

CS310 Automata Theory – 2016-2017

Nutan Limaye

Indian Institute of Technology, Bombay

nutan@cse.iitb.ac.in

Lecture 33: Effective computation

April 06, 2017

Module IV: Effective computation

Turing machines with resource constraints.

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Resources for computation.

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Many different ways exist. ...

Time complexity and complexity classes

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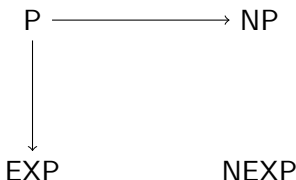
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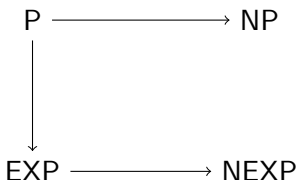
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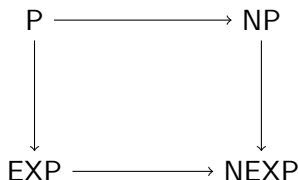
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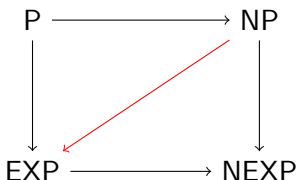
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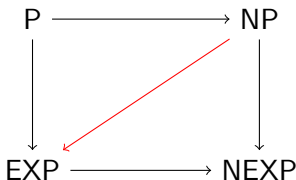
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