Relationships between complexity classes

How are P, NP, EXP, and NEXP related?

- P $\subseteq$ NP by definition.
- P $\subseteq$ EXP again by definition.
- Similarly, NP $\subseteq$ NEXP by definition.
- Finally, NP $\subseteq$ EXP due to the previous lemma.

$$\begin{tikzcd}
\mathrm{P} \arrow[r] \arrow[d] & \mathrm{NP} \arrow[d] \arrow[dl, red] \\
\mathrm{EXP} \arrow[r] & \mathrm{NEXP}
\end{tikzcd}$$

P vs. NP

P	the class of languages where membership can be decided quickly.
NP	the class of languages where membership can be verified quickly.

Examples

SAT $= \{\phi \mid \phi$ is satisfiable$\}$. in NP (and not known to be in P)

Reach $= \{(G, s, t) \mid t$ is reachable from s in $G\}$. in P

3-SAT $= \{\phi \mid \phi$ is a 3-CNF and satisfiable$\}$. in NP (and not known to be in P)

2-SAT $= \{\phi \mid \phi$ is a 2-CNF and satisfiable$\}$. in P

Factoring $= \{(k, n) \mid n$ has a factor $\leq k\}$. Google it!

Clique $= \{(G, k) \mid G$ has a clique of size $\geq k\}$. in NP (and not known to be in P)

Time heirarchy theorem

How do we separate NP from P?

To prove	Method used
not regular	pumping lemma for REG
non-context-free	pumping lemma or CFLs
not recognizable	diagonalization
not decidable	Rice's theorem or diagonalization and reductions
not in P	???

Finer structure inside P

Definition

A function $t : \mathbb{N} \to \mathbb{N}$ is said to be time constructible if the there exists a TM that on input 1^n, it outputs $t(n)$ in time $O(t(n))$.

Examples

n^2, $n \log n$.

Theorem

Let $t : \mathbb{N} \to \mathbb{N}$ be a time constructible function. There exists a language L such that $L \in TIME(t(n)^2)$, but $L \notin TIME(o(t(n)))$.