CS310 Automata Theory – 2016-2017

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Lecture 19: Extensions of DFA/NFAs
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Last class

Pushdown automata: NFA + Stack.

Formal definition of non-deterministic pushdown automata (NPDA).

Different acceptance conditions for NPDA.

Pumping lemma for CFLs.

Context-free grammars

Inductive definition of PAL.

$\epsilon, 0, 1$ are in PAL.

If w is in PAL then $0 \cdot w \cdot 0 \in$ PAL.

If w is in PAL then $1 \cdot w \cdot 1 \in$ PAL.

Context-free grammar for PAL.

$S \rightarrow \epsilon$.

$S \rightarrow 0$.

$S \rightarrow 1$.

$S \rightarrow 0S0$.

$S \rightarrow 1S1$.

Context-free grammar

Definition

A context-free grammar (CFG) G is given by (V, T, P, S_0), where

- V is a set of variables,
- T is a set of terminal symbols or the alphabet,
- P is a set of productions, $P \subseteq V \times (V \cup T)^*$,
- $S_0 \in V$, a start symbol.

Example: Grammar for PAL.

$S \to \epsilon.$

$S \to 0.$

$S \to 1.$

$S \to 0S0.$

$S \to 1S1.$

$$\begin{aligned} G_{pal} &= (V, T, P, S_0) \text{ such that} \\ V &= \{S\}, \\ T &= \{0, 1\}, \\ P &= \{S \to \epsilon, S \to 0, S \to 1, S \to 0S0, S \to 1S1\}, \\ S_0 &= S. \end{aligned}$$

Drivations of a CFG

Definition

Let G be a CFG given by (V, T, P, S_0).

Let $w, w' \in (V \cup T)^*$,

let $A \in V$ and let $(A \to v) \in P$ be a production in the grammar, where $v \in (V \cup T)^*$.

Then we say that $w \cdot A \cdot w'$ **derives** $w \cdot v \cdot w'$ **in one step**.

We denote it as follows: $w \cdot A \cdot w' \Rightarrow w \cdot v \cdot w'$.

Definition ($\Rightarrow^*$)

Let G be a CFG given by (V, T, P, S_0).

For all $\alpha \in (V \cup T)^*$, we say that $\alpha \Rightarrow^* \alpha$.

For all $\alpha, \beta, \gamma \in (V \cup T)^*$,

if $\alpha \Rightarrow^* \beta$ and $\beta \Rightarrow \gamma$ then $\alpha \Rightarrow^* \gamma$.

Language of a CFG

Definition

Let G be a CFG given by (V, T, P, S_0). The **language of** G, $L(G)$, is the set of all the strings over T which can be derived from S_0, i.e.

$$L(G) = \{w \in T^* \mid S \Rightarrow^* w\}.$$

Lemma

$L(G_{pal})$ is equal to PAL.
$\forall w \in \{0,1\},^* \; w \in PAL$ if and only if $w = w^R$.

Proof.

By Induction on $|w|$. DIY! □

Examples

Give context-free grammars for the following languages.

$\{a^i \cdot b^j \cdot c^k \mid \text{ either } i \neq j \text{ or } j \neq k\}$.

$\{a^i b^j \mid \text{either } i = j \text{ or } i = 2j\}$.

$\{x \mid x \neq w \cdot w, \text{ where } w \in \{a, b\}^*\}$.

$\{a^i \cdot b^j \cdot c^k \mid i = j + k\}$.

The grammars of all the above languages and the correctness for the last one were discussed in class.