Problem Set 1

1. Consider the following puzzle.
   Three boxes are presented to you. One contains gold, the other two are empty. Each box has printed on it, a clue at to its contents. The clues are:

   (**Box1**) *The gold is not here*
   (**Box2**) *The gold is not here*
   (**Box3**) *The gold is in Box 2*

   Only one message is true; the other two are false. Which box has the gold? Formalise the puzzle in propositional logic and find the solution.

2. In April 2012, I along with a friend visited the island of *Azbycxdwevfu*, known for its great natural beauty. The local inhabitants are all tribals, and there are 3 tribes : *Angels, Vampires* and *Normals*. The *Angels* are known to be *always* truthful, and are peace-loving; the *Vampires* are known to *always* lie and are cruel; the *Normals* are like normal human beings, they sometimes lie, and sometimes speak the truth. Their actions are also unpredictable. The members of each tribe have distinct tattoos on their bodies, and this was the only way we (my friend and I) could say whether two people belonged to the same tribe or not. Fortunately for us, all the *Vampires* had gone to the adjacent island to celebrate a festival.

   One day, I went out all by myself to a nearby rivulet. By the time I started back, it was quite dark, and I lost my way. After wandering for a long time, I saw two people A, B standing on a hillock; the inscriptions on their body told me that they belonged to different tribes. I wanted to find out which one belonged to the tribe of *Angels*, so that he could help me find my way back. I went and asked A whether B belonged to the *Normals*. He answered me yes or no, and from that I figured out who was an *Angel*. Who is who?

   Formalise the above in propositional logic and find the solution.

3. In a banquet hall, there are $n(> 1)$ people including you. Each person including you is either (i) a liar, and always gives wrong answers to questions, or (ii) truthful, and always gives right answers to questions. Each person has an identity card that clearly indicates whether he/she is a liar/truthful. However, these identity cards are not publicly visible, and one must ask a person to show his/her identity card to find out whether he/she is a liar/truthful. You are told that the total numebr of liars in the hall is even. Of course, you know whether you are a liar or truthful, and this immediately tells you whether the remaining number of liars/truthfuls are even/odd.

The banquet hall has 2 exit doors numbered 1,2. One of these doors leads to a beautiful bed of flowers and the other leads to a torture chamber. You wish to know which door leads to the bed of flowers without opening any of the doors. Every person other than you in the hall knows which door leads where, so it would be prudent to ask one or more of the others about this. The catch however is that you do not know apriori who among the others is a liar or truthful.

Suppose you are allowed to ask atmost one question to every other person in the hall. Thus, you may or may not choose to ask a person. However, if you choose to ask a person, you can only ask the following question:

Q1: Does door 1 lead to the flower bed?

In addition, you are allowed to see atmost one person's identity card. However, if you see a person's identity card, you can no longer ask question Q1 to him/her.

Use propositional logic to come up with a strategy such that you can correctly find out which door leads to a flower bed.

4. The Pigeon Hole Principle states that if there are $n+1$ pigeons sitting amongst n holes then there is atleast one hole with more than one pigeon sitting in it. For $i \in \{1, 2, .., n+1\}$ and $j \in \{1, 2, ..., n\}$, let the atomic proposition $P(i, j)$ indicate that the i-th pigeon is sitting in the j-th hole. Write out a propositional logic formula that states the Pigeon Hole Principle.

5. Prove formally $\vdash [(p \to q) \to q] \to [(q \to p) \to p]$

6. Let $\mathcal{H}$ be a given set of premises. If $\mathcal{H} \vdash (A \to B)$ and $\mathcal{H} \vdash (C \vee A)$, then show that $\mathcal{H} \vdash (B \vee C)$ where A, B, C are wffs.

7. Let $\mathcal{H}$ be a given set of premises. If $\mathcal{H} \vdash (A \to C)$ and $\mathcal{H} \vdash (B \to C)$, then show that $\mathcal{H} \vdash ((A \vee B) \to C)$. Here, A, B and C are wffs.

8. Let $\mathcal{L}$ be a formulation of propositional logic in which the sole connectives are negation and disjunction. The rules of natural deduction corresponding to disjunction and negation (also includes double negation) are available. For any wffs A, B and C, let $\neg(A \vee B) \vee (B \vee C)$ be an axiom of $\mathcal{L}$. Show that any wff of $\mathcal{L}$ is a theorem of $\mathcal{L}$.