Regularity checking

Lemma

$REG_{TM} = \{\langle M\rangle \mid L(M)$ is regular$\}$ is undecidable.

Assume for the sake of contradiction that a TM R is a TM that decides REG_{TM}.

Let $R'_{M,w}$ be s.t.

$$L(R'_{M,w}) = \begin{cases} \{0^n1^n \mid n \geq 0\} & \text{if } M \text{ rej } w \\ \Sigma^* & \text{if } M \text{ acc } w \end{cases}$$

If we get such an $R'_{M,w}$ we can design A as a follows.

Let A be as follows:

On input M, w

{

Create machine $R'_{M,w}$.

If R on $\langle R'_{M,w}\rangle$ accepts

then accept

else reject

}

Regularity checking

Lemma

$REG_{TM} = \{\langle M \rangle \mid L(M)$ is regular$\}$ is undecidable.

Assume for the sake of contradiction that a TM R be a TM that decides REG_{TM}.

Let $R'_{M,w}$ be as follows:

On input x

{

if $x = 0^n 1^n$

then accept

else run M on w and

if M acc w then acc

else rej

}

Let A be as follows:

On input M, w

{

Create machine $R'_{M,w}$.

If R on $\langle R'_{M,w} \rangle$ accepts

then accept

else reject

}

Universality of CFLs

Lemma

$ALL_{CFL} = \{\langle M \rangle \mid M$ is a PDA and $L(M) = \Sigma^*\}$ is undecidable.

Proof Strategy

$$\text{Input } (M, w) \longrightarrow N$$

$$\text{if } w \in L(M) \longrightarrow \exists x \in \Sigma^*, \text{ s.t. } x \notin L(N)$$

$$\text{if } w \notin L(M) \longrightarrow L(N) = \Sigma^*$$

Universality of CFLs

Lemma

$ALL_{CFL} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) = \Sigma^*\}$ is undecidable.

Design of N

if x does not encode a run of M on w

then accept

else

{

if M accepts w
then reject
else accept

}

$$\text{valid}_{M,w}(x) \quad \begin{aligned} &= 1 && \text{if } x \text{ is a valid encoding of a run of } M \text{ on } w \\ &= 0 && \text{otherwise} \end{aligned}$$

$$L_{M,w} = \{x \mid \overline{\text{valid}_{M,w}(x)} \text{ or } M \text{ rej } w\}.$$

We will design N s.t. $L(N) = L_{M,w}$.

If M acc w then $\exists x$, $x \notin L_{M,w}$, where x encodes the run of M on w.

If M rej w then $\forall x$, $x \in L_{M,w}$, i.e. $L_{M,w} = \Sigma^*$.