Outline of Selected Formulae

Instructions:

(1) Not all formulae that are used in the examination are listed here. Some easy-to-remember formulae are not listed.

(2) Also not all formulae listed here are used in the examination. Some may not be used at all, but still listed here in order to make the list complete.

(3) The formulae are only outlined without any relevant details.

- Different forms of interpolating polynomial are

  (1) $p_n(x) = \sum_{k=0}^{n} f(x_k) l_k(x)$, where $l_k(x)$ are appropriately defined polynomials.

  (2) $p_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \cdots, x_k] \prod_{i=0}^{k-1}(x - x_i)$ where $f[x_0, x_1, \cdots, x_k]$ are given by

  $$f[x_0, x_1, \cdots, x_k] = \frac{f[x_1, x_2, \cdots, x_k] - f[x_0, x_1, \cdots, x_{k-1}]}{x_k - x_0}$$

  or

  $$f[x_0, x_1, \cdots, x_k] = \int \cdots \int_{\tau_k} f^{(k)}(t_0 x_0 + t_1 x_1 + \cdots + t_k x_k)\, dt_1 \cdots dt_k$$

  where $\tau_k \subset \mathbb{R}^k$ is an appropriately defined set and t_0 is given in terms of t_j, $j = 1, 2, \cdots, k$.

  The mathematical error is

  $$\mathrm{ME}_n(x) = \frac{f^{(n+1)}(\xi_x)}{(n+1)!} \prod_{i=0}^{n}(x - x_i).$$

- Formulae for some quadrature rules and their mathematical errors:

Formula	**Mathematical Error**
$I_R(f) := (b-a)f(a)$	$\frac{f'(\eta)(b-a)^2}{2}$
$I_T(f) := (b-a)\left(\frac{f(a)+f(b)}{2}\right)$	$-\frac{f''(\eta)(b-a)^3}{12}$
$I_S(f) := \frac{b-a}{6}\left\{f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right\}$	$-\frac{f^{(4)}(\eta)(b-a)^5}{2880}$

- Some ODE Methods are as follows:

  (1) $y_{j+1} = y_{j-1} + 2hf(x_j, y_j)$

  (2) $y_{j+1} = y_j + \frac{h}{2}(f(x_j, y_j) + f(x_{j+1}, y_{j+1}))$

  (3) $y_{j+1} = y_j + hf(x_j, y_j)$

  (4) $y_{j-1} = y_j - hf(x_j, y_j)$

  (5) $y_{j+1} = y_j + \frac{h}{2} f(x_j, y_j) + \frac{h}{2}\left[f(x_{j+1}, y_j + hf(x_j, y_j))\right]$

  (6) $y_{j+1} = y_j + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where $k_1 = hf(x_j, y_j)$, $k_2 = hf\left(x_j + \frac{h}{2}, y_j + \frac{k_1}{2}\right)$, $k_3 = hf\left(x_j + \frac{h}{2}, y_j + \frac{k_2}{2}\right)$, and $k_4 = hf(x_j + h, y_j + k_3)$.

- Infinite norm definition: $\|f\|_{\infty, I} = \max_{x \in I} |f(x)|$.