Problem Set 9

1. Formulas in Presburger Arithmetic (PA) use first-order variables $x, y, \ldots$ that are evaluated over the structure $\mathcal{A} = (N, +)$, where $N = \{0, 1, \ldots\}$ is the set of natural numbers, and, $+$ is the ternary addition predicate. Here is a valid sentence in this logic:

$$\forall\ x\ \forall\ y\ \exists\ z\ [(x + z = y) \vee (y + z = x)]$$

   (a) Show that the relations $x < y, S(x, y)$ can be defined in PA.
   (b) Is the satisfiability problem decidable for PA? That is, given a formula φ in PA, can you decide if φ is satisfiable?

2. Let $AP = \{p, q, r\}$. Formulate the following as LT properties:
   (a) Eventually false
   (b) p occurs exactly twice, and q never occurs between two occurrences of p
   (c) If r occurs only finitely often, then p continously occurs from some point
   (d) r is true continously upto somepoint, and at the next point, both p, q hold, and then q and r alternate infinitely often
   (e) Infinitely often there are two consecutive occurrences of p
   (f) Between every consecutive occurrences of p, there is a q, and there is a prefix of r's of even length

3. Let TS and TS' be two transition systems without terminal states on the same set of atomic propositions AP. Then show that $Traces(TS) = Traces(TS')$ iff TS and TS' satisfy the same set of LT properties.

4. Consider a set of atomic propositions AP. Consider the following logic $\mathcal{X}$ defined as follows:

$$\varphi ::== (a \in AP) | \varphi \wedge \varphi | \neg\varphi | \varphi \Delta \varphi$$

   with semantics as follows:
   Given a word $w = A_0 A_1 \ldots$ over 2^{AP} and a position $i \in \mathbb{N}$, we define
   (a) $w, i \models a$ iff $a \in A_i$ for $a \in AP$
   (b) $w, i \models \varphi_1 \wedge \varphi_2$ iff $w, i \models \varphi_1$ and $w, i \models \varphi_2$
   (c) $w, i \models \neg\varphi$ iff $w, i \not\models \varphi$
   (d) $w, i \models \varphi \Delta \psi$ iff $\exists j > i,\ w, j \models \psi$ and $\forall i < k < j,\ w, k \models \varphi$.
   Comment on the equivalence of LTL and $\mathcal{X}$.

5. Exercises 5.1, 5.2, 5.5, 5.6 and 5.7 from Baier-Katoen.