

Quiz 2

[Total: 25]

Instructions

- If you use a result proved in class, state it completely.
- Results proved in the tutorial sheets **cannot be used** without proofs.
- Be precise in your writing.

Notation For $a \in \Sigma$ and $w \in \Sigma^*$ let $\#_a(w)$ denote the number of occurrences of the letter a in the string w . Let $L^2 := L \circ L = \{w.w' \mid \forall w, w' \in L\}$, let L^i be $L^{i-1} \circ L$ and let $L^* = \cup_{i \geq 0} L^i$.

1. Prove or disprove that the following languages are regular. (3+3+3)
 - (a) $\{x \cdot \# \cdot y \mid x, y \in \{a, b\}^*, \#_a(x) = \#_b(y)\}$.
 - (b) $\{x \cdot y \mid x, y \in \{a, b\}^*, \#_a(x) = \#_b(y)\}$.
 - (c) $\{w \in \{0, 1\}^* \mid w \text{ has equal number of occurrences of the strings } 01 \text{ and } 10\}$.
2. Let $L \subseteq \{0, 1\}^*$. Let us define $\text{Rot}(L) = \{y \cdot x \mid x \cdot y \in L\}$. For example, $L = \{001, 10\}$, then $\text{Rot}(L) = \{001, 010, 100, 10, 01\}$. (2+2+3+3+1)
 - (a) Let $L_{a,b} = \{a^n b^n \mid n \geq 0\}$. Write down the language $\text{Rot}(L_{a,b})$.
 - (b) Show that for any language L , $\text{Rot}(\text{Rot}(L)) = L$.
 - (c) Say L is regular and let M be a DFA recognizing L . Give an informal description of an NFA N for $\text{Rot}(L)$.
 - (d) Write down the formal construction of N .
 - (e) Say M has k states. Then give a bound on the number of states of the NFA that you constructed in part (d) above.
3. Let $\text{EQ} = \{w \cdot w \mid w \in \Sigma^*\}$. Prove or disprove that $\Sigma^* \setminus \text{EQ}$ is context-free. (5)