Applications of pumping lemma

Let $L = \{ww^R \mid w \in \Sigma^*\}$

- ☹ For any chosen p,
- ☺ let $x = \epsilon$, $y = 0^p$, $z = 110^p$.
- ☹ For any split of y into u, v, w
- ☺ $xuv^iwz = 0^q110^p$, as long as $i > 0$.
  In particular, $xuv^2wz \notin L$.

We say that a language L satisfies **Property-NR**
if the following conditions hold:

- ☹ $\forall p \geq 0$,
- ☺ $\exists x, y, z$ such that $x \cdot y \cdot z \in L$ and $|y| \geq p$,
- ☹ $\forall u, v, y$ such that $|v| > 0$, $y = u \cdot v \cdot w$,
- ☺ $\exists i \; x \cdot u \cdot v^i \cdot w \cdot z \notin L$.

If L satisfies Property-NR then L is not regular.

Applications of pumping lemma

$L = \{a^q \mid q \text{ is a prime number }\}$

- ☹ For any chosen p,
- ☺ let $x, z = \epsilon$, $y = a^n$, $n \geq p$ and a prime.
- ☹ For any split of y into u, v, w
- ☺ $xuv^{n+1}wz = a^{n(k+1)}$, where $k := |v|$.
  That is, $xuv^{n+1}wz = a^{n(k+1)} \notin L$.

We say that a language L satisfies **Property-NR** if the following conditions hold:

- ☹ $\forall p \geq 0$,
- ☺ $\exists x, y, z$ such that $x \cdot y \cdot z \in L$ and $|y| \geq p$,
- ☹ $\forall u, v, y$ such that $|v| > 0$, $y = u \cdot v \cdot w$,
- ☺ $\exists i \; x \cdot u \cdot v^i \cdot w \cdot z \notin L$.

If L satisfies Property-NR then L is not regular.

Building on pumping lemma

The following language is not regular:

$$EQ = \{w \in \{a, b\}^* \mid \#_a(w) = \#_b(w)\}$$

Suppose D is regular.

$D \cap L(a^* b^*)$ is also regular, as the intersection of two regular languages is regular and any regular expression defines a regular language.

But $D \cap L(a^* b^*) = \{a^n b^n \mid n \geq 0\}$ is not regular, which we proved using the pumping lemma.

Pumping down

Let $L = \{0^i 1^j \mid i, j \in \mathbb{N} \text{ and } i > j\}$.

For any choice of $p \geq 0$,

Let $x = \epsilon$, $y = 0^{p+1}$, $z = 1^p$.

Then $x \cdot y \cdot z \in L$.

Now for any choice of u, v, w, s.t $u \cdot v \cdot w = y$ and $|v| > 0$

$x \cdot u \cdot v^0 \cdot w \cdot z = 0^{p'} 1^p$, where $p' \leq p$.

$\therefore x \cdot u \cdot v^0 \cdot w \cdot z \notin L$.

Relations on Σ

Let R be an equivalence relation on the set Σ^*, i.e. $R \subseteq \Sigma^* \times \Sigma^*$ such that

REFLEXIVE	$\forall x \in \Sigma^* \; R(x,x)$ holds.
SYMMETRIC	$\forall x,y \in \Sigma^* \; R(x,y) = R(y,x)$ hold.
TRANSITIVE	$\forall x,y,z \in \Sigma^*$ if $R(x,y), R(y,z)$ hold then $R(x,z)$ also holds.

Relation of Σ^*

Let L be a regular language recognized by a DFA $A = (Q, \Sigma, \delta, q_0, F)$.

We say that $\forall x, y \in \Sigma^*$

$$x \equiv_A y \quad \text{iff} \quad \underbrace{\tilde{\delta}(q_0, x)}_{\substack{\text{state}\\ \text{reached}\\ \text{on } x\\ \text{from } q_0}} \quad = \quad \underbrace{\tilde{\delta}(q_0, y)}_{\substack{\text{state}\\ \text{reached}\\ \text{on } y\\ \text{from } q_0}}$$

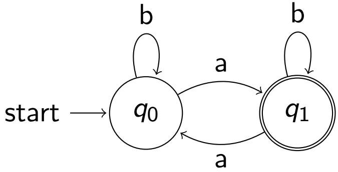
Assume that the auomaton is complete.

Observe that $\equiv_A$ is an equivalence relation.

Example

Example of an equivalence relation.

Consider the following automaton, say A.



$aab \equiv_A abababa$.

$aabaaa \equiv_A a$.

The words with even number of a's form one equivalence class.

The words with odd number of a's form the other equivalence class.

There are no other equivalence classes.