Module IV: Effective computation

Turing machines with resource constraints.

- Resources for computation.
  - Time: the number steps for which the TM runs
  - Space: the number of different cells on which the TM writes
  - The number of times an input bit can be read
  - The amount of energy used
  - ⋮
- Why bound resorces?
  - Viewing TM as algorithms.
  - TM to help in computation of important problems.
  - Finer study of decidable languages.
- How should we bound the resources?
  - Many different ways exist. ...

Time complexity and complexity classes

Let $t : \mathbb{N} \to \mathbb{N}$.

Definition

A language $L \subseteq \Sigma^*$ is said to be in class $\mathrm{TIME}(t(n))$ if there exists a deterministic Turing machine M such that $\forall x \in \Sigma^*$,

- M halts on x in time $O(t(|x|))$, where $|x|$ indicates the length of x.
- if $x \in L$ then M accepts x.
- if $x \notin L$ then M rejects x.

$$P = \bigcup_k \mathrm{TIME}(n^k)$$

$$EXP = \bigcup_k \mathrm{TIME}(2^{n^k})$$

Time complexity and complexity classes

Let $t : \mathbb{N} \to \mathbb{N}$.

Definition

A language $L \subseteq \Sigma^*$ is said to be in class $\mathrm{NTIME}(t(n))$ if there exists a non-deterministic Turing machine M such that $\forall x \in \Sigma^*$,

- each run of M halts on x in time $O(t(|x|))$, where $|x|$ indicates the length of x.
- if $x \in L$ then M accepts x on at least one run.
- if $x \notin L$ then M rejects x on all runs.

$$NP = \bigcup_k \mathrm{NTIME}(n^k)$$