Tutorial 9

1. Prove that any language accepted by an NPDA is Turing decidable.
2. Prove that the following language is decidable: $\{\langle M\rangle \mid M \text{ does not move left on any input}\}$.
3. Prove that for any non-deterministic TM there is an equivalent deterministic TM.
4. In class we proved that Halt $= \{(M, w) \mid M \text{ halts on } w\}$ is undecidable by giving a reduction (from A_{TM}). Prove that Halt is undecidable by giving a proof by the diagonalization argument.
5. Prove that $\overline{\text{Halt}}$ is not Turing recognizable.
6. A function $f : \Sigma^* \to \Gamma^*$ is called a **computable function** if there is a single tape TM M such that on every input $w \in \Sigma^*$ it halts with exactly $f(w)$ on the tape.

   We say that a language $L \subseteq \Sigma^*$ is **reducible** to language $L' \subseteq \Gamma^*$, which we denote by $L \leq L'$, if there is a computable function $f : \Sigma^* \to \Gamma^*$, where for every w, $w \in L \Leftrightarrow f(w) \in L'$. The function f of this form is called a **reduction**. Prove or disprove the following statements:

   (a) If $L \leq L'$ and L' is decidable then L is also decidable.

   (b) If $L \leq L'$ and L is undecidable then L' is also undecidable.

   (c) If $L \leq L'$ and L' is a CFL then so is L.

   (d) If L is Turing recognizable and $L \leq \overline{L}$ then L is decidable.

   (e) L is Turing recognizable if and only if $A \leq A_{TM}$.

   (f) There is a laguage L such that L is undecidable and $L \leq \overline{L}$.