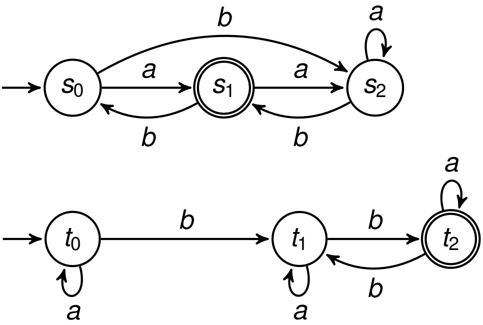
CS 228 : Logic in Computer Science

Krishna. S

Recap

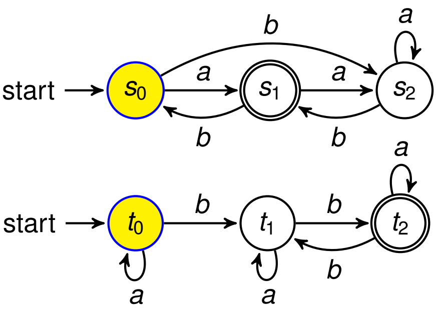
- Deterministic Finite Automata
- Closure under complementation
- Closure under Intersection

Closure under Intersection



Closure under Intersection

- *aaab*



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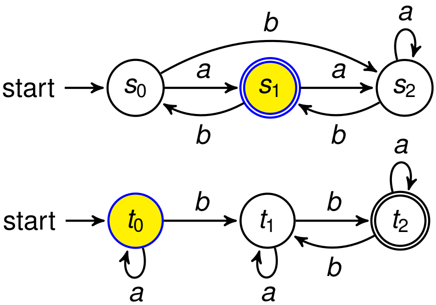

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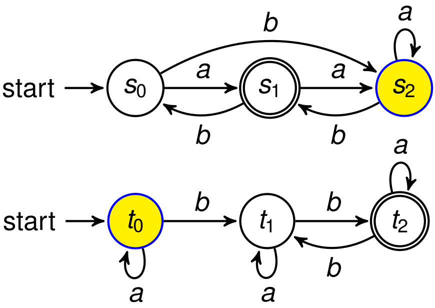

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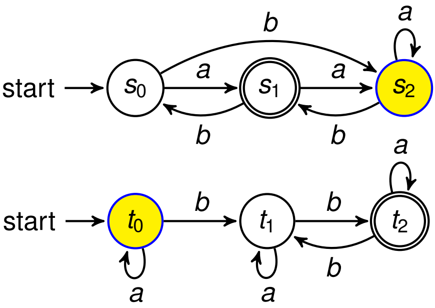

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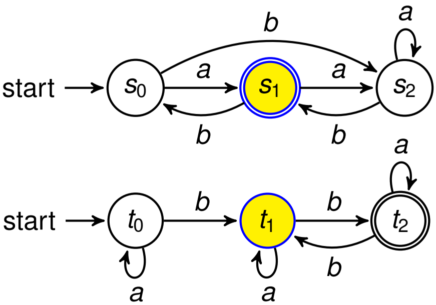

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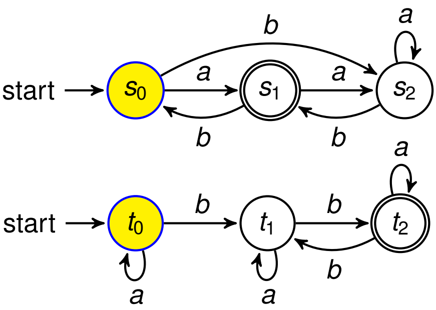

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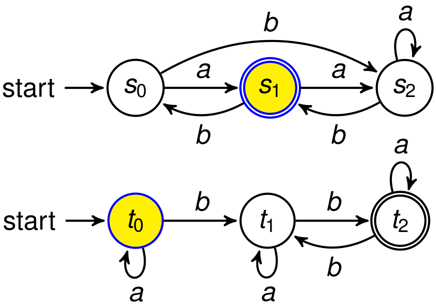

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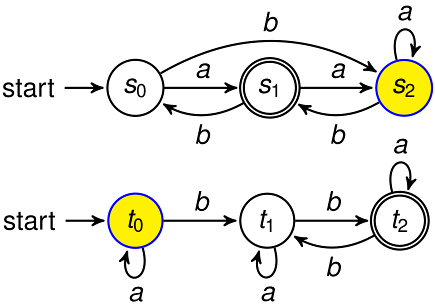

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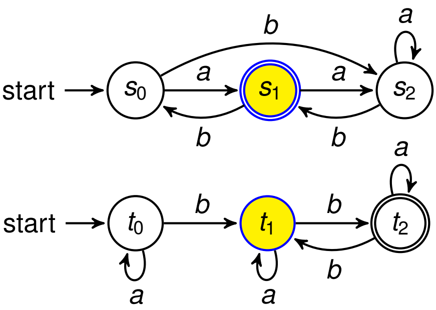

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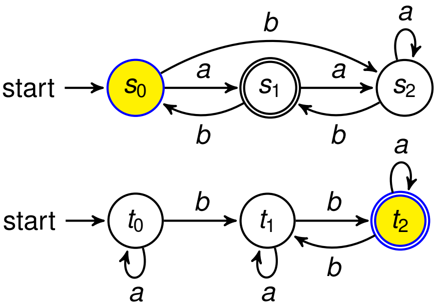

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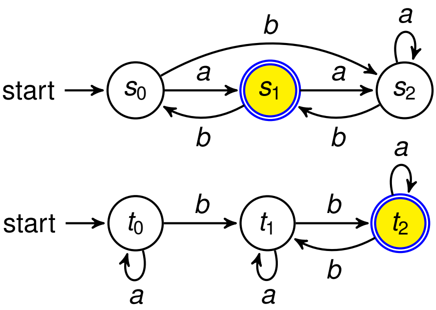

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$

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- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$

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Closure under Union

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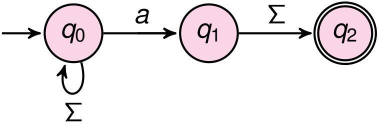
  - $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta}_1(p, x), \hat{\delta}_2(q, x))$

$x \in L(A)$ iff $x \in L(A_1)$ or $x \in L(A_2)$

Moving on to Non-determinism

- We looked at DFA
- Showed closure under union, intersection and complementation
- Before we examine closure under concatenation, we look at a more relaxed model, which is as good as a DFA

The Comfort of Non-determinism



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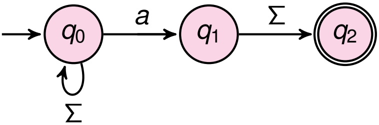

- Assume we relax the condition on transitions, and allow
  - $\delta : Q \times \Sigma \rightarrow 2^Q$
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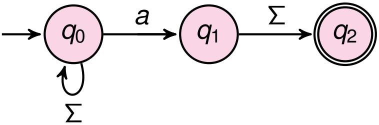

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  - Is *aabb* accepted?

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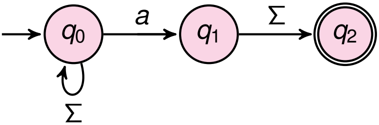

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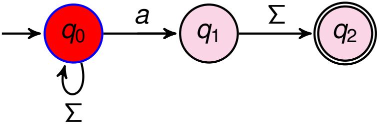
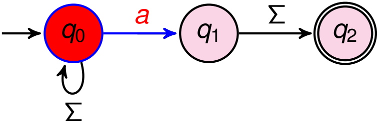


One run of *aabb*

Is *aabb* accepted?



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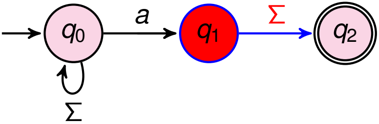

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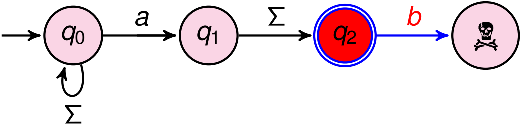

- A non-accepting run for *aabb*

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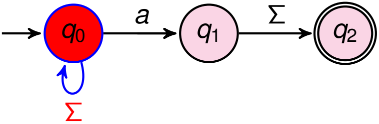

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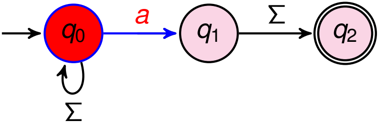

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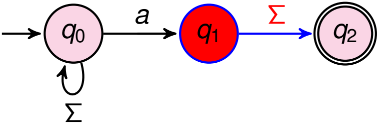

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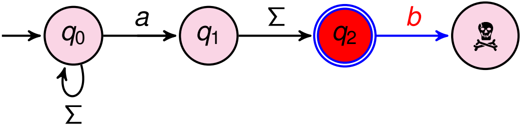

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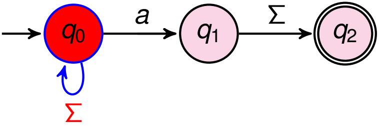

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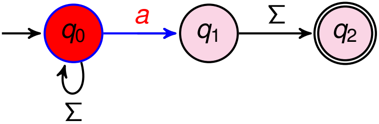

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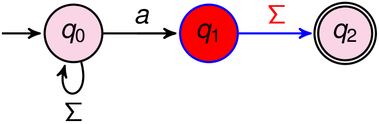

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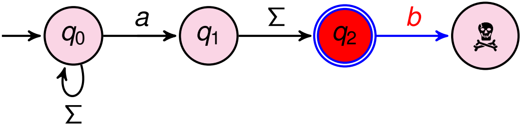

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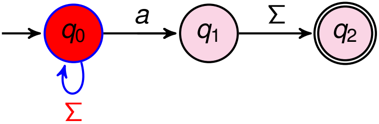

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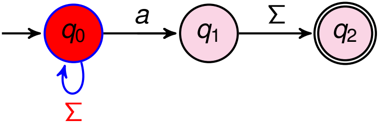

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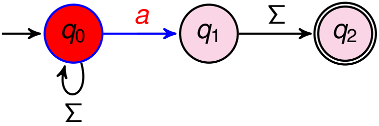

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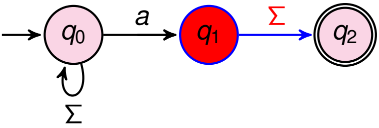
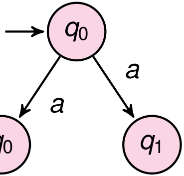


- An accepting run for *aaab*

Nondeterministic Finite Automata(NFA)

- $N = (Q, \Sigma, \delta, Q_0, F)$
  - Q is a finite set of states
  - $Q_0 \subseteq Q$ is the set of initial states
  - $\delta : Q \times \Sigma \rightarrow 2^Q$ is the transition function
  - $F \subseteq Q$ is the set of final states
- Acceptance condition : A word w is accepted iff it has atleast one accepting path

Run Tree of *aaab*



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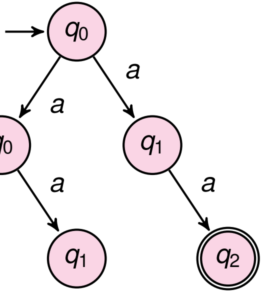

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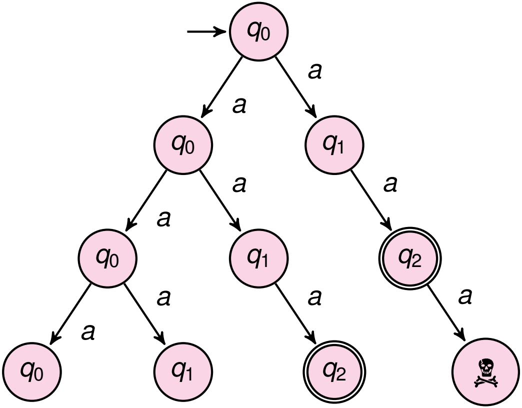

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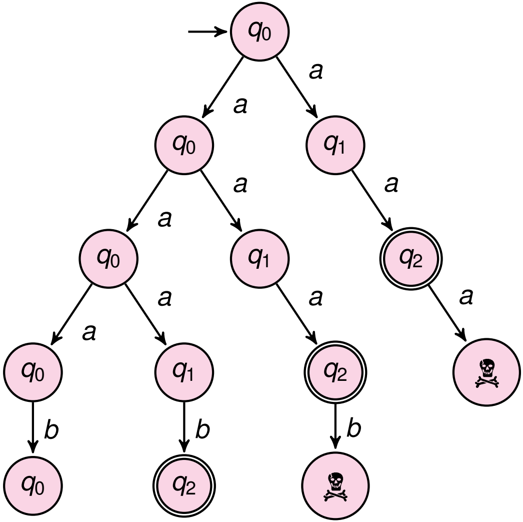

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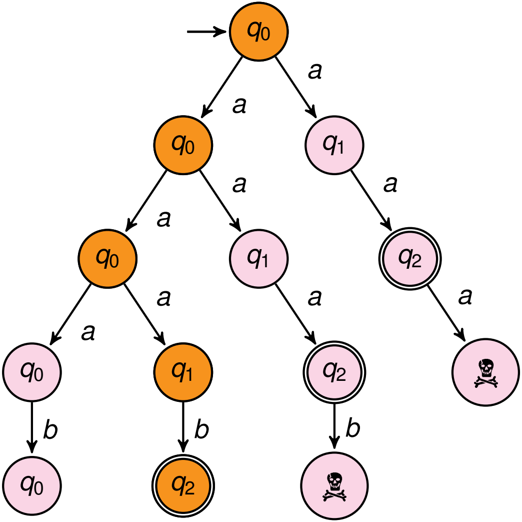

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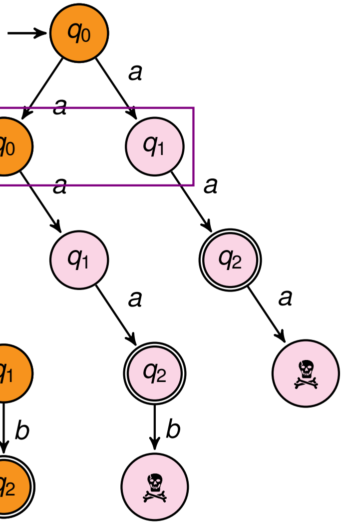

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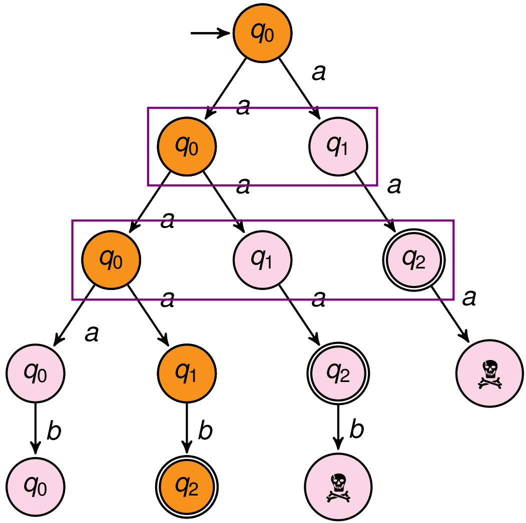

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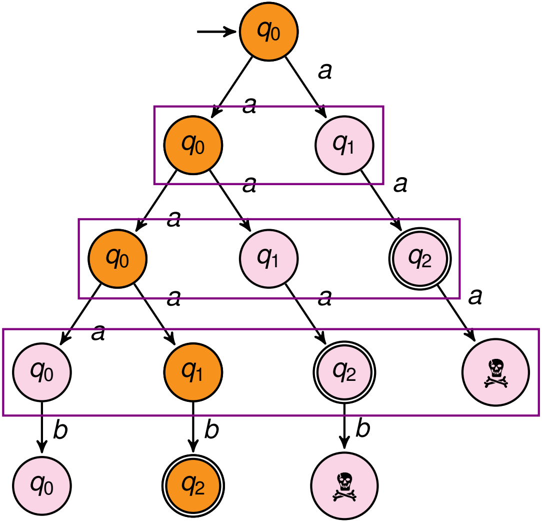

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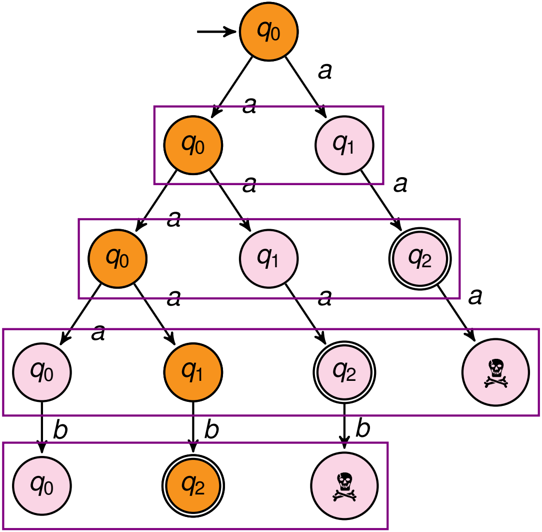

The Single Run

$$
\begin{tikzcd}
\arrow[r] & \{q_0\} \arrow[d, "a"] \\
& \{q_0, q_1\} \arrow[d, "a"] \\
& \{q_0, q_1, q_2\} \arrow[d, "a"] \\
& \boxed{\{q_0, q_1, q_2, ☠\}} \arrow[d, "b"] \\
& \boxed{\{q_0, q_2, ☠\}}
\end{tikzcd}
$$

NFA and DFA

- Any DFA is also an NFA

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NFA and DFA

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  - Combine all the runs of w in the NFA into a single run in the DFA
  - Combine states occurring in various runs to obtain a set of states
  - A set of states evolves into another set of states
  - Use $\delta : Q \times \Sigma \rightarrow 2^Q$, obtain $\Delta : 2^Q \times \Sigma \rightarrow 2^Q$
  - Δ is an extension of δ
  - Accept if the obtained set of states contains a final state

NFA and DFA

Given NFA $N = (Q, \Sigma, Q_0, \delta, F)$, obtain the DFA $D = (2^Q, \Sigma, Q_0, \Delta, F')$

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NFA and DFA

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- $\Delta : 2^Q \times \Sigma \to 2^Q$ is defined by $\Delta(A, a) = \bigcup_{q \in A} \delta(q, a)$
- $F' = \{S \in 2^Q \mid S \cap F \neq \emptyset\}$

NFA and DFA

Given NFA $N = (Q, \Sigma, Q_0, \delta, F)$, obtain the DFA $D = (2^Q, \Sigma, Q_0, \Delta, F')$

- $\Delta : 2^Q \times \Sigma \to 2^Q$ is defined by $\Delta(A, a) = \bigcup_{q \in A} \delta(q, a)$
- $F' = \{S \in 2^Q \mid S \cap F \neq \emptyset\}$

Note that $\hat{\delta}(A, a) = \bigcup_{q \in A} \delta(q, a) = \Delta(A, a)$

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NFA = DFA

$$x \in L(D) \leftrightarrow \hat{\Delta}(Q_0, x) \in F'$$

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$$\hat{\delta}(Q_0, x) \in F'$$

$$\leftrightarrow$$

$$\hat{\delta}(Q_0, x) \cap F \neq \emptyset$$

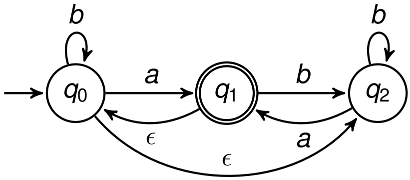
$$\leftrightarrow$$

$$x \in L(N)$$

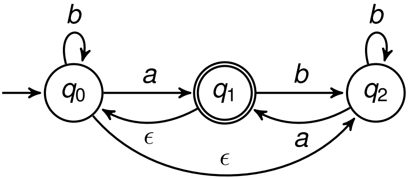
Regularity

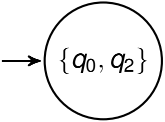
A language L is regular iff there exists an NFA A such that $L = L(A)$

ϵ-NFA



ϵ-NFA





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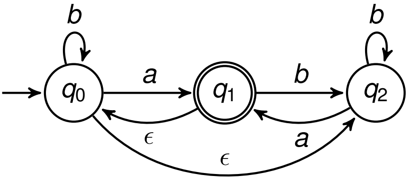

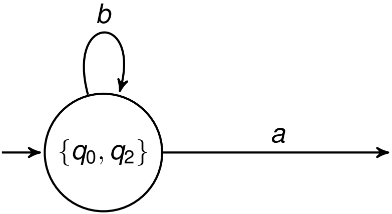

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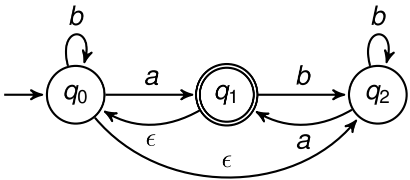

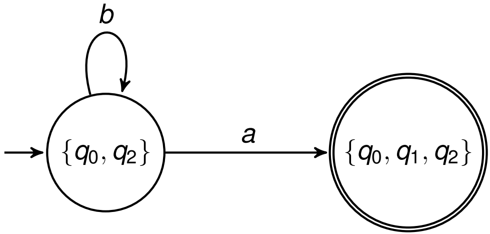

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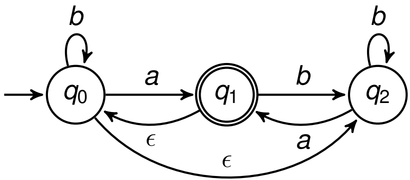
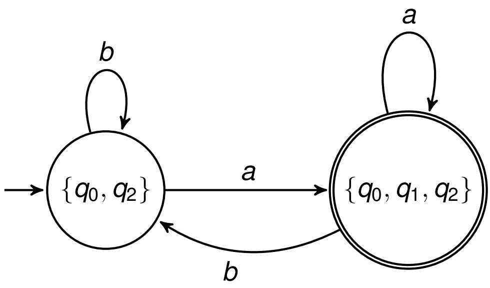


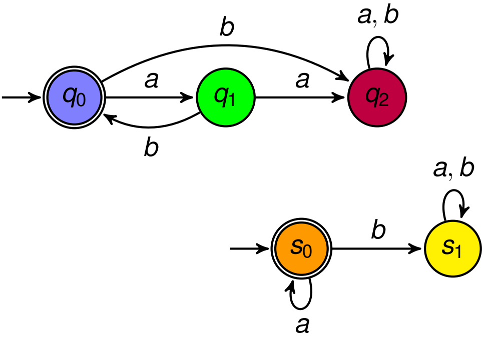


ϵ-NFA and DFA

- ϵ-close the initial states of the ϵ-NFA to obtain initial state of DFA
- From a state S, compute $\Delta(S, a)$ and ϵ-close it
- All states in the DFA are ϵ-closed
- Final states are those which contain a final state of the ϵ-NFA

Closure under Concatenation

- Given regular languages L_1, L_2, is $L_1.L_2$ regular



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