# Acceptance by NFA

## Definition (Acceptance by NFA)

A non-deterministic finite state automaton (NFA) $A = (Q, \Sigma_\epsilon, \delta, q_0, q_f)$, is said to accept a word $w \in \Sigma^*$, where $w = w_1 w_2 \dots w_n$ if

$w$ can be written as $y_1 y_2 \dots y_m$, where each $y_i \in \Sigma_\epsilon$ and $m \geq n$

there exists a sequence of states $p_0, p_1, \dots p_m$ s.t.

$p_0 = q_0$,

$p_m \in F$,

$p_{i+1} \in \delta(p_i, y_{i+1})$ for all $0 \leq i \leq m-1$.

An NFA $A$ is said to recognize a language $L$ if $L = \{w \mid A \text{ accepts } w\}$.

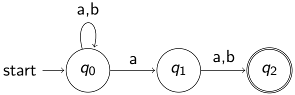
Notation: Let $A$ be an NFA/DFA. We use $L(A)$ to denote the language recognized by $A$.

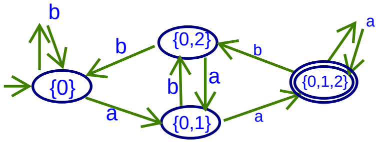
# Power of NFAs

**Lemma**

*Let $A$ be an NFA. Then $L(A)$ is a regular language. That is, NFA and DFA accept the same set of languages.*

We will work it out for an example.




| States of DFA -> | $\emptyset$ | $\{0\}$ | $\{1\}$ | $\{2\}$ | $\{0,1\}$ | $\{0,2\}$ | $\{1,2\}$ | {0, 1, 2} $\{\cancel{1,2,3}\}$ |
|---|---|---|---|---|---|---|---|---|
| $a$ | $\emptyset$ | $\{0,1\}$ | $\{2\}$ | $\emptyset$ | $\{0,1,2\}$ | $\{0,1\}$ | $\{2\}$ | $\{0,1,2\}$ |
| $b$ | $\emptyset$ | $\{0\}$ | $\{2\}$ | $\emptyset$ | $\{0,2\}$ | $\{0\}$ | $\{2\}$ | $\{0,2\}$ |

# Subset construction

From now on we will not distinguish between $\Sigma$ and $\Sigma_\epsilon$.

## Definition

Let $A = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Let $\hat{\delta} : 2^Q \times \Sigma \to 2^Q$ be defined as follows:

Let $S \subseteq Q$

$\hat{\delta}(S, \epsilon) := S$ If $A$ has epsilon transitions, then $\hat{\delta}(S, \epsilon)$ will be defined accordingly

$\hat{\delta}(S, xa) := \bigcup_{q \in \hat{\delta}(S,x)} \delta(q, a)$

## Definition

An NFA $A$ is said to accept a word $w \in \Sigma^*$ if $\hat{\delta}(\{q_0\}, w) \cap F \neq \emptyset$.

# Subset construction

## Lemma

*Let $A$ be an NFA. Then $L(A)$ is a regular language. That is, NFA and DFA accept the same set of languages.*

## Proof.

Let $A = (Q, \Sigma, \delta, q_0, F)$. We will construct a DFA $B = (Q', \Sigma, \delta', q_0', F')$ such that $L(A) = L(B)$.

### Subset construction

$Q' = 2^Q$,

$\delta'(S, a) = \hat{\delta}(S, a)$, where $S \subseteq Q$ and $a \in \Sigma$,

$q_0' = q_0$,

$F' = \{S \subseteq Q \mid S \cap F \neq \emptyset\}$.

### Correctness

Try to prove it yourself. The definition of $\hat{\delta}$ will be useful here.

□