Emptiness problem for TM

Lemma

The emptiness problem for TMs, $E_{TM} = \{\langle M \rangle \mid L(M) = \varnothing\}$, is undecidable.

Assume for the sake of contradiction that it is decidable. Let T be a machine that decides E_{TM}.

Let $T'_{M,w}$ be as follows:

On input x

{

if $w \neq x$ then reject

else do as per M

}

Let A be as follows:

On input M, w

{

Create machine $T'_{M,w}$.

If T on $\langle T'_{M,w} \rangle$ rejects

then accept

else reject

}

Equality for TM

Lemma

The equality problem for TMs, $EQ_{TM} = \{(M_1, M_2) \mid L(M_1) = L(M_2)\}$, is undecidable.

Assume for the sake of contradiction that EQ_{TM} is decidable. Let M be the TM for it.

Let M_1 be a machine that rejects all strings. That is, $L(M_1) = \varnothing$.

Given a machine M_2 as an input, use M to check whether $L(M_2) = L(M_1)$, i.e. to check whether $L(M_2) = \varnothing$ or not.

This implies that if EQ_{TM} is decidable then E_{TM} is decidable.

But from the previous result we know that E_{TM} is undecidable.