Tutorial 1

- Collaboration is encouraged.

- You may benefit from solving the problems on your own or in collaboration with others rather than reading solutions given in books or sources online.

- For the sake of getting better at solving problems, start writing down your thoughts and go over them repeatedly, even if you do not get the full solution.

- No credit is attached to solving this tutorial. However, solving these problems may help in being able to perform better in quizzes and exams which count towards the overall credits.

Notation: For $w \in \Sigma^*$ let w^R be the reverse of the string w, i.e. if $|w| = 1$ then $w^R = w$ and for $|w| > 1$ and $w = u \cdot a$ then $w^R = a \cdot u^R$.

1. Give an automaton that accepts all strings over $\{0,1\}^*$ such that the string is the binary representation of a number which is 3 (mod 7).

2. After the Chennai floods, a relief camp was set up near Adyar. Each person could donate 1kg rice, 1kg tur daal or 3/4 ltr oil. As soon as there were 3 kg rice, 2kg tur daal and 1ltr. oil they were put into one box and shipped out. In order to automate the process of shipping, the students of IIT Madras designed an automaton. Can you reproduce that automaton?

3. Mr. Mehta is an avid Star Wars fan but for some reasons, he dislikes Star Trek series. He has a huge collection of sci-fi fan fiction in digital form some of which involve a mash-up of both Star Wars and Star Trek Universes. In order to help him cater to his taste, design a non-deterministic finite automaton which accepts only those pieces of text which contain the string sequence 'Star Wars' but not 'Star Trek'.

4. Let L be any regular language. Design finite state automata for the following languages.

   (a) $L_1 = \{a^k \mid \exists w \in L \text{ such that } |w| = k\}$.

   (b) $L_2 = \{w \mid w \cdot w \in L\}$.

   (c) $L_3 = \{w \mid w^R \in L\}$.

5. Let $\Sigma = \{a,b\}$. Construct a DFA with as few states as possible for the language $L = \{w \mid w \text{ contains atleast two a's and at most one b}\}$.