Op-Amp Circuits: Feedback

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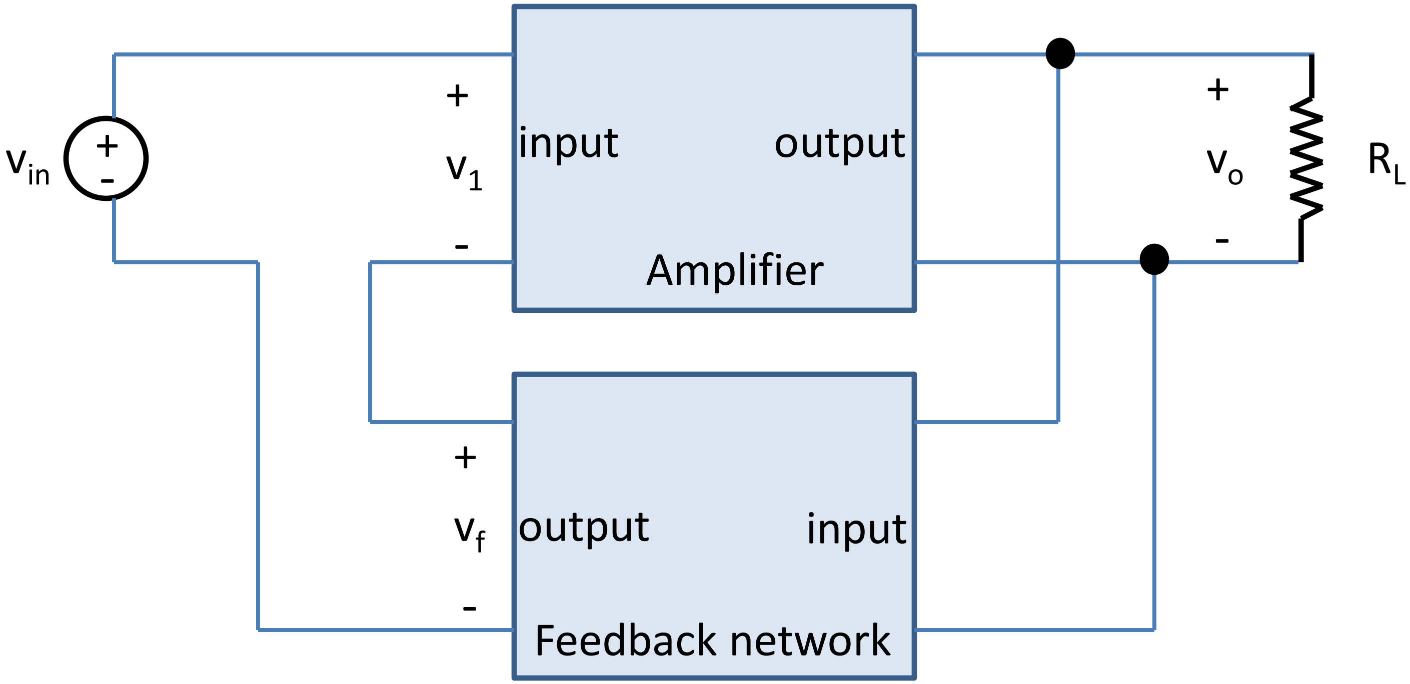
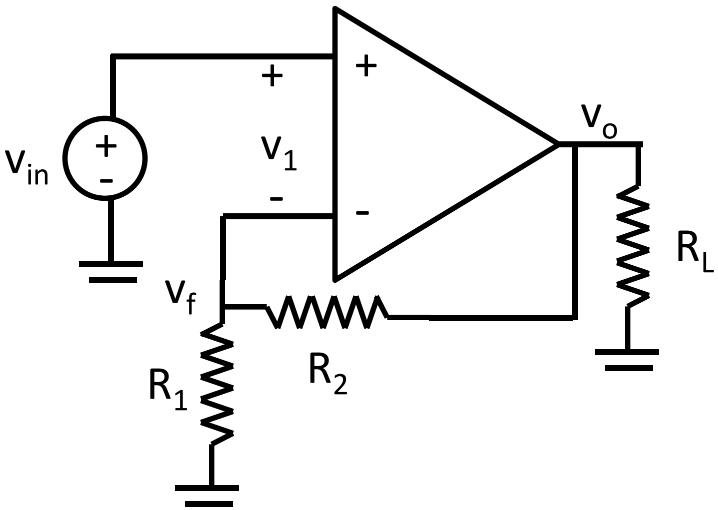
References: L. Bobrow's book and Prof. M. B. Patil's slides

What is feedback?



- Output of circuit/system returned back to input
- Four possible connections
  - Above is an example of ***series-parallel*** feedback
  - Output of feedback in ***series*** with input of amp, input of feedback in ***parallel*** with output of amp
  - Most beneficial for voltage amplification applications

Example of series-parallel feedback



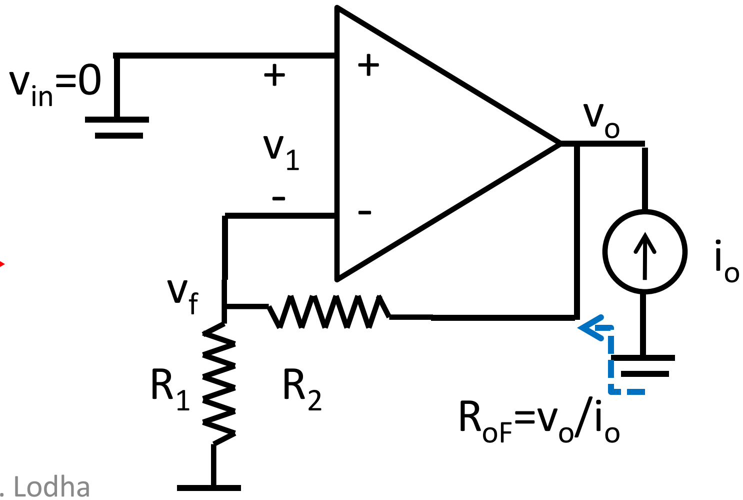
Negative feedback

$$v_1 = v_{in} - v_f$$

$$A_F = \frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1}$$

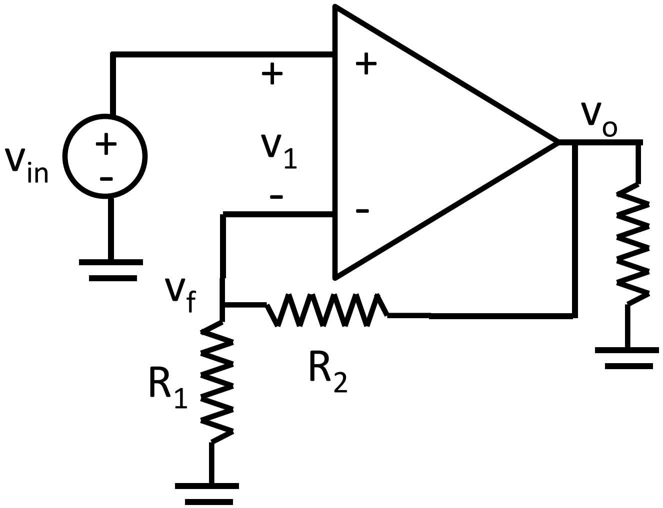
$$B = \frac{v_f}{v_o} = \frac{R_1}{R_1 + R_2} = \frac{1}{A_F}$$

For Ideal Op-Amp
-Infinite gain
-Infinite Input Resistance
-Zero Output Resistance



Feedback with non-ideal Op Amp

Assume A is finite, but $R_{in}=\infty$ and $R_o=0$



$$v_f = Bv_o$$

$$v_o = A(v_{in} - v_f) = Av_{in} - ABv_o$$

$$\boxed{A_F = \frac{v_o}{v_{in}} = \frac{A}{1+AB}}$$

$$A_F = \frac{A}{1+AB} \approx \frac{A}{AB} = \frac{1}{B} = 1 + \frac{R_2}{R_1}$$

For large A

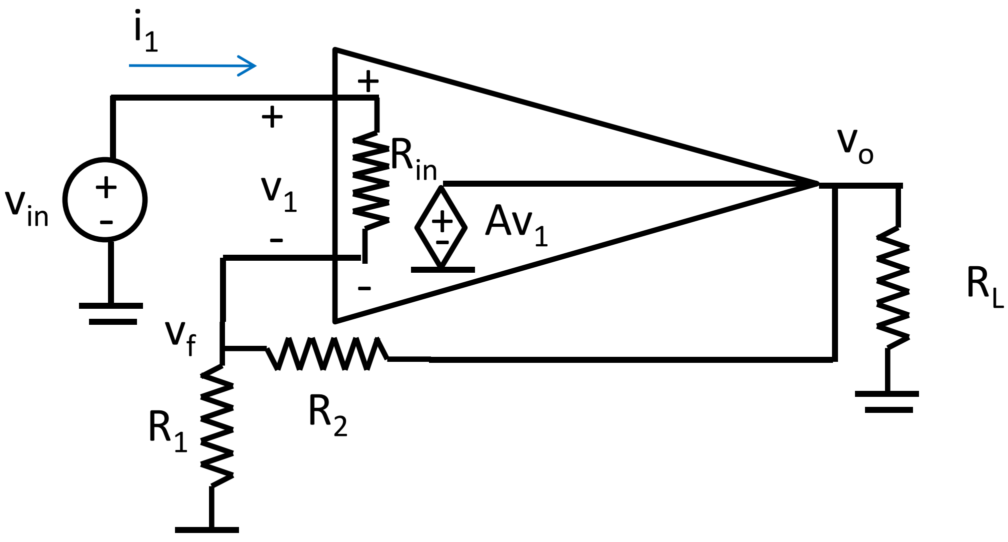
Trade-off gain with increase in stability!

A=200000, R_1=1 kΩ, R_2=100 kΩ → A_F=100.959

Suppose A changes by 10% to 220,000 → A_F=100.964 Only 0.005% change!

Feedback effect on input R

Assume A and R_{in} are both finite, but R_o=0



$$R_{iF} = \frac{v_{in}}{i_1} = \frac{v_{in}}{v_1} R_{in} = \frac{v_{in}}{v_o} AR_{in}$$

$$R_{in} >> R_1$$

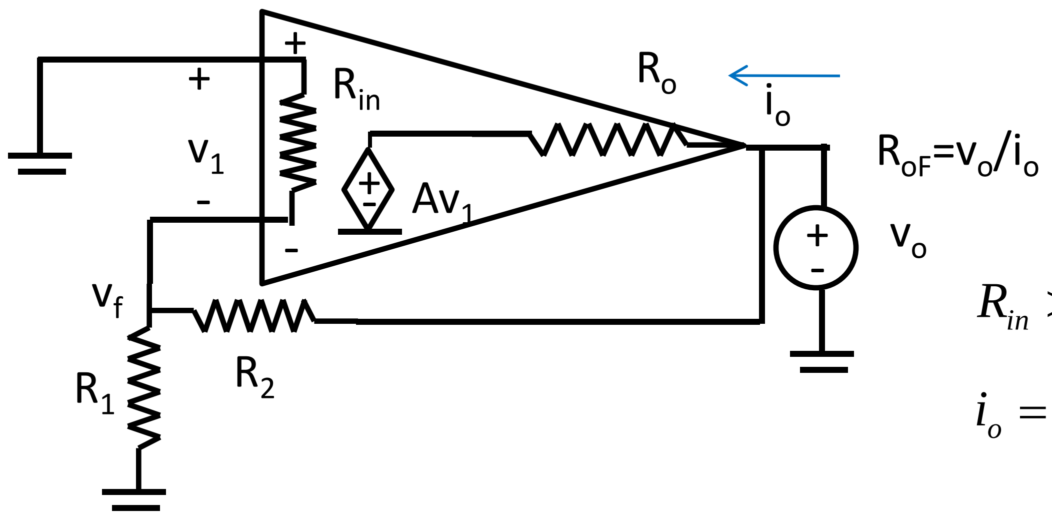
$$\frac{v_o}{v_{in}} = \frac{A}{1+AB}$$

$$\boxed{R_{iF} = (1+AB)R_{in}}$$

Negative feedback can be used to increase the input resistance of an amplifier.

Feedback effect on output R

Assume A, R_{in} and R_o are all finite



$$R_{in} >> R_1$$

$$i_o = \frac{1+AB}{R_o} v_o + \frac{1}{R_1+R_2} v_o$$

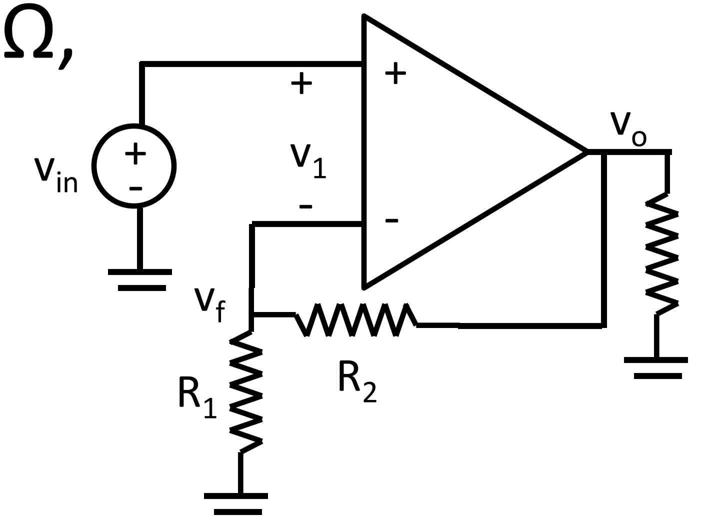
$$\Rightarrow \frac{1}{R_{oF}} = \frac{i_o}{v_o} = \frac{1+AB}{R_o} + \frac{1}{R_1+R_2}$$

$$\Rightarrow \boxed{R_{oF} = \frac{R_o}{1+AB} \parallel (R_1+R_2) \approx \frac{R_o}{1+AB}}$$

Negative feedback can be used to decrease the output resistance of an amplifier.

Example

- $A=200000$, $R_{in}=2\ M\Omega$, $R_o=75\ \Omega$, $R_1= 1\ k\Omega$, $R_2= 100\ k\Omega$
  - $B=R_1/R_1+R_2=0.0099$
  - $R_{iF}=(1+AB)R_{in}=3960\ M\Omega$
  - $R_{oF}=R_o/1+AB=0.038\ \Omega$



Frequency Response

$$\mathbf{A} = \frac{A}{1 + j\omega/\omega_H}$$

A is called the "open-loop" gain (frequency dependent), A is the dc gain

$$\mathbf{A_F} = \frac{\mathbf{A}}{1+\mathbf{A}B} = \frac{\left[\dfrac{A}{1+j\omega/\omega_H}\right]}{1+\left[\dfrac{A}{1+j\omega/\omega_H}\right]B}$$

For the example, A_F=A/1+AB

$$\mathbf{A_F} = \frac{A}{1+AB}\frac{1}{1+j\left[\dfrac{\omega}{(1+AB)\omega_H}\right]} = \frac{A_F}{1+j\omega/\omega_{HF}}$$

$$\omega_{HF} = (1+AB)\omega_H$$

Upper cut-off frequency

- Lower cut-off frequency is 0
- Internal transistor capacitances determine upper cut-off frequency
  - Acts like a low-pass filter
- With feedback, upper-cut off frequency multiplied by (1+AB)
  - Large increase in bandwidth

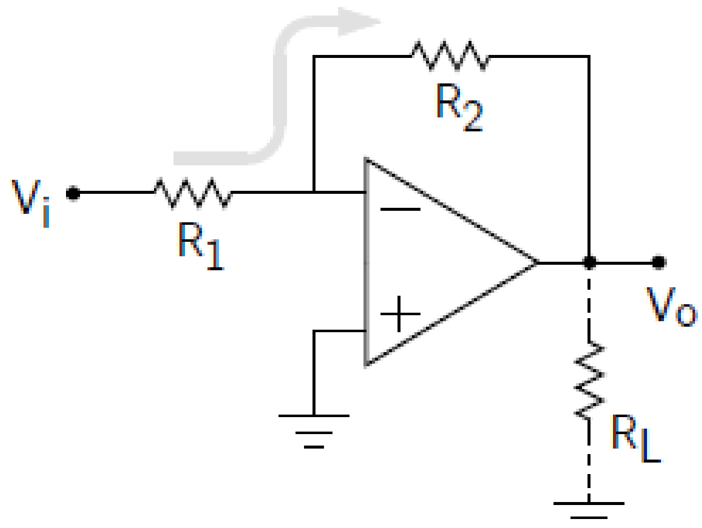
Gain-Bandwidth Product

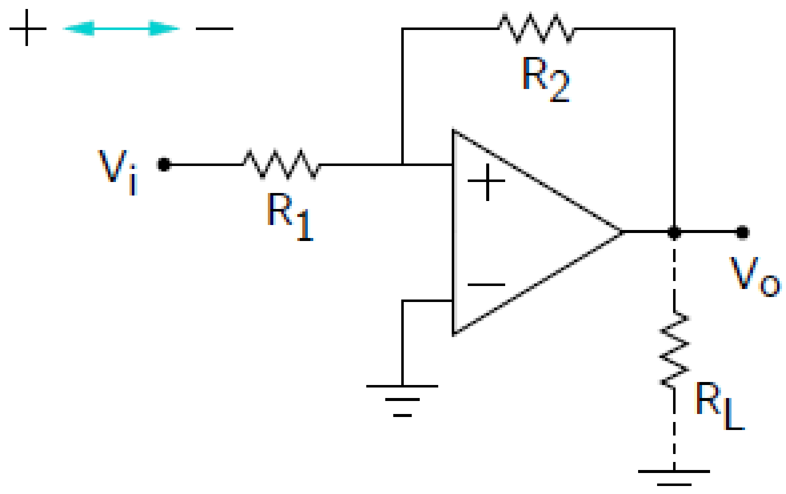
$$f_T = Af_H$$ Product of dc gain with upper cut-off frequency in Hz

$$f_{HF} = A_F f_{HF} = \frac{A}{1+AB}(1+AB) f_H = Af_H = f_T$$

- Addition of feedback does not change the gain-bandwidth product
  - Feedback amplifier and op-amp have the same product
  - What you lose in gain is made up in bandwidth

Feedback: Inverting Amplifier





$$V_o = A_V (V_+ - V_-) \qquad \text{Eq. 1}$$

$$V_- = V_i \frac{R_2}{R_1 + R_2} + V_o \frac{R_1}{R_1 + R_2} \qquad \text{Eq. 2}$$

$$V_i \uparrow \rightarrow \boxed{V_- \uparrow} \rightarrow V_o \downarrow \rightarrow \boxed{V_- \downarrow}$$

Eq. 2 Eq. 1 Eq. 2

Stable equilibrium is reached

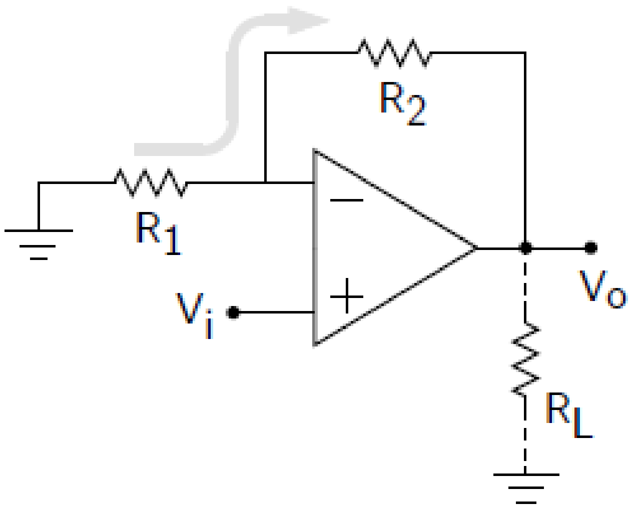
$$V_+ = V_i \frac{R_2}{R_1 + R_2} + V_o \frac{R_1}{R_1 + R_2} \qquad \text{Eq. 3}$$

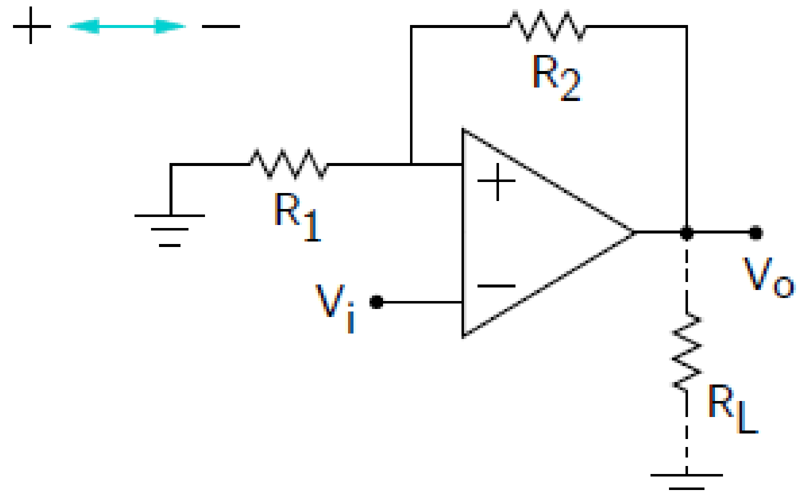
$$V_i \uparrow \rightarrow \boxed{V_+ \uparrow} \rightarrow V_o \uparrow \rightarrow \boxed{V_+ \uparrow}$$

Eq. 3 Eq. 1 Eq. 3

V_o rises (or falls) indefinitely till saturation→ positive feedback

Feedback: Non-Inverting Amplifier





$$V_o = A_V(V_+ - V_-) \qquad \text{Eq. 1}$$

$$V_- = V_o \frac{R_1}{R_1 + R_2} \qquad \text{Eq. 2}$$

$$V_i \uparrow \rightarrow \boxed{V_o \uparrow} \rightarrow V_- \uparrow \rightarrow \boxed{V_o \downarrow}$$

Eq. 1 Eq. 2 Eq. 1

Stable equilibrium is reached

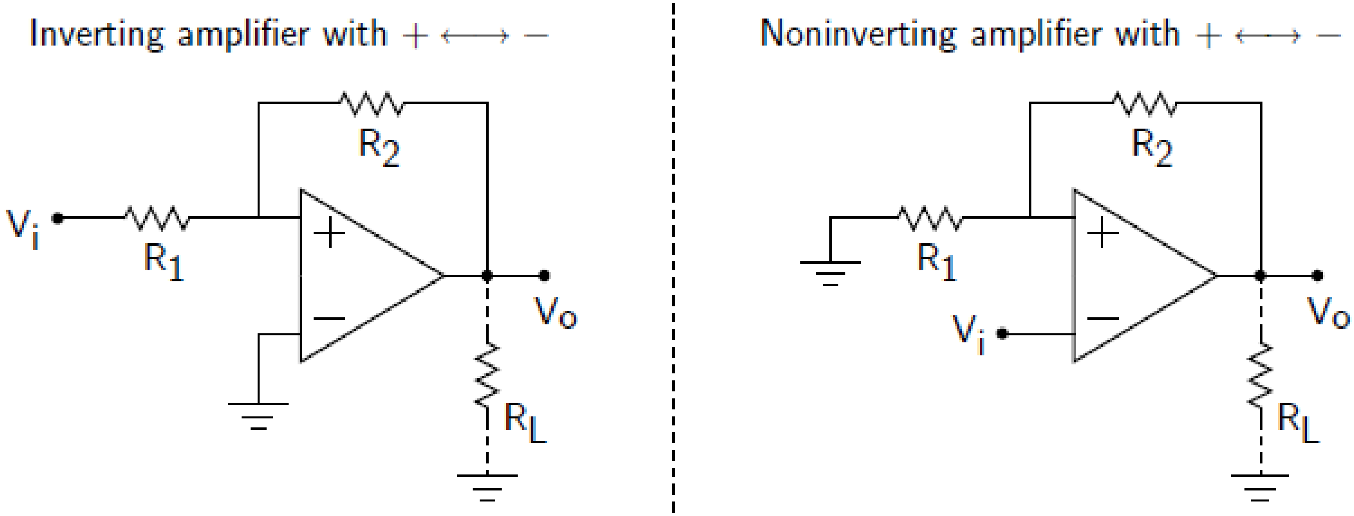
$$V_+ = V_o \frac{R_1}{R_1 + R_2} \qquad \text{Eq. 3}$$

$$V_i \uparrow \rightarrow \boxed{V_o \downarrow} \rightarrow V_+ \downarrow \rightarrow \boxed{V_o \downarrow}$$

Eq. 1 Eq. 3 Eq. 1

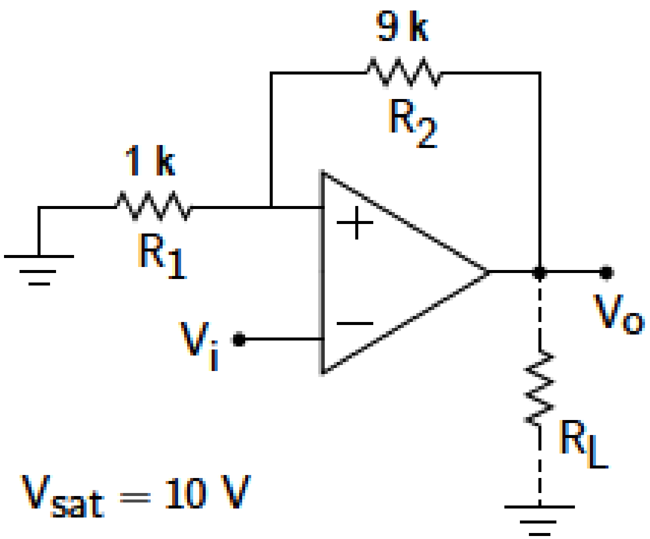
V_o rises (or falls) indefinitely till saturation→ positive feedback

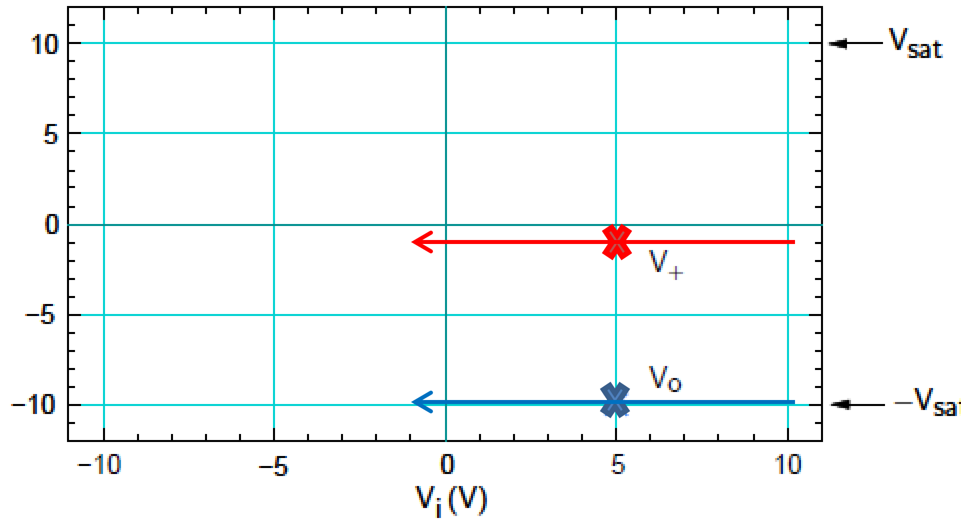
Feedback



- Both circuits exhibit positive feedback
- Output is limited by saturation, i.e. $V_o=\pm V_{sat}$

Inverting Schmitt Trigger





V_o is either +10V ($V_+>V_-$) or -10V ($V_+<V_-$) because of positive feedback

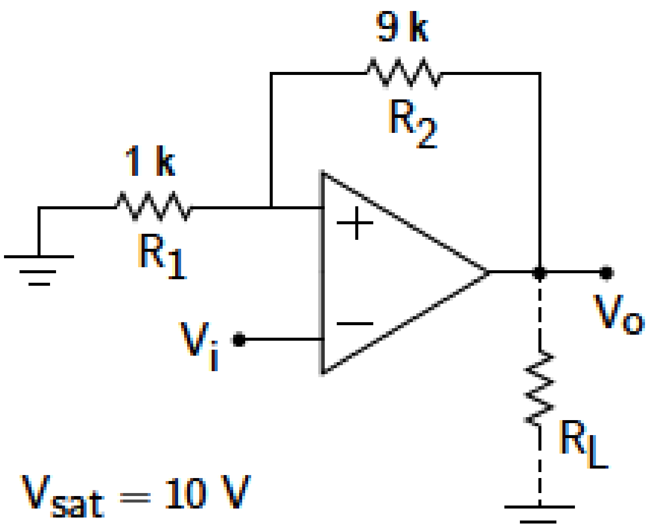
Case 1

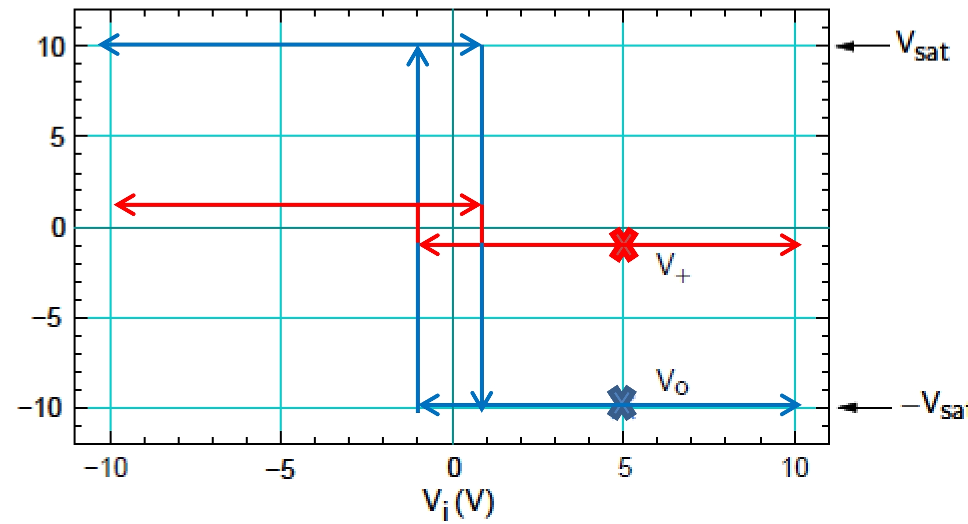
V_i=5V, Assume V_o=10V, V_+=1V → V_+-V_-=1-5=-4 V → V_o=-10V Inconsistent!

Case 2

V_i=5V, Assume V_o=-10V, V_+=-1V → V_+-V_-=-1-5=-6 V → V_o=-10V consistent!

Inverting Schmitt Trigger





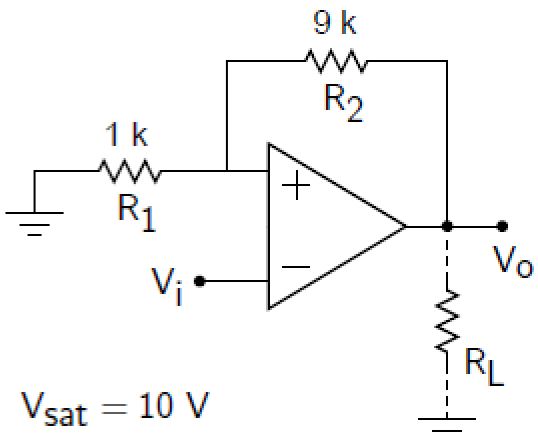
For decreasing values of V_i, V_o=-10V and V_+=-1V till V_i goes below -1V

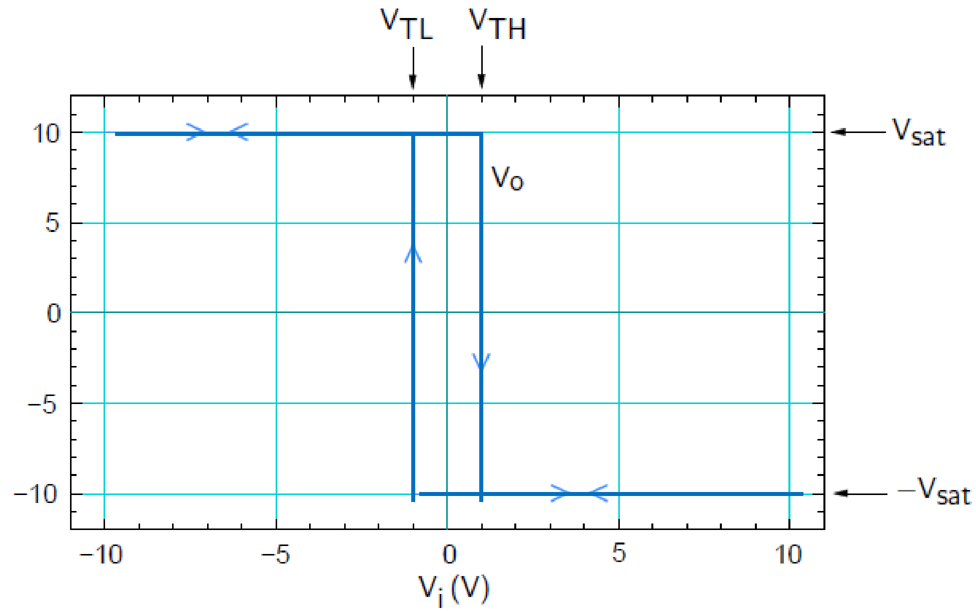
When V_i=V_-<V_+=-1V V_o=+V_{sat}=10V V_+ becomes +1V

Decreasing V_i further does not change V_o, since V_+-V_-=1-V_i>0

Coming back (increasing V_i) threshold voltage for flipping is +1V.

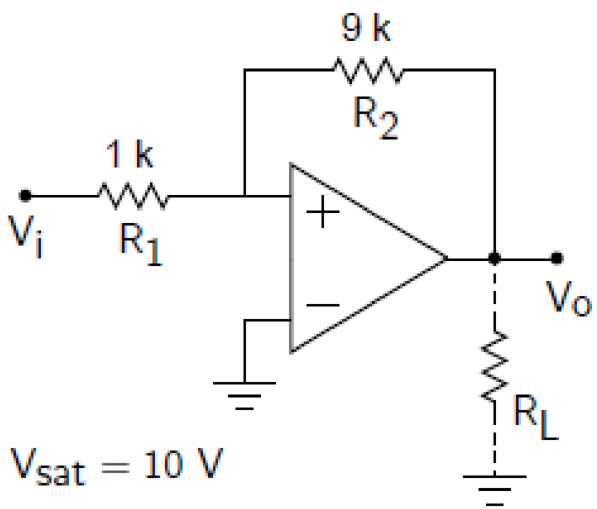
Inverting Schmitt trigger

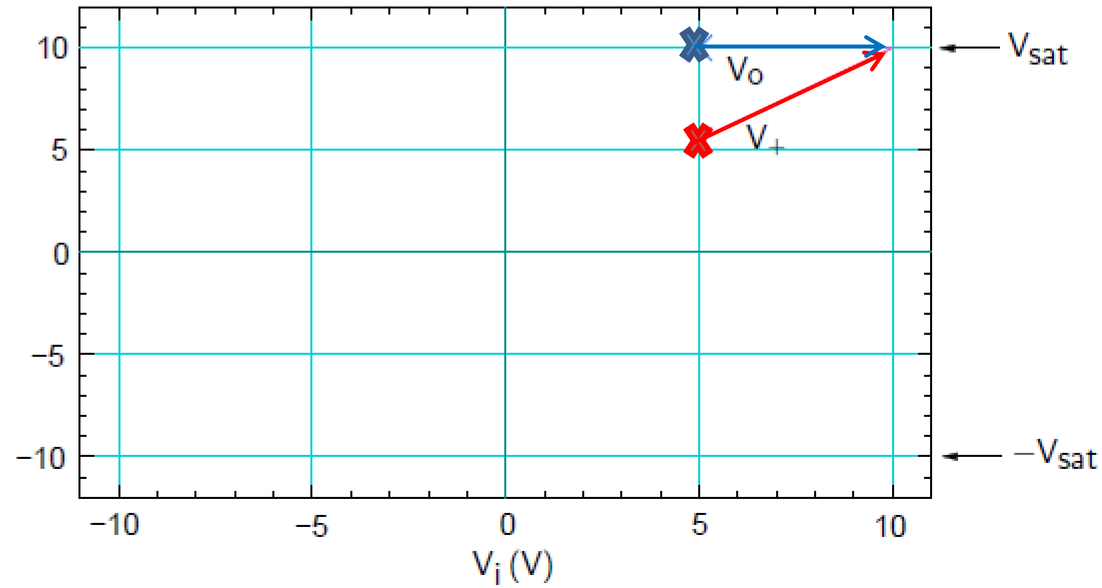




- The threshold (tripping) voltages V_{TL} and V_{TH} are $\pm\left(\frac{R_1}{R_1+R_2}\right)V_{sat}$
- Tripping point depends on position on V_o axis → MEMORY!
- $\Delta V_T = V_{TH} - V_{TL}$ is called hysteresis width

Non-inverting Schmitt Trigger





V_o is either +10V ($V_+ > V_-$) or -10V ($V_+ < V_-$) because of positive feedback

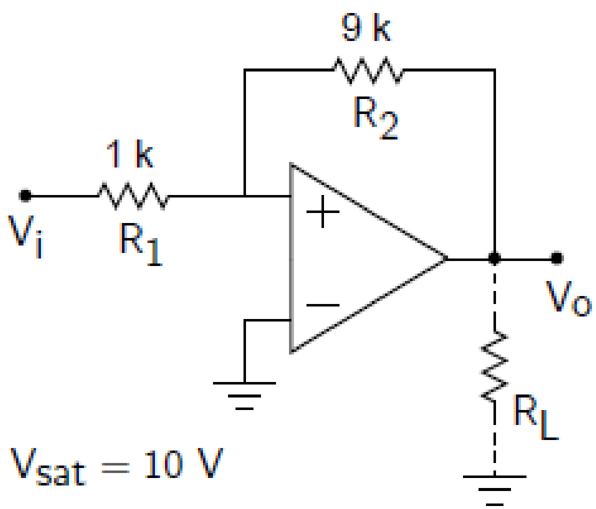
Case 1

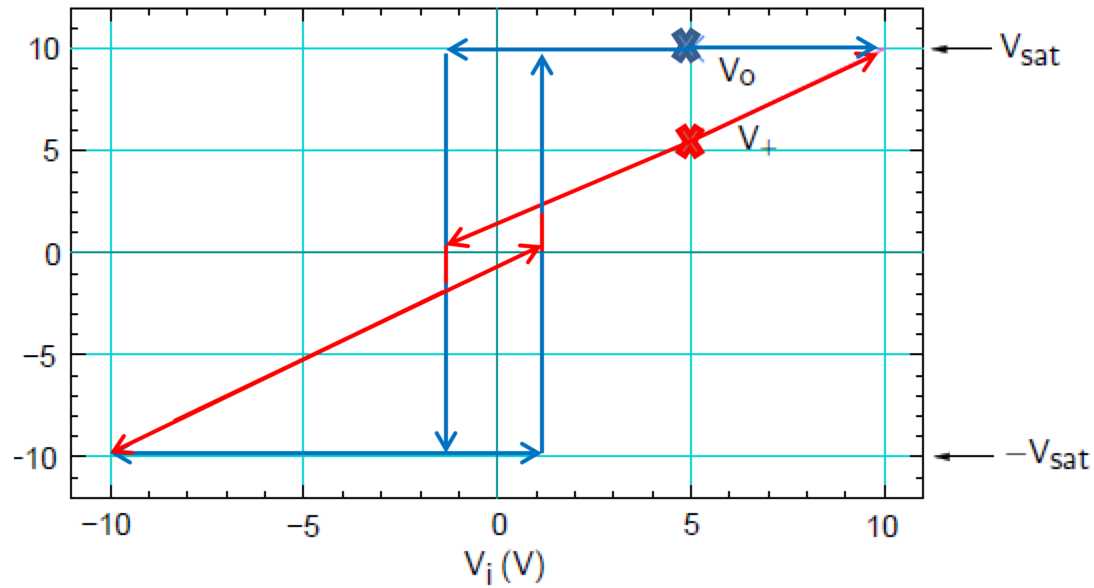
V_i=5V, Assume V_o=-10V, $V_+=(R_2/R_1+R_2)V_i+V_o(R_1/R_1+R_2)$=3.5V → V_+-V_-=3.5-0=3.5 V → V_o=-10V Inconsistent!

Case 2

V_i=5V, Assume V_o=+10V, V_+=5.5V → V_+-V_-=5.5-0=5.5 V → V_o=+10V consistent!

Non-inverting Schmitt Trigger





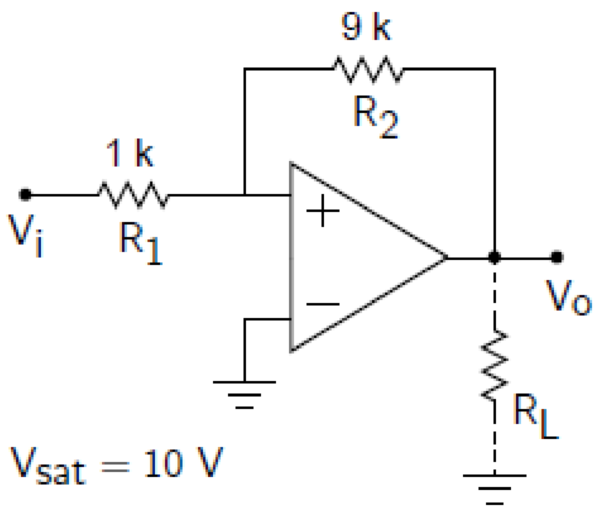
$$V_+ = \frac{R_2}{R_1+R_2}V_i + \frac{R_1}{R_1+R_2}V_o = \frac{9}{10}V_i + \frac{1}{10}V_{sat}$$

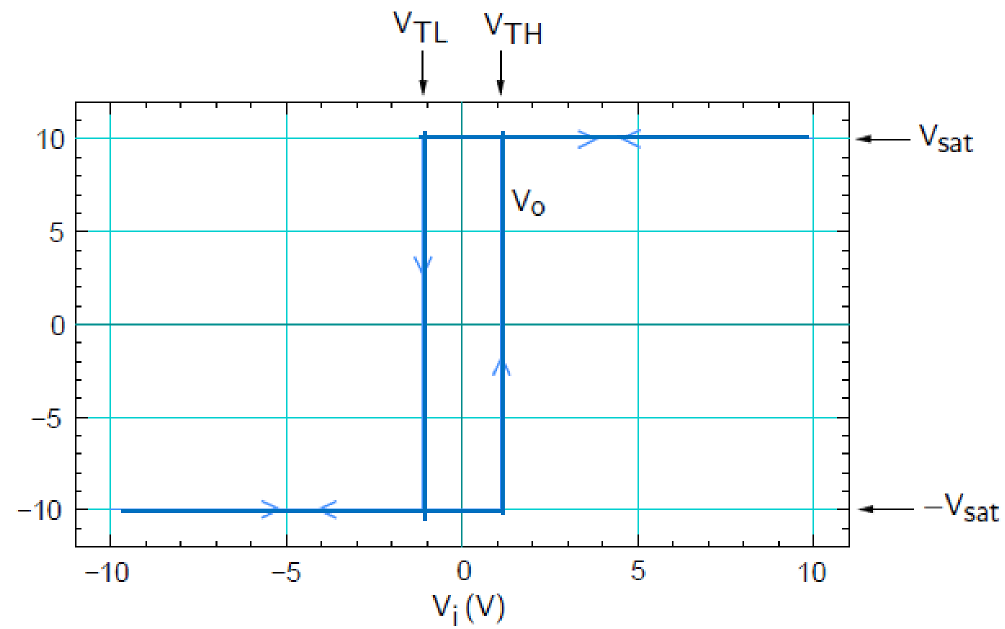
As V_i decreases and till $V_+>0$ $V_o=V_{sat}$
For $V_+=0V$, $V_i=-(R_1/R_2)V_{sat}=-1.11$ V, $V_o=-V_{sat}$

$$V_+ = \frac{R_2}{R_1+R_2}V_i + \frac{R_1}{R_1+R_2}V_o = \frac{9}{10}V_i - \frac{1}{10}V_{sat}$$

Further reduction of V_i does not change $V_o=-V_{sat}$, since $V_+<0$
V_o again flips to $+V_{sat}$ when $V_+=0V$, when $V_i=1.11V$

Non-inverting Schmitt Trigger





- The threshold (tripping) voltages V_{TL} and V_{TH} are $\pm\left(\frac{R_1}{R_2}\right)V_{sat}$
- Tripping point depends on position on V_o axis → MEMORY!
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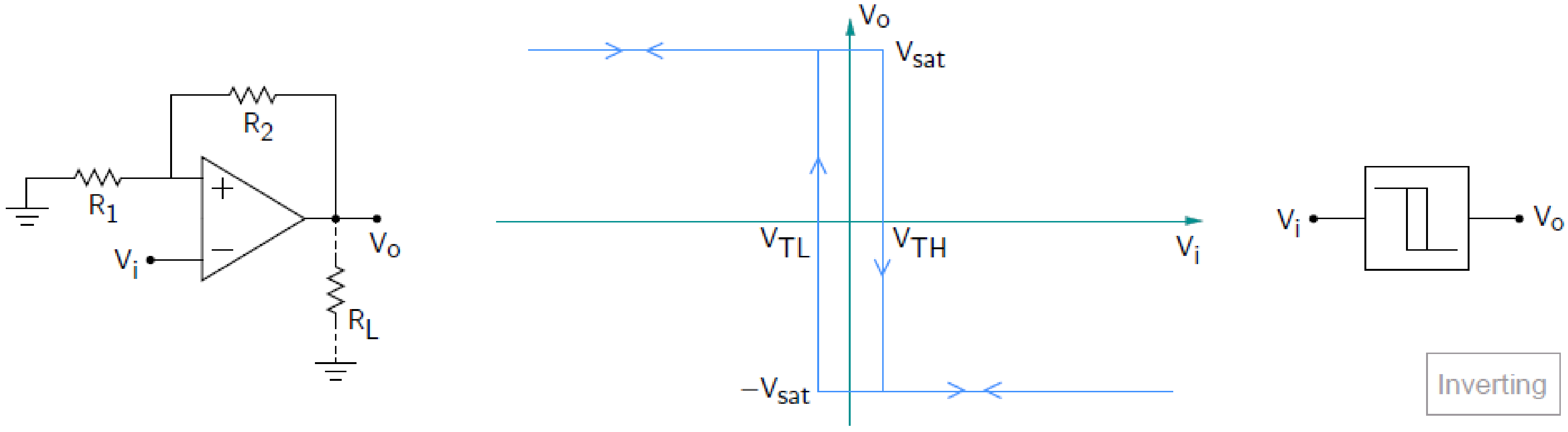
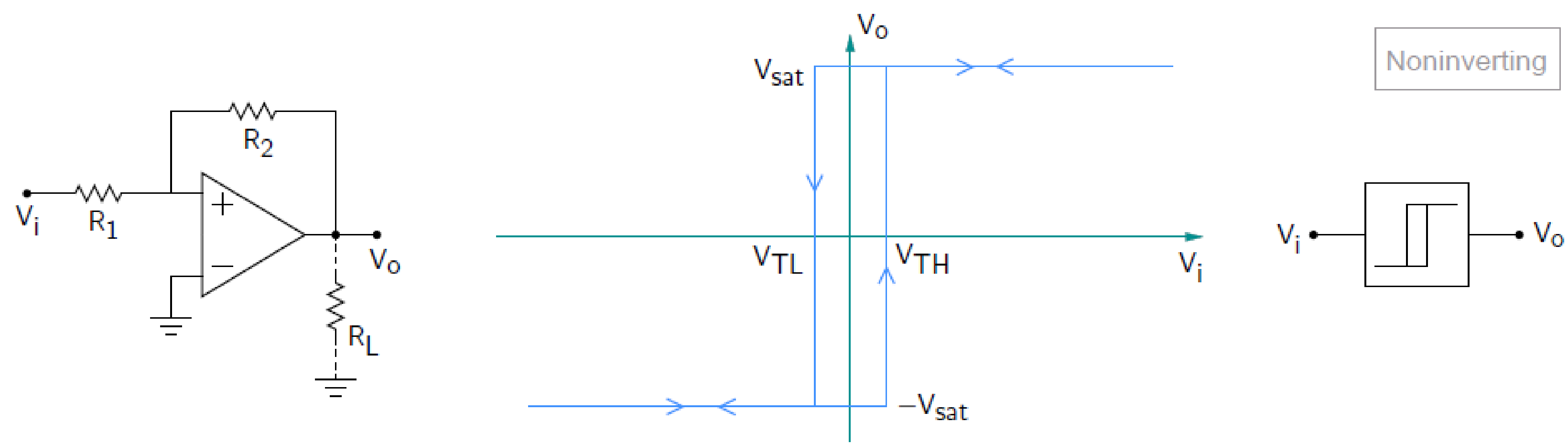
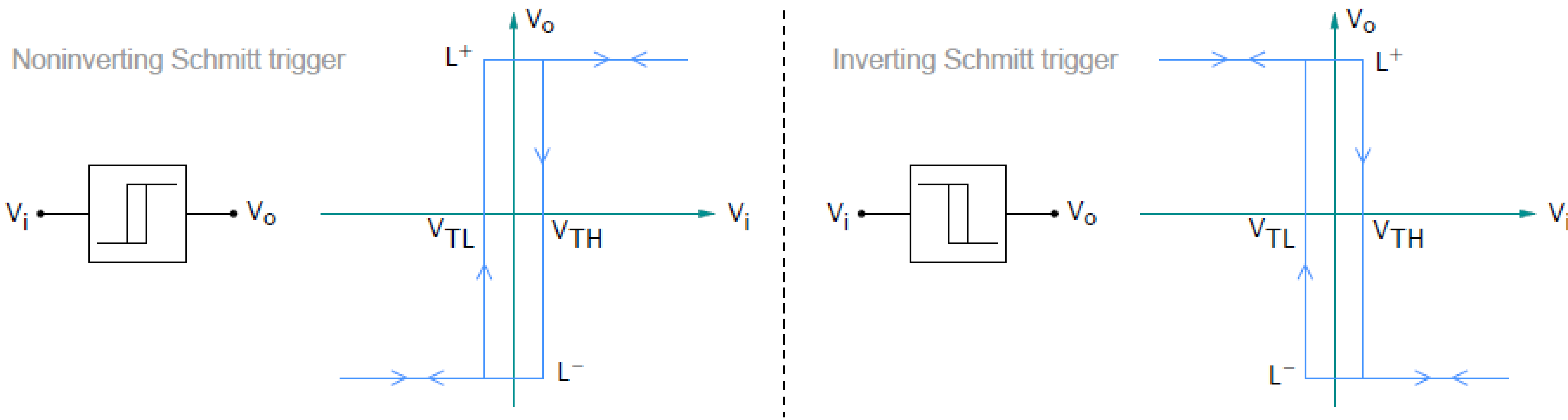
Schmitt Triggers



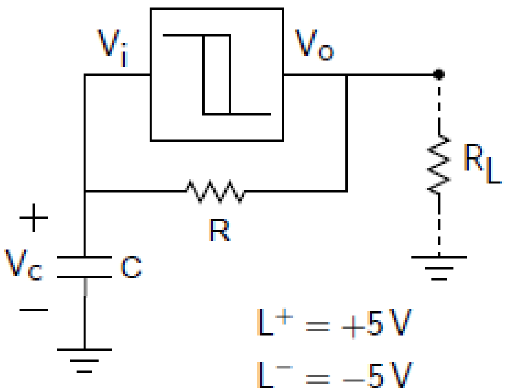


Schmitt Trigger: Application→ Astable multivibrator



- With a suitable RC circuit, Schmitt trigger can be made to freely oscillate between L^+ and L^-
  - Called an "astable multivibrator" (oscillator, wave-form generator)
- Produces oscillations where the frequency is controlled by component values (f_{max}~10 kHz)
- Other vibrator circuits
  - Monoshot (Timer)
  - Bistable (Flip-flop)

Astable MV

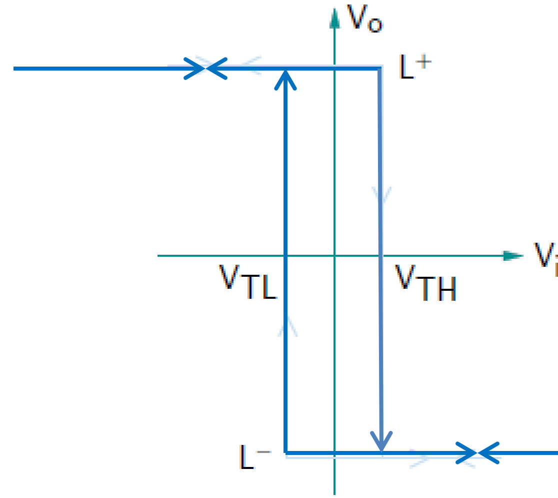


$L^+ = +5\,V$

$L^- = -5\,V$

$R = 2\,k$ $\quad V_{TH} = +1\,V$

$C = 1\,\mu F$ $\quad V_{TL} = -1\,V$



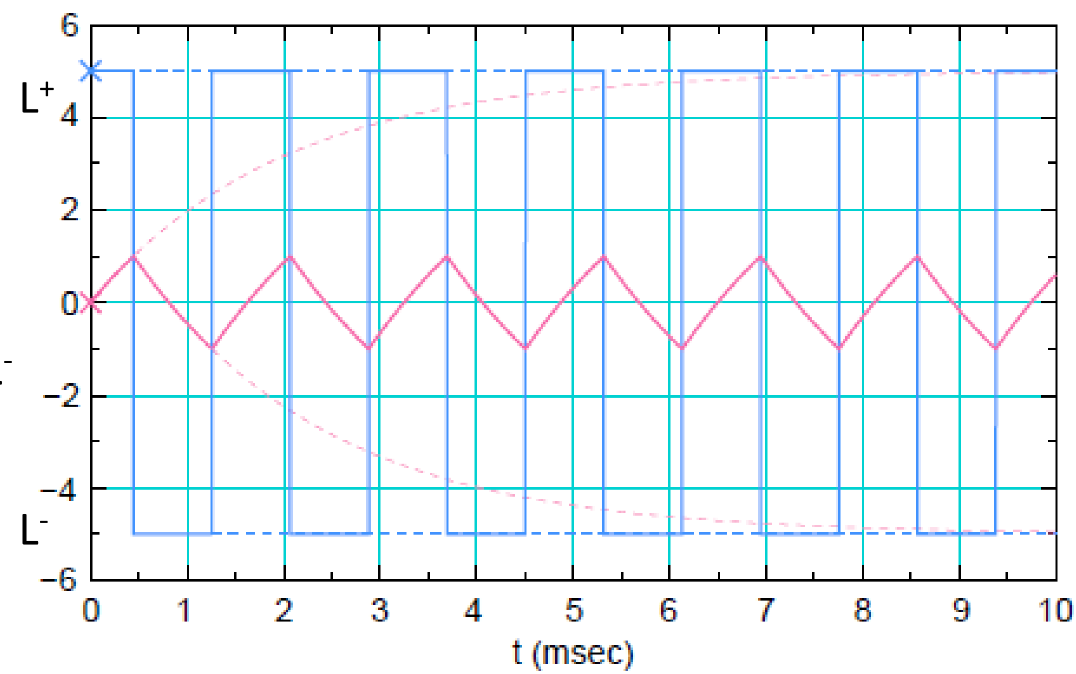
At t=0, V_o=L^+ and V_c=0

Capacitor starts charging towards L^+
As V_c (=V_i) crosses V_{TH}, V_o flips to L^-

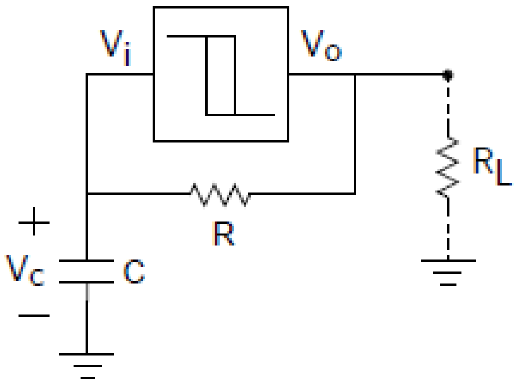
Capacitor starts discharging towards L^-
As V_c(=V_i) crosses V_{TL}, V_o flips to L^+

Circuit oscillates on its own.

Also called a "relaxation oscillator".



What is T (=1/f)?



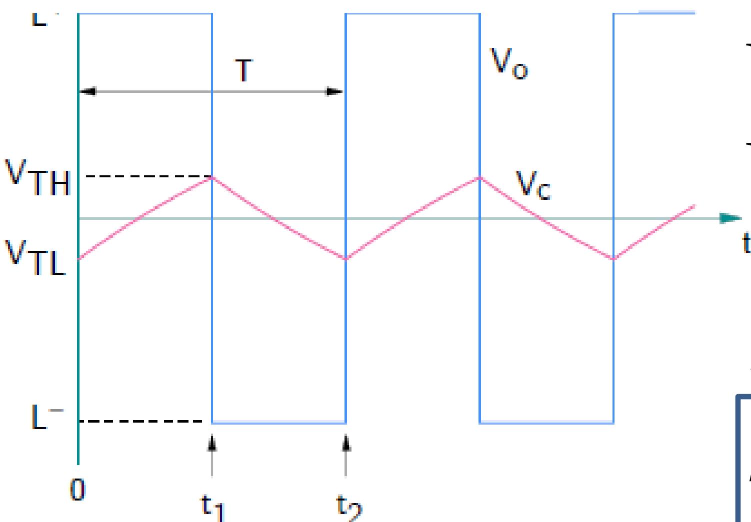
$$V_c(t) = A_1 e^{-t/\tau} + B_1$$

$$V_c(0) = V_{TL}, V_c(\infty) = L^+$$

$$V_{TH} = A_1 e^{-t_1/\tau} + B_1$$

Find A_1 and B_1

Find t_1



$$V_c(t) = A_2 e^{-(t-t_1)/\tau} + B_2$$

$$V_c(t_1) = V_{TH}, V_c(\infty) = L^-$$

$$V_{TL} = A_2 e^{-(t_2-t_1)/\tau} + B_2$$

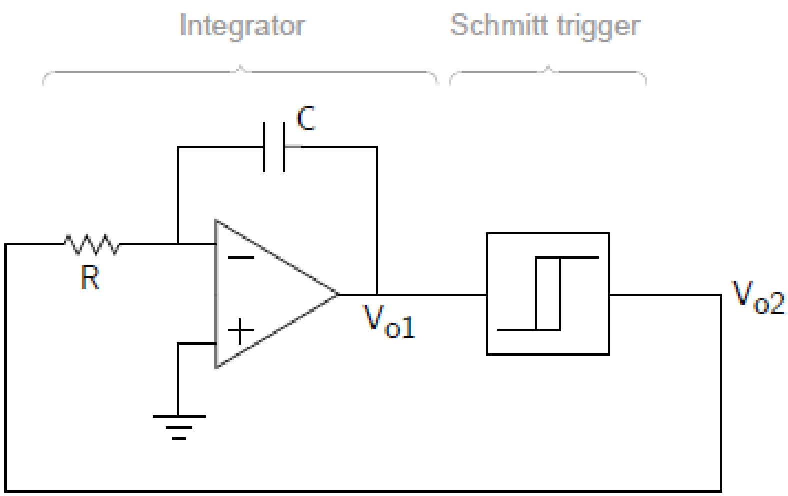
Find A_2 and B_2

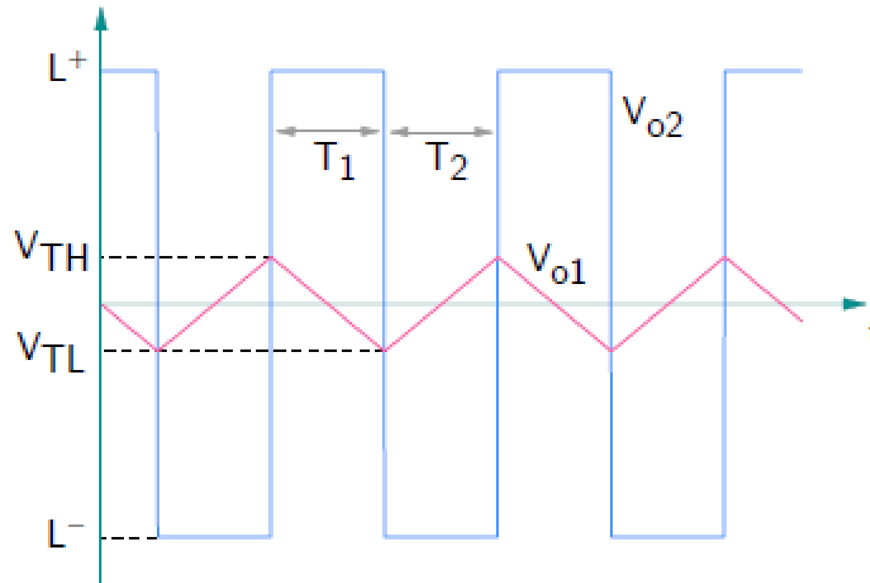
Find t_2

$$L^+ = L, L^- = -L, V_{TH} = V_T, V_{TL} = -V_T$$

$$\boxed{T = 2RC \ln\left(\frac{L+V_T}{L-V_T}\right)}$$

Astable MV





$$V_{o1} = -\frac{1}{RC}\int V_{o2}dt$$

$V_{o2} = L^+ \rightarrow V_{o1}$ Decreases linearly

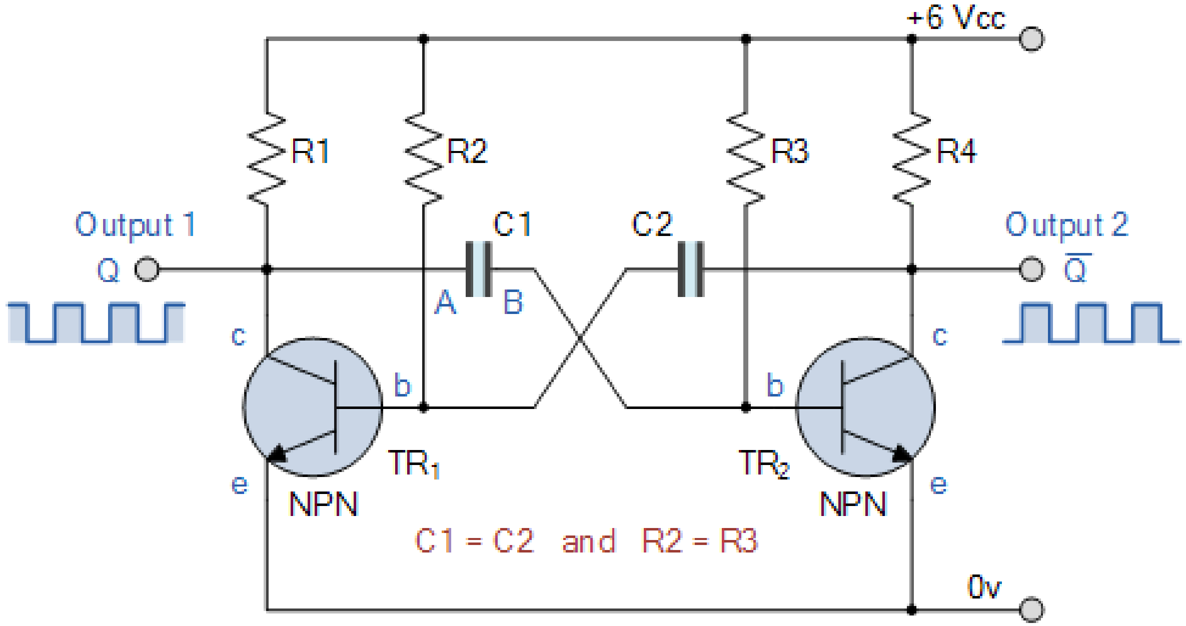
$V_{o2} = L^- \rightarrow V_{o1}$ Increases linearly

- Plot V_{o2} and V_{o1} vs t
- Find the time period

$$T_1 = \frac{V_{TH} - V_{TL}}{L^+ / RC} = RC\frac{V_{TH} - V_{TL}}{L^+}$$

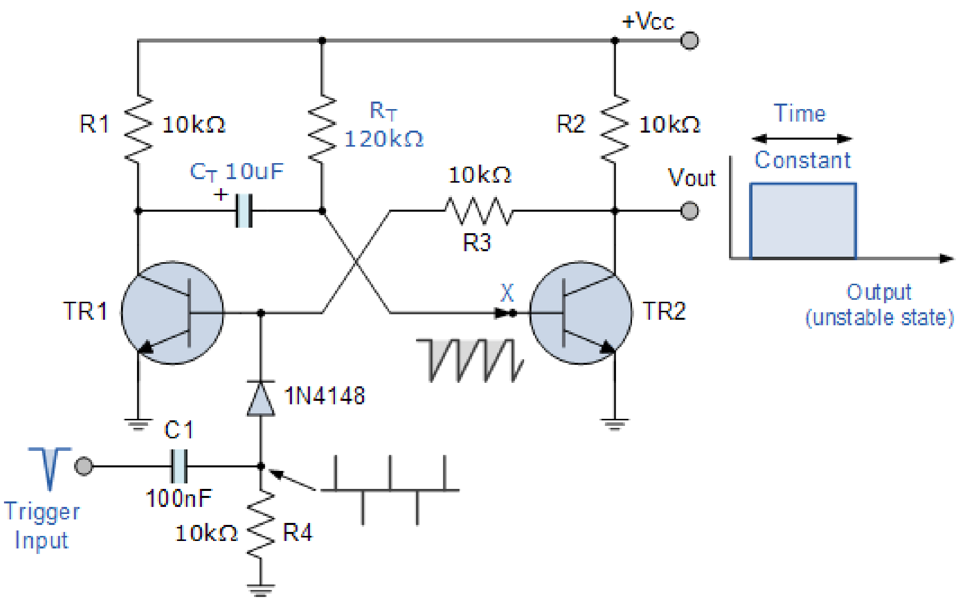
$$T_2 = \frac{V_{TH} - V_{TL}}{-L^- / RC} = RC\frac{V_{TH} - V_{TL}}{-L^-}$$

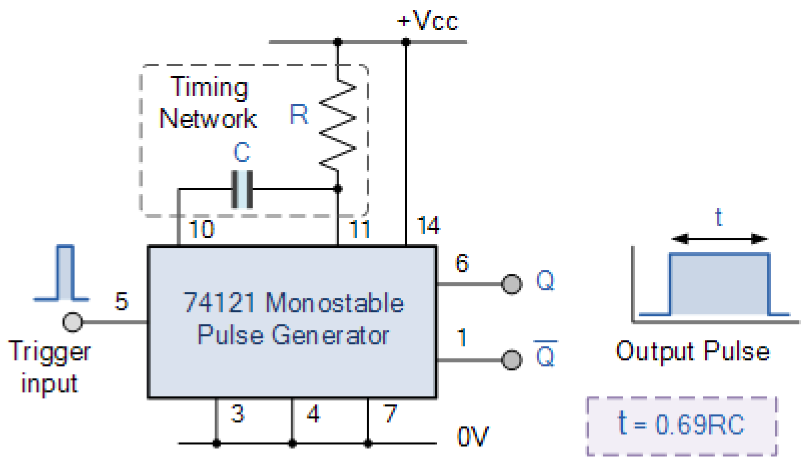
Vibrator Circuit: Examples



- Astable multivibrator
  - No stable state
  - Keeps oscillating

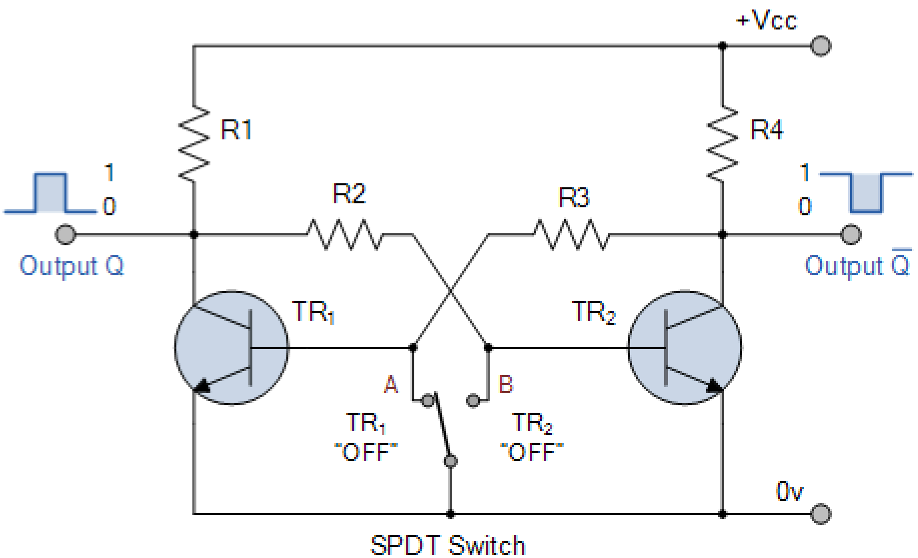
Vibrator Circuit: Examples

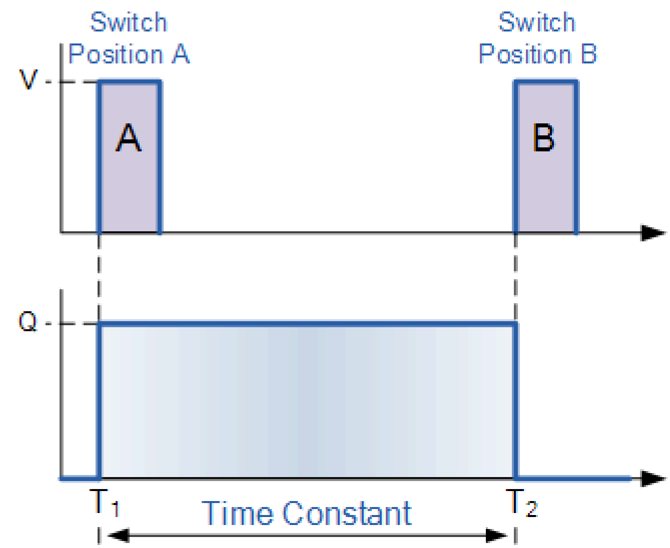




- Monostable multivibrator
  - No stable state
  - Produce a pulse with a trigger input

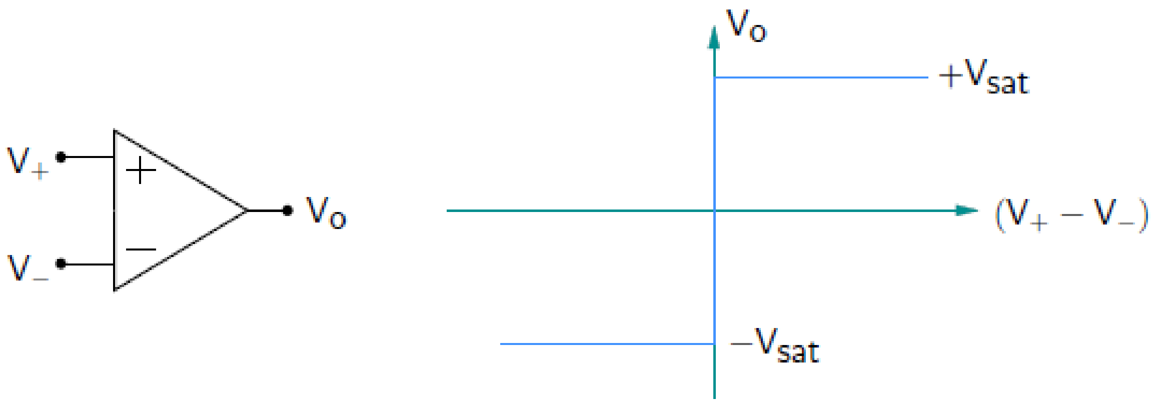
Vibrator Circuit: Examples





- Bistable multivibrator
  - Two stable states
  - Trigger/switch needed to move from one to another state

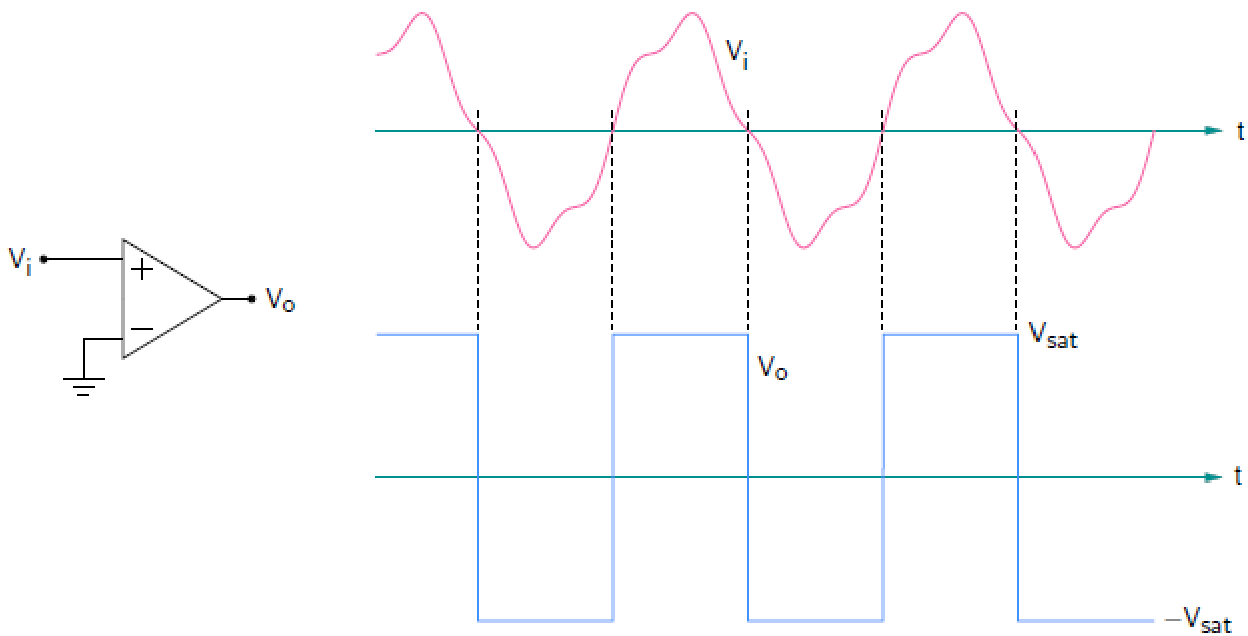
Comparators



$$\boxed{\begin{aligned} V_+ > V_- &\Rightarrow V_o = V_{sat} \\ V_+ < V_- &\Rightarrow V_o = -V_{sat} \end{aligned}}$$

- Width of the linear region ~0.1 mV can be neglected
  - High gain in linear region
- "Compare" V_+ with V_-

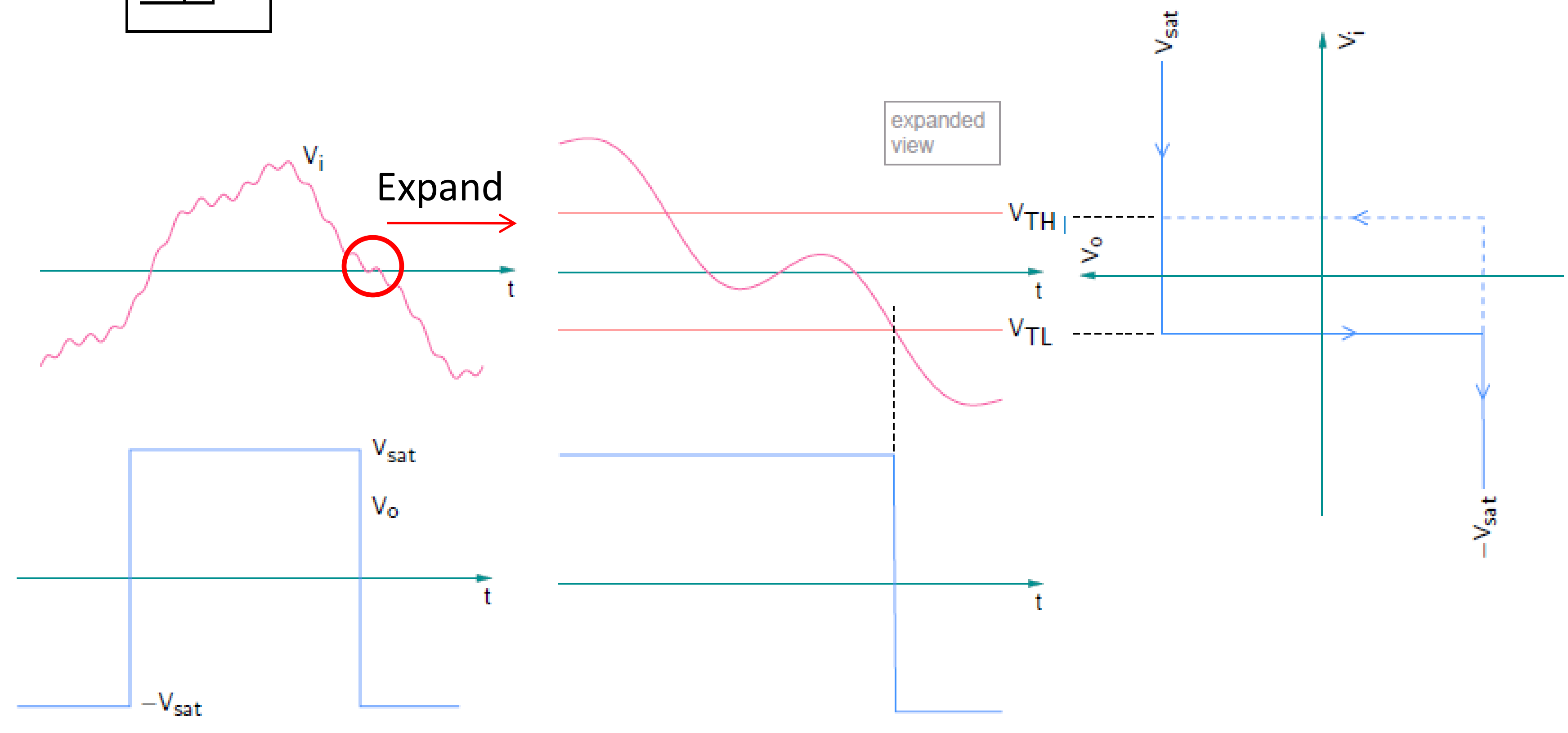
Comparator



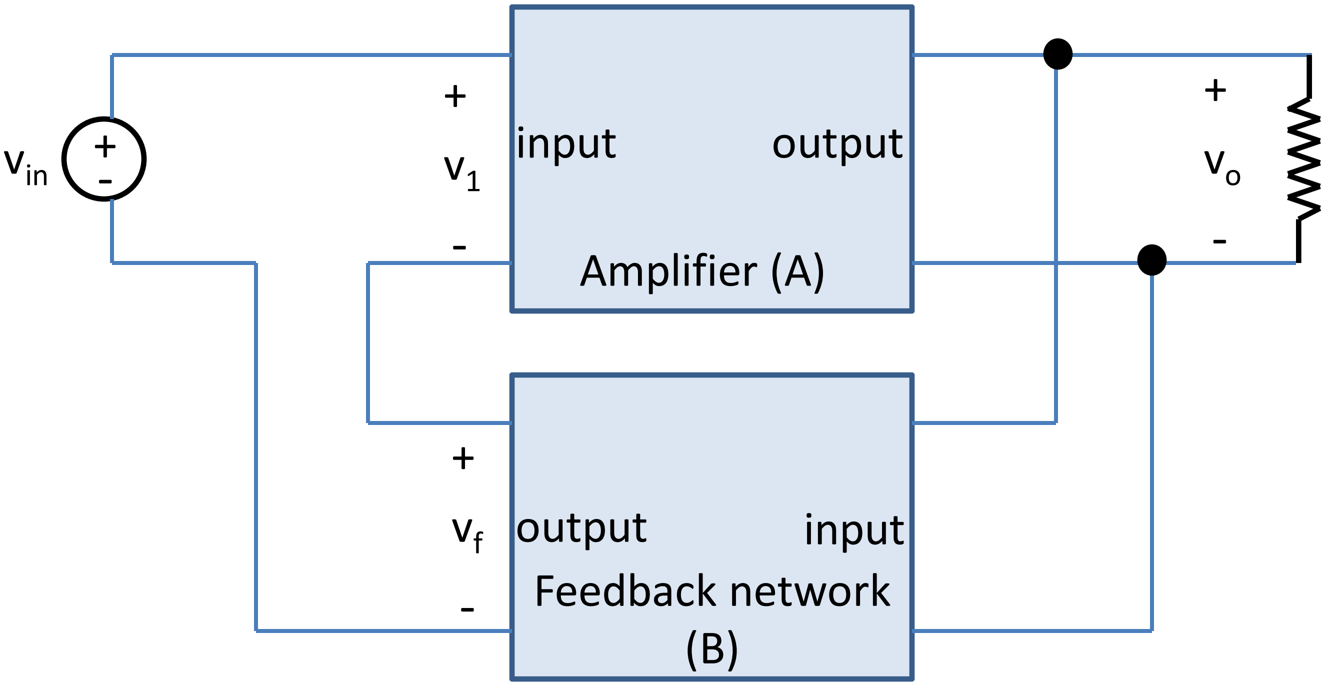
- An analog signal can be converted into digital signal (high/low) for further processing by digital circuits
- Also called a level detector if the reference voltage is not zero
- Zero-crossing detector if reference voltage is zero (above)

Schmitt Trigger based Comparator

- Output flips at V_{TH} and V_{TL} crossing, not zero
- Input signal noise will not affect output



Sinusoidal (Linear) Oscillators



$$v_{in} = v_1 + v_f$$

$$v_o = Av_1$$

$$v_f = Bv_o$$

$$\Rightarrow v_{in} = \frac{v_o}{A} + Bv_o$$

$$\Rightarrow \boxed{A_F = \frac{v_o}{v_{in}} = \frac{A}{1+AB}}$$

If AB>0, A_F<A→ negative (degenerative) feedback

If A_F>A → positive (regenerative) feedback

Barkhausen Condition (Oscillation)

$v_{in} = v_o = v_f = 0$ Initial Condition

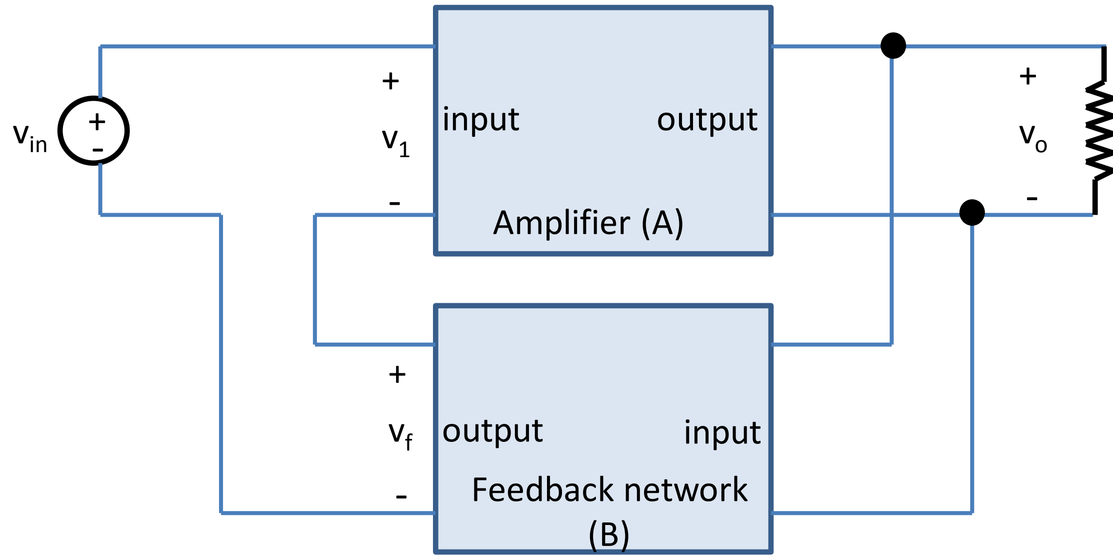
$v_o = A(v_{in} - v_f) = Av_{in}$ Apply v_{in}

$v_f = Bv_o = BAv_{in}$ Apply v_{in}

$v_{in} = 0$ Make input zero

$v_1 = -v_f = -BAv_{in}$

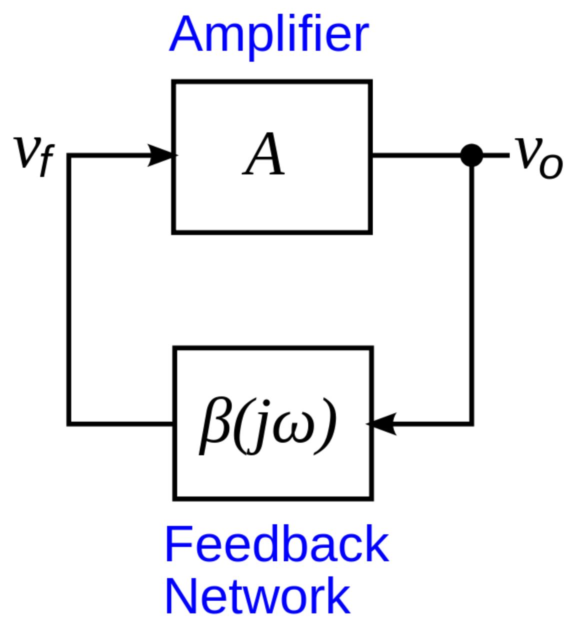
$BA = -1 \Rightarrow v_1 = v_{in}$ Original input



- Feedback network allows maintaining original input voltage and same output voltage
- Non-zero output without external input → oscillator
- Loop gain → product of individual gains around loop without external input
- Condition for oscillation → $\boxed{\text{-AB=1 (Barkhausen condition)}}$
  - Unity loop gain
  - A_F → infinite (non-zero output with no input)

$$\frac{v_o}{v_1} \times \frac{v_f}{v_o} \times \frac{v_1}{v_f} = A \times B \times -1 = -AB$$

Sinusoidal (Linear) Oscillators

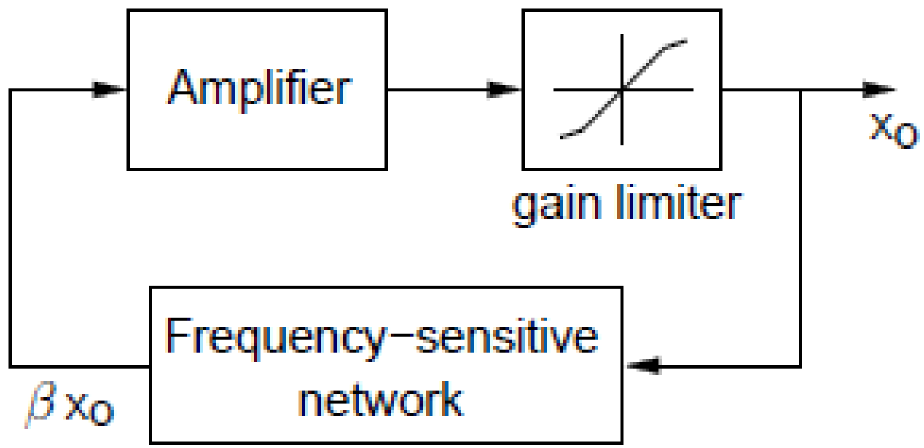


- In general, Barkhausen condition for loop gain βA states that the oscillator will sustain steady-state oscillations only for those frequencies for which:

$$|\beta A| = 1$$

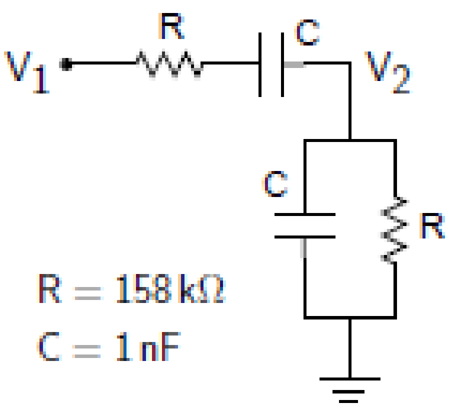
$$\angle \beta A = 2\pi n, n = 0,1,2,\ldots$$

Sinusoidal Oscillators

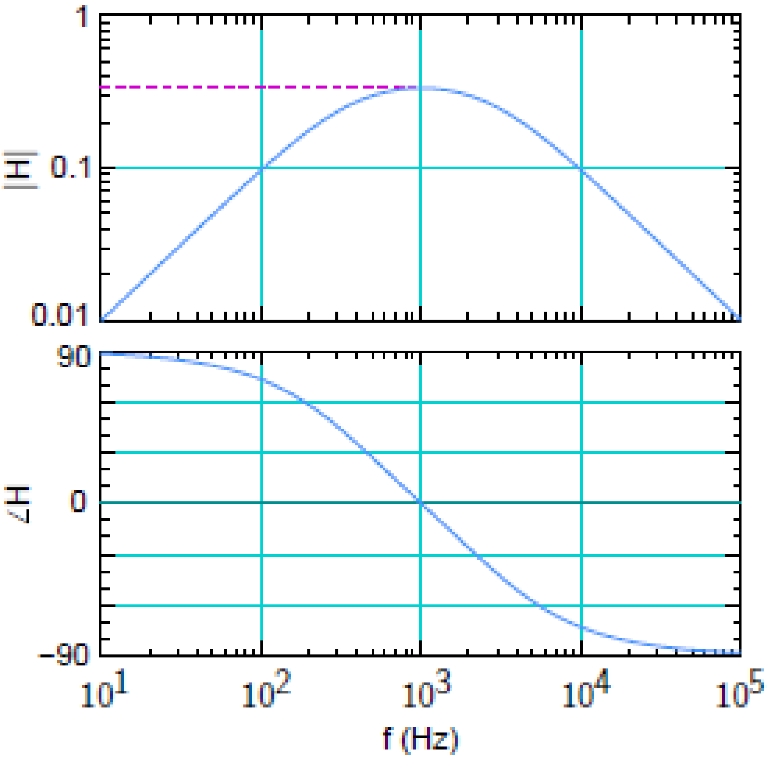


- Gain limiting circuit needed to limit amplitude of oscillations
  - Op-Amps (+/- V_{sat})
  - Diode resistor networks
- Upto ~100 kHz, op-amps and RC feedback networks are ok
- For high frequencies, gain and slew rate limitations
  - Transistor amplifiers, LC feedback

Wein-bridge Oscillator

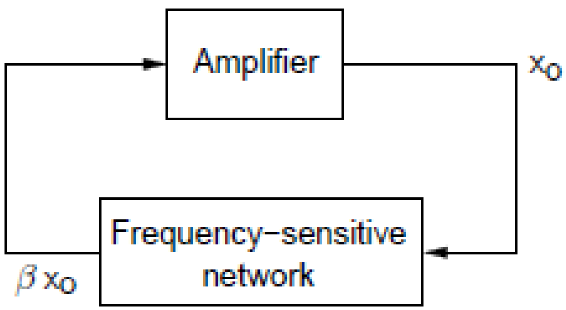


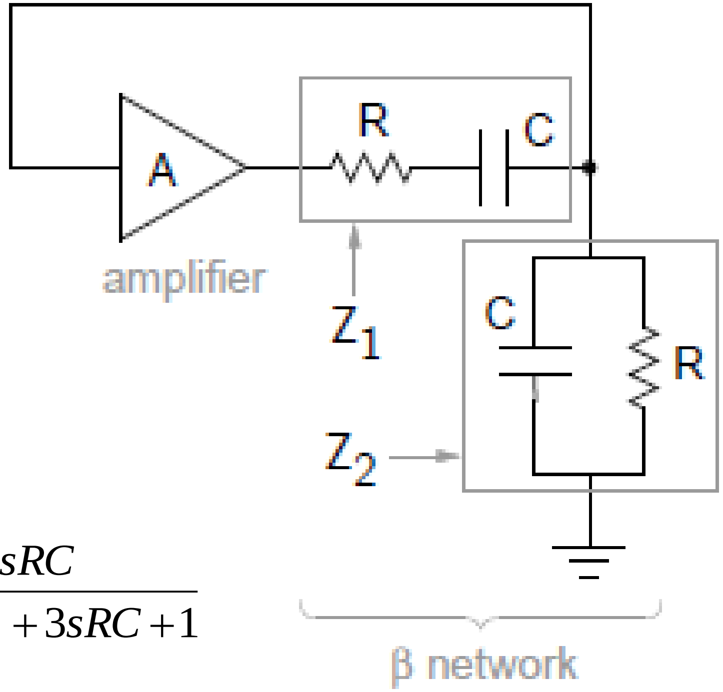
$$H(j\omega)=\frac{V_2(j\omega)}{V_1(j\omega)}=\frac{j\omega RC}{-(\omega RC)^2+3j\omega RC+1}$$



- The condition that $\angle\beta A=2\pi n, n=0,1,2,\ldots$ is satisfied at only one frequency ω_o=1/RC, i.e. f_o=1kHz
- For this frequency, $\beta(j\omega_o)=H(j\omega_o)=1/3$

Wein-bridge Oscillator





Assuming $R_{in}=\infty$ for the amplifier (β-network not loaded by amplifier)

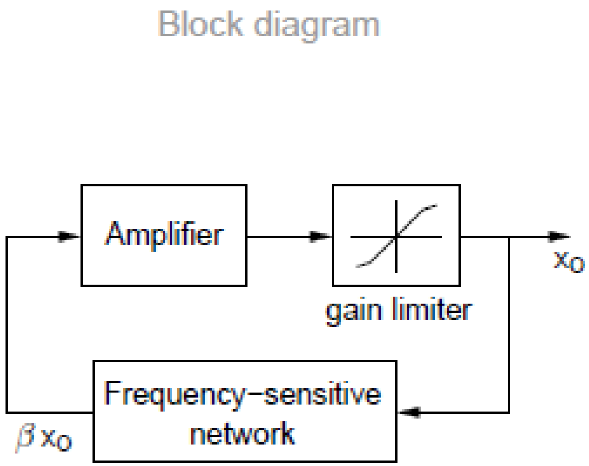
$$A(s)\beta(s) = A\frac{Z_2}{Z_1+Z_2} = A\frac{R\,\|\,(1/sC)}{R+(1/sC)+R\,\|\,(1/sC)} = A\frac{sRC}{(sRC)^2+3sRC+1}$$

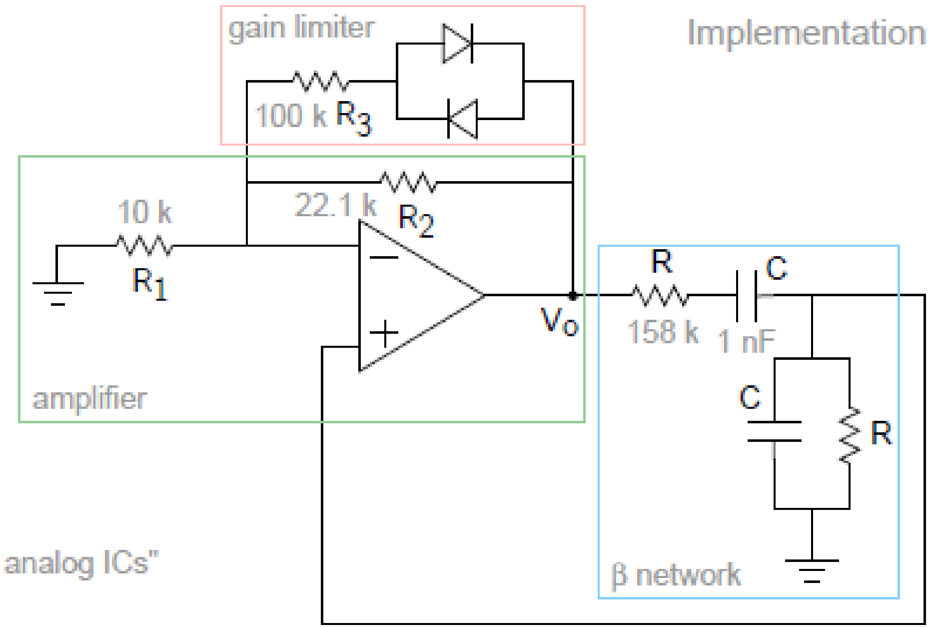
For $|A\beta| = 1$ (and with A equal to a real positive number)

$$\frac{j\omega RC}{-(\omega RC)^2+3j\omega RC+1}$$ Must be real and and equal to 1/A

$$\boxed{\Rightarrow \omega_o = 1/RC, |\beta(j\omega_o)| = \frac{1}{3} \Rightarrow A = 3}$$

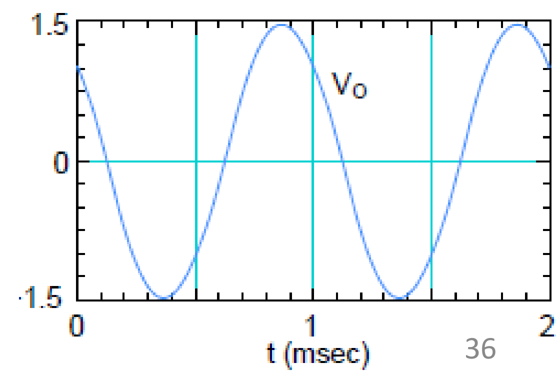
Wein-bridge Oscillator



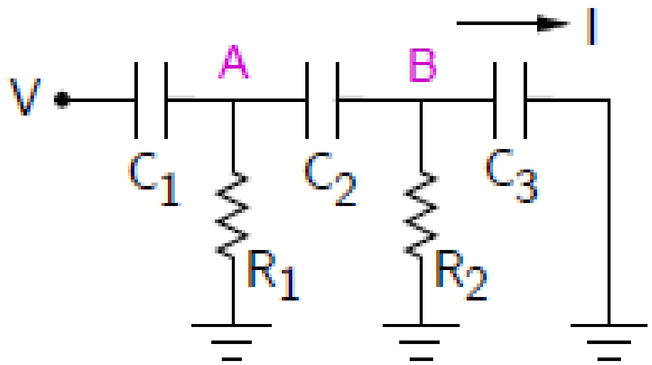


Ref.: S. Franco, "Design with Op Amps and analog ICs"
SEQUEL file: wien_osc_1.sqproj

- Frequency ω_o=1/RC, i.e. f_o=1kHz
- For amplifier gain A=3, 1+R_2/R_1=3 → R_2=2R_1
- For limiting gain, diodes are used. When one of the two conducts, R_2 → $R_2||R_3$ and gain reduces



Phase-shift oscillator



$R_1 = R_2 = R$, $C_1 = C_2 = C_3 = C$

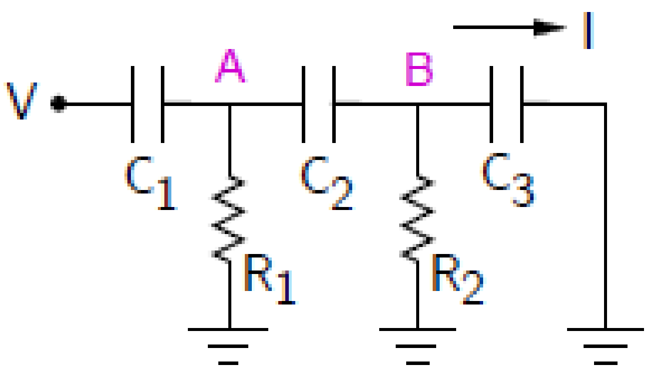
$$sC(V_A - V) + GV_A + sC(V_A - V_B) = 0 \qquad \textbf{(1)}$$

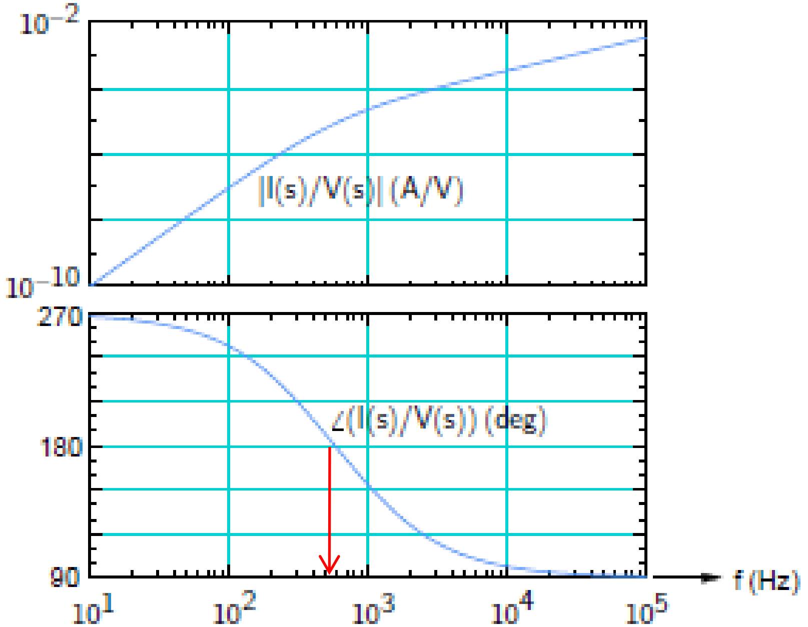
$$sC(V_B - V_A) + GV_B + sCV_B = 0 \qquad \textbf{(2)}$$

Solving (1) and (2)

$$I = \frac{1}{R}\frac{(sRC)^3}{3(sRC)^2 + 4sRC + 1}V$$

Phase-shift oscillator





$$\beta(j\omega)=\frac{I(j\omega)}{V(j\omega)}=\frac{1}{R}\frac{(j\omega RC)^3}{3(j\omega RC)^2+4j\omega RC+1}$$

$R_1=R_2=R=10k$, $C_1=C_2=C_3=C=16nF$

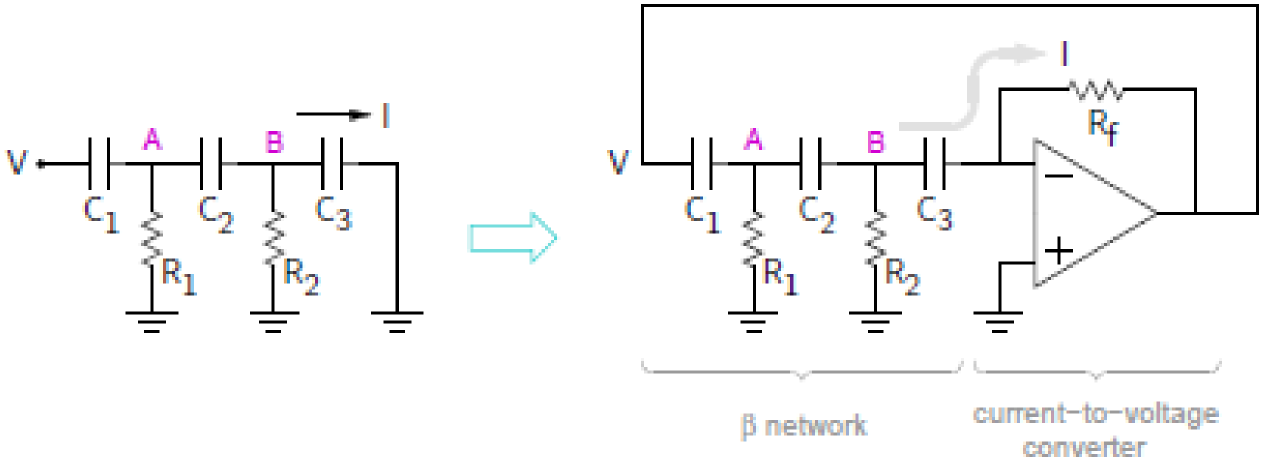
For β(jω) to be a real number, denominator should be purely imaginary

Phase angle=180 deg=π at ω_o

$$-3(\omega RC)^2+1=0\Rightarrow\omega\equiv\omega_o=\boxed{\frac{1}{\sqrt{3}}\frac{1}{RC}}$$

$$\boxed{\beta(j\omega_o)=\frac{1}{R}\frac{(j/\sqrt{3})^3}{4j/\sqrt{3}}=-\frac{1}{12R}}$$

Phase-shift oscillator



Note that virtual ground of the op-amp provides the ground of the β-network on the left

$$A\beta(j\omega) = -R_f \frac{I(j\omega)}{V(j\omega)} = -\frac{R_f}{R} \frac{(j\omega RC)^3}{3(j\omega RC)^2 + 4j\omega RC + 1}$$

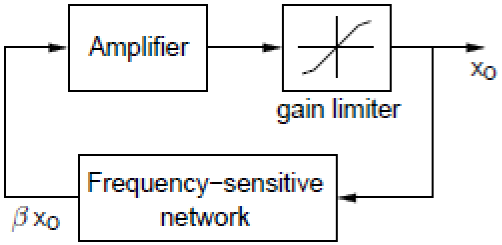
Phase angle=180 deg=π, $\angle \beta A = \pi + \pi = 2\pi$

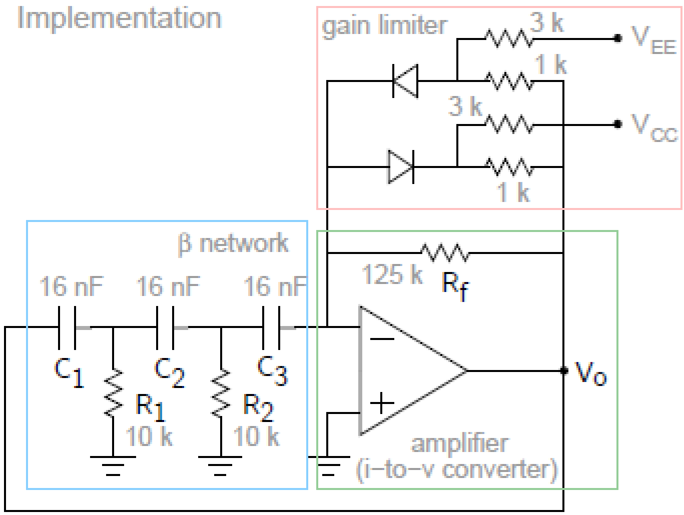
For the circuit to oscillate at ω_o (phase is 2π)

$$|A\beta(j\omega_o)| = 1 \Rightarrow R_f \times \frac{1}{12R} = 1 \Rightarrow \boxed{R_f = 12R}$$

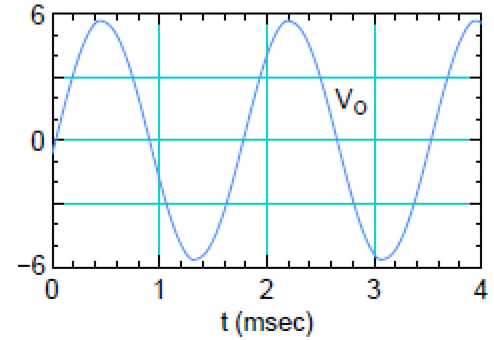
Phase-shift oscillator

Block Diagram





Output



$$\omega_o = \frac{1}{\sqrt{3}} \frac{1}{RC} \Rightarrow f_o = 574Hz$$