CS 228 : Logic in Computer Science

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Summarizing Construction

$\varphi = a \,\mathsf{U} [(b \,\mathsf{U} \bigcirc [\Box c])] = a \,\mathsf{U} \psi$ with
$\psi = b \,\mathsf{U} \zeta, \zeta = \bigcirc \chi, \chi = \neg(\mathit{true} \,\mathsf{U} \neg c)$

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- Formulae affecting the truth of φ are $\zeta, \neg\zeta, \chi, \neg\chi, a, \neg a, b, \neg b, c, \neg c, \mathit{true}, \mathit{false}$
- Consider all possible scenarios involving the above formulae
- Each consistent scenario is a state
- For example, $\{\varphi, a, \neg\psi, b, \neg\zeta, \neg\chi, c\}$ is a state. This is a scenario under which φ can be true.

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- Consider all possible scenarios involving the above formulae
- Each consistent scenario is a state
- For example, $\{\varphi, a, \neg\psi, b, \neg\zeta, \neg\chi, c\}$ is a state. This is a scenario under which φ can be true.
- Among the infinite words that originate from this state, those words which satisfy all formulae written in the state must be accepted.
- What qualifies a run $B_0 B_1 B_2 \ldots$ as “good"?
  - Anytime $\varphi_1 \,\mathsf{U} \varphi_2, \neg\varphi_2$ is in some B_i, there must be a $B_j, j \geqslant i$ with $\varphi_1 \,\mathsf{U} \varphi_2, \varphi_2 \in B_j$. B_i not good enough for $\varphi_1 \,\mathsf{U} \varphi_2$.
  - No such requirements if $\varphi_2, \varphi_1 \,\mathsf{U} \varphi_2 \in B_i$. Such B_i are good.
  - No such requirements if $\neg(\varphi_1 \,\mathsf{U} \varphi_2) \in B_i$. Such B_i are good.
  - Is $\{\varphi, a, \neg\psi, b, \neg\zeta, \neg\chi, c\}$ good? Good for whom?

LTL to GNBA

- Let $\varphi = a \,\mathsf{U}(\neg a \,\mathsf{U} c)$. Let $\psi = \neg a \,\mathsf{U} c$
- Subformulae of φ : $\{a, \neg a, c, \psi, \varphi\}$. Let $B = \{a, \neg a, c, \neg c, \psi, \neg\psi, \varphi, \neg\varphi\}$.
- Possibilities at each state : consistent subsets of B
  - $\{a, c, \psi, \varphi\}$
  - $\{\neg a, c, \psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \neg\varphi\}$
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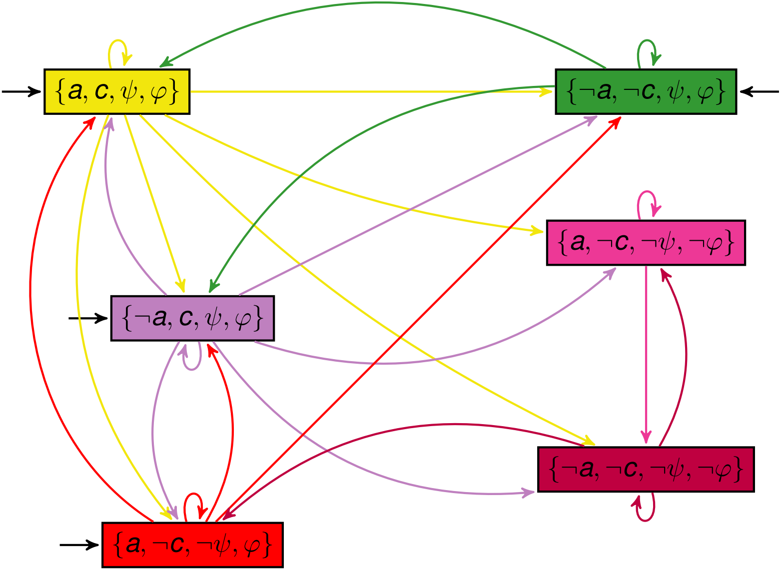

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GNBA Acceptance Condition

- $\psi = \neg a \,\mathsf{U} c$
- $\varphi = a \,\mathsf{U} \psi$
- $F_\psi = \{B \mid \psi \in B \to c \in B\}$
- $F_\varphi = \{B \mid \varphi \in B \to \psi \in B\}$
- $\mathcal{F} = \{F_\psi, F_\varphi\}$

Final States

$\{a, c, \psi, \varphi\} \in F_\psi, F_\varphi$

$\{\neg a, \neg c, \psi, \varphi\} \in F_\varphi$

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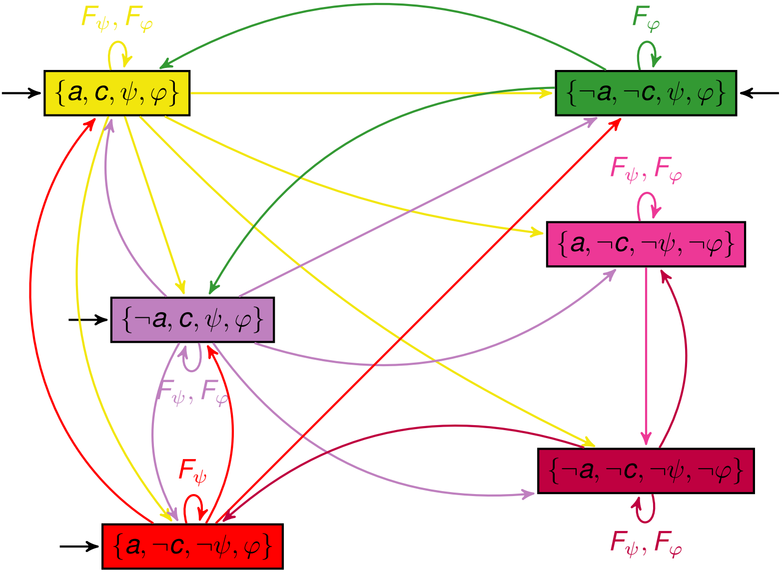

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  - $\psi \in B \rightarrow \neg\psi \notin B$ and $\psi \notin B \rightarrow \neg\psi \in B$
  - Whenever $\psi_1 \, \mathsf{U} \psi_2 \in Cl(\varphi)$,
    - $\psi_2 \in B \rightarrow \psi_1 \, \mathsf{U} \psi_2 \in B$
    - $\psi_1 \, \mathsf{U} \psi_2 \in B$ and $\psi_2 \notin B \rightarrow \psi_1 \in B$

Building the Automaton

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

- $Q = \{B \mid B \subseteq Cl(\varphi)$ is consistent $\}$
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- $\delta : Q \times 2^{AP} \to 2^{Q}$ is such that
  - For $C = B \cap AP$, $\delta(B, C)$ is enabled and is defined as :
  - If $\bigcirc\psi \in Cl(\varphi)$, $\bigcirc\psi \in B$ iff $\psi \in \delta(B, C)$
  - If $\varphi_1 \,\mathsf{U} \varphi_2 \in Cl(\varphi)$,
    $\varphi_1 \,\mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \,\mathsf{U} \varphi_2 \in \delta(B, C)))$

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- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
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  $F_{\varphi_1 \mathsf{U} \varphi_2} = \{B \in Q \mid \varphi_1 \mathsf{U} \varphi_2 \in B \to \varphi_2 \in B\}$
- Prove that $L(\varphi) = L(A_\varphi)$

$L(\varphi) \subseteq L(\mathcal{A}_\varphi)$

Let $\sigma = A_0 A_1 A_2 \cdots \in L(\varphi)$. Show that there is an accepting run $B_0 A_0 B_1 A_1 B_2 A_2 \ldots$ in $\mathcal{A}_\varphi$ for σ, B_i are the states, such that $B_i = \{\psi \mid A_i A_{i+1} \ldots \models \psi\}$.

- If $\varphi = a$. All starting states contain a, and can go to all successor states with all combinations of propositions.
- If $a \in B_i$, every run starting at B_i starts with a. Hence, $A_i A_{i+1} \ldots \models a$

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- If $\varphi = \bigcirc a$, then all initial states contain $\bigcirc a$, and all successor states contain a. The initial states can contain any set of propositions.
- If $\bigcirc a \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, B_i \cap AP)$ iff $a \in B_{i+1}$, for every successor B_{i+1}. Then $A_{i+1} \ldots \models a$, and hence $A_i A_{i+1} \ldots \models \bigcirc a$.

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- If $\varphi_1 \, \mathsf{U} \varphi_2 \in B_i$, then either $\varphi_2 \in B_i$ or $\varphi_1, \varphi_1 \, \mathsf{U} \varphi_2 \in B_i$. If $\varphi_2 \in B_i$ then $A_i A_{i+1} \ldots \models \varphi_1 \, \mathsf{U} \varphi_2$.

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- When is $B_i B_{i+1} B_{i+2} \ldots$ an accepting run?
- $B_j \in F_{\varphi_1 \mathsf{U} \varphi_2}$ for infinitely many $j \geqslant i$.

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- When is $B_i B_{i+1} B_{i+2} \ldots$ an accepting run?
- $B_j \in F_{\varphi_1 \,\mathsf{U} \varphi_2}$ for infinitely many $j \geqslant i$.
- $\varphi_2 \notin B_j$ or $\varphi_2, \varphi_1 \,\mathsf{U} \varphi_2 \in B_j$ for infinitely many $j \geqslant i$.
- By construction, there is an accepting run where $\varphi_2 \in B_k$ for some $k \geqslant i$. Hence, $A_i A_{i+1} \ldots \models \varphi_1 \,\mathsf{U} \varphi_2$.

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- If $\neg(\varphi_1 \, \mathsf{U} \varphi_2) \in B_i$, then either $\neg\varphi_1, \neg\varphi_2 \in B_i$ or $\varphi_1, \neg\varphi_2 \in B_i$. If $\neg\varphi_1, \neg\varphi_2 \in B_i$ then $A_i A_{i+1} \ldots \models \neg(\varphi_1 \, \mathsf{U} \varphi_2)$.

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- If $\varphi_1, \neg\varphi_2 \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, B_i \cap AP)$ iff $\varphi_1, \neg\varphi_2 \in B_{i+1}$ or $\neg\varphi_1, \neg\varphi_2 \in B_{i+1}$.

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- If $\neg(\varphi_1 \mathsf{U} \varphi_2) \in B_i$, then either $\neg\varphi_1, \neg\varphi_2 \in B_i$ or $\varphi_1, \neg\varphi_2 \in B_i$. If $\neg\varphi_1, \neg\varphi_2 \in B_i$ then $A_i A_{i+1} \ldots \models \neg(\varphi_1 \mathsf{U} \varphi_2)$.
- If $\varphi_1, \neg\varphi_2 \in B_i$, then by construction, $B_{i+1} \in \delta(B_i, B_i \cap AP)$ iff $\varphi_1, \neg\varphi_2 \in B_{i+1}$ or $\neg\varphi_1, \neg\varphi_2 \in B_{i+1}$.
- Either case, $A_i A_{i+1} \ldots \models \neg(\varphi_1 \mathsf{U} \varphi_2)$

$L(\mathcal{A}_\varphi) \subseteq L(\varphi)$

For a sequence $B_0 B_1 B_2 \dots$ of states satisfying

- $B_{i+1} \in \delta(B_i, A_i)$,
- $\forall F \in \mathcal{F}$, $B_j \in F$ for infinitely many j,

we have $\psi \in B_0 \leftrightarrow A_0 A_1 \dots \models \psi$

- Structural Induction on ψ. Interesting case : $\psi = \varphi_1 \, \mathsf{U} \varphi_2$
- Assume $A_0 A_1 \dots \models \varphi_1 \, \mathsf{U} \varphi_2$. Then $\exists j \geqslant 0$, $A_j A_{j+1} \dots \models \varphi_2$ and $A_i A_{i+1} \dots \models \varphi_1$ for all $i \leqslant j$.
- By induction hypothesis, $\varphi_2 \in B_j$ and $\varphi_1 \in B_i$ for all $i \leqslant j$
- By construction, $\varphi_1 \, \mathsf{U} \varphi_2 \in B_j, \dots, B_0$.

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For a sequence $B_0 B_1 B_2 \ldots$ of states satisfying

(a) $B_{i+1} \in \delta(B_i, A_i)$,

(b) $\forall F \in \mathcal{F}$, $B_j \in F$ for infinitely many j,

we have $\psi \in B_0 \leftrightarrow A_0 A_1 \ldots \models \psi$

- Conversely, assume $\varphi_1 \mathsf{U} \varphi_2 \in B_0$. Then $\varphi_2 \in B_0$ or $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_0$.
- If $\varphi_2 \in B_0$, by induction hypothesis, $A_0 A_1 \ldots \models \varphi_2$
- If $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_0$. Assume $\varphi_2 \notin B_j$ for all $j \geqslant 0$. Then $\varphi_1, \varphi_1 \mathsf{U} \varphi_2 \in B_j$ for all $j \geqslant 0$.
- As $B_0 B_1 \ldots$ satisfies (b), $B_j \in F_{\varphi_1 \mathsf{U} \varphi_2}$ for infinitely many $j \geqslant 0$, a contradiction.
- Thus, $\varphi_2 \in B_k$ for some smallest index k. Then by induction hypothesis, $A_i A_{i+1} \ldots \models \varphi_1$ and $A_k A_{k+1} \models \varphi_2$ for all $i < k$
- Hence, $A_0 A_1 \ldots \models \varphi_1 \mathsf{U} \varphi_2$.

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- LTL $\varphi \rightsquigarrow$ NBA A_φ : Number of states in $A_\varphi \leqslant |\varphi|.2^{|\varphi|}$
- Lower Bound : Find a family of LTL formulae φ_n such that the state space of $A_{\varphi_n} \geqslant \mathcal{O}(2^{|\varphi|})$

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- $\varphi_n = \Diamond[a \wedge \bigcirc^n \Box \phi]$ over $AP = \{a\}$.

Satisfiability Checking of LTL

- Translate LTL formula φ into GNBA A_φ.
- Size of equivalent NBA is $\mathcal{O}(2^{|\varphi|})$. Lower bound as well.
- Emptiness check of NBA entails satisfiability check of φ.