Tutorial 2

Notation: Let $\Sigma = \{a, b\}$. For $w \in \Sigma^*$ let $|w|$ denote the length of w. Let $\#_a(w)$ denote the number of as in w and let $\#_b(w)$ denote the number of bs in w. Let w^R be the reverse of the string w.

1. Let L_1, L_2 be two regular languages. Show that $L_1 - L_2 = \{w \mid w \in L_1 \text{ and } w \notin L_2\}$ is also regular.

2. Let $\Sigma = \{a_1, a_2, \ldots, a_k\}$. Draw an NFA with $k + 1$ states for the following language:

$$L := \{w \mid \exists i \text{ s. t. } 1 \leq i \leq k \text{ and } a_i \text{ does not appear in } w\}.$$

   Can there be an NFA for this with only k states?

3. A homomorphism on a set Σ is a map from Σ to another set Σ'^* such that each letter in Σ is mapped to a string over Σ'. For example, say $\Sigma = \{0, 1\}$ and a function h is defined as follows $h(0) := aaab$ and $h(1) := aba$ then h is a homomorphism on $\{0, 1\}$. Let L be a regular language. Show that the following language is also regular

$$h(L) := \{h(w) \mid w \in L\}.$$

4. Recall the definition of $\hat{\delta}$ (Lecture 3, slide 3). Prove property (P_3) about $\hat{\delta}$ (Lecture 3, Slide 6.)

5. Recall the construction we used to convert an NFA with ε moves to an equivalent NFA without ε moves. (Lecture 5, Slide 12.) Prove the correctness of this construction.

6. Let L, L' be two regular languages. Let us define $L||L'$ as follows:

$$L||L' := \{x_1y_1x_2y_2 \ldots x_ny_n \mid x_1x_2 \ldots x_n, y_1y_2 \ldots y_n \in \Sigma^*, x_1x_2 \ldots x_n \in L \text{ and } y_1y_2 \ldots y_n \in L_2\}$$

   Prove that $L||L'$ is regular by explicitly constructing an NFA/DFA for the language.

7. Let L be a regular language. Show that the following language is also regular.

$$\frac{L}{2} = \{w \mid \exists w' \text{ s.t.} |w| = |w'| \text{ and } w \cdot w' \in L\}.$$