Mid-Sem Examination Practice Questions

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1. (a) You want to smooth / blur an image without causing any other effects or artifacts. If you are restricted to use a filter defined in the (Fourier) frequency domain, give an example of a filter you would use ?

   (b) What is the intensity transformation function underlying histogram equalization for a continuous-domain image $I(x)$ with probability density function (PDF) $P(f)$? Is this a linear function ? Why or why not ? What will the PDF of the transformed image be, if the image is: (i) continuous, (ii) discrete.

   (c) What is the problem with adaptive histogram equalization (AHE), which contrast-limited AHE (CLAHE) intends to reduce ? Describe clearly and precisely (i) the motivation behind CLAHE and (ii) the CLAHE algorithm.

   (d) Consider a continuous-domain bandlimited function $f(x)$ (say, in 1D), whose Fourier transform has frequency components lying within $[-L, L]$. Consider measurements of this function in the form of its discrete samples $f(x_i)$ at a rate above the Nyquist rate. Can we hope to reconstruct this continuous-domain function from the given discrete-domain data ? If not, why not ? If so, describe a way to estimate the continuous-domain function $f(x)$, given its sampled values $f(x_i)$.

   (e) What is the effect of processing a color image with the mean-shift algorithm, applied on the color and spatial feature, on the histogram of the image ? That is, describe how the image histogram changes after applying the mean-shift algorithm. Can this algorithm be applied for denoising a piecewise-smooth color image corrupted with additive independent and identically-distributed noise ? Why or why not ?

   (f) An infinitely-differentiable function having finite support is bandlimited. Prove or disprove.

   (g) Consider a 1D discrete image that is a box function. What will be the image resulting from an application of the median filter (1 pass over the image) ? Consider a 2D image that is a box function (in 2D). What will be the resulting image after application of the median filter ? Is the median filter linear ? Is the median filter edge preserving ?

2. Suppose you want to convolve a 2D image of size $N \times N$ with a large 2D mask of size $M \times M$, which isn't separable, a large number (T) of times. How can you exploit Fourier analysis to design an alternative algorithm that is more efficient (i.e., with lower algorithmic complexity) for the same task ? Derive the algorithmic complexity for each approach.

3. Prove, or disprove, that the convolution operation is: (i) symmetric, (ii) linear, (iii) associative, (iv) space invariant.

4. Prove, or disprove, that the Fourier transform is: (i) linear, (ii) space invariant.

5. Define the structure tensor, in the context of a 2D image. Is the the structure tensor symmetric ? Prove or disprove. Is the structure tensor a positive-definite matrix ? Prove or disprove. What will be the eigenvectors of the structure tensor for a pixel on an image edge ? Assume all other objects / edges lie are far from this edge.

6. Consider an image $f(x)$. Blur the image $f(x)$ by convolving it with a Gaussian $g(x; 0, a^2)$ (with mean 0 and variance a^2) to produce $f'(x)$. Blur the resulting image $f'(x)$ by convolving it with another Gaussian $g(x; 0, b^2)$ to produce $f''(x)$. Show that the final blurred image $f''(x)$ could have been obtained using a single convolution of $f(x)$ with a Gaussian $g(x; 0, c^2)$. Find the relationship between the standard deviations a, b, and c.

7. Consider two images A and B that are of the same size. Image A has intensities $x \in [0, 1]$. Assume that image B corresponds to independent and identically-distributed zero-mean noise. Let the probability

mass functions (PMFs), i.e., histograms, of the intensities in A and B be $P_A(x)$ and $P_B(x)$, respectively. Construct a third image C, where the intensity at each pixel n is the sum of the intensities of the n-th pixels in A and B, i.e., $C(n) := A(n) + B(n), \forall n$. In image C, let the PMF of intensities be $P_C(x)$. Assuming that images A and B have a very large number of pixels, give an expression that well approximates the PMF $P_C(\cdot)$ in terms of the PMFs $P_A(\cdot)$ and $P_B(\cdot)$.

8. Consider a linear system whose output is defined as the discrete Fourier transform of the input, i.e., the input and output are one period of the associated periodic discrete signals. Is this system time invariant (or shift invariant) ? Provide a proof or disprove using a counter example.

9. Prove or disprove (e.g., using a counter example) the following statements:
   (a) If a function $f(x)$ is bandlimited, then $f(x)$ is infinitely differentiable (i.e., derivatives of all orders exist).
   (b) If a function $f(x)$ is infinitely differentiable, then $f(x)$ is bandlimited.

10. (a) To convolve a $2D$ image $f(x_1, x_2)$ of size $A \times A$ pixels, where $A := 2^a$, with an arbitrary $2D$ mask $g(x_1, x_2)$ of size $B \times B$ pixels, where $B < A$, what is the order of (arithmetic) operations required, in terms of the image size ?
   (b) If $g(x_1, x_2)$ is a 2D Gaussian filter mask, can the number of operations be reduced ? If so, describe the process and the reduced number of operations. If not, show why this is impossible ?
   (c) If $g(x_1, x_2)$ is a binary circular / disc filter mask (where the mask value equals 1 for all pixels within a distance of $B/2$ from the center pixel, and 0 otherwise), can the number of operations be reduced ? If so, describe the process and the reduced number of operations. If not, show why this is impossible ?