CS 207: Discrete Structures

Graph theory

Cut-edges, connected components, matchings

Lecture 28

Sept 29 2015

Topic 3: Graph theory

Recap of last five lectures:

1. Basics: graphs, paths, cycles, walks, trails; connected graphs.
2. Eulerian graphs and a characterization in terms of degrees of vertices.
3. Bipartite graphs and a characterization in terms of odd length cycles.

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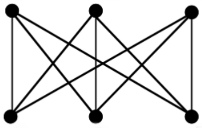
5. A proof and algo for large bipartite subgraphs of a graph.
6. Graph representation (as matrices).
7. Graph isomorphisms and automorphisms.
8. Connected components of a graph.

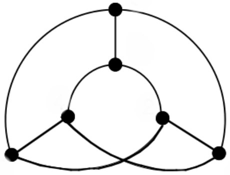
Reference: Most of Chapter 1 from Douglas West.

Recap: Yesterday's Pop Quiz

An isomorphism from simple graph G to H is a bijection $f : V(G) \to V(H)$ such that $uv \in E(G)$ iff $f(u)f(v) \in E(H)$.
An automorphism of G is an isomorphism from G to itself.



$K_{3,3}$



ML_3

1. Are the above graphs isomorphic? If yes, provide the edge-preserving bijection between their vertices. Else describe a property that differentiates them.
2. Can a graph have zero automorphisms? Find a graph on 6 vertices, that has exactly one automorphism. Is this the smallest such graph?

Recap: Connected components

- Connectedness is an equivalence relation.

Definition

A (connected) component of G is maximal connected subgraph of G, i.e., a subgraph that is connected and is not contained in any other connected subgraph of G.

- A component with no edges is called trivial. Thus isolated vertices form trivial components.
- Components are pairwise disjoint.

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Theorem: Characterize cut-edges using cycles

An edge is a cut-edge iff it belongs to no cycle.

This lecture, we will start a new topic:

Matchings

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- Suppose m people are applying for n different jobs. But not all applicants are qualified for all jobs, and each can hold at most one job.
- Then can you find a unique way to match jobs to applicants?

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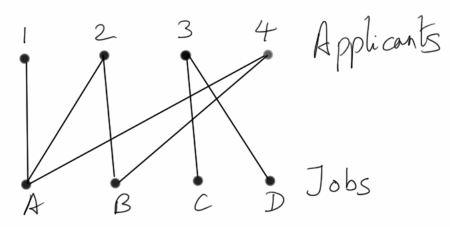

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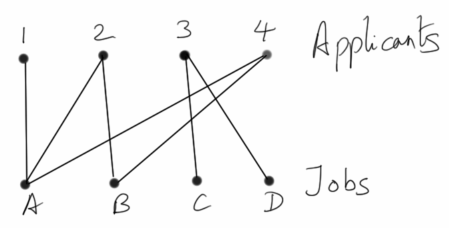

- What are the properties of such an assignment?
- Another practical example: the dating scene!

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- Is there a perfect matching if everyone is fully qualified/likes everyone?
- How many perfect matchings are possibly in $K_{n,n}$?