Introduction to Semiconductors
P-N Junction Diode

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EE 101

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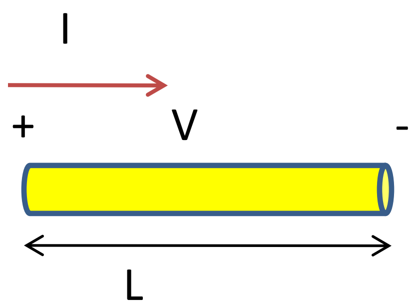
Conductivity of metals



$$I = \frac{Nq}{T} = \frac{Nqv}{L}$$

$$J = \frac{Nqv}{AL} = nqv = nq\mu E$$

$$J = \sigma E \Rightarrow \boxed{\sigma = nq\mu}$$

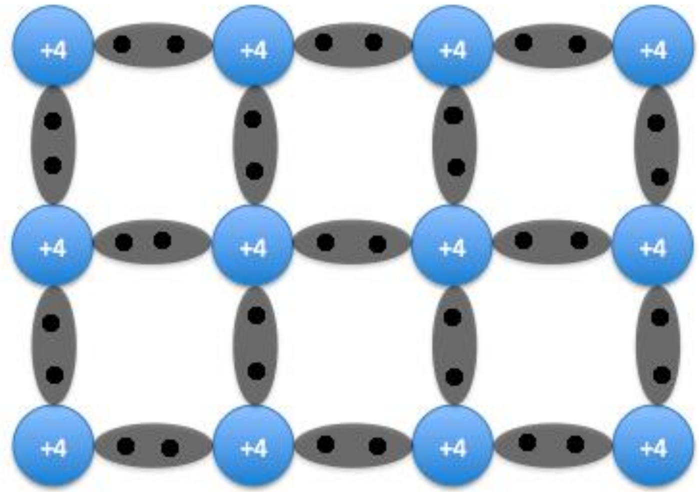
<u>Ohm's Law</u>

$$J = \sigma E \Rightarrow \boxed{I = \frac{\sigma A}{L} V}$$

$$\boxed{R = \frac{\sigma A}{L}}$$

- Metal→ free valence electrons
- Without applied voltage → random thermal motion, avg current=0
- With applied voltage → electrons go from high to low potential
- v=drift velocity=μE
- μ is mobility in cm^2/Vs
- Typically for metals
  - Cu, n~10^{28}-10^{29} e/m^3
  - High conductivity (σ)
- For insulators, n~10^7 e/m^3

Semiconductors



- $n \sim 10^{16}$-10^{19} e/m^3
- For example,
  - Si, $n=1.5 \times 10^{16}$ m^{-3}
  - Ge, $n=2.6 \times 10^{19}$ m^{-3}
- Two types of mobile charge carriers
  - Electrons and holes
  - In Si, 4 valence electrons, covalent bond
  - Thermal energy at room T excites electrons to break away
  - Vacated state → hole, +q
  - In pure intrinsic semiconductors, $n=p=n_i$
- In Si, 1.1 eV to form an electron – hole pair, E_g=bandgap=1.1 eV
- In Ge, E_g=0.67 eV

E_g=1.1 eV for Si
E_g=0.67 eV for Ge

Semiconductor conductivity

- Metals → unipolar (only electrons)
- Semiconductor → bipolar conductivity (electrons and holes)

$$J = nq\mu_n E + nq\mu_p E$$

$$J = \sigma E \Rightarrow \sigma = nq\mu_n + nq\mu_p$$

$$\text{In Si, at RT, } \mu_n = 0.13, \mu_p = 0.05\,\text{cm}^2/\text{Vs}$$

$$\text{n}_\text{i} = 1.5 \times 10^{16}\ \text{m}^{-3}$$

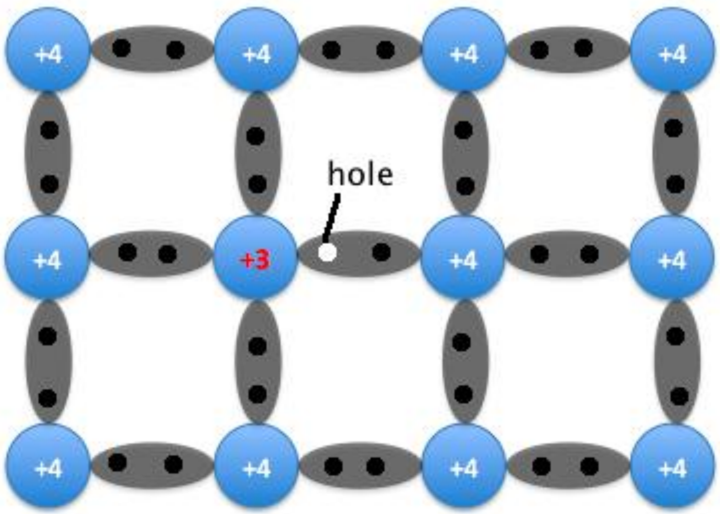
$$\Rightarrow \sigma = nq\mu_n + nq\mu_p = n_i q(\mu_n + \mu_p) = 4.32 \times 10^{-4}\ \text{mho/m}$$

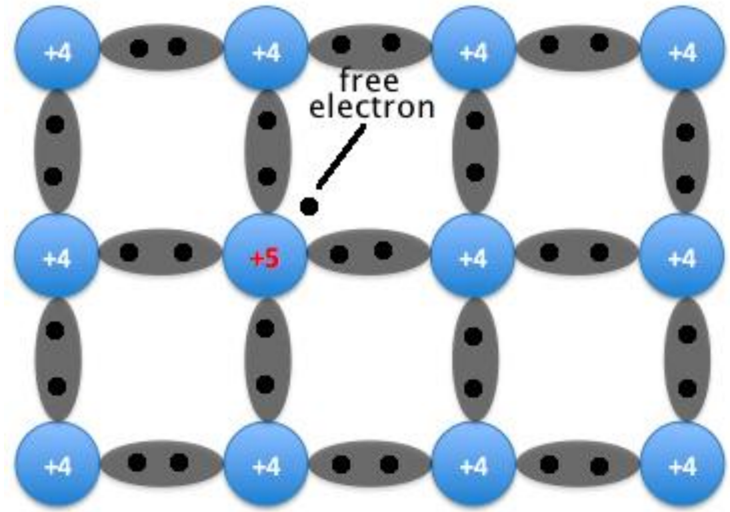
$$\text{Cu at RT, } \sigma = 5.8 \times 10^{+7}\ \text{mho/m}$$

Other semiconductors

- Elemental (Group IV): Si, Ge
- Compound
  - IV-IV: SiC
  - III-V: GaAs, GaN, GaP, In P, AlAs
  - II-VI: ZnO, ZnS, CdSe, CdTe
- Alloyed: $Si_{i-x}Ge_x$, $Al_xGa_{1-x}As$, $Ga_xIn_{1-x}As_{1-y}P_y$
- Used for high speed, optoelectronic, high power and high temperature devices

Doping of semiconductors





Group III impurity → B, Ga (3 valence e^-)
Holes that can "accept" electrons
Acceptor impurity
Positively charged h^+ →p-type semiconductor

Group V impurity → P, As, Sb (5 valence e^-)
"Donate" electrons
Donor impurity
Negatively charged e^- →n-type semiconductor

- Doping → modulate conductivity by introducing impurities

Doping → concentrations

Under thermal equilibrium,

$$np = n_i^2$$

For pure (intrinsic) semiconductor

$$n = p = n_i$$

N-type Doped

Charge neutrality implies

$$N_D + p = N_A + n$$

For n-type, $N_A = 0$

$$N_D + p = n$$

$$\boxed{n \rangle\rangle p,\ n \approx N_D,\ p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D}}$$

P-type Doped

Charge neutrality implies

$$N_D + p = N_A + n$$

For p-type, $N_D = 0$

$$N_A + n = p$$

$$\boxed{p \rangle\rangle n,\ p \approx N_A,\ n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A}}$$

Doping → conductivity

In Si, at RT, $\mu_n = 0.13, \mu_p = 0.05\,\mathrm{cm}^2/\mathrm{Vs}$

$\mathrm{n_i} = 1.5 \times 10^{16}\ \mathrm{m}^{-3}$

2 parts per 10^8 of donor impurity

$$\mathrm{N_D} = (5 \times 10^{28})\left(\frac{2}{10^8}\right) = 10^{21}\ \mathrm{m}^{-3}$$

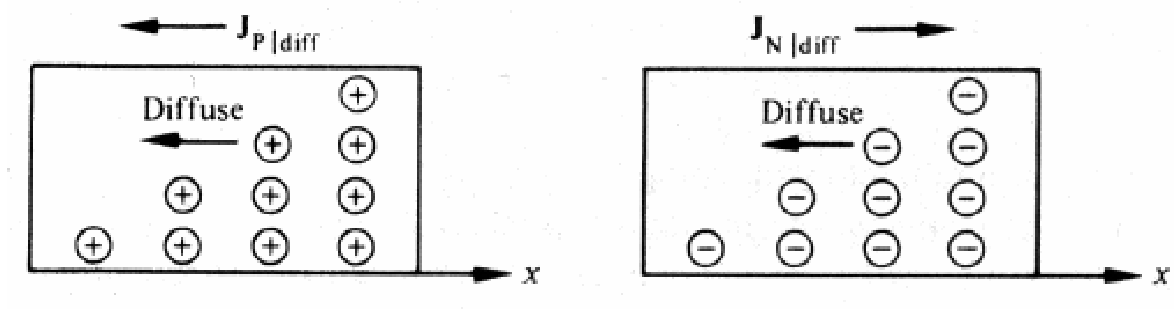
$$n \sim 10^{21}\ \mathrm{m}^{-3} \text{ and } \mathrm{p} = \frac{\mathrm{n_i^2}}{\mathrm{n}} = 2.25 \times 10^{11}\ \mathrm{m}^{-3}$$

$$\Rightarrow \sigma = nq\mu_n + nq\mu_p = q(n\mu_n + p\mu_p) = 20.8\ \mathrm{mho/m}$$

A factor of 50000 increase w.r.t undoped ($4.3\mathrm{x}10^{-4}$) case

- Small percentage of impurity can drastically alter conductivity

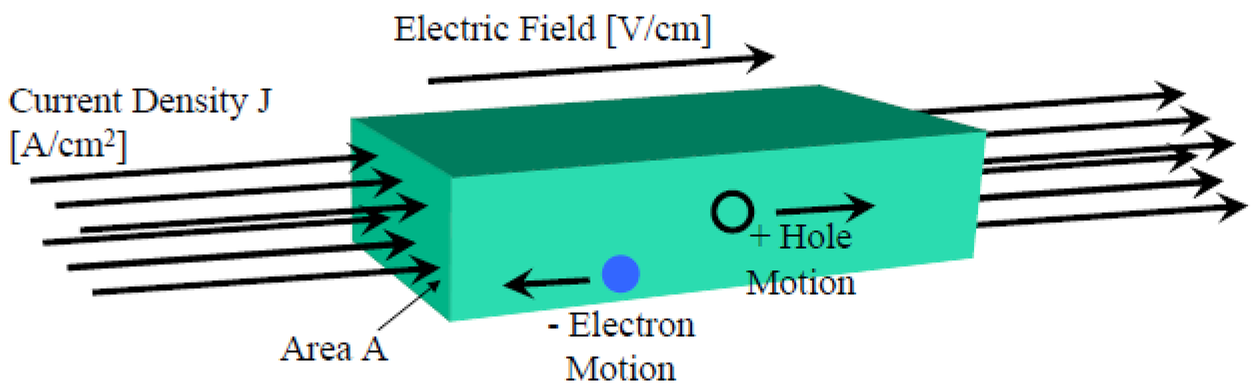
How does current flow in semiconductors? – Diffusion Current



$$J_p\,|_{Diffusion} = -qD_p \nabla p \quad or \quad J_n\,|_{Diffusion} = qD_n \nabla n$$

- Diffusion current:“flow of carriers” from one region of higher concentration to lower concentration results in a “diffusion current”.

How does current flow in semiconductors? – Drift Current



$J_p|_{Drift} = q\,p\,v_d$ and $J_n|_{Drift} = q\,n\,v_d$

Hole Drift current density

Electron Drift current density

Under low electric field,

$J_p = qp\mu_p E$ and $J_n = qn\mu_n E$

- Current due to motion of carrier under the influence of an electric field

Total current

$$J_p = J_p|_{Drift} + J_p|_{Diffusion} = q\mu_p pE - qD_p\nabla p$$

and

$$J_n = J_n|_{Drift} + J_n|_{Diffusion} = q\mu_n nE + qD_n\nabla n$$

and

$$J = J_p + J_n$$

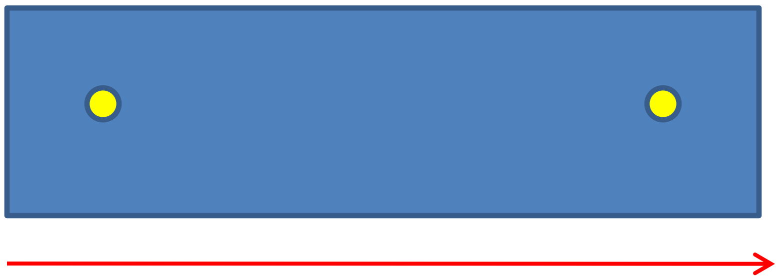
$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Einstein's relationship

- Summation of drift and diffusion currents of both holes and electrons

Graded Semiconductor

In general,



$$J = J_p(x) = J_{pdrift}(x) + J_{pdiff}(x)$$

$$J = qp\mu_p E - qD_p \frac{dp}{dx} = 0$$

$$\Rightarrow E(x) = \frac{D_p}{\mu_p}\frac{1}{p}\frac{dp}{dx} = \frac{kT}{q}\frac{1}{p}\frac{dp}{dx}$$

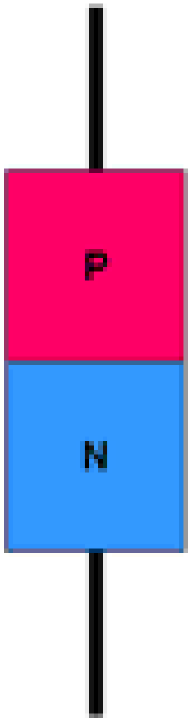
$$\int_{V_1}^{V_2} dV = -\frac{kT}{q}\int_{p_1}^{p_2}\frac{dp}{p}$$

$$\boxed{V_{21} = \frac{kT}{q}\ln\frac{p_1}{p_2}}$$

$$\Rightarrow p_1 = p_2 e^{\frac{qV_{21}}{kT}} \text{ and } n_1 = n_2 e^{-\frac{qV_{21}}{kT}}$$

$$\Rightarrow p_1 n_1 = p_2 n_2 = n_i^2$$

The p-n junction diode



$$p_p = N_A \qquad n_n = N_D$$

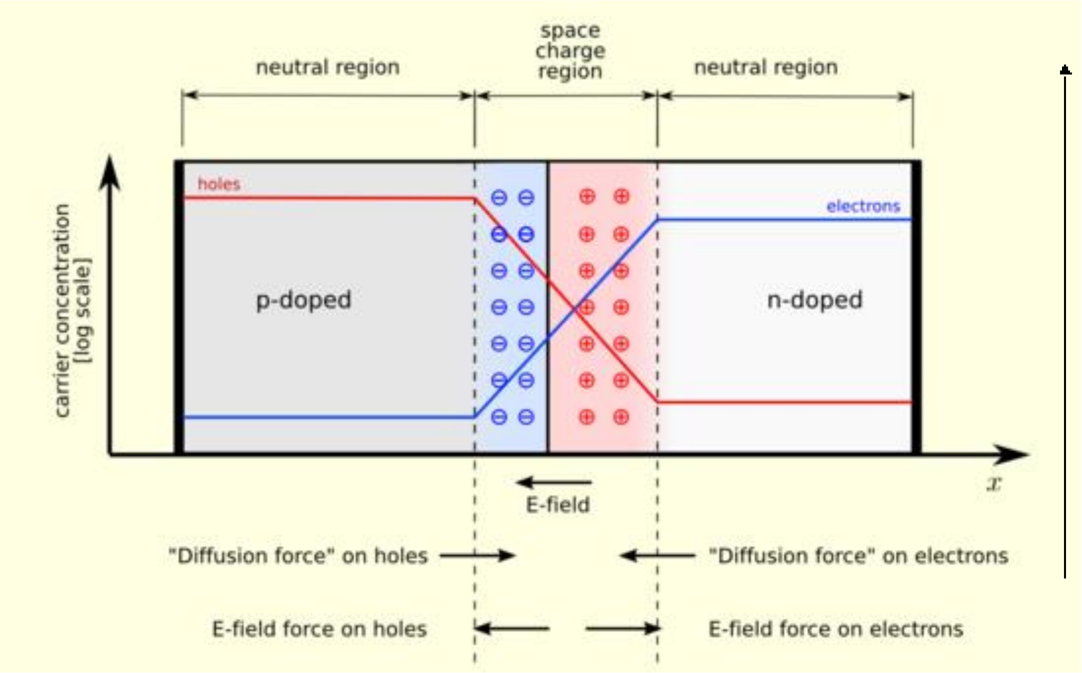
$$n_p = \frac{n_i^2}{N_A} \qquad p_n = \frac{n_i^2}{N_D}$$

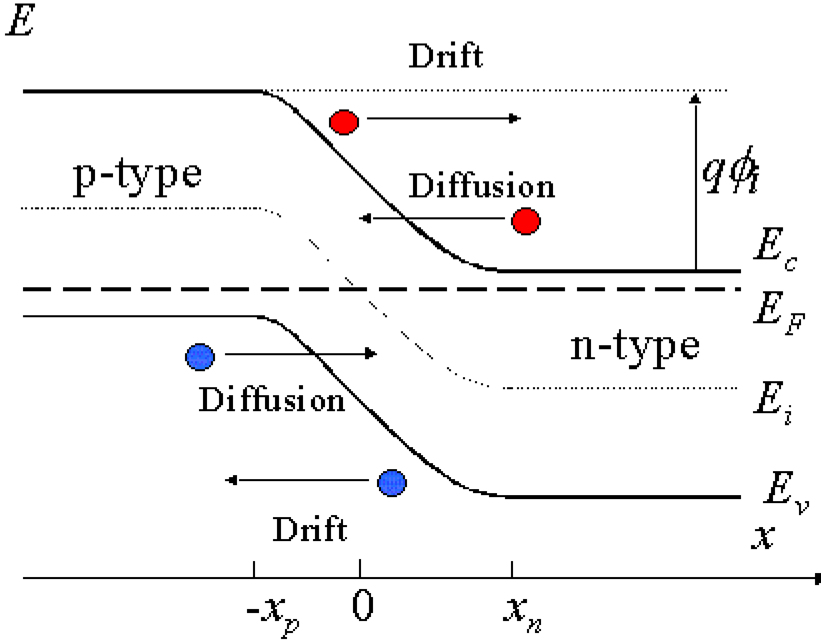
Barrier potential or built-in voltage

$$V_o = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{n_n}{n_p} = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$$

Equilibrium picture (no voltage applied)

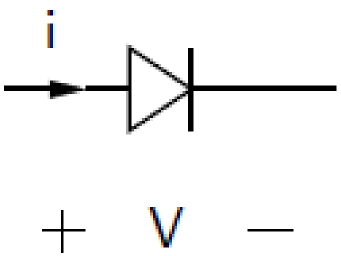
$$\phi_i = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$



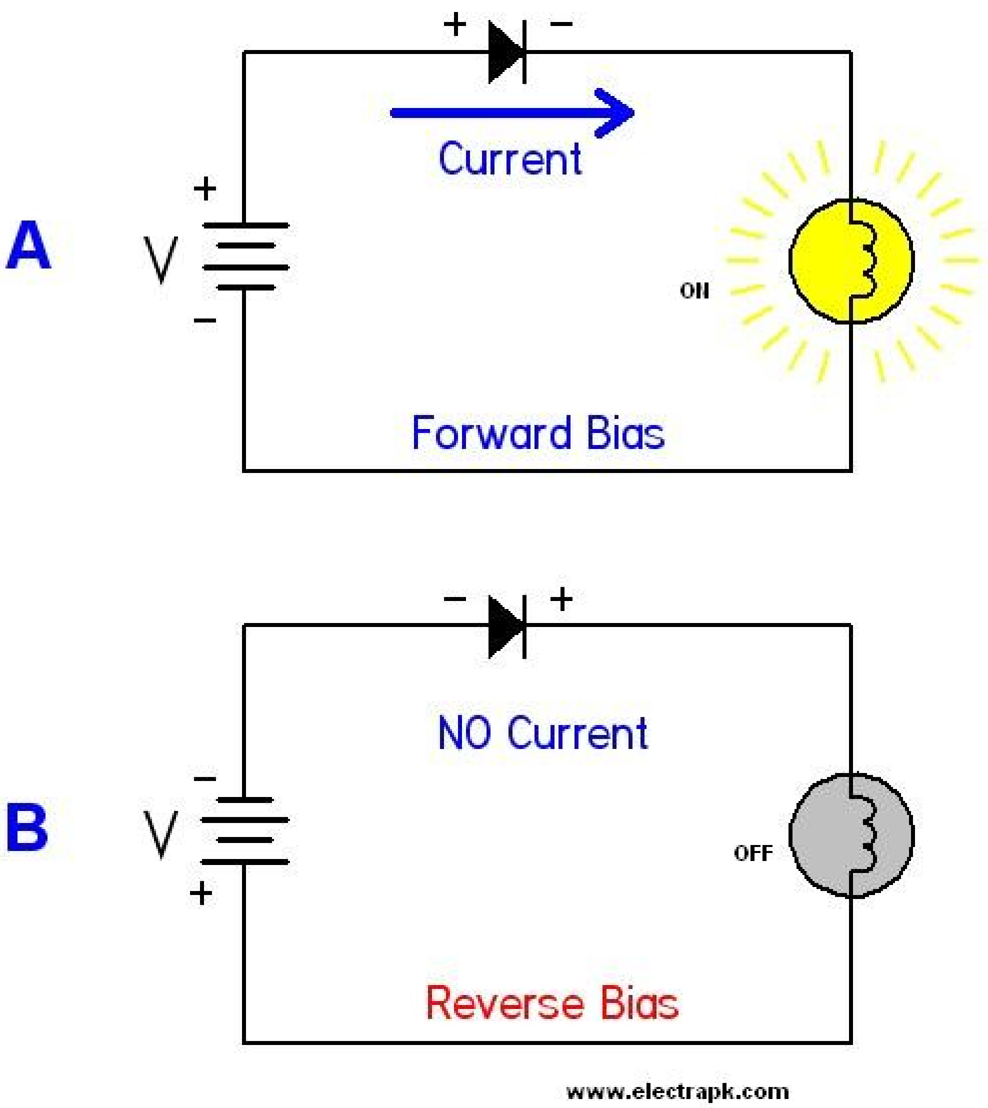


- J_{drift} and J_{diff} for each carrier type cancel out
- Net current is zero

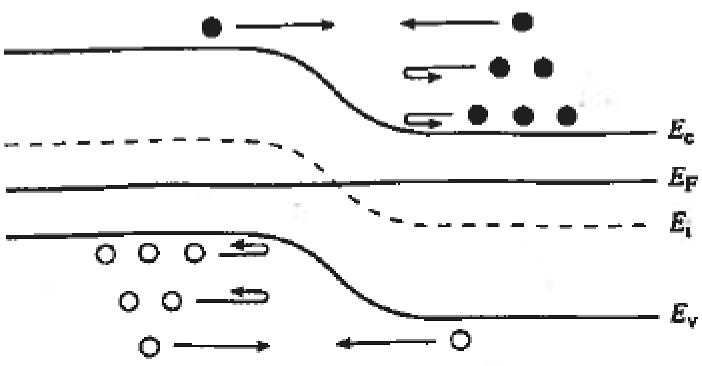
The Biased Diode



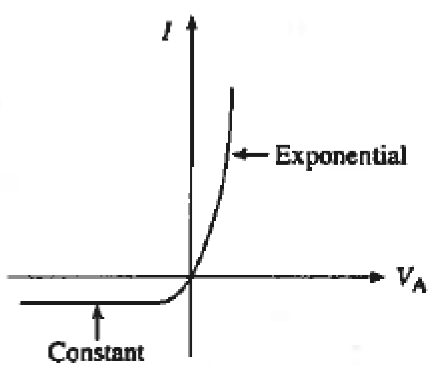
- Low resistance state in forward bias
- High resistance state in reverse bias
- Check Valve → diode

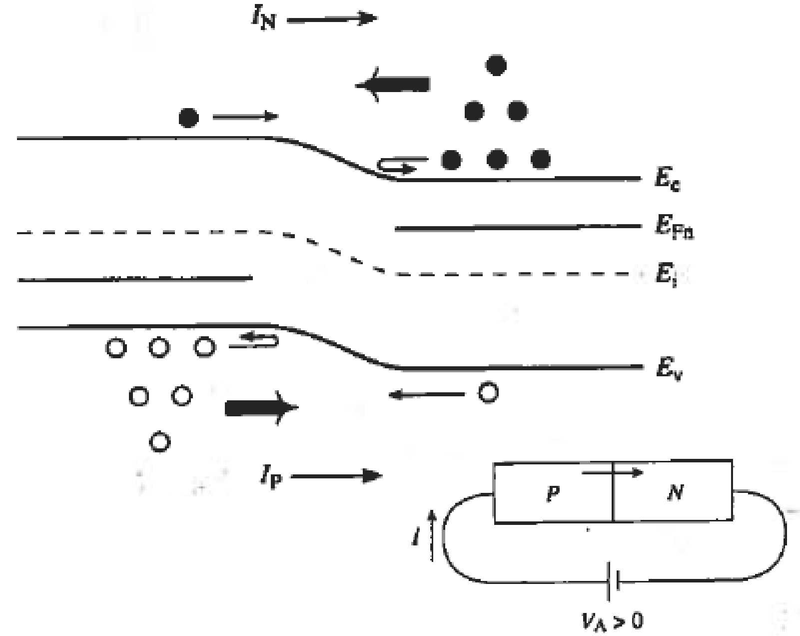


Current flow (Band diagrams)

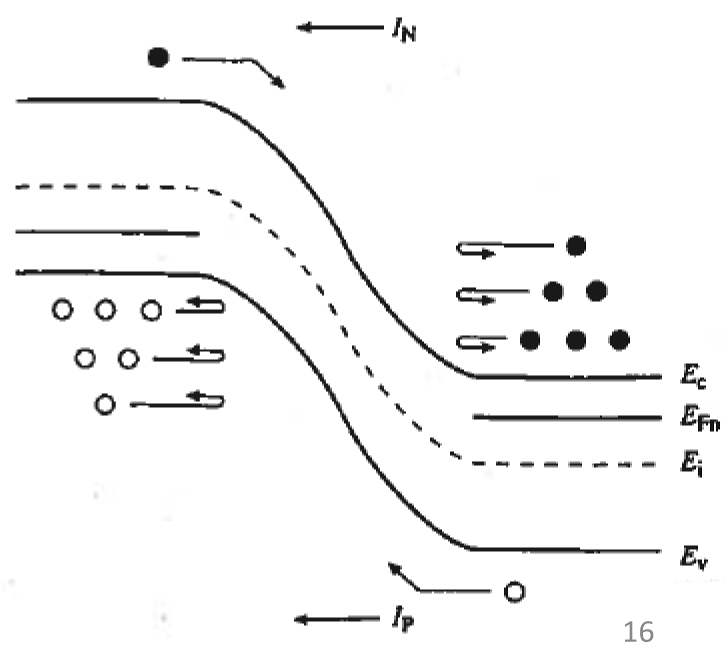


(a) Equilibrium ($V_A = 0$)





(b) Forward bias ($V_A > 0$)



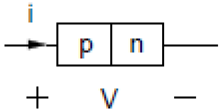
(c) Reverse bias ($V_A < 0$)

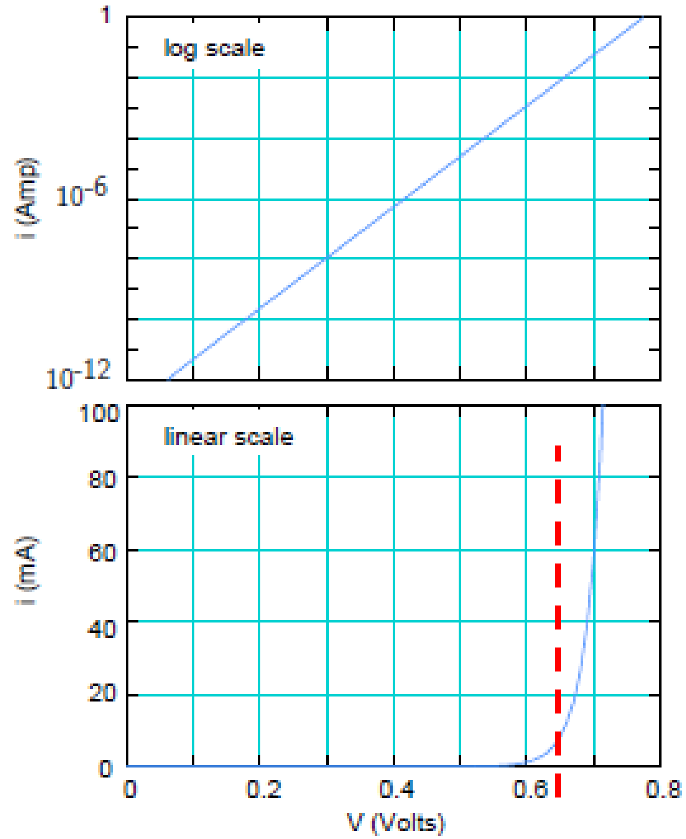
Forward Bias Diode I-V

$$i = I_s \left[\exp\left(\frac{V}{V_T} \right) - 1 \right]$$

$I_s = 10^{-13}$ A, V_T= 25 mV

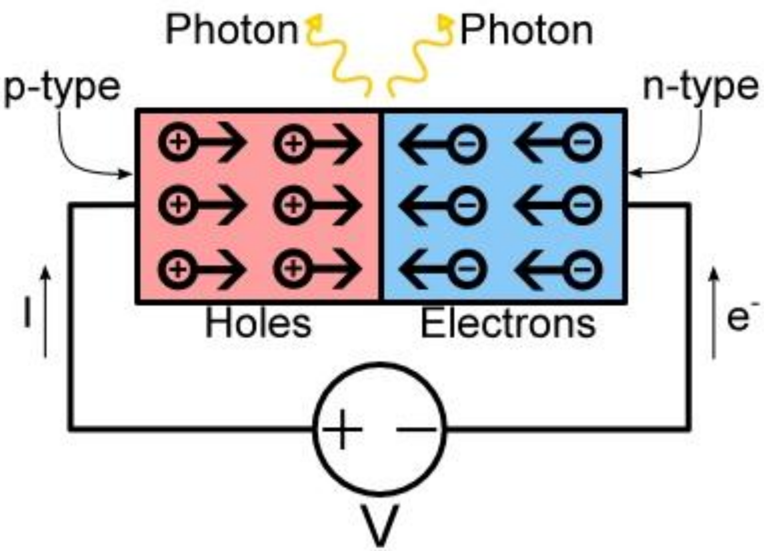
V	$x = V/V_T$	e^x	i (Amp)
0.1	3.87	0.479×10^{2}	0.469×10^{-11}
0.2	7.74	0.229×10^{4}	0.229×10^{-9}
0.3	11.6	0.110×10^{6}	0.110×10^{-7}
0.4	15.5	0.525×10^{7}	0.525×10^{-6}
0.5	19.3	0.251×10^{9}	0.251×10^{-4}
0.6	23.2	0.120×10^{11}	0.120×10^{-2}
0.62	24.0	0.260×10^{11}	0.260×10^{-2}
0.64	24.8	0.565×10^{11}	0.565×10^{-2}
0.66	25.5	0.122×10^{12}	0.122×10^{-1}
0.68	26.3	0.265×10^{12}	0.265×10^{-1}
0.70	27.1	0.575×10^{12}	0.575×10^{-1}
0.72	27.8	0.125×10^{13}	0.125





- Note that I increases significantly beyond 0.65 V → 10's of mA
- 0.65 is the "cut-in" voltage of the diode → I_s dependent
- For Si, typically, cut-in voltage → 0.6 – 0.7 V, Ge → 0.2 – 0.3 V, GaAs → 1.1 V

Types and Applications of Diodes



(source: wikipedia)

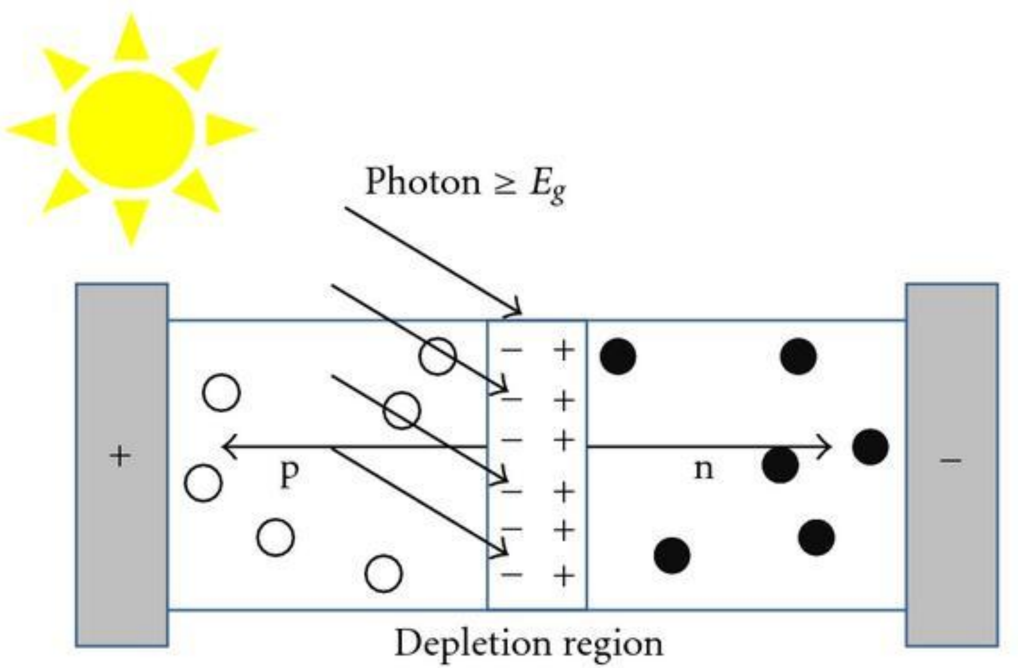
- Light-emitting diodes (LEDs)
  - Electrical → Optical conversion
  - Made of III-V semiconductors
  - Light of specific wavelength (red, blue, green, yellow)
  - Semiconductor lasers → Coherent light

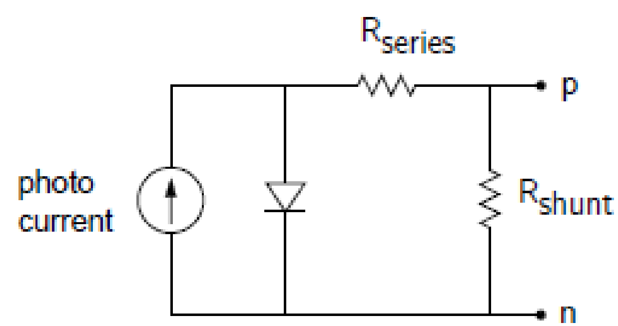
Parameter	Type of Light Bulb			
	Incandescent	Fluorescent	White LED	
			Circa 2010	Circa 2025
Luminous Efficiency (Lumens/W)	~12	~40	~70	~150
Useful Lifetime (hours)	~1000	~20,000	~60,000	~100,000
Purchase Price	~$1.50	~$5	~$10	~$5
Estimated Cost over 10 Years	~$410	~$110	~$100	~$40

Courtesy of: "Fundamentals of Applied Electromagnetics" by Fawwaz T. Ulaby, Eric Michielssen, Umberto Ravaioli. Page 33, Figure TF1-7

Types and Applications of Diodes

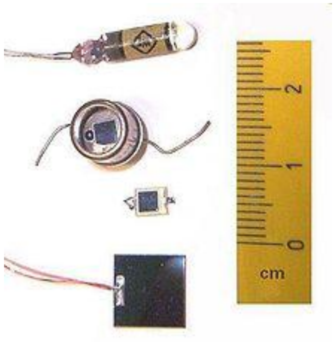
- Solar cells
  - Optical → Energy
  - Solar panel → Multpile solar cells in series/parallel





Types and Applications of Diodes

- Photodiodes
  - Detect optical signals (DC or time-varying) → Electrical signal
  - Similar to solar cell → Optimized for high sensitivity, low noise and high frequency applications
  - Consumer electronics
    - CD players, infrared remote control receivers, cameras
  - Fibre-optic communications → high frequency application
  - Spectroscopy, night vision, astronomy → high sensitivity app



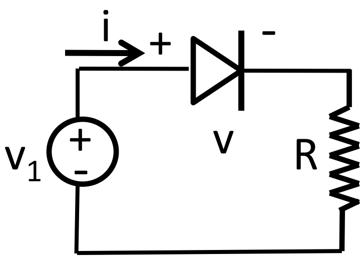
Discrete

Chip

Source: wikipedia

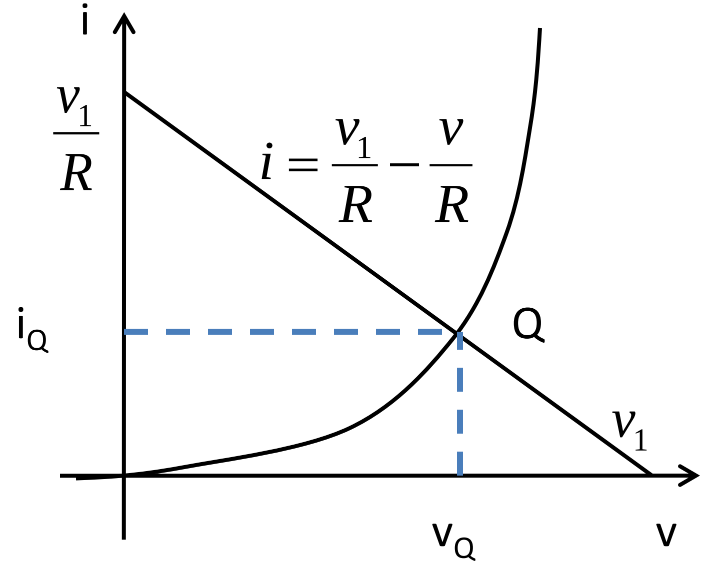
Material	Electromagnetic spectrum wavelength range (nm)
Silicon	190–1100
Germanium	400–1700
Indium gallium arsenide	800–2600
Lead(II) sulfide	<1000–3500

Diode circuits



$$i = I_S\left(e^{\frac{v}{\eta V_T}} - 1\right)$$

$$v_1 = v + Ri \Rightarrow i = \frac{v_1}{R} - \frac{v}{R}$$



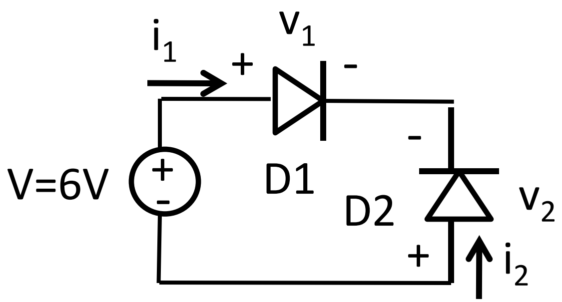
Transcendental equation -> no analytical equation

Graphical solution

i=(v_1-v)/R is called the load line → external constraint on the diode

Q → Quiescent operating point, or Q point or operating point

Diode circuits



The battery will forward bias D1 and reverse bias D2

Hence $i_2 = -I_S = -i_1$

$$i_1 = I_S\left(e^{\frac{v_1}{\eta V_T}} - 1\right) \Rightarrow v_1 = \eta V_T \ln\left(\frac{i_1}{I_S} + 1\right)$$

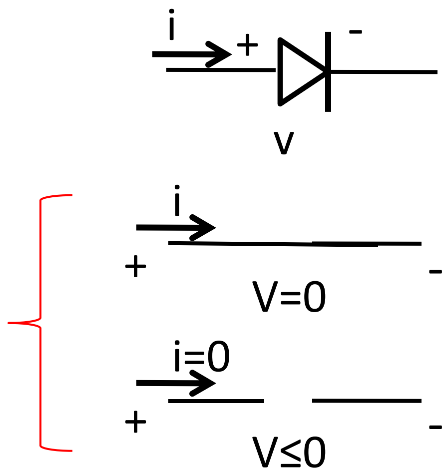
$$v_1 = \eta V_T \ln\left(\frac{I_S}{I_S} + 1\right) = 0.036V$$

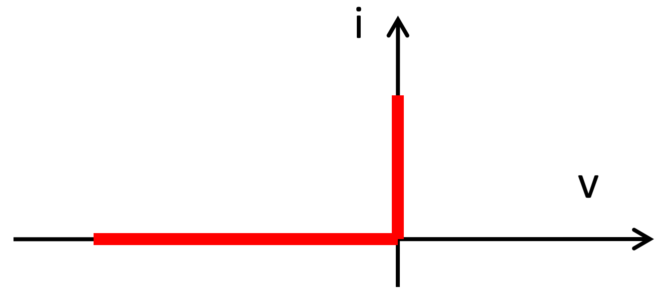
$$v_2 = -6 + v_1 = -5.964V$$

- Assume both diodes have the same saturation current I_S and η

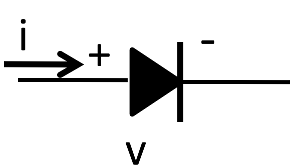
DC Diode Models

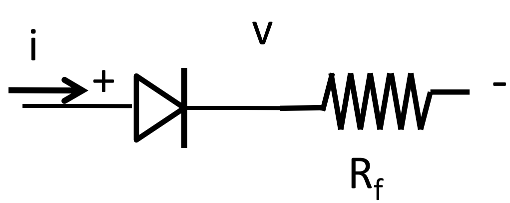
(i) Ideal diode

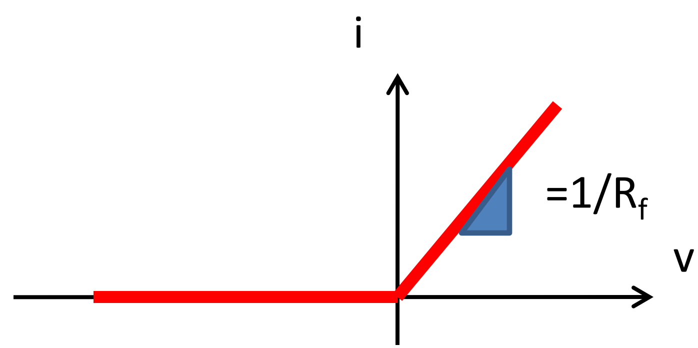




(ii) Non-ideal diode Model 1

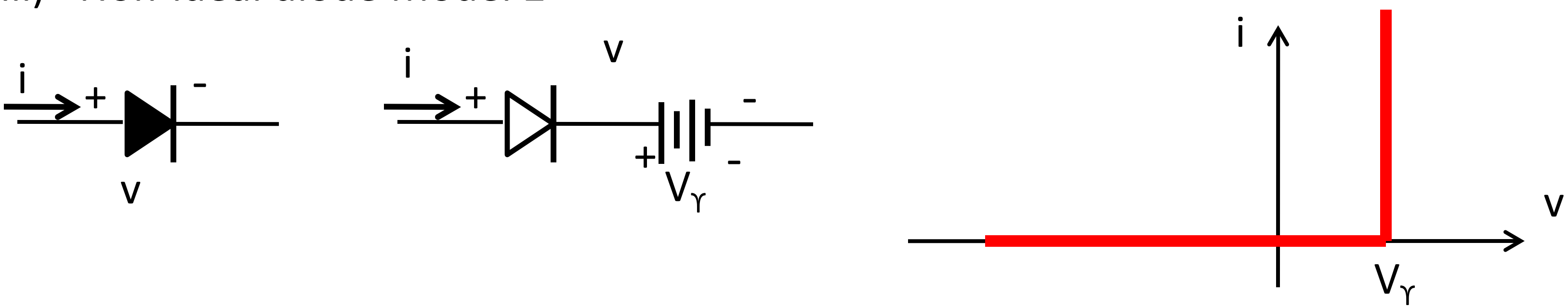




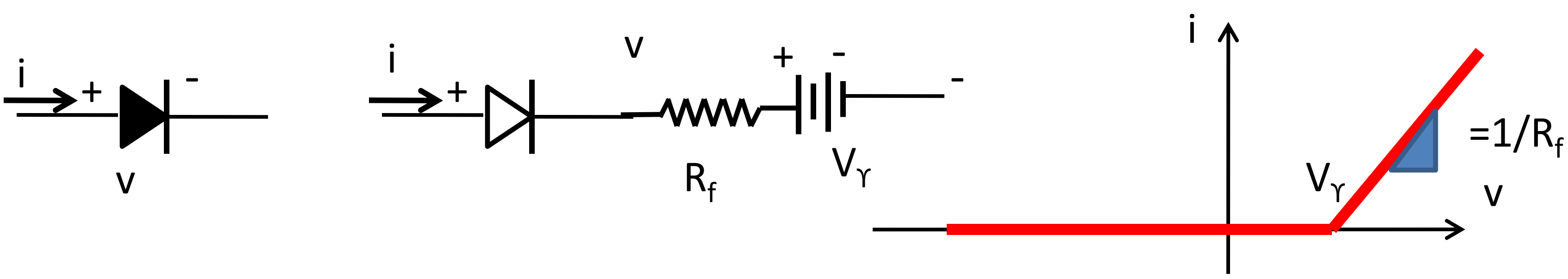


DC Diode Models

(iii) Non-ideal diode Model 2



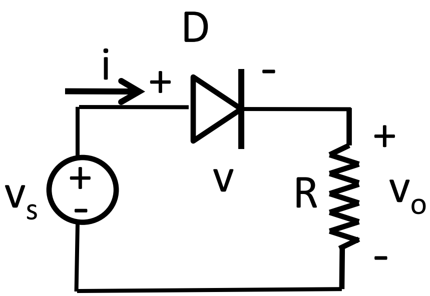
(iv) Non-ideal diode Model 3



Algorithm for diode circuits (Ideal diode model)

- Assume forward bias (ON) or reverse bias (OFF)
- Replace the diode by a short circuit or open circuit
- Solve the circuit using known techniques
- If inconsistent, e.g. $I<0$ for forward bias, then change assumption and resolve

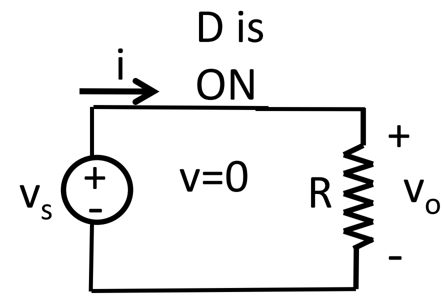
Example: Half-wave rectifier



For $v_s > 0$, assume D is ON

$v_o = v_s$

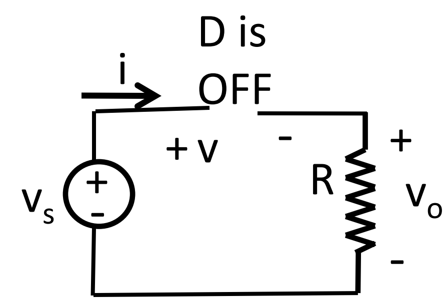
$i = v_s/R > 0$ → D is ON is correct



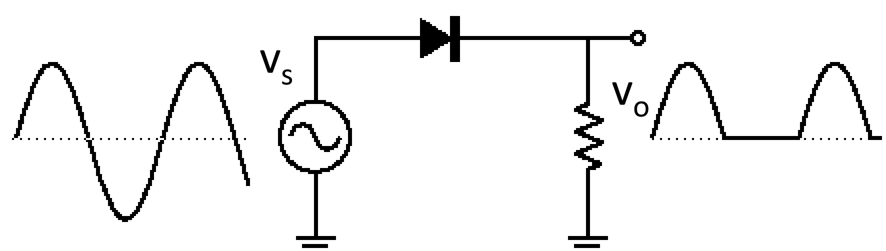
For $v_s \le 0$, assume D is OFF

$i = 0$

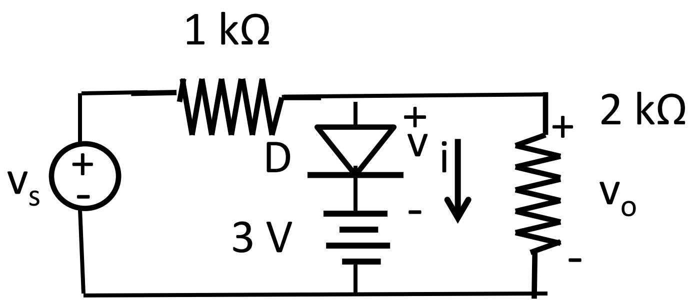
$v = v_s - Ri = v_s \le 0$ → D is OFF is correct



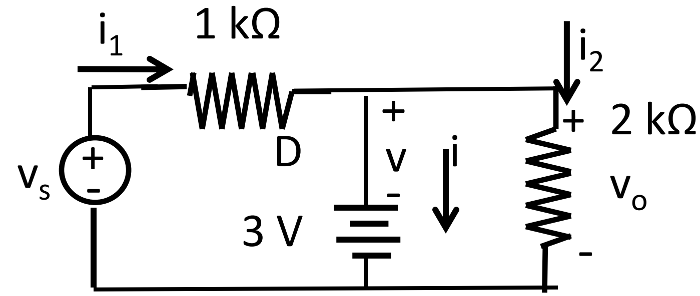
$$v_o = \begin{cases} 0\text{ V for } v_s \le 0 \\ v_s \quad \text{for } v_s > 0 \end{cases}$$



Example: Diode clipper circuit



For v_s>0 assume that D is ON



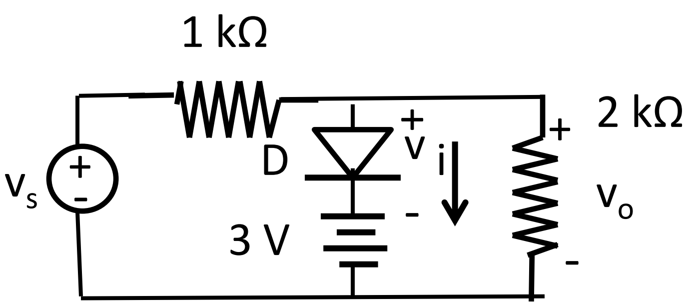
$$i_1 = \frac{v_s - 3}{1000}, i_2 = \frac{3}{2000} = 1.5\,\text{mA}$$

$$i = i_1 - i_2 = \frac{v_s}{1000} - 3\times10^{-3} - 1.5\times10^{-3}$$

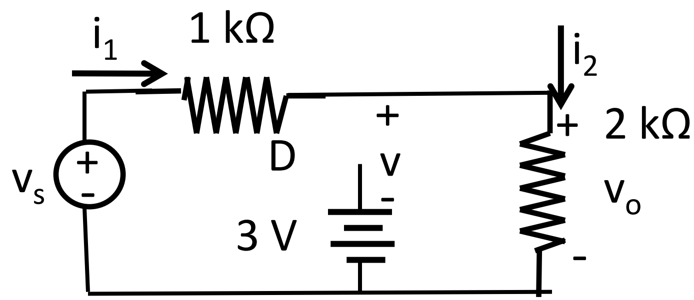
$$i > 0 \Rightarrow v_s > 4.5V$$

$$v_o = 2000\times1.5\times10^{-3} = 3V$$

Example: Diode clipper circuit



For $v_s \leq 4.5$ 0 assume that D is OFF

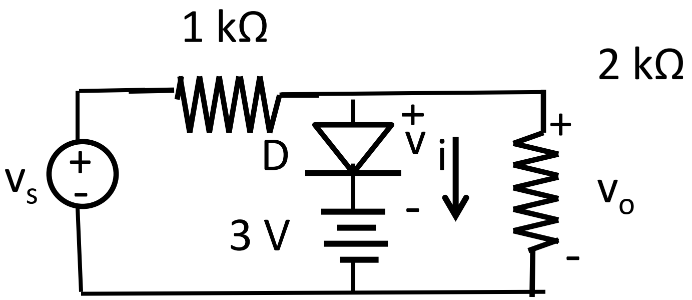


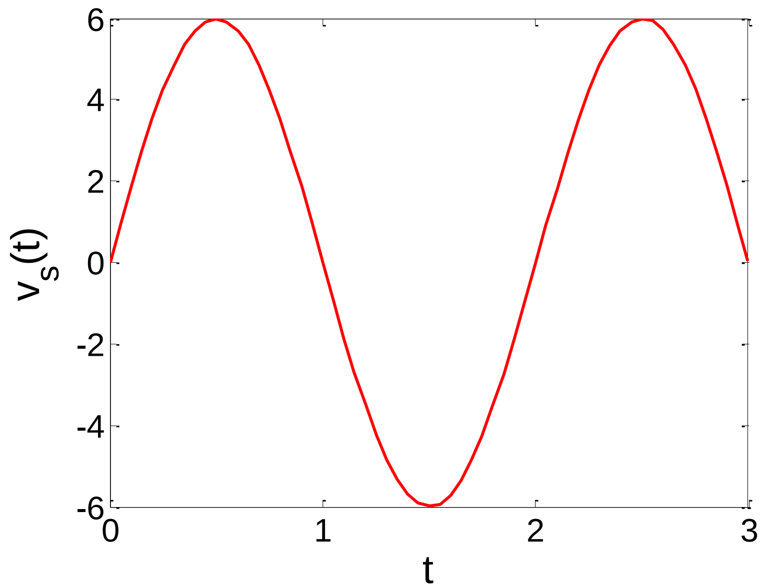
$$v_o = \frac{2}{1+2} \times v_s = \frac{2}{3} v_s$$

$$v = v_o - 3 = \frac{2}{3} v_s - 3 \leq 0$$

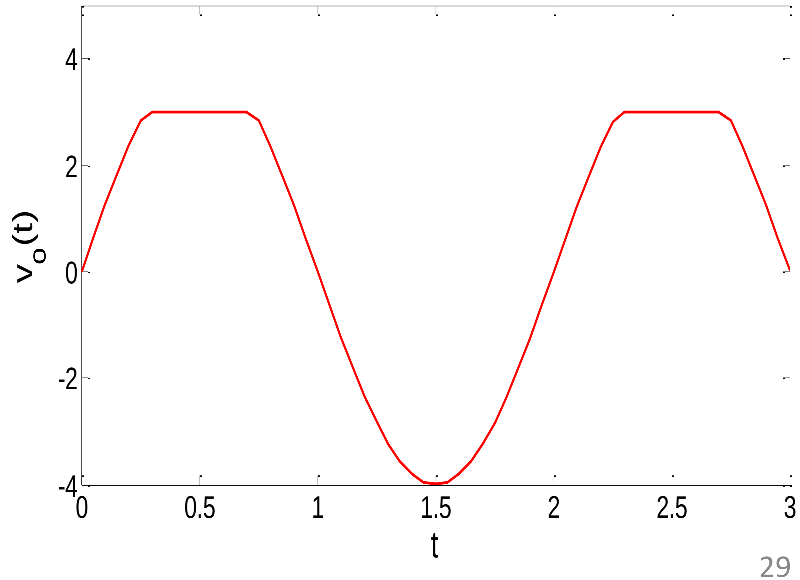
Hence assumption that D is OFF is correct

Example: Diode clipper circuit

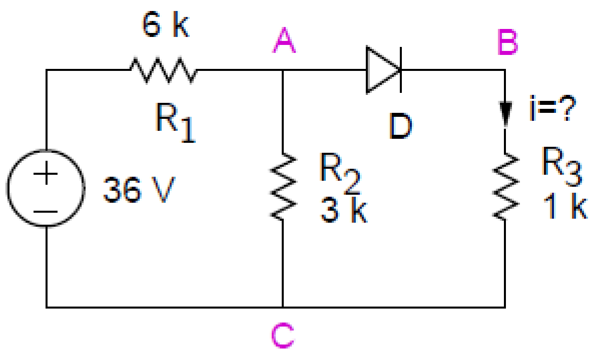


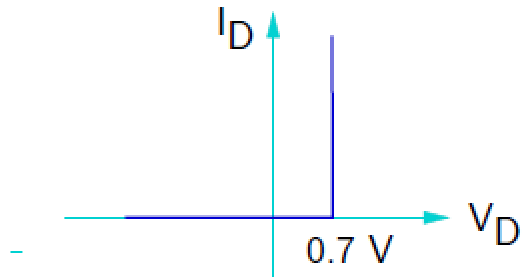


$$v_o = \begin{cases} \frac{2}{3} v_s & \text{for } v_s \le 4.5V \\ 3V & \text{for } v_s > 4.5V \end{cases}$$

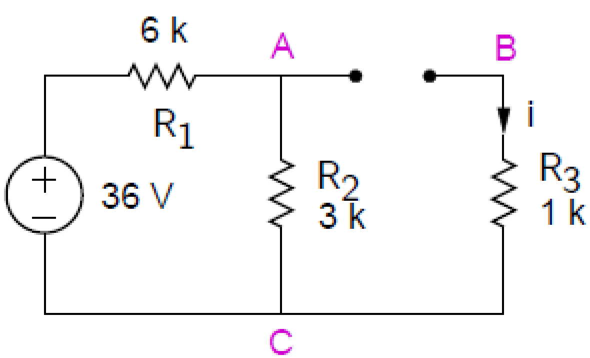


Example



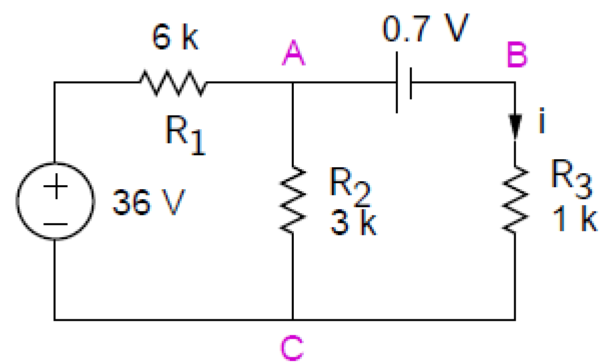


Case 1: D is off.



$V_{AB} = V_{AC}$ =(3x36)/9=12 V
which is not consistent with our assumption of D being off.
→ D must be on.

Case 2: D is on.

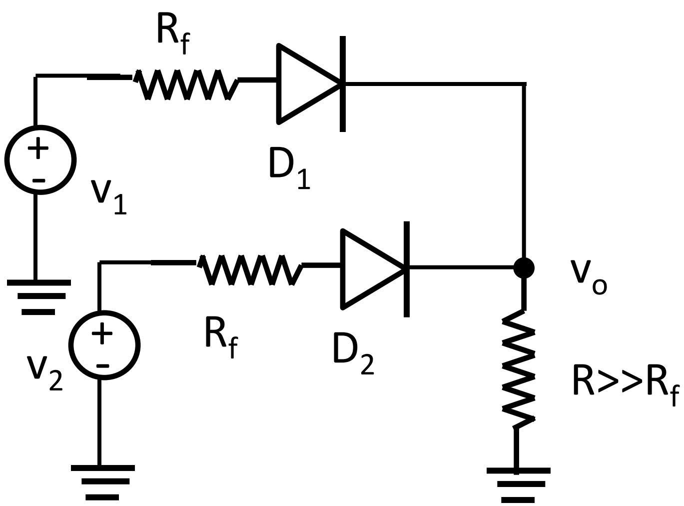


Taking $V_C = 0$ V,

$$\frac{V_A - 36}{6\ \text{k}} + \frac{V_A}{3\ \text{k}} + \frac{V_A - 0.7}{1\ \text{k}} = 0\,,$$

$\rightarrow\ V_A = 4.47$ V, $i = 3.77$ mA .

Application to Digital Logic



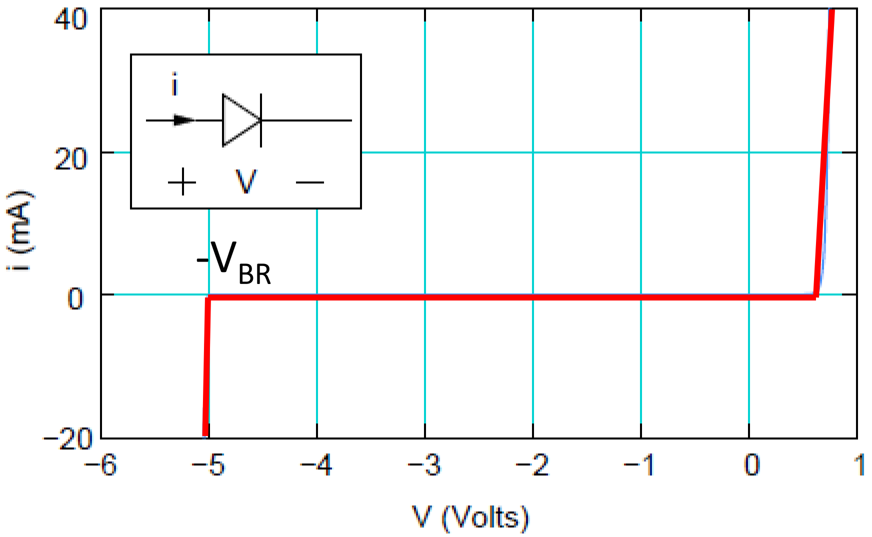
V_1	V_2	D_1	D_2	Vo
0	0	OFF	OFF	0
0	V_H	OFF	ON	$(R/R_f+R)V_H \sim V_H$
V_H	0	ON	OFF	$(R/R_f+R)V_H \sim V_H$
V_H	V_H	ON	ON	$(2R/R_f+2R)V_H \sim V_H$

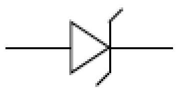
- Assume high voltage=V_H>0 represents "1"
- Assume low voltage=0V represents "0"
- Assume ideal diodes

V_1	V_2	Vo
0	0	0
0	1	1
1	0	1
1	1	1

OR Gate

Zener Diodes

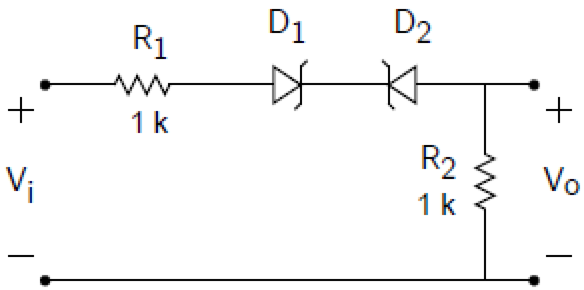




Symbol for a Zener diode

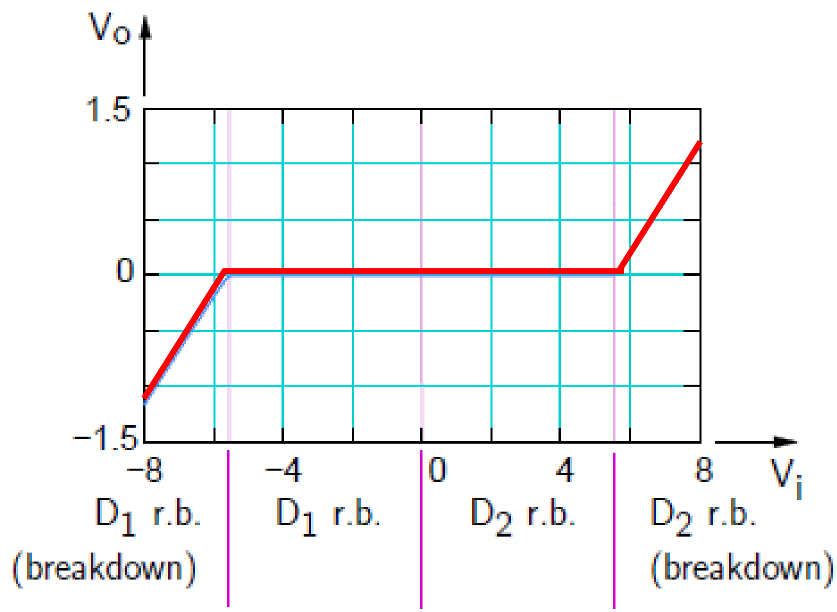
- At high reverse biases, diodes "break down"
  - Large current flow, voltage "clamped" at $-V_{BR}$
- Breakdown occurs due to
  - (i) Avalanche of carriers generated by collisions → typically in lightly doped diodes, large $|V_{BR}|$ → tens to hundreds of V
  - (ii) Zener tunneling of carriers → typically in heavily doped diodes, small $|V_{BR}|=V_Z$ → 5-6 V in Si
- Zener diodes are used to limit voltage swing in electronic circuits

Example



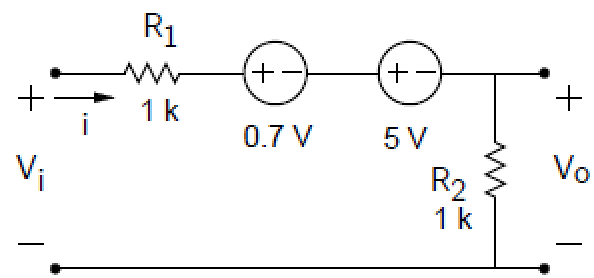
$V_{on} = 0.7$ V, $V_Z = 5$ V.

Plot V_o versus V_i.



For a current to flow, we have two possibilities:

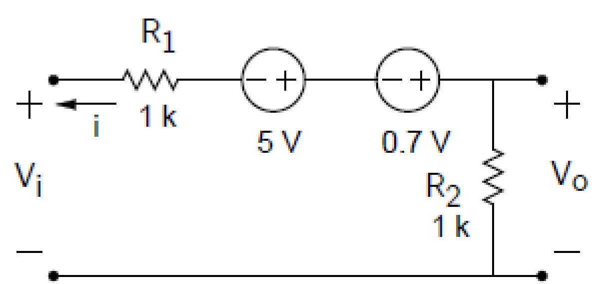
D_1 on (forward), D_2 in reverse breakdown



$$V_o = i\,R_2 = \frac{V_i - 5.7}{R_1 + R_2}\,R_2$$

Since $i > 0$, this can happen only when $V_i > 5.7$ V.

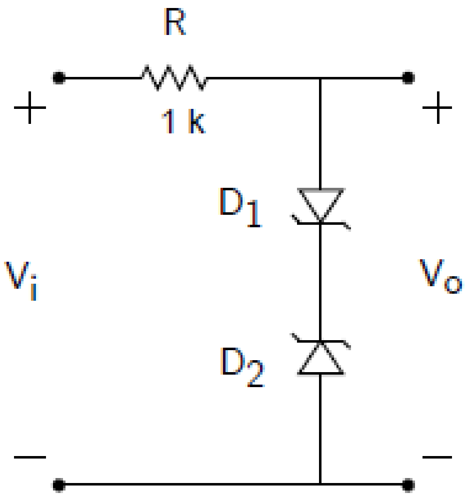
D_2 on (forward), D_1 in reverse breakdown



$$V_o = -i\,R_2 = \frac{V_i + 5.7}{R_1 + R_2}\,R_2$$

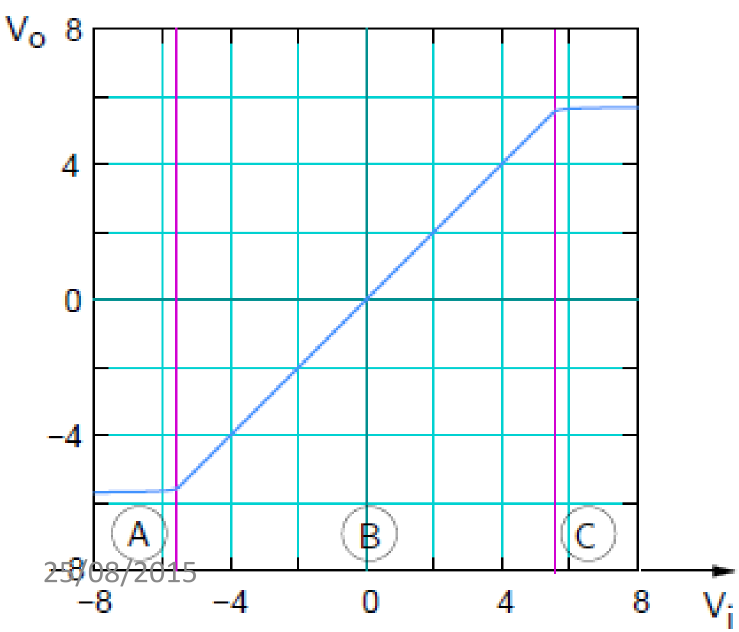
Since $i > 0$, this can happen only when $V_i <$ -5.7 V.

Example: Voltage Limiter



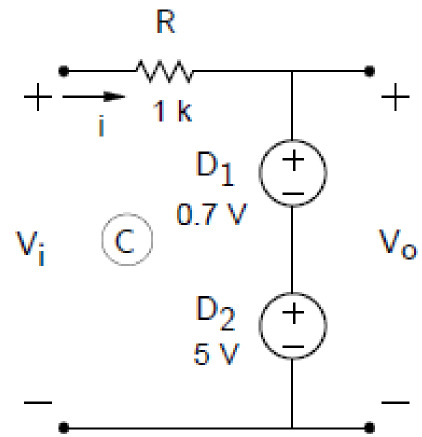
$V_{on} = 0.7$ V, $V_Z = 5$ V.

Plot V_o versus V_i.



For a current to flow, we have two possibilities:

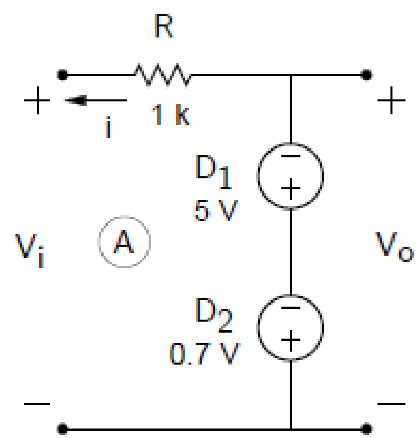
D_1 on (forward), D_2 in reverse breakdown



$V_o = 5.7$ V

$V_i > 5.7$ V

D_2 on (forward), D_1 in reverse breakdown

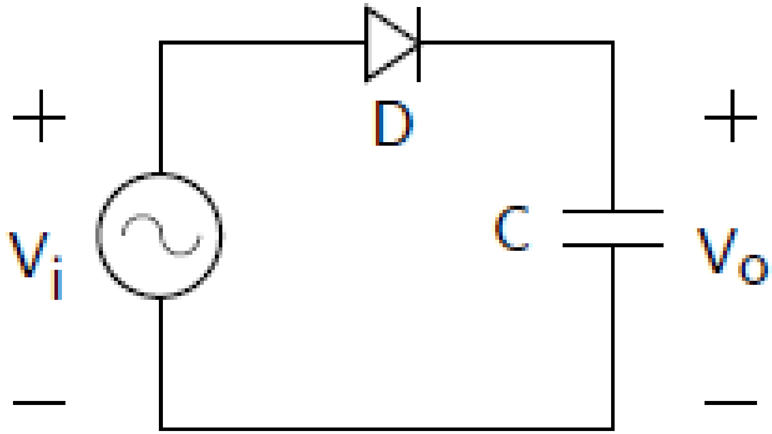


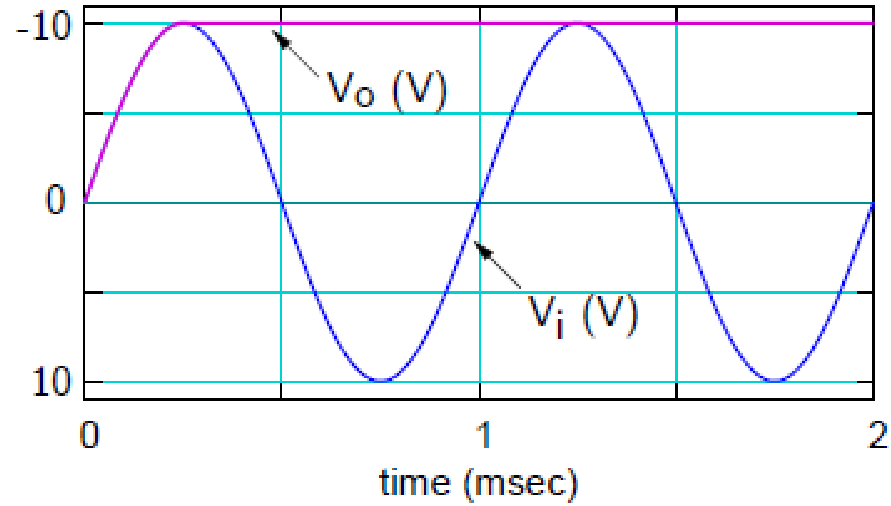
$V_o = -5.7$ V

$V_i < -5.7$ V

In the range, $-5.7\text{ V} < V_i < 5.7\text{ V}$, no current flows, and $V_o = V_i$. B

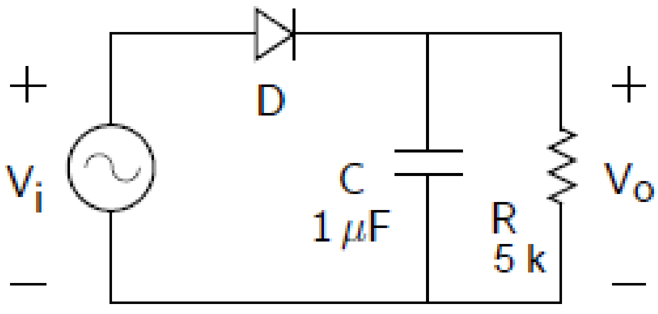
Peak Detector

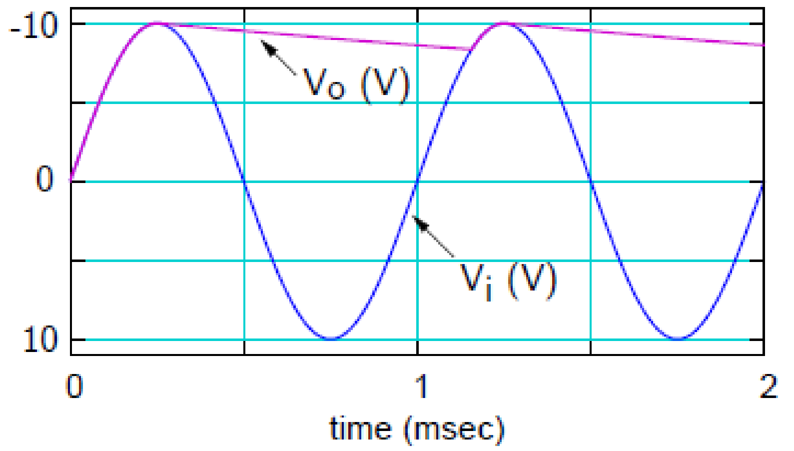




- Let $V_o(t)$=0 V at t = 0, and assume the diode to be ideal, with V_{on}=0 V.
- For 0 < t < T/4, V_i rises from 0 to V_m. As a result, the capacitor charges.
- Since the on resistance of the diode is small, time constant τ<<T/4; therefore the charging process is instantaneous → $V_o(t) = V_i(t)$.
- For t > T/4, V_i starts falling. The capacitor holds the charge it had at t=T/4 since the diode prevents discharging.

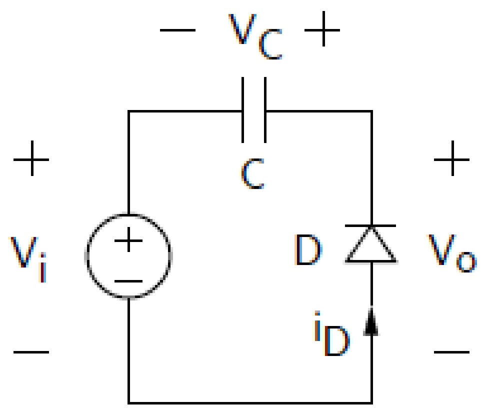
Peak Detector (Contd.)

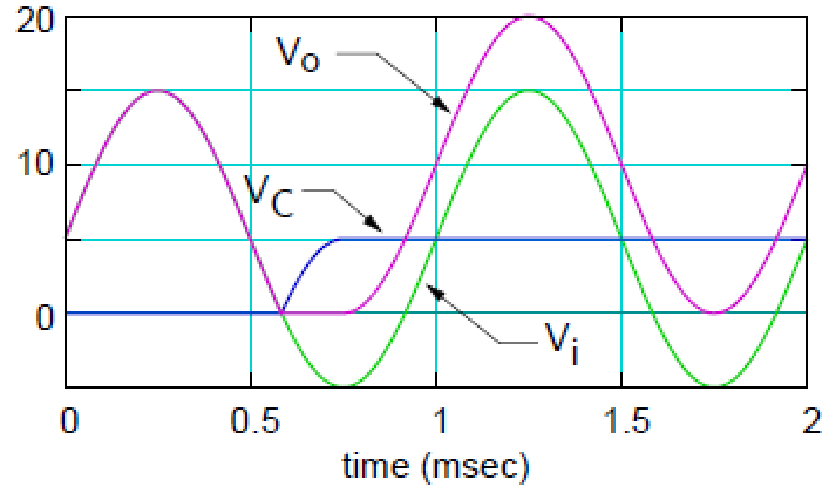




- Capacitor can discharge through R when diode is off
- When $V_i > V_o$, the capacitor charges again.
- The time constant for the charging process is $\tau = R_{Th}C$, where $R_{Th} = R || R_{on}$ is the Thevenin resistance seen by the capacitor, R_{on} being the on resistance of the diode.
- Since $\tau << T$, the charging process is instantaneous.
- Used to demodulate AM signals → envelope detector

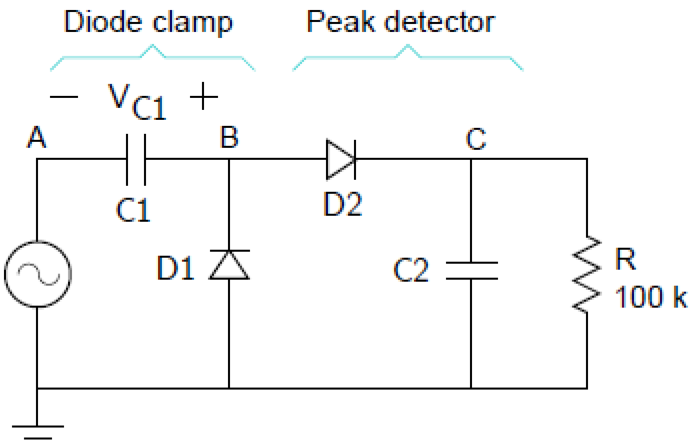
Clamping Circuit (DC restorer)

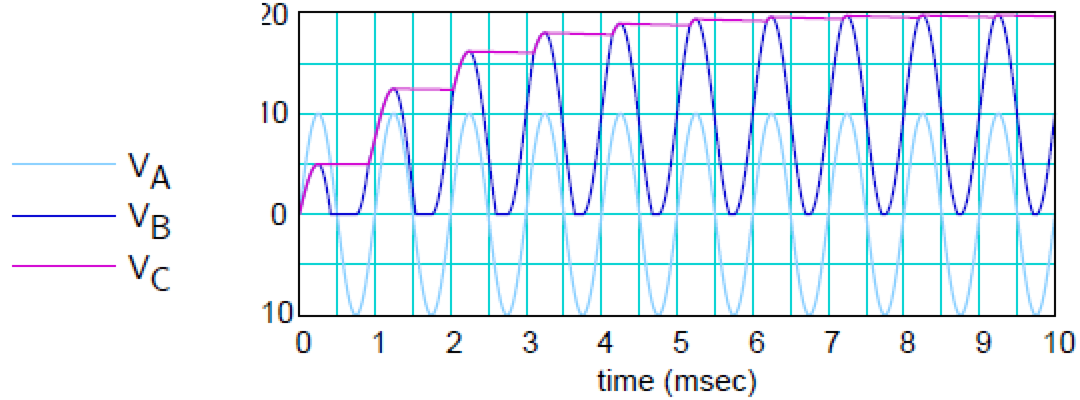




- Assume V_{on} = 0 V for the diode.
- When D conducts, $V_D = -V_o = 0 \rightarrow V_C + V_i = 0$, i.e., $V_C = -V_i$.
- V_C can only increase with time (or remain constant) since i_D can only be positive.
- The net result is that the capacitor gets charged to a voltage $V_C = -V_i$ corresponding to the maximum negative value of V_i, and holds that voltage thereafter. Let us call this voltage V_C^0 (a constant).
- $V_o(t) = V_C(t) + V_i(t) = V_C^0 + V_i(t)$, which is a "level-shifted" version of V_i.

Voltage doubler (Peak-to-peak detector)

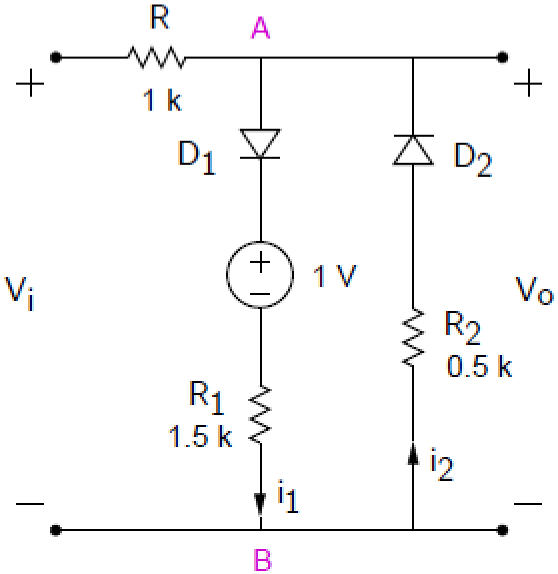


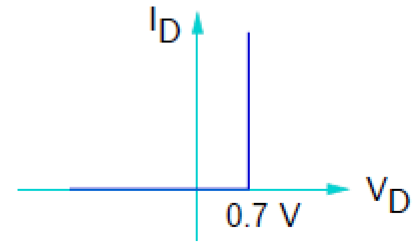


- The diode clamp shifts V_A up by V_m (the amplitude of the AC source), making V_B go from 0 to $2V_m$.
- The peak detector detects the peak of V_B ($2V_m$ w.r.t. ground), and holds it constant.
- Note that it takes a few cycles to reach steady state.

Backup

Example





(a) Plot V_o versus V_i for $-5\text{ V} < V_i < 5\text{ V}$.

- First, let us show that D_1 on → D_2 off, and D_2 on → D_1 off.
- Consider D_1 to be on → $V_{AB} = 0.7 + 1 + i_1 R_1$.
  - Note that $i_1 > 0$, since D_1 can only conduct in the forward direction.
  - $V_{AB} > 1.7$ V → D_2 cannot conduct.
- Similarly, if D_2 is on, $V_{BA} > 0.7$ V, i.e., $V_{AB} < -0.7$ V → D_1 cannot conduct.
- Clearly, D_1 on => D_2 off, and D_2 on => D_1 off.

Example

- For −0.7 V < V_i < 1.7 V, both D_1 and D_2 are off.
  - no drop across R, and $V_o = V_i$.
- For $V_i < -0.7$ V, D_2 conducts → $V_o = -0.7 - i_2R_2$

Use KVL to get i_2: $V_i + i_2R_2 + 0.7 + Ri_2 = 0$.

$$\rightarrow i_2 = -\frac{V_i + 0.7}{R + R_2}\text{, and}$$

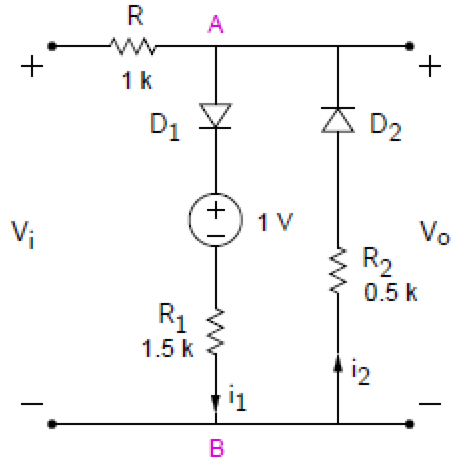
$$V_o = -0.7 - R_2 i_2 = \frac{R_2}{R + R_2} V_i - 0.7 \frac{R}{R + R_2} .$$

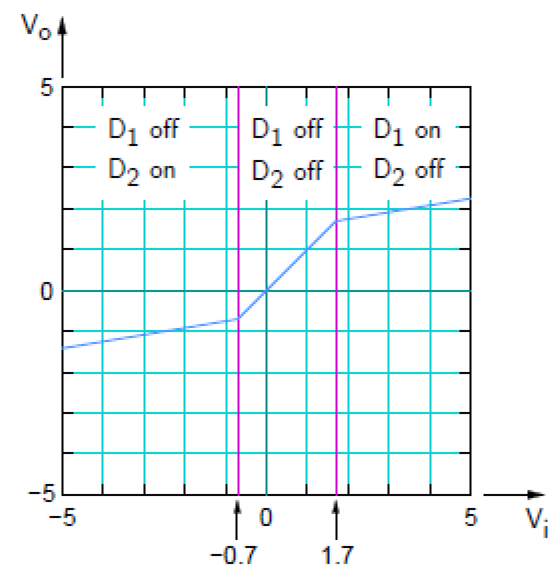
- For $V_i > 1.7$ V, D_1 conducts → $V_o = 0.7 + 1 + i_1R_1$

Use KVL to get i_1: $-V_i + i_1R + 0.7 + 1 + i_1R_1 = 0$.

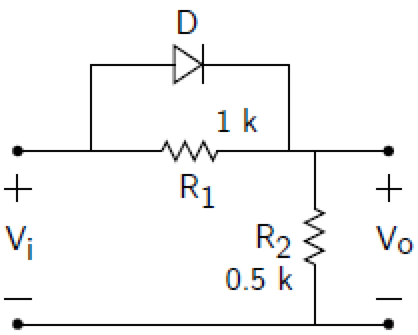
$$\rightarrow i_1 = \frac{V_i - 1.7}{R + R_1}\text{, and}$$

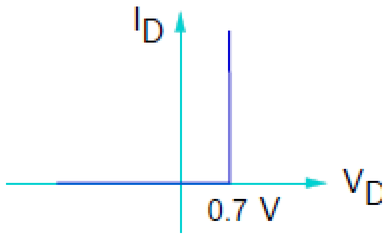
$$V_o = 1.7 + R_1 i_1 = \frac{R_1}{R + R_1} V_i + 1.7 \frac{R}{R + R_1} .$$



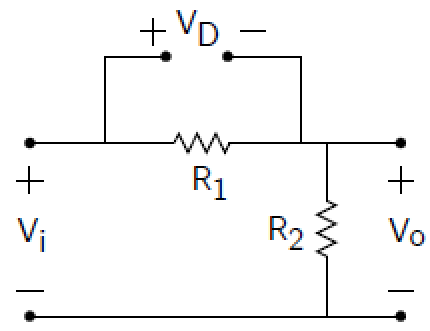


Example



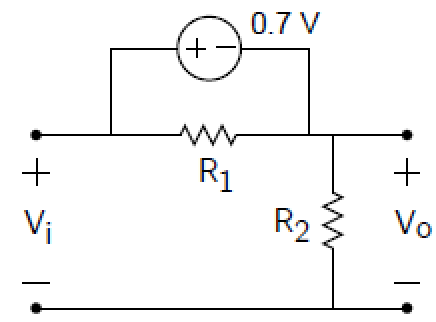


Plot V_o versus V_i for $-5\ V < V_i < 5\ V$.



D off

$$V_o = \frac{R_2}{R_1 + R_2} V_i$$



D on

$$V_o = V_i - 0.7$$

- At what value of V_i will the diode turn on?

In the off state, $V_D = \dfrac{R_1}{R_1 + R_2} V_i$.

For D to change to the on state, $V_D \models\ 0.7\ V$.

i.e., $V_i = \dfrac{R_1 + R_2}{R_1} \times 0.7 = 1.05\ V$.