CS310 Automata Theory – 2016-2017

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Lecture 5: Finite state automata
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Last class

On power of NFAs

Correctness of subset construction.

NFA with ϵ moves equivalent to NFA with no ϵ moves.

Hanlding the ϵ moves

Lemma

For any NFA A with ϵ transitions, there is another NFA, say B, such that B has no ϵ transitions and $L(A) = L(B)$.

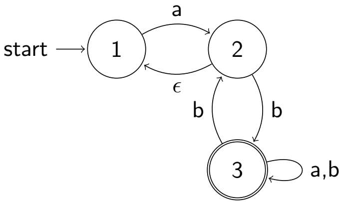
Proof Idea

Let $S \subseteq Q$.

Let

$$E(S) = \left\{ q \;\middle|\; \begin{array}{l} q \text{ is reachable from some state in } S \\ \text{with zero or more } \epsilon \text{ transitions} \end{array} \right\}$$

Example



$$\begin{aligned} E(\{1\}) &= \{1\} \\ E(\{2\}) &= \{1,2\} \\ E(\{3\}) &= \{3\} \end{aligned}$$

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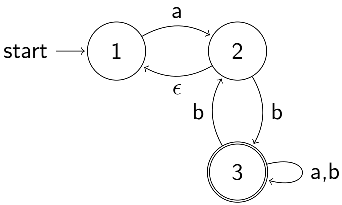

$$\begin{aligned} \delta'(1, a) &= E(\delta(1, a)) \\ &= E(\{2\}) \\ &= \{1, 2\} \end{aligned}$$

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Proof Idea

Let $S \subseteq Q$.

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$$E(S) = \left\{ q \;\middle|\; \begin{array}{l} q \text{ is reachable from some state in } S \\ \text{with zero or more } \epsilon \text{ transitions} \end{array} \right\}$$

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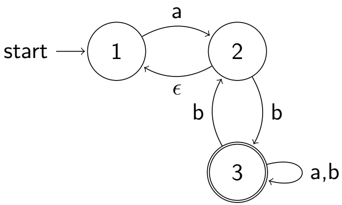

$$\begin{aligned} \delta'(3, b) &= E(\delta(3, b)) \\ &= E(\{1\}) \\ &= \{1\} \end{aligned}$$

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Proof Idea

Let $S \subseteq Q$.

Let

$$E(S) = \left\{ q \;\middle|\; \begin{array}{l} q \text{ is reachable from some state in } S \\ \text{with zero or more } \epsilon \text{ transitions} \end{array} \right\}$$

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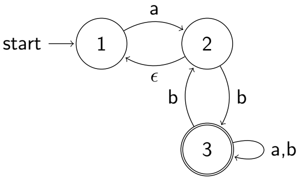

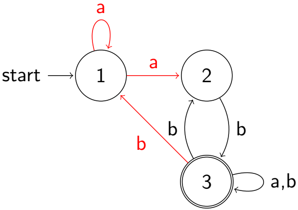

Hanlding the ϵ moves

Lemma

For any NFA A with ϵ transitions, there is another NFA, say B, such that B has no ϵ transitions and $L(A) = L(B)$.

Proof.

Let $A = (Q, \Sigma, \delta, q_0, F)$ be an NFA with ϵ transitions. We construct NFA, say B as follows:

Construction

$Q' = Q$,

Σ' same as Σ, but no ϵ used anywhere,

$\delta'(q, a) = E(\delta(q, a))$,

$q_0' = q_0$,

$F' = F$.

Wha if ϵ transitions from the start/to the final state in A? □

Hanlding the ϵ moves

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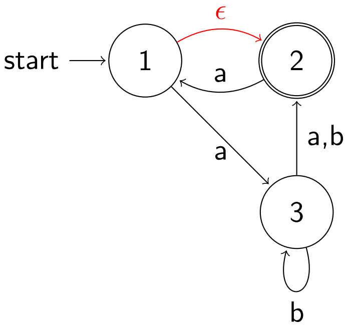
Example



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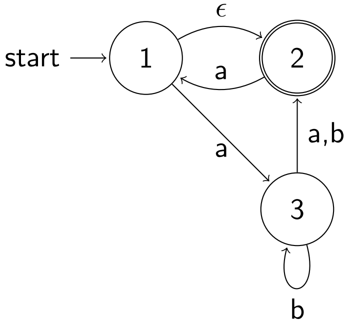

Add a new start state $\tilde{q}_0$.

Consider $\hat{\delta}(E(q_0), c)$ for every $c \in \Sigma$.

Add an edge from $\tilde{q}_0$ to $q \in Q$ with label c if

$$q \in E\left(\hat{\delta}(E(q_0), c)\right).$$

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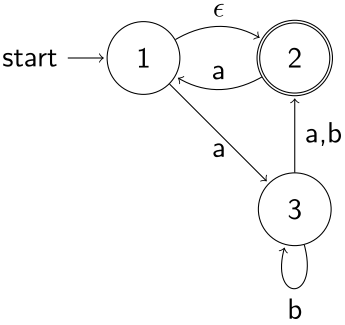
Example



$$
\begin{aligned}
& E(\hat{\delta}(E(1), a)) \\
= \;& E(\hat{\delta}(\{1, 2\}, a)) \\
= \;& E(\delta(1, a) \cup \delta(2, a)) \\
= \;& E(\{3, 1\}) \\
= \;& \{1, 2, 3\}
\end{aligned}
$$

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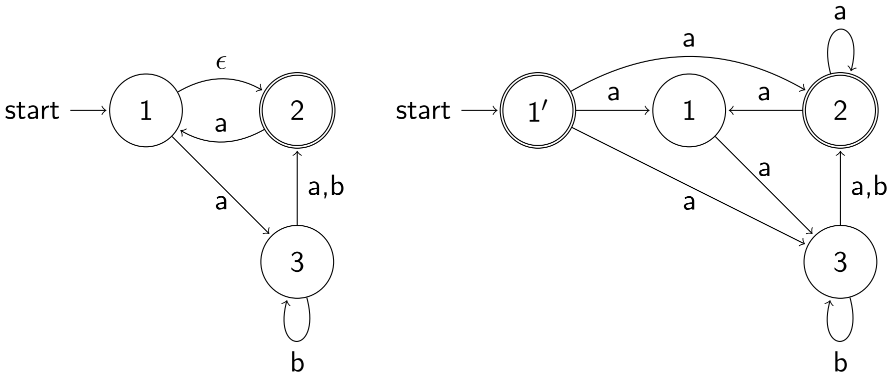
Example



Hanlding the ϵ moves

Lemma

For any NFA A with ϵ transitions, there is another NFA, say B, such that B has no ϵ transitions and $L(A) = L(B)$.

Proof.

Let $A = (Q, \Sigma, \delta, q_0, F)$ be given. We construct $B = (Q', \Sigma', \delta', q_0, F')$ as follows:

Construction

$Q' = Q \cup \{\tilde{q}_0\}$, $q'_0 = \tilde{q}_0$, Σ' same as Σ but no ϵ,

$$F' = \begin{cases} F \cup \{\tilde{q}_0\} & \text{if } E(\{q_0\}) \cap F \neq \emptyset \\ F & \text{otherwise} \end{cases}$$

$$\delta'(q, a) = \begin{cases} E(\delta(E(q_0), a)) & \text{if } q = \tilde{q}_0 \\ E(\delta(q, a)) & \text{otherwise} \end{cases}$$

Correctness

Tutorial 2.