Regular languages

Definition

A language $L \subseteq \Sigma^*$ is a said to be recognized by a DFA A if $L = \{w \mid w \text{ is accepted by } A\}$.

Definition (REG)

A language is said to be a regular language if it is recognized by some DFA.

Examples

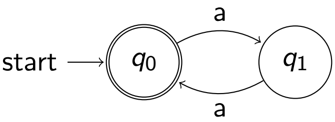
$$
\begin{aligned}
L &= \{w \in \{a, b\}^* \mid w \text{ ends with aa}\} \\
L' &= \{w \in \{a, b\}^* \mid w \text{ contains aa}\} \\
L_{odd} &= \{w \in \{a, b\}^* \mid w \text{ contains odd number of a}\} \\
L_3 &= \{w \in \{0, 1\}^* \mid w \text{ encodes a number in binary divisible by 3}\}
\end{aligned}
$$

Closure properties of regular languages

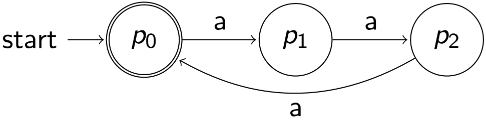
Example

Let $\Sigma = \{a\}$ for this example.

Let $L_1 = \{w \mid |w| \equiv 0 \ (mod\ 2)\}$



Let $L_2 = \{w \mid |w| \equiv 0 \ (mod\ 3)\}$



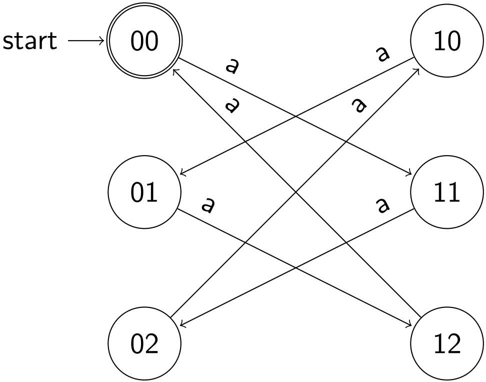
What is $L_1 \cap L_2$?

$L_1 \cap L_2 = \{w \mid |w| \equiv 0 \ (mod\ 6)\}$

Closure properties of regular languages

Example continued

$$L_1 \cap L_2 = \{w \mid |w| \equiv 0 \ (mod\ 6)\}$$



Closure properties of regular languages

Lemma

Let $L_1, L_2 \subseteq \Sigma^*$ be two regular languages, then $L_1 \cap L_2$ is also a regular language.

Proof.

Product construction

Let $A_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$ and $A_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$ be the automata recognizing L_1, L_2, respectively.

Let A be a finite state automaton $(Q, \Sigma, \delta, q_0, F)$ s.t.

$$
\begin{array}{rcl}
Q & = & \{(q, q') \mid q \in Q_1, q' \in Q_2\} \\
\delta((q, q'), a) & = & (\delta_1(q, a), \delta_2(q', a)) \\
q_0 & = & (q_0^1, q_0^2) \\
F & = & \{(q, q') \mid q \in F_1, q' \in F_2\}
\end{array}
$$

Correctness

$\forall w \in \Sigma^*$, w is accepted by A iff w is accepted by both A_1 and A_2.

□

Closure properties of regular languages

Lemma

Let $L_1, L_2 \subseteq \Sigma^*$ be two regular languages, then $L_1 \cap L_2$ is also a regular language.

Lemma

Let $L_1, L_2 \subseteq \Sigma^*$ be two regular languages, then $L_1 \circ L_2$ is also a regular language, where $L_1 \circ L_2 = \{w \cdot w' \mid w \in L_1, w' \in L_2\}$ and $\cdot$ represents the concatenation operation.

Non-deterministic finite state automata

Informal description: A finite state automaton in which can branch out on different states on the same letter.

Definition (NFA)

A non-deterministic finite state automaton (NFA) $A = (Q, \Sigma_\epsilon, q_0, F, \delta)$, where

Q is a set of states,

$\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$ is the input alphabet,

q_0 is the initial state,

$F \subseteq Q$ is the set of final states,

δ is a set of transitions, i.e. $\delta \subseteq Q \times \Sigma_\epsilon \times 2^Q$

$\cancel{\forall q \in Q, \forall a \in \Sigma, |\delta(q, a)| \leq 1}$.

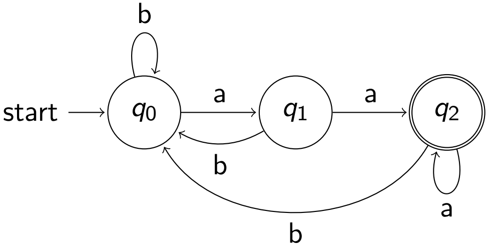
$\forall q \in Q, \forall a \in \Sigma_\epsilon,$ $\delta(q, a) \subseteq Q$.

Non-deterministic finite state automata

Example

Input: Text file over the alphabet $\{a, b\}$

Check: does the file end with the string 'aa'



Non-deterministic finite state automata

Example

Input: $w \in \{a, b\}$

Check: Is a the second-last letter of w?