Configuration

Definition

The configuration of a TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_f, q_{rej})$ is given by

$$\Gamma^* \times Q \times \Gamma^*$$

A configuration need not include blank symbols.

Let $u, v \in \Gamma^*$, $a, b, c \in \Gamma$ and $q, q' \in Q$.

Suppose $(q', c, L) \in \delta(q, b)$ is a transition in M,
then starting from $u \cdot a \cdot q \cdot b \cdot v$ in one step we get $u \cdot q' \cdot a \cdot c \cdot v$.

We say that $u \cdot a \cdot q \cdot b \cdot v$ **yields** $u \cdot q' \cdot a \cdot c \cdot v$.
We denote it by $u \cdot a \cdot q \cdot b \cdot v \mapsto u \cdot q' \cdot a \cdot c \cdot v$.

Special configurations

Start configuration

We assume that the head is on the left of the input in the beginning.

Therefore, $q_0 \cdot w$ is the start configuration.

Accepting configuration

Any configulation that contains q_{acc} is an accepting configuration.

Rejecting configuration

Any configulation that contains q_{rej} is a rejecting configuration.

Halting configurations: if a configuration is accepting or rejecting then it is called a halting configuration.

A TM may not halt!

Acceptance by a TM

A TM M is said to accept a word $w \in \Sigma^*$ if there exists a sequence of configurations $C_0, C_1, \ldots, C_k$ such that

C_0 is a start configuration,

$C_i \mapsto C_{i+1}$ for all $0 \leq i \leq k-1$,

C_k is an accepting configuration.

The notion of rejection by TM is not as straightforward!

Turing recognizable languages

Definition

A language L is said to be Turing recognizable if there is a Turing machine M such that $\forall w \in L$, M reaches an accepting configuration on w.

We say that M recognizes L.

For words not in L

the machine may run forever,

or may reach q_{rej},

both are valid outcomes,

and the machine is allowed to do either of the two.

Turning decidable languages

Definition

A language L is said to be Turing decidable if there is a Turing machine M such that

$\forall w \in L$, M reaches the accepting configuration on w.

$\forall w \notin L$, M reaches the rejecting configuration on w.

We say that M decides L.

If a language L is Turing decidable then

- the TM deciding L always halts.
- L is also Turing recognizable.
- Turing decidable languages form a subclass of Turing recognizable languages.