Example of a language in NL

$$\text{Reach} = \{(G = (V, E), s, t) \mid \text{there is a path in } G \text{ from } s \text{ to } t\}$$

current $\leftarrow s$; count $\leftarrow 0$;

while count $< n+1$ or current $\neq t$;

{

next $\leftarrow$ non-det. guess a vertex from neighbors of current;
current $\leftarrow$ next;
count $++$;

}

if current $= t$ then accept;

else reject;

NL is contained in P

Configurations of a non-deterministic space bounded machine.

Configuration of a space bounded Turing machine M

index: input head position (uses $O(\log n)$ bits)

data: the working space bits (uses $O(s(n))$ bits)

S_M: machine related information (Q, δ) (uses $O(1)$ bits)

A typical configuration $\langle \text{index, data}, S_M \rangle$

Let $\mathcal{C}_M$ be the set of all possible configuration of M.

Let C_0 be the initial configuration.

Let C_{acc} be the accepting configuration.

NL is contained in P

Definition

Let L be a language in $NSPACE(\mathrm{s(n)})$ with TM M. Let C, C' be two configurations in $\mathcal{C}_M$. We say that a configuration C yields C' on input w if the machine M in one step goes from C to C' on input w.

Configurations Graph of M on input w.

Let $\mathcal{E}_{M,w} = \{(C, C') \mid C, C' \in \mathcal{C}_{\mathcal{M}}$ and C yields C' on input $w\}$

Let $\mathcal{G}_{M,w} = (\mathcal{C}_M, \mathcal{E}_{M,w})$

Let $\mathcal{G}_{M,w}$ be the configuration graph of M on w.

NL is contained in P

Theorem

If L is in $NSPACE(s(n))$ then L is in $TIME(2^{O(s(n))})$.

We know that $L \in NSPACE(\mathrm{s(n)})$. Let M be the machine.

First note that, $w \in L$ if and only if C_{acc} is reachable from C_0 in $\mathcal{G}_{M,w}$.

On any input w, the graph $\mathcal{G}_{M,w}$ can be computed in time $TIME(2^{O(s(n))})$.

- $|\mathcal{C}_M| = 2^{O(s(n))}$.
- Given C, C', checking whether $(C, C') \in \mathcal{E}_{M,w}$ or not is checkable in time $2^{O(s(n))}$.

NL is contained in P

Theorem

If L is in $NSPACE(s(n))$ then L is in $TIME(2^{O(s(n))})$.

We know that $L \in NSPACE(\mathrm{s(n)})$. Let M be the machine.

First note that, $w \in L$ if and only if C_{acc} is reachable from C_0 in $\mathcal{G}_{M,w}$.

On any input w, the graph $\mathcal{G}_{M,w}$ can be computed in time $TIME(2^{O(s(n))})$.
Checking whether C_{acc} is reachable from C_0 can be checked in time $2^{O(s(n))}$.

Reachability in a graph of size $2^{O(s(n))}$.

Corollary

NL is contained in P.