CS 207: Discrete Structures

Instructor : S. Akshay

July 20, 2015

Lecture 01 – Introduction

Logistics

Course hours: Slot 3;
Mon 10:35-11:35, Tue 11:35-12:30, Thu 8:30-9:25
Office hours: By email appointment
Tutorial hours: One hour per week (to be decided)

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- max{tutorial participation, home assignments}: 5%

Logistics

Course hours: Slot 3;
Mon 10:35-11:35, Tue 11:35-12:30, Thu 8:30-9:25
Office hours: By email appointment
Tutorial hours: One hour per week (to be decided)

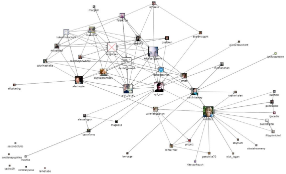
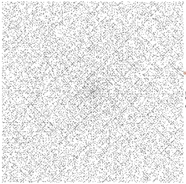
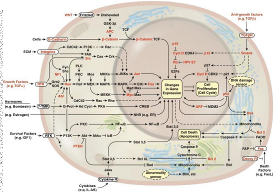
Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- max{tutorial participation, home assignments}: 5%

Course material, references will be posted at

- `http://www.cse.iitb.ac.in/~akshayss/teaching.html`
- piazza (will be set up by TAs soon)

Goal


First things first...

- What are discrete structures?
- Why are we interested in them?

Course Outline

What we will broadly cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. Introduction to abstract algebra and number theory

Course Outline

What we will broadly cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. Introduction to abstract algebra and number theory

What we don't cover

1. Logic : predicate, first-order logic– CS228
2. Discrete probability – IC102
3. Algorithms – CS218
4. Finite automata – CS310
5. Details and applications of everything above – rest of your (academic) life!

Course Outline

What we will cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. Introduction to abstract algebra and number theory

Textbooks

- Discrete Mathematics and its Applications with Combinatorics and Graph Theory, by Kenneth H Rosen.
- Discrete Mathematics by Norman Biggs.
- Introduction to Graph theory by Douglas B West.
- More will be listed on webpage as we go along.

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To write proofs and convey your ideas formally.
4. To develop skills to read/understand/solve new material in the future.

Chapter 1: Proofs and Structures

Outline of next few classes

- Propositions, statements
- What/why of proofs and some generic proof strategies
- Mathematical induction
- Notions and properties of sets, functions, relations

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n+1)(n-1) = (n^2 - 1)$ for any integer n

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

What is common between these statements?

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

A proposition is a statement that is either true or false (but not both).

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

A **proposition** is a statement that is either true or false (but not both).

Give an example of a statement that is not a proposition.

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n+1)(n-1) = (n^2 - 1)$ for any integer n

A proposition is a statement that is either true or false (but not both).

Give an example of a statement that is not a proposition.

- $x + 1 = 8$

Propositional calculus

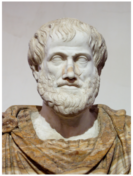


Figure: Aristotle (384 – 322 BCE)

- propositions are statements that are either true or false.
- Just as we use variables $x, y, \ldots$ for numbers, we will use variables $p, q, \ldots$ for propositions.
- "if it rains, it will be wet" : $p \to q$
- combining propositions: $\neg p, p \vee q, p \wedge q, p \to q$, p iff q.
- Can all mathematical statements be written this way?

Predicates and quantifiers

Consider again...

$$(n+1)(n-1) = (n^2-1)$$
$$x = y + 8$$

Predicates and quantifiers

Consider again...

- $\forall n \qquad (n+1)(n-1) = (n^2 - 1)$
- $\forall x, \exists y, \qquad\qquad x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n

Predicates and quantifiers

Consider again...

- $\forall n \in \mathbb{N}\ (n+1)(n-1) = (n^2-1)$
- $\forall x, \exists y, x, y \in \mathbb{Z}\ x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n
- $\in$ is the element of symbol
- $\mathbb{N}$ stands for all natural numbers
- $\mathbb{Z}$ stands for all integers
- $\mathbb{R}$, $\mathbb{Q}$, ...

Some propositions are not so easy to "determine"...
– e.g., $2^{67} - 1$ is not a prime.

Theorems and proofs

A theorem is a proposition which can be shown true

Classwork: Prove the following theorems.

1. For all $a, b, c \in \mathbb{R}^{\geq 0}$, if $a^2 + b^2 = c^2$, then $a + b \geq c$
2. If 6 is prime, then $6^2 = 30$.
3. x is an even integer iff $x + x^2 - x^3$ is even.

4. There are infinitely many prime numbers.
5. There exist irrational numbers x, y such that x^y is rational.
6. For all $n \in \mathbb{N}$, $n! \leq n^n$.
7. There does not exist a (input-free) C-program which will always determine whether an arbitrary (input-free) C-program will halt.