

Relationships between complexity classes

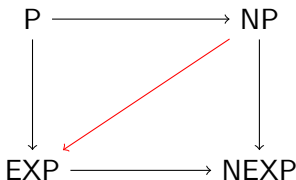
How are P, NP, EXP, and NEXP related?

$P \subseteq NP$ by definition.

$P \subseteq EXP$ again by definition.

Similarly, $NP \subseteq NEXP$ by definition.

Finally, $NP \subseteq EXP$ due to the previous lemma.



P vs. NP

P the class of languages where membership can be decided quickly.

NP the class of languages where membership can be verified quickly.

Examples

$\text{SAT} = \{\phi \mid \phi \text{ is satisfiable}\}$. in NP (and not known to be in P)

$\text{Reach} = \{(G, s, t) \mid t \text{ is reachable from } s \text{ in } G\}$. in P

$\text{3-SAT} = \{\phi \mid \phi \text{ is a 3-CNF and satisfiable}\}$. in NP (and not known to be in P)

$\text{2-SAT} = \{\phi \mid \phi \text{ is a 2-CNF and satisfiable}\}$. in P

$\text{Factoring} = \{(k, n) \mid n \text{ has a factor } \leq k\}$. Google it!

$\text{Clique} = \{(G, k) \mid G \text{ has a clique of size } \geq k\}$. in NP (and not known to be in P)

Time heirarchy theorem

How do we separate NP from P?

| To prove | Method used |
|------------------|--|
| not regular | pumping lemma for REG |
| non-context-free | pumping lemma or CFLs |
| not recognizable | diagonalization |
| not decidable | Rice's theorem or diagonalization and reductions |
| not in P | ??? |

Finer structure inside P

Definition

A function $t : \mathbb{N} \rightarrow \mathbb{N}$ is said to be time constructible if there exists a TM that on input 1^n , it outputs $t(n)$ in time $O(t(n))$.

Examples

$$n^2, n \log n.$$

Theorem

Let $t : \mathbb{N} \rightarrow \mathbb{N}$ be a time constructible function. There exists a language L such that $L \in \text{TIME}(t(n)^2)$, but $L \notin \text{TIME}(o(t(n)))$.