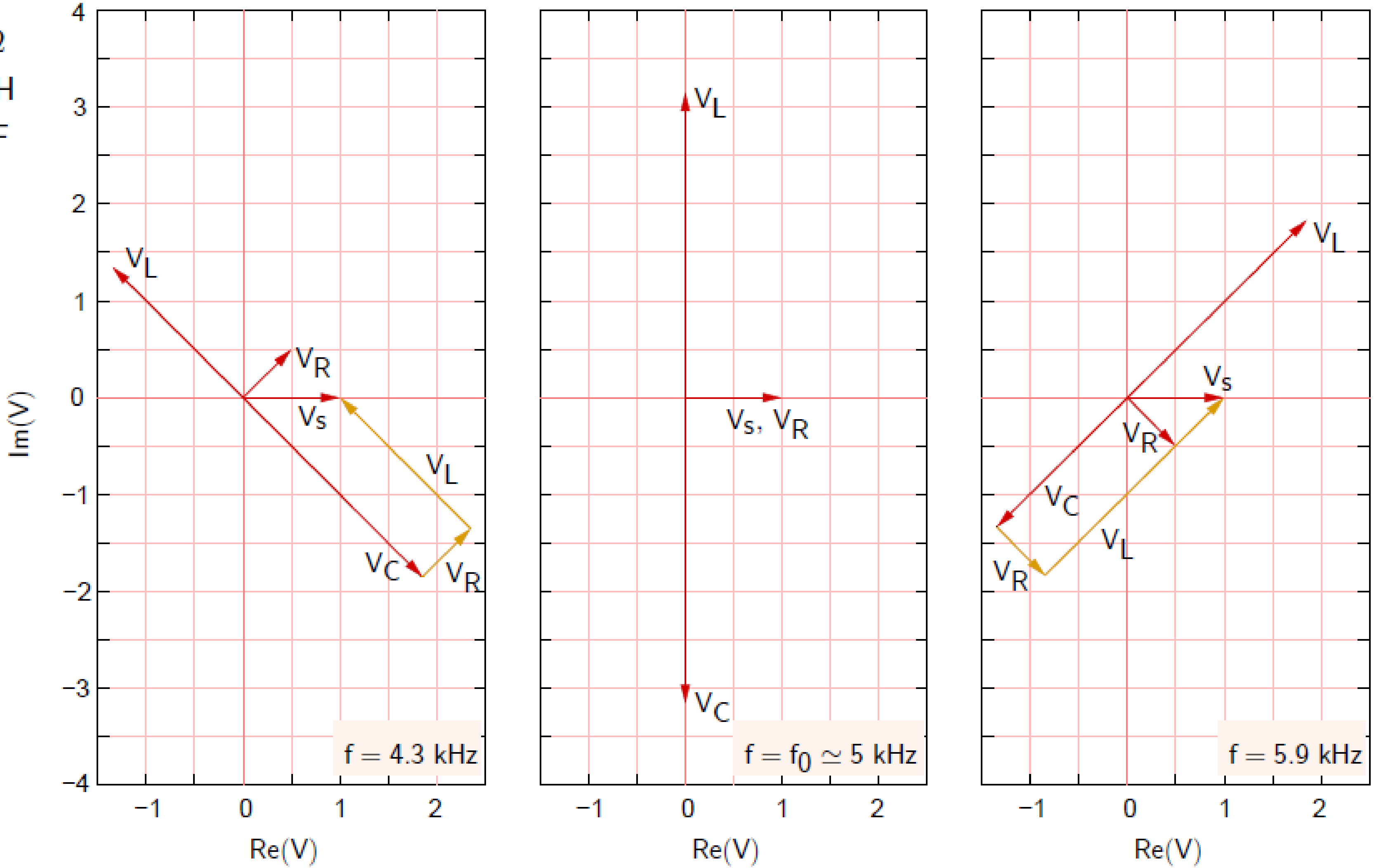
Example (contd.)



$R = 10\,\Omega$

$L = 1$ mH

$C = 1\ \mu F$



Series and Parallel Ckts

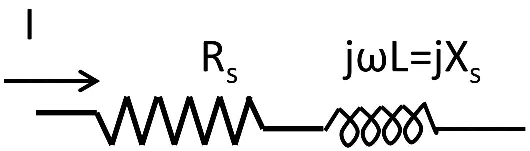
$$\text{Series } RLC \text{ circuit: } I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$$

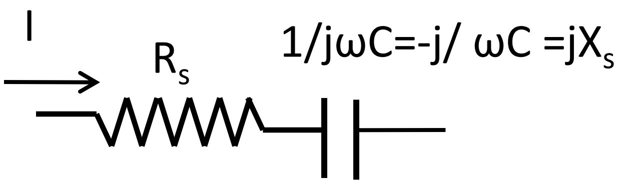
$$\text{Parallel } RLC \text{ circuit: } V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$$

The two situations are identical if we make the following substitutions:

- I → V,
- R →1/R,
- L →C.

Q factor of Coils (Series Reactances)





- For high Q-coils you need $|X_s|>>R_s$

$$Q = 2\pi\left(\frac{\text{max energy stored}}{\text{total energy lost in a period}}\right)$$

$$Q = \frac{2\pi[\mathrm{W_L}(t)]_{\max}}{P_R T}$$

$$i(t) = I\cos\omega t$$

$$\mathrm{W_L}(t) = \frac{1}{2}Li^2(t) = \frac{L^2 I^2}{2}\cos^2(\omega t)$$

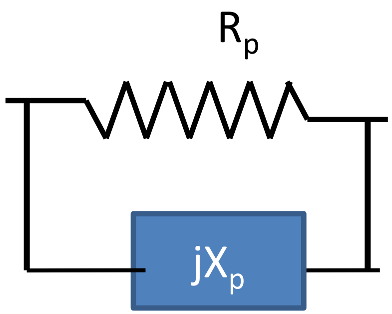
$$[\mathrm{W_L}(t)]_{\max} = \frac{L^2 I^2}{2} = \frac{1}{2}\left(\frac{X_s}{\omega}\right)I^2$$

$$P_R T = \frac{1}{2}R_s I^2 T = \frac{1}{2}R_s I^2\left(\frac{2\pi}{\omega}\right) = \frac{\pi R_s I^2}{\omega}$$

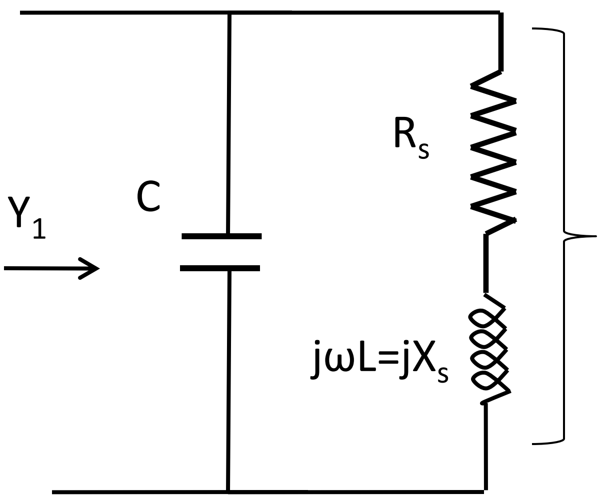
$$\boxed{Q = \frac{2\pi\left(\frac{1}{2}\left(\frac{X_s}{\omega}\right)I^2\right)}{\frac{\pi R_s I^2}{\omega}} = \frac{X_s}{R_s}}$$

$$\Rightarrow \text{In general, } Q = \frac{|X_s|}{R_s}$$

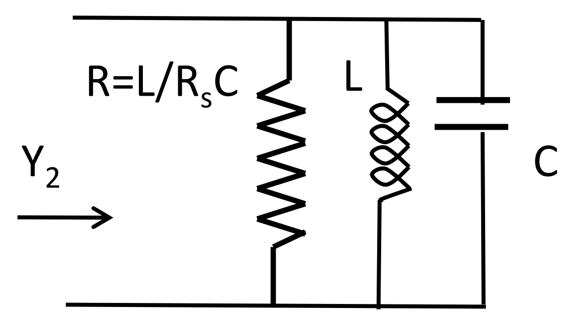
Q factor of Coils (Parallel Reactances)



Show that, $\boxed{Q = \dfrac{R_p}{\left|X_p\right|}}$



High Q coil, $X_s/R_s >> 1$



$$\boxed{\begin{gathered} Y_1 \approx Y_2 \\ \omega_r = \frac{1}{\sqrt{LC}} \end{gathered}}$$