CS 228 : Logic in Computer Science

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LTL to GNBA

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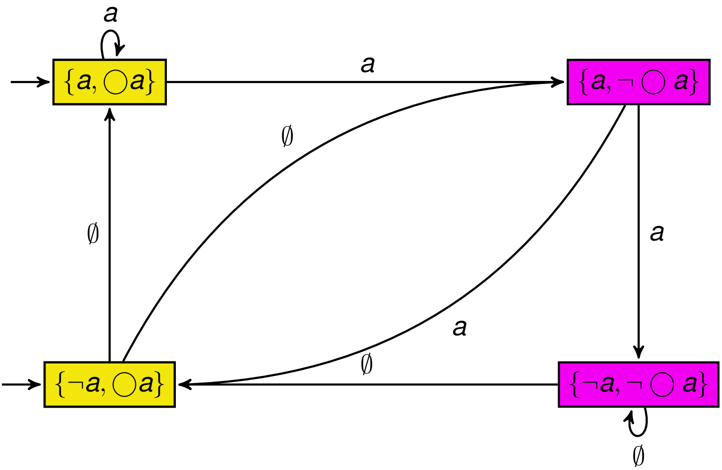
- Each state s of the automaton constructed gives some guarantees about the truth of some subformulae
- The initial states give guarantees about the truth of φ
  - Identify states of A_φ with various sets of subformulae of φ
  - Think of this as some labelling of the states
  - If B is a label for state s, and if $B = \{\varphi_1, \psi_1, \neg a\}$, then every infinite accepted string w starting at state s is such that $w \models \varphi_1, \psi_1, \neg a$.
  - The initial state(s) of A_φ must be such that all accepting paths beginning from them satisfy φ

LTL to GNBA

- Let $\varphi = \bigcirc a$.
- Subformulae of φ : $\{a, \bigcirc a\}$. Let $B = \{a, \bigcirc a, \neg a, \neg \bigcirc a\}$.
- Possibilities at each state : some consistent subset of B holds
  - $\{a, \bigcirc a\}$
  - $\{\neg a, \bigcirc a\}$
  - $\{a, \neg \bigcirc a\}$
  - $\{\neg a, \neg \bigcirc a\}$
- Our initial state(s) must guarantee truth of $\bigcirc a$. Thus, initial states: $\{a, \bigcirc a\}$ and $\{\neg a, \bigcirc a\}$

LTL to GNBA

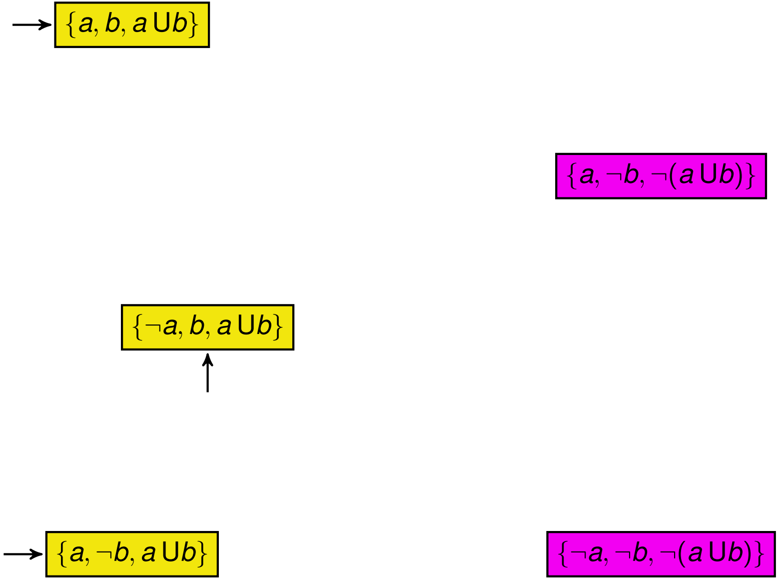


For a run $B_0 A_0 B_1 A_1 \ldots$, we have $B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

LTL to GNBA

- Let $\varphi = a \, \mathsf{U} b$.
- Subformulae of φ : $\{a, b, a \, \mathsf{U} b\}$. Let $B = \{a, \neg a, b, \neg b, a \, \mathsf{U} b, \neg(a \, \mathsf{U} b)\}$.
- Possibilities at each state : some consistent subset of B holds
  - $\{a, \neg b, a \, \mathsf{U} b\}$
  - $\{\neg a, b, a \, \mathsf{U} b\}$
  - $\{a, b, a \, \mathsf{U} b\}$
  - $\{a, \neg b, \neg(a \, \mathsf{U} b)\}$
  - $\{\neg a, \neg b, \neg(a \, \mathsf{U} b)\}$
- Our initial state(s) must guarantee truth of $a \, \mathsf{U} b$. Thus, initial states: $\{a, b, a \, \mathsf{U} b\}$ and $\{\neg a, b, a \, \mathsf{U} b\}$ and $\{a, \neg b, a \, \mathsf{U} b\}$.

LTL to GNBA


$\{a, b, a\,\mathsf{U}\,b\}$
$\{a, \neg b, \neg(a\,\mathsf{U}\,b)\}$
$\{\neg a, b, a\,\mathsf{U}\,b\}$
$\{a, \neg b, a\,\mathsf{U}\,b\}$
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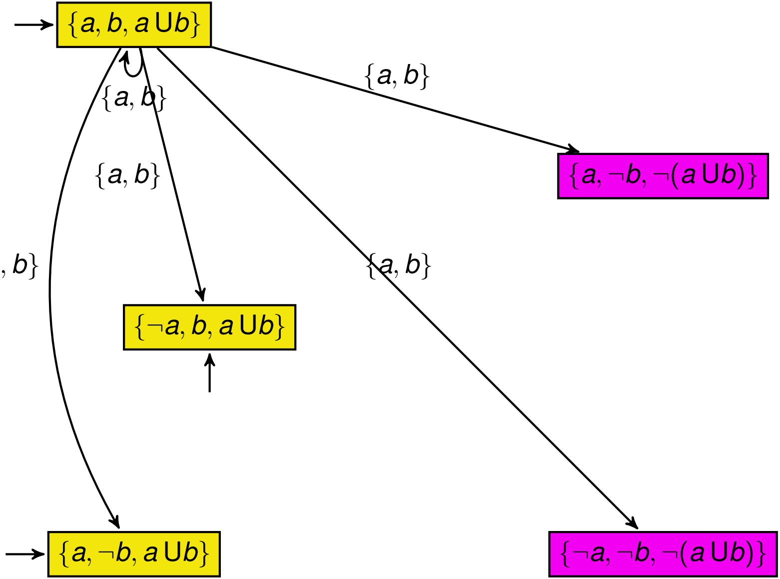
LTL to GNBA



$\{a, b, a\,\mathsf{U}\,b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
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$\{a, b\}$
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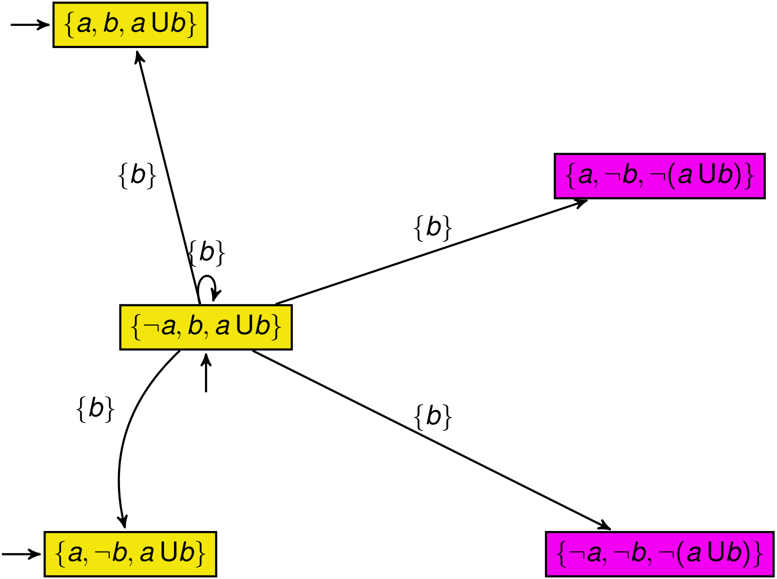

$\{a, b, a\,\mathsf{U}\,b\}$
$\{b\}$
$\{a, \neg b, \neg(a\,\mathsf{U}\,b)\}$
$\{b\}$
$\{b\}$
$\{\neg a, b, a\,\mathsf{U}\,b\}$
$\{b\}$
$\{b\}$
$\{a, \neg b, a\,\mathsf{U}\,b\}$
$\{\neg a, \neg b, \neg(a\,\mathsf{U}\,b)\}$

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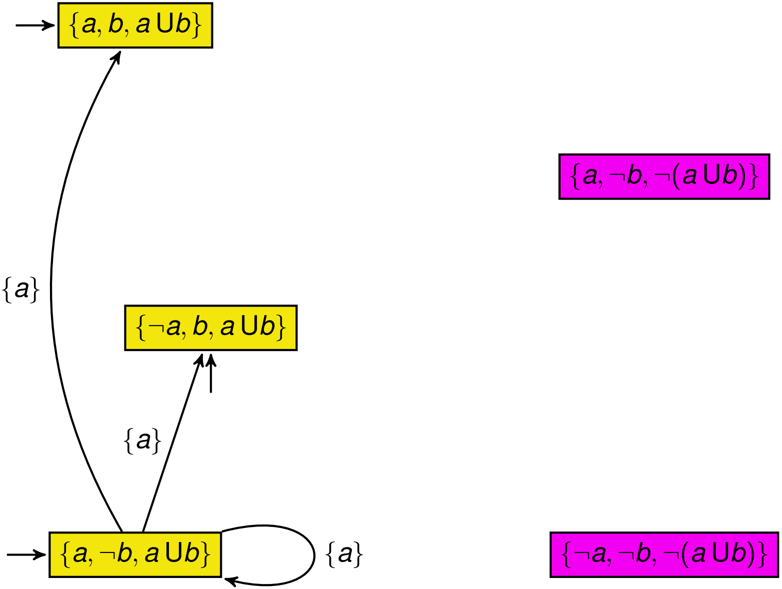

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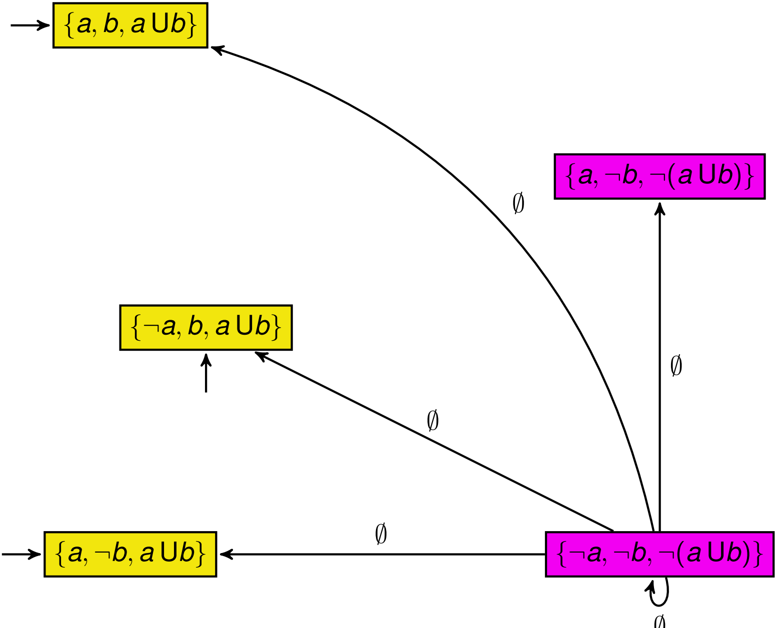

$\{a, b, a\,\mathsf{U}\,b\}$
$\{a, \neg b, \neg(a\,\mathsf{U}\,b)\}$
$\emptyset$
$\emptyset$
$\{\neg a, b, a\,\mathsf{U}\,b\}$
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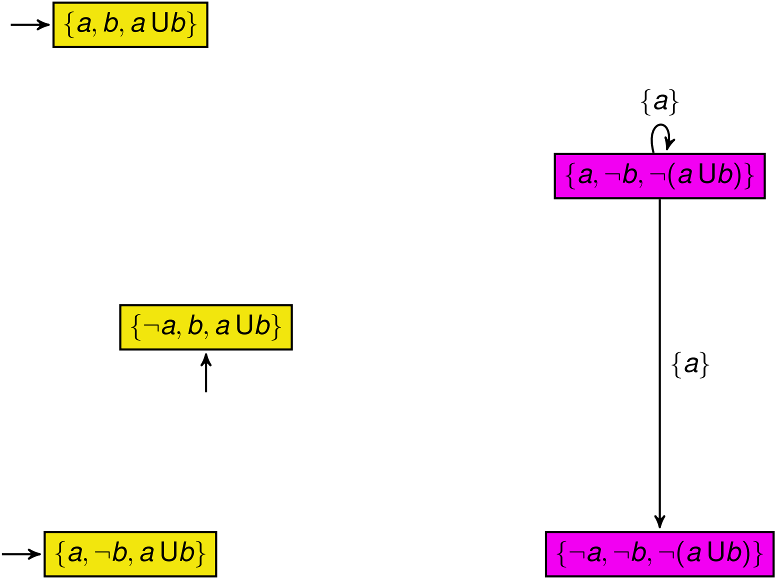
$\emptyset$

LTL to GNBA



$\{a, b, a\,\mathsf{U}\,b\}$
$\{a\}$
$\{a, \neg b, \neg(a\,\mathsf{U}\,b)\}$
$\{\neg a, b, a\,\mathsf{U}\,b\}$
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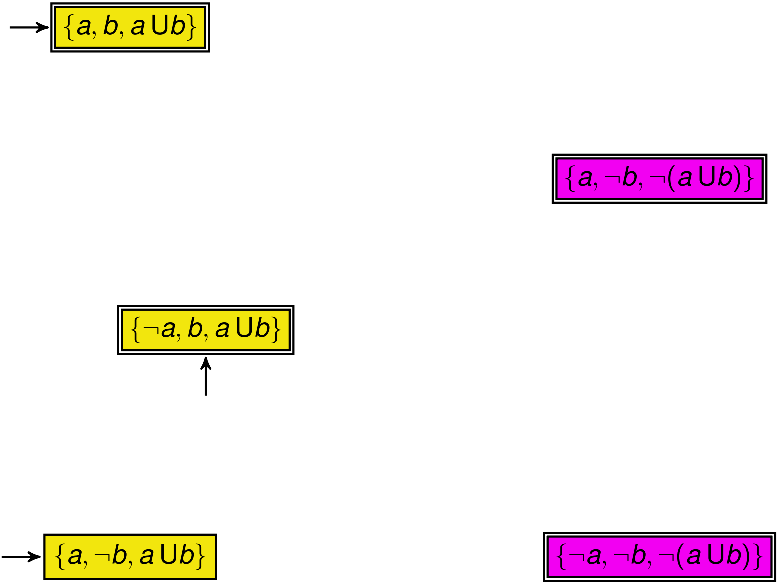
LTL to GNBA : Accepting States



LTL to GNBA

For an accepting run $B_0 A_0 B_1 A_1 \ldots$, show that
$B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

LTL to GNBA

- Let $\varphi = a \,\mathsf{U} (\neg a \,\mathsf{U} c)$. Let $\psi = \neg a \,\mathsf{U} c$
- Subformulae of φ : $\{a, \neg a, c, \psi, \varphi\}$. Let $B = \{a, \neg a, c, \neg c, \psi, \neg\psi, \varphi, \neg\varphi\}$.
- Possibilities at each state : consistent subsets of B
  - $\{a, c, \psi, \varphi\}$
  - $\{\neg a, c, \psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \varphi\}$
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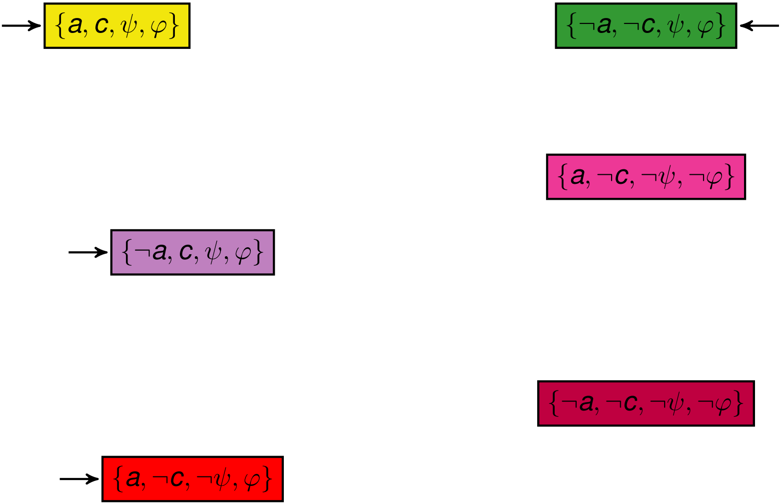
LTL to GNBA


$\{a, c, \psi, \varphi\}$
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$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
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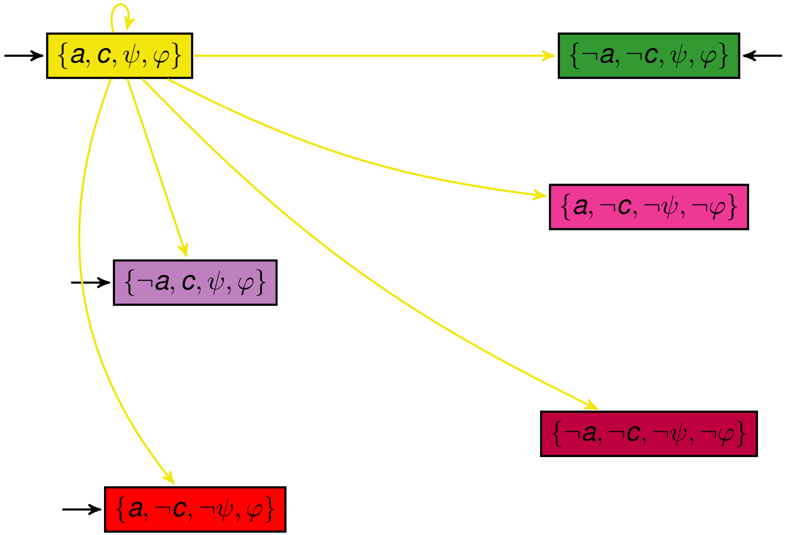
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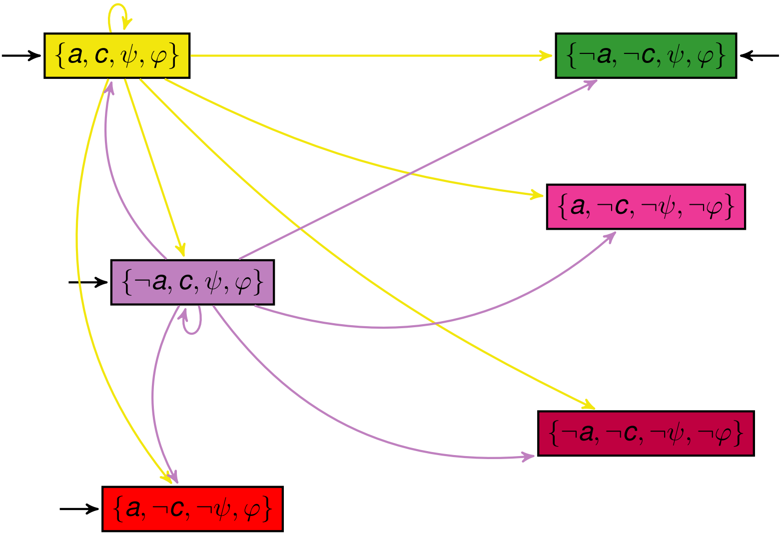
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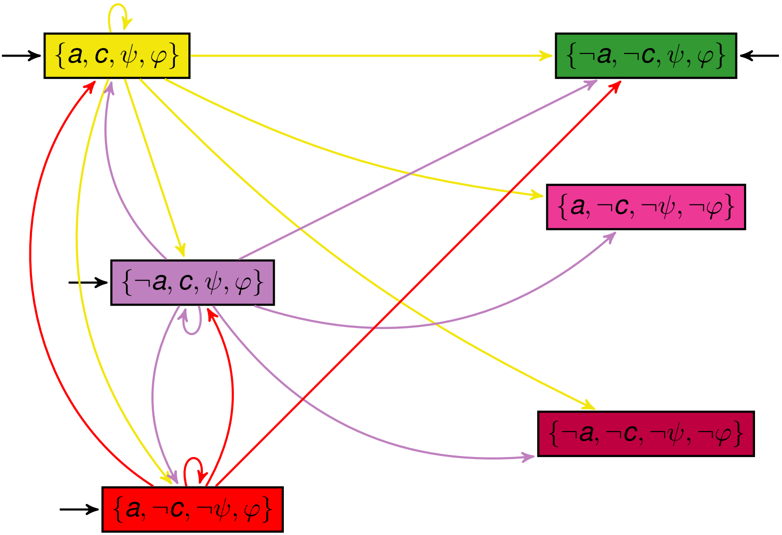
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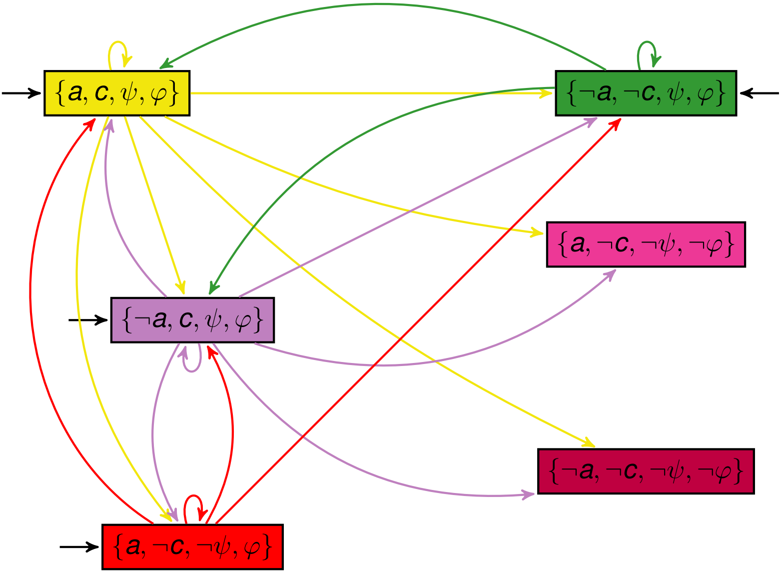

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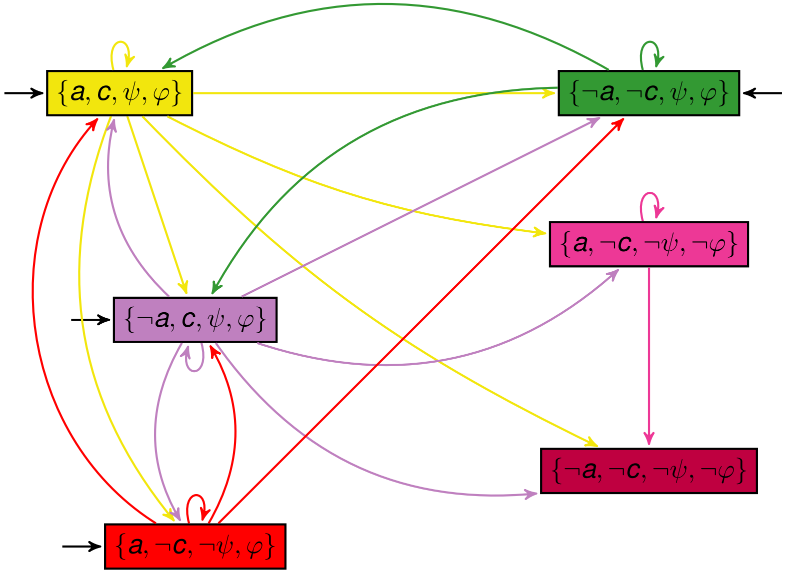

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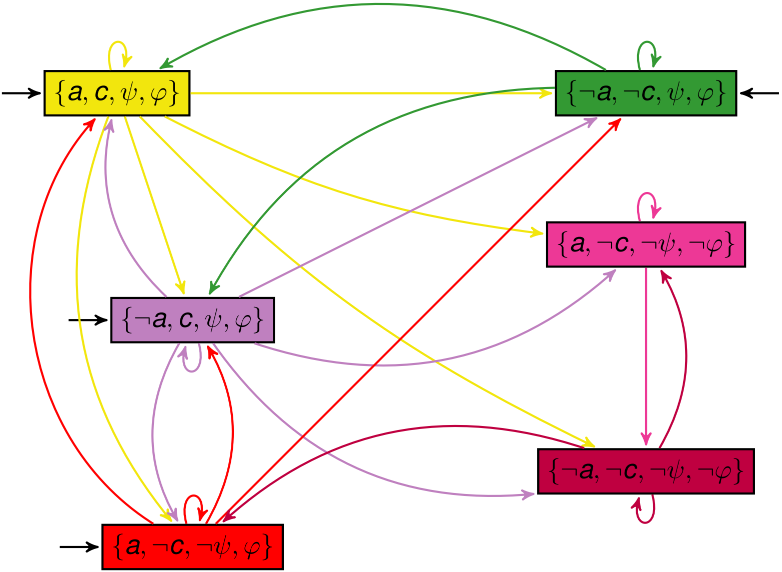
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GNBA Acceptance Condition

- $\psi = \neg a \, \mathsf{U} c$
- $\varphi = a \, \mathsf{U} \psi$
- $F_1 = \{B \mid \psi \in B \rightarrow c \in B\}$
- $F_2 = \{B \mid \varphi \in B \rightarrow \psi \in B\}$
- $\mathcal{F} = \{F_1, F_2\}$

Final States

$\{a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \psi, \varphi\} \in F_1$

$\{a, \neg c, \neg\psi, \neg\varphi\} \in F_1, F_2$

$\{\neg a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \neg\psi, \neg\varphi\} \in F_1, F_2$

$\{a, \neg c, \neg\psi, \varphi\} \in F_2$

Consistent Sets as States

- Given φ, build $Cl(\varphi)$, the set of all subformulae of φ and their negations

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- Consider those $B \subseteq Cl(\varphi)$ which are maximal consistent
  - $\varphi_1 \wedge \varphi_2 \in B \leftrightarrow \varphi_1 \in B$ and $\varphi_2 \in B$

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Consistent Sets as States

- Given φ, build $Cl(\varphi)$, the set of all subformulae of φ and their negations
- Consider those $B \subseteq Cl(\varphi)$ which are maximal consistent
  - $\varphi_1 \wedge \varphi_2 \in B \leftrightarrow \varphi_1 \in B$ and $\varphi_2 \in B$
  - $\psi \in B \rightarrow \neg\psi \notin B$ and $\psi \notin B \rightarrow \neg\psi \in B$
  - Whenever $\psi_1 \,\mathrm{U} \psi_2 \in Cl(\varphi)$,
    - $\psi_2 \in B \rightarrow \psi_1 \,\mathrm{U} \psi_2 \in B$
    - $\psi_1 \,\mathrm{U} \psi_2 \in B$ and $\psi_2 \notin B \rightarrow \psi_1 \in B$

Building the Automaton

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

- $Q = \{B \mid B \subseteq Cl(\varphi) \text{ is consistent } \}$
- $Q_0 = \{B \mid \varphi \in B\}$
- $\delta : Q \times 2^{AP} \rightarrow 2^Q$ is such that

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- $Q_0 = \{B \mid \varphi \in B\}$
- $\delta : Q \times 2^{AP} \to 2^Q$ is such that
  - For $C = B \cap AP$, $\delta(B, C)$ is enabled and is defined as :
  - If $\bigcirc\psi \in Cl(\varphi)$, $\bigcirc\psi \in B$ iff $\psi \in \delta(B, C)$
  - If $\varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)$,
    $\varphi_1 \mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \mathsf{U} \varphi_2 \in \delta(B, C)))$

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    $\varphi_1 \mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \mathsf{U} \varphi_2 \in \delta(B, C)))$
- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
  $F_{\varphi_1 \mathsf{U} \varphi_2} = \{B \in Q \mid \varphi_1 \mathsf{U} \varphi_2 \in B \to \varphi_2 \in B\}$

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- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \,\mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
  $F_{\varphi_1 \mathsf{U} \varphi_2} = \{B \in Q \mid \varphi_1 \,\mathsf{U} \varphi_2 \in B \rightarrow \varphi_2 \in B\}$
- Prove that $L(\varphi) = L(A_\varphi)$

$L(\varphi) \subseteq L(\mathcal{A}_\varphi)$

Let $\sigma = A_0 A_1 A_2 \cdots \in L(\varphi)$. Show that there is an accepting run $B_0 A_0 B_1 A_1 B_2 A_2 \ldots$ in $\mathcal{A}_\varphi$ for σ, B_i are the states, such that $B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

- If $a \in B_i$, every run starting at B_i starts with a. Hence, $A_i A_{i+1} \ldots \models a$

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- If $\bigcirc a \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, -)$ iff $a \in B_{i+1}$. Then $A_{i+1} \ldots \models a$, and hence $A_i A_{i+1} \ldots \models \bigcirc a$.

$L(\varphi) \subseteq L(\mathcal{A}_\varphi)$

Let $\sigma = A_0A_1A_2 \cdots \in L(\varphi)$. Show that there is an accepting run $B_0A_0B_1A_1B_2A_2 \ldots$ in $\mathcal{A}_\varphi$ for σ, B_i are the states, such that $B_i = \{\psi \mid A_iA_{i+1} \ldots \models \psi\}$.

- If $\varphi_1 \,\mathsf{U} \varphi_2 \in B_i$, then either $\varphi_2 \in B_i$ or $\varphi_1, \varphi_1 \,\mathsf{U} \varphi_2 \in B_i$. If $\varphi_2 \in B_i$ then $A_iA_{i+1} \ldots \models \varphi_1 \,\mathsf{U} \varphi_2$.

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Let $\sigma = A_0 A_1 A_2 \cdots \in L(\varphi)$. Show that there is an accepting run $B_0 A_0 B_1 A_1 B_2 A_2 \ldots$ in $\mathcal{A}_\varphi$ for σ, B_i are the states, such that $B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

- If $\varphi_1 \mathsf{U} \varphi_2 \in B_i$, then either $\varphi_2 \in B_i$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_i$. If $\varphi_2 \in B_i$ then $A_i A_{i+1} \ldots \models \varphi_1 \mathsf{U} \varphi_2$.
- If $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, -)$ iff $\varphi_2 \in B_{i+1}$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_{i+1}$.

$L(\varphi) \subseteq L(\mathcal{A}_\varphi)$

Let $\sigma = A_0 A_1 A_2 \cdots \in L(\varphi)$. Show that there is an accepting run $B_0 A_0 B_1 A_1 B_2 A_2 \ldots$ in $\mathcal{A}_\varphi$ for σ, B_i are the states, such that $B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

- If $\varphi_1 \mathsf{U} \varphi_2 \in B_i$, then either $\varphi_2 \in B_i$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_i$. If $\varphi_2 \in B_i$ then $A_i A_{i+1} \ldots \models \varphi_1 \mathsf{U} \varphi_2$.
- If $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, -)$ iff $\varphi_2 \in B_{i+1}$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_{i+1}$.
- When is $B_i B_{i+1} B_{i+2} \ldots$ an accepting run?
- $B_j \in F_{\varphi_1 \mathsf{U} \varphi_2}$ for infinitely many $j \geqslant i$.

$L(\varphi) \subseteq L(\mathcal{A}_\varphi)$

Let $\sigma = A_0 A_1 A_2 \cdots \in L(\varphi)$. Show that there is an accepting run $B_0 A_0 B_1 A_1 B_2 A_2 \ldots$ in $\mathcal{A}_\varphi$ for σ, B_i are the states, such that $B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

- If $\varphi_1 \mathsf{U} \varphi_2 \in B_i$, then either $\varphi_2 \in B_i$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_i$. If $\varphi_2 \in B_i$ then $A_i A_{i+1} \ldots \models \varphi_1 \mathsf{U} \varphi_2$.
- If $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, -)$ iff $\varphi_2 \in B_{i+1}$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_{i+1}$.
- When is $B_i B_{i+1} B_{i+2} \ldots$ an accepting run?
- $B_j \in F_{\varphi_1 \mathsf{U} \varphi_2}$ for infinitely many $j \geqslant i$.
- $\varphi_2 \notin B_j$ or $\varphi_2, \varphi_1 \mathsf{U} \varphi_2 \in B_j$ for infinitely many $j \geqslant i$.
- By construction, there is an accepting run where $\varphi_2 \in B_k$ for some $k \geqslant i$. Hence, $A_i A_{i+1} \ldots \models \varphi_1 \mathsf{U} \varphi_2$.

$L(\mathcal{A}_\varphi) \subseteq L(\varphi)$

For a sequence $B_0 B_1 B_2 \ldots$ of states satisfying

- $B_{i+1} \in \delta(B_i, A_i)$,
- $\forall F \in \mathcal{F}$, $B_j \in F$ for infinitely many j,

we have $\psi \in B_0 \leftrightarrow A_0 A_1 \ldots \models \psi$

- Structural Induction on ψ. Interesting case : $\psi = \varphi_1 \mathsf{U} \varphi_2$
- Assume $A_0 A_1 \ldots \models \varphi_1 \mathsf{U} \varphi_2$. Then $\exists j \geqslant 0$, $A_j A_{j+1} \ldots \models \varphi_2$ and $A_i A_{i+1} \ldots \models \varphi_1$ for all $i \leqslant j$.
- By induction hypothesis, $\varphi_2 \in B_j$ and $\varphi_1 \in B_i$ for all $i \leqslant j$
- By construction, $\varphi_1 \mathsf{U} \varphi_2 \in B_j, \ldots, B_0$.

$L(\mathcal{A}_\varphi) \subseteq L(\varphi)$

For a sequence $B_0 B_1 B_2 \dots$ of states satisfying

(a) $B_{i+1} \in \delta(B_i, A_i)$,

(b) $\forall F \in \mathcal{F}$, $B_j \in F$ for infinitely many j,

we have $\psi \in B_0 \leftrightarrow A_0 A_1 \dots \models \psi$

- Conversely, assume $\varphi_1 \mathsf{U} \varphi_2 \in B_0$. Then $\varphi_2 \in B_0$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_0$.
- If $\varphi_2 \in B_0$, by induction hypothesis, $A_0 A_1 \dots \models \varphi_2$
- If $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_0$. Assume $\varphi_2 \notin B_j$ for all $j \geqslant 0$. Then $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_j$ for all $j \geqslant 0$.
- As $B_0 B_1 \dots$ satisfies (b), $B_j \in F_{\varphi_1 \mathsf{U} \varphi_2}$ for infinitely many $j \geqslant 0$, a contradiction.
- Thus, $\varphi_2 \in B_k$ for some smallest index k. Then by induction hypothesis, $A_i A_{i+1} \dots \models \varphi_1$ and $A_k A_{k+1} \models \varphi_2$ for all $i < k$
- Hence, $A_0 A_1 \dots \models \varphi_1 \mathsf{U} \varphi_2$.

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- $\varphi_n = \Diamond[a \wedge \bigcirc^n \Box \phi]$ over $AP = \{a\}$.