CS310 Automata Theory – 2016-2017

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Lecture 16: Extensions of DFA/NFAs
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Last class

Pushdown automata: NFA + Stack.

Formal definition of non-deterministic pushdown automata (NPDA).

Different acceptance conditions for NPDA.

Pushdown automata

NFA + Stack

$$L_{a,b} = \{a^n b^n \mid n \geq 0\}.$$

$$PAL = \{w \cdot w^R \mid w \in \Sigma^*\}.$$

Pushdown automata: formal definition

Definition

A non-deterministic pushdown automaton (NPDA)
$A = (Q, \Sigma, \Gamma, \delta, q_0, \perp, F)$, where

Q:	set of states	Σ:	input alphabet
Γ:	stack alphabet	q_0:	start state
$\perp$:	start symbol	F:	set of final states

$\delta \subseteq Q \times \Sigma \times \Gamma \times Q \times \Gamma^*$.

Understanding δ

For $q \in Q, a \in \Sigma$ and $X \in \Gamma$, if $\delta(q, a, X) = (p, \gamma)$,
then p is the new state and γ replaces X in the stack.

if $\gamma = \epsilon$ then X is popped.
if $\gamma = X$ then X stays unchanges on the top of the stack.
if $\gamma = \gamma_1\gamma_2 \dots \gamma_k$ then X is replaced by γ_k
and $\gamma_1\gamma_2 \dots \gamma_{k-1}$ are pushed on top of that.

Configuration of an NPDA

Definition (Configurations)

A configuration of an NPDA $A = (Q, \Sigma, \Gamma, \delta, q_0, \perp, F)$ is a three tuple (q, w, γ), where $q \in Q$, $w \in \Sigma^*$, and $\gamma \in \Gamma^*$.

if $(p, \gamma) \in \delta(q, a, X)$ then $\forall w \in \Sigma^*$ and $\gamma' \in \Gamma^*$,

$$(q, a \cdot w, X\gamma) \ \vdash \ (p, w, \gamma \cdot \gamma')$$

Let I, J are two configurations of A.

We say that $I \vdash^k J$ iff $\exists I'$ such that $I \vdash I'$ and $I' \vdash^{k-1} J$.

Language recognized by pushdown automata

Definition

We say that a word is accepted by an NPDA A if $(q_0, w, \perp) \vdash^* (q, \epsilon, \epsilon)$, where $q \in Q$. acceptance by an empty stack.

A language L is said to be recognized by an NPDA A if the set $\{w \mid w \text{ is accepted by } A\}$ is the same as L.

The class of languages recognized by NPDAs is called Context-free languages.

Another notion of acceptance of words:

We say that a word is accepted by an NPDA A if $(q_0, w, \perp) \vdash^* (q, \epsilon, \gamma)$, where $q \in F$. acceptance by a final state.

Context-free languages

Examples

$\text{PAL} = \{w \cdot w^R \mid w \in \Sigma^*\}.$

$\text{Balanced} = \{w \in \{(,),[,]\} \mid w \text{ balanced string of paranthesis } \}.$

$L_{a/b/c} = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}.$

$\text{NEQ} = \{x \mid x \neq w \cdot w \text{ for any } w \in \Sigma^*\}.$

$L_{a/b/c} = \{a^i b^j c^k \mid i \neq j \text{ and } j \neq k\}.$?

$L_{a,b,c} = \{a^n \cdot b^n \cdot c^n \mid n \geq 0\}.$?

$\text{EQ} = \{w \cdot w \mid w \in \Sigma^*\}.$?

Non-context-free languages

Lemma (Pumping lemma for CFLs)

Say L is a language over the alphabet Σ^*. If

- ☹ *for all $n \in \mathbb{N}$,*
- ☺ *$\exists z \in \Sigma^*$, such that*
- ☹ *for all possible ways of breaking z into $z = u \cdot v \cdot w \cdot x \cdot y$, s.t. $|v \cdot w \cdot x| \le n$ and $|v \cdot x| > 0$,*
- ☺ *$\exists i \in \mathbb{N}$ s. t. $u \cdot v^i \cdot w \cdot x^i \cdot y \notin L$,*

then L is not a CFL.

Applications of the pumping lemma for CFLs

Let $L_{a,b,c} = \{a^n b^n c^n \mid n \geq 0\}$

- ☹ For any chosen n,
- ☺ let $z = a^n \cdot b^n \cdot c^n$
- ☹ For any split of z into u, v, w, x, y
- ☺ as $|v \cdot w \cdot x| \leq n$
  Either $v \cdot w \cdot x$ has no c's, or no a's.
  Therefore, $u \cdot v^0 \cdot w \cdot x^0 \cdot y \notin L$.

Say L is a language over the alphabet Σ^*. If

- ☹ for all $n \in \mathbb{N}$,
- ☺ $\exists z \in \Sigma^*$, such that
- ☹ for all possible ways of breaking z into $z = u \cdot v \cdot w \cdot x \cdot y$, s.t. $|v \cdot w \cdot x| \leq n$ and $|v \cdot x| > 0$,
- ☺ $\exists i \in \mathbb{N}$ s. t. $u \cdot v^i \cdot w \cdot x^i \cdot y \notin L$,

then L is not a CFL.

Applications of the pumping lemma for CFLs

Let $EQ = \{w \cdot w \mid w \in \{a, b\}^*\}$.

- ☹ For any chosen n,
- ☺ let $z = a^n \cdot b \cdot a^n \cdot b$
- ☹ For any split of z into u, v, w, x, y
- ☺ Note that $|v \cdot w \cdot x| \leq n$.
  (after some case analysis.)
  Therefore, $u \cdot v^0 \cdot w \cdot x^0 \cdot y \notin L$.

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