CS310 Automata Theory – 2016-2017

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Lecture 3: Finite state automata
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Last class

- Finite state automata
  - Examples of finite state automata
  - Definition of DFA
  - Definition of acceptance by a DFA.
- Closure properties of regular languages
  - Union, intersection, concatenation.
- Non-deterministic finite state automata
  - Definition, ϵ transitions
  - acceptance by NFA.

Non-deterministic finite state automata

Informal description: A finite state automaton in which can branch out on different states on the same letter.

Definition (NFA)

A non-deterministic finite state automaton (NFA) $A = (Q, \Sigma_\epsilon, q_0, F, \delta)$, where

Q is a set of states,

$\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$ is the input alphabet,

q_0 is the initial state,

$F \subseteq Q$ is the set of final states,

δ is a set of transitions, i.e. $\delta \subseteq Q \times \Sigma_\epsilon \times 2^Q$

$\cancel{\forall q \in Q, \forall a \in \Sigma, |\delta(q, a)| \leq 1}$.

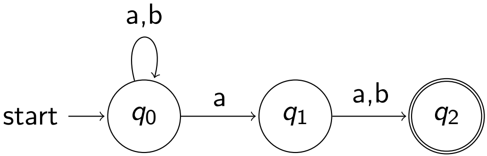
$\forall q \in Q, \forall a \in \Sigma_\epsilon, \delta(q, a) \subseteq Q$.

Non-deterministic finite state automata

Example

Input: $w \in \{a, b\}$

Check: Is a the second-last letter of w?



Acceptance by NFA

Definition (Acceptance by NFA)

A non-deterministic finite state automaton (NFA) $A = (Q, \Sigma_\epsilon, \delta, q_0, q_f)$, is said to accept a word $w \in \Sigma^*$, where $w = w_1 w_2 \dots w_n$ if

w can be written as $y_1 y_2 \dots y_m$, where each $y_i \in \Sigma_\epsilon$ and $m \geq n$

there exists a sequence of states $p_0, p_1, \dots p_m$ s.t.

$p_0 = q_0$,

$p_m \in F$,

$p_{i+1} \in \delta(p_i, y_{i+1})$ for all $0 \leq i \leq m-1$.

An NFA A is said to recognize a language L if $L = \{w \mid A \text{ accepts } w\}$.

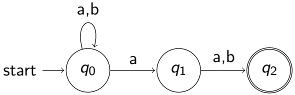
Notation: Let A be an NFA/DFA. We use $L(A)$ to denote the language recognized by A.

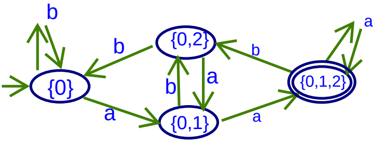
Power of NFAs

Lemma

Let A be an NFA. Then $L(A)$ is a regular language. That is, NFA and DFA accept the same set of languages.

We will work it out for an example.





States of DFA ->	$\emptyset$	$\{0\}$	$\{1\}$	$\{2\}$	$\{0,1\}$	$\{0,2\}$	$\{1,2\}$	{0, 1, 2} $\{\cancel{1,2,3}\}$
a	$\emptyset$	$\{0,1\}$	$\{2\}$	$\emptyset$	$\{0,1,2\}$	$\{0,1\}$	$\{2\}$	$\{0,1,2\}$
b	$\emptyset$	$\{0\}$	$\{2\}$	$\emptyset$	$\{0,2\}$	$\{0\}$	$\{2\}$	$\{0,2\}$

Subset construction

From now on we will not distinguish between Σ and Σ_ϵ.

Definition

Let $A = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Let $\hat{\delta} : 2^Q \times \Sigma \rightarrow 2^Q$ be defined as follows:

Let $S \subseteq Q$

$\hat{\delta}(S, \epsilon) := S$ If A has epsilon transitions, then $\hat{\delta}(S, \epsilon)$ will be defined accordingly

$\hat{\delta}(S, xa) := \bigcup_{q \in \hat{\delta}(S,x)} \delta(q, a)$

Definition

An NFA A is said to accept a word $w \in \Sigma^*$ if $\hat{\delta}(\{q_0\}, w) \cap F \neq \emptyset$.

Subset construction

Lemma

Let A be an NFA. Then $L(A)$ is a regular language. That is, NFA and DFA accept the same set of languages.

Proof.

Let $A = (Q, \Sigma, \delta, q_0, F)$. We will construct a DFA $B = (Q', \Sigma, \delta', q_0', F')$ such that $L(A) = L(B)$.

Subset construction

$Q' = 2^Q$,

$\delta'(S, a) = \hat{\delta}(S, a)$, where $S \subseteq Q$ and $a \in \Sigma$,

$q_0' = q_0$,

$F' = \{S \subseteq Q \mid S \cap F \neq \emptyset\}$.

Correctness

Try to prove it yourself. The definition of $\hat{\delta}$ will be useful here.

$\square$