Finer structure inside P

Definition

A function $t : \mathbb{N} \to \mathbb{N}$ is said to be time constructible if the there exists a TM that on input 1^n, it outputs $t(n)$ in time $O(t(n))$.

Examples

n^2, $n \log n$.

Theorem

Let $t : \mathbb{N} \to \mathbb{N}$ *be a time constructible function. There exists a language* L *such that* $L \in TIME(t(n)^2)$*, but* $L \notin TIME(o(t(n)))$.

As a result of the theorem we have

For any $i \geq 2$ and $1 > \epsilon > 0$, $TIME(n^i) \subsetneq TIME(n^{2i+\epsilon})$.

Polynomial time reductions and NP-hardness

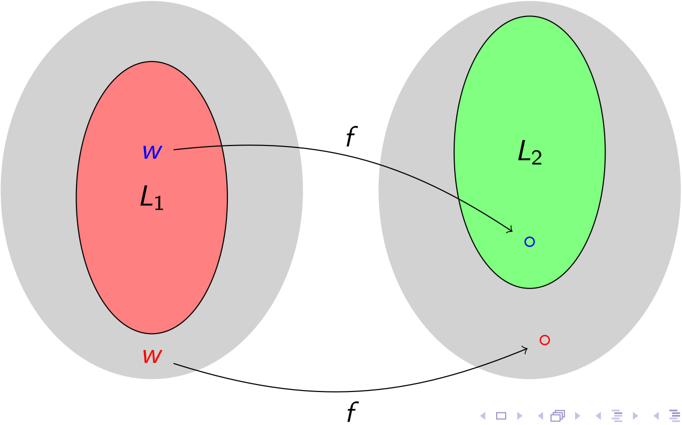
Definition

A function $f : \Sigma^* \to \Sigma^*$ is polynomial time computable if there is a polynomial time Turing machine TM, say M, such that on any input $w \in \Sigma^*$, M stops with only $f(w)$ on its tape.

Polynomial time reductions and NP-hardness

Definition

A language L_1 is said to be polynomial time reducible to another language L_2, denoted as $L_1 \leq_m L_2$, if there exists a polynomial time computable function f such that for all $w \in \Sigma^*$, $w \in L_1 \Leftrightarrow f(w) \in L_2$.



Polynomial time reductions and NP-hardness

Definition

A language L is said to be NP-hard if for every language $L' \in$ NP, there is a polynomial time reduction such that $L' \leq_m L$.

Definition

A language L is said to be NP-complete if the following two conditions hold:

- L is in NP.
- L is NP-hard.

Theorem ([Cook-Levin, 1970])

SAT is NP-complete.

Space bounded Turing Machines

The Turing Machine model with space bounds

- The input tape is assumed to be read-only.
- The space required to write down the input is not counted towards the space of the machine.
- The output tape assumed to be write-only.
- The space required to write down the output is not counted towards the space of the machine.

Space complexity and complexity classes

Let $s : \mathbb{N} \to \mathbb{N}$.

Definition

A language $L \subseteq \Sigma^*$ is said to be in class SPACE($s(n)$) if there exists a deterministic Turing machine M such that $\forall x \in \Sigma^*$,

- M halts on x using at most space $O(s(|x|))$,
- where $|x|$ indicates the length of x.
- if $x \in L$ then M accepts x.
- if $x \notin L$ then M rejects x.

$$L = \text{SPACE}(\log n)$$

$$PSPACE = \bigcup_k \text{SPACE}(n^k)$$

Examples

Min $= \{(w_1, w_2, \ldots, w_n, i) \mid w_i$ is the minimum among $w_1 \ldots w_n\}$.

Deg $= \{(G = (V, E), d, i) \mid v_i$ has degree $d\}$.

ADD $= \{(u, v, i) \mid i$th bit of $u + v$ is $1\}$.

Verify-SAT

$= \{(\phi, a) \mid a = a_1, a_2, \ldots, a_n$ is an assignment satisfying $\phi\}$.