

## Problem Set 10

1. Let  $TS$  and  $TS'$  be two transition systems without terminal states on the same set of atomic propositions  $AP$ . Then show that  $Traces(TS) = Traces(TS')$  iff  $TS$  and  $TS'$  satisfy the same set of LT properties.
2. Consider a set of atomic propositions  $AP$ . Consider the following logic  $\mathcal{X}$  defined as follows:

$$\varphi ::= (a \in AP) \mid \varphi \wedge \varphi \mid \neg \varphi \mid \varphi \Delta \varphi$$

with semantics as follows:

Given a word  $w = A_0A_1 \dots$  over  $2^{AP}$  and a position  $i \in \mathbb{N}$ , we define

- (a)  $w, i \models a$  iff  $a \in A_i$  for  $a \in AP$
- (b)  $w, i \models \varphi_1 \wedge \varphi_2$  iff  $w, i \models \varphi_1$  and  $w, i \models \varphi_2$
- (c)  $w, i \models \neg \varphi$  iff  $w, i \not\models \varphi$
- (d)  $w, i \models \varphi \Delta \psi$  iff  $\exists j > i, w, j \models \psi$  and  $\forall i < k < j, w, k \models \varphi$ .

Comment on the equivalence of LTL and  $\mathcal{X}$ .

3. Exercises 5.1, 5.2, 5.5, 5.6 and 5.7 from Baier-Katoen.
4. Consider a  $\omega$ -automaton  $(Q, \Sigma, \delta, q_0, Acc)$ , and let  $\mathcal{G} \subseteq 2^Q$  be a set of good states. An  $\omega$ -word  $\alpha$  is said to be accepted iff there is a run  $\rho$  of  $\alpha$  such that  $Inf(\rho) \in \mathcal{G}$ .  $\delta : Q \times \Sigma \rightarrow 2^Q$  is the transition function.
  - Construct a deterministic  $\omega$ -automata with this acceptance condition that captures the language “Finitely many  $b$ ’s”.
  - Show that  $\omega$ -automata with this acceptance condition captures  $\omega$ -regular languages.
  - How do you complement a deterministic  $\omega$ -automata with this acceptance condition?