Assignment 4: CS 663

Due: 30th September before 11:55 pm

Remember the honor code while submitting this (and every other) assignment. You may discuss broad ideas with other students or ask me for any difficulties, but the code you implement and the answers you write must be your own. We will adopt a zero-tolerance policy against any violation.

Submission instructions: Follow the instructions for the submission format and the naming convention of your files from `http://www.cse.iitb.ac.in/~suyash/cs663/submissionStyle.pdf`. However, please do not submit the face image databases in your zip file that you will upload on moodle. Please see `http://www.cse.iitb.ac.in/~ajitvr/CS663_Fall2016/HW4/assignment4_SVD_FaceRecognition.rar`. Upload the file on moodle before 11:55 pm on 30th September. Policy for late submissions will be the same as in the aforementioned guidelines document. Please preserve a copy of all your work until the end of the semester.

1. In this part, you will implement a mini face recognition system. Download the ORL face database from `http://www.cl.cam.ac.uk/Research/DTG/attarchive/pub/data/att_faces.zip`. It contains 40 sub-folders, one for each of the 40 subjects/persons. For each person, there are ten images in the appropriate folder. The images are of size 92 by 110 each. Each image is in the pgm format. You can view the images in this format, either through MATLAB or through image viewers like IrfanView on Windows, or xv/display/gimp on Unix. Though the face images are in different poses, expressions and facial accessories, they are all roughly aligned (the eyes are in roughly similar locations in all images). For the first part of the assignment, you will work with the images of the first 32 people. For each person, you will include the first six images in the training set and the remaining four images in the testing set (note: there are 10 images per person labeled 1.pgm to 10.pgm). You should create an eigen-space from the training set as described during the lectures without explicitly computing the covariance matrix (note, in this case, you do have $N \ll d$ where $d = 92 \times 110$ is the size of the image, and $N = 35 \times 6$ is the number of images in the training set). Record the recognition rate using squared difference between the eigencoefficients while testing on all the images in the test set, for $k \in \{1, 2, 3, 5, 10, 15, 20, 30, 50, 75, 100, 150, 170\}$. Plot the rates in your report in the form of a graph.

   Repeat the same experiment on the Yale Face database from `http://www.cse.iitb.ac.in/~ajitvr/CS663_Fall2016/HW4/CroppedYale.rar`. This database contains 60 images each of 38 individuals (labeled from 1 to 39, with number 14 missing). Each image is in pgm format and has size 192 by 168. The images are taken under different lighting conditions but in the same pose. Take the first 40 images of every person for training and test on the remaining 20 images (by first 30 images, I mean the first 30 images that appear in a directory listing as produced by the dir function of MATLAB). Plot in your report the recognition rates for $k \in \{1, 2, 3, 5, 10, 15, 20, 30, 50, 60, 65, 75, 100, 200, 300, 500, 1000\}$ based on squared difference between all the eigencoefficients and between all except the three eigencoefficients corresponding to the eigenvectors with the three largest eigenvalues. [40 points]
   Note: Use the MATLAB routine uigetdir() (only once per database) which prompts the user to browse to the top-most directory of the database folder, and then use paths relative to the chosen directory in your code to read the appropriate images, i.e., followed by a call to the dir() MATLAB function to get the listing of the image files in the chosen directory.

2. Display in your report the reconstruction of any one face image from the Yale database using $k \in \{2, 10, 20, 50, 75, 100, 125, 150, 175\}$ values. Plot the 25 eigenvectors (eigenfaces) corresponding to the 25 largest eigenvalues using the subplot or subimage commands in MATLAB. [10 points]
   Note: Use uigetdir() to prompt the user to choose the Yale database folder. Then, you can directly use the

relative path of any one image of your choice. You need not prompt to directly choose an image. In your report, display results for the same input image.

3. What will happen if you test your system on images of people which were not part of the training set? (i.e. the last 8 people from the ORL database). What mechanism will you use to report the fact that there is no matching identity? Work this out carefully and explain briefly in your report. Test whatever you propose on all the 32 remaining images (i.e. 8 people times 4 images per person), as also the entire test set containing 6 images each of the first 32 people. How many false positives/negatives did you get? [10 points]
Note: Same as Q.1.

4. Given a matrix $\mathbf{A}$ of size $m \times n$, write a MATLAB routine called MySVD which takes this matrix as input and outputs the left and right singular vectors (i.e. column vectors of $\mathbf{U}$ and $\mathbf{V}$ under usual notation) and the singular values (i.e. diagonal entries of $\mathbf{S}$) of $\mathbf{A}$. You are not allowed to use the svd or svds functions of MATLAB directly. You should use only the eigenvalue decomposition routines eig or eigs for this task. Cross-check your answer by verifying that $\mathbf{A} = \mathbf{USV}^T$ based on your computation. [10 points]
Note: The file 'matA.txt' in code/ folder for this question will contain the input matrix $\mathbf{A}$ for this question; each row of this matrix appears on a new line, where the elements in a row are separated by a space. Use the MATLAB routine dlmread() to read this matrix from the file, and then write your code as usual.

5. Consider a set of N vectors $\mathcal{X} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, ..., \boldsymbol{x_N}\}$ each in $\mathbb{R}^d$, with average vector $\bar{\mathbf{x}}$. We have seen in class that the direction $\mathbf{e}$ such that $\sum_{i=1}^{N} \|\boldsymbol{x_i} - \bar{\mathbf{x}} - (\mathbf{e} \cdot (\boldsymbol{x_i} - \bar{\boldsymbol{x}}))\mathbf{e}\|^2$ is minimized, is obtained by maximizing $\mathbf{e}^t\mathbf{C}\mathbf{e}$, where $\mathbf{C}$ is the covariance matrix of the vectors in $\mathcal{X}$. This vector $\mathbf{e}$ is the eigenvector of matrix $\mathbf{C}$ with the highest eigenvalue. Prove that the direction $\mathbf{f}$ perpendicular to $\mathbf{e}$ for which $\mathbf{f}^t\mathbf{C}\mathbf{f}$ is maximized, is the eigenvector of $\mathbf{C}$ with the second highest eigenvalue. For simplicity, assume that all non-zero eigenvalues of $\mathbf{C}$ are distinct and that rank$(\mathbf{C}) > 2$. [10 points]

6. Consider a set of N vectors $\mathcal{X} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, ..., \boldsymbol{x_N}\}$ each in $\mathbb{R}^d$ $(N > d)$. Assume their mean vector is $\mathbf{0}$. Let $\mathbf{V} \in \mathbb{R}^{d \times d}$ be the orthonormal matrix containing the principal components of this dataset arranged in descending order of the eigenvalues (assume all eigenvalues are distinct). Let us denote the order k $(k < d)$ linear approximation of vector $\boldsymbol{x_i}$ using $\mathbf{V}$ as $L(\boldsymbol{x_i}^{(k)}; \mathbf{V}) = \boldsymbol{V_k}\boldsymbol{\alpha_i^{(k)}}$ where $\boldsymbol{V_k}$ is a $d \times k$ matrix containing the first k columns of $\mathbf{V}$, and $\boldsymbol{\alpha_i^{(k)}} = \boldsymbol{V_k}^t\boldsymbol{x_i}$. Let us denote the order k $(k < d)$ non-linear approximation of vector $\boldsymbol{x_i}$ using $\mathbf{V}$ as $N(\boldsymbol{x_i}^{(k)}; \mathbf{V}) = \mathbf{V}\boldsymbol{\alpha_i}$ where $\boldsymbol{\alpha_i} = \arg\min_{\boldsymbol{c_i}} \|\boldsymbol{x_i} - \mathbf{V}\boldsymbol{c_i}\|^2$ subject to the constraint that vector $\boldsymbol{c_i}$ has at the most k non-zero elements. The total reconstruction errors for the linear and non-linear approximations are respectively $E_L(\mathbf{V}) = \sum_{i=1}^{N} \|\boldsymbol{x_i} - L(\boldsymbol{x_i}^{(k)}; \mathbf{V})\|^2$ and $E_N(\mathbf{V}) = \sum_{i=1}^{N} \|\boldsymbol{x_i} - N(\boldsymbol{x_i}^{(k)}; \mathbf{V})\|^2$. Which of the following statements is true and <u>why</u>:

   (a) $E_L(\mathbf{V}) \leq E_N(\mathbf{V})$
   (b) $E_L(\mathbf{V}) \geq E_N(\mathbf{V})$
   (c) $E_L(\mathbf{V}) = E_N(\mathbf{V})$
   (d) One cannot make a conclusion about which error is greater.

   Also devise an efficient algorithm to obtain the order k non-linear approximation of $\boldsymbol{x_i}$ given $\mathbf{V}$, and state its time complexity. Argue <u>why</u> your algorithm is correct.

   Based on what you have studied about PCA in class, can you conclude the following: There cannot exist an orthonormal basis $\mathbf{W}$ such that $E_N(\mathbf{W}) < E_N(\mathbf{V})$ for some fixed k. Justify your answer. [6 + 8 + 4 = 20 points]