CS310 Automata Theory – 2016-2017

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Lecture 20: Turing machines, computability

March 06, 2017

Last two modules

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Context-free grammars: Recursive programs. The class of languages generated by these grammars is CFLs.

Turing machines

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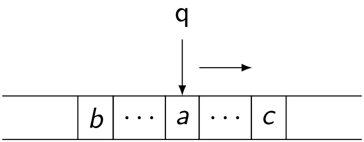

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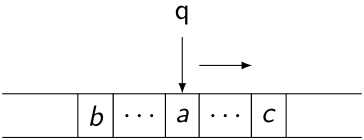

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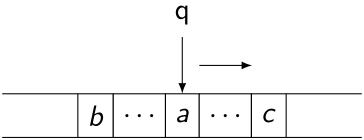

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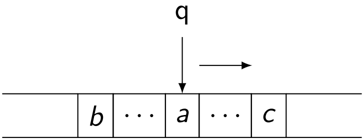

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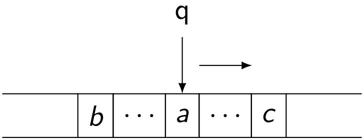

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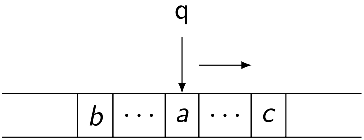

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Special states for accepting and rejecting.

Example

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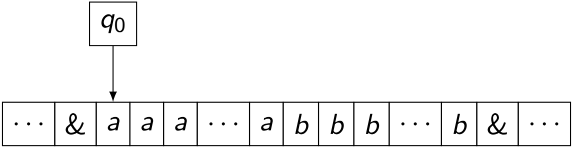

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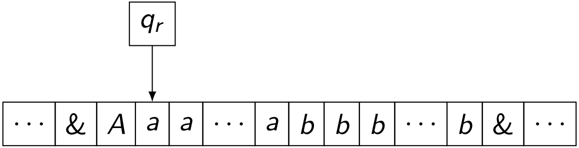

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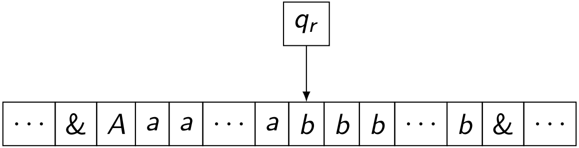

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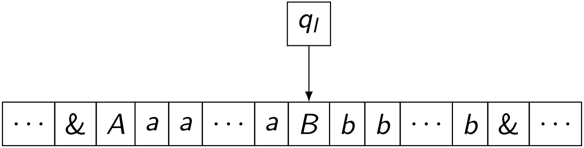

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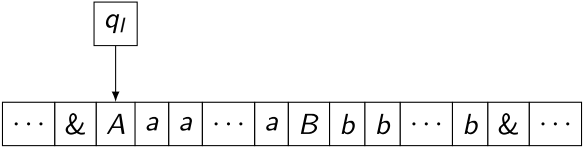

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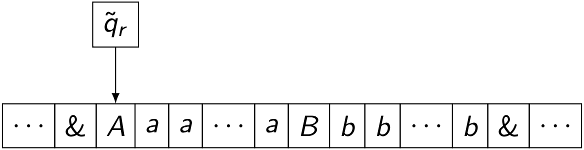

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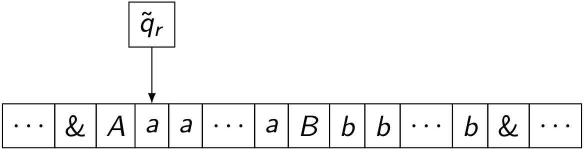

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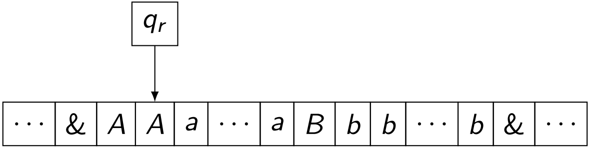

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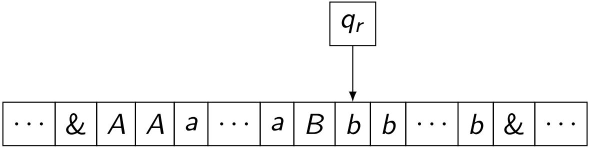

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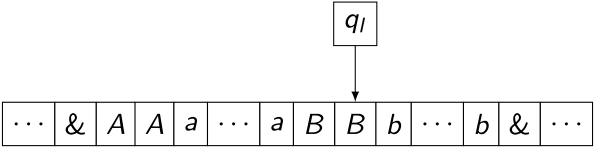

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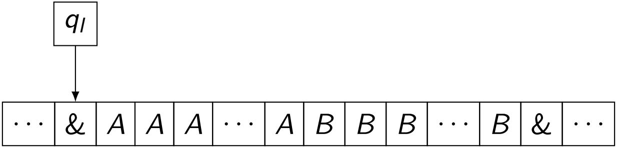

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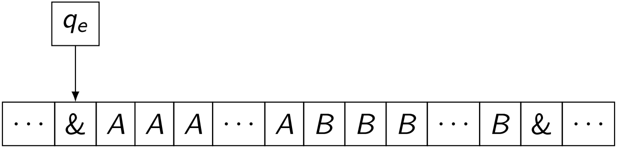

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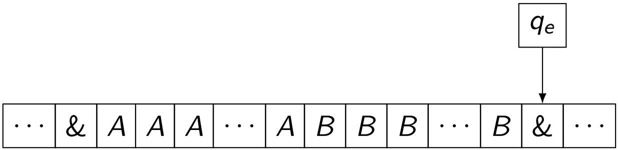

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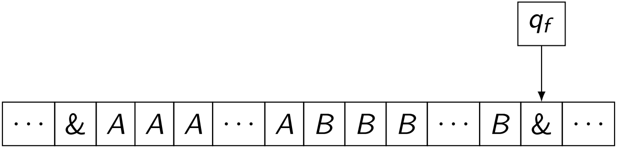

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A Turing machine (TM) is given by $M = (Q, \Sigma, \Gamma, \delta, q_0, q_f, q_{rej})$

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Turing machine for a non-context free language

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Give a full description of a Turing machine for the above language.