

# CS207 (Discrete Structures)

## Exercise problem set 5

Aug 26 2015

1. Consider the standard deck of 52 playing cards. A balanced hand is a subset of 13 cards containing four cards of one suit and three cards of each of the remaining three suits. Find  $N$ , the number of balanced hands. Find the number of ways of dealing the cards to four (distinguishable) players so that each player gets a balanced hand. Is this number equal to  $N(N-1)(N-2)(N-3)$ ?
2. Find the coefficients of  $x^{10}$  in
  - (a)  $(1+x)^{12}$
  - (b) the power series of  $x^4/(1-3x)^3$
3. Using generating functions, find the number of ways of selecting  $k$  objects from  $n$  different kinds of objects if repetitions are allowed, and we must select at least 2 objects of each kind?
4. Use generating functions to determine the number of different ways to give 15 (identical) chocolates to 6 children so that each child receives at least one chocolate but not more than three chocolates.
5. Using generating functions, prove Pascal's identity:  $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$  where  $r < n \in \mathbb{Z}^+$ .
6. Write a recurrence for the number of derangements. That is, no. of ways to arrange  $n$  letters into  $n$  addressed envelopes such that no letter goes to the correct envelope.
7. Solve the following recurrences:
  - (a)  $T(n) = 5T(n-1) - 6T(n-2)$  with  $T(0) = 6, T(1) = 30$ .
  - (b)  $T(n) = n(T(n/2))^2$  with  $T(1) = 6$ .
  - (c)  $T(n) = \sqrt{n}T(\sqrt{n}) + n$  with  $T(2) = 2$ .