Tutorial 4

Notation: Let $\Sigma = \{a, b\}$. For $w \in \Sigma^*$ let $|w|$ denote the length of w. Let $\#_a(w)$ denote the number of as in w and let $\#_b(w)$ denote the number of bs in w. Let w^R be the reverse of the string w.

1. **Pumping Lemma vs. Myhill-Nerode**

   (a) Give an example non-regular language L such that the pumping lemma cannot be used to prove that it is not regular.

   (b) Use the Myhill-Nerode theorem to prove that the language above is not regular.

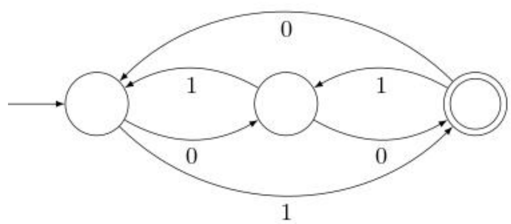
2. **DFA minimization**

   (a) Build an NFA for the following language:
   $L = \{w \in \{a, c, b, d\}^* \mid |w| \geq 1$ and the last letter in w does not appear anywhere else in w$\}$

   (b) Prove that the minimum DFA for the above language L must have at least 16 states.

3. **DFA to Regular expression**

   (a) Given below is the description of a DFA. Give equivalent regular expression for that DFA.



   (b) Let $x \in \Sigma^*$ be any string and $L_x = \{y \mid xy \in L\}$. How many distinct L_x languages are possible for the above DFA?

4. **Decision problems about finite state automata**

   (a) Let L be a regular language on Σ and $\hat{w}$ be any string in Σ^*. Give an algorithm to determine if L contains any w such that $\hat{w}$ is a substring of it, that is, such that $w = u\hat{w}v$ with $u, v \in \Sigma^*$. Analyse the run time complexity of your algorithm.

   (b) Let L be any regular language on $\Sigma = \{$a, b$\}$. Give an algorithm for determining if L contains any strings of even length. Analyse the run time complexity of your algorithm.