Comparing decidability and recognizability

Theorem

A language L is Turing decidable if and only if L and $\overline{L}$ are both Turing recognizable.

Proof.

($\Rightarrow$)

If L is Turing decidable then L is also Turing recognizable (as we just saw).

If L is Turing decidable, then $\overline{L}$ is also Turing decidable.

Therefore, $\overline{L}$ is also Turing recognizable.

($\Leftarrow$)

Let M_1, M_2 be two TMs recognizing $L, \overline{L}$, respectively.

We wish to come up with a TM M that will decide L.

Idea: on input w run both M_1, M_2, if M_1 reaches accepting configuration then accept.

Else M_2 will reach the accepting configuraion. In that case, reject.

□

Variants of Turing machines

k-tape Turing machines

Usual TM + Multiples tapes + independent tape-head for each tape.

$\delta \subseteq Q \times \Gamma^k \times Q \times \Gamma^k \times \{L, R, S\}^k$.

Example

Given: 1^n on the input tape

Output: 1^{n^2} on the same tape.

Are k-tape TMs more powerful than 1-tape TMs?

Theorem

Every k-tape Turing machine has an equivalent 1-tape Turing machine.

k-tape Turing machines

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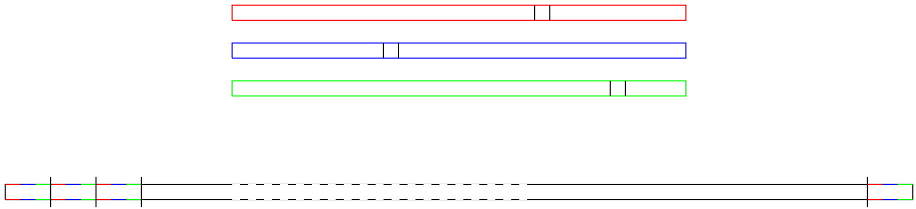
Proof sketch:



Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej},)$ be the k-tape Turing machine.

Let $M' = (Q', \Sigma, \Gamma', \delta', q_0, q_{acc}, q_{rej})$ be such that,

$\overline{\Gamma} = \{\overline{a} \mid a \in \Gamma\}$, $\Gamma = \Gamma \cup \overline{\Gamma} \cup \{\#\}$.

$\overline{\Gamma}$ symbols used to denote tape head positions.

k-tape Turing machines

Theorem

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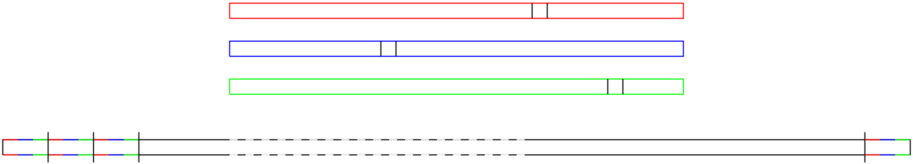
Proof sketch:



To simulate 1 step of M, M' works follows:

reads the tape left to right once, remembeing the marked symbols in its states,

uses δ to determine the next state,

sweeps the input left to right again to update marked symbols.