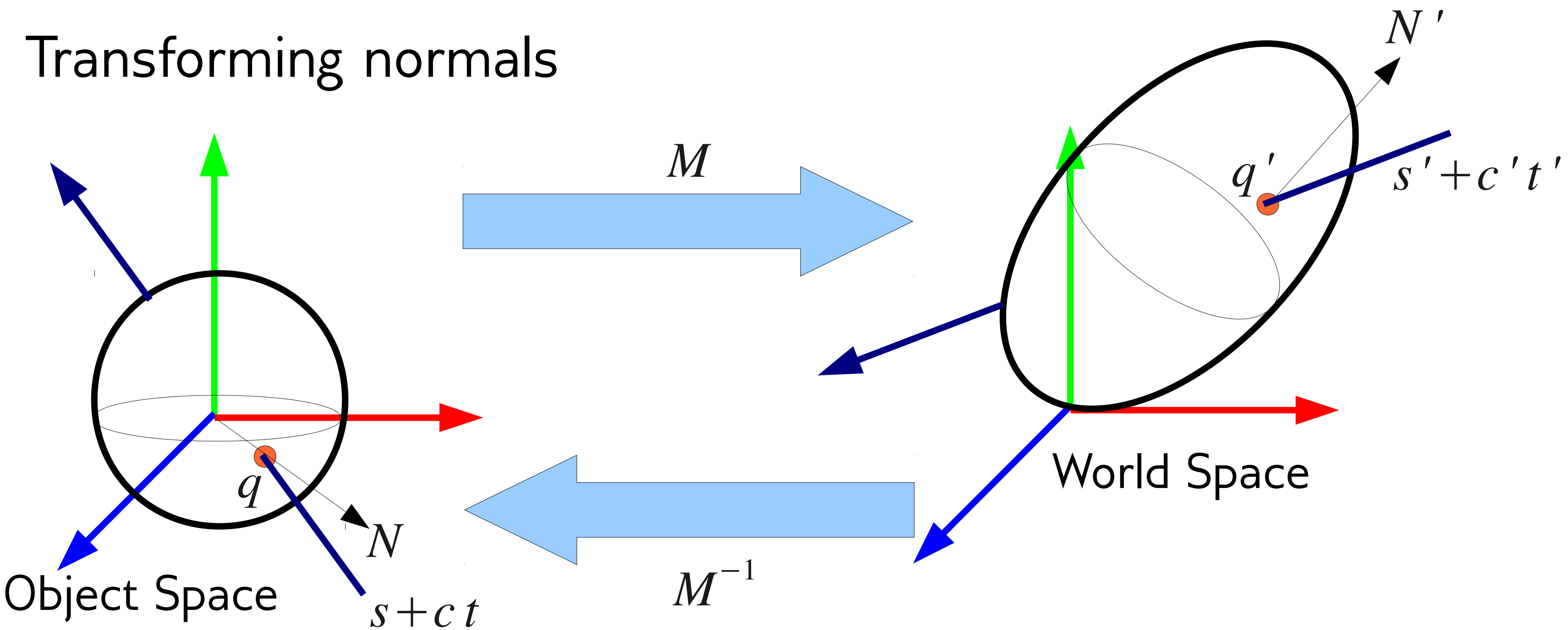
CS475/CS675
Computer Graphics

Ray Tracing 2

Ray Tracing

- Transforming normals

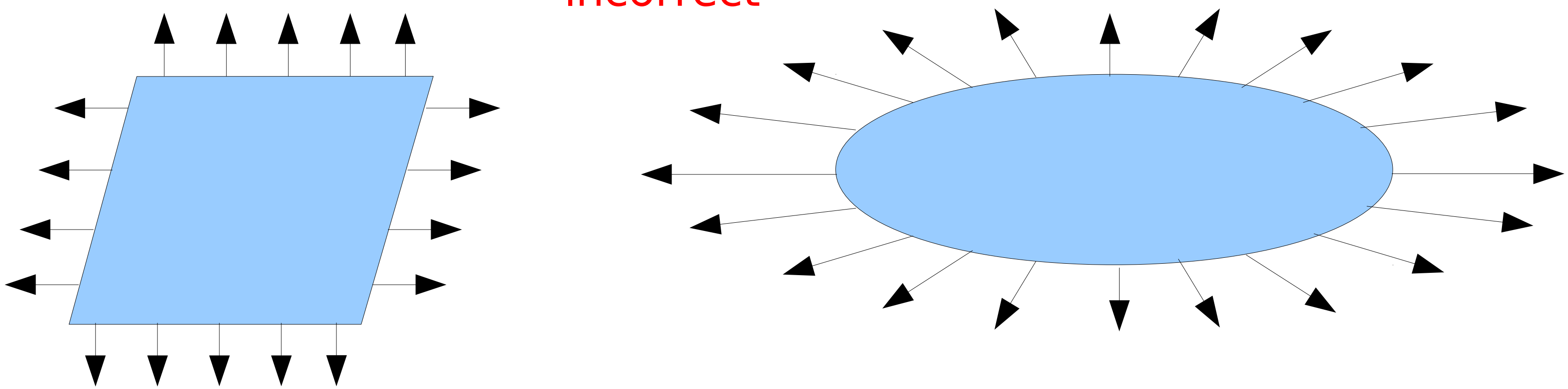


We also need the normal at the point of intersection. How does the normal transform when the object undergoes an affine transformation?

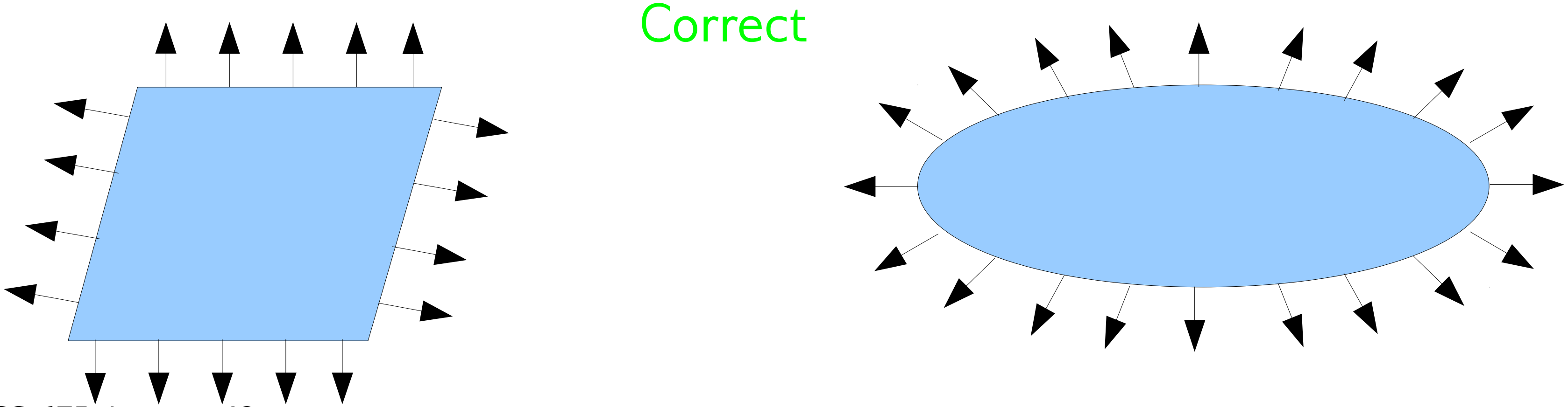
Ray Tracing

- Transforming normals

Incorrect

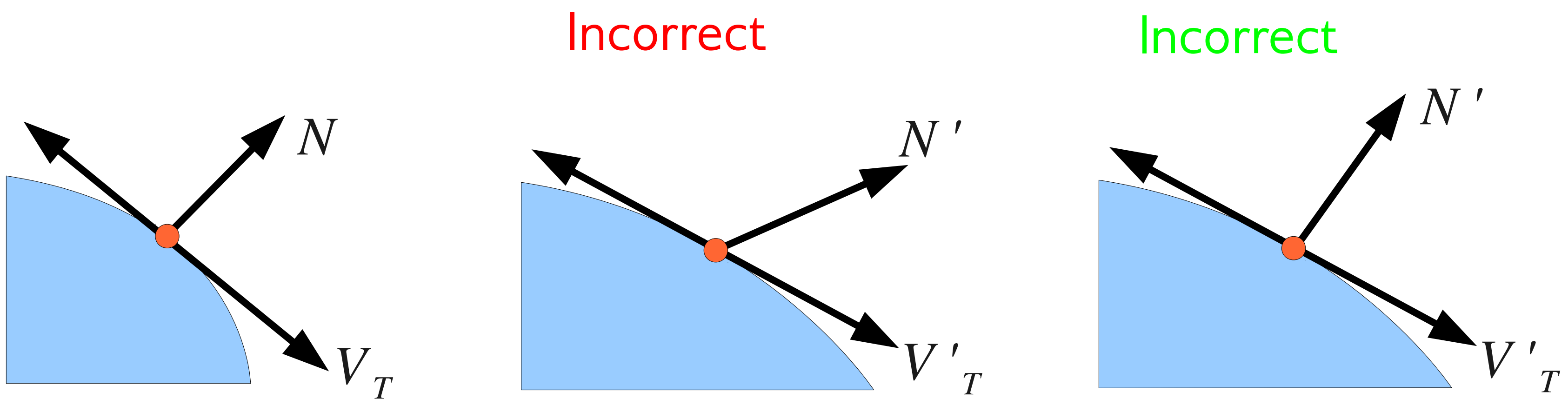


Correct



Ray Tracing

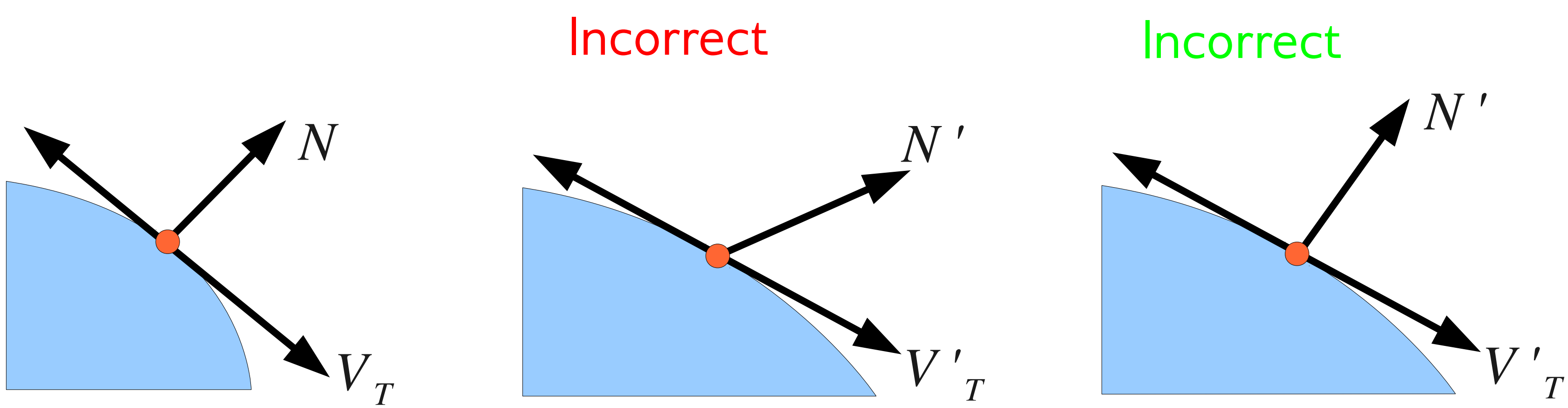
- Transforming normals – think about the tangents instead



- If V_T is an vector in the tangent plane then after the transformation it becomes $V'_T = M.V_T$
- The correct transformation to be applied to the normal should keep it perpendicular to the tangent vector.

Ray Tracing

- Transforming normals – think about the tangents instead



$$
\begin{aligned}
N^T V_T &= 0 \\
\Rightarrow N^T (M^{-1} M) V_T &= 0 \\
\Rightarrow (N^T . M^{-1})(M.V_T) &= 0 \\
\Rightarrow (N^T . M^{-1}) V'_T &= 0 \\
\Rightarrow N'^T V'_T &= 0
\end{aligned}
$$

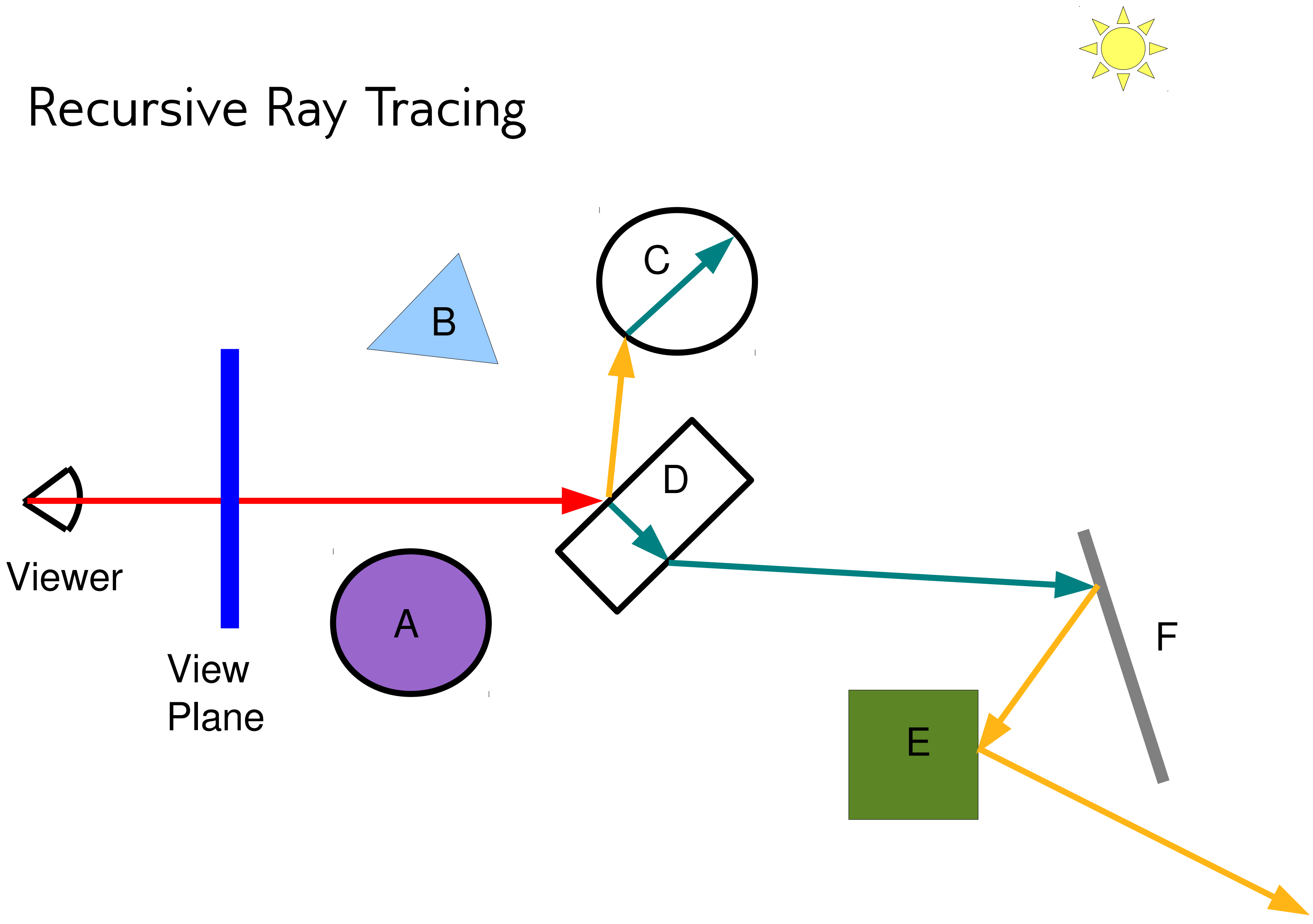
$$
\Rightarrow N' = (N^T . M^{-1})^T = (M^{-1})^T . N
$$

Ray Tracing

- Ray Casting – Single Step Ray Tracing
  - For every pixel in the image
    - Shoot a ray
    - Find closest intersection with object
    - Find normal at the point of intersection
    - Compute illumination at point of intersection
    - Assign pixel color

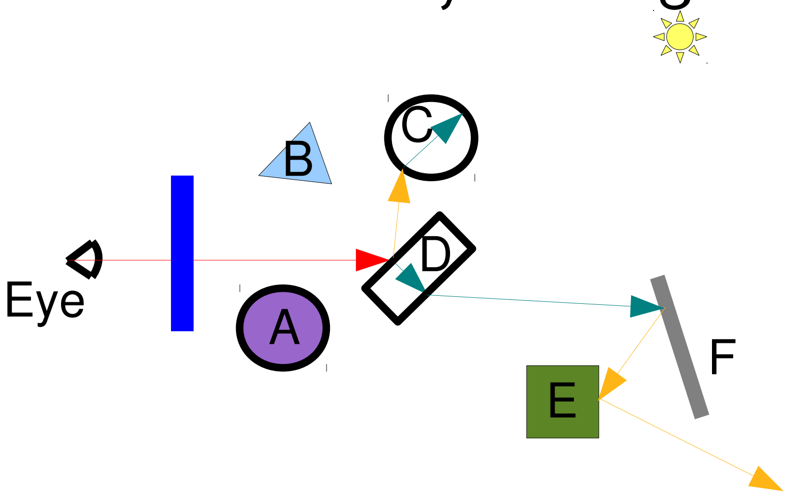
Ray Tracing

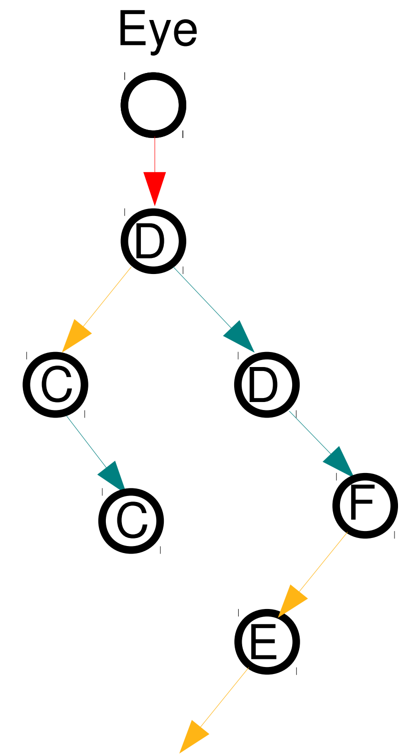
- Recursive Ray Tracing



Ray Tracing

- Recursive Ray Tracing

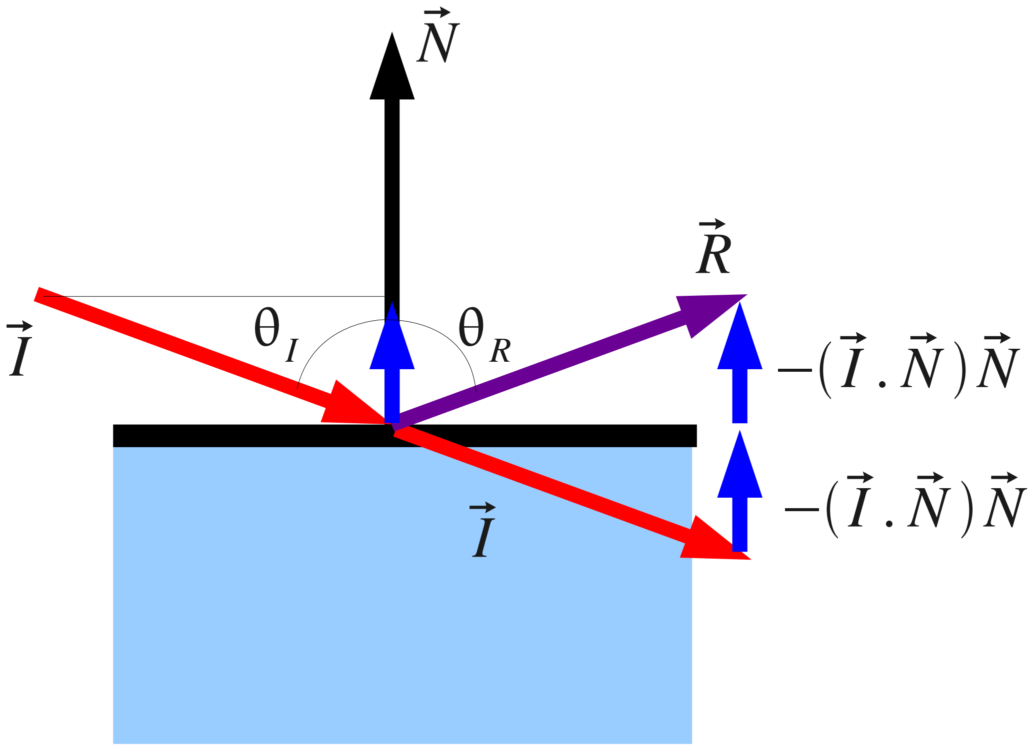




- Primary Rays - from the Eye
- Secondary Rays – Reflection, Refraction
- Shadow Rays

Ray Tracing

- Reflected Ray
  - Angle of Incidence = Angle of Reflection



$\vec{I}$ Incident Ray

$\vec{R}$ Reflected Ray

$\vec{N}$ Surface Normal

$$\vec{R}=\vec{I}-2(\vec{I}\,.\,\vec{N})\vec{N}$$

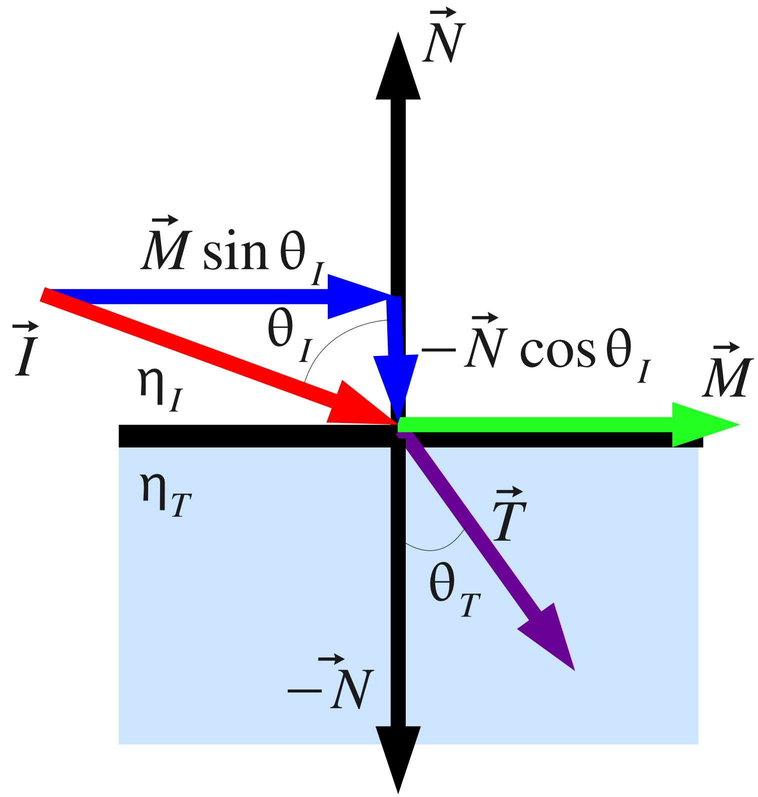
Ray Tracing

- Transmitted Ray

$$\eta_I \sin\theta_I = \eta_T \sin\theta_T$$

$$\frac{\eta_I}{\eta_T} = \frac{\sin\theta_T}{\sin\theta_I} = \eta_R$$

Snell-Descartes Law



$$\vec{I} = M\sin\theta_I - N\cos\theta_I$$ - Incident Ray

$$\Rightarrow \vec{M} = (I + N\cos\theta_I)/\sin\theta_I$$

So the transmitted ray is given by:

$$\begin{aligned}
\vec{T} &= -N\cos\theta_T + M\sin\theta_T \\
&= -N\cos\theta_T + (I + N\cos\theta_I)\frac{\sin\theta_T}{\sin\theta_I} \\
&= \eta_R \vec{I} + (\eta_R\cos\theta_I - \cos\theta_T)\vec{N} \\
&= \eta_R \vec{I} + \left(\eta_R\cos\theta_I - \sqrt{1 - \eta_R^2(1 - \cos^2\theta_I)}\right)\vec{N} \\
&= \eta_R \vec{I} + \left(\eta_R(-\vec{N}.\vec{I}) - \sqrt{1 - \eta_R^2(1 - (-\vec{N}.\vec{I})^2)}\right)\vec{N}
\end{aligned}$$

Normalize the result!

Ray Tracing

- Transmitted Ray

$$\eta_I \sin\theta_I = \eta_T \sin\theta_T$$

$$\frac{\eta_I}{\eta_T} = \frac{\sin\theta_T}{\sin\theta_I} = \eta_R$$

Snell-Descartes Law

$$\vec{I} = M\sin\theta_I - N\cos\theta_I$$ - Incident Ray

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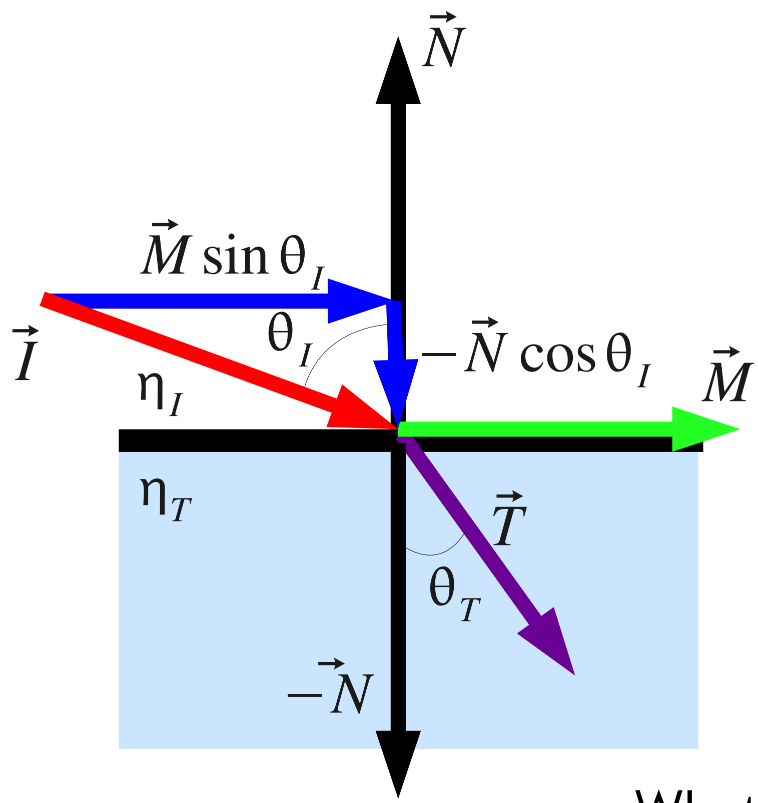


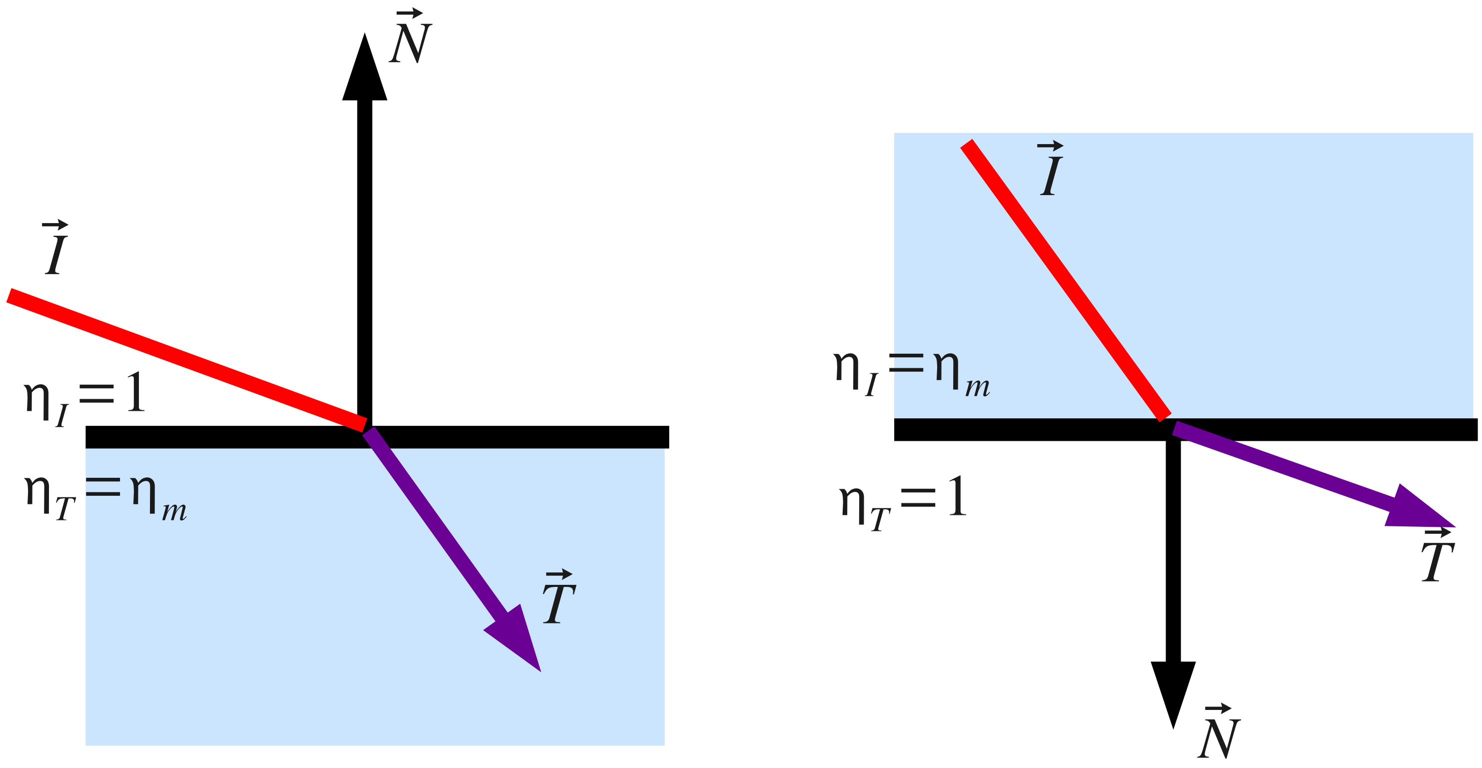
So the transmitted ray is given by:

$$\begin{aligned}
\vec{T} &= -N\cos\theta_T + M\sin\theta_T \\
&= -N\cos\theta_T + (I + N\cos\theta_I)\frac{\sin\theta_T}{\sin\theta_I} \\
&= \eta_R\vec{I} + (\eta_R\cos\theta_I - \cos\theta_T)\vec{N} \\
&= \eta_R\vec{I} + (\eta_R\cos\theta_I - \sqrt{1-\eta_R^2(1-\cos^2\theta_I)})\vec{N} \\
&= \eta_R\vec{I} + (\eta_R(-\vec{N}.\vec{I}) - \sqrt{1-\eta_R^2(1-(-\vec{N}.\vec{I})^2)})\vec{N}
\end{aligned}$$

What happens when the square root is imaginary?

Ray Tracing

- Transmitted Ray



Entering and Leaving transmissive material is different – check dot product with normal.

Ray Tracing

- Total Internal Reflection

ig. 3.7A The optical manhole. From under water, the entire celestial emisphere is compressed into a circle only 97.2° across. The dark boundary efining the edges of the manhole is not sharp due to surface waves. The rays re analogous to the crepuscular type seen in hazy air, Section 1.9. (Photo by ı. Granger)

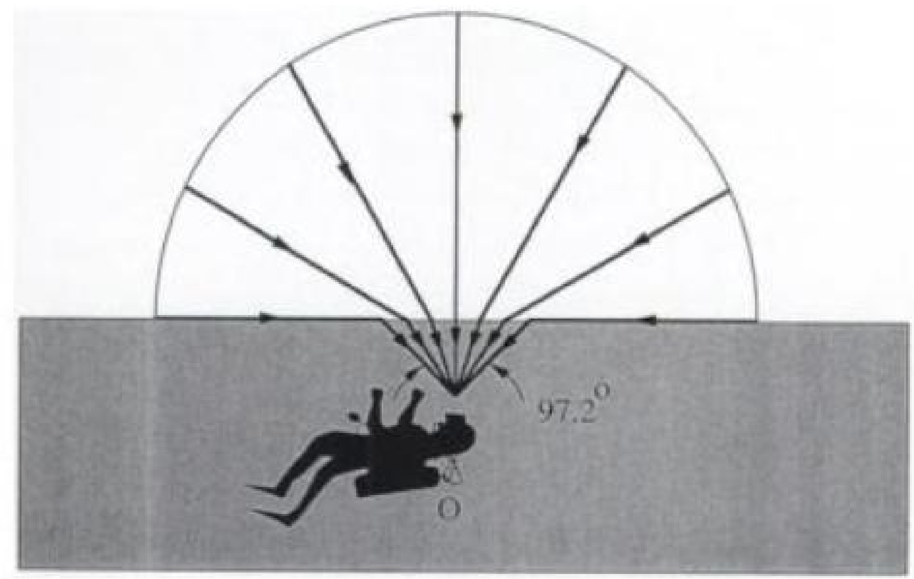
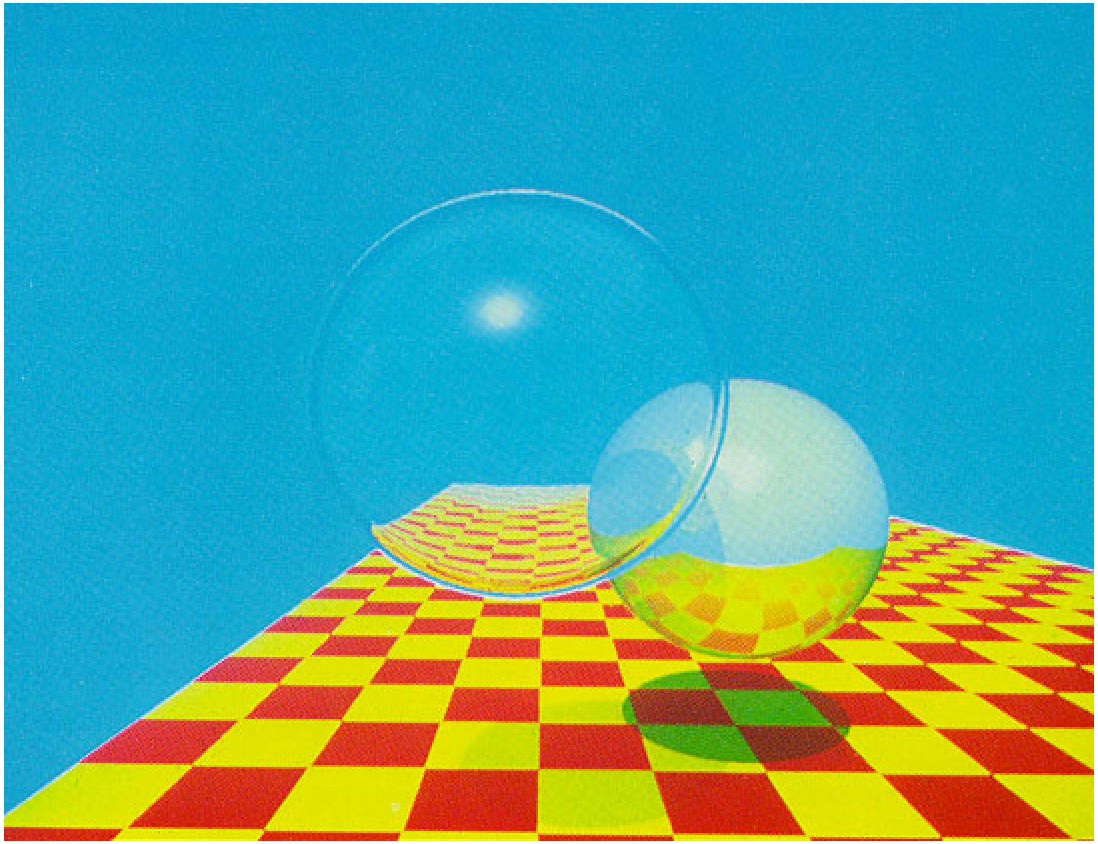


Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

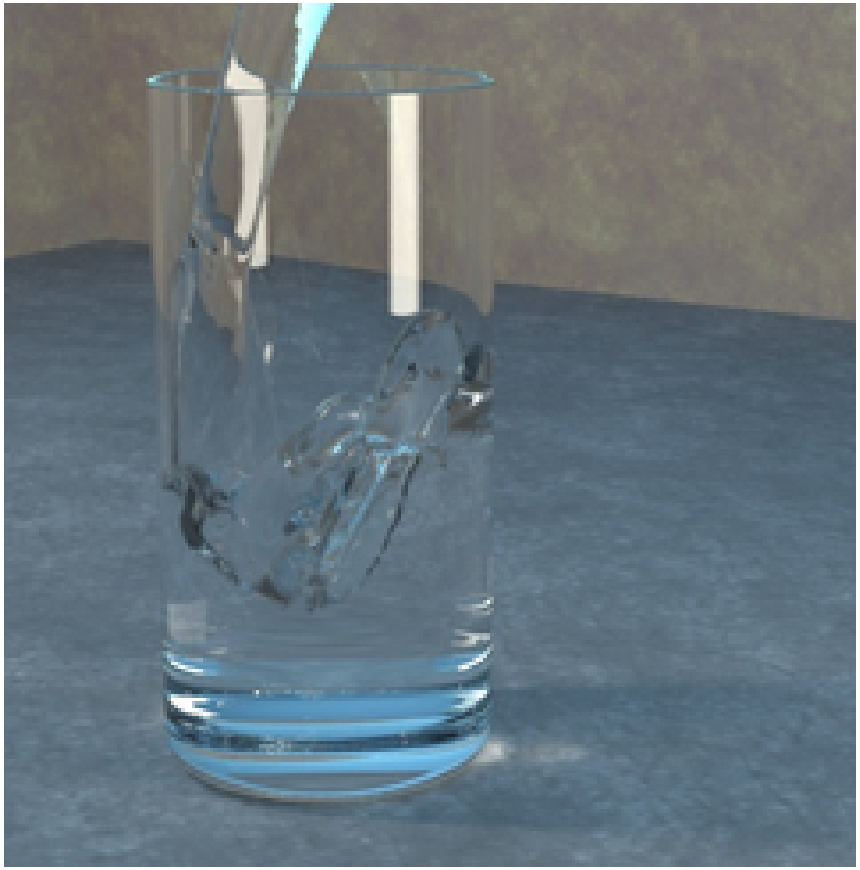
Image from Color and light in Nature by David K. Lynch, William Charles Livingston

Ray Tracing



Whitted, T. "An improved illumination model for shaded display", Communications of the ACM, 23(6):343-349, 1980.

Ray Tracing



Enright, D., Marschner, S. and Fedkiw, R., "Animation and Rendering of Complex Water Surfaces", SIGGRAPH 2002, ACM TOG 21, 736-744 (2002).

Ray Tracing

- Illumination : The Phong Model
  - For a single light source total illumination at any point is given by:

$$I = k_a I_a + k_d I_d + k_s I_s$$

where

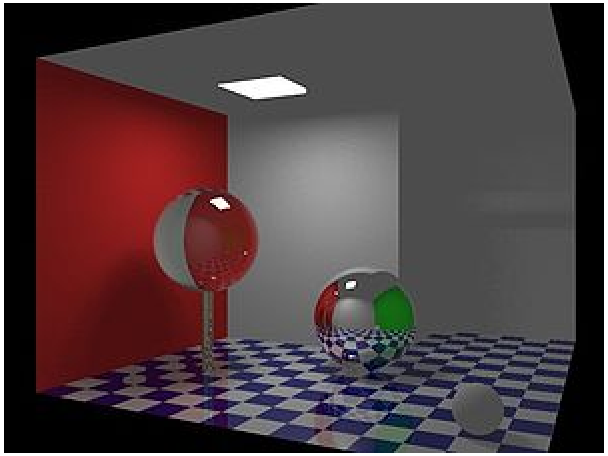
$k_a I_a$ is the contribution due to ambient reflection

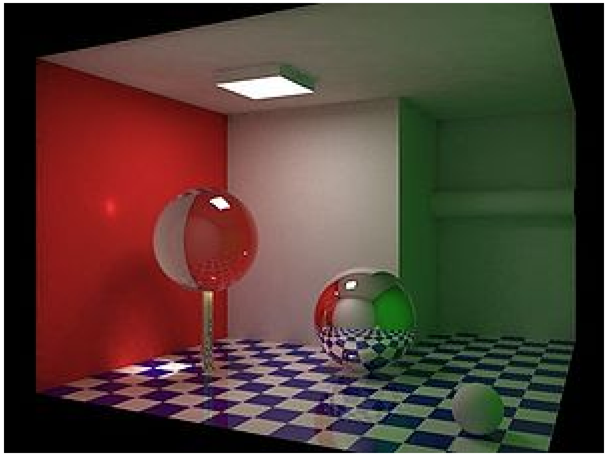
$k_d I_d$ is the contribution due to diffuse reflection

$k_s I_s$ is the contribution due to specular reflection

Ray Tracing

- Components of the Phong Model
- Ambient Illumination: I_a
  - Represents the reflection of all indirect illumination.
  - Has the same value everywhere.
  - Is an approximation to computing *Global Illumination*.





From http://en.wikipedia.org/wiki/Global_illumination (14/08/2009)

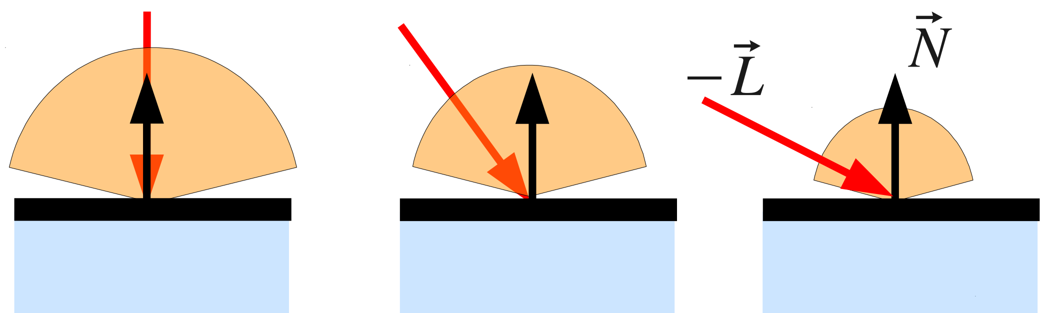
Ray Tracing

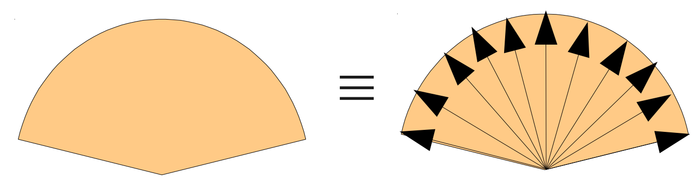
- Components of the Phong Model
- Diffuse Illumination: $I_d = I_L \cos\theta_L$
  - Assumes Ideal Diffuse Surface – that reflects light equally in all direction.
  - Surface is very rough at microscopic level. For e.g., Chalk and Clay.

Ray Tracing

- Components of the Phong Model
- Diffuse Illumination: $I_d = I_L \cos\theta_L$
  - Reflects light according to *Lambert's Cosine Law*

$$I_d = I_L \cos\theta_L$$
$$= I_L(\vec{L}.\vec{N})$$





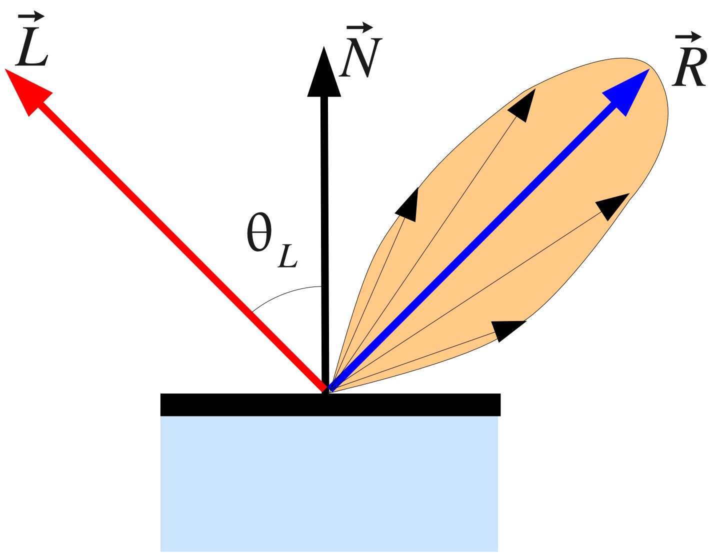
$\vec{L}$: vector to the light source

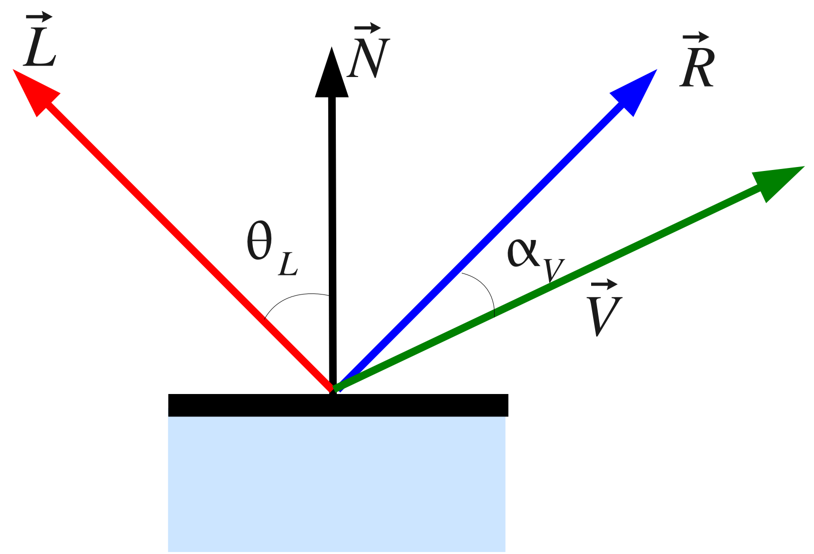
I_L : intensity of the light source

$\vec{N}$: surface normal

Ray Tracing

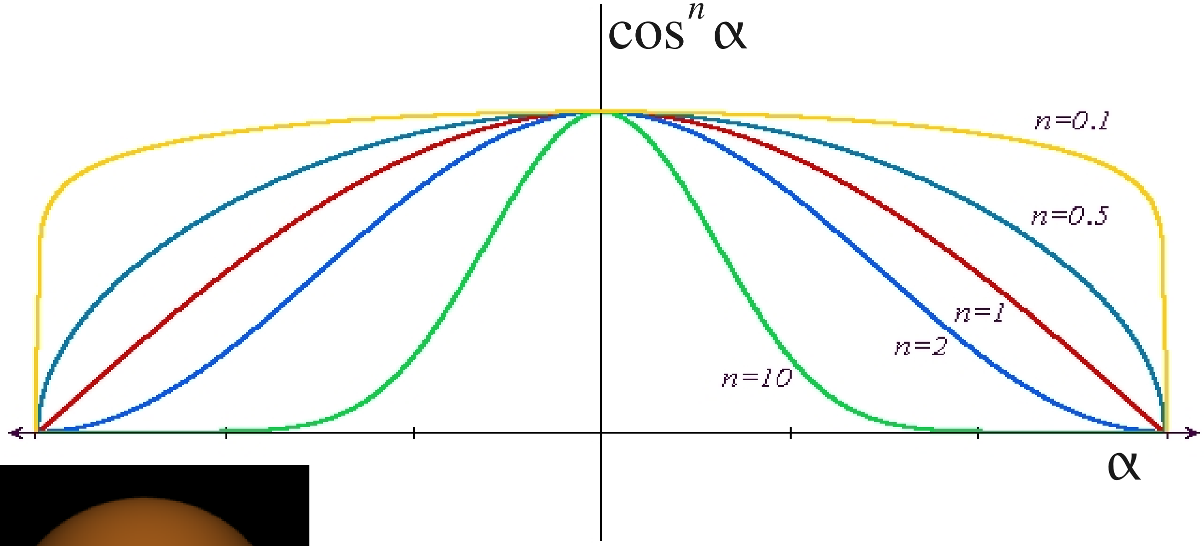
- Components of the Phong Model
- Specular Illumination: $I_s = I_L \cos^n \alpha_v = I_L (\vec{R} . \vec{V})^n$
  - Ideal specular surface reflects only along one direction.
  - Reflected intensity is view dependent – Mostly it is along the reflected ray but as we move away some of the reflection is slightly offset from the reflected ray due to microscopic surface irregularites.

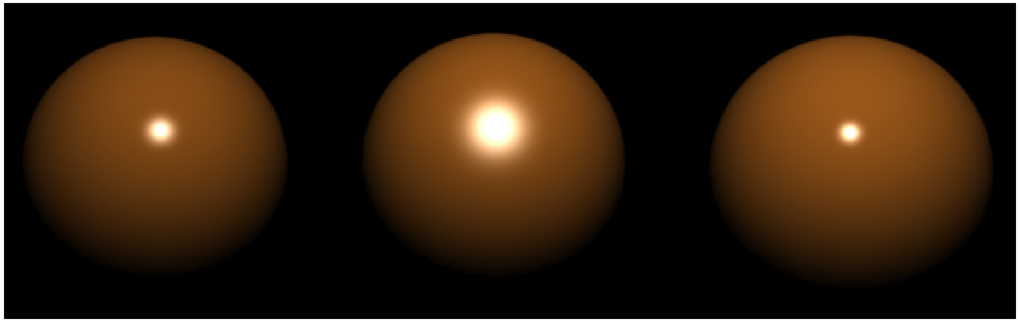




Ray Tracing

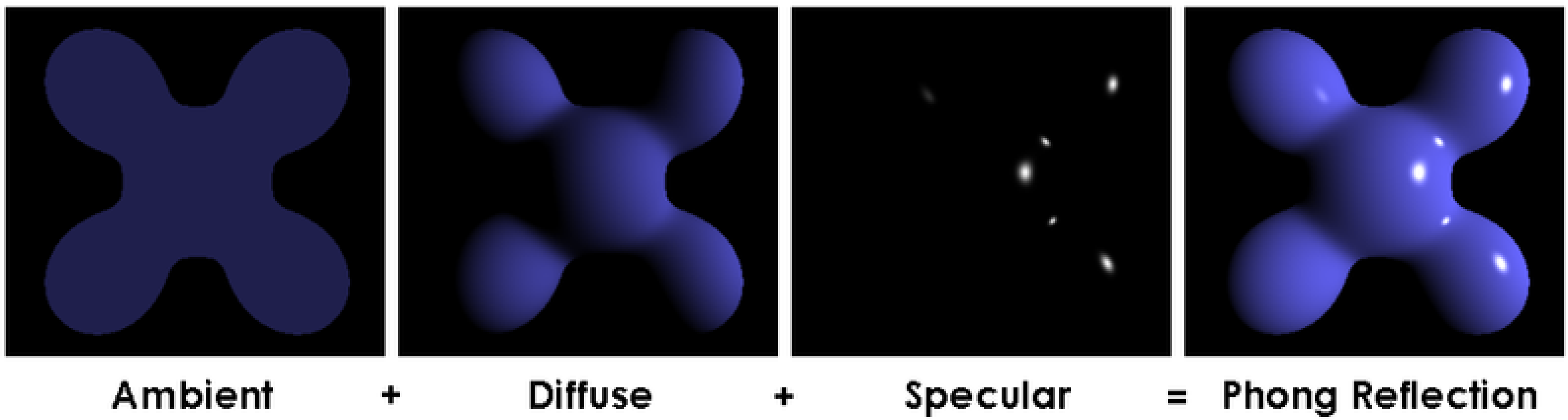
- Components of the Phong Model
- Specular Illumination: $I_s = I_L \cos^n \alpha_v = I_L (\vec{R} . \vec{V})^n$
  - n is called the coefficient of shininess and $I_L = I_t / r^2$





Ray Tracing

- The Phong Illumination Model



http://en.wikipedia.org/wiki/Phong_shading

$$I = k_a I_a + k_d I_d + k_s I_s$$

  - k_a, k_d, k_s are material constants defining the amount of light that is reflected as ambient, diffuse and specular. They may be defined in as three values with R, G, B components.

Ray Tracing

- Local Illumination Model

$$I_{local} = k_a I_a + \sum_{1 \le i \le m} (k_d I_{di} + k_s I_{si})$$

  Global Illumination Model

$$I_{global} = I_{local} + k_r I_{reflected} + k_t I_{transmitted}$$

- Reflected and transmitted components may also be attenuated based on distance the ray travels.

Ray Tracing

- Surface Material Properties
- Colour – For each object there can be a
  - Diffuse colour, Specular colour, Reflected colour and Transmitted colour
  - Remember differently coloured light is at different wavelength so:

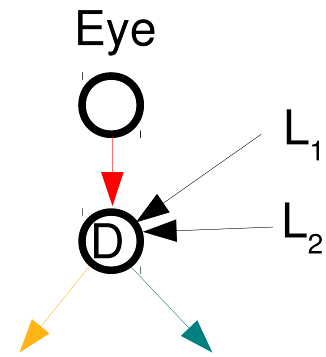
$$I_\lambda = k_a C_{d\lambda} I_a + \sum_{1 \le i \le m} \left(k_d C_{d\lambda} I_{di} + k_s C_{s\lambda} I_{si} \right) + k_r C_{r\lambda} I_r + k_t C_{t\lambda} I_t$$

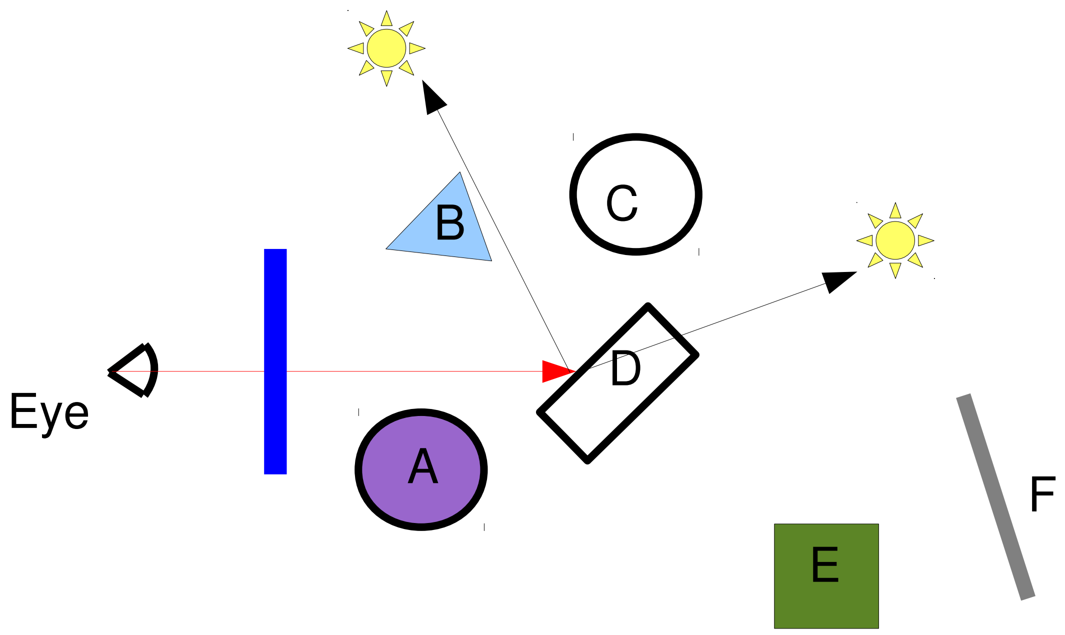
- Accounting for shadows:

$$I_\lambda = k_a C_{d\lambda} I_a + \sum_{1 \le i \le m} S_i \left(k_d C_{d\lambda} I_{di} + k_s C_{s\lambda} I_{si} \right) + k_r C_{r\lambda} I_r + k_t C_{t\lambda} I_t$$

Ray Tracing

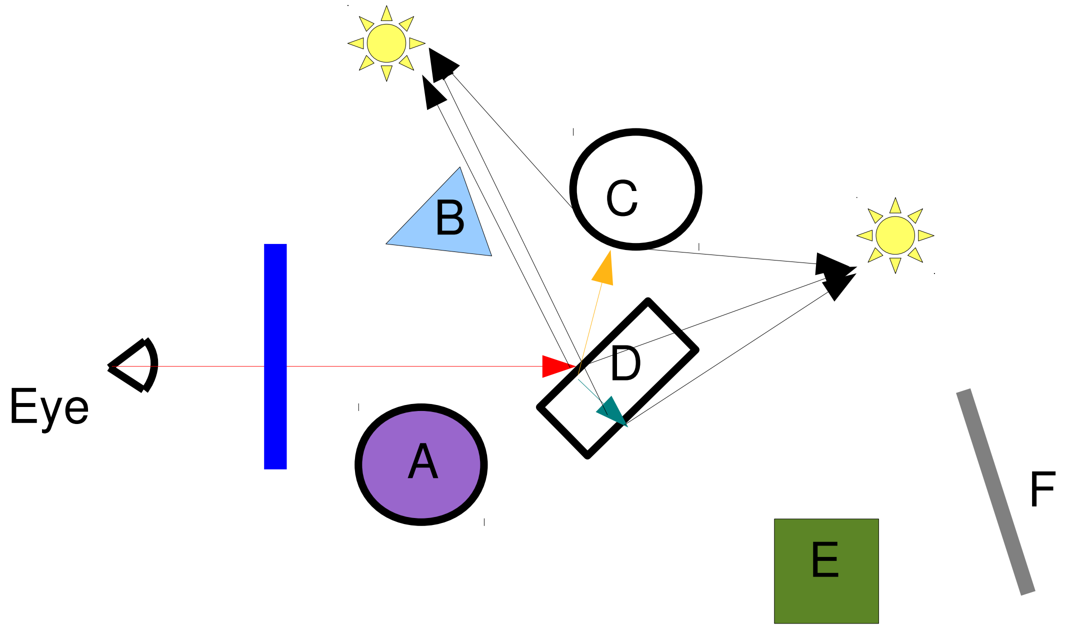
- Recursive Ray Tracing

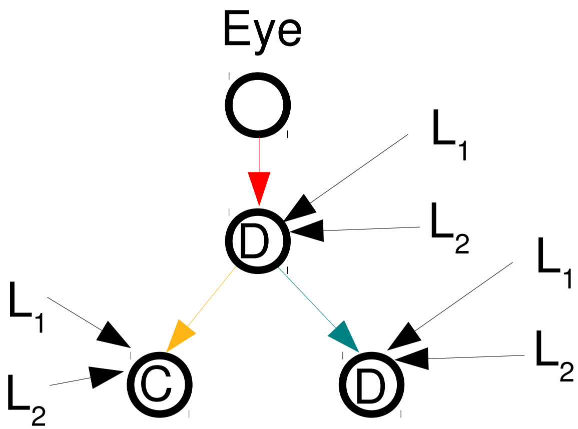




Ray Tracing

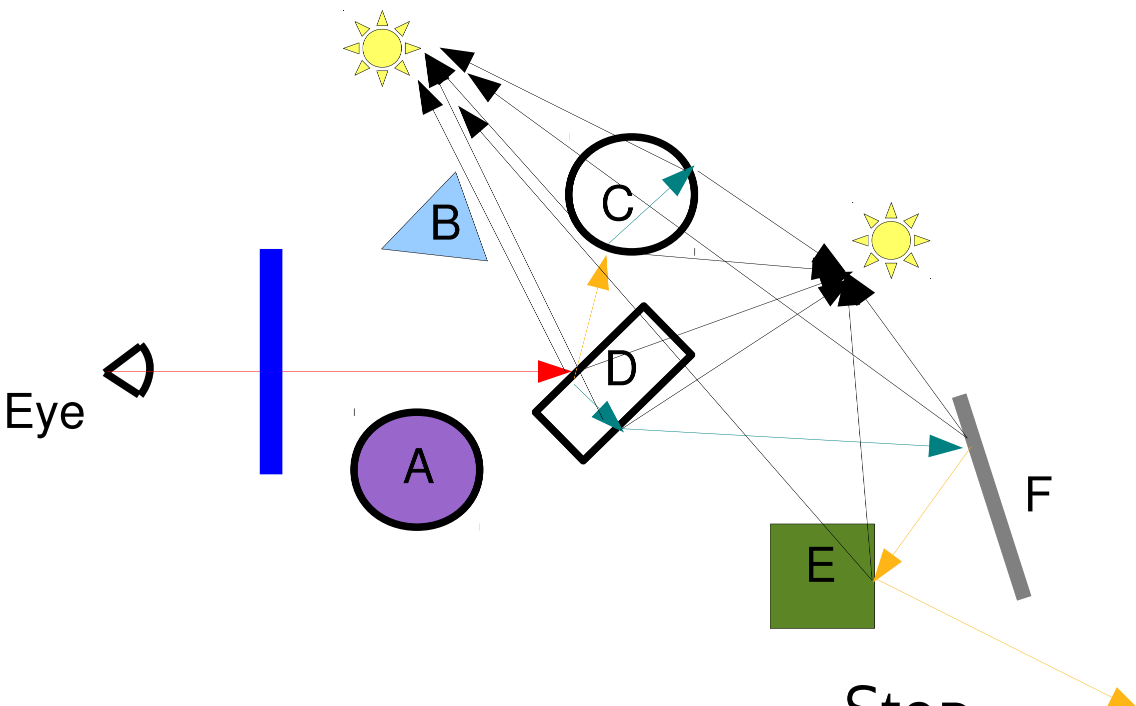
- Recursive Ray Tracing

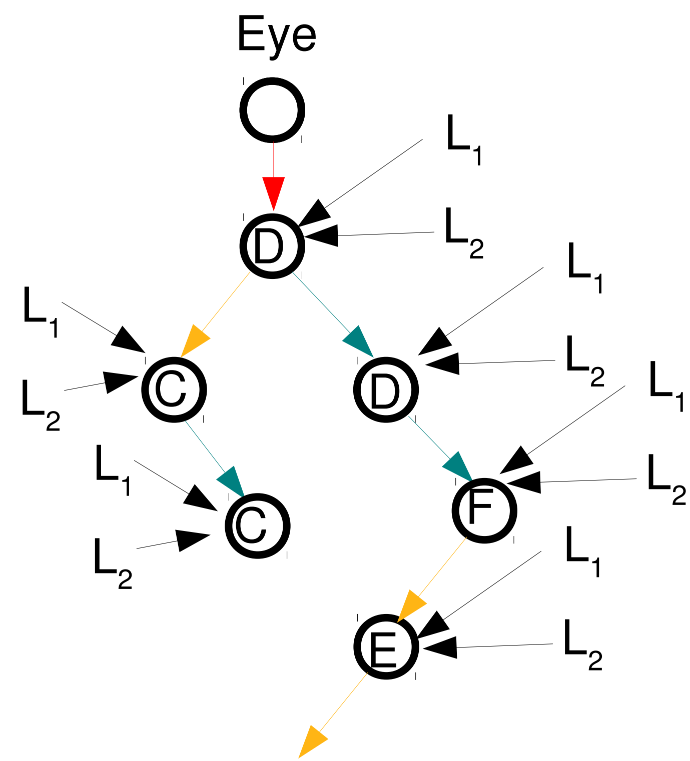




Ray Tracing

- Recursive Ray Tracing

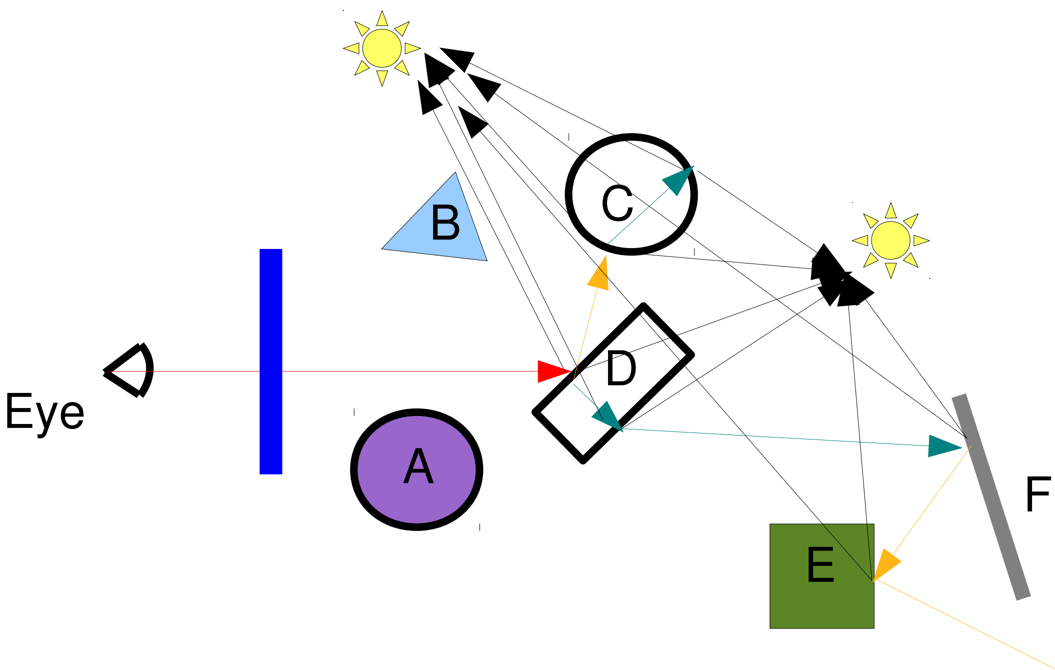


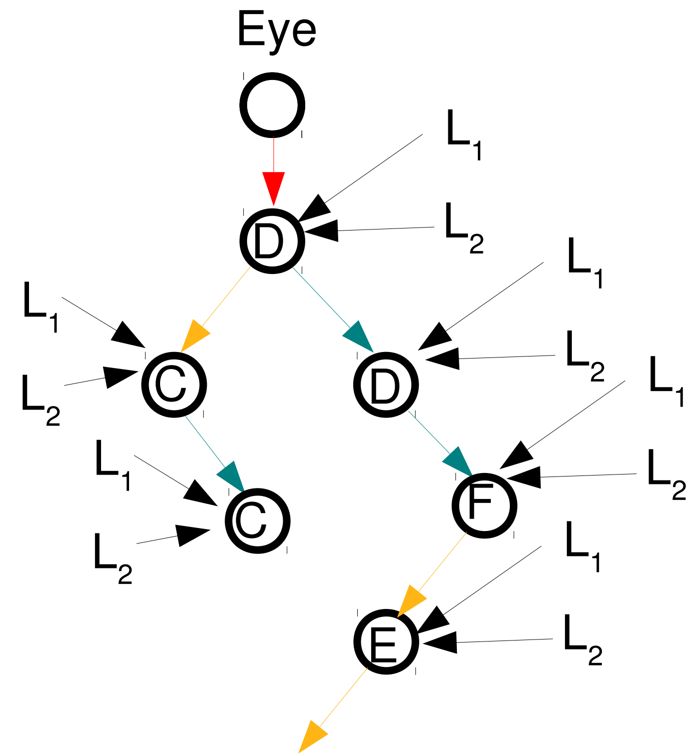


- Stop
  - When a ray leaves the scene
  - Contributed intensity is too less

Ray Tracing

- Recursive Ray Tracing

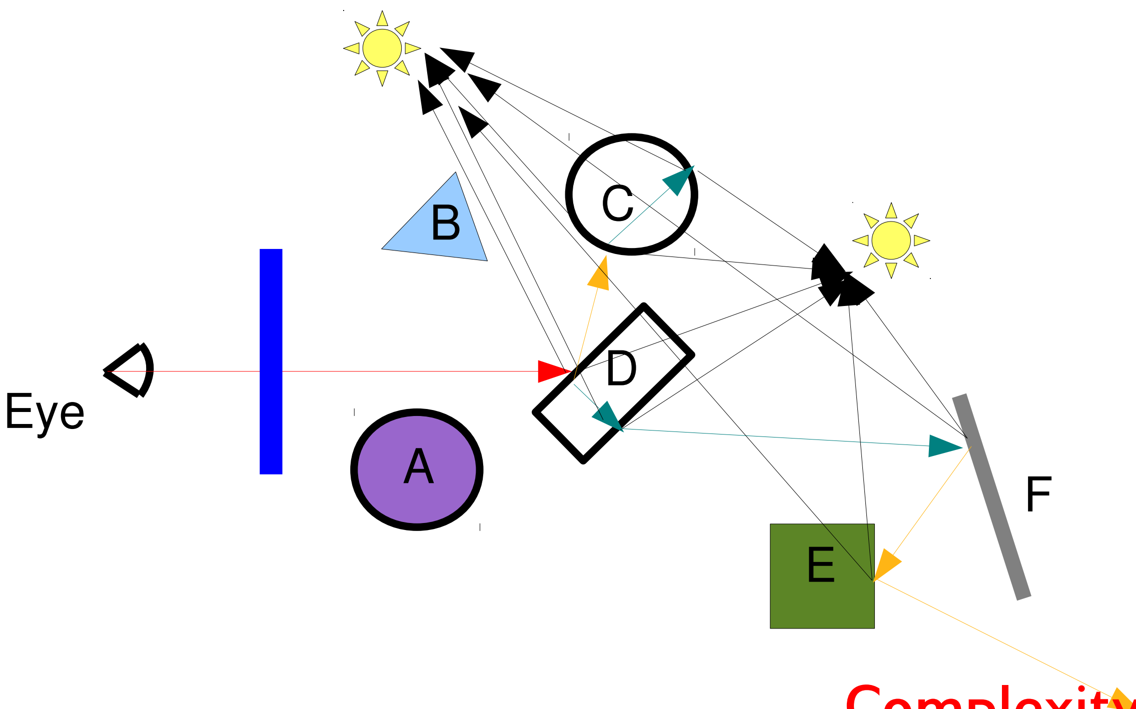


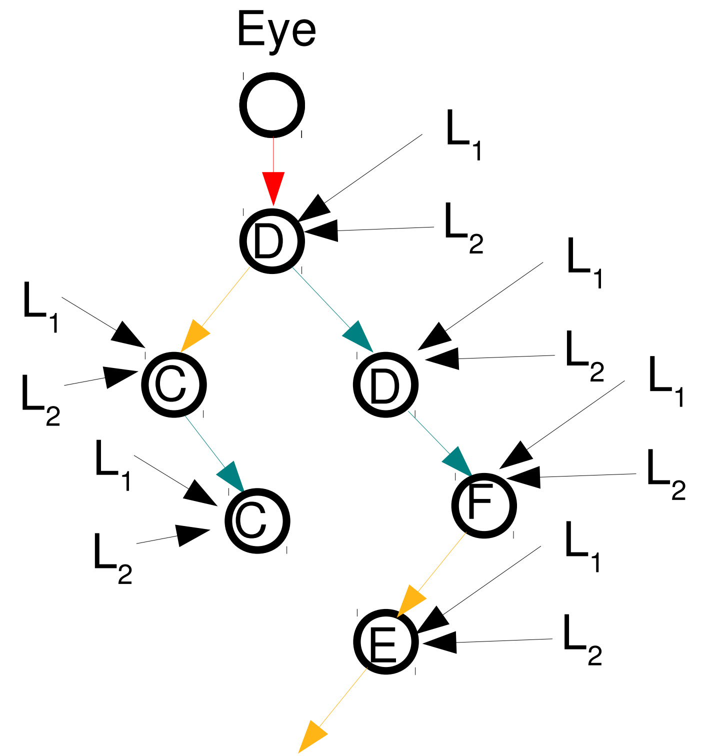


- **Complexity?**

Ray Tracing

- Recursive Ray Tracing

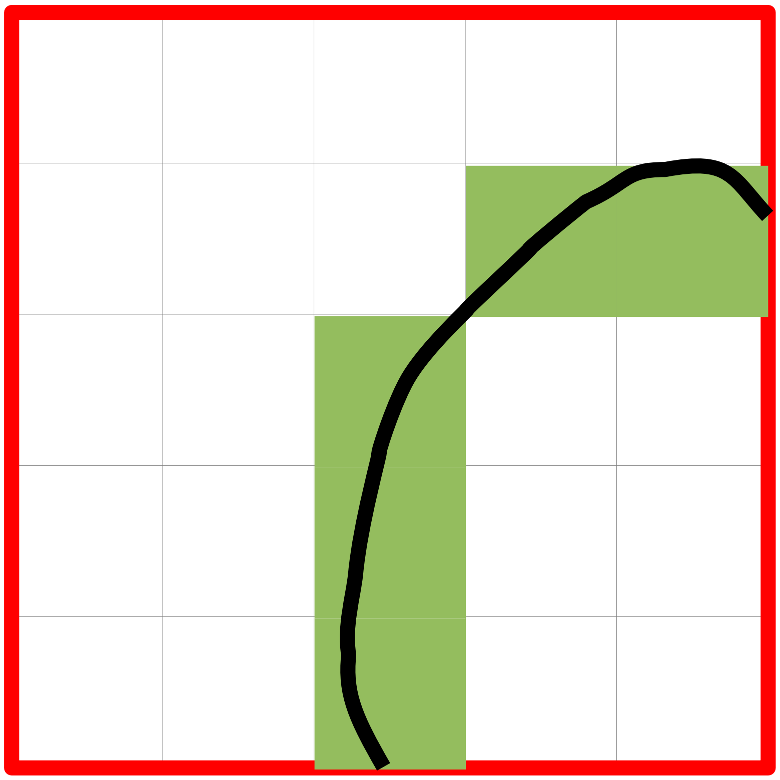




- Complexity = h x w x $N_{objects}$ x interesection cost x depth of recursion x N_{shadow_rays} x ...

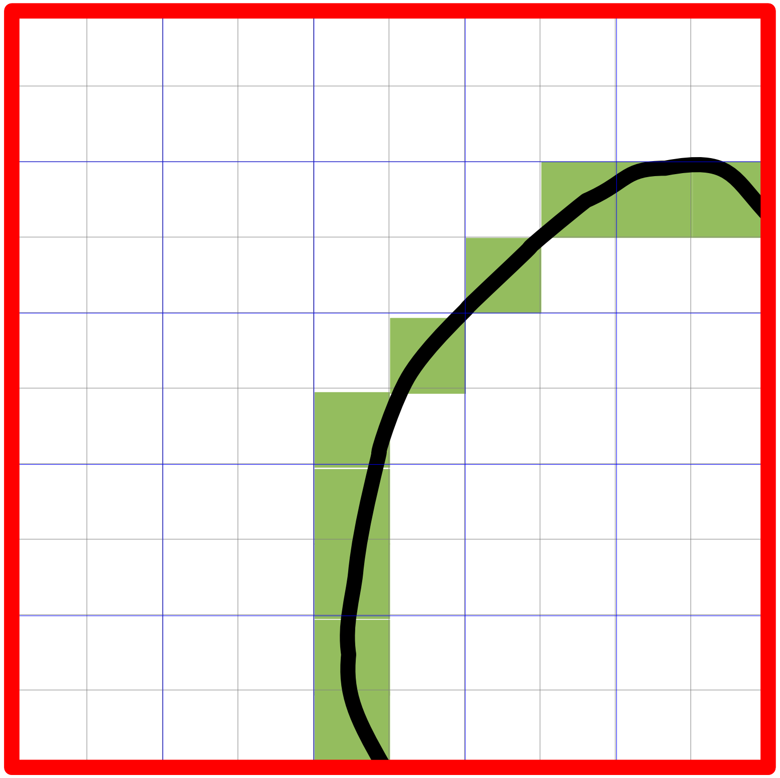
Ray Tracing

- Aliasing – Discrete samples of a continuous world



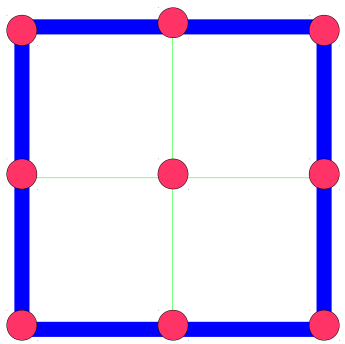
Ray Tracing

- Anti-aliasing – Shoot more rays per pixel - super sample!

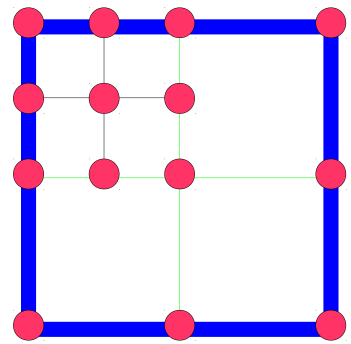


Ray Tracing

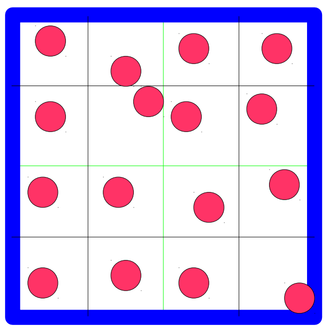
- Sampling strategies for anti-aliasing



Regular super sampling



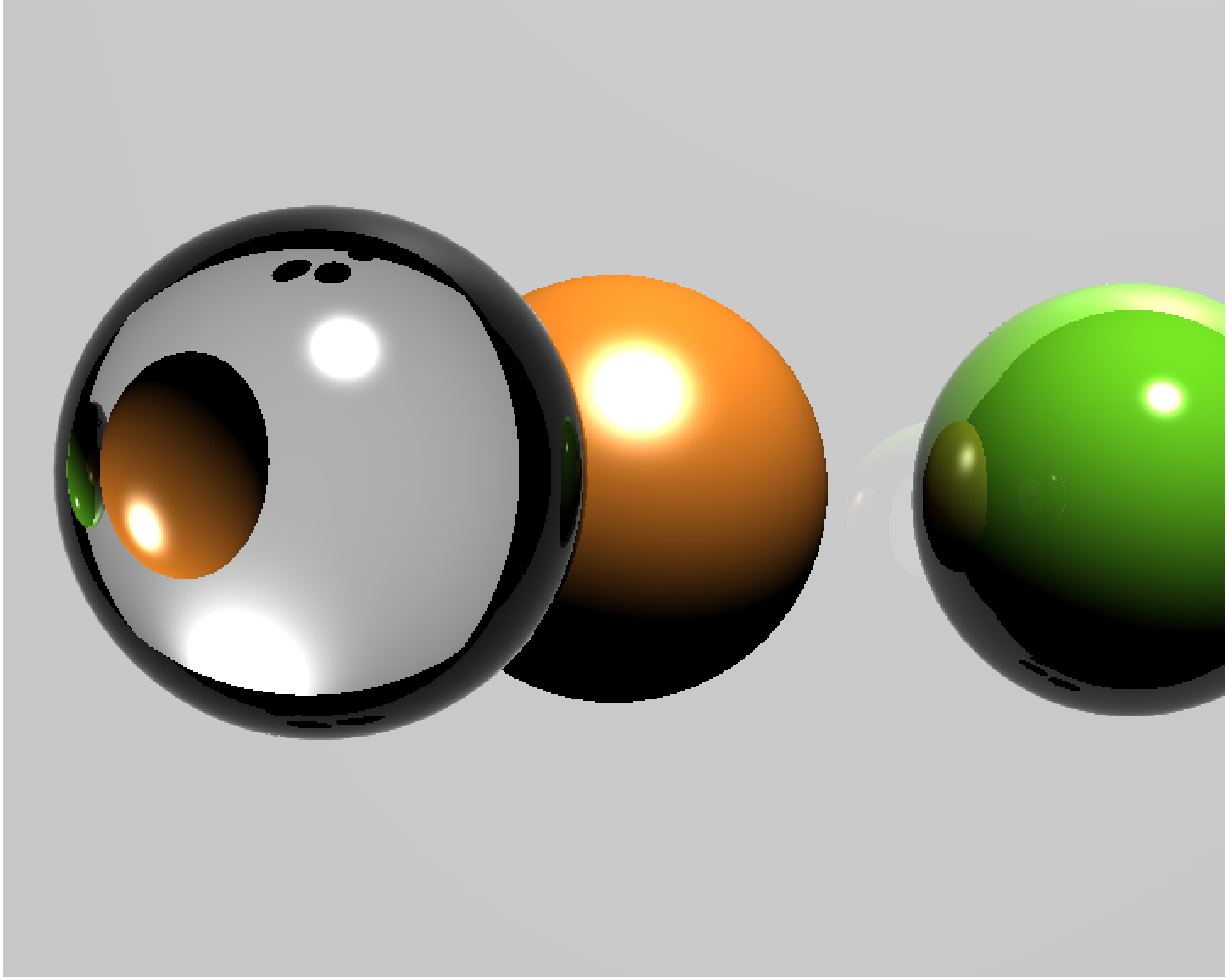
Adaptive super sampling



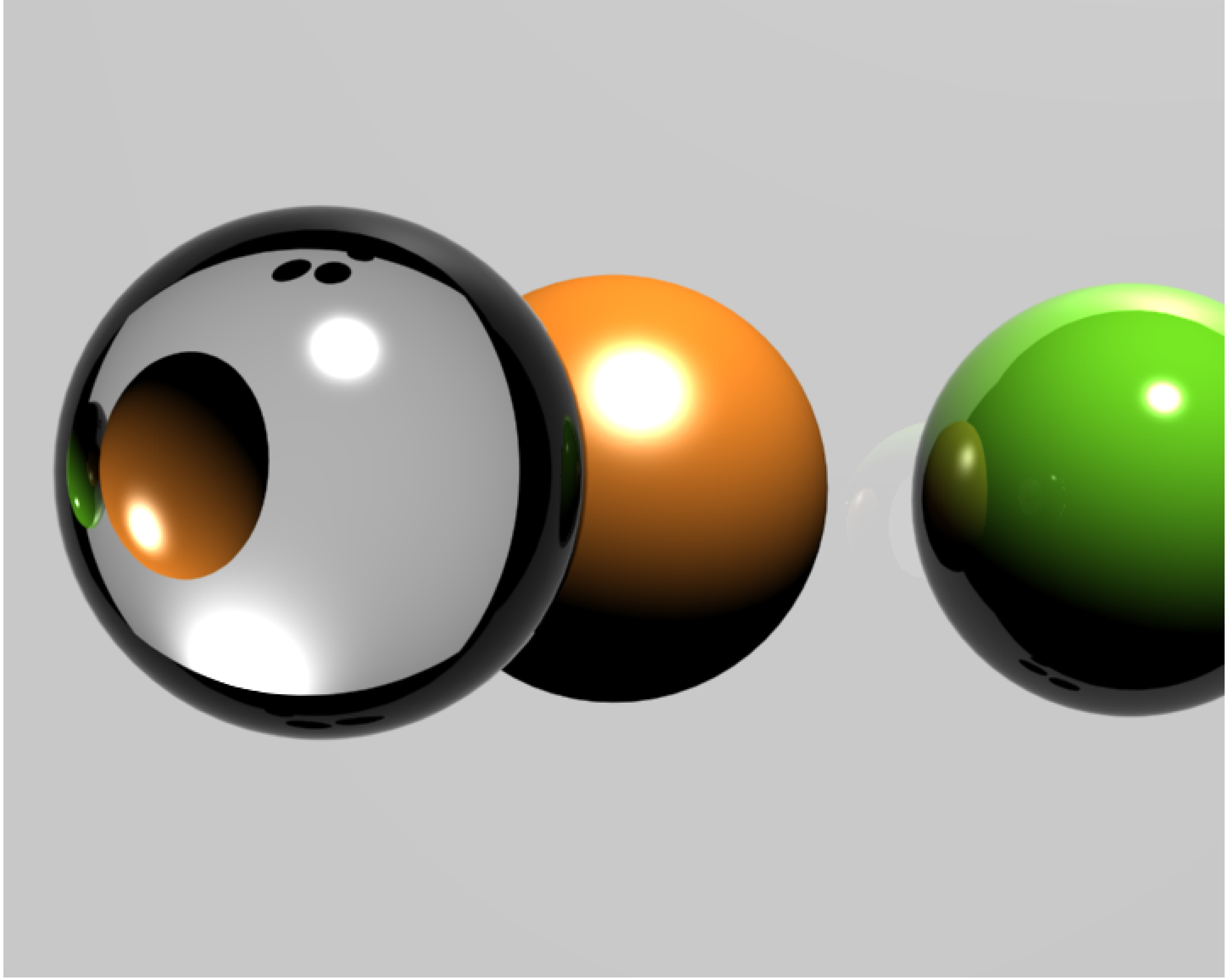
Stochastic or jittered super sampling

- Aliasing vs. Noise
- Aggregating the samples.

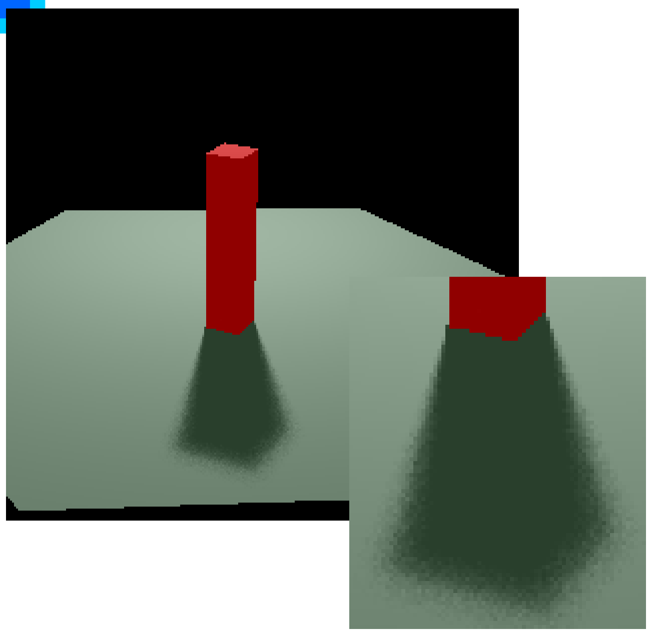
Ray Tracing

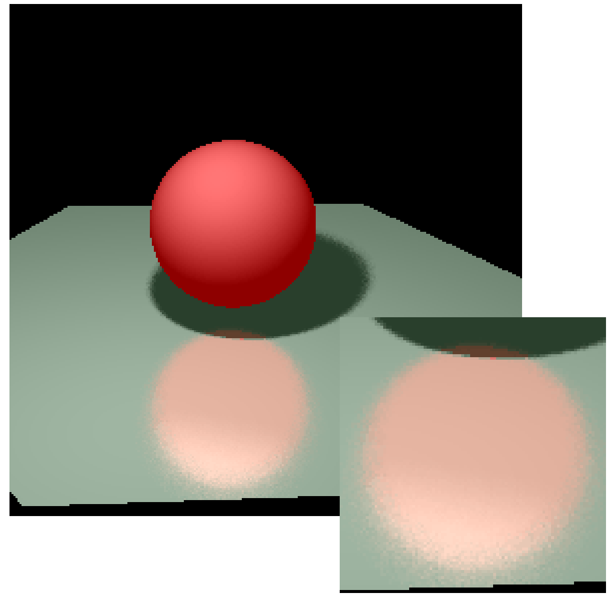


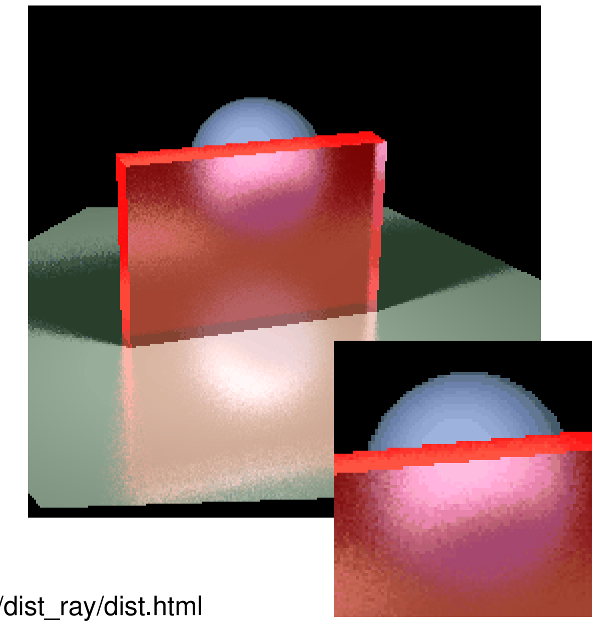
Ray Tracing



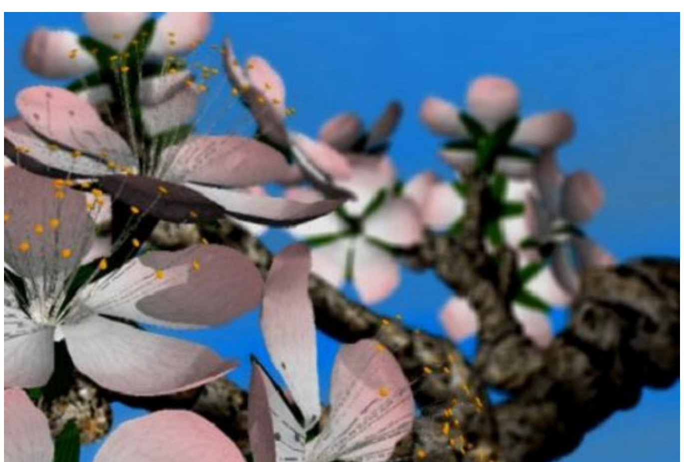
Distributed Ray Tracing

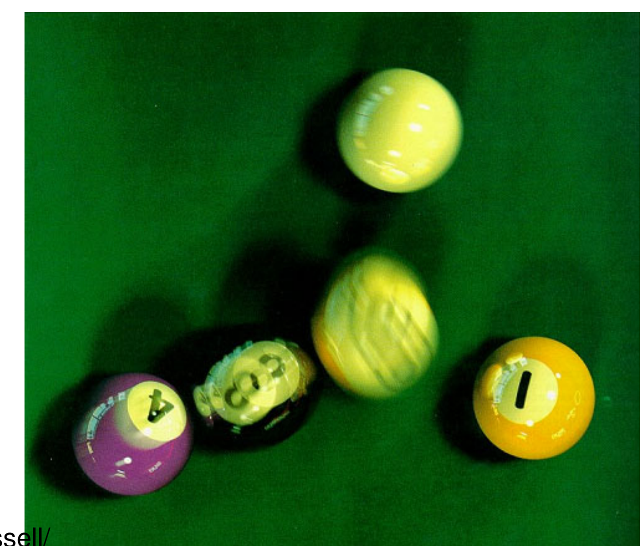






http://web.cs.wpi.edu/~matt/courses/cs563/talks/dist_ray/dist.html





http://www.cs.utexas.edu/~fussell/

The Rendering Equation

$$L_o(x,\omega,\lambda,t) = L_e(x,\omega,\lambda,t) + \int_\Omega f_r(x,\omega',\omega,\lambda,t)\, L_i(x,\omega',\lambda,t)\, (-\omega \cdot \boldsymbol{n})\, d\omega'$$

$L_o(x,\omega,\lambda,t)$ is the total amount of light of wavelength λ, directed outward along direction ω at time t, from a particular position x

$L_e(x,\omega,\lambda,t)$ is the emitted light.

$L_i(x,\omega',\lambda,t)$ is the light of wavelength λ, coming inward toward x from direction ω' at time t

$f_r(x,\omega',\omega,\lambda,t)$ is the bidirectional reflectance distribution function (BRDF), i.e., the proportion of light reflected from ω' to ω at position x, time t, and at wavelength λ

$(-\omega \cdot \boldsymbol{n})$ is the attenuation of incident light due to incident angle

$\int_\Omega \ldots d\omega'$ is the integral over a sphere of inward directions

The Rendering Equation

$$L_o(x,\omega,\lambda,t)=L_e(x,\omega,\lambda,t)+\int_{\Omega} f_r(x,\omega',\omega,\lambda,t)\,L_i(x,\omega',\lambda,t)(-\omega\cdot\boldsymbol{n})\,d\omega'$$

- Is this enough?
- BTDF - Refraction, BSDF – Sub surface scattering
- Phosphoresence
- Diffraction
- Atmospheric Scattering