# Digital Image Processing

## Image Enhancement : Gray-Level Transformations

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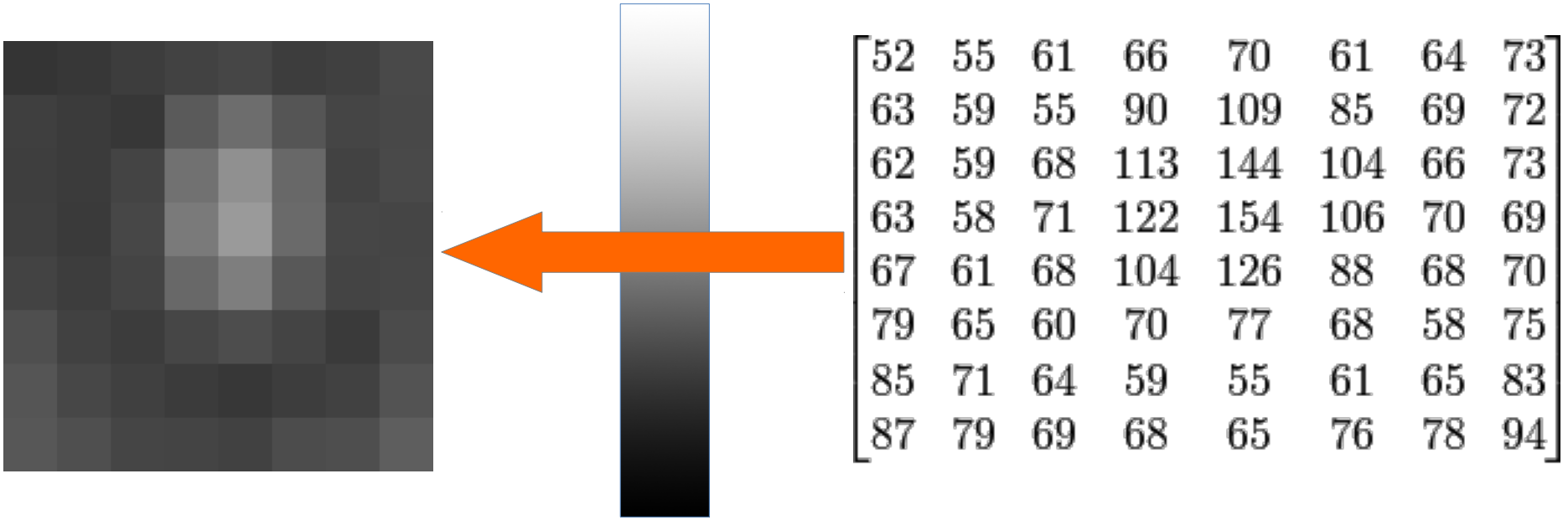
# Digital Image

- Assumptions
  - 8-bit per pixel
  - 256 intensity levels or gray levels
    - 0,1, ..., 255 : this is called the grayscale
    - 0 = black
    - 255 = white



$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

# Enhancement

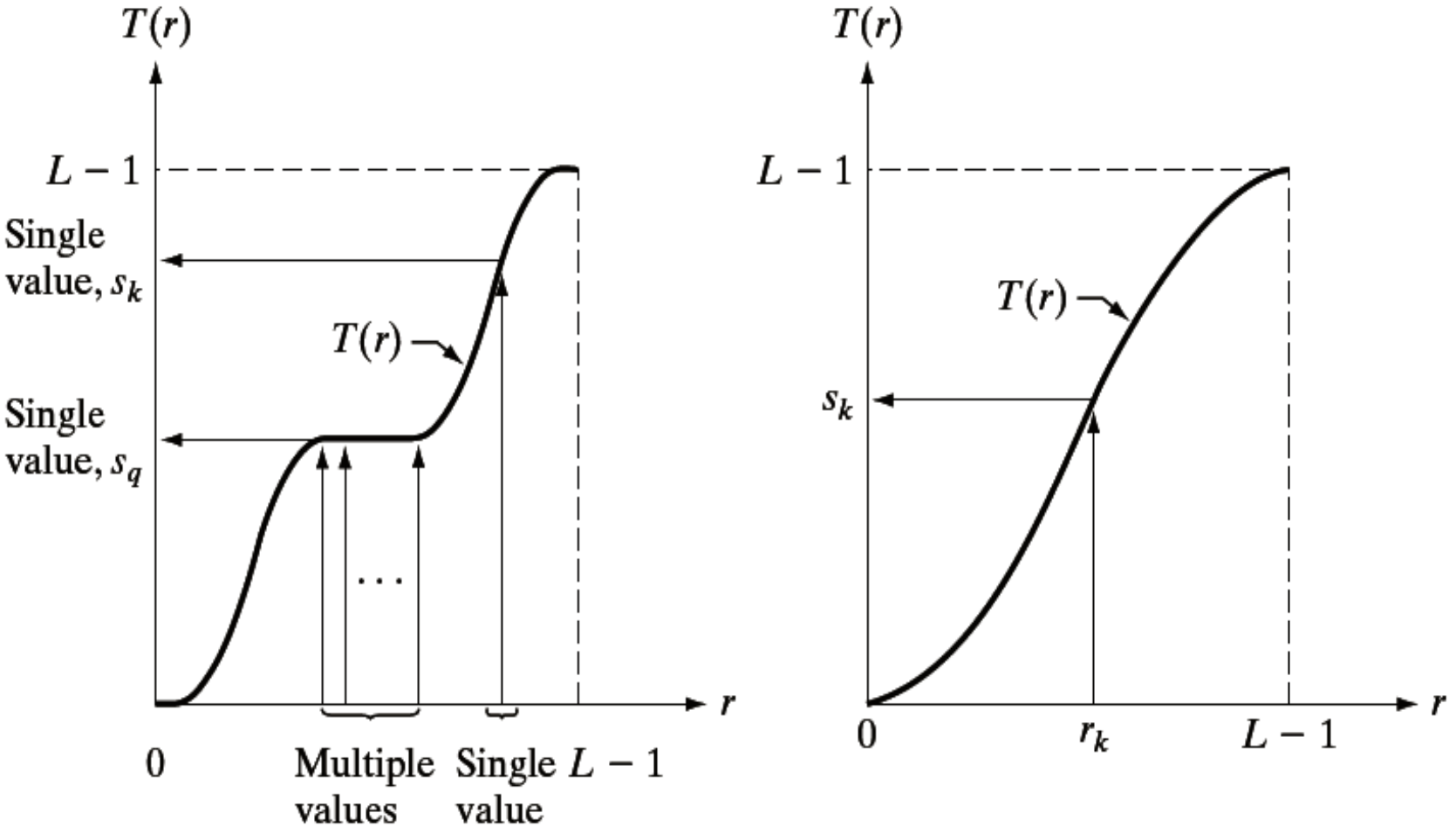
- What does it mean ?
  - Processing an image so that
    the result it more suitable (compared to the original image)
    for a **specific** application
- Evaluation
  - Visual, subjective
  - Quantitative

# Gray-Level Transformations

- Design **functions** to transform gray levels within an image by directly operating on them
  - Example
    - Input Image : f (x,y)
    - Transformation : T [.] operates on individual intensities
    - Output Image : g (x,y) = T [ f (x,y) ]
  - Another example
    - Transformation :
      U [.] operates on
      intensities in a neighborhood
    - Output Image :
      g (x,y) = U [ f (x,y), f(x+1,y), f(x-1,y) ]

# Gray-Level Transformations

- We want transformations to be **monotonic** functions
  - Monotonically increasing function: a > b → T(a) >= T(b)
  - Monotonically decreasing function: a > b → T(a) <= T(b)

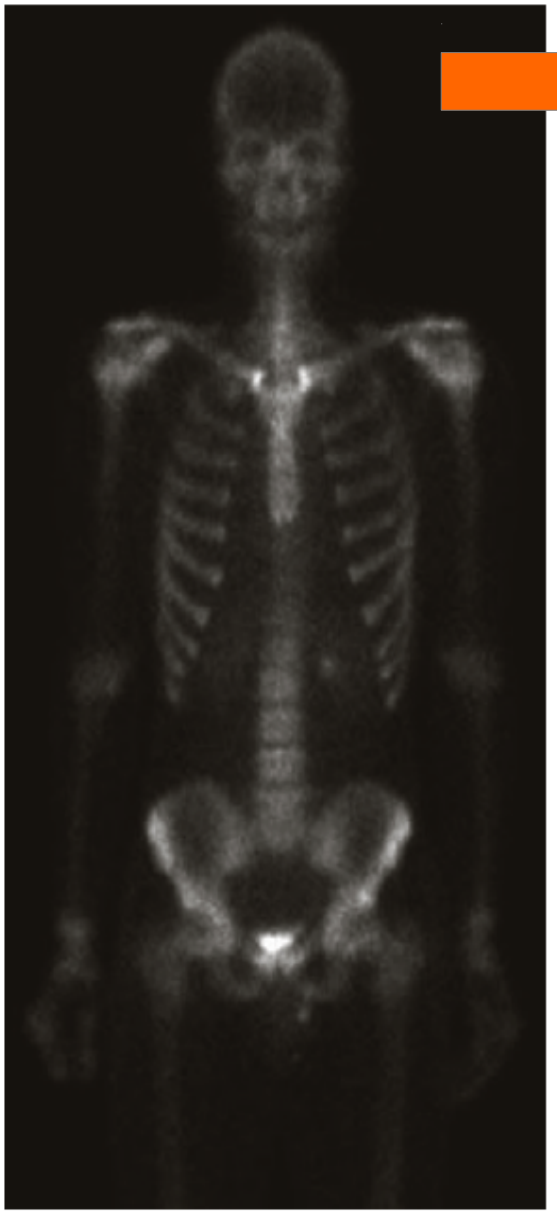


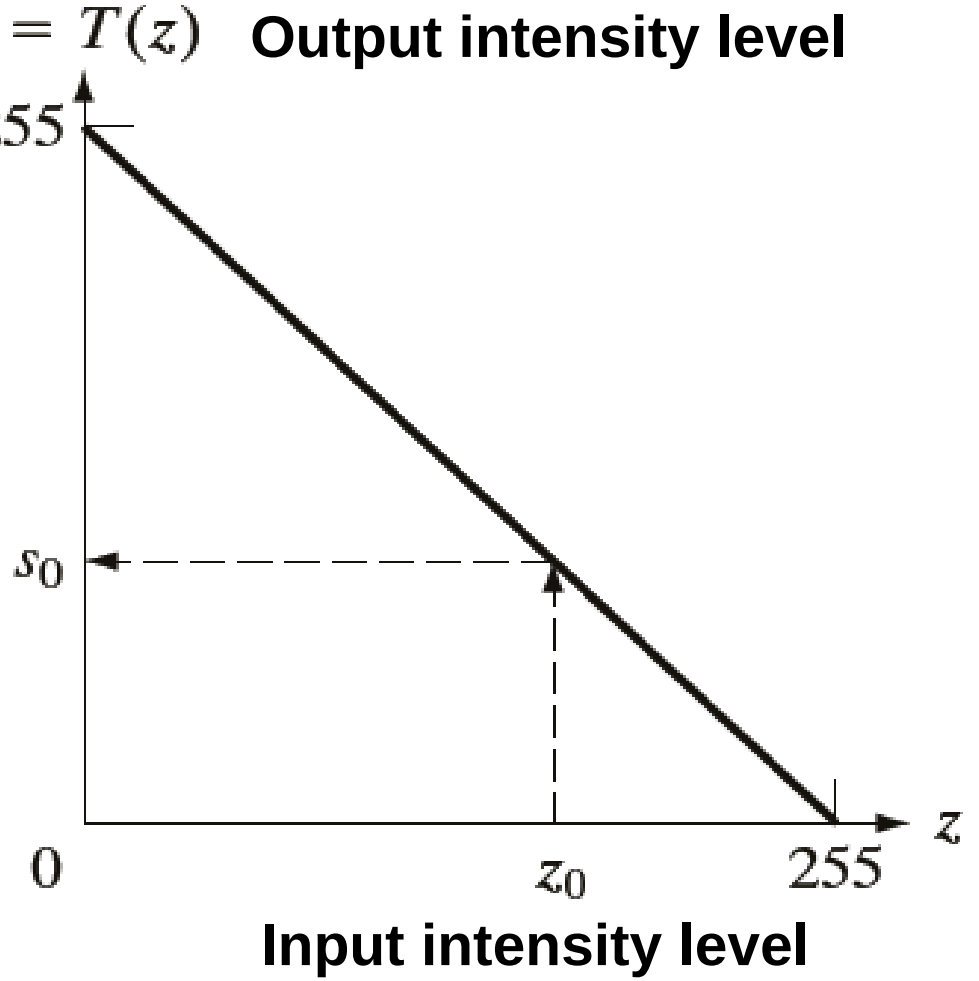

a b

**FIGURE 3.17**
(a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

# Gray-Level Transformations

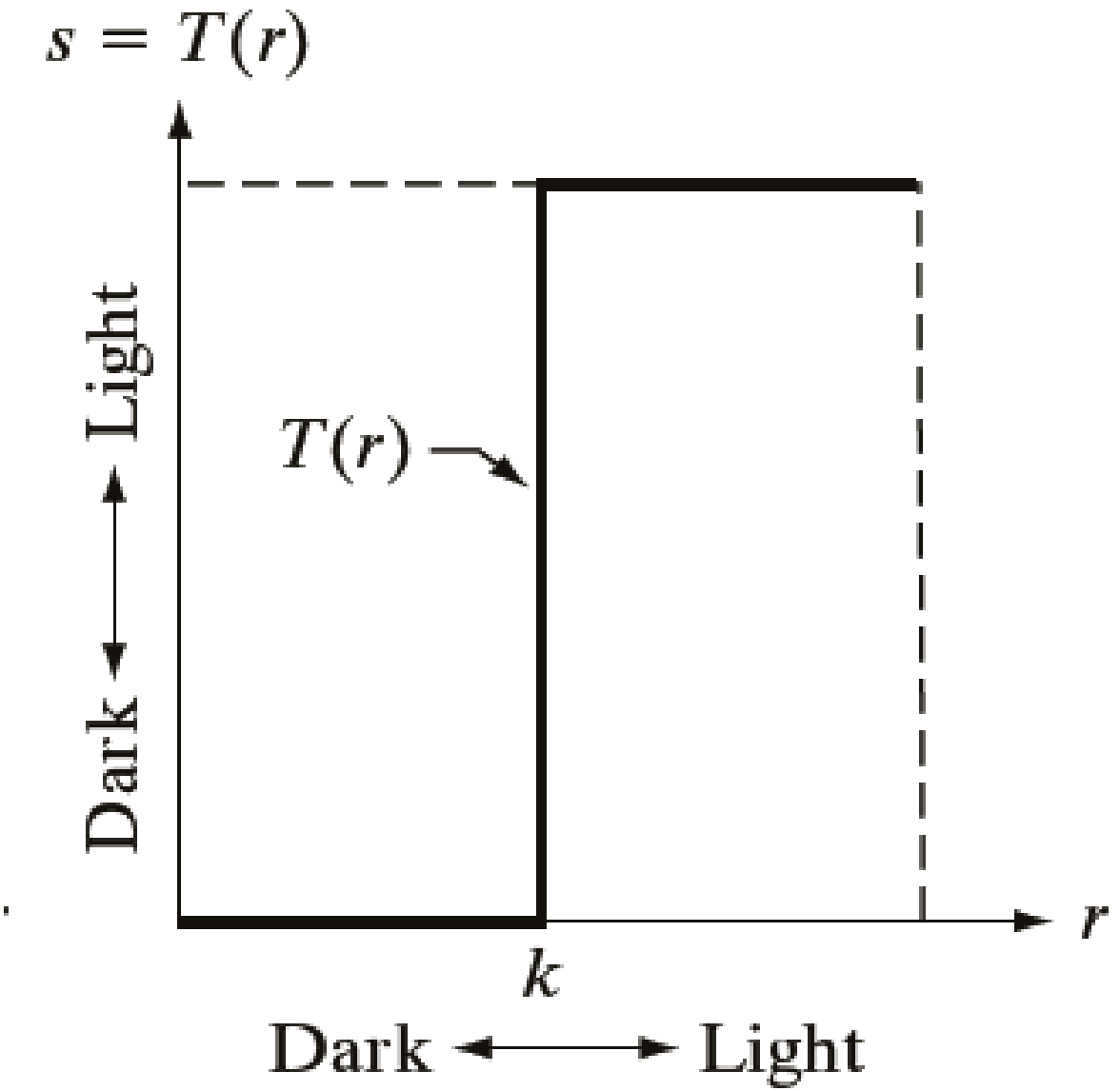
- Negative

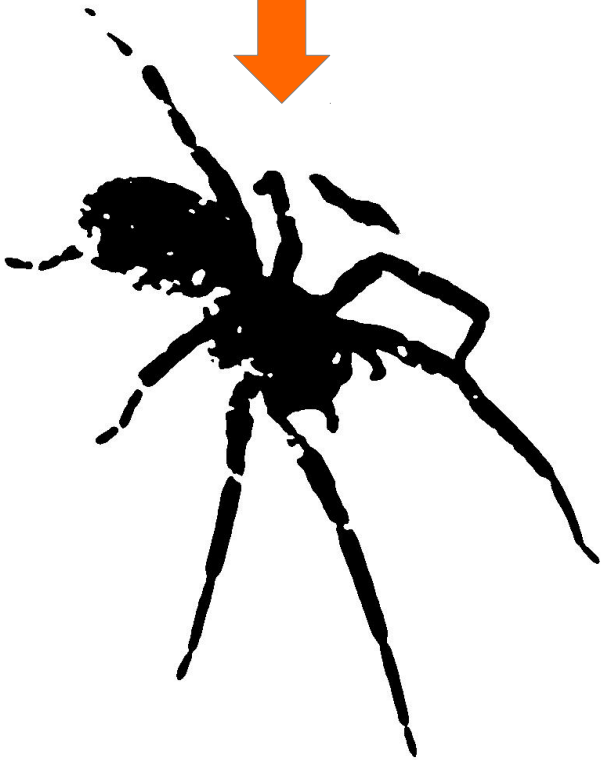




# Gray-Level Transfo

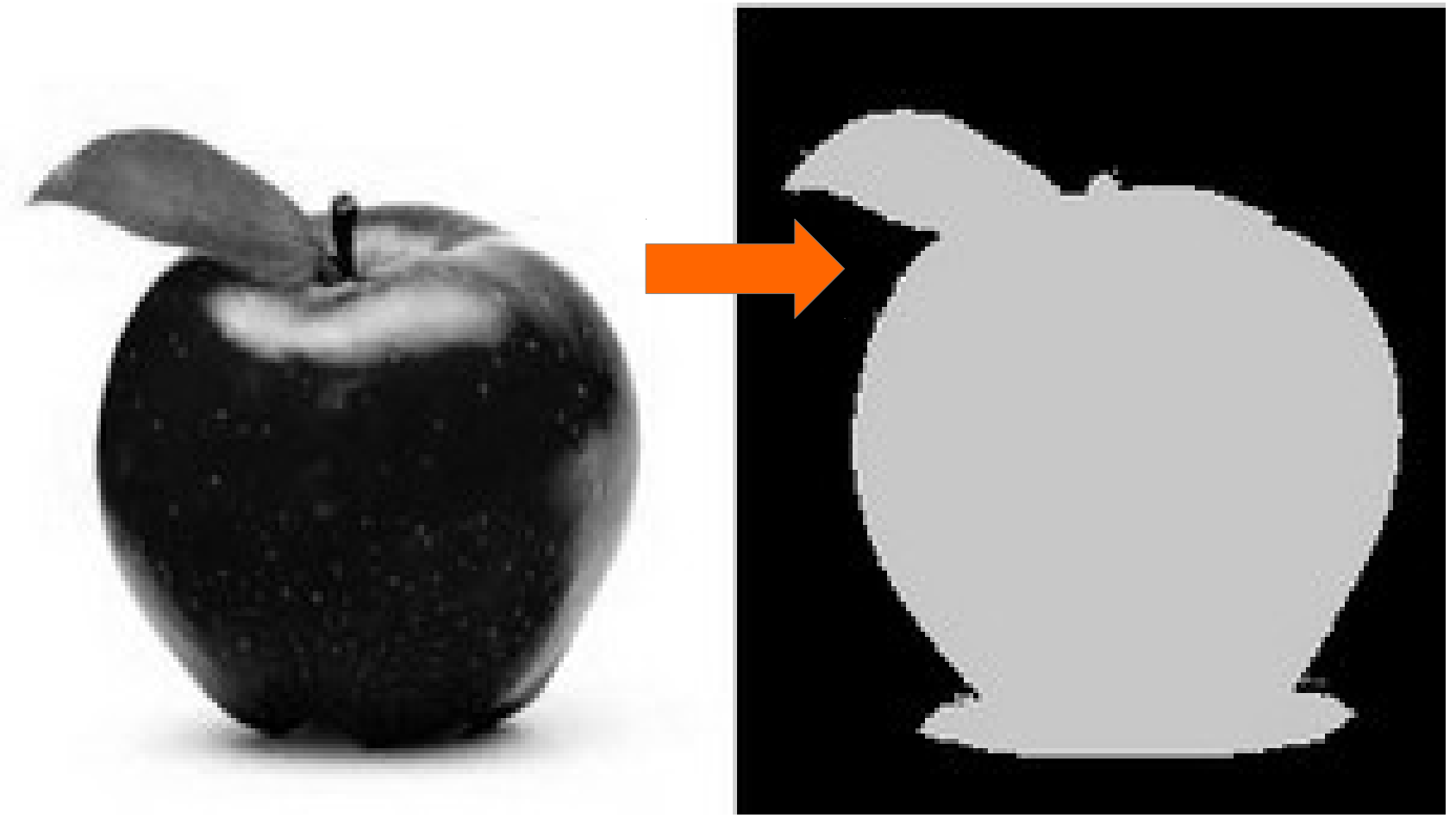
- Thresholding



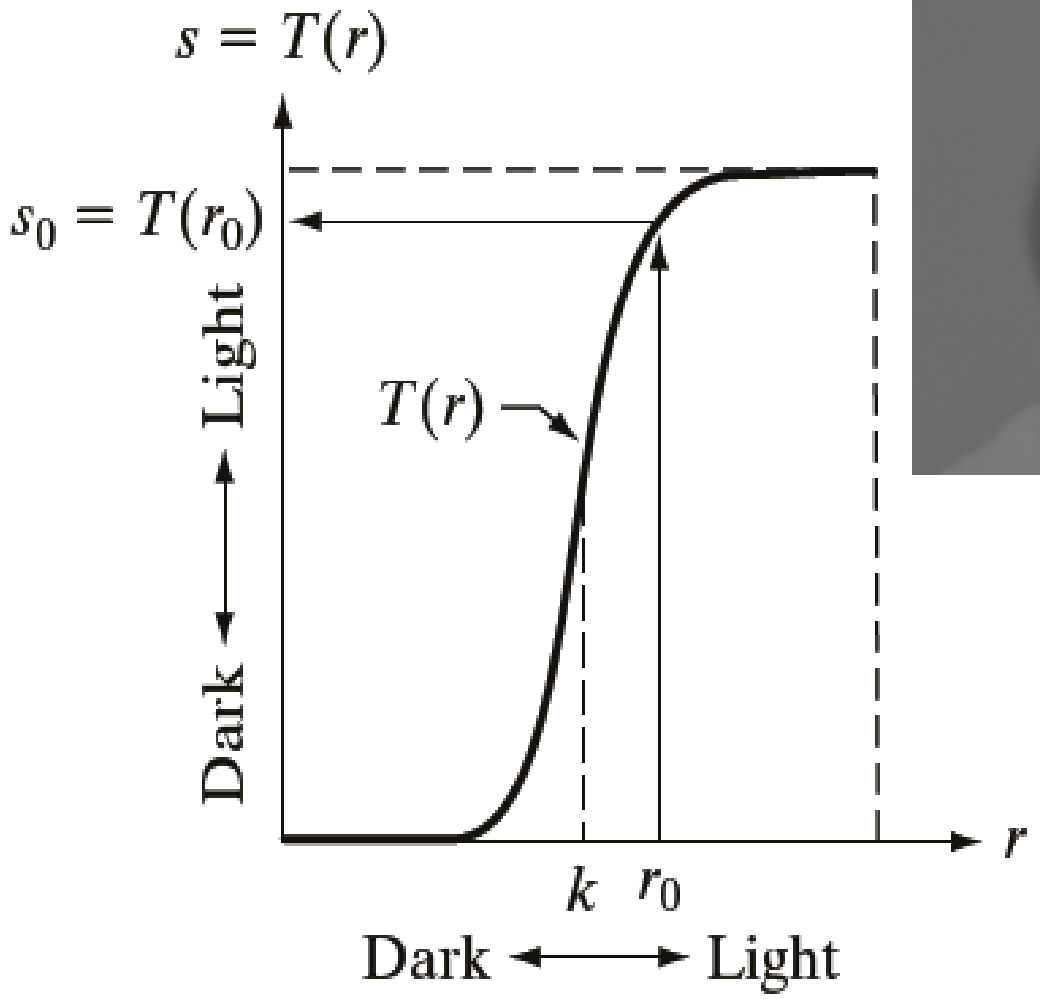
# Gray-Level Transformations

- Thresholding
  - What is the transformation function for this ?



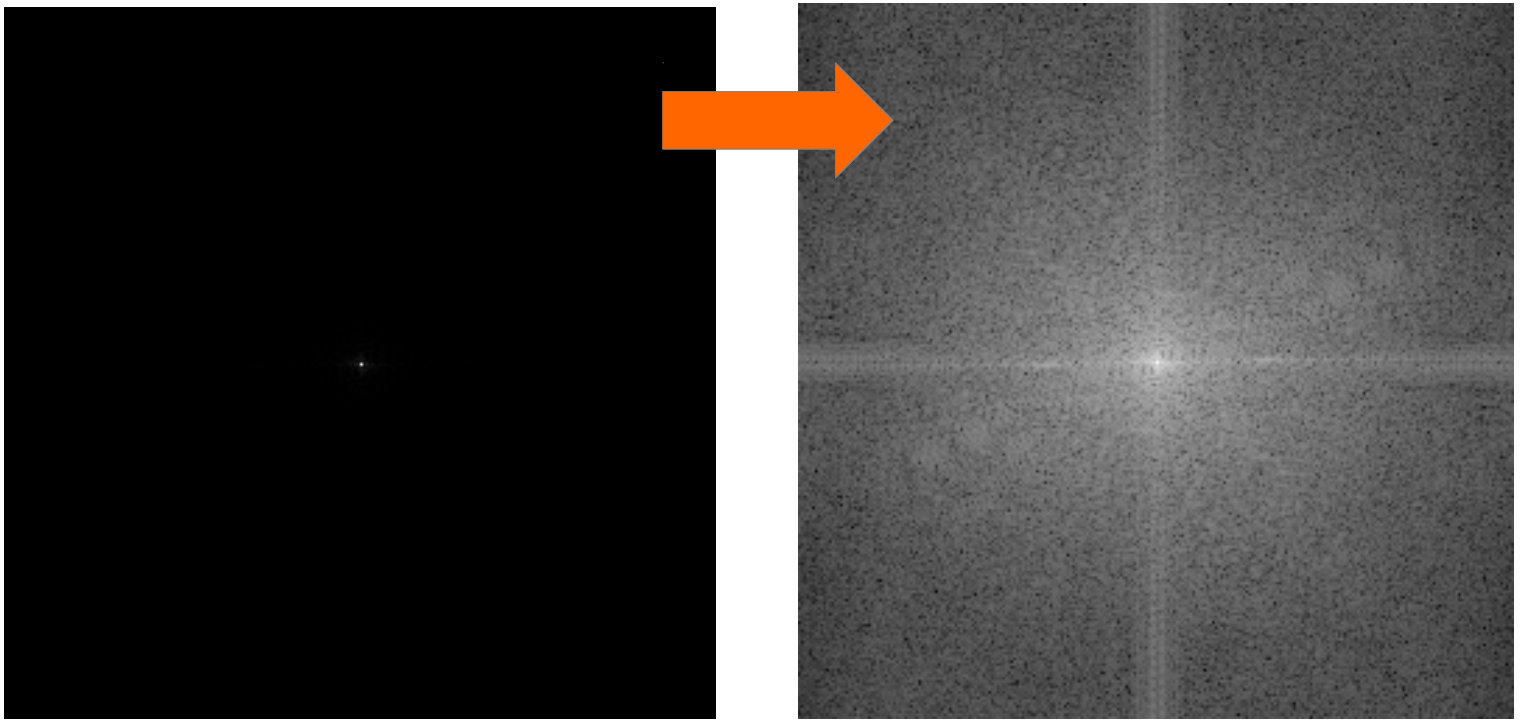
# Gray-Level Transformations

- Contrast stretching



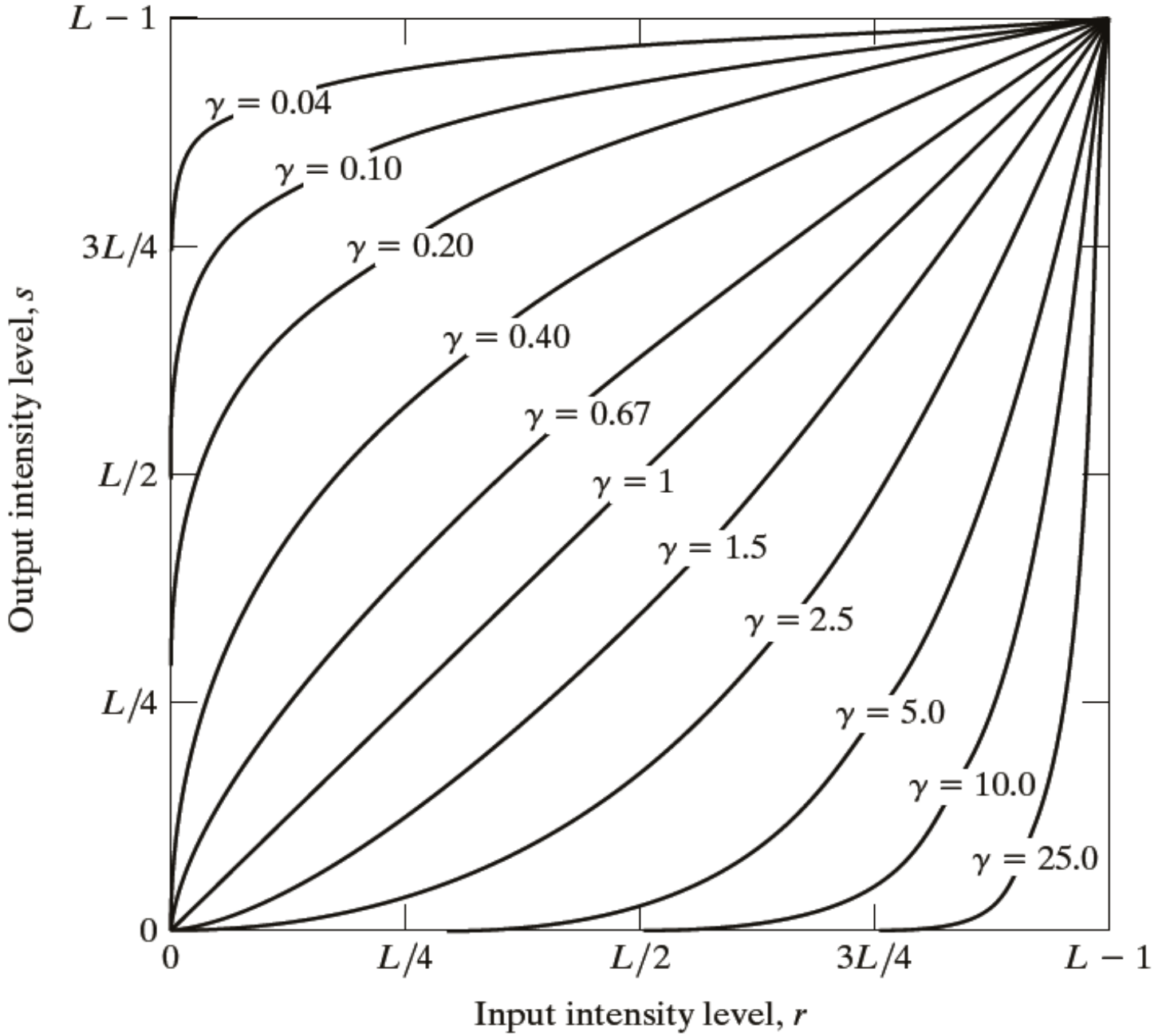

# Gray-Level Transformations

- Logarithm
  - g(x,y) = c log [ 1 + f(x,y) ], where c = constant
  - Input image has some values very large
    - Remaining values map to few colors (left) → details lost
  - Log → compress large values, spread small values



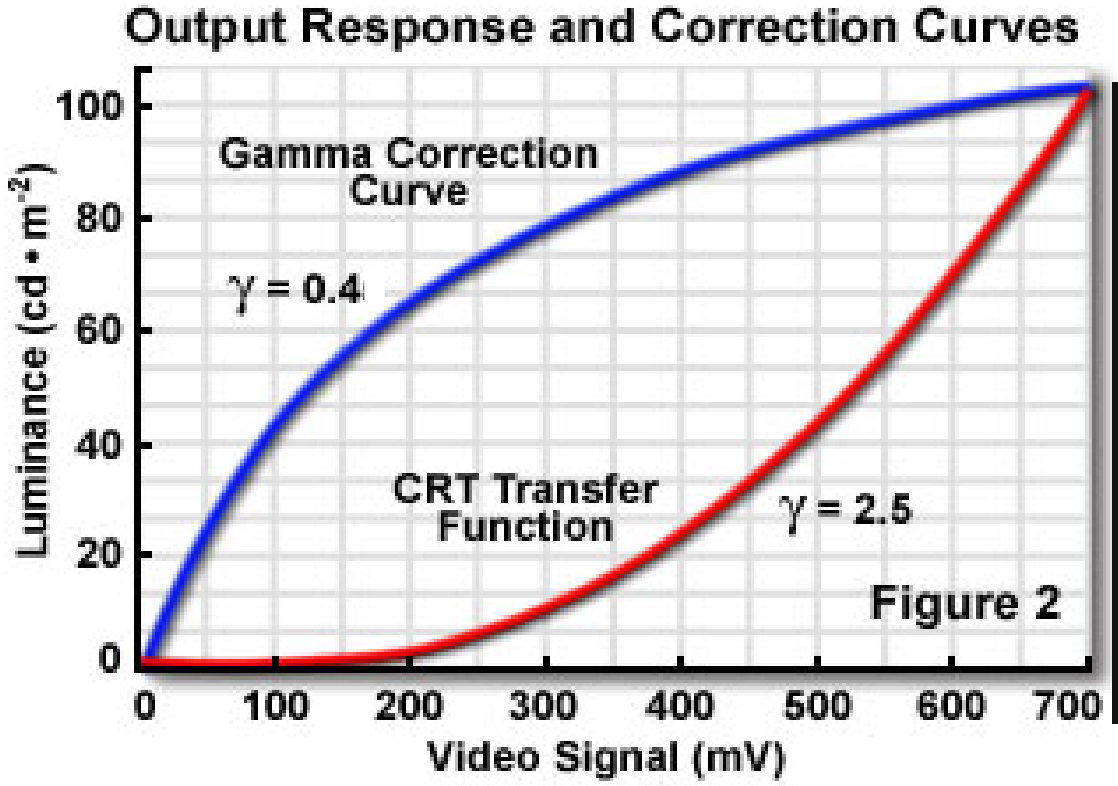
# Gray-Level Transformations

- Power and root transformations
  - s(x,y) = d ( r(x,y) / c)$^{\gamma}$
  - For graph →
    - r in [0,L-1]
    - c = L-1
    - d = L-1



# Gray-Level Transformations

- Gamma correction for display monitors
  - Mapping function from intensity (data) to voltage (screen appearance) is a power function
    - gamma within [1.8, 2.5]
  - Correction applies power transform: gamma' = 1 / gamma

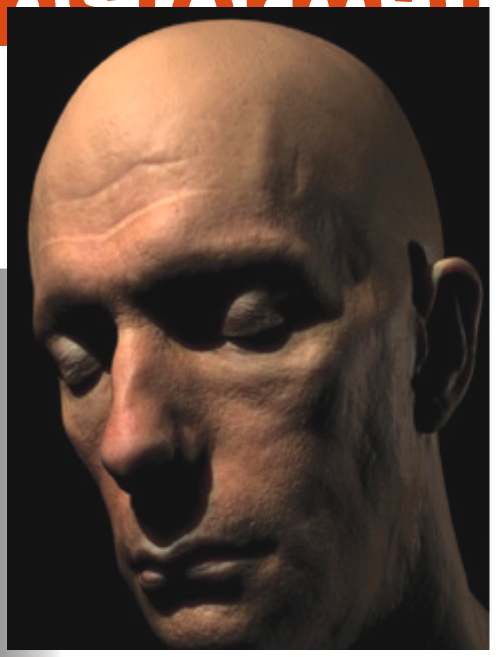



**Without Correction**

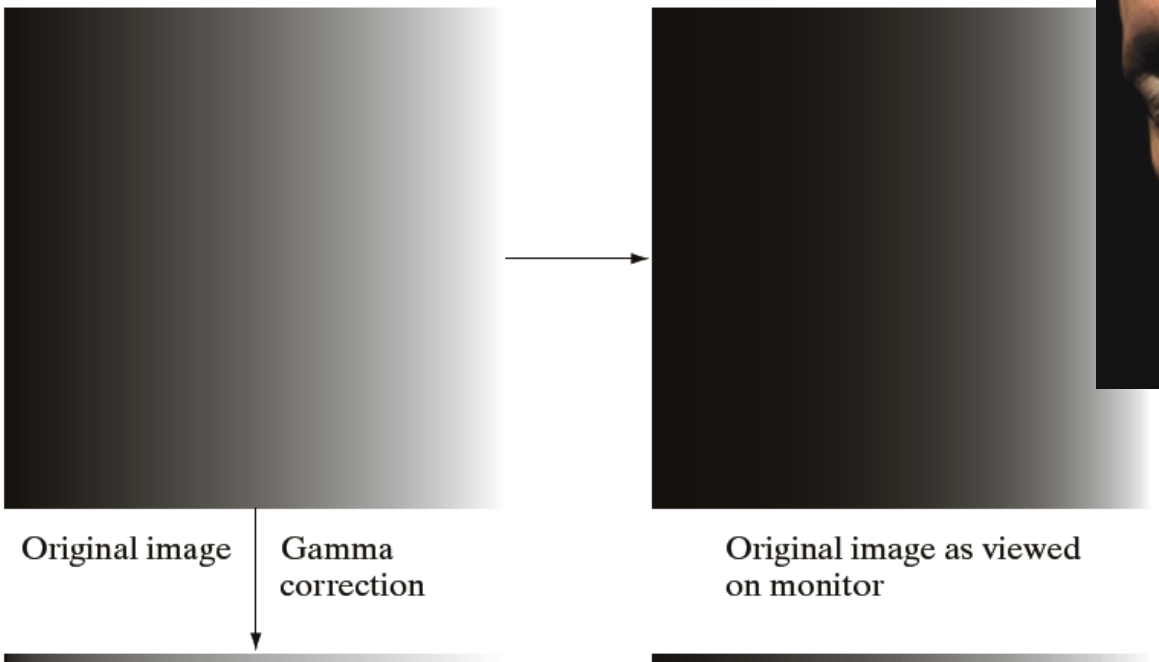
**With Correction**

# Gray-Level Transformations

- Gamma correction

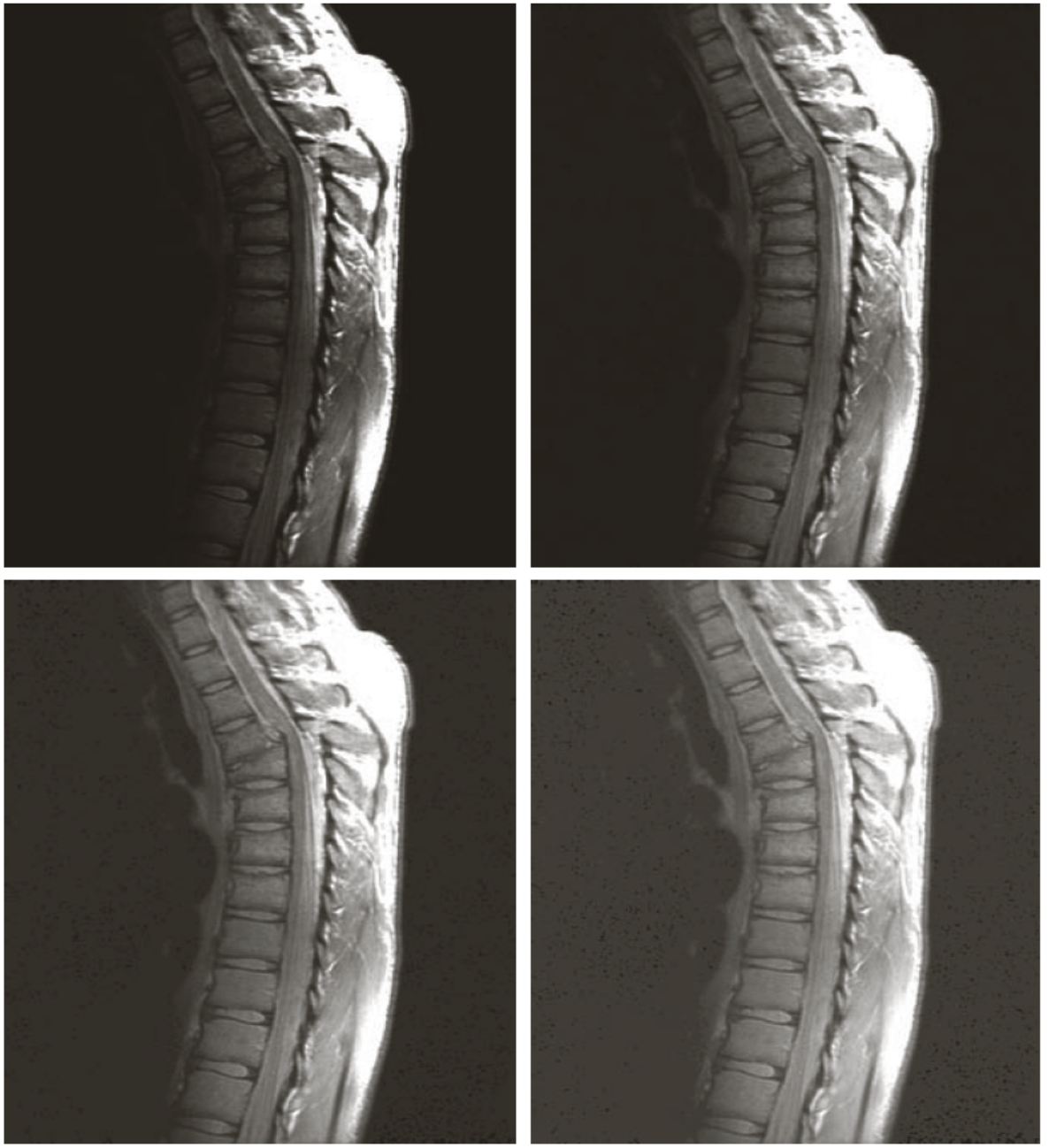






# Gray-Level Transformations

- Gamma correction in medical imaging (e.g., MRI)
  - gamma =
    1.0 , 0.6
    0.4 , 0.3

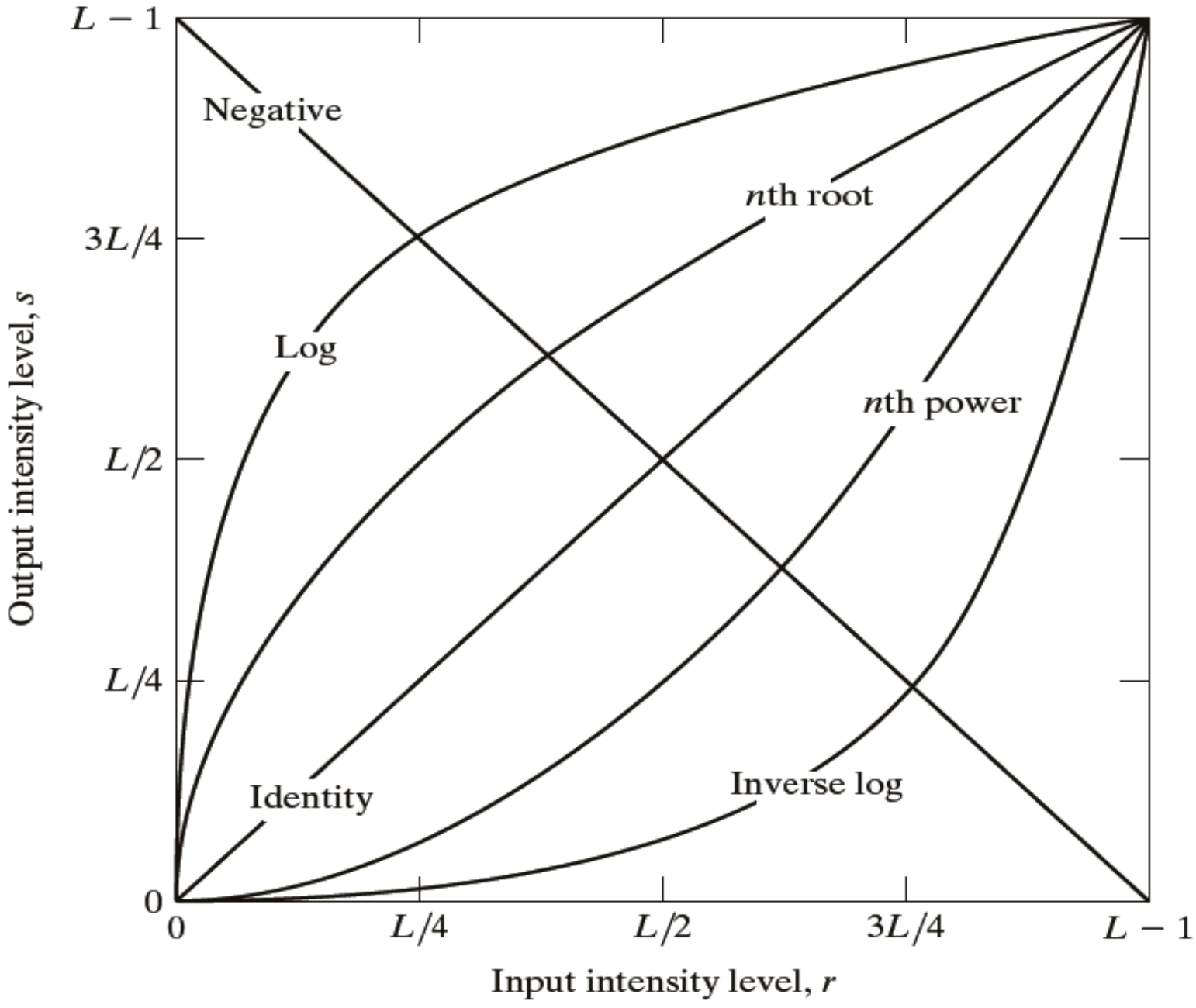


# Gray-Level Transformations

- Gamma correction to improve washed-out images
  - Input image = top left
  - For other images, is gamma more than 1 or less than 1 ?

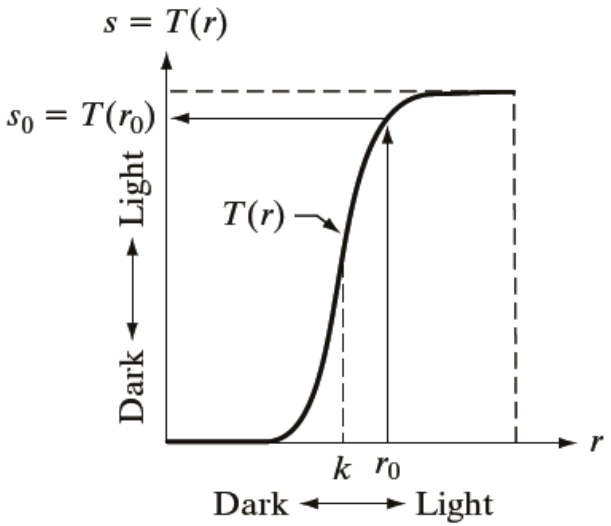
# Gray-Level Transformations

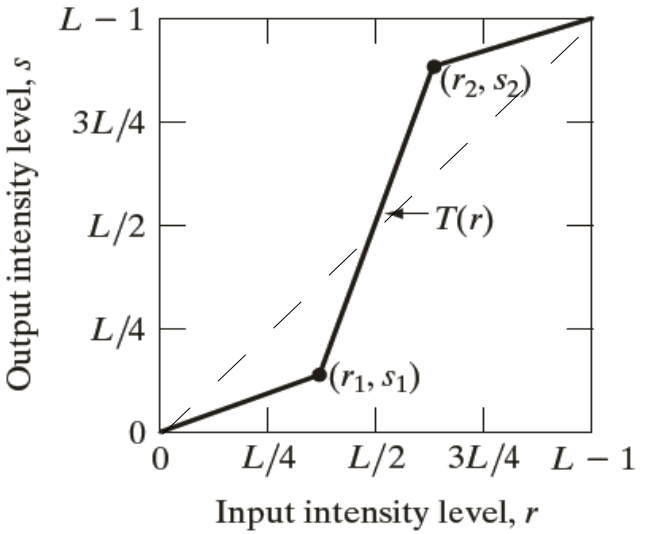
- Typical transformation functions
  - Log, exp
  - Power, root
    - n > 1



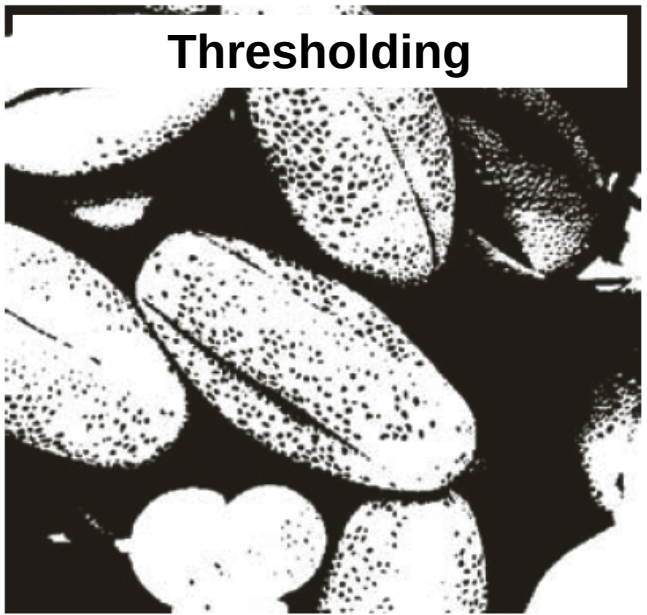
# Gray-Level Transformations

- **Piecewise-linear** transformation functions
  - Easier to design than complex nonlinear functions




  - Can model complex transformations
  - Contrast stretching

# Gray-Level Transformations

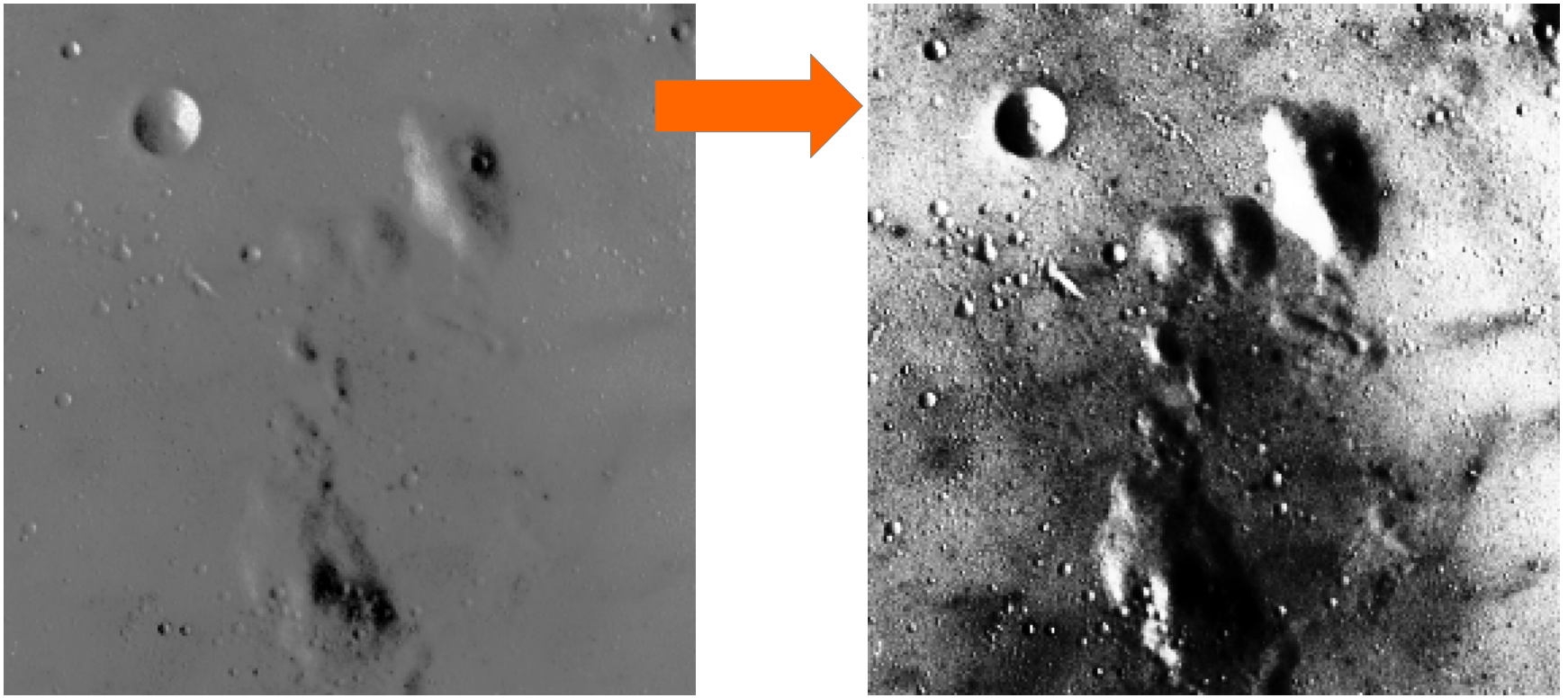
- Histogram processing
  - What is a histogram ?
    - Probability theory refresher
  - What can it achieve ?
    - Contrast enhancement
    - Automatic, without tuning parameters

# Gray-Level Transformations

- Histogram processing
  - What can it achieve ?

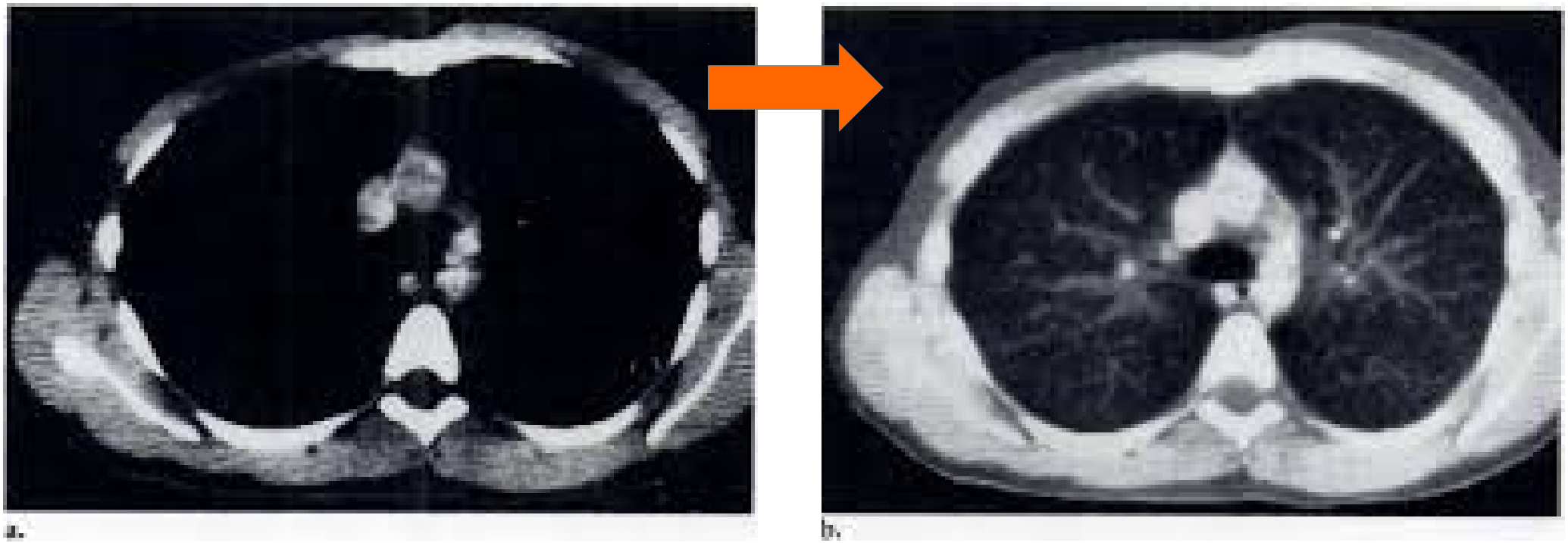
# Gray-Level Transformations

- Histogram processing
  - What can it achieve ?



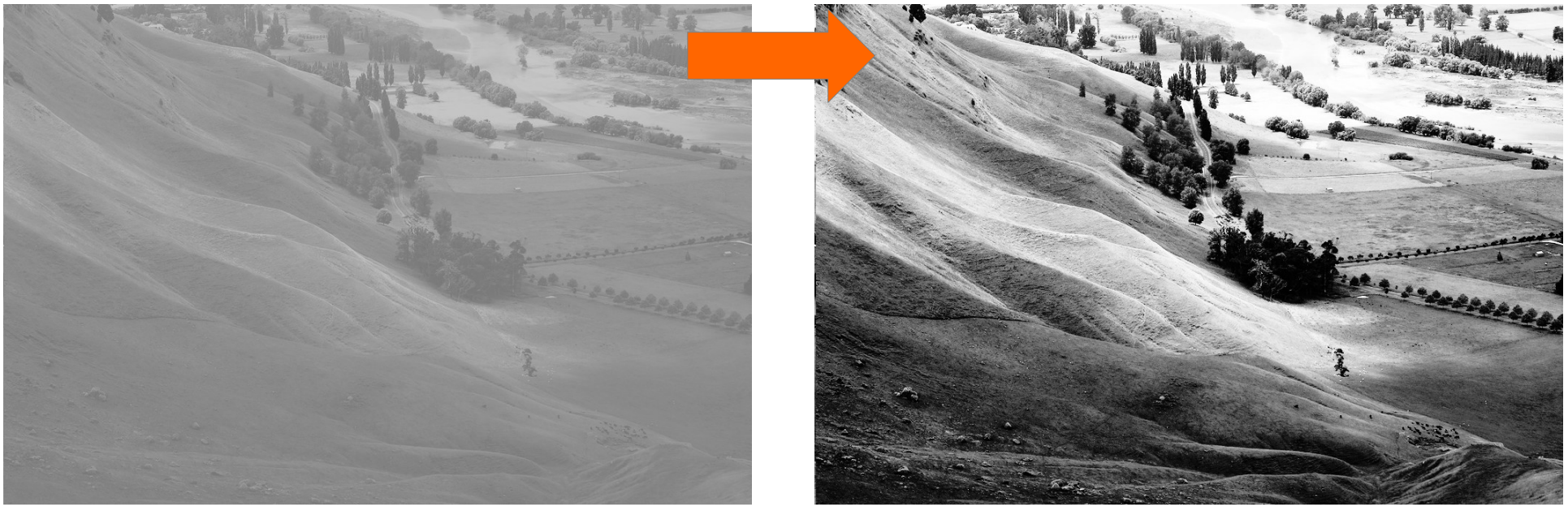
# Gray-Level Transformations

- Histogram processing
  - What can it achieve ?



# Gray-Level Transformations

- Histogram processing
  - What can it achieve ?



# Gray-Level Transformations

- Histogram processing
  - What is a histogram ?
    - Probability theory refresher (very brief)
      - Probability mass/density function
      - Random variables
      - Random experiments
      - Sample space, events, probability functions
      - Transformation of a random variable

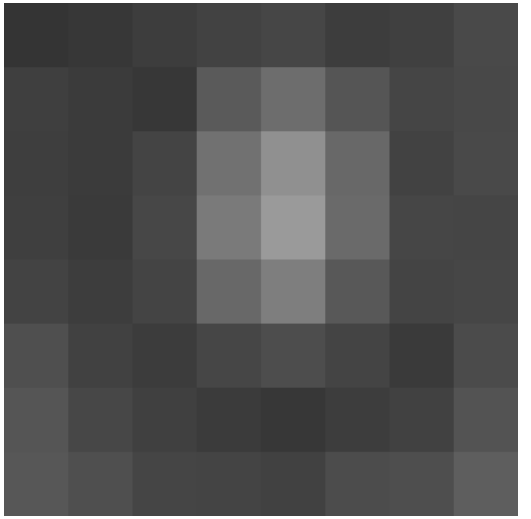
# Gray-Level Transformations

- **Random experiment** = an experiment whose outcome isn't certain
  - Flip of a coin
  - Throw of a die

- **Sample space** = set of all possible outcomes of a random experiment
  - Coin flip : { head, tail }
  - Die throw : { 1, 2, 3, 4, 5, 6 }

# Gray-Level Transformations

- Random experiment
  - Select a pixel location in an image
- Sample space
  - Set of all pixel locations
    { (1,1), (1,2), …, (1,8),
    (2,1), (2,2), …, (2,8),
    …
    (8,1), (8,2), …, (8,8) }



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# Gray-Level Transformations

- **Event** is a subset of a sample space
  - Event of getting a head on a coin toss = { head }
  - Event of getting an even number on a die roll = ?
    = { 2, 4, 6 }
  - Event of getting a pixel in the left half of the image

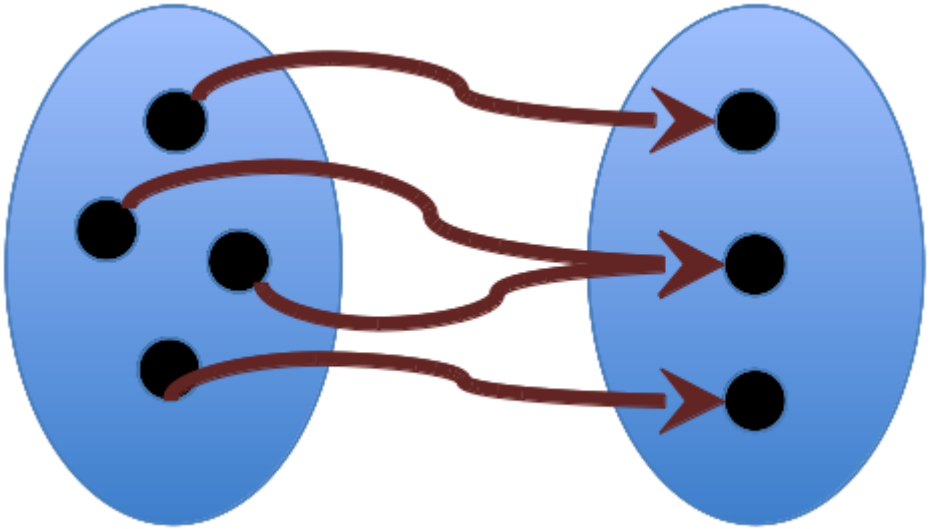
- **Event space** = set of all possible events

# Gray-Level Transformations

- **Probability function P(.)**
  - A probability function on **sample space Ω** **assigns every event A** in Ω a **number** in [0,1] s.t.
    - P (Ω) = 1
    - P (A U B) = P (A) + P (B) when A ∩ B = 0
  - P(A) is the **probability** that **event** A **occurs**
  - Examples
    - Probability function for coin toss = ?
    - Probability function for die roll = ?

# Gray-Level Transformations

- **Random variable** = a function
  - $X : \Omega \rightarrow R$
  - Domain = sample space
  - Range = set of real numbers



# Gray-Level Transformations

- Random variable is an **abstraction**
  - We don't care about outcomes (directly)
  - We only care about values mapped to outcomes
  - Die example
    - We don't care about die value, just even or odd
  - Image example
    - We don't care about pixel location, just pixel intensity

# Gray-Level Transformations

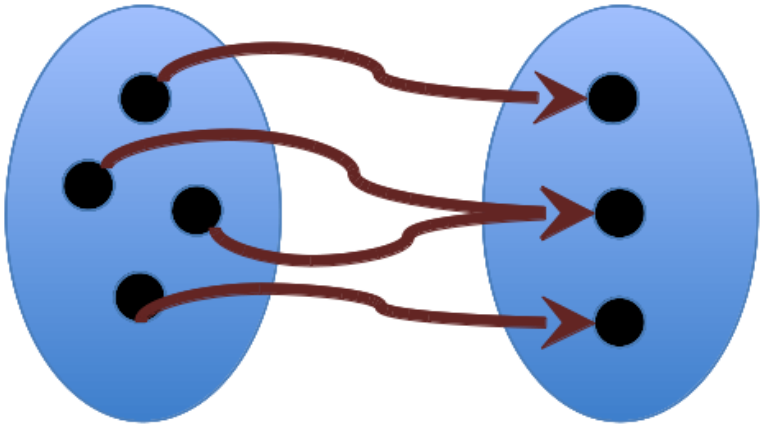
- Defining events using random variables
  - Example: Sum of a pair of dice
  - Sample Space: $\Omega = \{ (i,j) : 1 <= i,j <= 6 \}$
  - Probability function: $P ( (i,j) ) = 1/36$
  - Random variable: $X ( (i,j) ) = i + j$
  - Event $\{ X=a \} = \{ w \in \Omega : X(w)=a \}$
    - $P (X = 5)$
      $= P ( \{ (1,4), (2,3), (3,2), (4,1) \} )$
      $= P ((1,4)) + P ((2,3)) + P ((3,2)) + P ((4,1))$
      $= 4 / 36$

# Gray-Level Transformations

- Defining events using random variables
  - Example:
    Intensity at a randomly-chosen pixel in 8x8 image f(x,y)
  - Sample Space: $\Omega = \{ (x,y) : 1 <= x,y <= 8 \}$
  - Probability function: $P ( (x,y) ) = 1/64$
  - Random variable: $Z ( (x,y) ) = f (x,y)$
  - Event $\{ Z > 100 \} = \{ w \in \Omega : Z(w) > 100 \}$

# Gray-Level Transformations

- **Discrete** random variable (RV)
  - Maps outcomes to values in a countable set
  - **Probability mass function** $p : R \rightarrow [0,1]$
    - Probability that the RV takes a value 'a'
    - $p(a) = p(X=a) = \sum_{w : X(w) = a} P(w)$
  - **Cumulative distribution function** $F : R \rightarrow [0,1]$
    - Probability that the RV takes a value **less than** x
    - $F(a) = p(X <= a) = \sum_{w : X(w) <= a} P(w)$
    - $F(a) = p(X <= a) = \sum_{b : b <= a} p(X=b)$

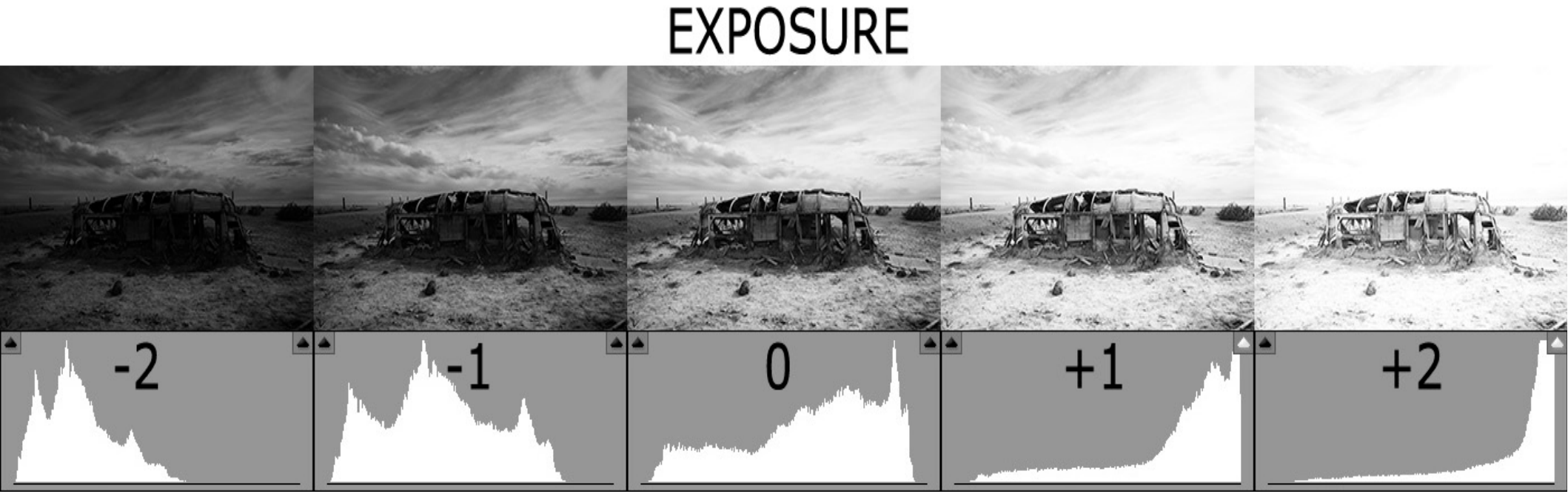


# Gray-Level Transformations

- **Continuous** random variable (RV)
  - Maps outcomes to values in an uncountable set
    - Sample space itself is uncountable
      - e.g., continuous image function X(.) defined on [0,1]x[0,1]
  - **Cumulative distribution function** $F : R \rightarrow [0,1]$
    - Probability that the RV takes a value less than c
      - $F(c) = p(x < c) = \int_{w : X(w) <= c} P(w)\, dw$
      - $F(c) = p(x < c) = \int_{x<c} p(x)\, dx$
  - **Probability density function** $p : R \rightarrow R$
    - p (.) is derivative of F (.)
    - p (x) is NOT probability (p(x) can be > 1)
    - Events, of interest, are subsets of the form { x : a < x < b }
    - $p(a < x < b) = \int_{a<x<b} p(x)\, dx$

# Gray-Level Transformations

- Histograms are related to photograph exposure
  - Aperture size (amount of light per unit time)
  - Shutter speed (time of exposure)



# Gray-Level Transformations

- Histograms are related to photograph exposure
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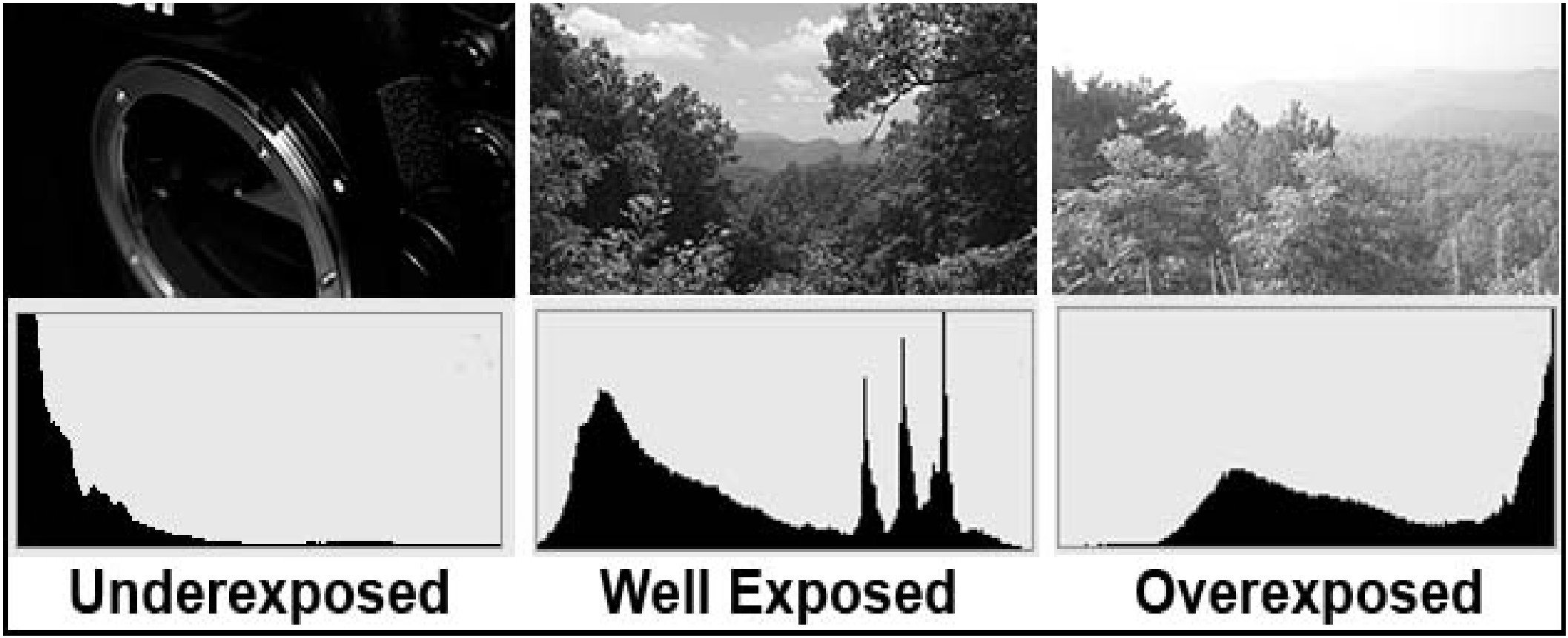
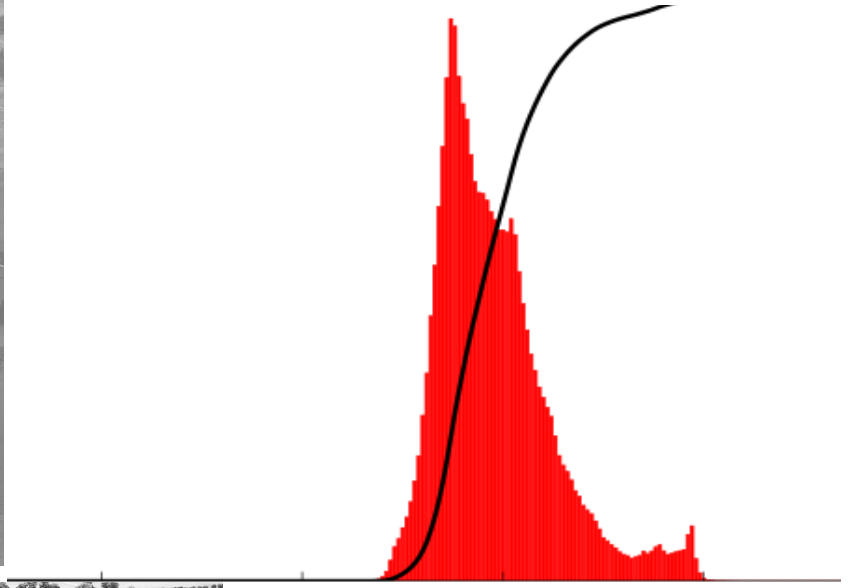
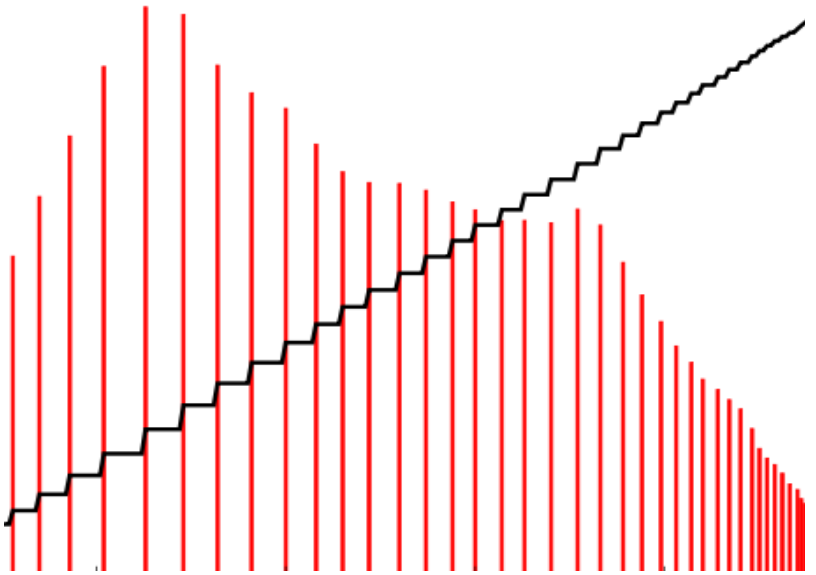
  - Shutter speed



# Gray-Level Transformations

- Histogram equalization : Motivation





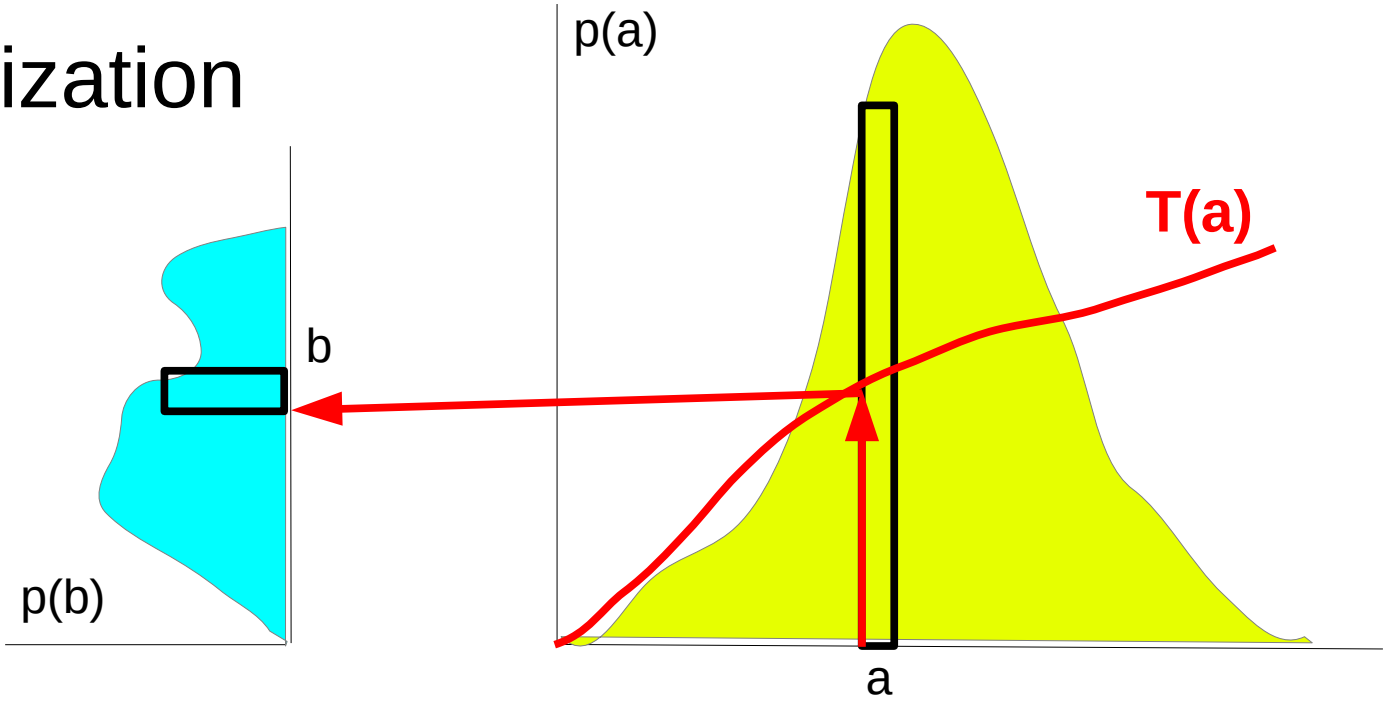
# Gray-Level Transformations

- Histogram equalization
  - Assume **continuous** distributions (and image)
  - Given histogram p (a)
  - **Design function T (a) = b**
    **such that**
    **p (b) = uniform distribution**
  - Assume $0 < a < 1$, $0 < b < 1$
  - Then, we want p (b) = 1

  - Consider a **monotonically increasing** intensity-transformation function T(.)

# Gray-Level Transformations

- Histogram equalization



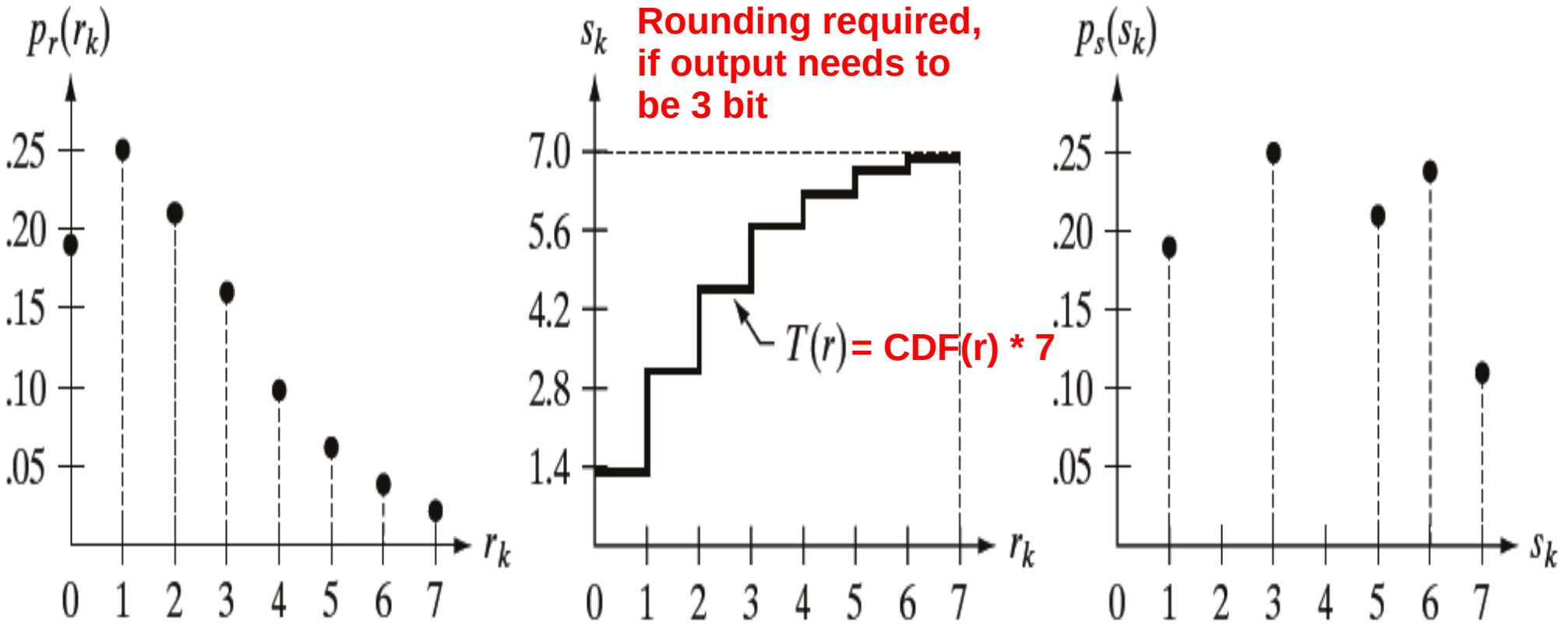

  - Let b = T(a)
  - **Mass conservation** implies: p(a) da = p(b) db
  - Integrate left hand side from 0 to a'
  - Integrate right hand side from 0 = T (a') = b'
  - Left hand side = $CDF_a$ (a')
  - Right hand side = $CDF_b$ (b')

# Gray-Level Transformations

- Histogram equalization
  - If we want p (b) = 1, then $CDF_b$ (b') = b'
  - So, $CDF_a$ (a') = $CDF_b$ (b') = b'
  - So, transformation function T (.) =
    CDF of intensities in original image $CDF_a$ (.)

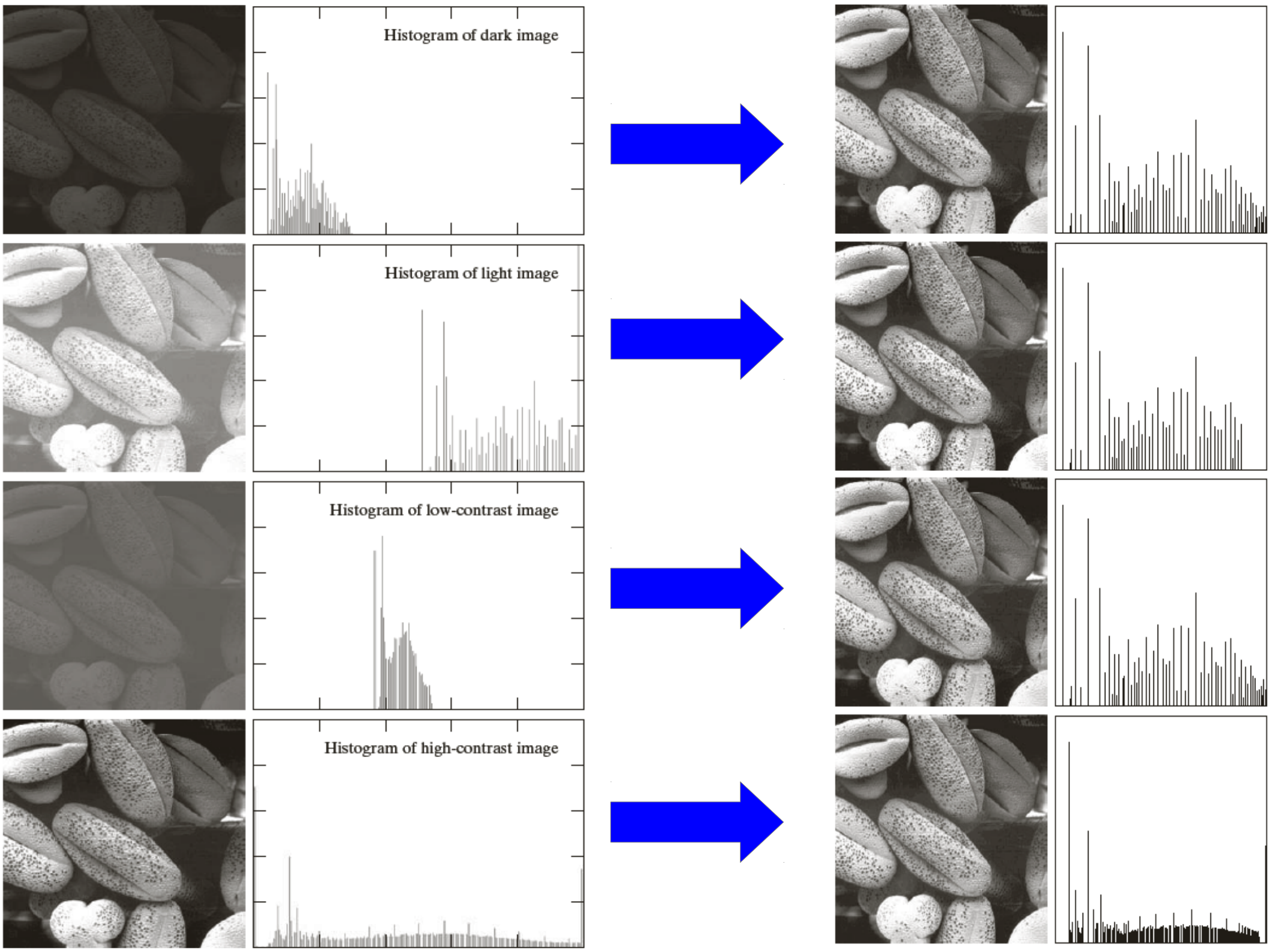
# Gray-Level Transformations

- Histogram equalization example (3-bit image)

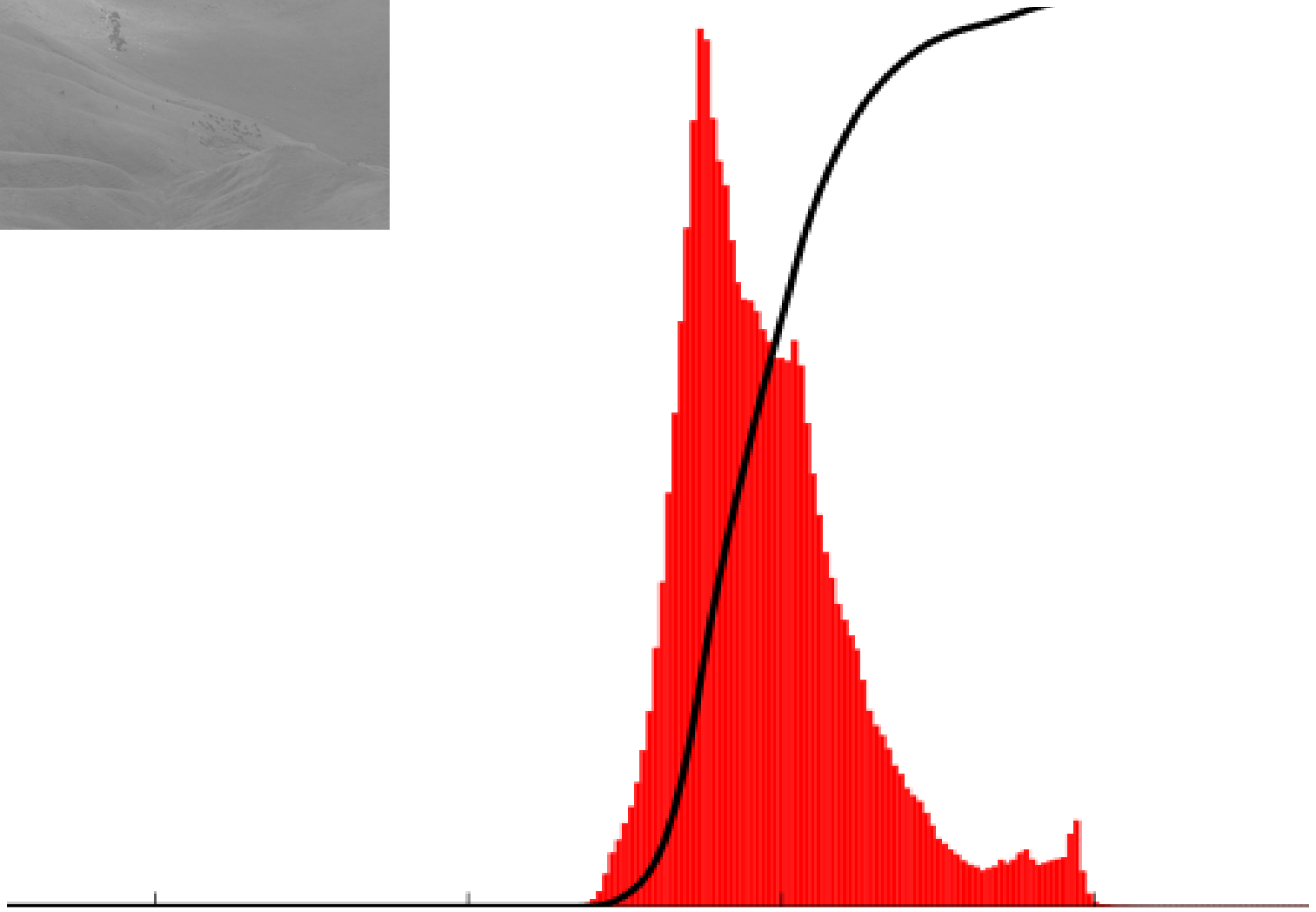



a b c

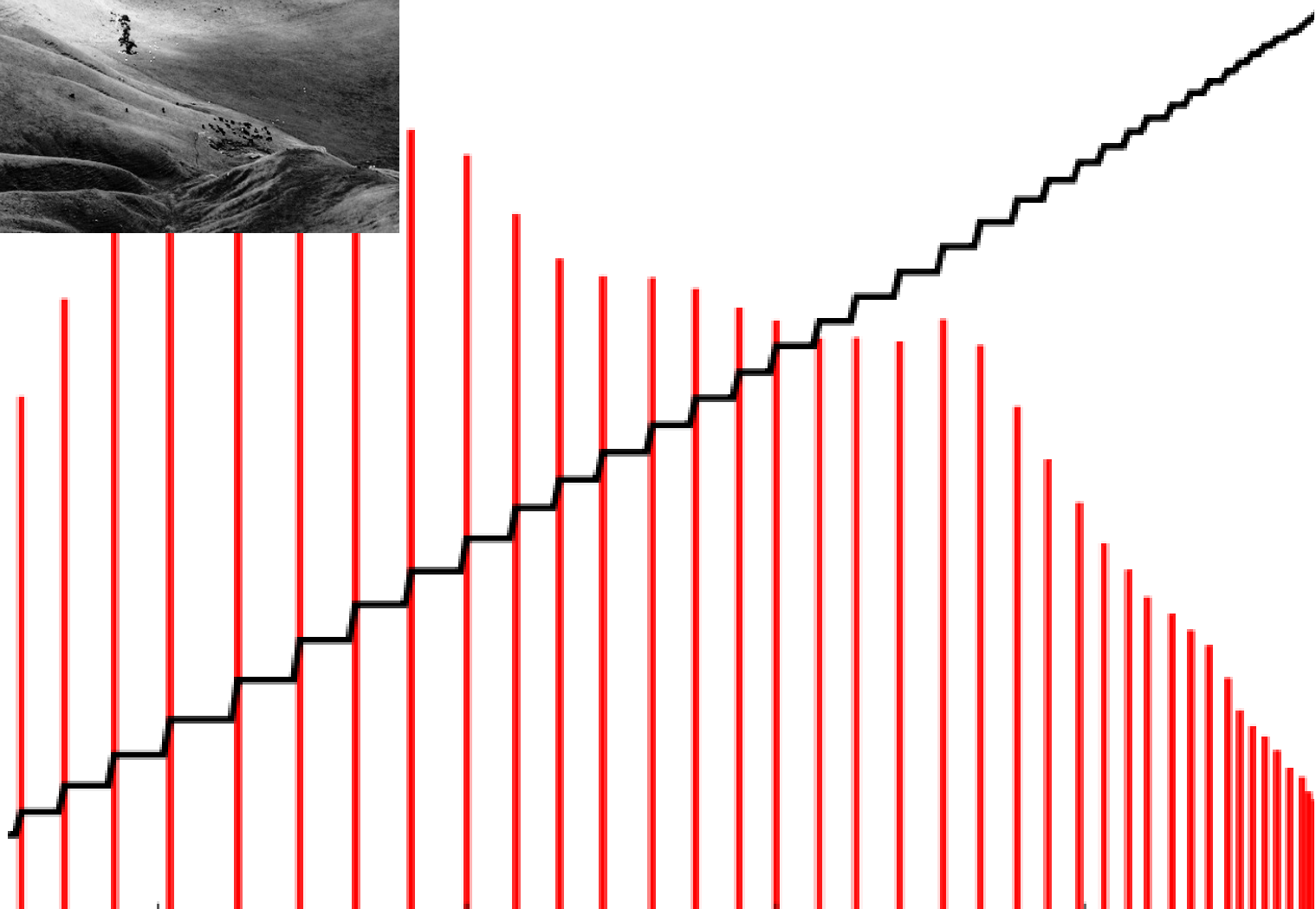
**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.


Histogram of dark image
Histogram of light image
Histogram of low-contrast image
Histogram of high-contrast image

# ansformations



# ansformations



# Gray-Level Transformations

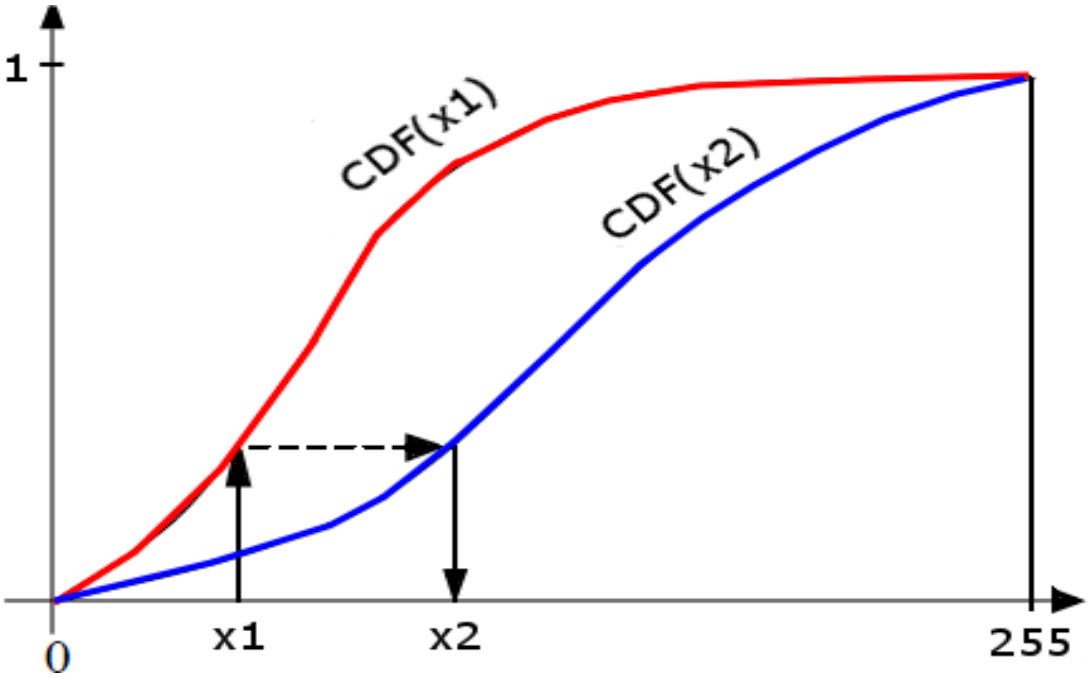
- Why is the output histogram NOT exactly uniform ?
  - Because of discretization of space and intensity
  - Theoretically
    - Contradicts the assumption of continuous distributions (i.e., continuous spatial domain, continuous intensities)
    - We used that to derive transformation for histogram equalization
  - Practical problem (1 of 2)
    - Imagine 8-bit images → 256 intensities
    - Imagine an input image containing only 100 different intensities
    - Uniform histogram must contain **some** pixels for each intensity
    - How can we map 100 values to 256 values ?
      - No systematic and straightforward way to do that

# Gray-Level Transformations

- Why is the output histogram NOT exactly uniform ?
  - Because the set of intensities in digital images is discrete
  - Practical problem (2 of 2)
    - Imagine 8-bit images → 256 intensities
    - Imagine an input image containing 256 different intensities and 2560 pixels
    - Uniform histogram must contain **an equal number of (10)** pixels for each intensity
    - If some intensity “a” in input image has 20 pixels, then intensity b=T(a) in output image will have >= 20 pixels
    - How can we decide which 10 of those >=20 pixels should be assigned intensity “b” ?
      - No systematic and straightforward way to do that

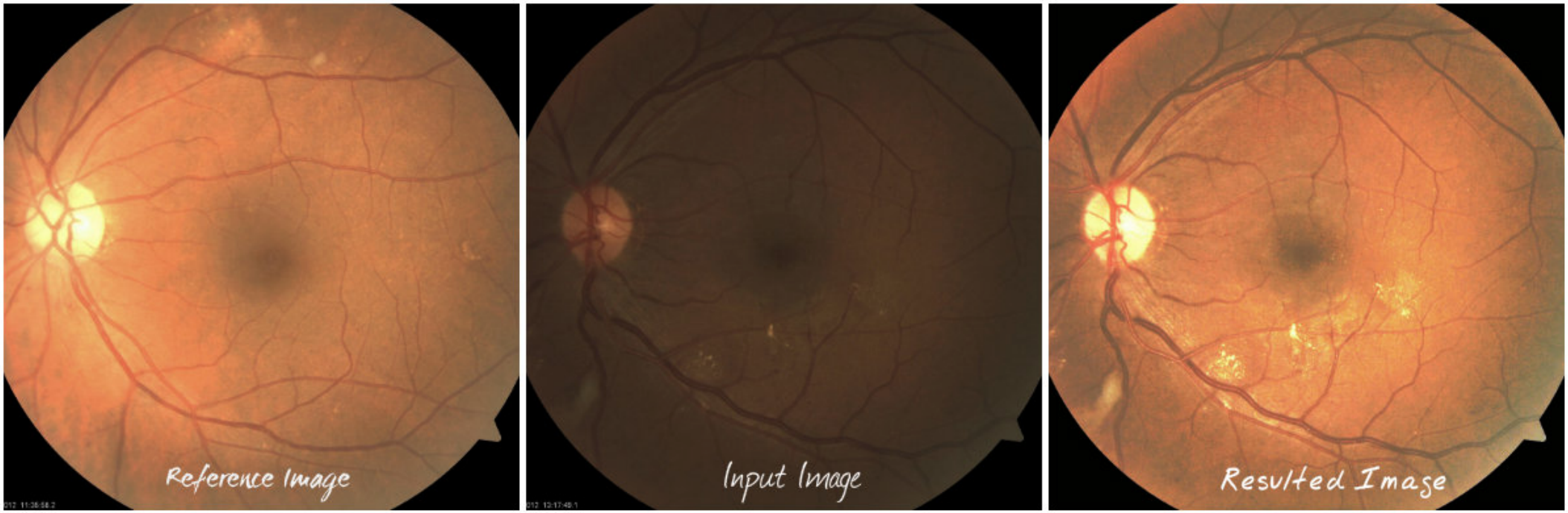
# Gray-Level Transformations

- Histogram **matching**
  - Sometimes, we don't want to equalize
  - But, we want to match the histogram of one image to another chosen histogram (NOT uniform)
    - e.g., for standardizing the intensity
  - In that case,
    $CDF_a(a) = CDF_b(b)$
    implies
    $b = CDF_b^{-1}(CDF_a(a))$



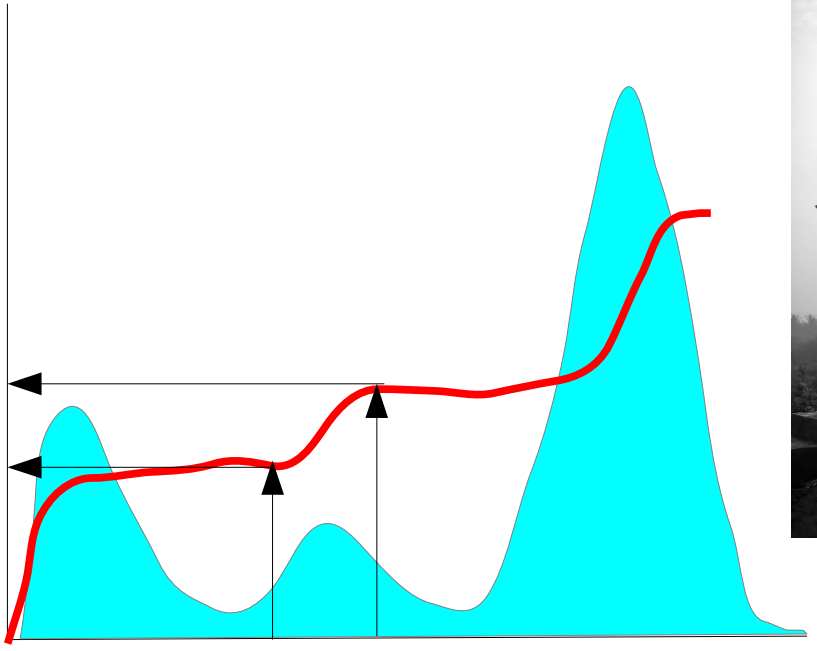
# Gray-Level Transformations

- Histogram **matching**



# Gray-Level Transformations

- Limitations of histogram equalization
  - Images has lots of dark pixels or / and lots of bright pixels
  - Then, the “middle” intensities don't get contrast enhanced



# Gray-Level Transformations

- Limitations of histogram equalization
  - Images has lots of dark pixels or / and lots of bright pixels
  - Then, the “middle” intensities don't get contrast enhanced
  - How can we get better output ? e.g., top right image

# Gray-Level Transformations

- **Adaptive histogram equalization (AHE)**
  - **Localized analysis**
  - Algorithm:

    For each pixel “p”
    - Construct a window of size NxN around the pixel
    - Perform histogram equalization within that window

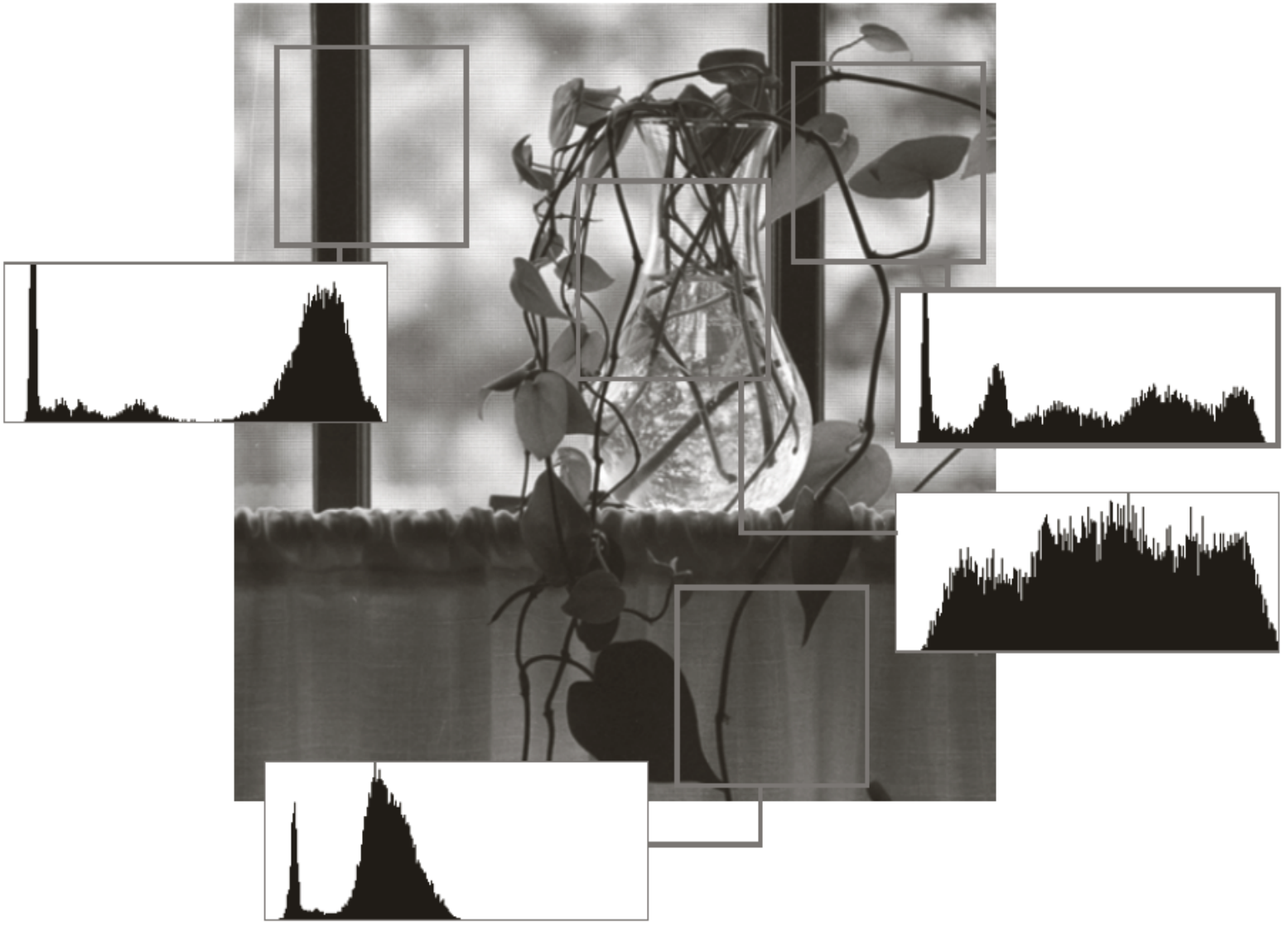
      (1) Compute histogram within window

      (2) Compute CDF within window

      (3) Map intensity of center pixel “p” based on the CDF
  - Window size N is a user-defined parameter
    - User-defined parameter is also called “free” parameter
    - How to choose N ?

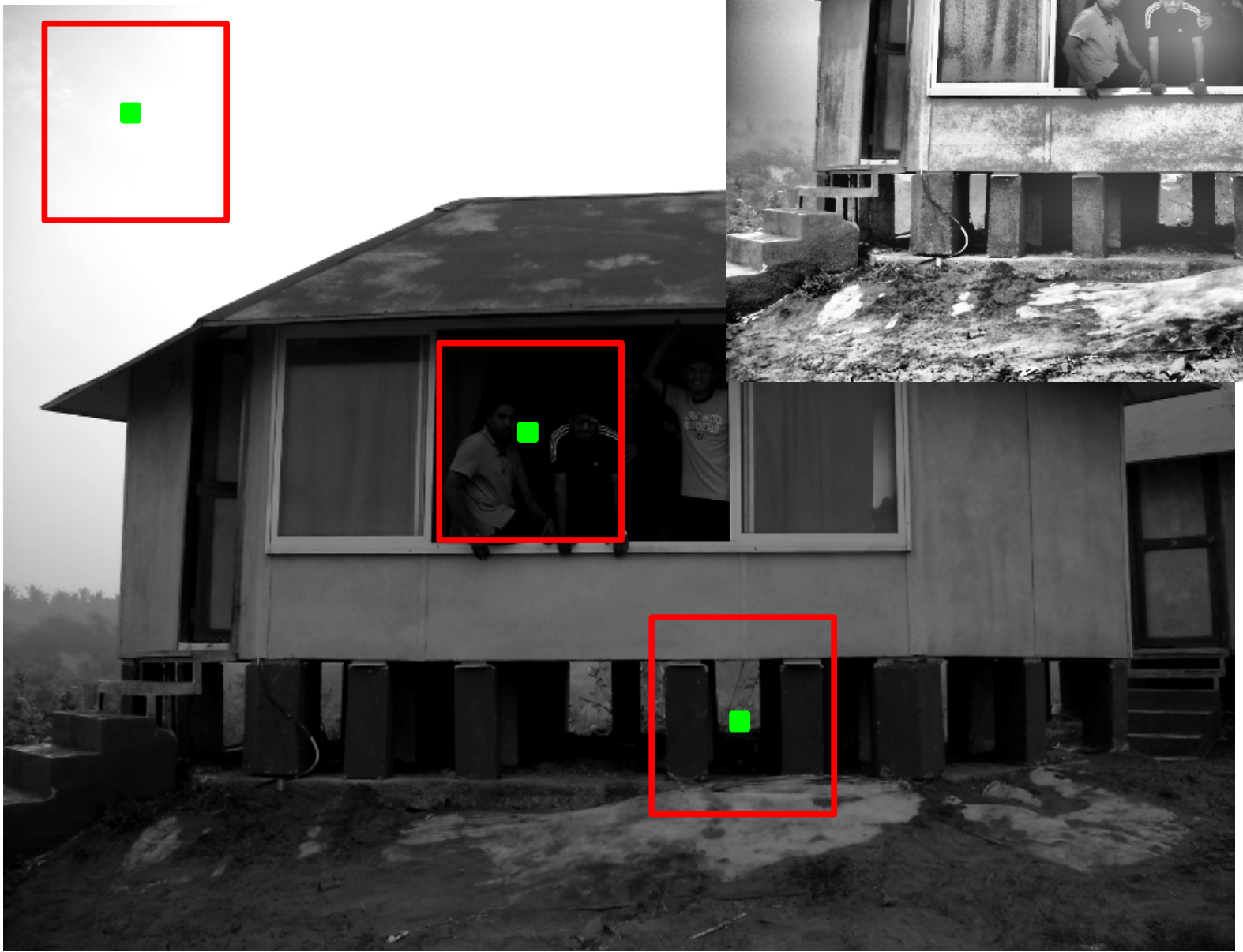
# Gray-Level Transformations

- Adaptive histogram equalization (AHE)
  - Variations in local histograms



# Gray-Level

- Adaptive histogram ec

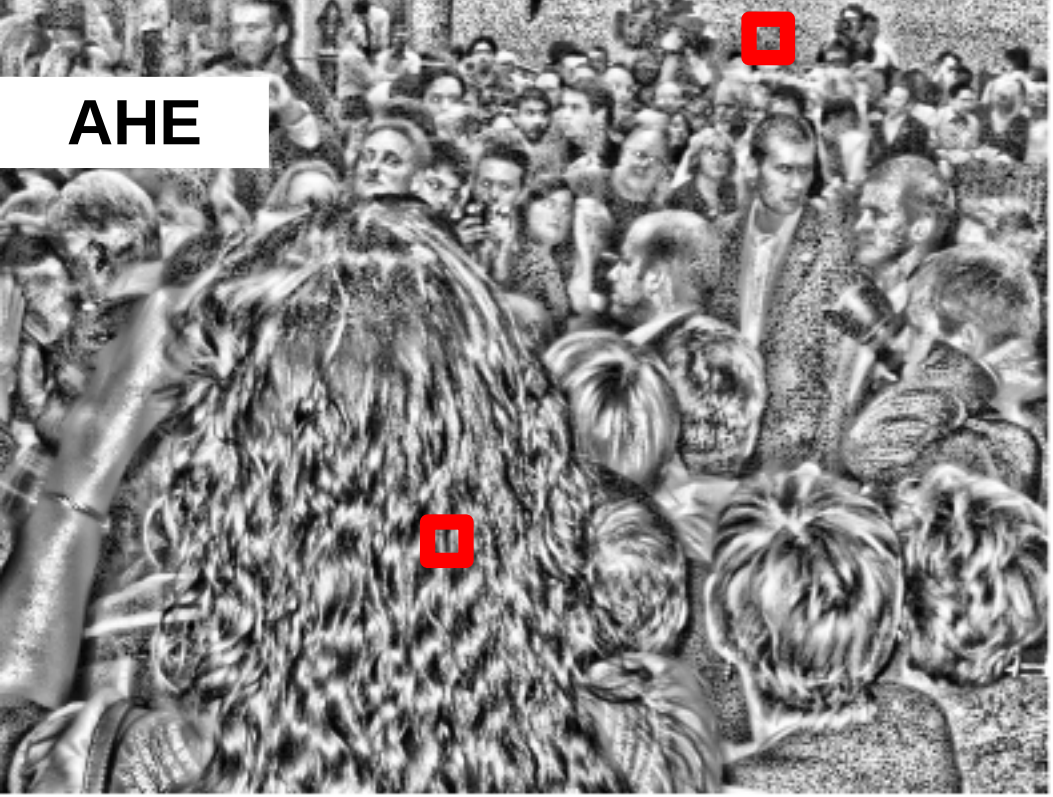
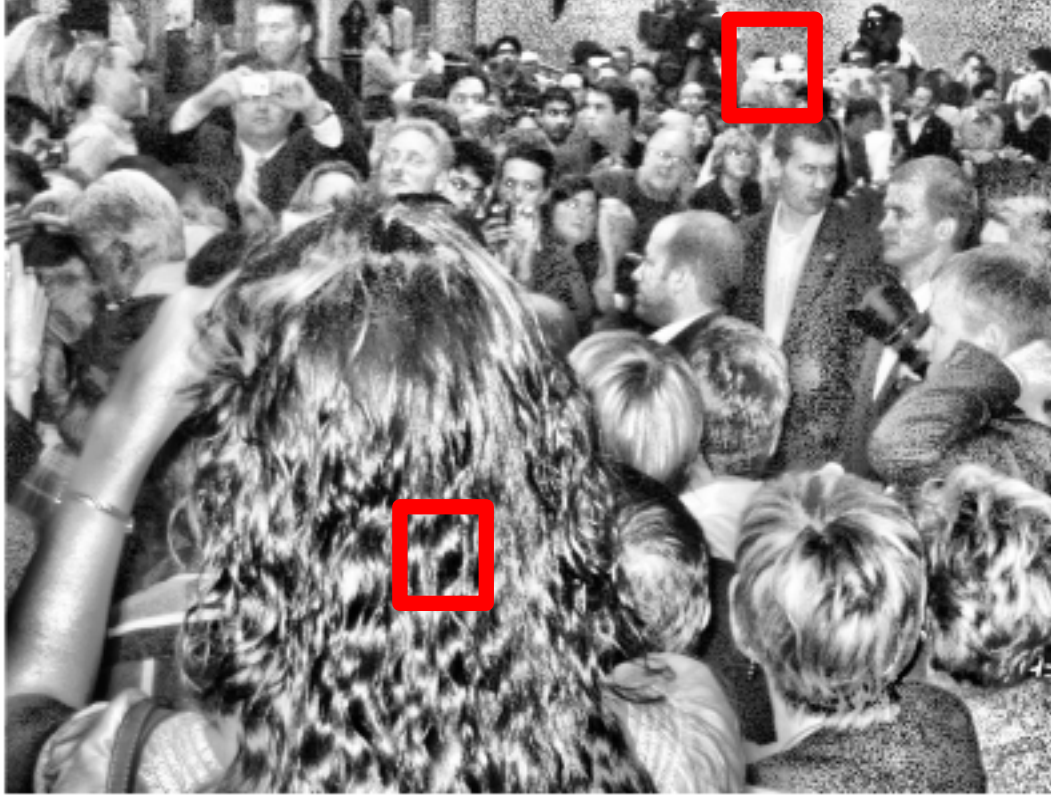
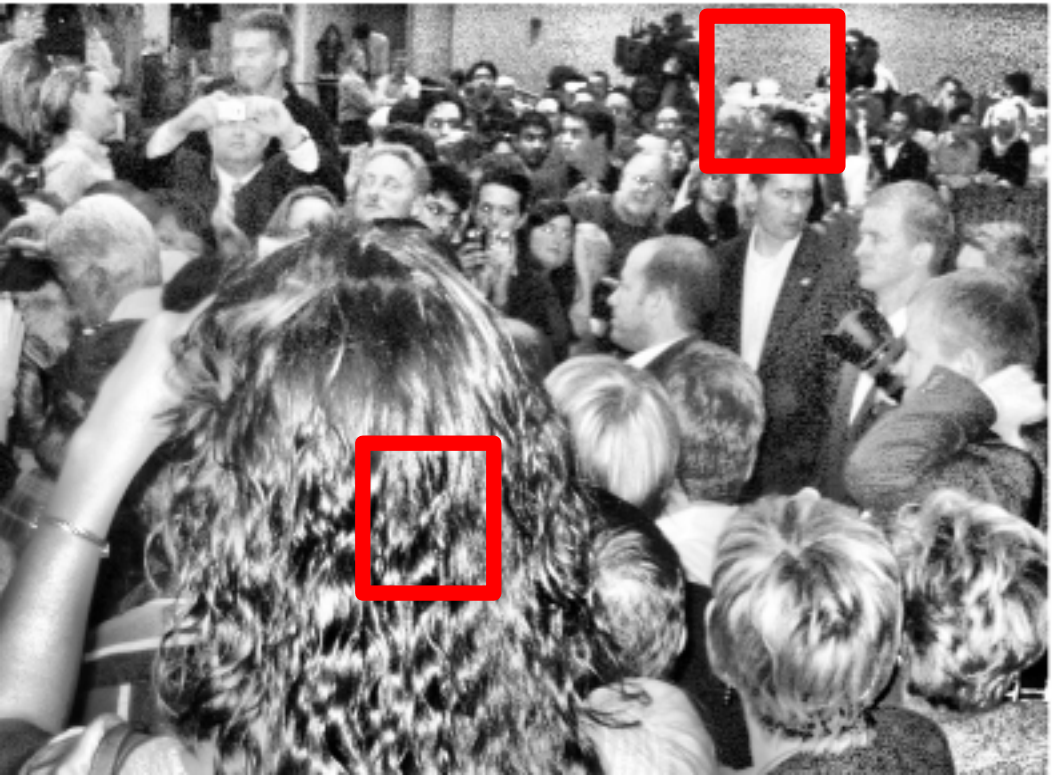
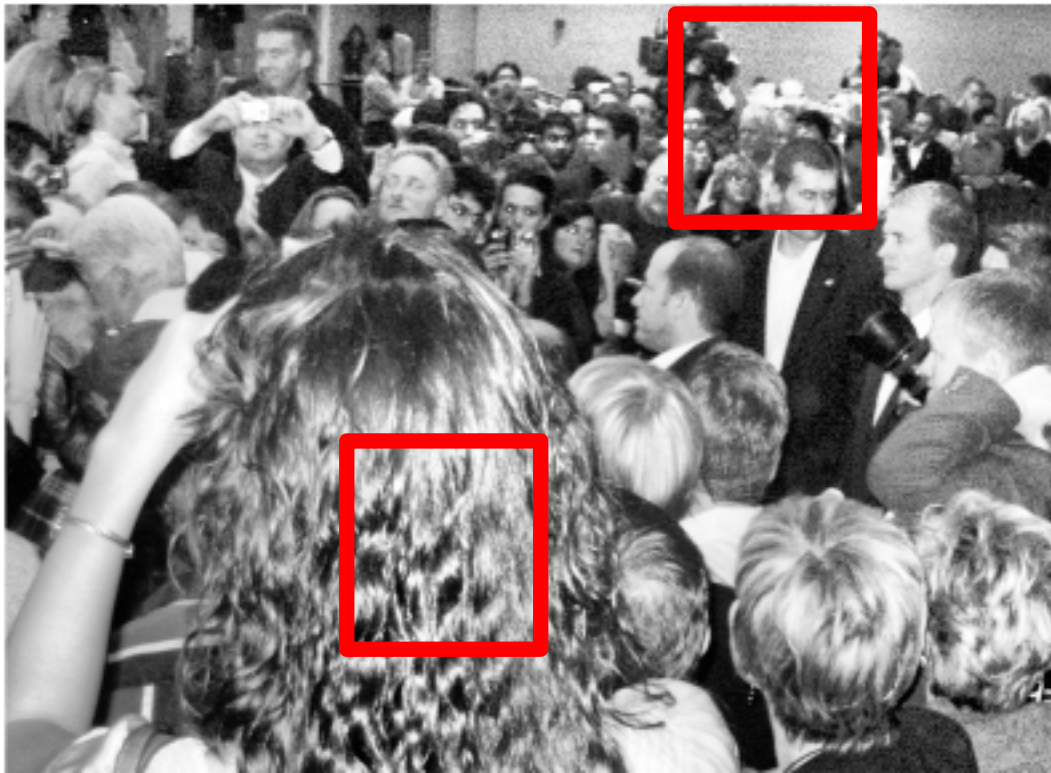


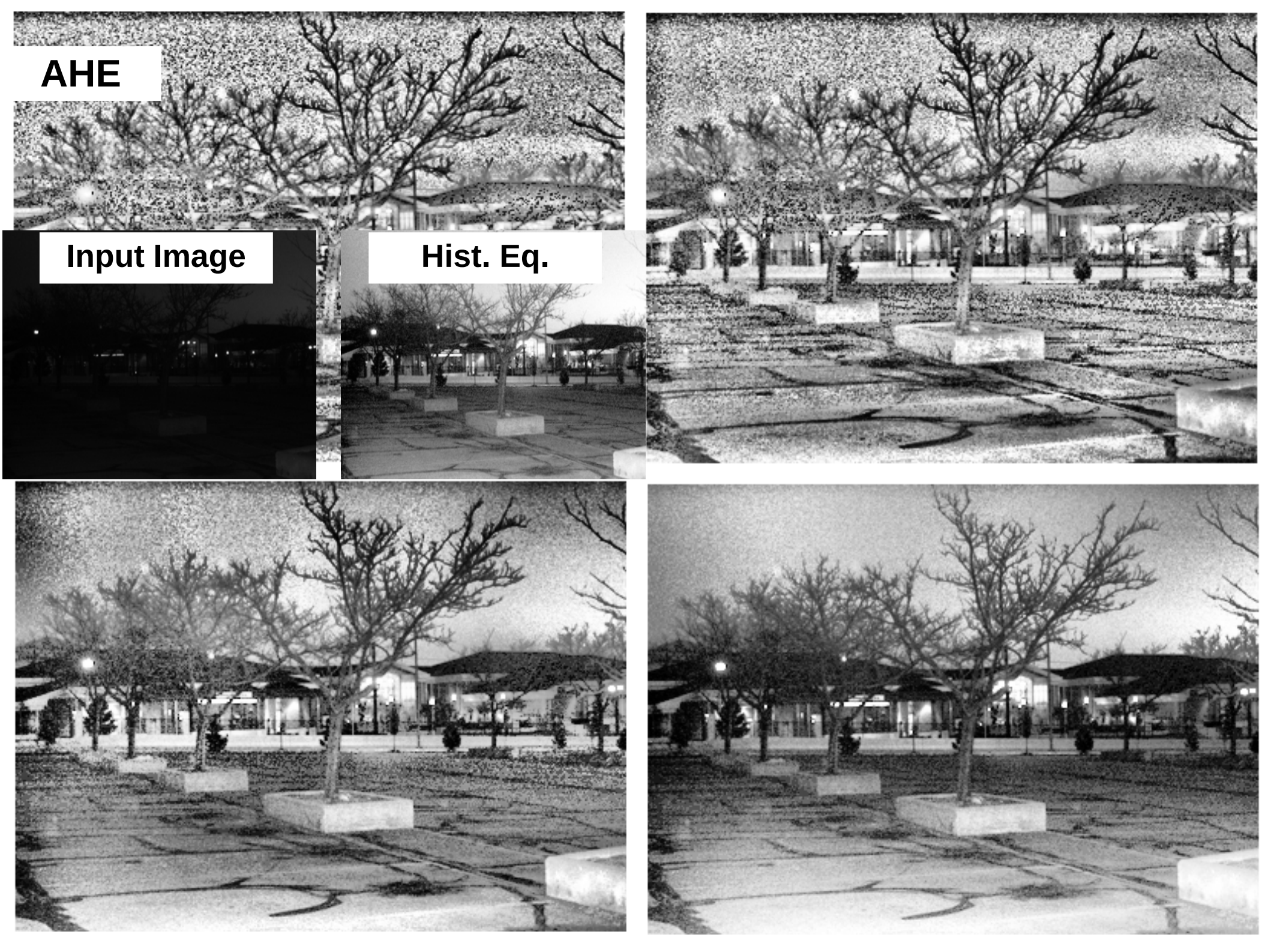
# Gray-Level Transformations

- Input image

Global Hist. Eq.

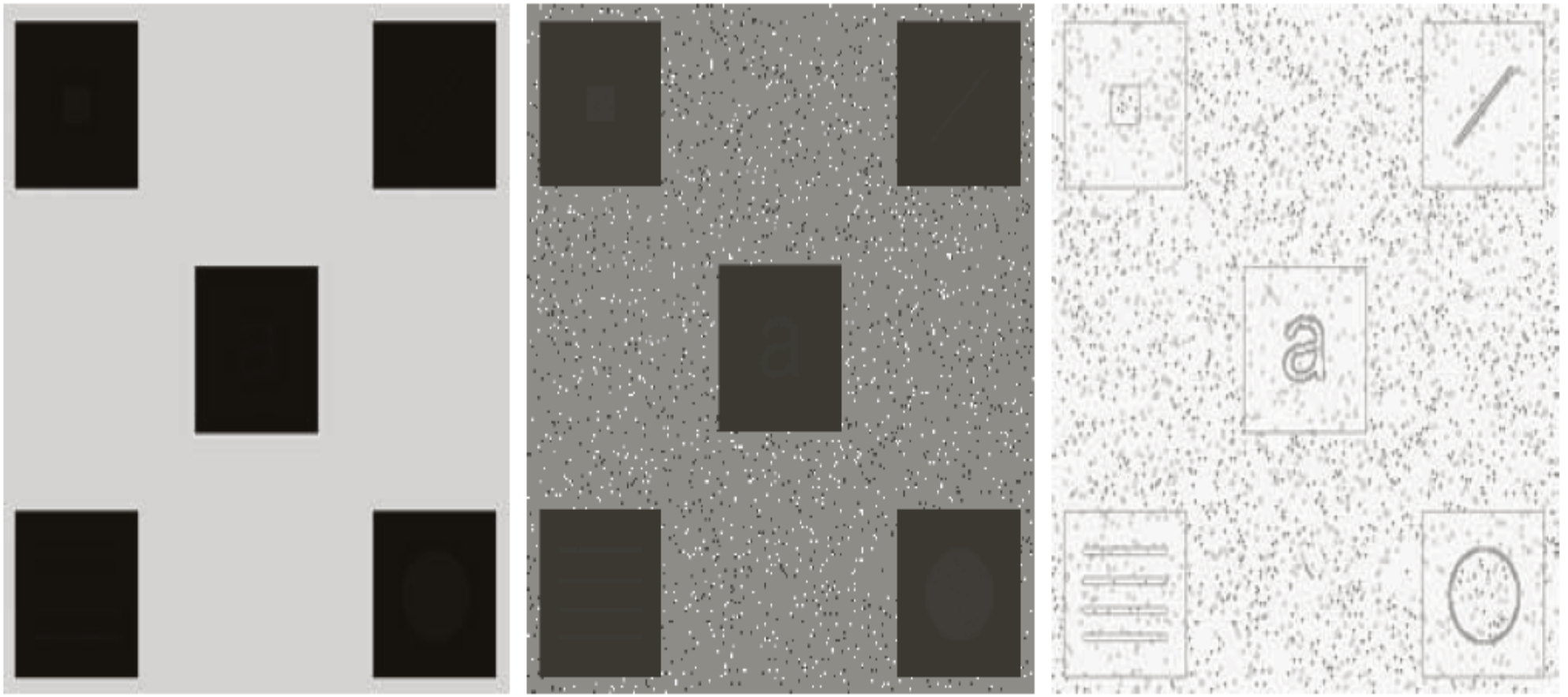
- On next slide:
  AHE with square windows of widths 25, 64, 100, 200


AHE





AHE
Input Image
Hist. Eq.

# Gray-Level Transformations

- Adaptive histogram equalization (AHE)



a b c

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

# Gray-Level Transformations

- Limitations of adaptive histogram equalization
  - Small window size → bring out more detail
  - Small window size → amplify noise in constant patches

- How to try to get the best of both worlds ?
  - We want to enhance texture, without amplifying noise

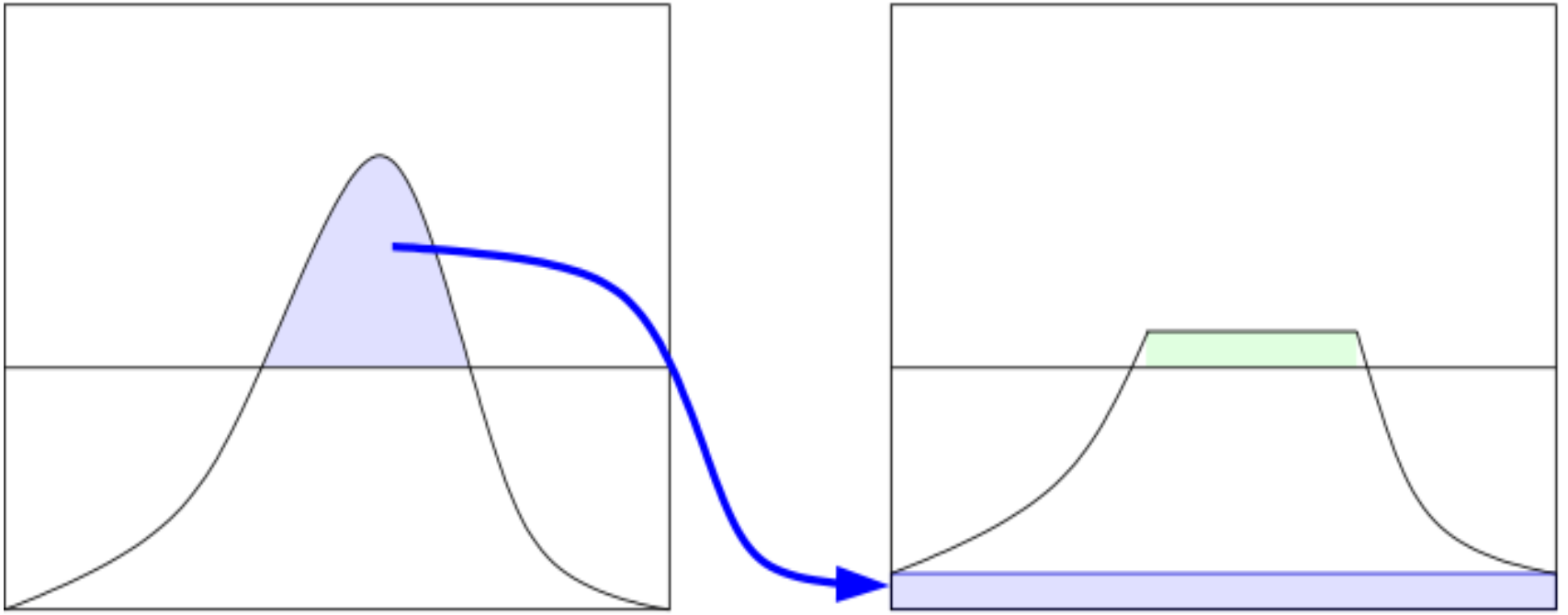
# Gray-Level Transformations

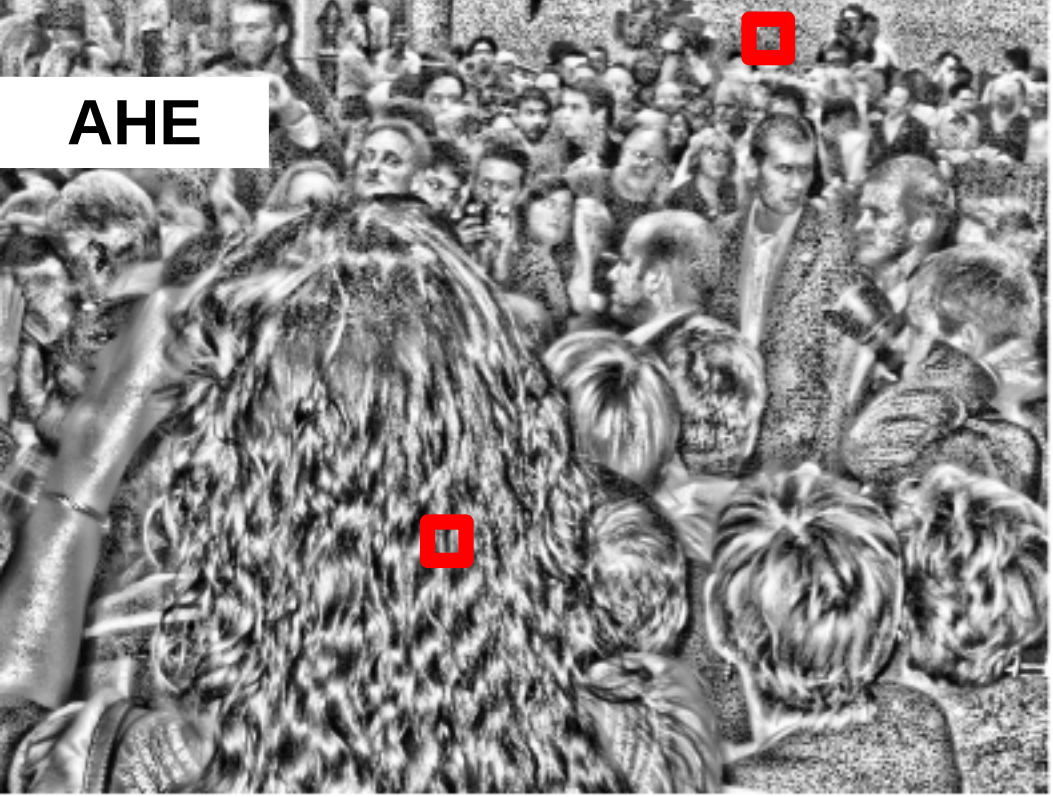
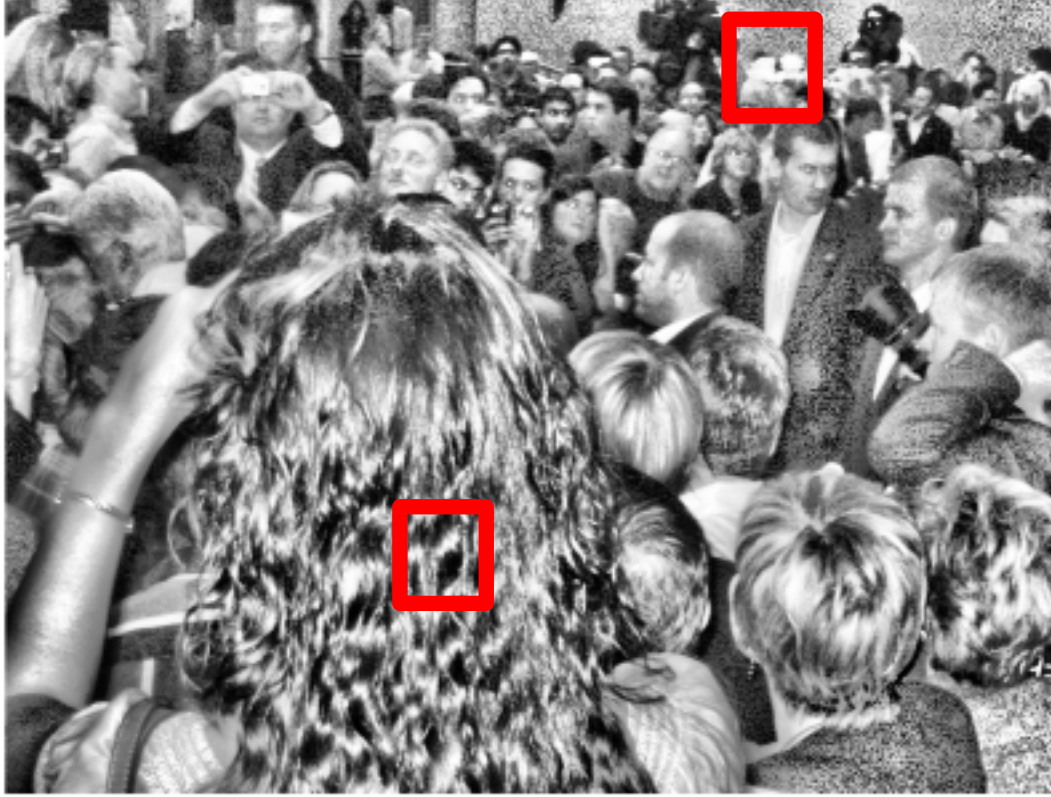
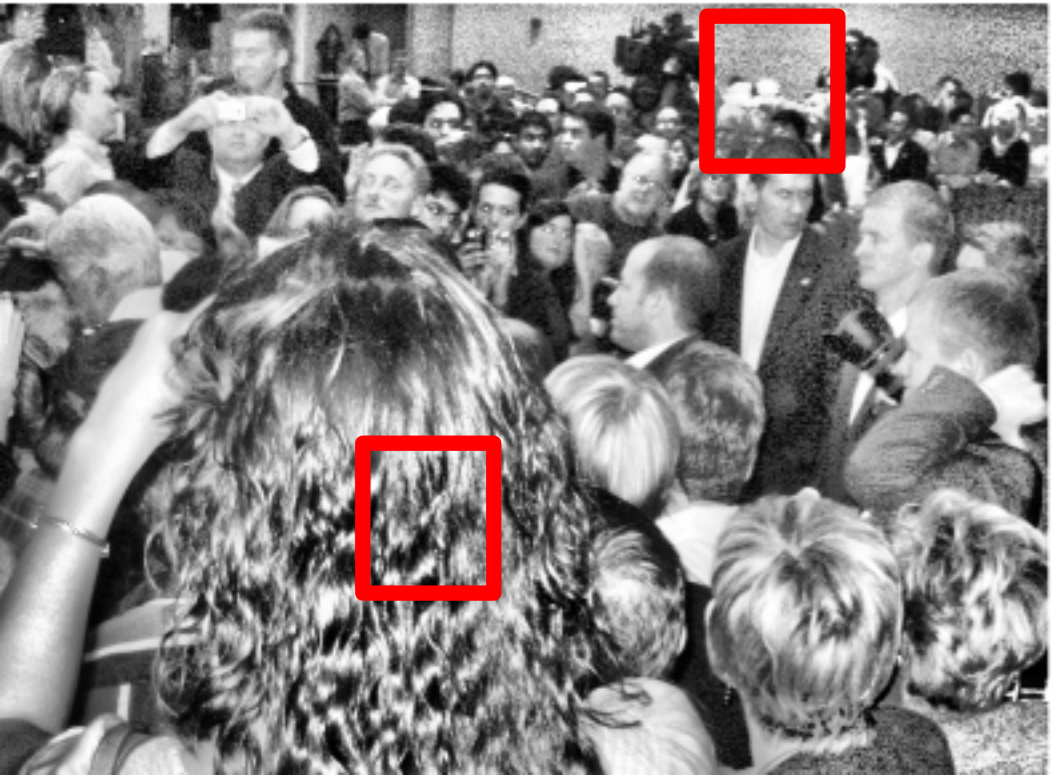
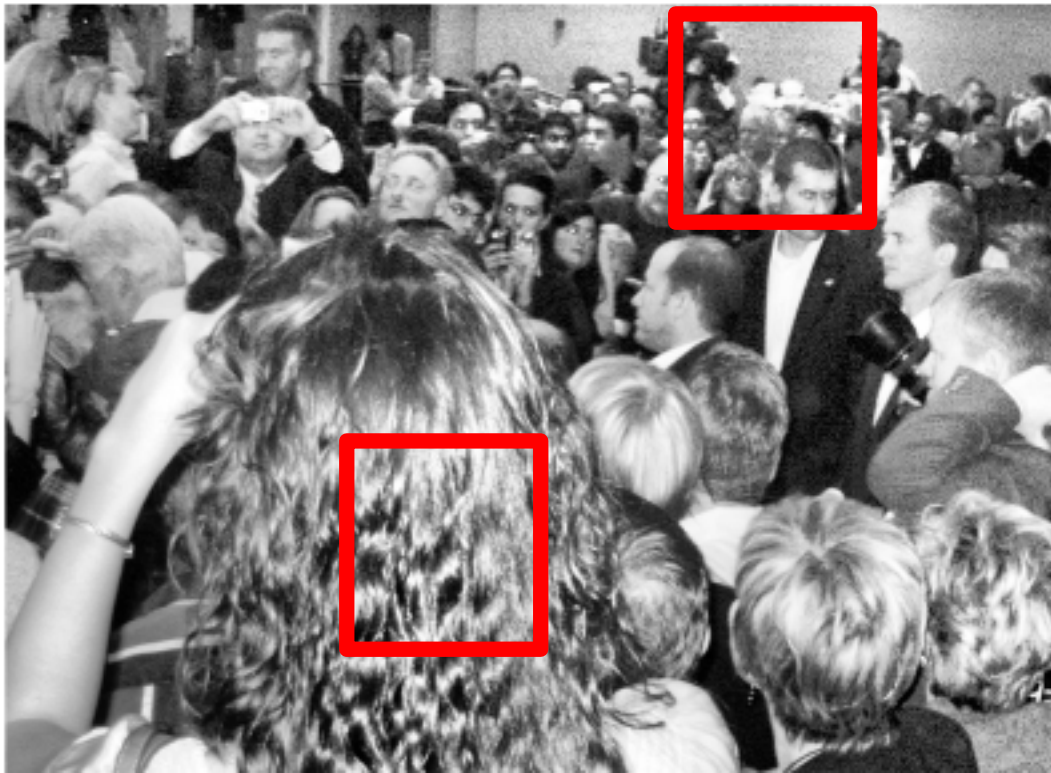
- **Contrast-limited adaptive histogram equalization (CLAHE)**
  - **Principle:** Contrast amplification in the vicinity of a given pixel value is given by:
    - **Slope** of the transformation function

      = Slope of the CDF
    - But CDF slope depends on histogram (PDF) value

# Gray-Level Transformations

- Contrast-limited adaptive histogram equalization (CLAHE)
  - Limits the amplification by:
    - (1) Clip the histogram at a predefined value (free parameter)
    - (2) Redistribute the mass uniformly throughout the range
    - (3) Then, compute the CDF




AHE





CLAHE

# Gray-Level Transformations

- (CL)AHE is computationally expensive
  - Computes PDF (+ CDF) at window around at **every** pixel
- Can update histograms in a smart way
  - As neighborhood shifts,
    only a small number of pixel change
- Approximations to (CL)AHE for speedup
  - See next slide ...

# Gray-Level Transformations

- Approximations to CLAHE for speedup

  (1) Split image into **MxM non-overlapping tiles**

  (2) Compute CDF for each tile

  (3) Bilinearly interpolate transformation for all pixels based on distances from tile centers