

# Power of 2DFAs

## Lemma

*The class of language recognized by 2DFAs is regular.*

## Proof.

Let  $T_x : Q \times \{\bowtie\} \rightarrow Q \times \{\perp\}$ , which is defined as follows:

$T_x(p) := q$  if whenever  $A$  enters  $x$  on  $p$   
it leaves  $x$  on  $q$ .

$T_x(\bowtie) := q$   $q$  is the state in which  $A$  emerges  
on  $x$  the first time.

$T_x(q) := \perp$  if  $A$  loops on  $x$  forever.



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Total number of functions of the type

$$T_x \leq (|Q| + 1)^{(|Q|+1)}$$

$T_x = T_y \Rightarrow \forall z (xz \in F \Leftrightarrow yz \in F)$ . Prove this.

$$T_x = T_y \Leftrightarrow x \equiv_A y$$



# Pushdown automata

NFA + Stack

$$L_{a,b} = \{a^n b^n \mid n \geq 0\}.$$

$$PAL = \{w \cdot w^R \mid w \in \Sigma^*\}.$$