NFA to regular expression

Lemma

Given any NFA A, we can obtain a regular expression, say R_A, such that $L(A) = L(R_A)$.

Examples in class

Limitations of NFA

Lemma

The number of regular languages is countable.

Proof.

By counting.

Every regular language is recognized by a DFA.

Every DFA has a finite description.

All DFAs can therefore be enumerated, i.e. there is a one-to-one mapping (bijection) from all DFAs to $\mathbb{N}$.

$\square$

This implies that there exist languages which are not accepted by any DFA.

Limitations of NFA

What are examples of languages not accepted by NFAs?

$$PAL = \{w \cdot w^R \mid w \in \Sigma^*\}.$$

$$EQ = \{w \cdot w \mid w \in \Sigma^*\}.$$

$$L_{a,b} = \{a^n \cdot b^n \mid n \geq 0\}.$$

Proving that PAL is not a regular language

Lemma

$\forall n \in \mathbb{N}$ let $PAL_n = \{w \cdot w^R \mid w \in \Sigma^*, |w| = n\}$. Any automaton accepting PAL_n must have $|\Sigma|^n$ states.

Proof.

By Pigeon Hole Principle.

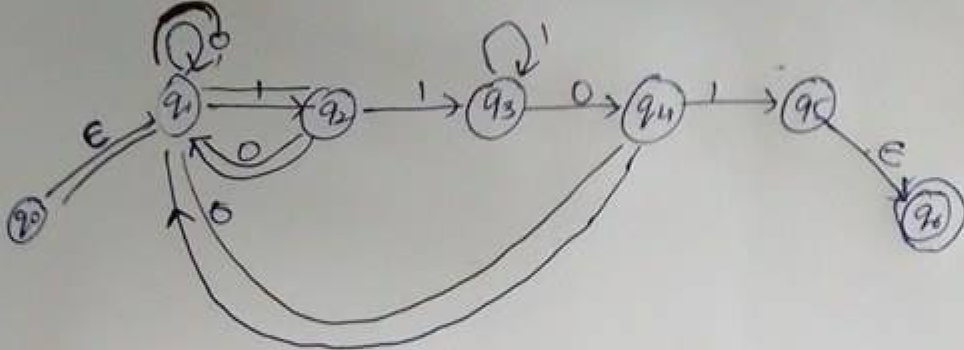
Suppose $\exists x, y \in \Sigma^n$ such that $x \neq y$,

automaton reaches the same state after reading both x, y.

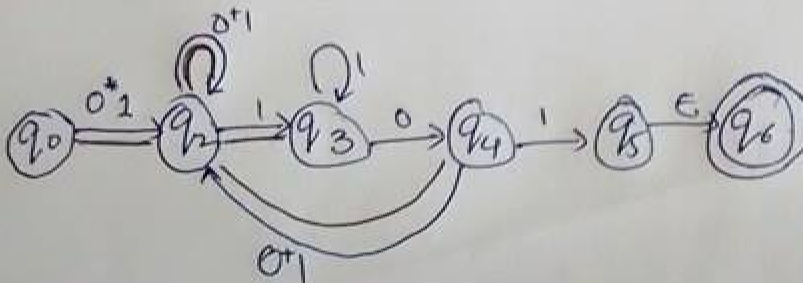
Then $x \cdot x^R$ and $y \cdot x^R$ are both accepted or both rejected,

which is a contradiction.

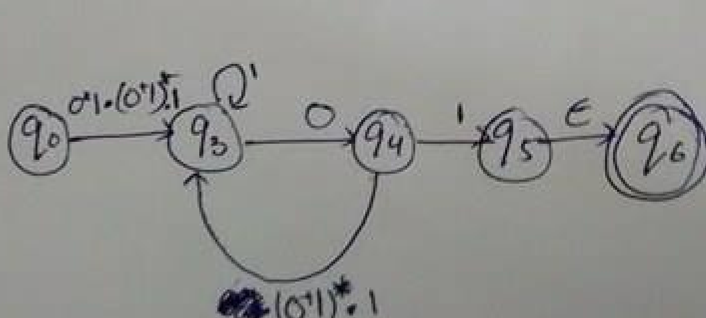
$\square$



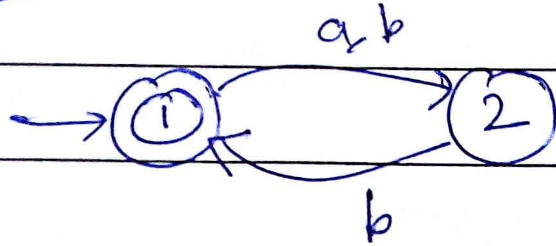
1) Removing q_1

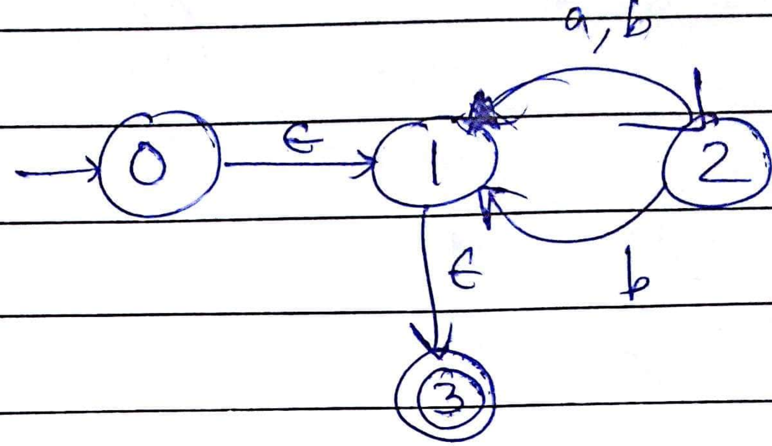


2) Removing q_2

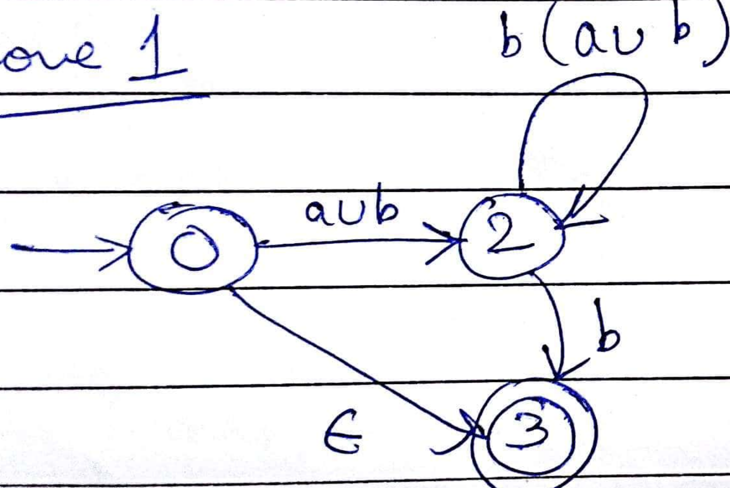


Example 2

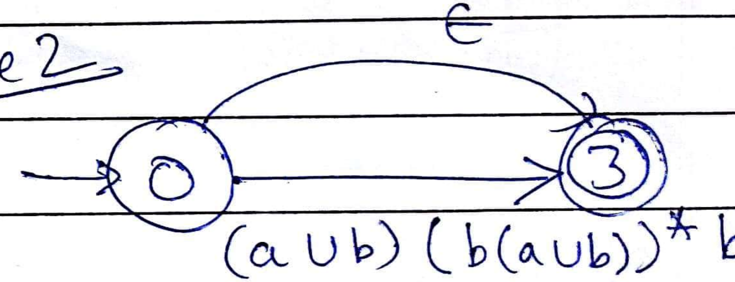




Remove 1



Remove 2



Reg. Ex : $\epsilon \cup (a \cup b)(b(a \cup b))^* b$