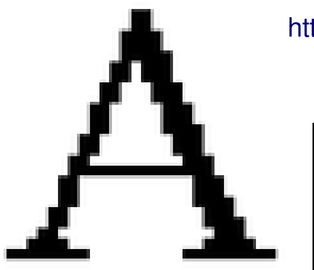
CS 775: Advanced Computer Graphics

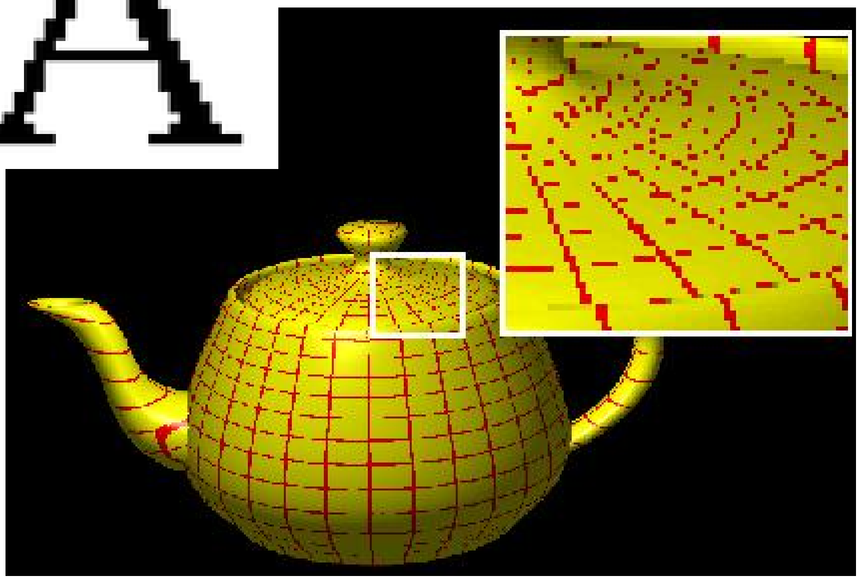
Lecture 12 : Understanding Aliasing

Aliasing

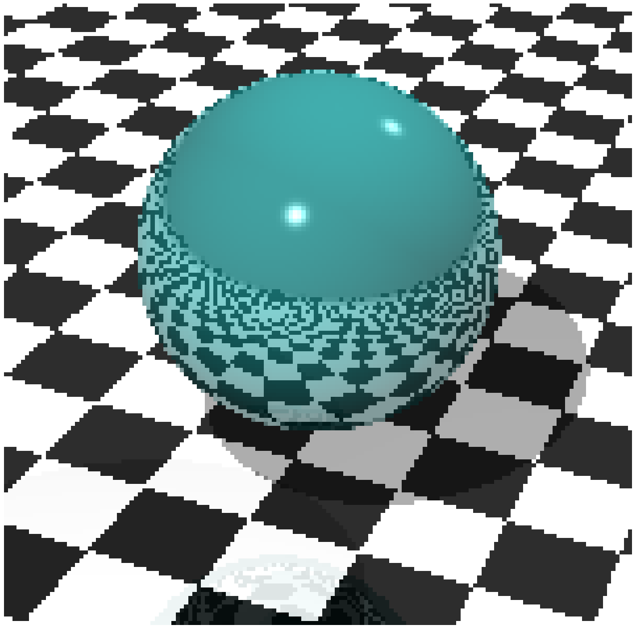
- Discrete samples of continuous information



http://en.wikipedia.org/wiki/Aliasing



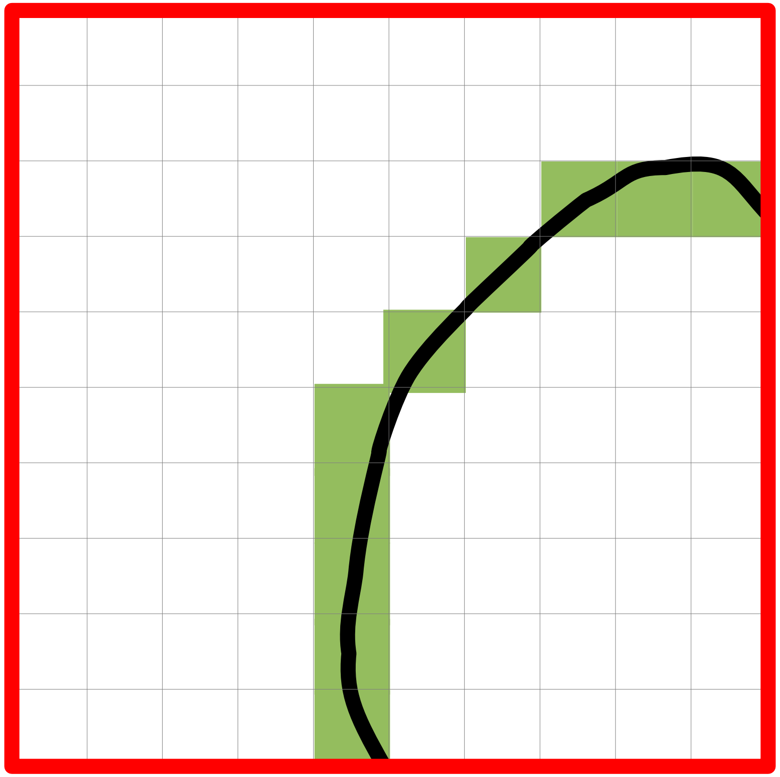
https://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/



http://www.codeproject.com/KB/graphics/RayTracerNet.aspx

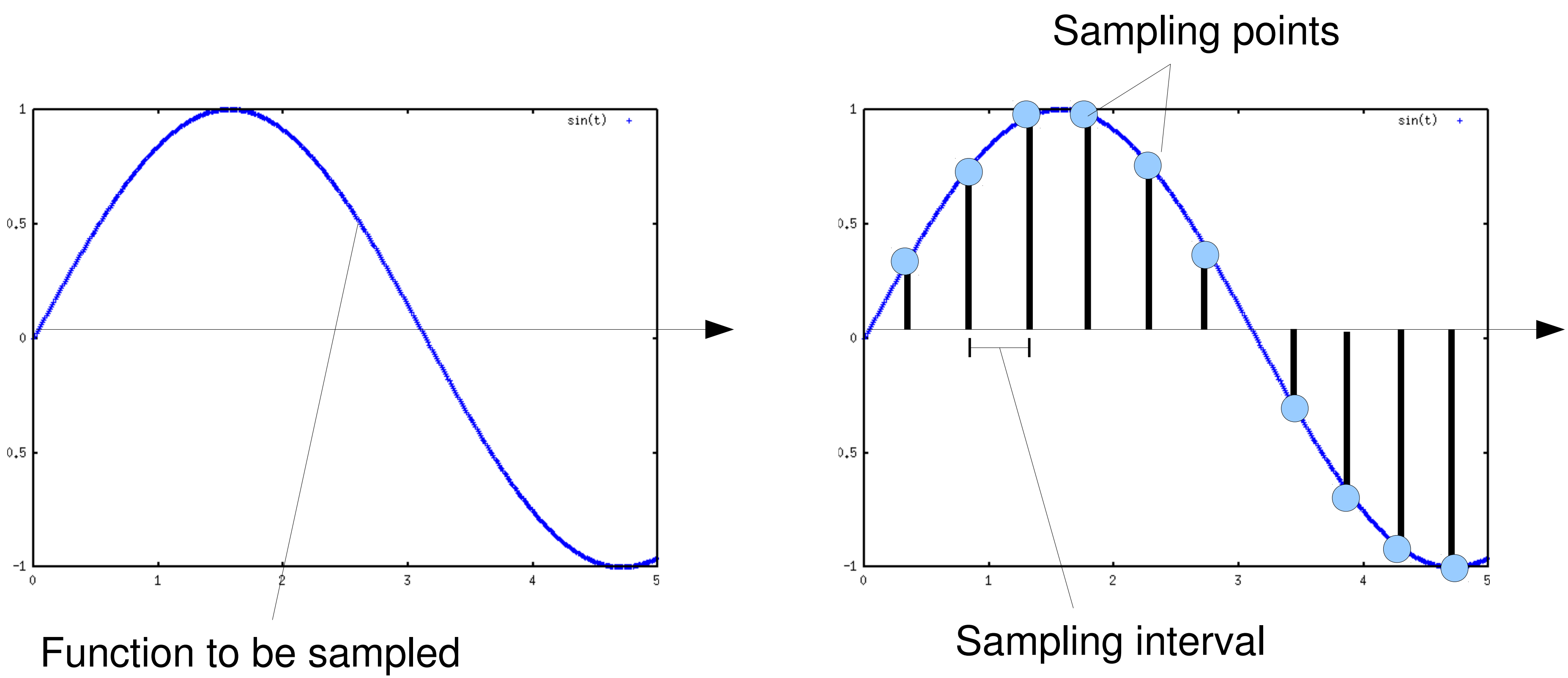
Aliasing

- Discrete samples of continuous information



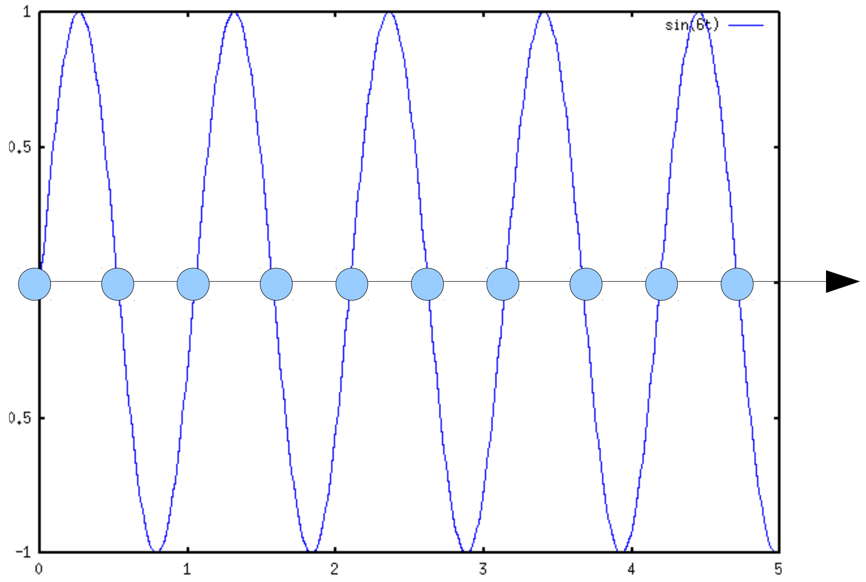
Aliasing

- A signal processing view : Aliasing is caused by inadequate sampling of continuous information.

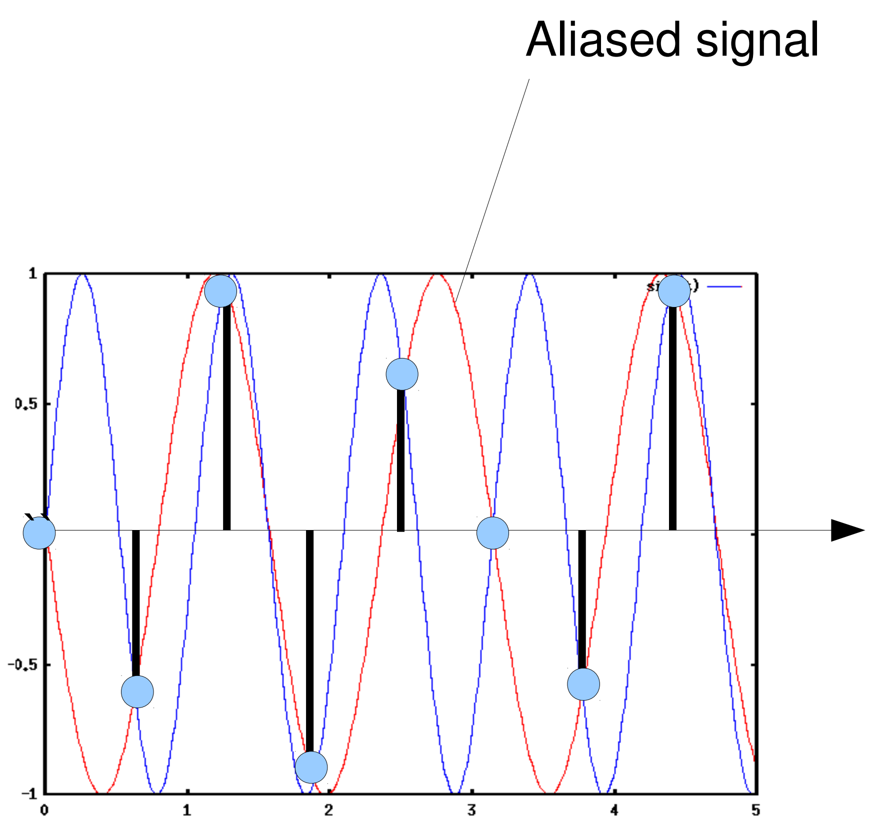


Aliasing

- The effect of sampling interval



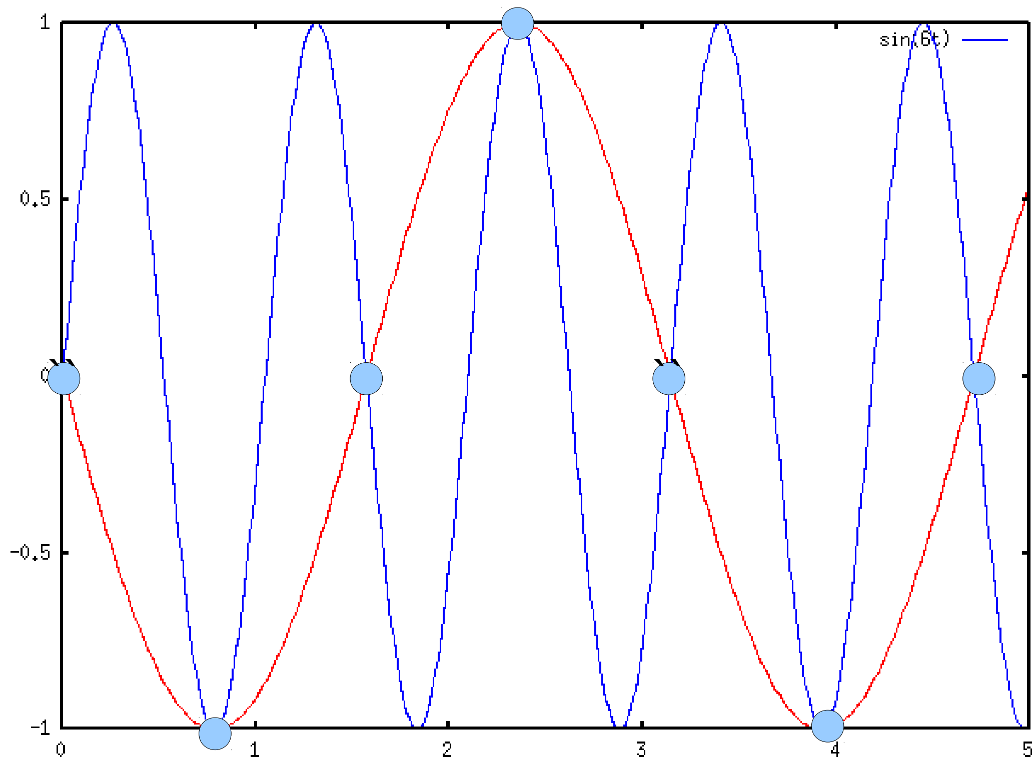
Sampling interval equal to one-half of the period.



Sampling interval more than one-half of the period.

Aliasing

- The samples seem to represent a signal at a lower frequency which are known as *aliases* (i.e., lost higher frequency information reappears as impersonating lower frequencies) – hence the name *"Aliasing."*



Aliasing

- Nyquist-Shannon Sampling Theorem:
  - A continuous bandlimited function of a single variable can be completely represented by a set of samples made at equally spaced intervals.
  - The intervals between such samples must be less than half the period (or greater than twice the frequency) of the highest frequency component in the function.

$$f_{max} < \frac{1}{2\Delta x} \quad \text{or} \quad f_s > 2 f_{max}$$

Fourier Theory

- The Fourier Transform – Any signal, $f(x)$ can be considered to be made up of a weighted sum of sine and cosine waves.
- The Fourier Transform is reversible

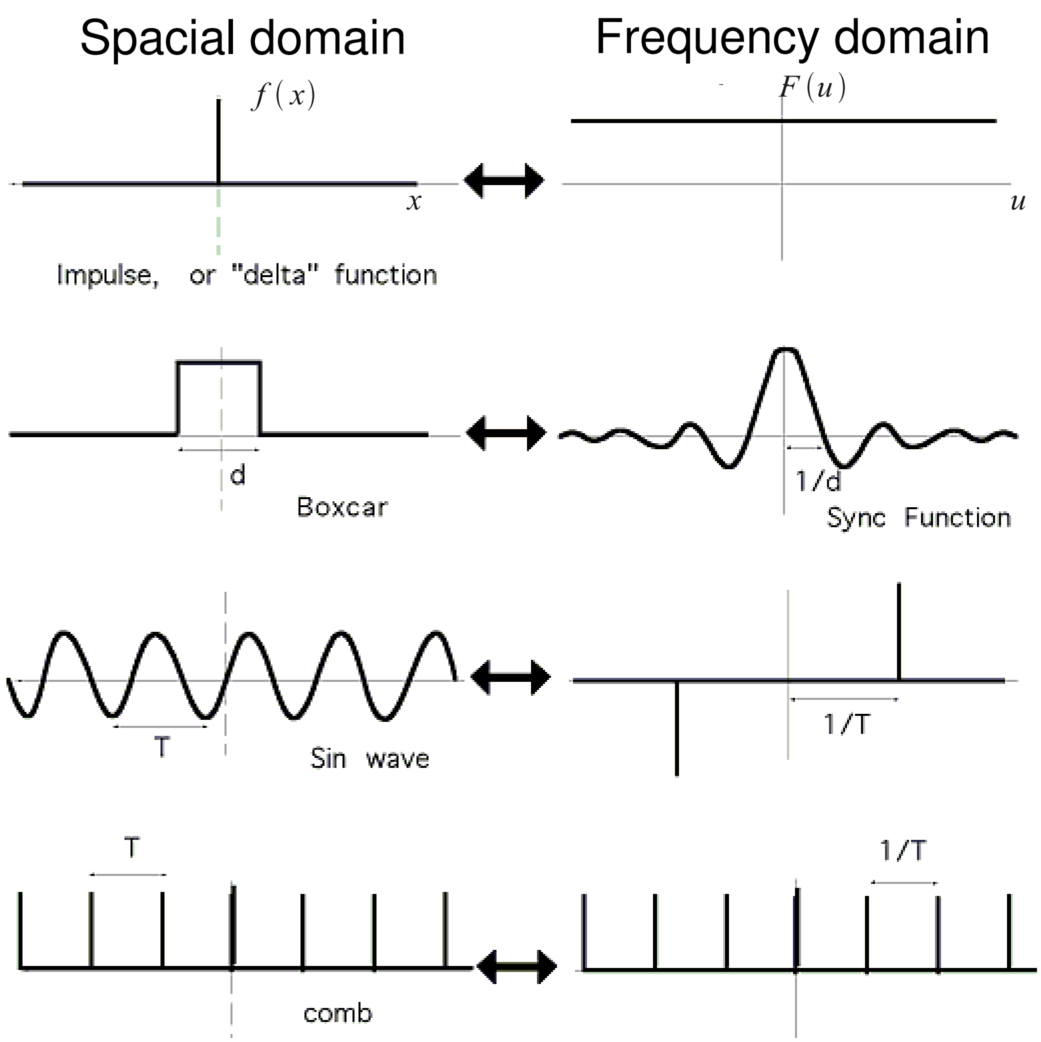
$$F(u) = \int_{-\infty}^{\infty} f(x) \mathrm{e}^{-i 2 \pi u x} \, dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) \mathrm{e}^{i 2 \pi u x} \, du$$

- A Fourier transform converts the function from a spacial domain representation to a spectral/frequency domain representation.

Fourier Theory

- Fourier Transform in 1D



Fourier Theory

- The Convolution Operator

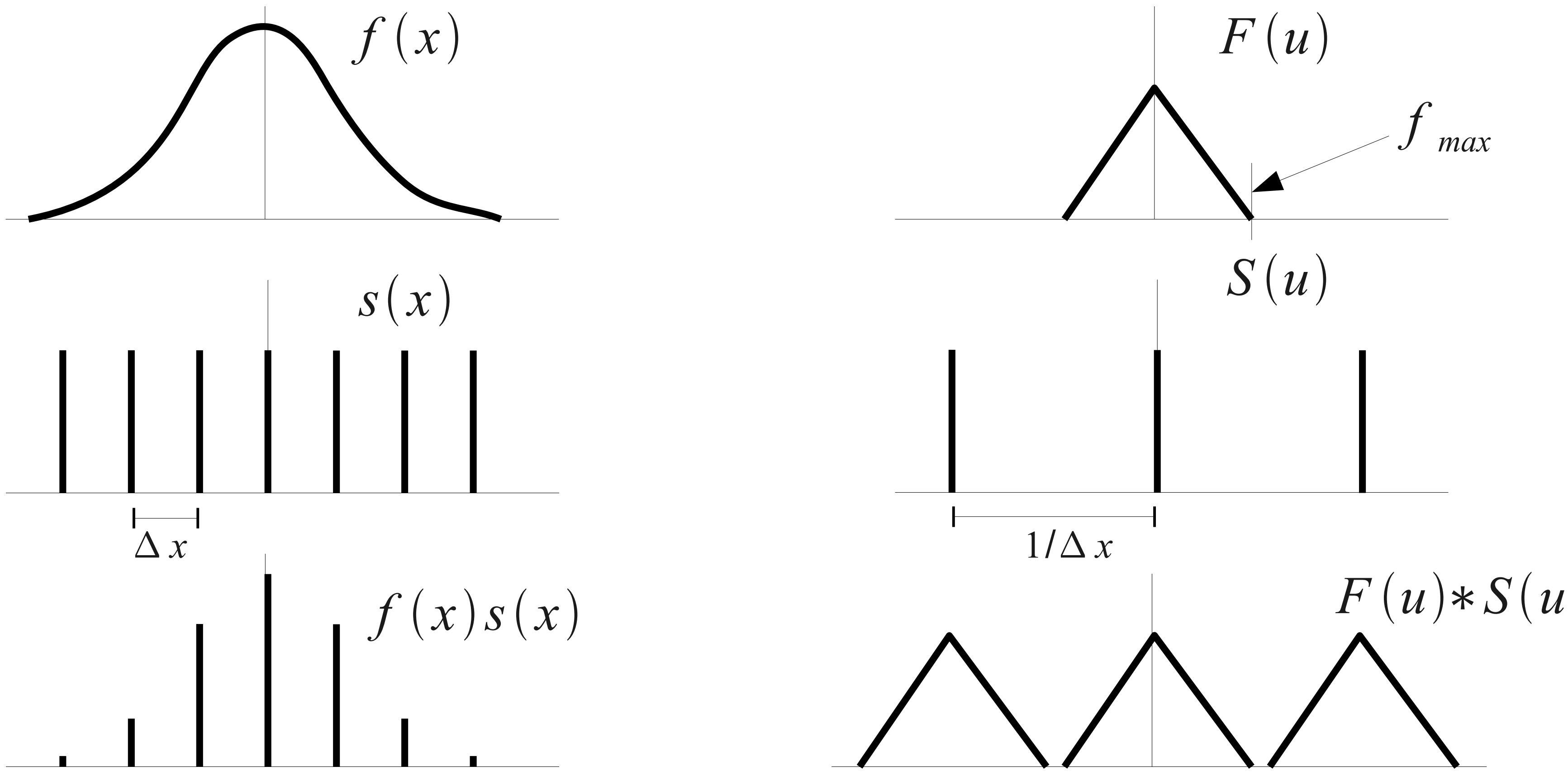
$$f(x) * g(x) = \int_{-\infty}^{\infty} f(\alpha) g(x-\alpha) d\alpha$$

- The Convolution Theorem

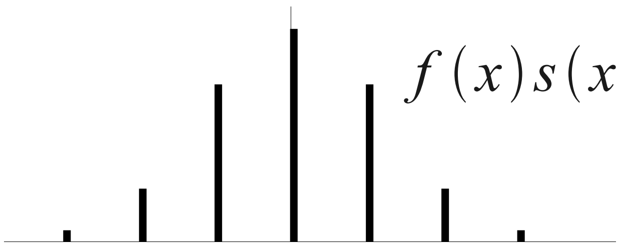
$$F(f(x) * g(x)) = F(u) G(u)$$

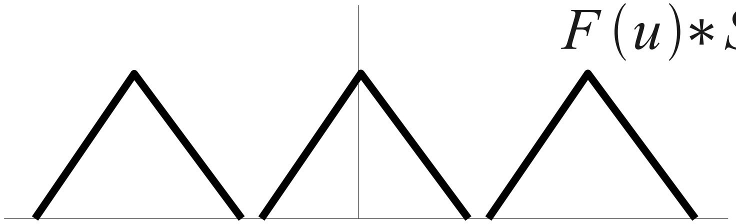
$$F(f(x) g(x)) = F(u) * G(u)$$

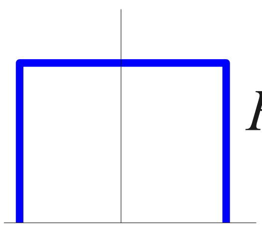
Aliasing



Aliasing

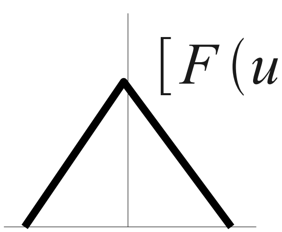






And this reconstruction of the original signal from samples works only when

$$f_{max} < \frac{1}{2\Delta x} \quad \text{or} \quad f_s > 2 f_{max}$$



Aliasing

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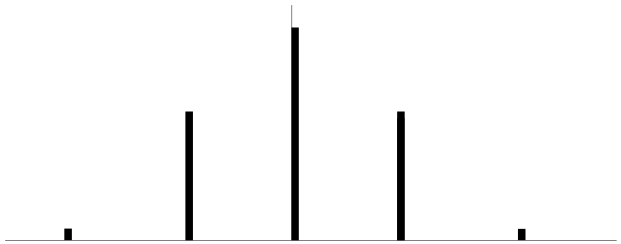
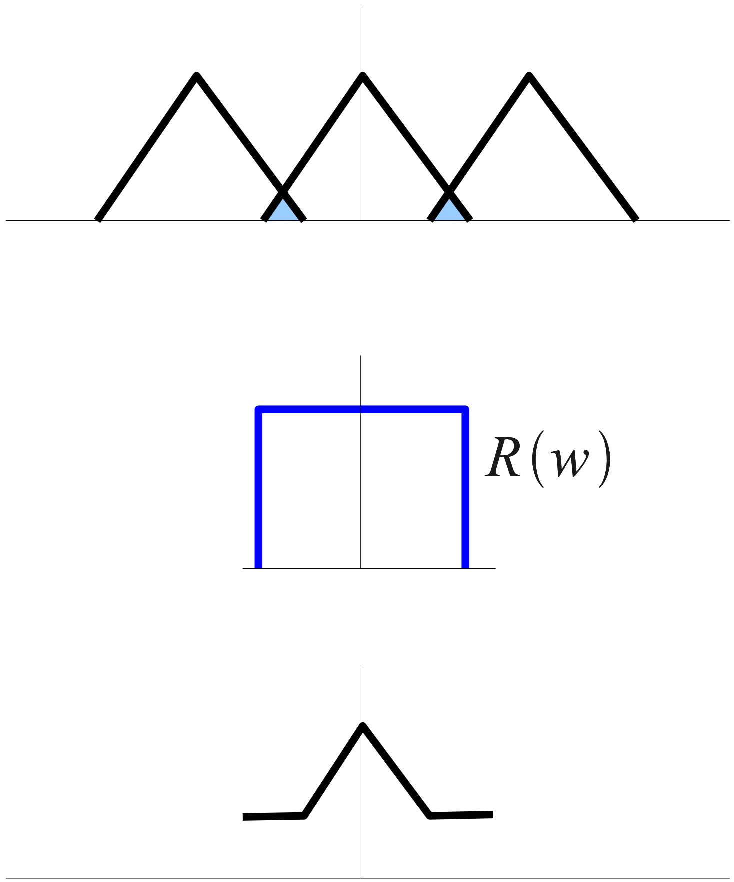
Aliasing

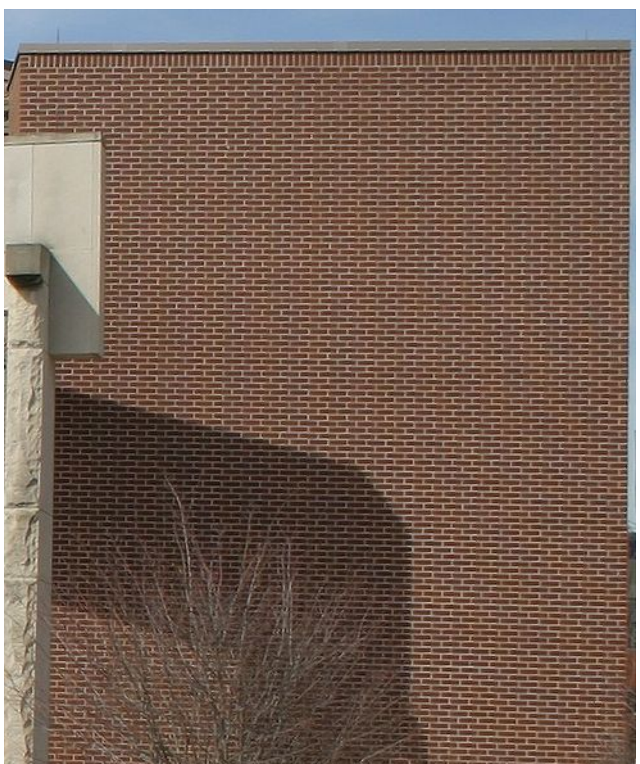




Lower sampling rates cause aliasing.

Aliasing in images

- Images can be treated as 2D signals.
- Aliasing in images also happens when information at a higher frequency is sampled at less than the Nyquist limit.



A pixel of side Δx has a Nyquist limit of $1/2\Delta x$.

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