c] $\Box a \rightarrow \underbrace{\neg \bigcirc [\neg a \wedge \Box \neg a]}$
$\hookrightarrow \bigcirc [a \vee \Diamond a]$

$\therefore$ if $\Box a$ is true $\bigcirc [a \vee \Diamond a]$ is true $\Rightarrow$ valid.

d] $(\Box a) \, U (\Diamond b) \longrightarrow \Box (a \vee \Diamond b)$
Note it is satisfiable for a^ω.
It is not valid: consider $b \cdot (\phi)^\omega$
$\therefore$ $(\Box a) U (\Diamond b)$ is true for model $b \cdot \phi^\omega$ ~~thus~~ but false for
$\Box (a \, U (\Diamond b))$

e] $\Diamond b \rightarrow a \, U b$. obviously satisfiable.
not valid. Consider $\phi^+ \cdot b^\omega$ as counterexample.

Tut - 12.

1] DMA: Acc: $F = \{F_1 \ldots F_k\}$ where $F_i \in 2^Q$, accepting run $\Rightarrow$
$\exists F_i$, $\inf(\rho) = F_i$.

Let DMA $= (Q_1, q_0, \delta, \Sigma, F)$.
where $\delta: Q_1 \times \Sigma \longrightarrow Q_1$.

$\therefore$ To show DMA $\subseteq$ NBA we just have to show every DMA can be written as an NBA accepting the same language.

$\therefore$ We define an NBA $N_i = (Q, q, \delta', \Sigma, F)$
where $Q: Q_1 \times \{fin, inf\} \times \mathbb{N}_{|F_i|} \times \{0,1\}$

~~Late~~ ie. We define $|F|$ NBA and then take their union.
Thus for each F_i we have an NBA accepting the same language as that of DMA with $\inf(\rho) =$ ~~having~~ F_i.

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$\times\{0,$

$\therefore$ We define the states as a triplets. of Q_1, {inf, fin}, $\{1, \dots k\}$, where fin and inf indicate the fact if we can allow to see the states not in F_i. If current state is ~~inf~~ (s, inf, t) $t \in \mathbb{N}$, then ~~$\delta(s, inf, t, \Sigma)$~~

$$\delta'((s, inf, t), \Sigma) \subseteq \bigcup_{s_2 \in F_i} (s_2, inf, t) \cup \{(s_2, inf, t+1) \text{ for } s_2$$

i.e. If we are in inf marked state then all transitions go to ~~all~~ only states in F_i.

Since it is non-deterministic, we can call this as 2 layers/levels ~~indepen~~ with transition only from (fin) top layer to bottom layer (inf). Good state in lower layer. This ensures that $inf(\rho) \subseteq F_i$.

Now, we maintain flags so as to see if $inf(\rho)$ has exactly F_i. $\exists$

Let $F_i = \{B_1, B_2 \dots B_k\}$.

We establish 2 sub layers ~~such that~~ of $(B_i, inf, i, 1)$ & $(B_j, inf, i, 0)^{i \neq j}$..

Here we ~~have transitions~~ want to have the case that all B_i occur infinitely. ~~thus a good sta~~

Thus.

$\forall i$, $\forall a \in \Sigma$, ~~$\delta'((B_i, inf, i, 1))$~~ if $\delta(B_i, a) = B_{i+1}$, ~~then $\delta'((B_{i+1}, inf, i+1, 1))$~~

then $\delta'((B_i, inf, i, 1), a) = (B_{i+1}, inf, i+1, 1)$.

$\forall i, \forall a \in \Sigma$, if $\delta(B_i, a) = B_j \wedge j \neq i+1$,

$\Rightarrow \delta'(B_i, inf, i, 1) = (B_j, inf, i, 0)$.

$\forall i, \forall a \in \Sigma$, ~~$\delta(B_j, a) =$~~ $\forall t$ $\delta(B_t, a) = B_{i+1} \Rightarrow \delta'(B_t, inf, i, 0) = (B_{i+1}, inf, i+1)$

Similarly, for rest transition amongst F_i.

$$\delta'((B_j, inf, i, 0)) = (\delta(B_j, a), inf, i, 0)$$

Our good state will be any of $(B_i, inf, i, 1)$

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2) DMA's : ~~can be represe~~ $(Q, q_0, \delta, \Sigma, \mathcal{F})$.

Union of 2 DMA's :- Consider each state as ~~product~~ tuple of 2 states (q_1, q_2) $q_1 \in Q_1$, $q_2 \in Q_2$. ~~For the~~

$\therefore$ ~~$q_0 : (q_{0_1}, q_{0_2})$~~.

For the new DMA.

$Q : Q_1 \times Q_2$, $q_0 = (q_{0_1}, q_{0_2})$, $\delta((q_1, q_2)) = ((\delta_1, a), (\delta_2, a))$

$\Sigma = \Sigma_1 \cup \Sigma_2$.

~~$\mathcal{F} = \{ \}$~~

~~For the $\mathcal{F}$, we have $\mathcal{F} = \{F_i\}$~~

The 2 DMAs had $\mathcal{F}^1, \mathcal{F}^2$ ~~s.t. $\mathcal{F}^1 = \{\{f^1_{11}, f^1_{12}, \dots f^1_{1k}\}, \dots\}$~~

$\therefore$ $\mathcal{F}^0$ is new a set.

$F_i^0 = \{ F_i^0 \mid \forall j\ \exists q \in Q_2,\ (f^1_{ij}, q) \in F_i^0 \}$ ~~and~~

f^1_{ij} j^{th} element in F_i in $\mathcal{F}^1$.

$\mathcal{F}^{0'} = \{ F_i^{0'} \mid \forall j, \exists q \in Q_1,\ (q, f_{ij}) \in F_i^{0'} \}$ and

$f_{ij} \Rightarrow j^{th}$ element in F_i in $\mathcal{F}$

$\mathcal{F} = \mathcal{F}^0 \cup \mathcal{F}^{0'}$.

Informally all sets such that first ~~pair~~ element is in F_i and second is in set Q_2 ~~and~~ and all elements in F_i are covered or else, vice versa.

3) ~~Union~~

Consider a DMA: $(Q, q_0, \delta, \Sigma, \mathcal{F})$. where $\mathcal{F} = \{F_1, F_2 \dots F_n\}$.

each $F_i = \{f_{i1}, f_{i2} \dots f_{ik}\}$.

$\therefore$ ~~L(DMA)~~ Construct a DBA such that. $(Q, q_0, \delta, \Sigma, f_{ij})$

Let language accepted by such DBA be $L(i,j)$.

$$\therefore L(DMA) = \bigcup_{i=1}^{n} \left[\left(\bigcap_{j=1}^{k} L(i,j) \right) \cap \left(\bigcup_{s \notin F_i}^{s \in Q} L(Q, q_0, \delta, \Sigma, s) \right)' \right]$$

$\hookrightarrow$ all f_{ij} are accepted in inf(p)

$\hookrightarrow$ no other is accepted in inf(p)

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4) a) $\forall i, [Q_a(i) \to \exists j((j>i) \wedge Q_a(j))]$

b) can't be captured in LTL or FO.

MSO.

~~These~~

~~second. let~~

Define ~~second(x,y)~~ $second(x,y) = \exists z,\ S(x,z) \wedge S(z,y)$

$$\exists X. [\forall x (first(x) \Rightarrow X(x)) \wedge \forall x,y\ [X(x) \wedge second(x,y) \to X(y)] \wedge \forall y [X(y) \to Q_a(y)]]$$

c) Muller Acceptance: $inf(\rho) \in F$.

~~$\exists F_1 \exists F_2 \ldots \exists F_n$~~ $fin(X) = \neg inf(X) = \exists x \forall y [y > x \to \neg X(y)]$

We write ~~inf(x)~~ ~~$inf(X) = \forall x (X(x) \to \exists y, (y > x \wedge X($~~

$\therefore$ ~~Must~~ MAC : $\underset{X_i \in Q}{\exists X_i} \ldots \bigvee_{i=1}^{n}$ ~~inf(X)~~ $\left[\bigwedge_{X \in F_i} inf(X) \wedge \bigwedge_{X \notin F_i} \neg(inf(X))\right]$

muller acc. cond^n.