CS 228 : Logic in Computer Science

Krishna. S

GNBA

- Generalized NBA, a variant of NBA
- Only difference is in acceptance condition
- Acceptance condition in GNBA is a set $\mathcal{F} = \{F_1, \ldots, F_k\}$, each $F_i \subseteq Q$
- An infinite run ρ is accepting in a GNBA iff

$$\forall F_i \in \mathcal{F}, \mathit{Inf}(\rho) \cap F_i \neq \emptyset$$

- Note that when $\mathcal{F} = \emptyset$, all infinite runs are accepting
- GNBA and NBA are equivalent in expressive power.

LTL to GNBA

- Given φ, consider all possible subformulae of φ, their negations

LTL to GNBA

- Given φ, consider all possible subformulae of φ, their negations
- Each state *s* of the automaton constructed gives some guarantees about the truth of some subformulae

LTL to GNBA

- Given φ, consider all possible subformulae of φ, their negations
- Each state *s* of the automaton constructed gives some guarantees about the truth of some subformulae
- The initial states give guarantees about the truth of φ

LTL to GNBA

- Given φ, consider all possible subformulae of φ, their negations
- Each state s of the automaton constructed gives some guarantees about the truth of some subformulae
- The initial states give guarantees about the truth of φ
  - Identify states of A_φ with various sets of subformulae of φ
  - Think of this as some labelling of the states
  - If B is a label for state s, and if $B = \{\varphi_1, \psi_1, \neg a\}$, then every infinite accepted string w starting at state s is such that $w \models \varphi_1, \psi_1, \neg a$.
  - The initial state(s) of A_φ must be such that all accepting paths beginning from them satisfy φ

LTL to GNBA

- Let $\varphi = \bigcirc a$.
- Subformulae of φ : $\{a, \bigcirc a\}$. Let $B = \{a, \bigcirc a, \neg a, \neg \bigcirc a\}$.
- Possibilities at each state : some consistent subset of B holds
  - $\{a, \bigcirc a\}$
  - $\{\neg a, \bigcirc a\}$
  - $\{a, \neg \bigcirc a\}$
  - $\{\neg a, \neg \bigcirc a\}$
- Our initial state(s) must guarantee truth of $\bigcirc a$. Thus, initial states: $\{a, \bigcirc a\}$ and $\{\neg a, \bigcirc a\}$

LTL to GNBA

$\{a, \bigcirc a\}$

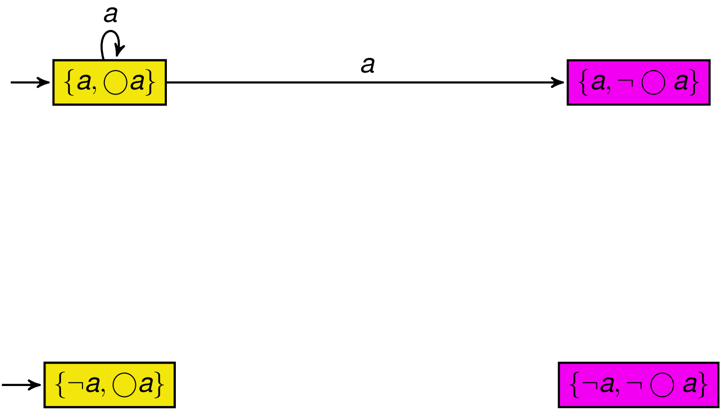
$\{a, \neg \bigcirc a\}$

$\{\neg a, \bigcirc a\}$

$\{\neg a, \neg \bigcirc a\}$

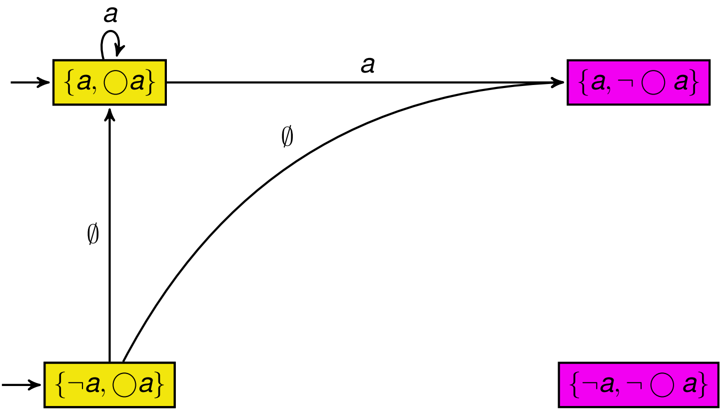
LTL to GNBA

LTL to GNBA



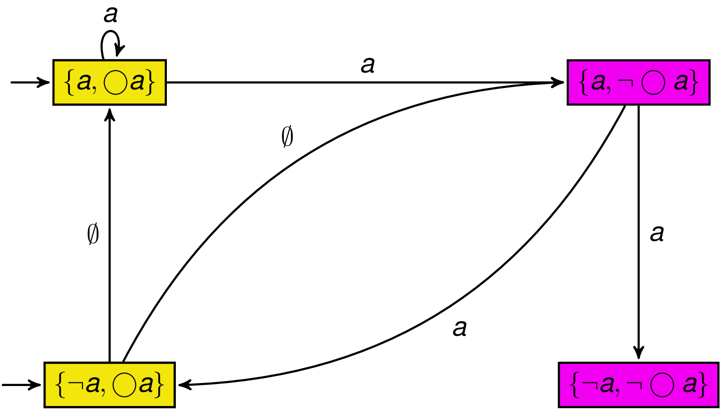
a
a
$\{a, \bigcirc a\}$
$\{a, \neg \bigcirc a\}$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



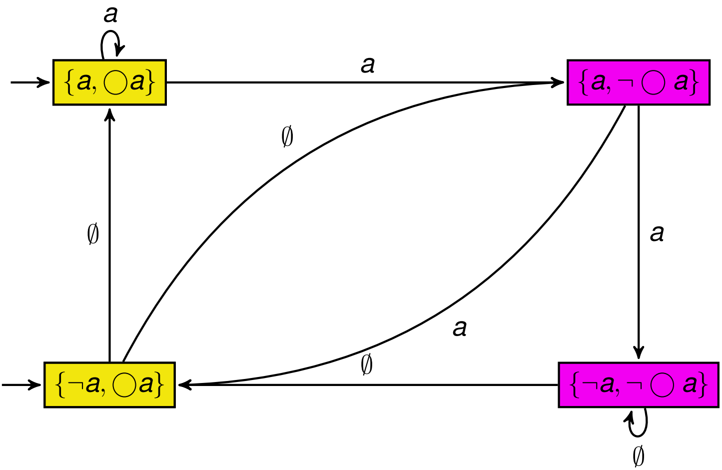
a
a
$\{a, \bigcirc a\}$
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



a
$\{a, \bigcirc a\}$
a
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
a
a
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



a
$\{a, \bigcirc a\}$
a
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
a
a
$\emptyset$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$
$\emptyset$

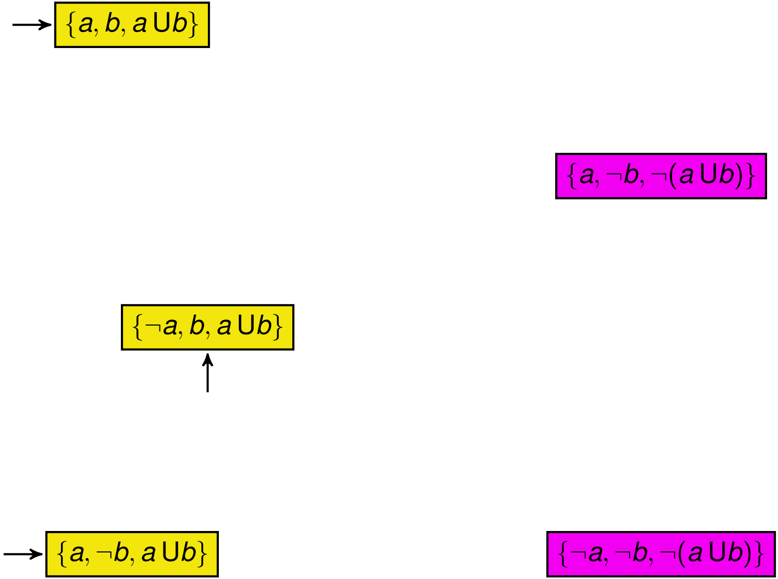
LTL to GNBA

- Claim : Runs from a state labelled set B indeed satisfy B
- No good states. All strings accepted.

LTL to GNBA

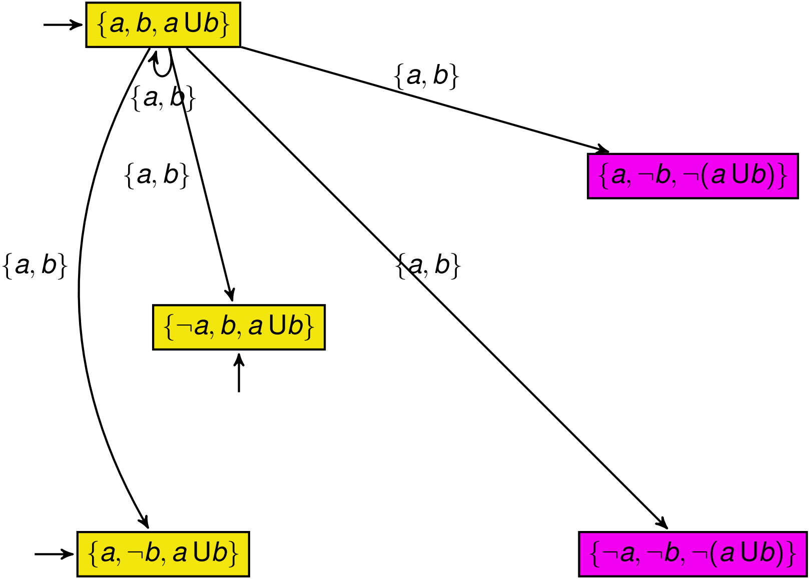
- Let $\varphi = a \, \mathsf{U} b$.
- Subformulae of φ : $\{a, b, a \, \mathsf{U} b\}$. Let $B = \{a, \neg a, b, \neg b, a \, \mathsf{U} b, \neg(a \, \mathsf{U} b)\}$.
- Possibilities at each state : some consistent subset of B holds
  - $\{a, \neg b, a \, \mathsf{U} b\}$
  - $\{\neg a, b, a \, \mathsf{U} b\}$
  - $\{a, b, a \, \mathsf{U} b\}$
  - $\{a, \neg b, \neg(a \, \mathsf{U} b)\}$
  - $\{\neg a, \neg b, \neg(a \, \mathsf{U} b)\}$
- Our initial state(s) must guarantee truth of $a \, \mathsf{U} b$. Thus, initial states: $\{a, b, a \, \mathsf{U} b\}$ and $\{\neg a, b, a \, \mathsf{U} b\}$ and $\{a, \neg b, a \, \mathsf{U} b\}$.

LTL to GNBA

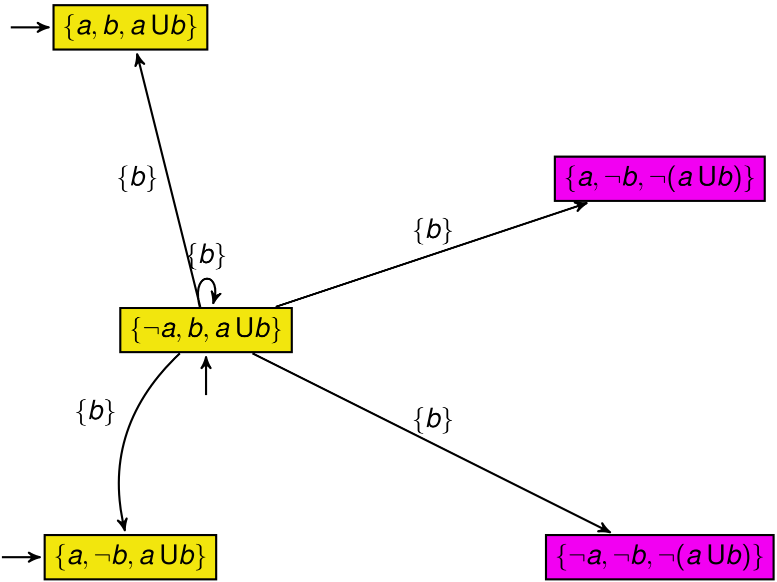


$\{a, b, a\,\mathsf{U}\,b\}$
$\{a, \neg b, \neg(a\,\mathsf{U}\,b)\}$
$\{\neg a, b, a\,\mathsf{U}\,b\}$
$\{a, \neg b, a\,\mathsf{U}\,b\}$
$\{\neg a, \neg b, \neg(a\,\mathsf{U}\,b)\}$

LTL to GNBA

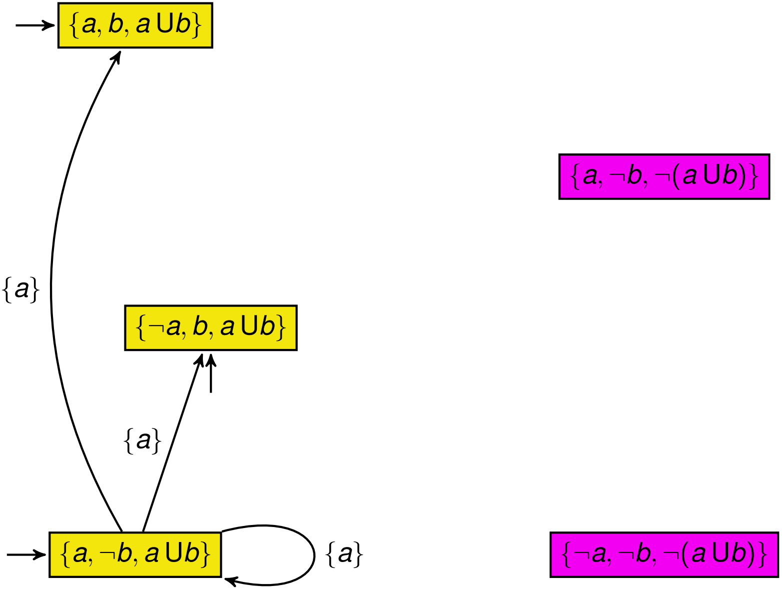

$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA

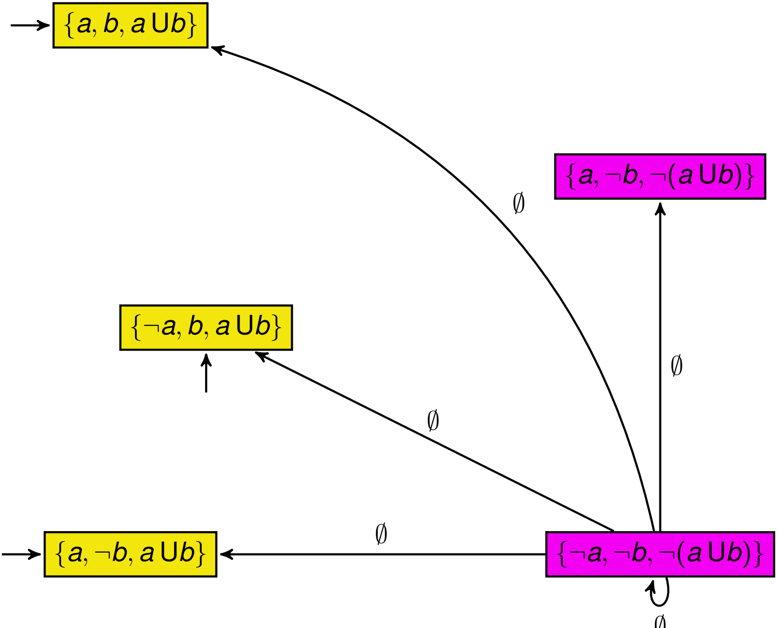


$\{a, b, a \,\mathsf{U}\, b\}$
$\{b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{b\}$
$\{b\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{b\}$
$\{b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA

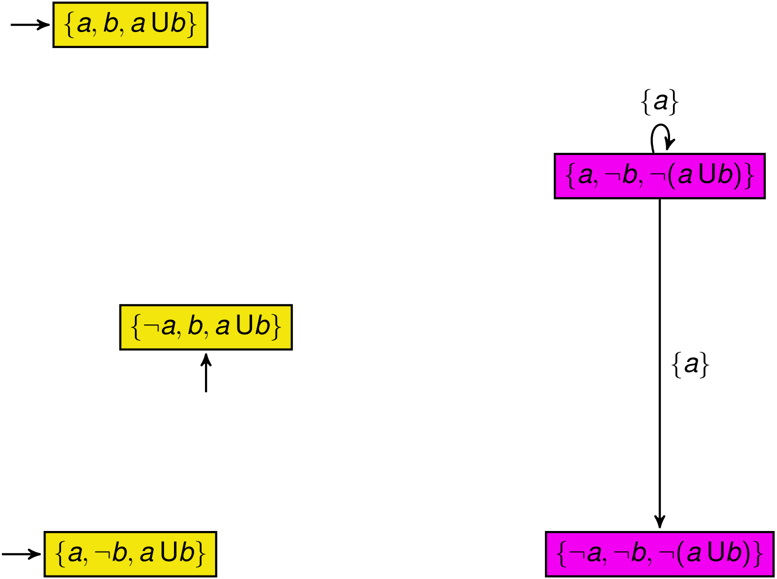

$\{a, b, a \mathsf{U} b\}$
$\{a, \neg b, \neg(a \mathsf{U} b)\}$
$\{a\}$
$\{\neg a, b, a \mathsf{U} b\}$
$\{a\}$
$\{a, \neg b, a \mathsf{U} b\}$
$\{a\}$
$\{\neg a, \neg b, \neg(a \mathsf{U} b)\}$

LTL to GNBA



$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\emptyset$
$\emptyset$
$\emptyset$
$\emptyset$
$\emptyset$

LTL to GNBA



$\{a, b, a \mathsf{U} b\}$
$\{a\}$
$\{a, \neg b, \neg(a \mathsf{U} b)\}$
$\{\neg a, b, a \mathsf{U} b\}$
$\{a\}$
$\{a, \neg b, a \mathsf{U} b\}$
$\{\neg a, \neg b, \neg(a \mathsf{U} b)\}$

LTL to GNBA : Accepting States