Tutorial 3

Notation: Let $\Sigma = \{a, b\}$. For $w \in \Sigma^*$ let $|w|$ denote the length of w. Let $\#_a(w)$ denote the number of as in w and let $\#_b(w)$ denote the number of bs in w.

1. Prove or disprove that the following languages are regular.
   (a) $EQ = \{w \cdot w \mid w \in \Sigma^*\}$.
   (b) $Twice = \{w \in \{a, b\}^* \mid \#_a(w) = 2 \cdot \#_b(w)\}$.
   (c) $Prod = \{w \in \{a, b\}^* \mid \#_a(w) \cdot \#_b(w) \text{ is even}\}$.
   (d) $Len = \{w1^n \mid |w| = n\}$.
   (e) $NEQ = \{0^i 1^j \mid i \neq j\}$.
   (f) $L = \{a^n b^m c^{n-m} \mid n \geq m \geq 0\}$.

2. Let L be a regular language. One of the following languages is regular and the other is not. Give a proof and provide a counterexample, respectively.
   (a) $\{w \in \{a, b\}^* \mid \exists n \geq 0, \exists x \in L, x = w^n\}$
   (b) $\{w \in \{a, b\}^* \mid \exists n \geq 0, \exists x \in L, w = x^n\}$

3. Let L be any language (not necessarily regular) over a unary alphabet, i.e. $L \subseteq \{a\}^*$. Show that L^* is regular.

4. For each of the language L below suppose there is a relation $\equiv_L$ on Σ^* such that $x \equiv_L y$ if and only if $\forall z \in \Sigma^* \ (xz \in L \Leftrightarrow yz \in L)$. What is the number of equivalence classes such a relation must have in each of the following cases?
   (a) $L = \{a^n b^n \mid n \geq 0\}$
   (b) Language consisting of all w such that $\#_a(w) \pmod 3 = 1$ OR $\#_b(w) \pmod 3 = 2$.
   (c) $L = b^* a (a + b)^*$
   (d) $L = \{a^n b^m a^n b^m | m, n \geq 0\}$
   (e) $L = \{w \mid w \in \Sigma^*, 2 \leq \#_a(w) \leq 5, 3 \leq \#_b(w) \leq 4\}$