A few notable things

Note the following about the proof.

H accepts $\langle M, w \rangle$ when M accepts w.

D rejects $\langle M \rangle$ when M accepts $\langle M \rangle$.

D rejects $\langle D \rangle$ when D accepts $\langle D \rangle$.

Note also the following things about similar problems.

$A_{\text{DFA}} = \{(M, w) \mid \text{ DFA } M \text{ accepts } w\}$ is decidable.

Similarly, $A_{\text{PDA}} = \{(M, w) \mid \text{ PDA } M \text{ accepts } w\}$ is also decidable.

Diagonalization inside the proof

Behaviour of D on itself.

	$\langle M_1 \rangle$	$\langle M_2 \rangle$	$\langle M_3 \rangle$	$\ldots \langle D \rangle \ldots$	$\ldots\ldots$
M_1	$\cancel{\checkmark}$ $\times$	$\times$	$\checkmark$	$\checkmark \ldots$	$\ldots\ldots$
M_2	$\checkmark$	$\cancel{\times}$ $\checkmark$	$\times$	$\times \ldots$	$\checkmark \ldots \times \checkmark \ldots$
M_3	$\times$	$\times$	$\cancel{\checkmark}$ $\times$	$\ldots \times$	$\checkmark \ldots\ldots$
$\vdots$					
$\vdots$					
D				$\ldots$? $\ldots$	$\ldots\ldots$

Other undecidable problems and reducibility

Reducing A_{TM} to another problem to prove undecidibility.

$$\text{Halt} = \{(M, w) \mid M \text{ halts on } w\}$$

We would like to show that Halt is undecidable.

Assume that Halt is decidable. Let $\mathcal{H}$ be the TM deciding Halt.

$\mathcal{A}$: Run $\mathcal{H}$ on (M, w). If it rejects then reject, else do as per M on w.

$\mathcal{A}$ accepts (M, w) if M accepts w and rejects it if either M rejects w or M loops forever on w.

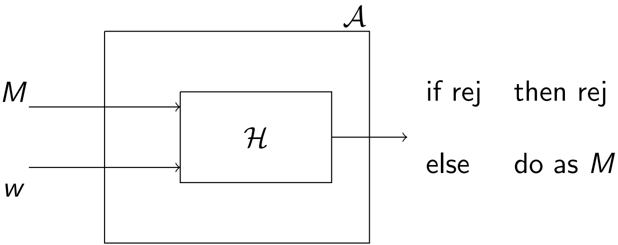
$\mathcal{H}$ decides Halt if and only if $\mathcal{A}$ decides A_{TM}.

The halting problem

Lemma

The halting problem, $Halt = \{(M, w) \mid M \text{ halts on } w\}$, is undecidable.

Another way to describe the same proof.



If Halt is decidable then $\mathcal{A}$ decides A_{TM}, which is a contradiction.