CS310 Automata Theory – 2016-2017

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Lecture 34: Effective computation
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Time complexity and complexity classes

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Proof idea: Analyze the construction of Problem 3 from Tutorial 9.

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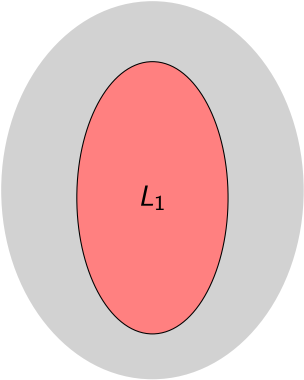

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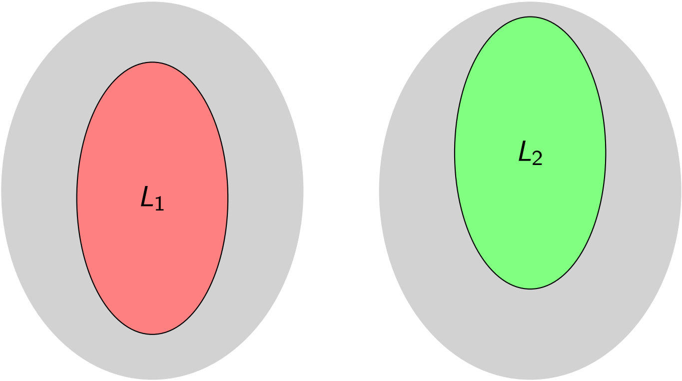

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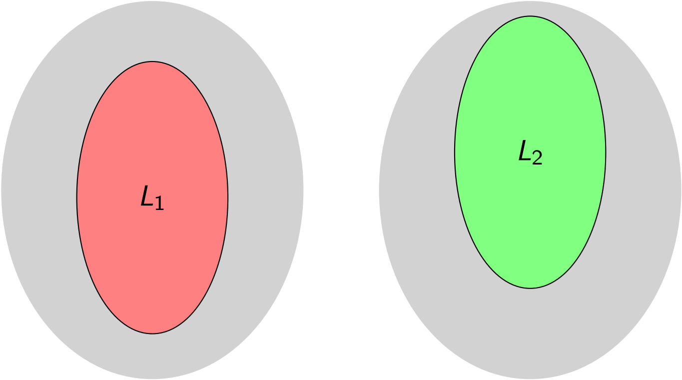

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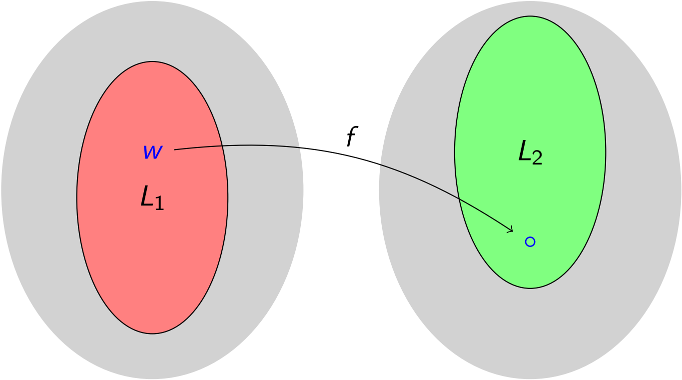

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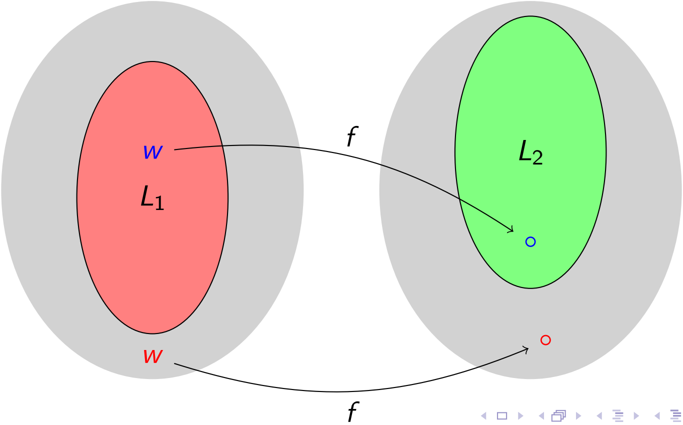

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Theorem ([Cook-Levin, 1970])

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Space complexity and complexity classes

Let $s : \mathbb{N} \to \mathbb{N}$.

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