CS310 Automata Theory – 2016-2017

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Lecture 14: Extensions of DFA/NFAs
February 02, 2017

Last class

Different models of computation: 2DFA.

Notion of acceptance/rejection by a 2DFA.

Examples of languages recognized by 2DFA.

Main claim: The class of languages recognized by 2DFAs is exactly REG.

Module - II: Different models of computation

2DFA: Two-way deterministic finite state automata.

$\#$	w_1	w_2	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	w_n	$\$$

Input head moves left/right on this tape.

It does not go to the left of $\#$.

It does not go to the right of $\$$.

Can potentially get stuck in an infinite loop!

Formal definition of 2DFA

Definition

A 2DFA $A = (Q, \Sigma \cup \{\#, \$\}, \delta, q_0, q_{acc}, q_{rej})$, where

Q :	set of states,	Σ:	input alphabet
$\#$:	left endmarker	$\$$:	right endmarker
q_0:	start state		
q_{acc}:	accept state	q_{rej}:	reject state

$$\delta : Q \times (\Sigma \cup \{\#, \$\} \to Q \times \{L, R\}$$

The following conditions are forced:

$\forall q \in Q$, $\exists q', q'' \in Q$ s.t. $\delta(q, \#) = (q', R)$ and $\delta(q, \$) = (q'', L)$.

2DFA: Two-way deterministic finite state automata

Examples

Let $\Sigma = \{a, b\}$ and L be a regular language.

$L_1 = \{w \in \Sigma^* \mid \text{second letter from the end if } a\}$.

$L_2 = \{w \in \Sigma^* \mid w \cdot w \in L\}$

$L_2 = \{w \in \Sigma^* \mid w^{\leq |w|} \in L\}$

Acceptance by 2DFA

Definition

Let A be a 2DFA.

A word w is said to be accepted by A if A reaches q_{acc} on w.

A word w is said to be rejected by A if A reaches q_{rej} on w.

A is said to recognize a language L if $\forall w \in L$, A reaches q_{acc}.

2DFA may loop forever if $w \notin L$ or may enter q_{rej}.

Power of 2DFAs

Lemma

The class of language recognized by 2DFAs is regular.

Proof.

Let $T_x : Q \times \{\bowtie\} \rightarrow Q \times \{\perp\}$, which is defined as follows:

$$
\begin{array}{lll}
T_x(p) := & q & \text{if whenever } A \text{ enters } x \text{ on } p \\
 & & \text{it leaves } x \text{ on } q. \\
 & & \\
T_x(\bowtie) := & q & q \text{ is the state in which } A \text{ emerges} \\
 & & \text{on } x \text{ the first time.} \\
 & & \\
T_x(q) := & \perp & \text{if } A \text{ loops on } x \text{ forever.}
\end{array}
$$

$\square$

Power of 2DFAs

Lemma

The class of language recognized by 2DFAs is regular.

Proof.

Let $T_x : Q \times \{\bowtie\} \to Q \times \{\perp\}$, which is defined as follows:

$T_x(p) := q$ if whenever A enters x on p it leaves x on q.

$T_x(\bowtie) := q$ q is the state in which A emerges on x the first time.

$T_x(q) := \perp$ if A loops on x forever.

Total number of functions of the type

$T_x \leq (|Q| + 1)^{(|Q|+1)}$

$T_x = T_y \Rightarrow \forall z(xz \in F \Leftrightarrow yz \in F)$. Prove this.

$T_x = T_y \Leftrightarrow x \equiv_A y$

Pushdown automata

NFA + Stack

$$L_{a,b} = \{a^n b^n \mid n \geq 0\}.$$

$$PAL = \{w \cdot w^R \mid w \in \Sigma^*\}.$$