BJT Amplifiers (Analog Circuits)

EE 101

S. Lodha

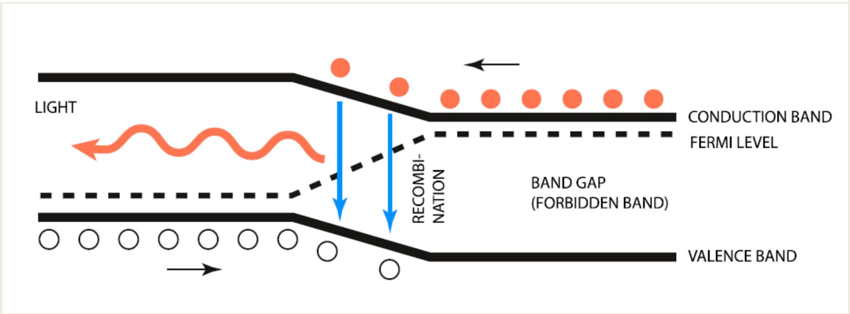
Reference material: L. Bobrow's Book

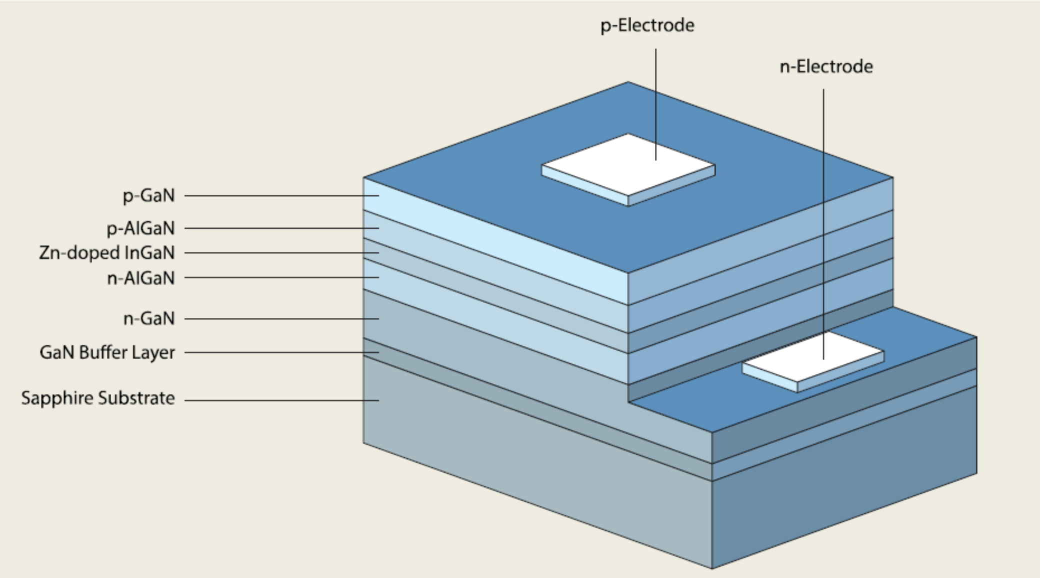
Nobel Prize in Physics 2014

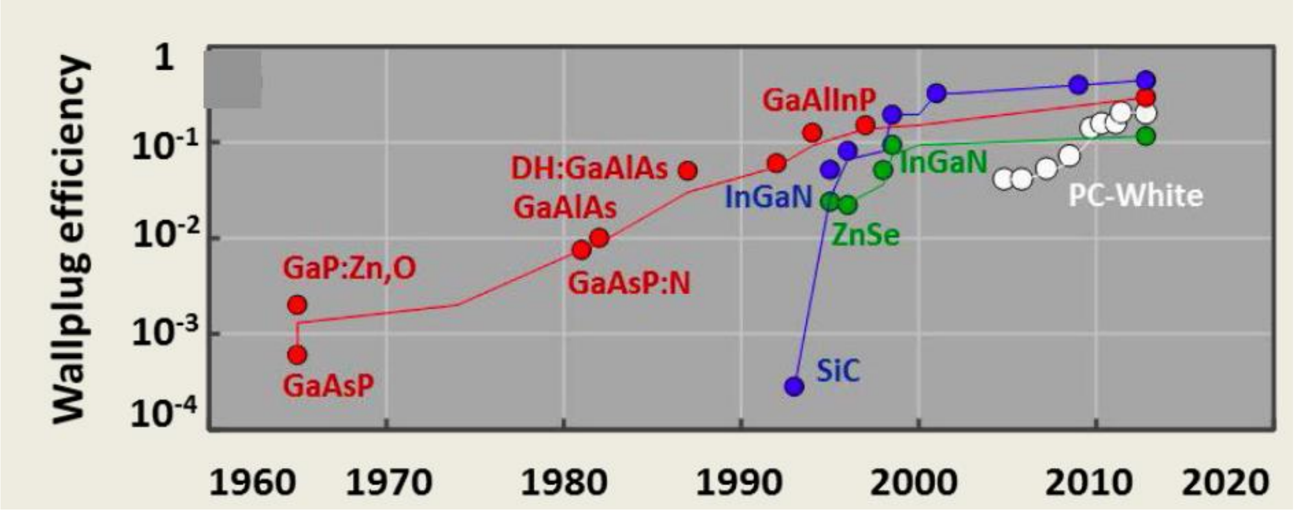
Isamu Akasaki, Hiroshi Amano and Shuji Nakamura

EFFICIENT BLUE LIGHT-EMITTING DIODES LEADING
TO BRIGHT AND ENERGY-SAVING WHITE LIGHT SOURCES

Blue LED


LIGHT
CONDUCTION BAND
FERMI LEVEL
RECOMBI-
NATION
BAND GAP
(FORBIDDEN BAND)
VALENCE BAND

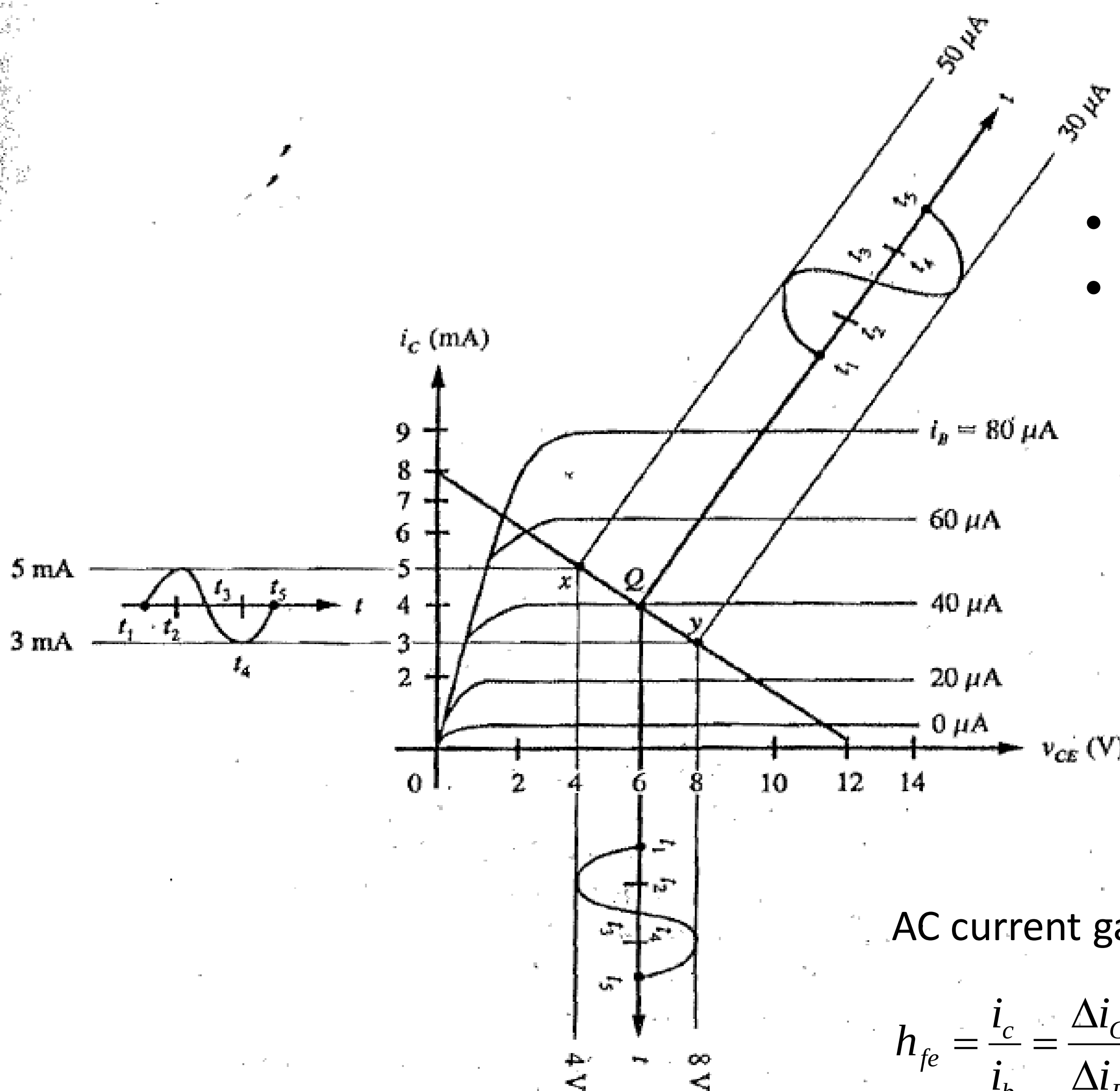

p-Electrode
n-Electrode
p-GaN
p-AlGaN
Zn-doped InGaN
n-AlGaN
n-GaN
GaN Buffer Layer
Sapphire Substrate


Wallplug efficiency
1
10-1
10-2
10-3
10-4
1960
1970
1980
1990
2000
2010
2020
GaAllnP
DH:GaAlAs
GaAlAs
GaAsP:N
GaP:Zn,O
GaAsP
InGaN
InGaN
ZnSe
SiC
PC-White

Impact

- Transformed entire lighting industry
  - 20-30% of electricity is used for lighting
  - Standard bulb → 16 lm/W (4% efficiency)
  - LED → 300 lm/W (50% efficiency)
    - 100,000 hour lifetime
- Other applications
  - Displays in electronics
  - Sensors etc.

BJT Amplifier: Collector Characteristics



- Active region operation
- Q → quiescent operating point

$$i_C = I_{CQ} + i_c = 4 + \sin \omega t \text{ mA}$$

$$i_B = I_{BQ} + i_b = 40 + 10 \sin \omega t \text{ μA}$$

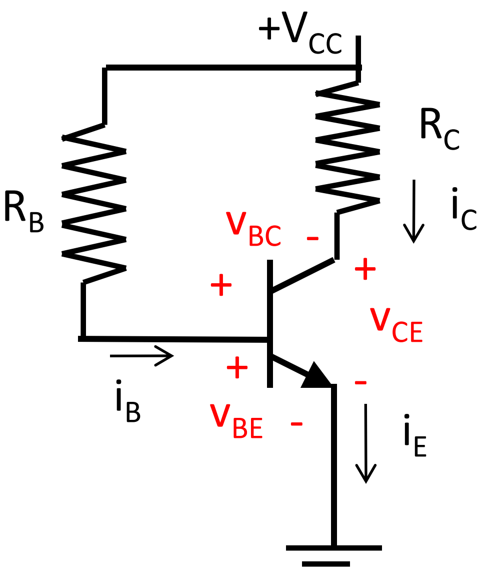
$$v_{CE} = V_{CE} + v_{ce} = 6 - 2 \sin \omega t \text{ V}$$

AC current gain $h_{fe}=\beta_{ac}$

$$h_{fe} = \frac{i_c}{i_b} = \frac{\Delta i_C}{\Delta i_B} = \frac{i_{Cx} - i_{Cy}}{i_{Bx} - i_{By}} = \frac{(5-3)\times 10^{-3}}{(50-30)\times 10^{-6}} = 100$$

DC biasing of BJT: Fixed Bias

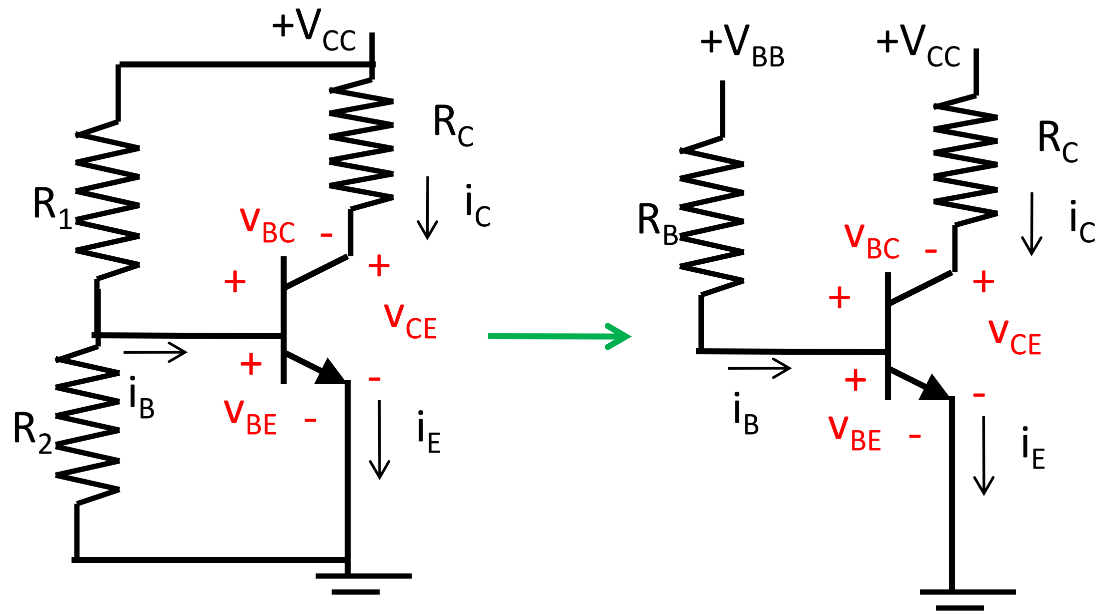
Example 1



$$i_B = \frac{V_{CC} - v_{BE}}{R_B} \qquad i_C = h_{FE} i_B$$

$$v_{CE} = -R_C h_{FE} i_B + V_{CC}$$

Example 2 (Voltage divider bias)



Thevenin Equivalent $\longrightarrow$

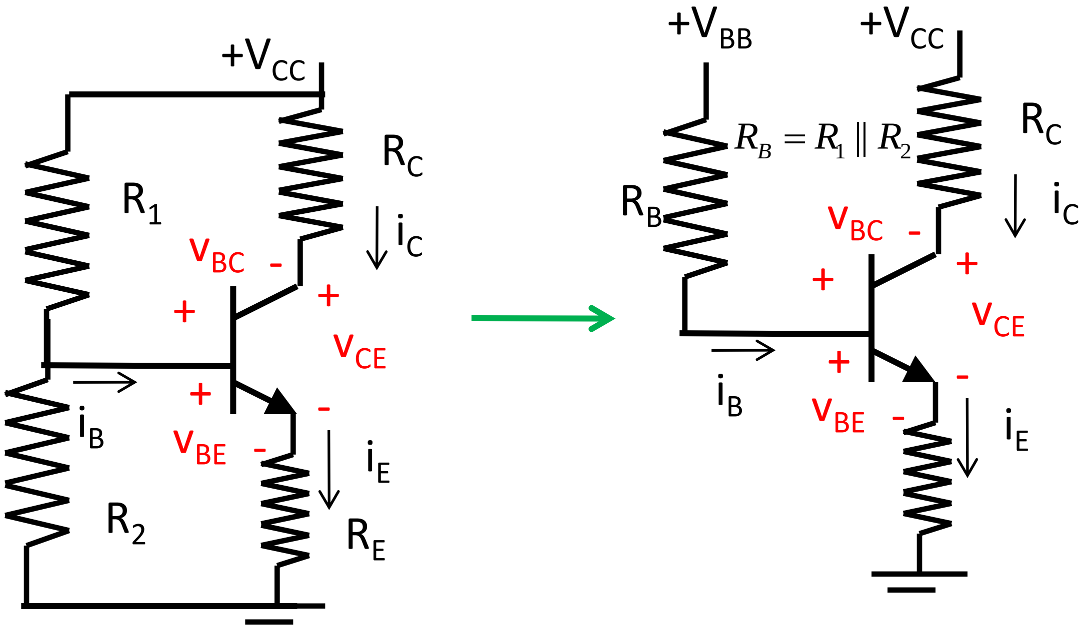
$$R_B = R_1 \parallel R_2$$

$$V_{BB} = V_{CC}(R_2 / R_1 + R_2)$$

- In fixed bias, variations (temperature, transistor-to-transistor) in h_{FE} can affect Q point
  - h_{FE} increases, i_C increases

DC biasing of BJT: Self-Bias

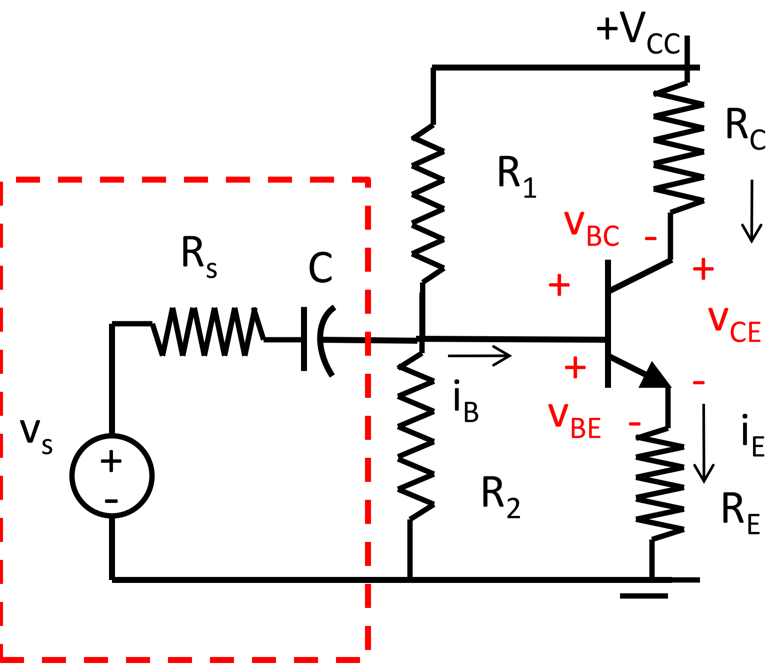
$$V_{BB} = V_{CC}(R_2 / R_1 + R_2)$$

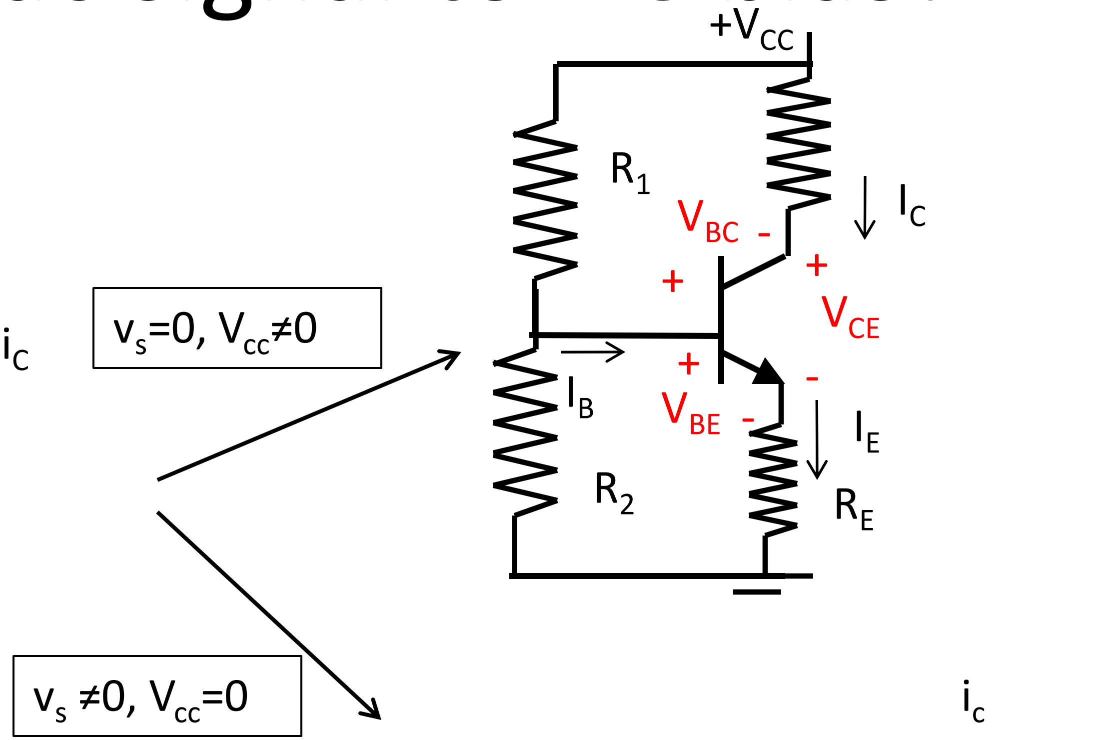


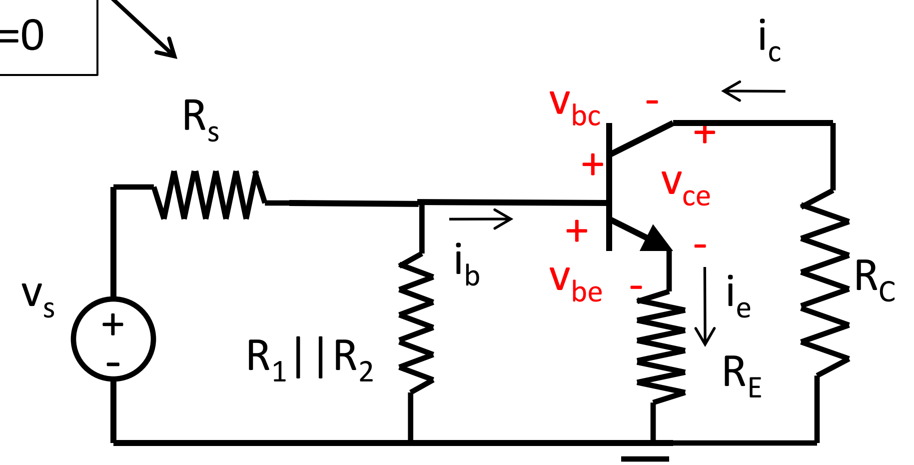
$$V_{BB} = R_B i_B + v_{BE} + R_E(i_B + i_C) \Rightarrow i_B = \frac{V_{BB} - v_{BE}}{R_B + (1 + h_{FE})R_E}$$

- If h_{FE} increases → i_C tends to increase but i_B decreases and tends to decrease i_C
- ***More stable*** Q point, also called self-biasing circuit

How to add ac signal to DC bias?



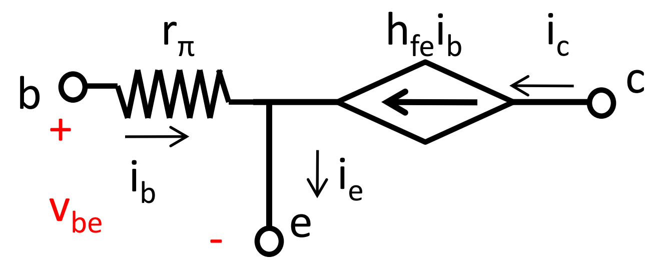


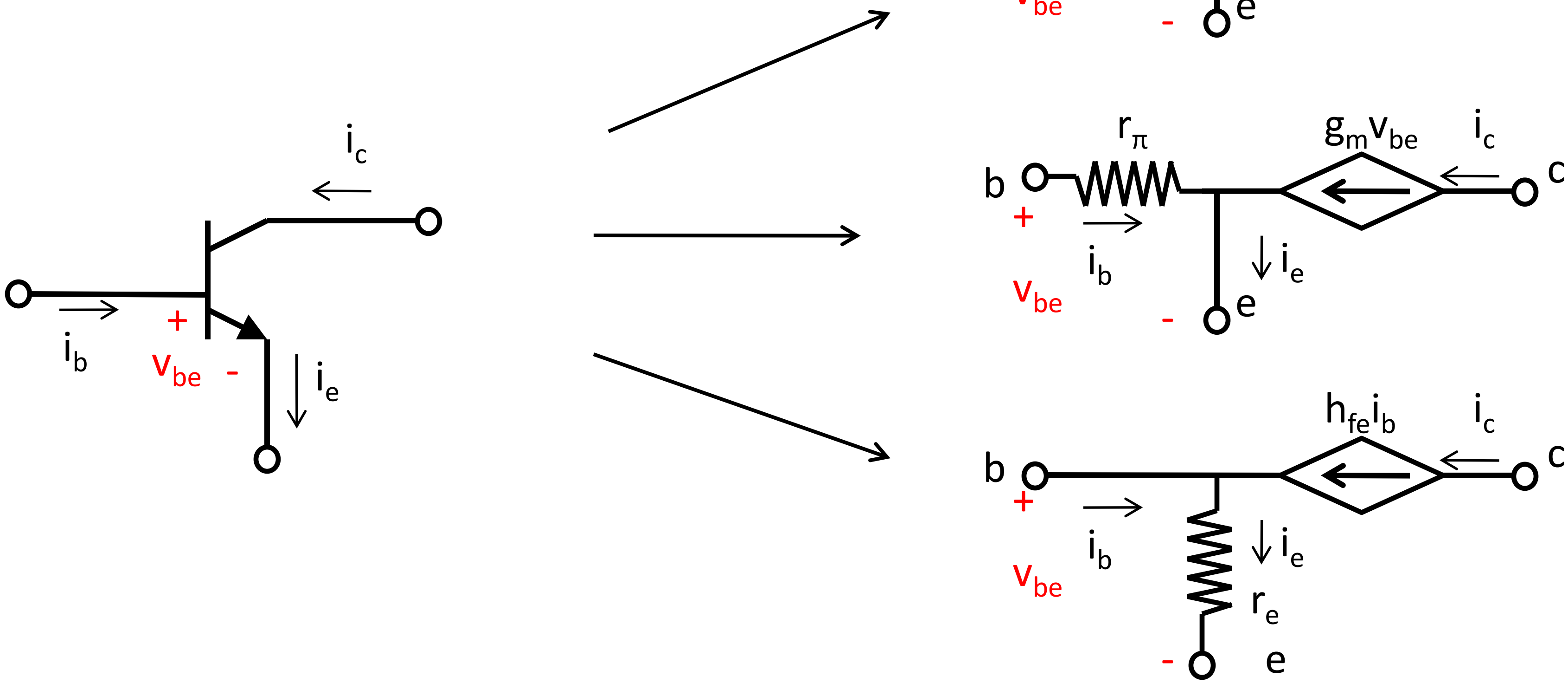


- Coupling capacitor C → open circuit for DC → no effect on dc Q point
- C large enough → short circuit for ac source v_s
- With active (linear) region biasing → use principle of superposition to analyse

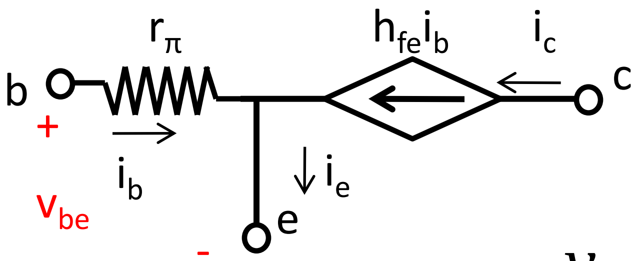
Small signal ac models

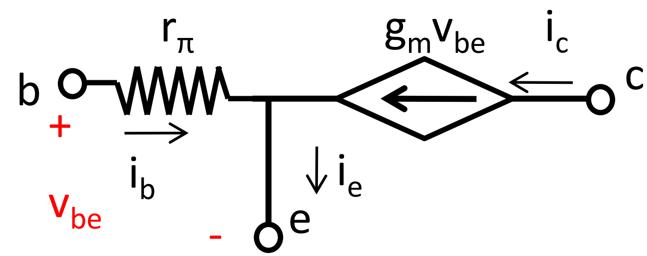
- Ac signal is small such that
  - transistor operates in active mode
  - Non-linear effects/distortions can be ignored
    - o/p characteristics are linear and parallel





Small signal model



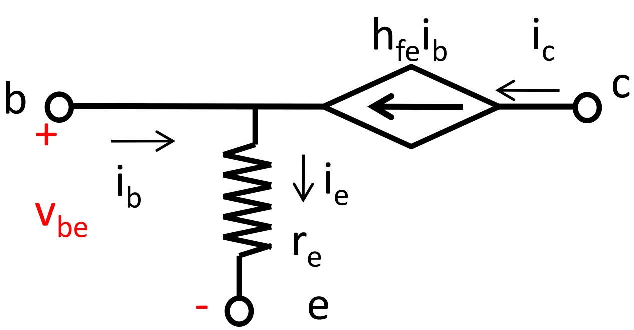


$$i_b = \frac{v_{be}}{r_\pi}$$

Ac base resistance= r_π

$$\boxed{h_{fe}i_b = h_{fe}\frac{v_{be}}{r_\pi} = g_m v_{be}}$$

Transconductance $g_m = h_{fe}/r_\pi$



$$r_e = \frac{v_{be}}{i_b + h_{fe}i_b} = \frac{v_{be}}{(1+h_{fe})i_b} = \frac{r_\pi}{1+h_{fe}} \approx \frac{r_\pi}{h_{fe}} = \frac{1}{g_m}$$

$$\boxed{r_\pi = (1+h_{fe})r_e \approx h_{fe}r_e}$$

g_m, r_π, r_e

For npn transistor in active mode

$$i_C = I_S(e^{v_{BE}/V_T} - 1) \approx I_S e^{v_{BE}/V_T}$$

$$di_C = d[I_S(e^{v_{BE}/V_T} - 1)] = \frac{1}{V_T}(I_S e^{v_{BE}/V_T})dv_{BE} \approx \frac{i_C}{V_T}dv_{BE}$$

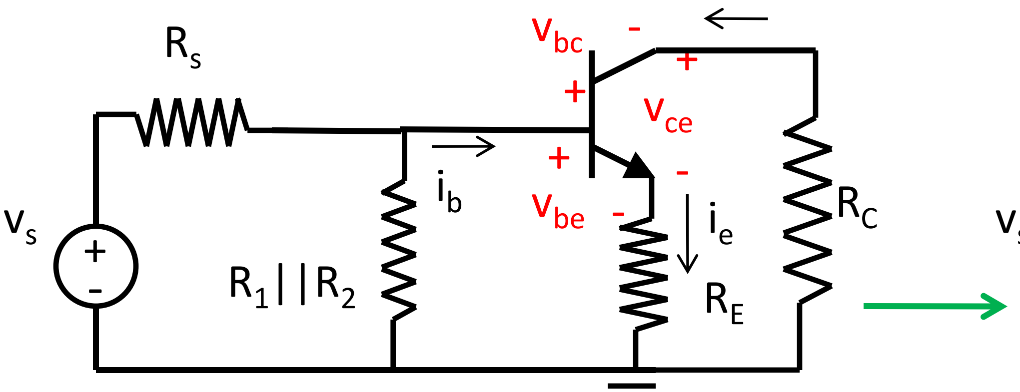
$$g_m = \frac{i_c}{v_{be}} = \frac{di_C}{dv_{BE}} = \frac{i_C}{V_T}$$

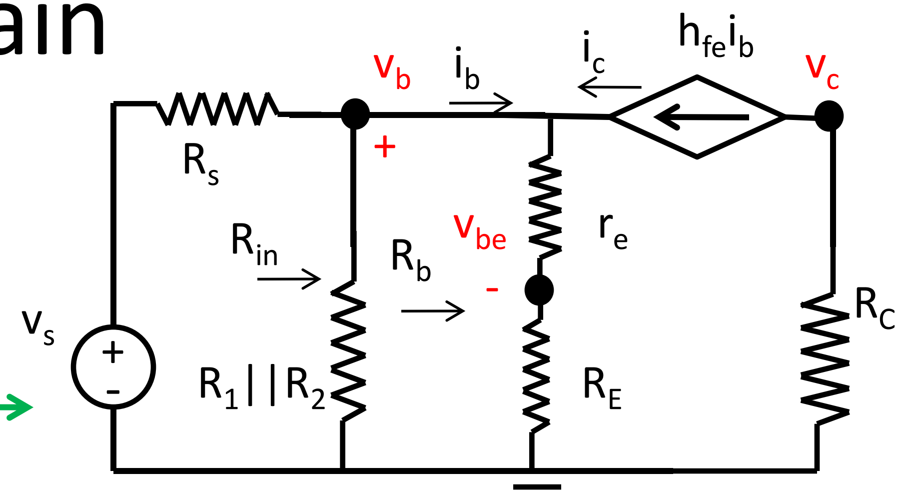
Biased at $i_C = I_{CQ}$,

$$\boxed{\begin{aligned} g_m &= \frac{I_{CQ}}{V_T} \\ r_e &= \frac{1}{g_m} = \frac{V_T}{I_{CQ}} \\ r_\pi &= h_{fe} r_e = \frac{h_{fe} V_T}{I_{CQ}} \end{aligned}}$$

At room temperature, $V_T = kT/q = 0.026$ V

CE Amplifier: AC gain





$$v_b = r_e(i_b + h_{fe} i_b) + R_E(i_b + h_{fe} i_b)$$

$$\Rightarrow R_b = \frac{v_b}{i_b} = (1 + h_{fe})(r_e + R_E) \approx h_{fe}(r_e + R_E)$$

R_b is the resistance between base and reference looking into the transistor

$$R_{in} = (R_1 \parallel R_2) \parallel R_b$$

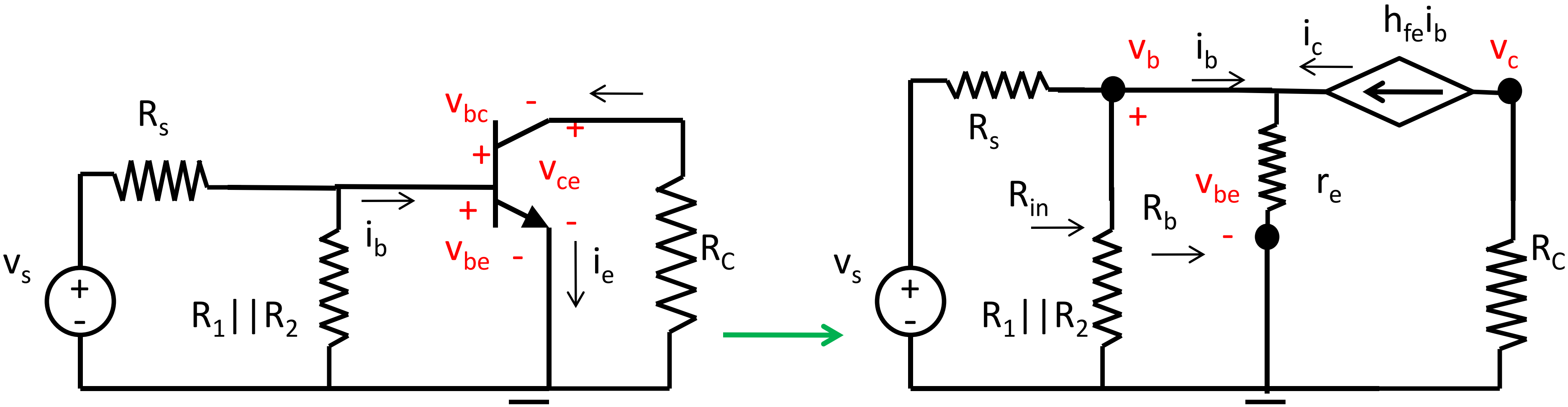
R_{in} is the resistance between base and reference looking into the amplifier → ac input resistance

$$A_v = \frac{v_c}{v_b} = \frac{-R_C h_{fe} i_b}{R_b i_b} = \frac{-R_C h_{fe}}{(1 + h_{fe})(r_e + R_E)} \approx \frac{-R_C}{r_e + R_E}$$

ac voltage gain A_v, v_c is 180 degress out of phase w.r.t. v_b

- r_e is determined by Q point
- R_E can be decreased to increase gain, but will compromise stability

CE Amplifier: Bypass capacitor



A bypass capacitor added in parallel to R_E with negligible impedance w.r.t R_E

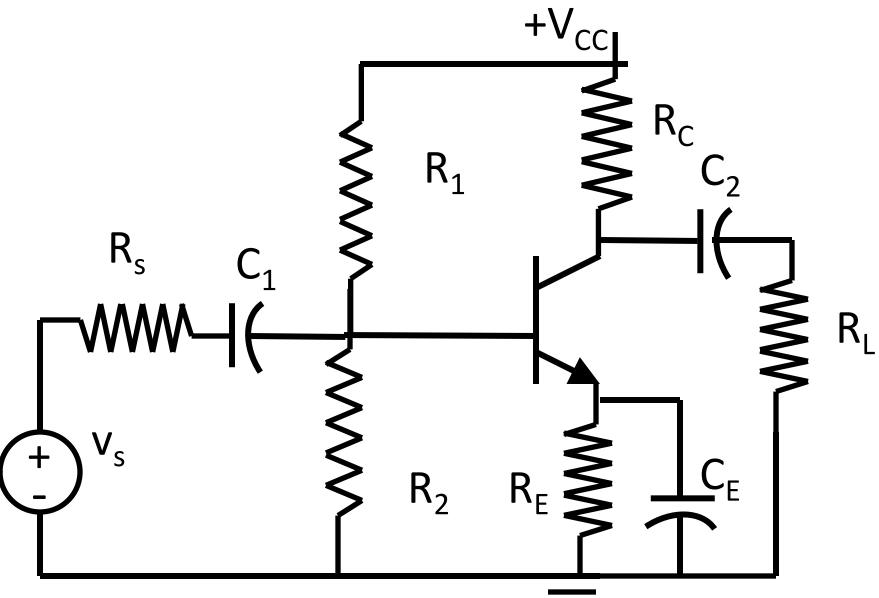
-Does not affect dc conditions (open circuit, r_e unchanged)

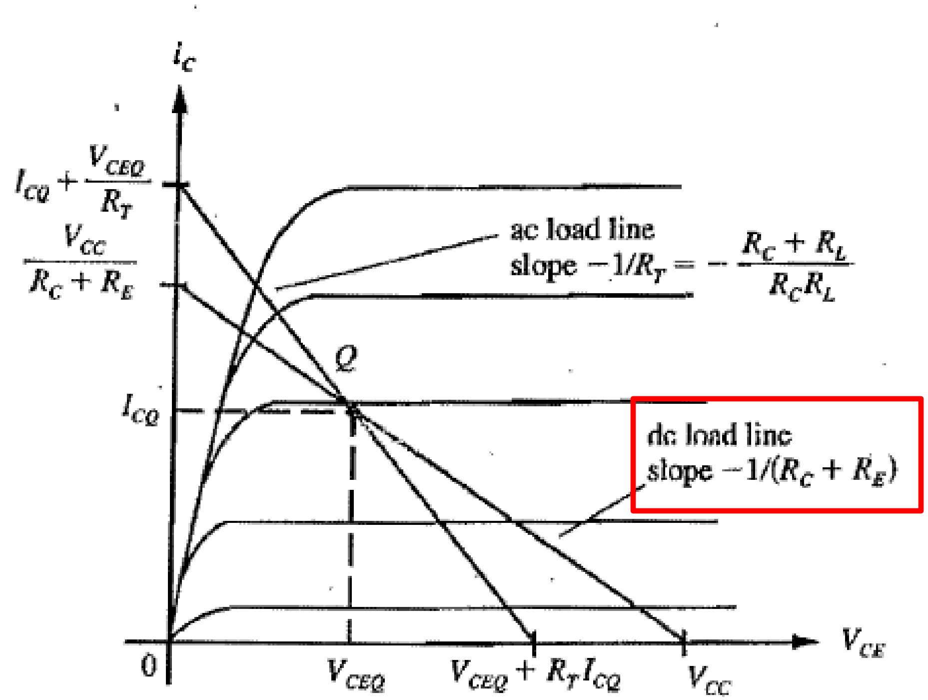
-Short circuit for ac

$$A_v = \frac{v_c}{v_b} = \frac{-R_C h_{fe}}{(1+h_{fe})r_e} \approx \frac{-R_C}{r_e}$$

$$A_{vs} = \frac{v_c}{v_s} = \frac{v_c}{v_b}\frac{v_b}{v_s} = \frac{R_{in}}{R_{in}+R_s}A_v$$ ac voltage gain A_{vs} from source to output

CE Amplifier with load



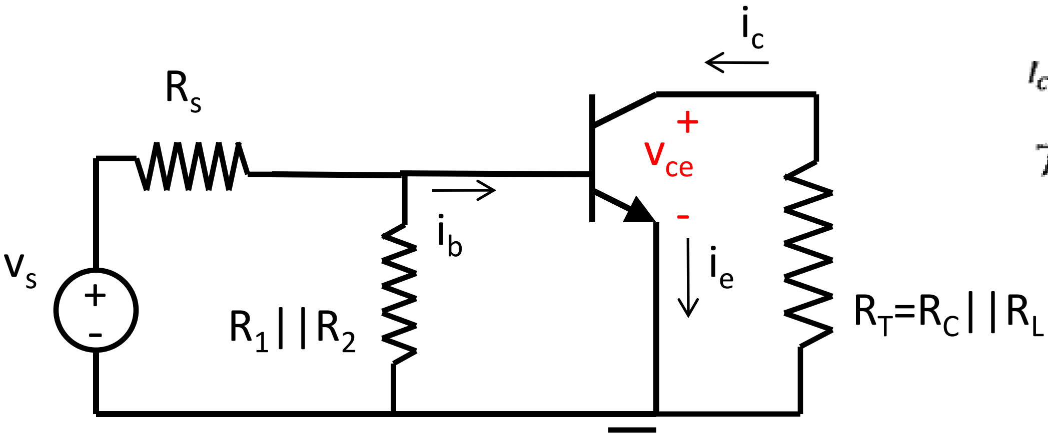


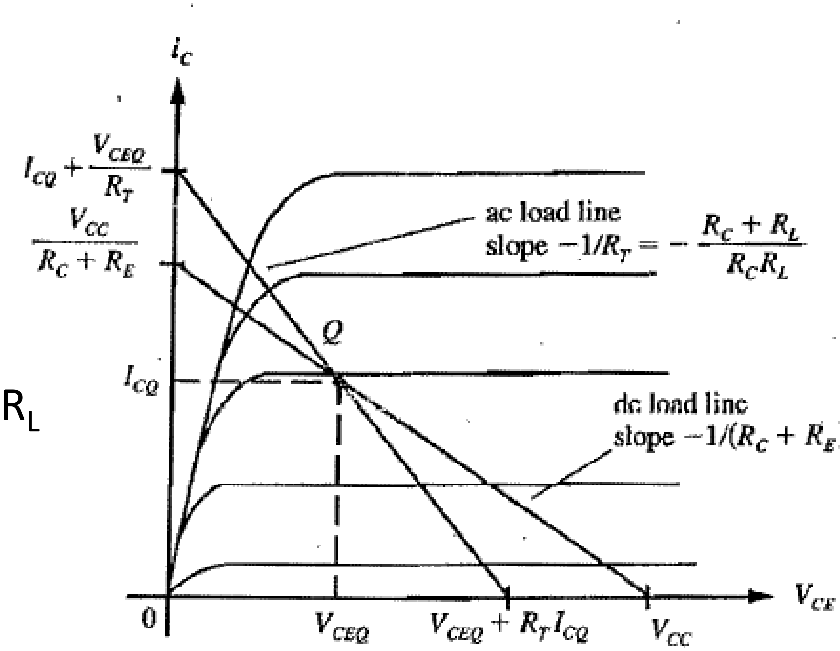
$$V_{CC} = R_C i_C + v_{CE} + R_E(i_B + i_C) \approx R_C i_C + v_{CE} + R_E i_C$$

$$\Rightarrow \boxed{i_C = -\frac{1}{R_C + R_E} v_{CE} + \frac{V_{CC}}{R_C + R_E}}$$

DC load line

AC load line



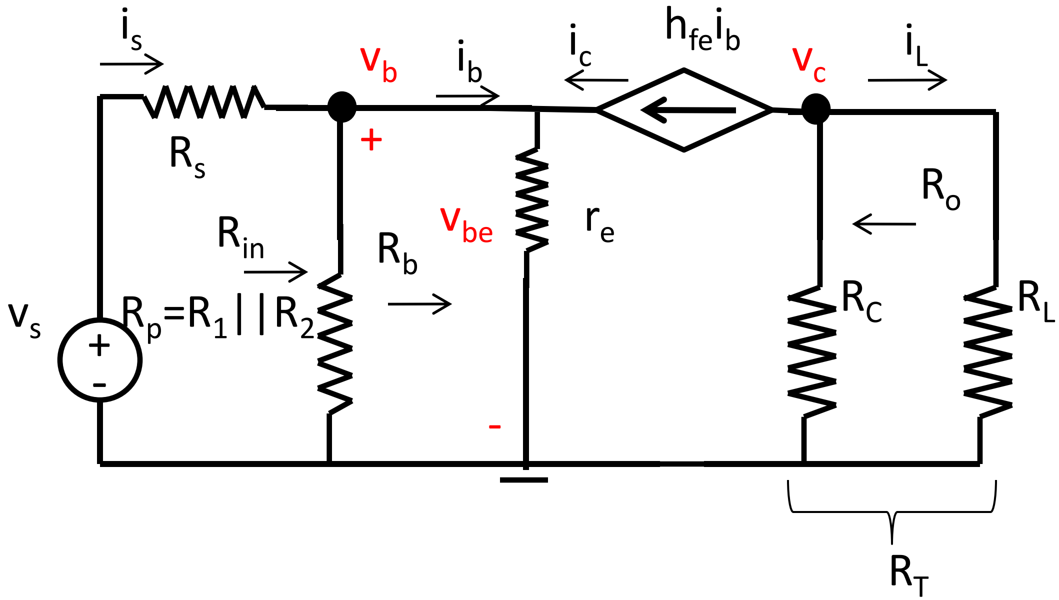


$$v_{CE} = V_{CEQ} + v_{ce} \qquad i_C = I_{CQ} + i_c \Rightarrow i_c = i_C - I_{CQ}$$

$$v_{ce} = -R_T i_c \qquad v_{CE} = V_{CEQ} - R_T(i_C - I_{CQ}) \Rightarrow \boxed{i_C = -\frac{1}{R_T} v_{CE} + (I_{CQ} + \frac{V_{CEQ}}{R_T})}$$

AC load line

Current gain



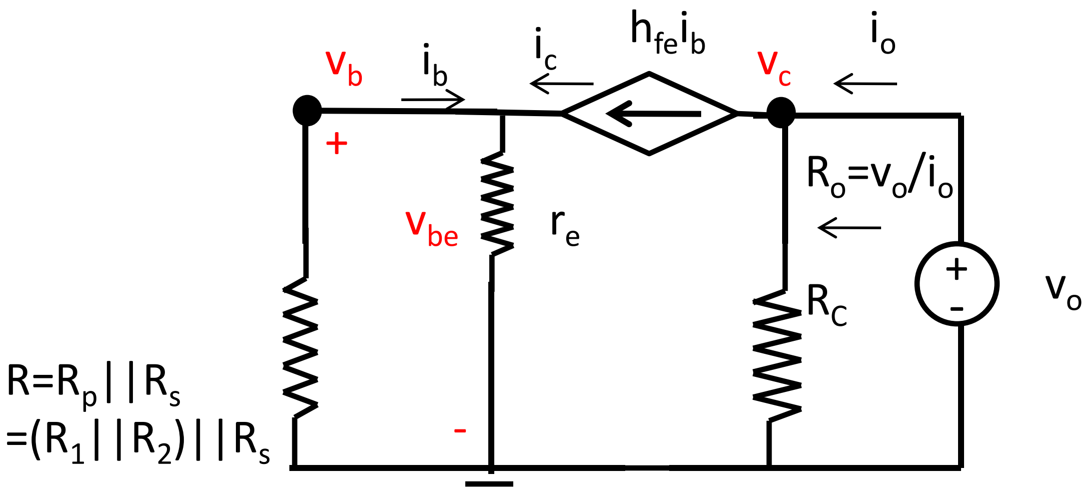
$$R_b = (1+h_{fe})r_e \approx h_{fe}r_e \qquad R_{in} = (R_1 \parallel R_2) \parallel R_b$$

$$A_v = -\frac{h_{fe}R_T}{(1+h_{fe})r_e} \approx -\frac{R_T}{r_e} \qquad A_i = \frac{i_L}{i_b} = \frac{-h_{fe}R_C}{R_C+R_L}$$

AC current gain

$$A_{is} = \frac{i_L}{i_s} = \frac{i_L}{i_b}\frac{i_b}{i_s} = \frac{A_i R_p}{R_p+R_b}$$

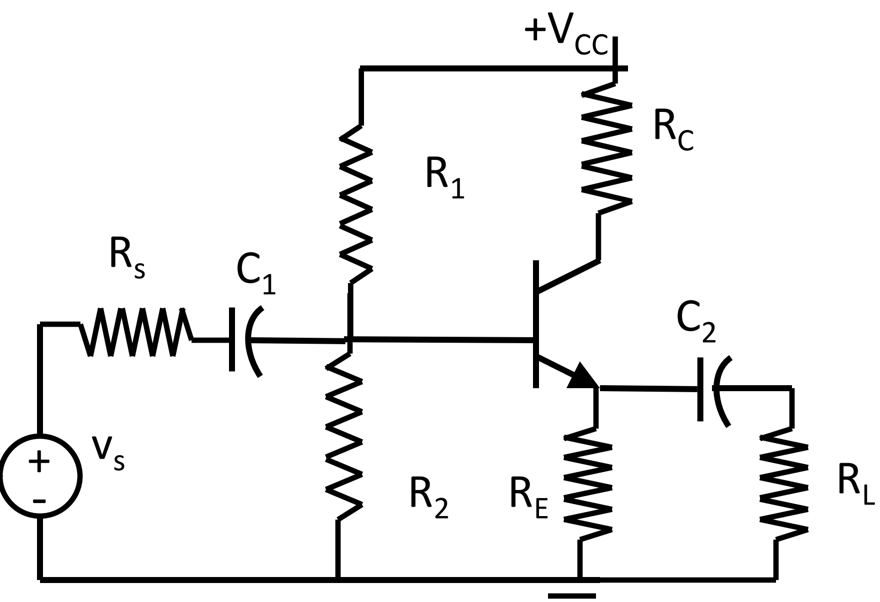
Output Resistance: R_o



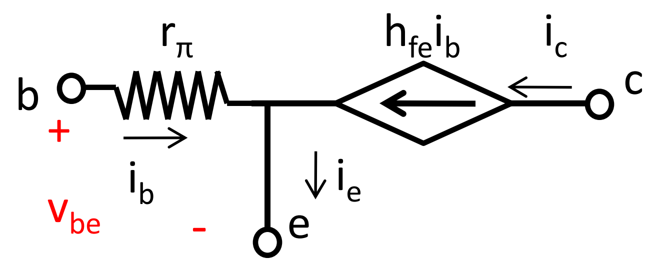
$$i_b + h_{fe}i_b = \frac{v_b}{r_e} \Rightarrow i_b = \frac{v_b}{(1+h_{fe})r_e} = \frac{-Ri_b}{(1+h_{fe})r_e} \Rightarrow i_b = 0$$

$$i_o = h_{fe}i_b + \frac{v_o}{R_C} = \frac{v_o}{R_o} \Rightarrow \boxed{R_o = R_C}$$

Example: Emitter Follower (common collector amplifier)



For $R_T = R_E || R_L$, and using,



Show that,

1. $$A_v = \frac{v_e}{v_b} \approx \frac{R_T}{r_e + R_T}$$ If $R_T >> r_e$, $A_v=1$, $v_e=v_b$, **emitter "follows" base voltage with unity gain**

2. $$R_{in} = R_p \| R_b$$ For large h_{fe}, R_T, R_1 and R_2, **R_{in} can be made large**

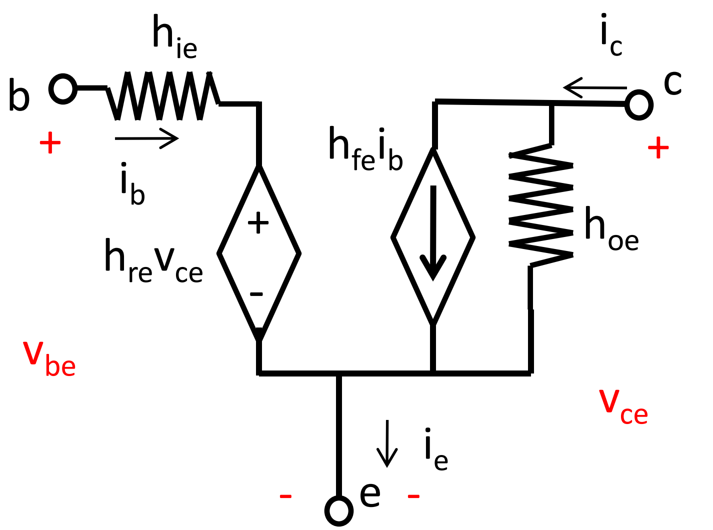
$$R_b = \frac{v_b}{i_b} = r_\pi + (1+h_{fe})R_T \approx r_\pi + h_{fe}R_T$$

3. $$R_o = R_E \| \left(\frac{r_\pi + R}{1+h_{fe}}\right)$$ For large h_{fe}, **R_{out} can be made small**

$$R = R_s \| (R_1 \| R_2)$$

Buffer or isolation amplifier

Accurate Hybrid *(h)* parameter model



$$h_{ie} = \left.\frac{v_{be}}{i_b}\right|_{v_{ce}=0}$$

Short circuit input resistance

$$h_{re} = \left.\frac{v_{be}}{v_{ce}}\right|_{i_b=0}$$

open circuit reverse voltage gain

$$h_{oe} = \left.\frac{i_c}{v_{ce}}\right|_{i_b=0}$$

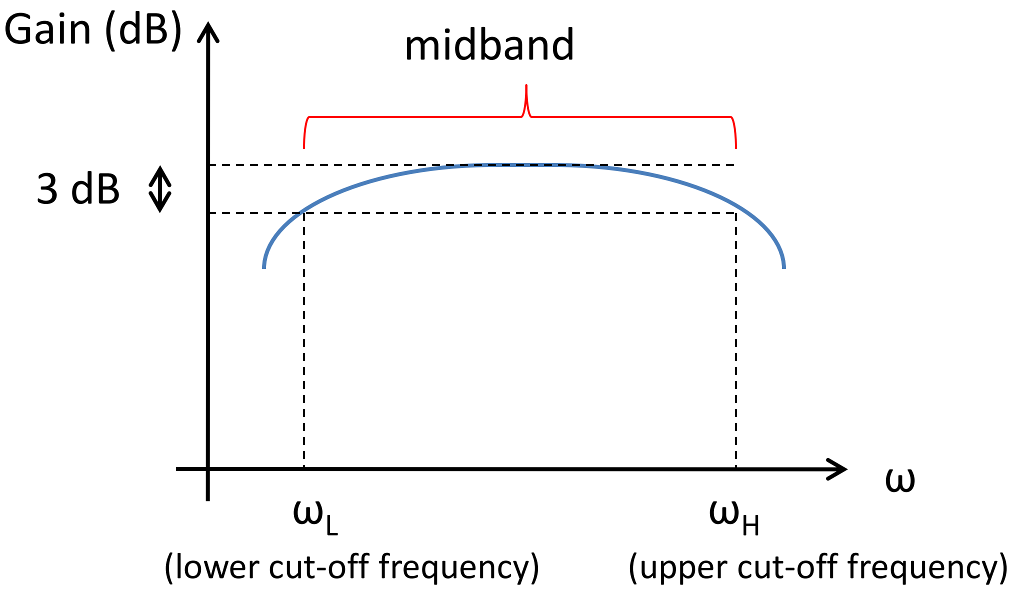
Open circuit output conductance

$$h_{fe} = \left.\frac{i_c}{i_b}\right|_{v_{ce}=0}$$

short circuit forward current gain

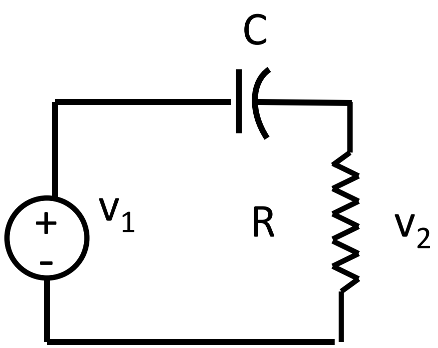
Typical values, h_{ie}=1.5kΩ, $h_{re}=10^{-4}$, h_{fe}=100, $h_{oe}=10^{-5}\ \Omega^{-1}$

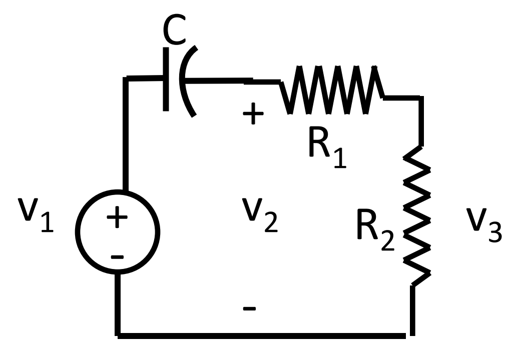
Frequency response



- So far we have ignored capacitive effects, in general gain is frequency dependent
- $\omega_L \rightarrow$ coupling and bypass capacitors
- $\omega_H \rightarrow$ determined by internal capacitances of the BJT

Recall: High pass filters





$$V_2 = \frac{j\omega RC}{1+j\omega RC} V_1$$

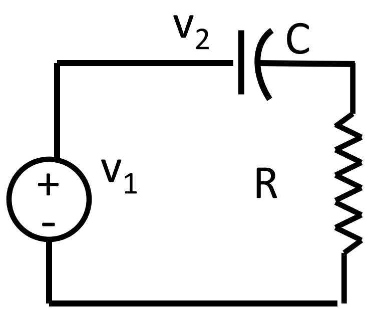
$$ang(V_2) = 90^0 - \tan^{-1}(\omega RC) + ang(V_1)$$

$$0 < \tan^{-1}(\omega RC) < 90^0 \Rightarrow ang(V_2) > ang(V_1)$$

V_2 leads V_1 → lead network
High pass filter, ω_c=1/RC

V_3 leads V_1 → also lead network
High pass filter, ω_c=1/(R_1+R_2)C

Recall: Low pass filter



$$V_2 = \frac{1}{1 + j\omega RC} V_1$$

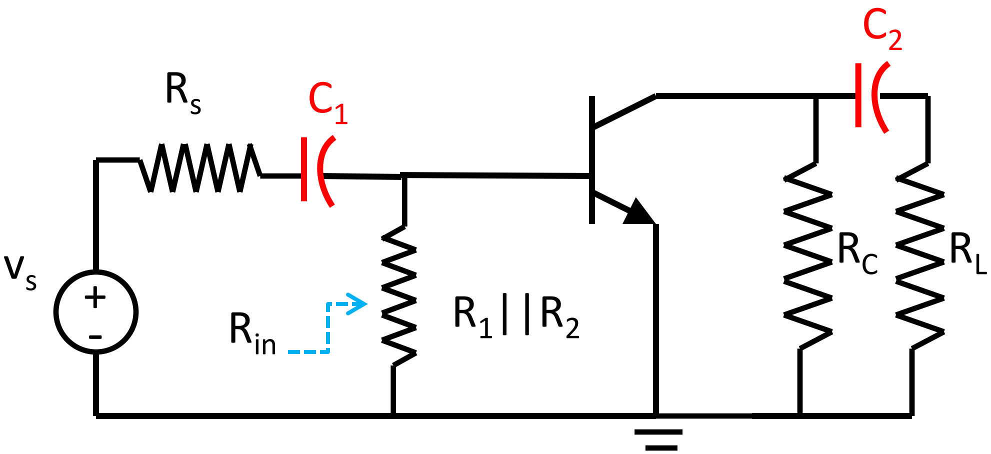
$$ang(V_2) = ang(V_1) - \tan^{-1}(\omega RC)$$

$$0 < \tan^{-1}(\omega RC) < 90^0 \Rightarrow ang(V_2) < ang(V_1)$$

V_2 lags V_1 → lag network

Low pass filter, ω_c=1/RC

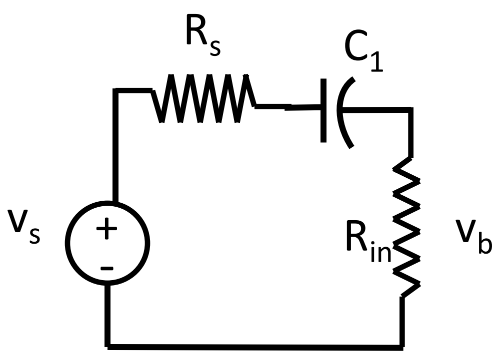
CE amplifier: Effect of coupling capacitor C_1



Ignoring, C_2

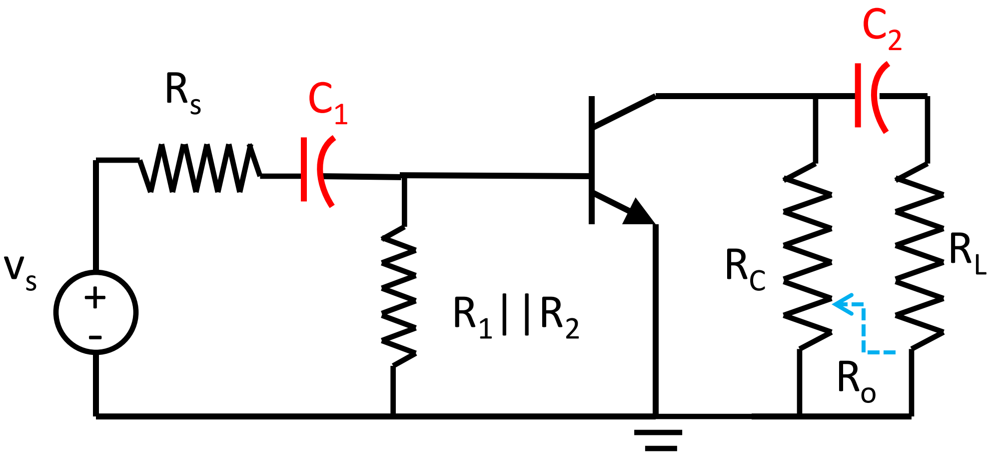
$$\boxed{\omega_1 = \frac{1}{(R_s + R_{in})C_1}}$$

$$R_{in} = (R_1 \parallel R_2) \parallel R_b \qquad R_b = (1 + h_{fe})(r_e)$$



Base-equivalent lead network

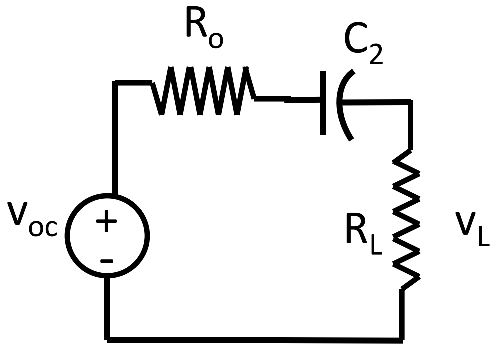
CE amplifier: Effect of coupling capacitor C_2



Ignoring, C_1

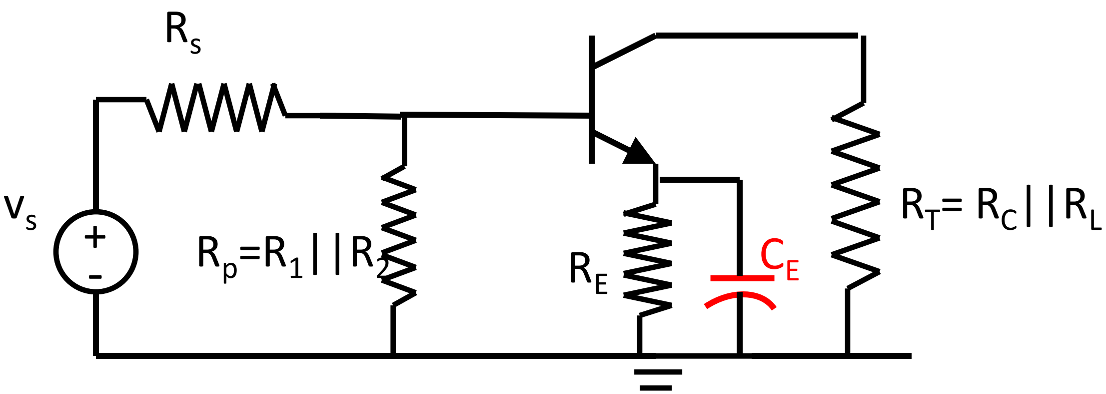
$$\omega_2 = \frac{1}{(R_o + R_L)C_2}$$

$$R_o = R_C$$



Collector-equivalent lead network

CE amplifier: Effect of bypass capacitor C_E

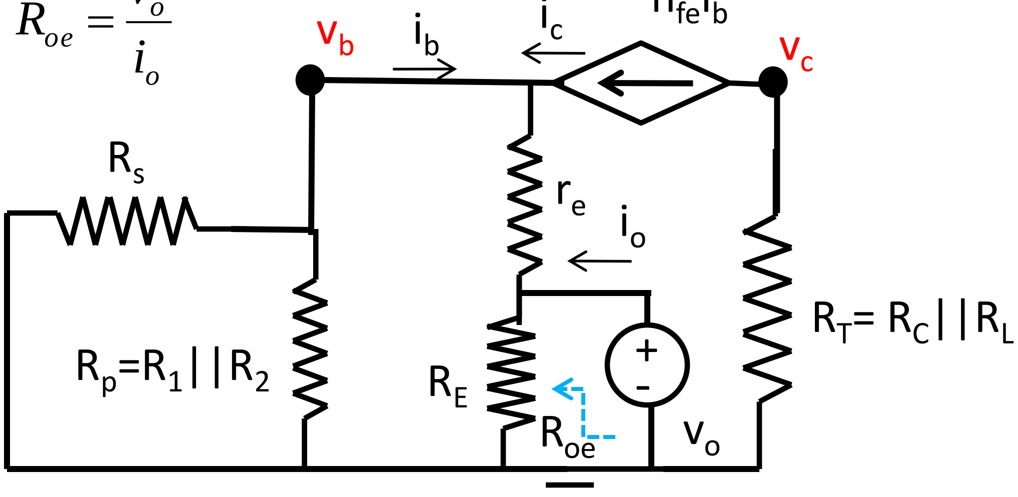


KCL at v_b

$$\frac{v_b}{R}+\frac{v_b-v_o}{r_e}=h_{fe}i_b=\frac{h_{fe}(v_b-v_o)}{(1+h_{fe})r_e}$$

$$\Rightarrow v_b=\frac{R/(1+h_{fe})}{r_e+R/(1+h_{fe})}v_o$$

$$R_{oe}=\frac{v_o}{i_o}$$



Circuit to determine R_{oe} in series with C_E

Substituting this for v_b in,

$$i_o=\frac{v_o}{R_E}+\frac{v_o-v_b}{r_e}$$

$$R_{oe}=\frac{v_o}{i_o}=R_E\parallel(r_e+\frac{R}{1+h_{fe}})$$

$$\boxed{\omega_3=\frac{1}{R_{oe}C_E}}$$

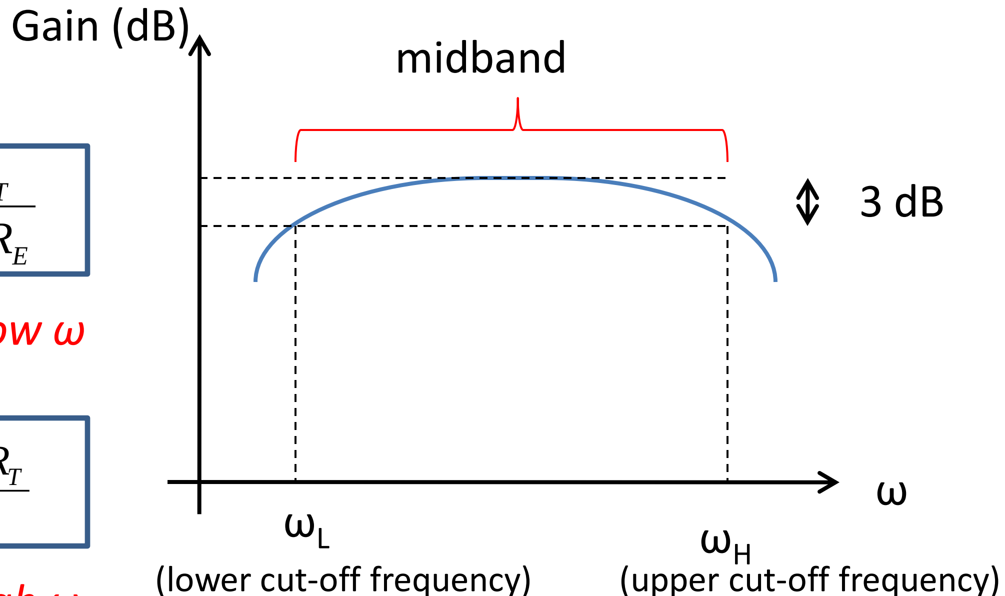
Frequency response

$$A_v = \frac{v_c}{v_b} = \frac{-R_T}{r_e + R_E}$$

Low ω

$$A_v = \frac{v_c}{v_b} = \frac{-R_T}{r_e}$$

High ω



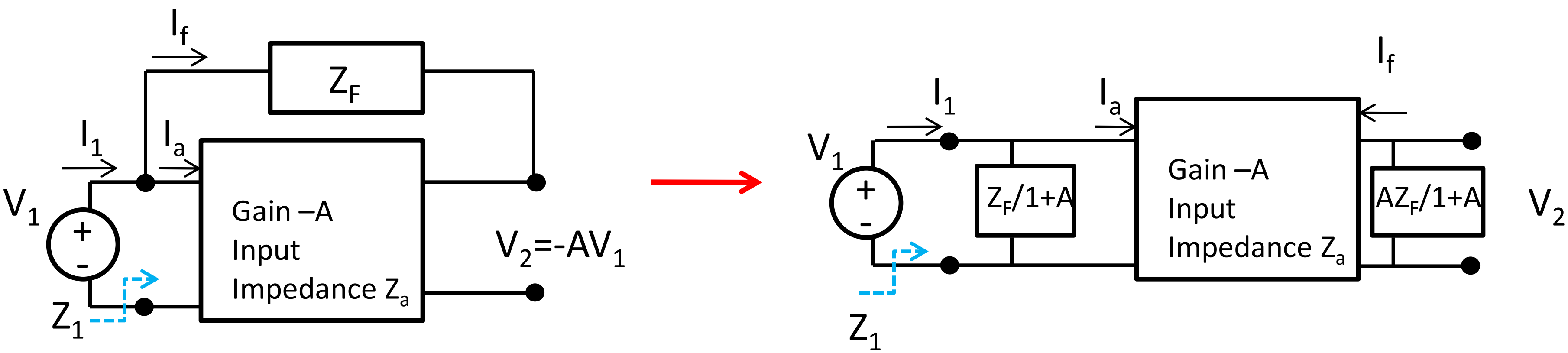
$$\omega_1 = \frac{1}{(R_s + R_{in})C_1}$$

$$\omega_2 = \frac{1}{(R_o + R_L)C_2}$$

$$\omega_3 = \frac{1}{R_{oe}C_E}$$

- The lower cut-off frequency is determined by which has the higher value amongst ω_1,ω_2,$\boldsymbol{\omega_3}$
- Generally ω_3 is the largest of all three
  - At high ω, C_E behaves as a short circuit and gain increases
  - At low ω, C_E acts less and less as a short circuit and gain decreases

Miller's theorem



$$I_1 = I_a + I_f = \frac{V_1}{Z_a} + \frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_a} + \frac{(1+A)V_1}{Z_F}$$

$$\Rightarrow \frac{I_1}{V_1} = \frac{1}{Z_a} + \frac{(1+A)}{Z_F}$$

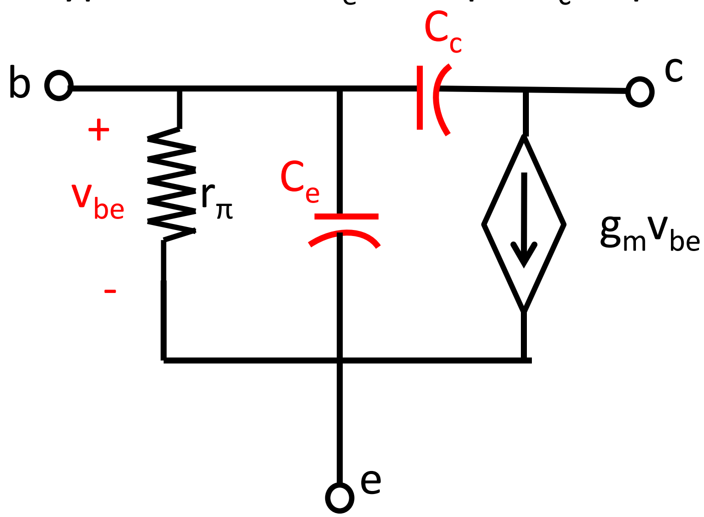
$$\boxed{\Rightarrow Z_1 = Z_a \parallel \frac{Z_F}{1+A}}$$

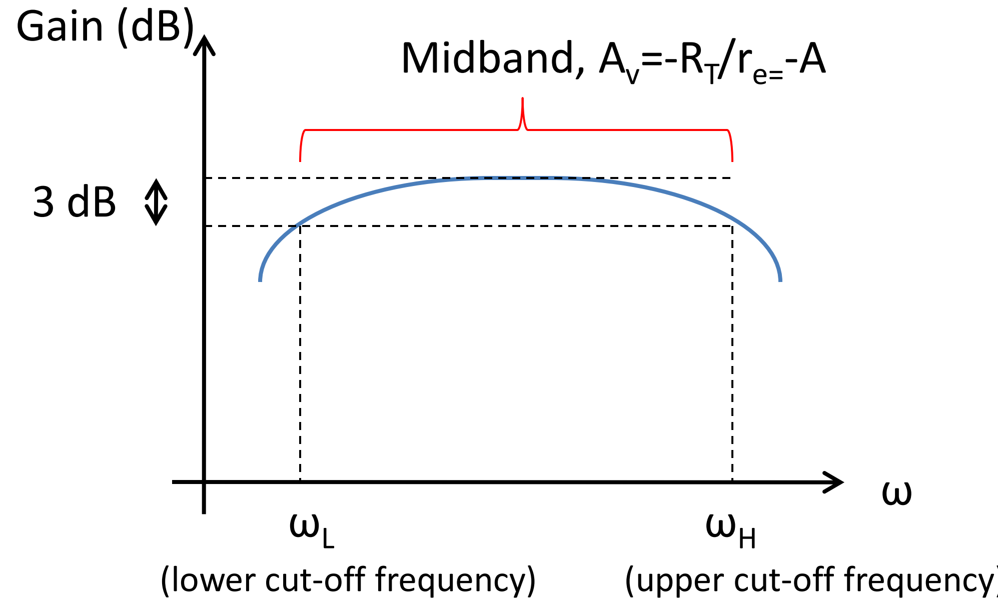
$$\boxed{I_f = \frac{V_1 - V_2}{Z_F} = \frac{(-V_2 / A) - V_2}{Z_F} = \frac{V_2}{AZ_F /(1+A)}}$$

Miller's theorem → reflects feedback impedance across to input and output

High frequency Response

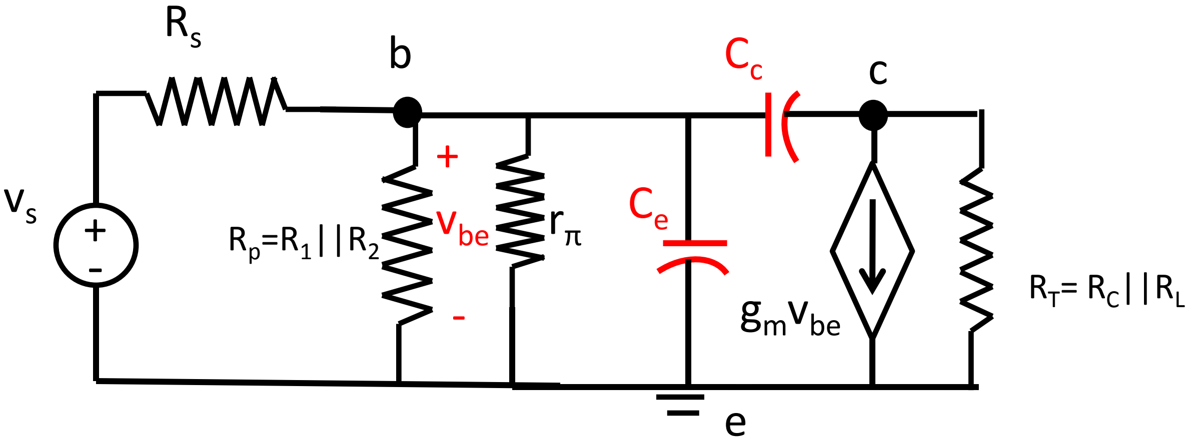
Typical values C_e=100 pF, C_c=5 pF





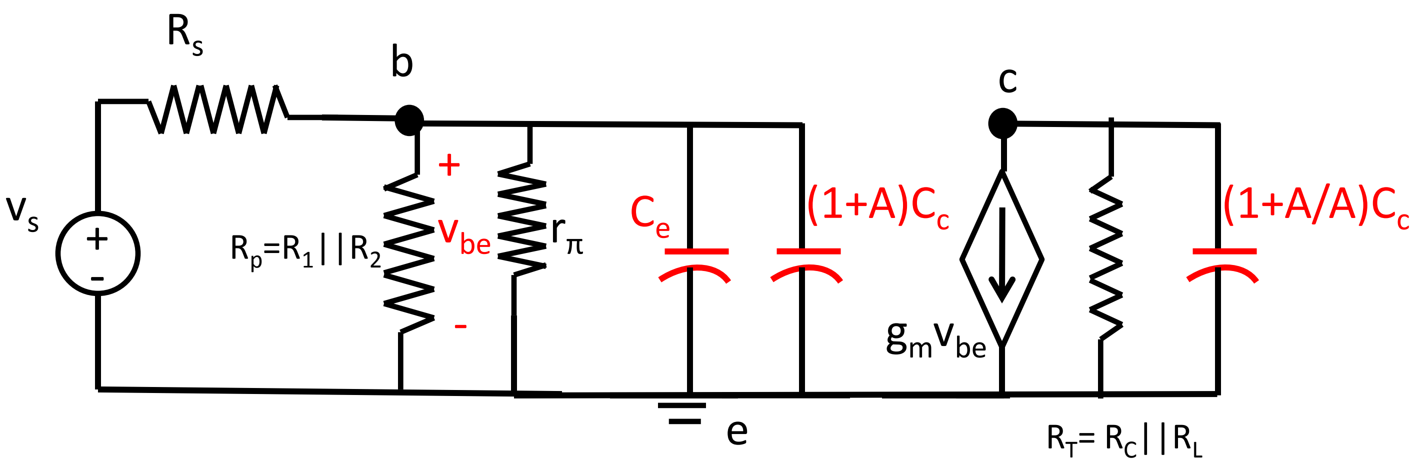
- At low ω, C_1, C_2 and C_E need to be accounted for
- At mid ω, C_1, C_2 and C_E are short circuits and C_c and C_e are open, A_v=-R_T/r_e
- At high ω, C_e and C_c need to be accounted for

High frequency response



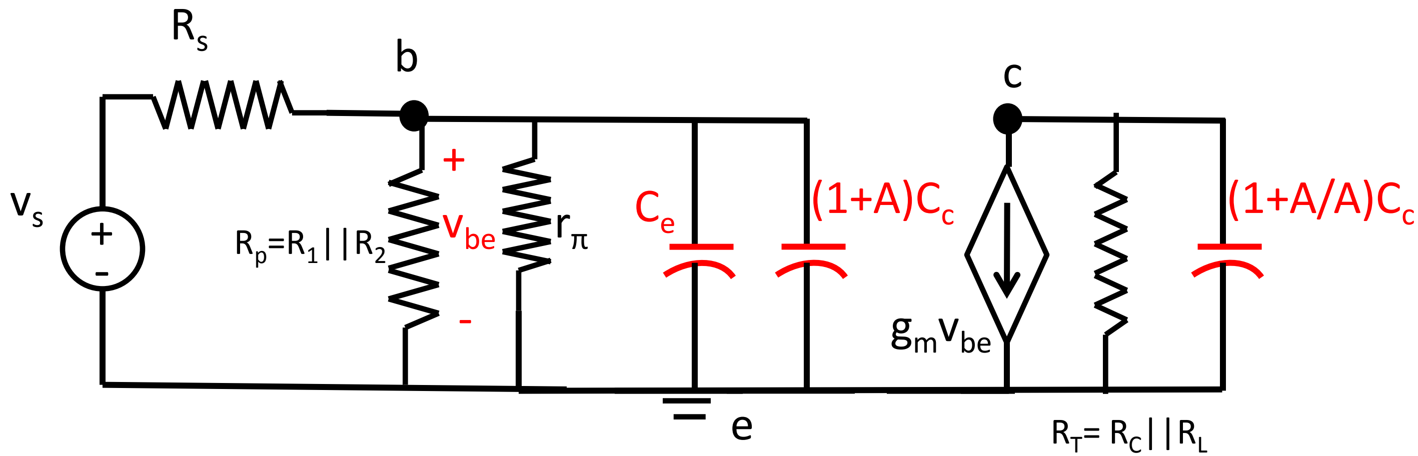
$$\frac{Z_F}{1+A}=\frac{1}{j\omega C_c(1+A)}$$ **Input**

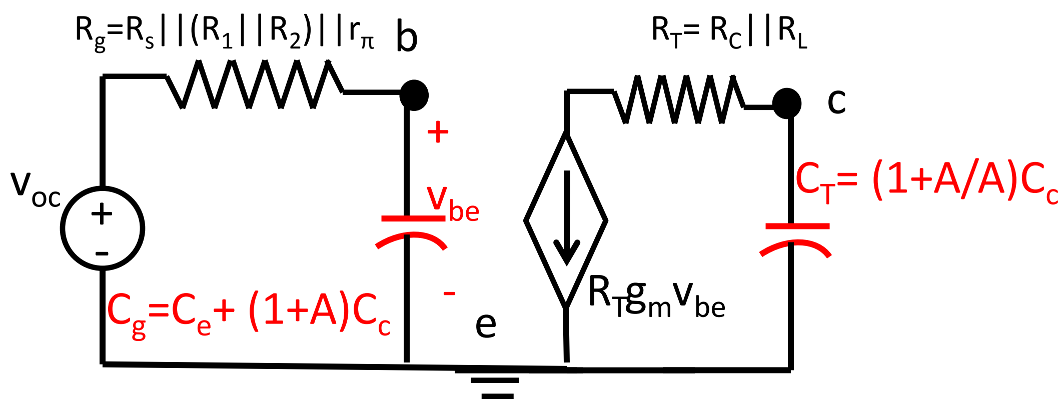
$$\frac{AZ_F}{1+A}=\frac{1}{j\omega C_c(1+A)/A}$$ **Output**



- C_c can be reflected using Miller's theorem

High frequency response



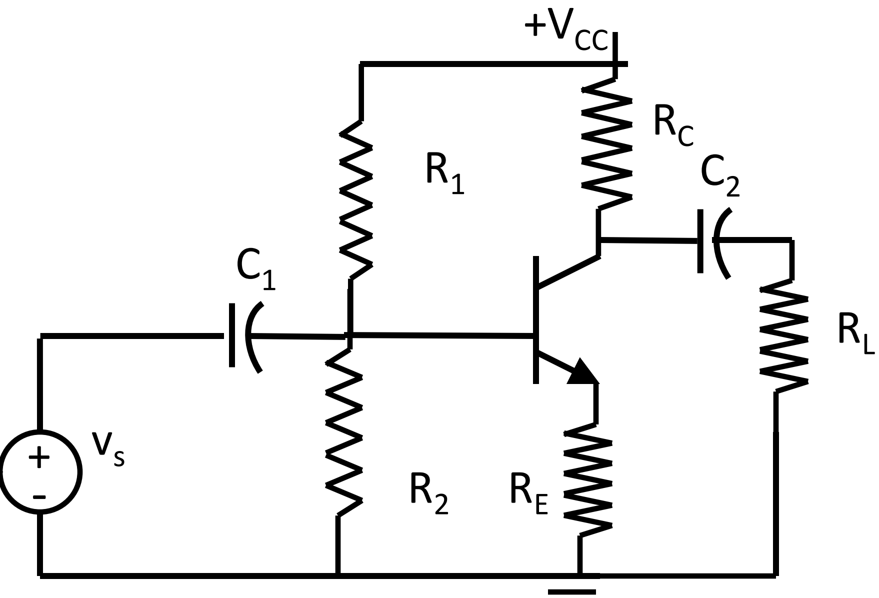


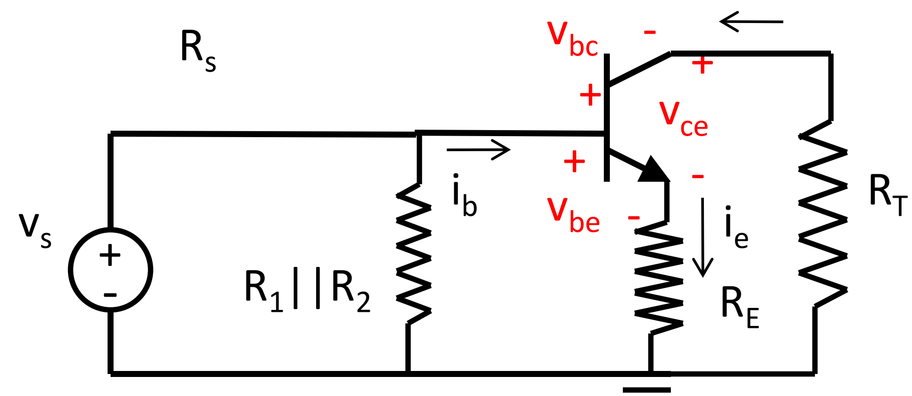
$$\omega_g = \frac{1}{R_g C_g}$$

$$\omega_T = \frac{1}{R_T C_T}$$

- Base and collector “lag” networks
- Upper cut-off frequency is the lower of the two → ω_g, ω_T

Amplifier Power Analysis





$$v_{CE} = V_{CEQ} + v_{ce}$$

$$v_{ce} = -(R_T + R_E) i_c$$

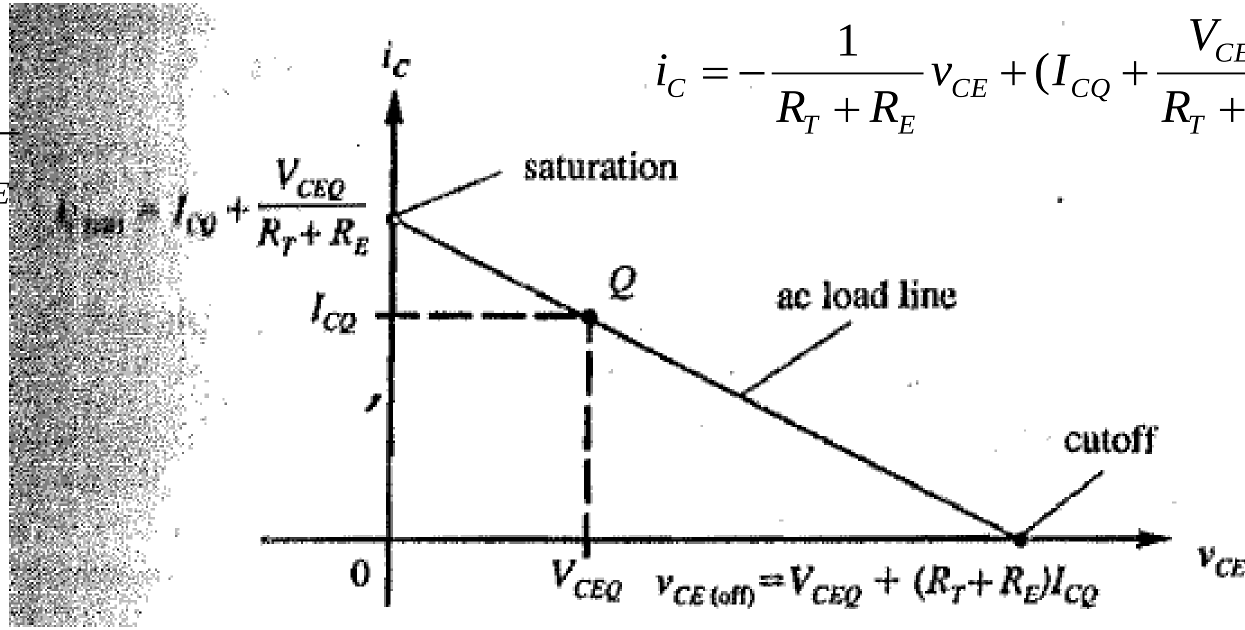
AC load line

$$i_C = I_{CQ} + i_c \Rightarrow i_c = i_C - I_{CQ}$$

$$v_{CE} = V_{CEQ} - (R_T + R_E)(i_C - I_{CQ}) \Rightarrow \boxed{i_C = -\frac{1}{R_T + R_E} v_{CE} + (I_{CQ} + \frac{V_{CEQ}}{R_T + R_E})}$$

Amplifier AC Load Line

$$i_{C(sat)} = I_{CQ} + \frac{V_{CEQ}}{R_T + R_E}$$



$$i_C = -\frac{1}{R_T + R_E} v_{CE} + (I_{CQ} + \frac{V_{CEQ}}{R_T + R_E})$$

Assume i_C(off)=0A, v_{CE}(sat)=0V

$$i_{C(sat)} = I_{CQ} + \frac{V_{CEQ}}{R_T + R_E}$$

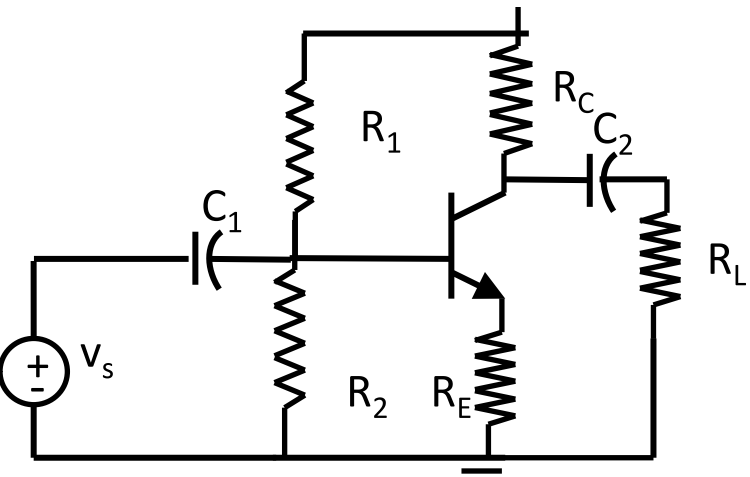
$$v_{CE(off)} = V_{CEQ} + (R_T + R_E) I_{CQ}$$

For largest possible ac signal, Q-point should be at the center of load line

$$i_{C(sat)} = I_{CQ} + \frac{V_{CEQ}}{R_T + R_E} = 2I_{CQ} \Rightarrow \boxed{\frac{V_{CEQ}}{I_{CQ}} = R_T + R_E}$$

Amplifier Power

Q-point at the center of load line gives maximum sinusoidal collector current amplitude I_{CQ}



Average ac power absorbed by R_T $\quad P_T = R_T\left(\frac{I_{CQ}}{\sqrt{2}}\right)^2 = \frac{1}{2} I_{CQ}^2 R_T$

Average ac power absorbed by R_E $\quad P_E = R_E\left(\frac{I_{CQ}}{\sqrt{2}}\right)^2 = \frac{1}{2} I_{CQ}^2 R_E$

Total ac output power

$$P_o = P_T + P_E = \frac{1}{2} I_{CQ}^2 (R_T + R_E)$$

Maximum output power

$$\boxed{P_o = \frac{1}{2} I_{CQ} V_{CEQ}}$$

Efficiency of the amplifier (output signal power/dc power)
(max efficiency = ¼)

$$\eta = \frac{P_o}{P_{dc}} = \frac{P_o}{V_{cc} I_{CQ}}$$

Instantaneous power dissipation of the BJT
(max at center of load line)

$$p = v_{CE} i_C$$

Quiescent power dissipation of the BJT (no ac signal)
(max at center of load line)

$$P_{DQ} = V_{CEQ} I_{CQ}$$