Light and Computer Vision: Shape from Shading

CS 763

(Sharat@iitb.ac.in)

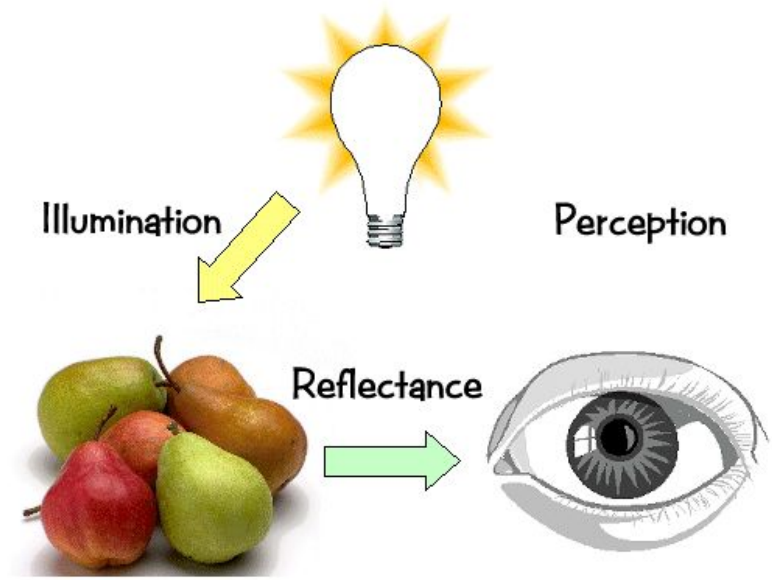
Computer Vision is an old (new) topic. Slides for these have been taken with permission from various sources and compiled for this edition of cs 763

Agenda

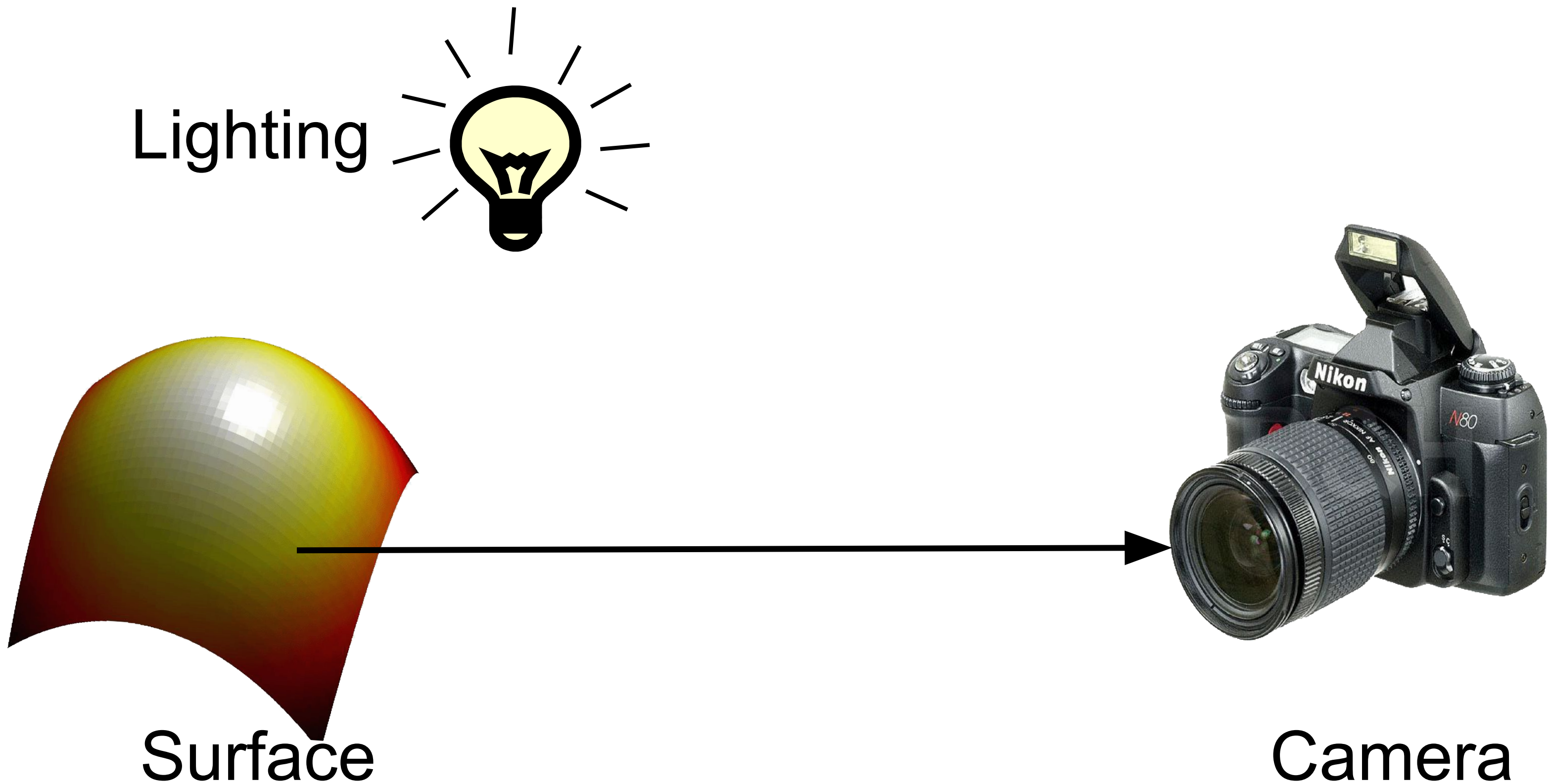
- Want: Reconstruct three dimensional structure from a photo
- Why (consider face as an example)
  - Relighting, generate new view, generate expressions
  - Fundamental problem in vision
- How we will approach this
  - We will take a “first principles approach”
  - Some (possibly) new mathematical concepts

Image Understanding: First Principles

- Able to see things
  - because there is light
- Humans able to interpret
  - perception
- Computer vision
  - camera
  - software

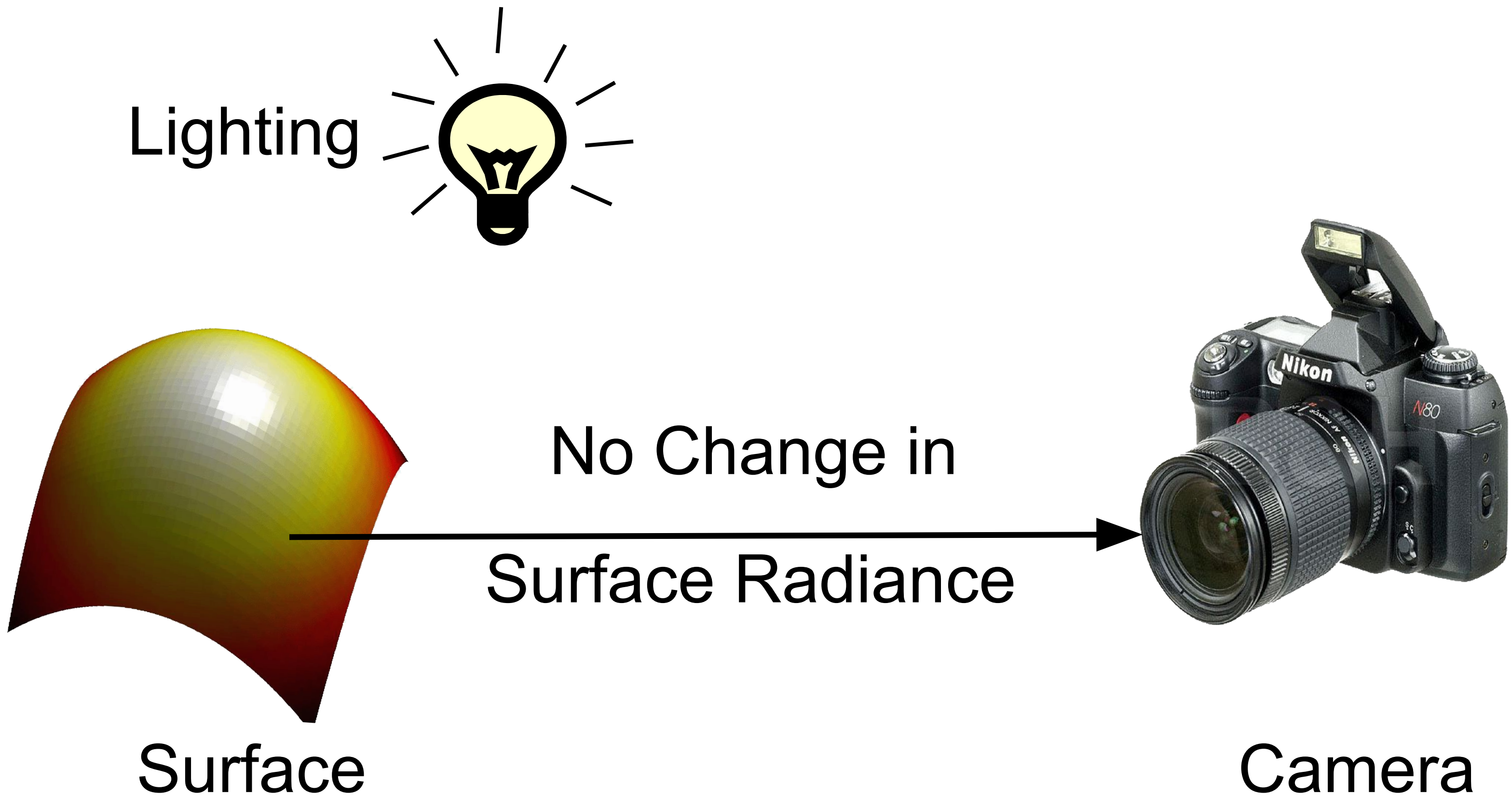


Fundamental Assumption



- Image captured by the camera
- In vision, what assumption are we making
  - about photons emerging
  - single unoccluded scene?

Fundamental Assumption in Vision



- What we capture is what is out there
- We need to define what is “out there”

Contents

- Introduction
- Concept of scene and image irradiance
  - Proving (falsifying) a fundamental assumption
- Problem Definition
- Reflectance function
- Shape from shading
- Photometric stereo

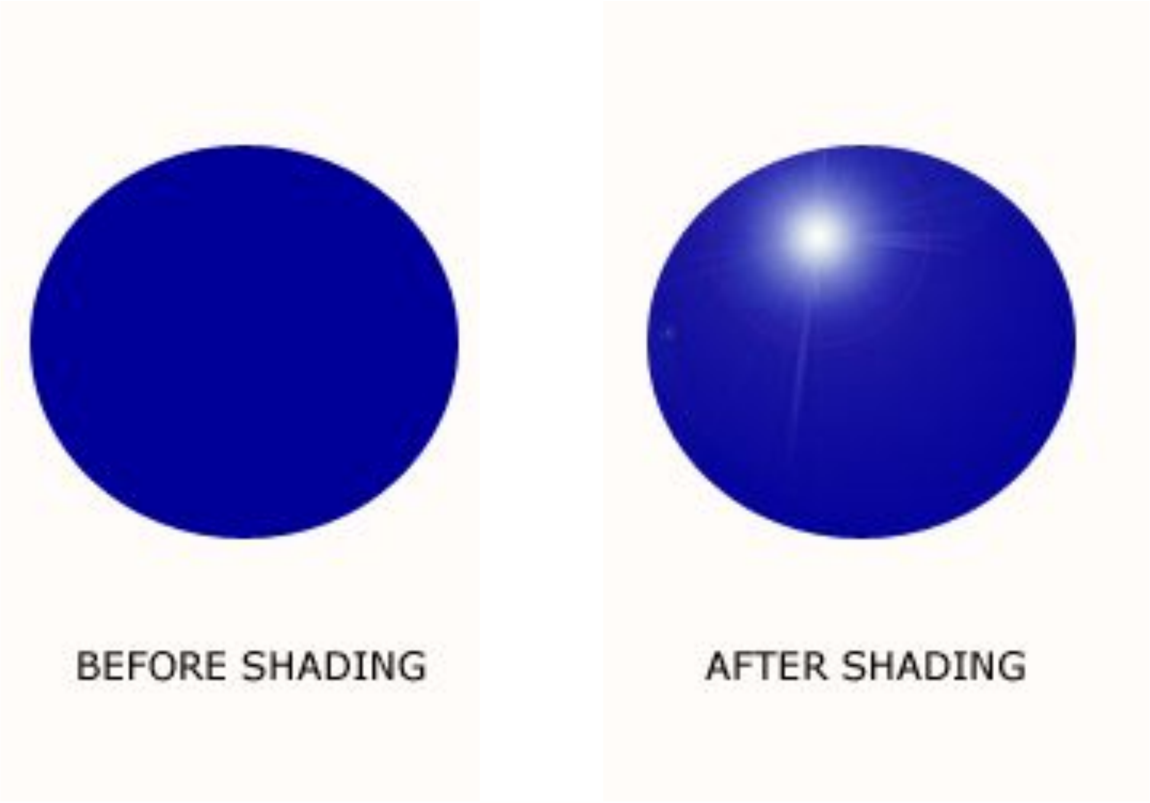
Introduction

Big Picture View

From 2D to 3D

- The human eye records 2D images on the retina, but the brain is able to infer the 3D shape of the scene underlying the image
- The brain uses several "cues" for this conversion from 2D to 3D – shading, defocus, texture, motion, shadows, contours and so on
- The 2D to 3D conversion is the direct opposite of computer graphics
- Here we focus on shading

Baby Steps

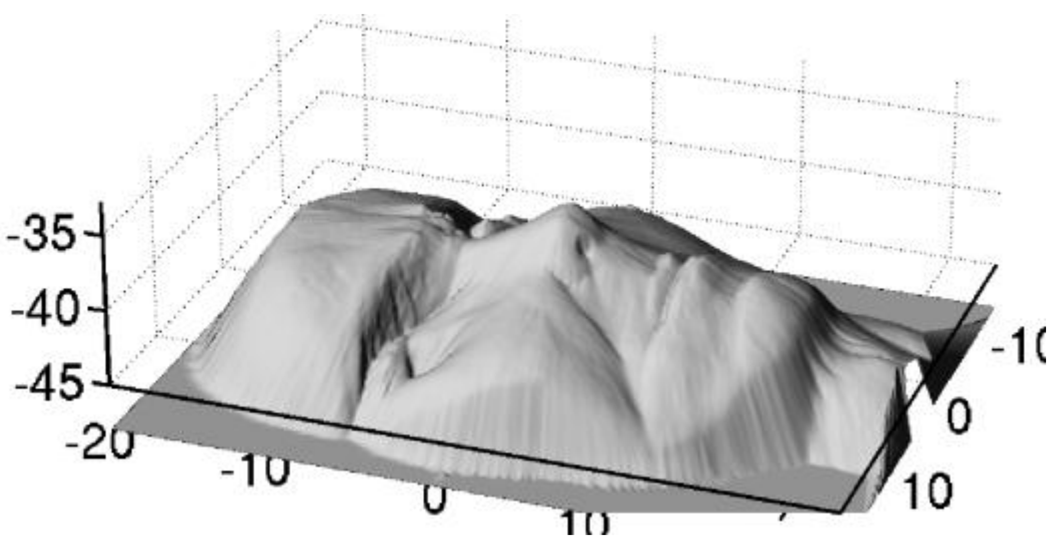


http://www.famouslogos.org/the-basics-of-three-dimensional-design

- Understand reflectance

Next Steps

- Build 3D model



Bigger Steps

Don’t Expect This

Haze

De-hazed

… Or This

Contents

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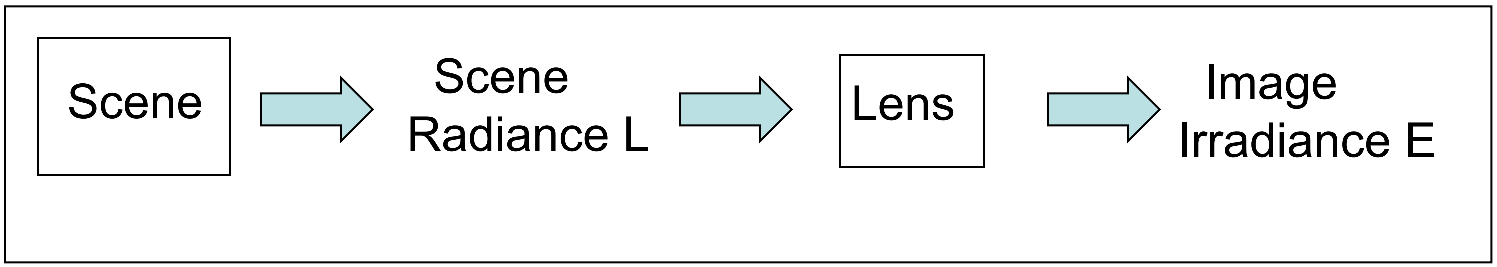
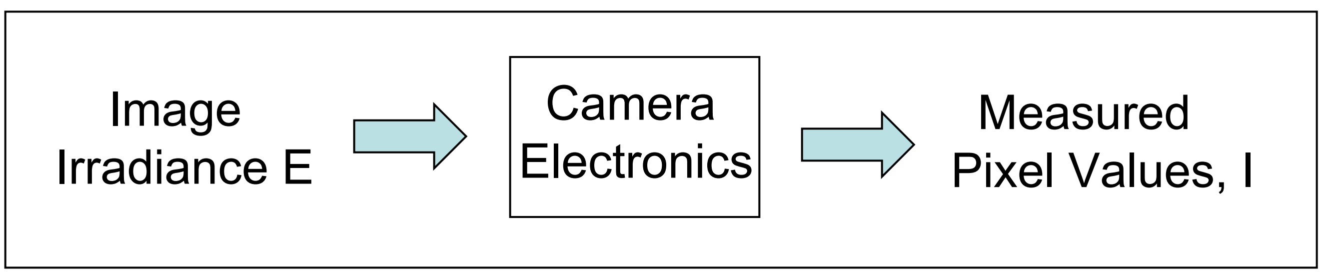
- Reflectance function
- Photometric stereo
- Shape from shading

Scene and image: Relation

Before light hits the image plane



After light hits the image plane

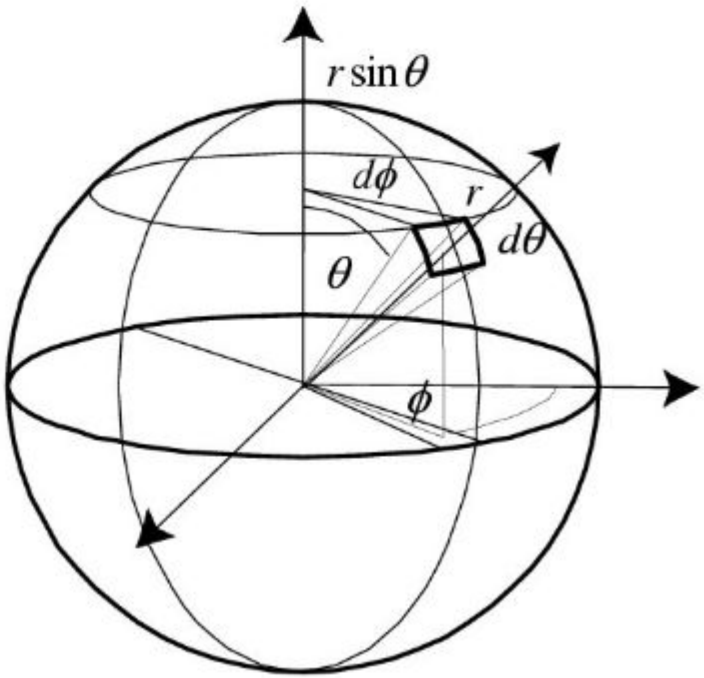


Goal: Measured $I(x,y)$ should be “what is out there?”

Radiometric Terms

- Boring but (some are) necessary
- Mathematical notation (terms)
  - Solid angle
  - Radiant intensity of source ← Graphics
  - Surface Irradiance
  - Surface Radiance
  - Image Irradiance
  - Pixel value ← Vision

Recall Solid Angle



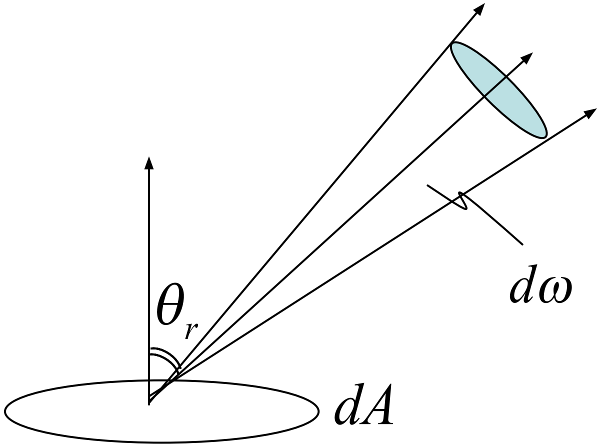
$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

Radiance

- Radiance is **the** quantity of interest
- Compare definition of mass
- The scene radiance is the
  - power of light per unit **foreshortened** area in a given direction
- Why: Radiance remains conserved in space as it propagates along ray

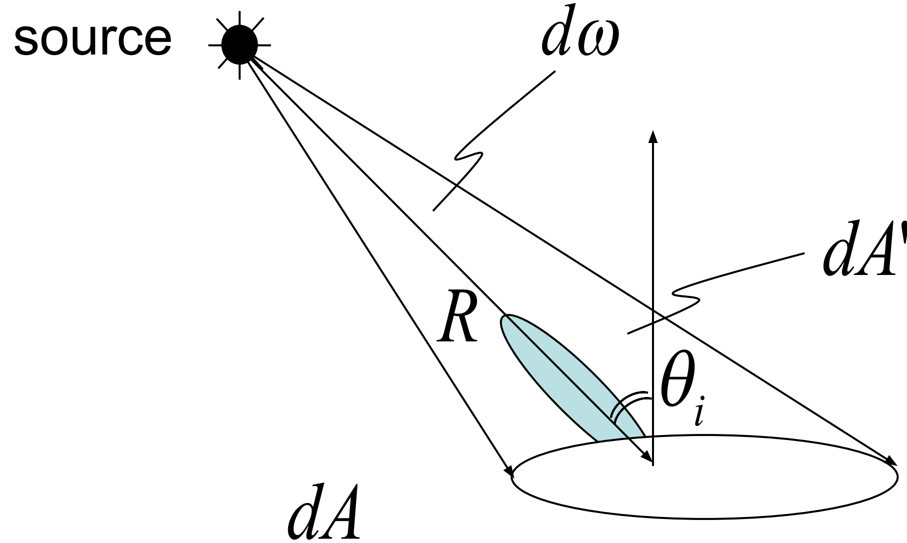


$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ \ d\omega}$$

(watts / m^2 steradian)

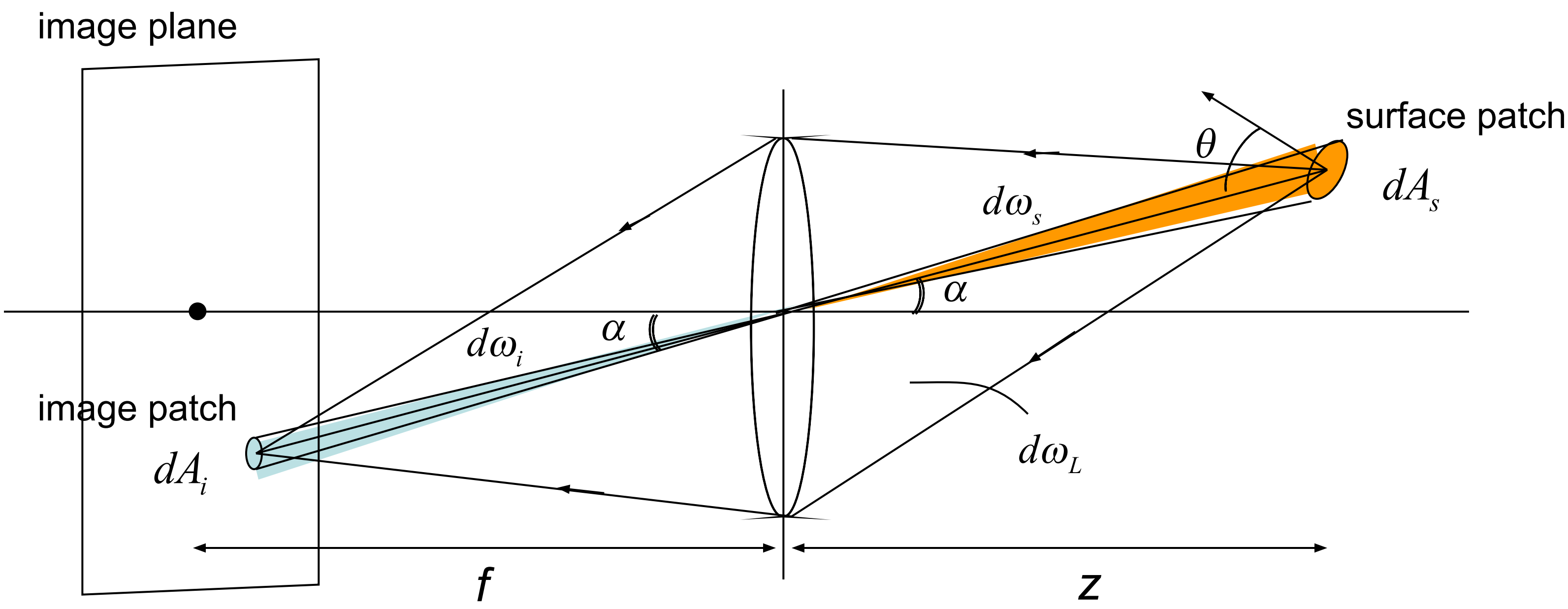
Image Irradiance

- Irradiance is the power per unit area
- Does not depend from where the light is coming from in general, but for us, it is going to come from the scene



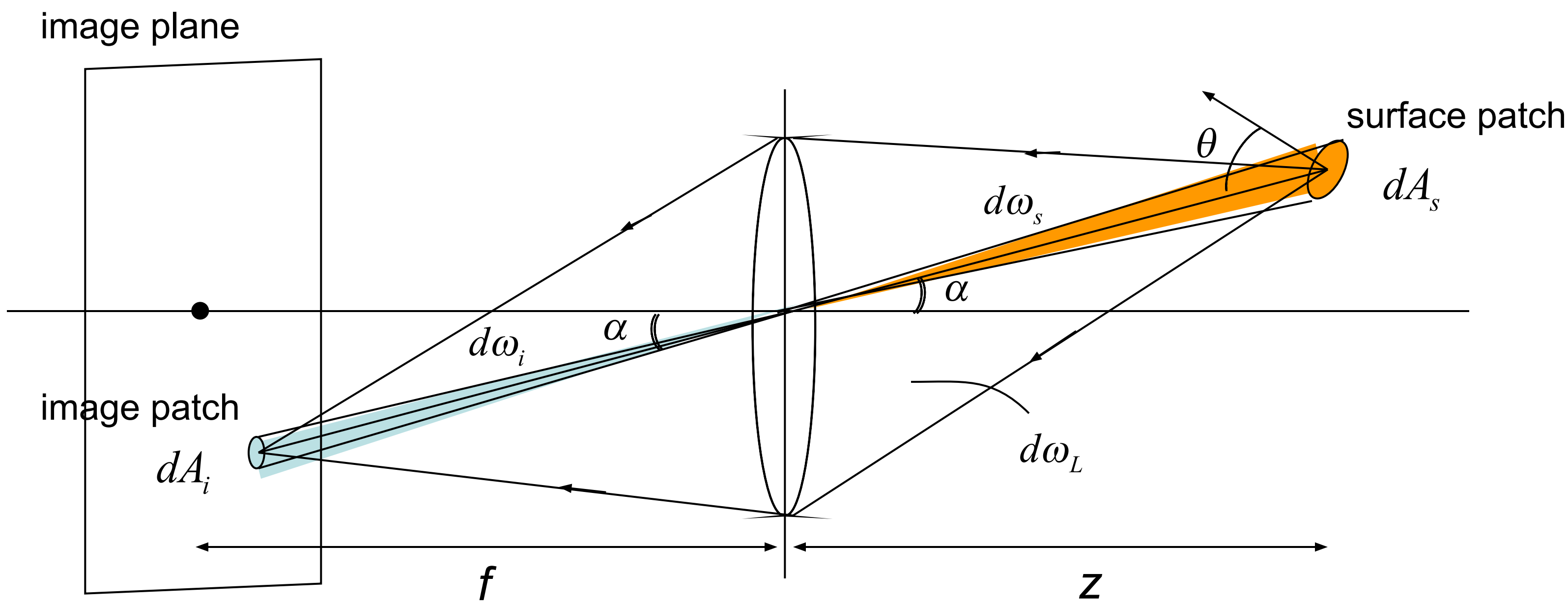
$$E = \frac{d\Phi}{dA}$$

Relation between E and L



- Three solid angles of interest
  - orange and green (equal)
  - subtended by lens

Relation between E and L

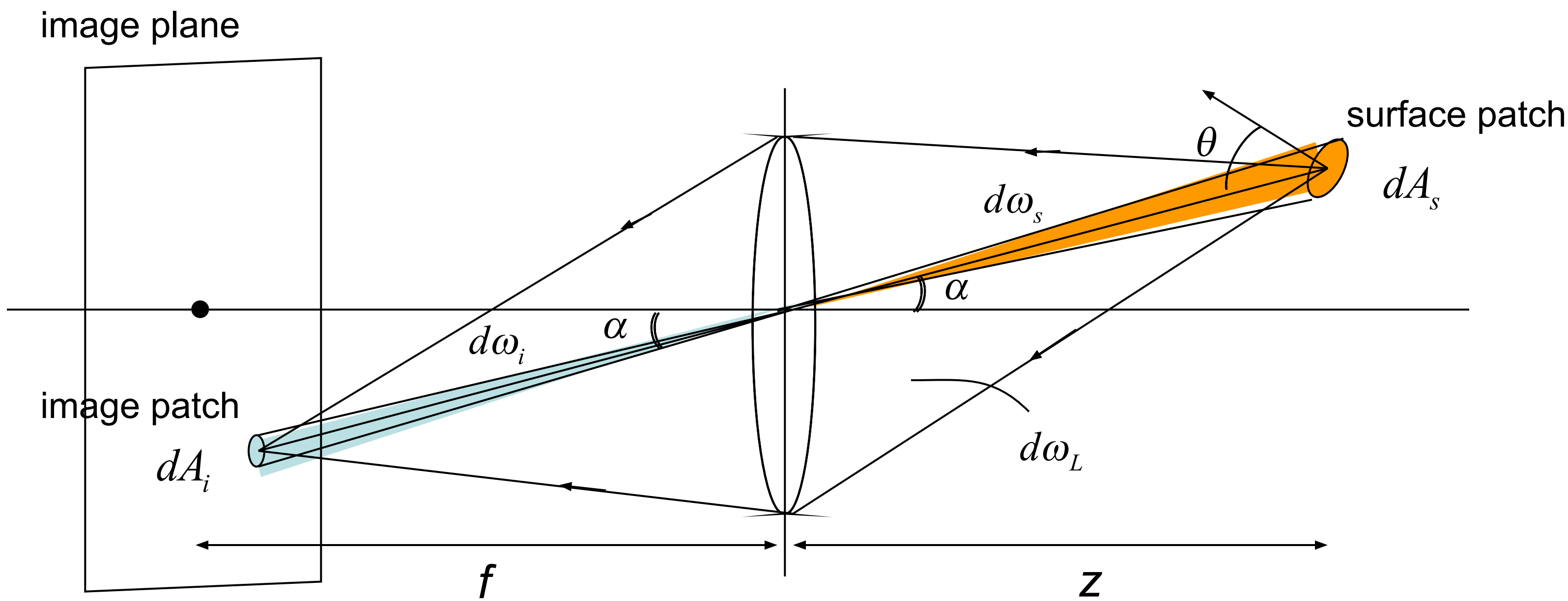


$$d\omega_i = d\omega_s \qquad \frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2}$$

$$\boxed{\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2} \rightarrow (1)$$

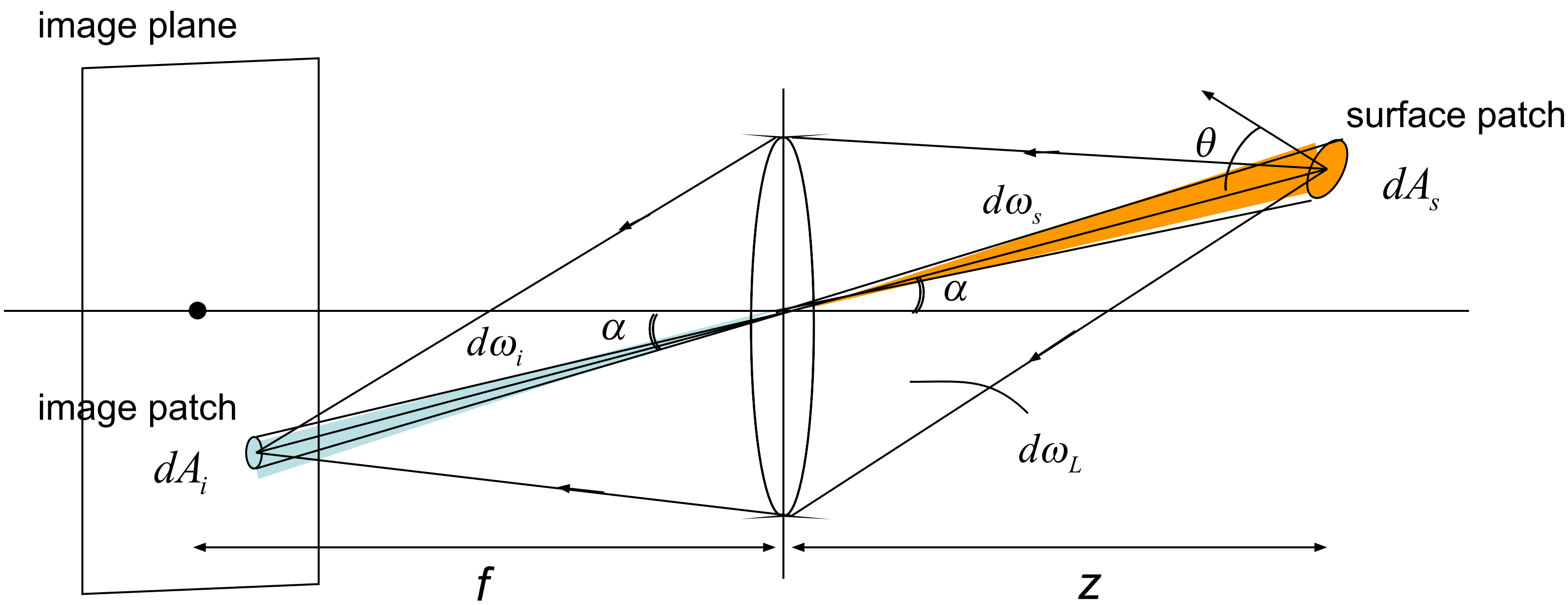
$$\boxed{d\omega_L = \frac{\pi d^2}{4}\frac{\cos\alpha}{(z/\cos\alpha)^2}} \rightarrow (2)$$

Relation between E and L



- Power received by lens from surface patch equals power received by image patch
  - We use scene **radiance** to determine power sent by patch
  - We use image **irradiance** to determine power received

Relation between E and L



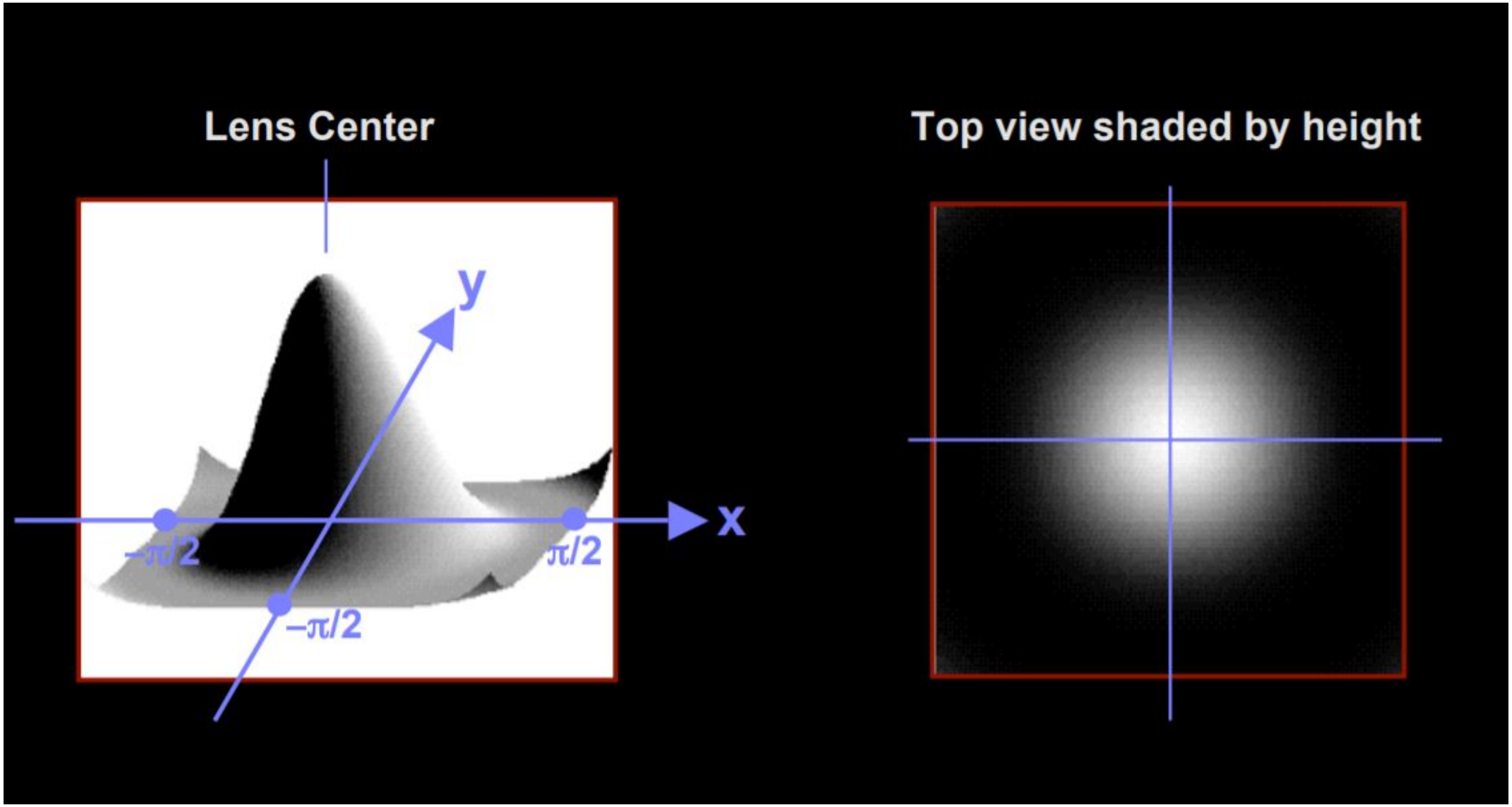
$$L\,(dA_s\,\cos\theta)\,d\omega_L \;=\; E\,dA_i$$ → (3)

From (1), (2), and (3):

$$\boxed{E \;=\; L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cos\alpha^4}$$

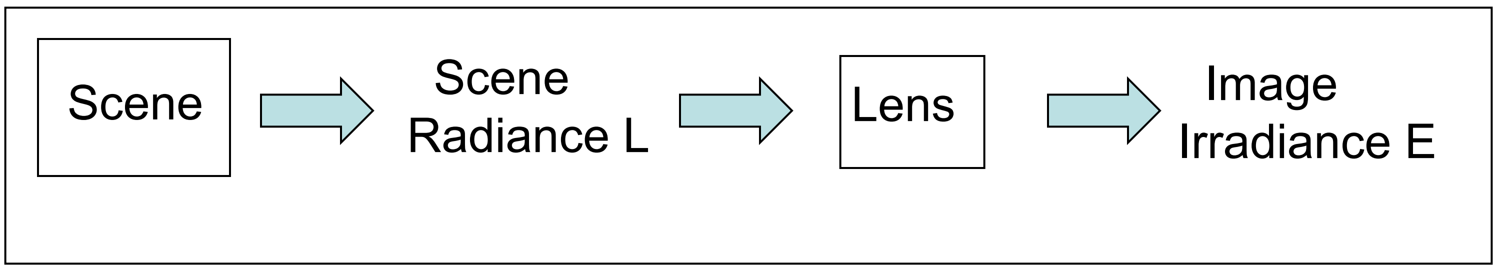
Linear relation?

Off axis cut off



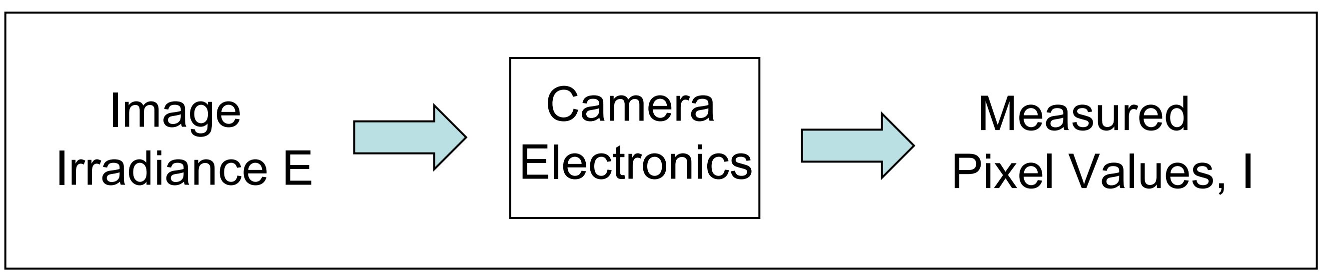
Pixel Value & Irradiance: Relation

Before light hits the image plane



Linear Mapping!

After light hits the image plane



Non-linear Mapping!

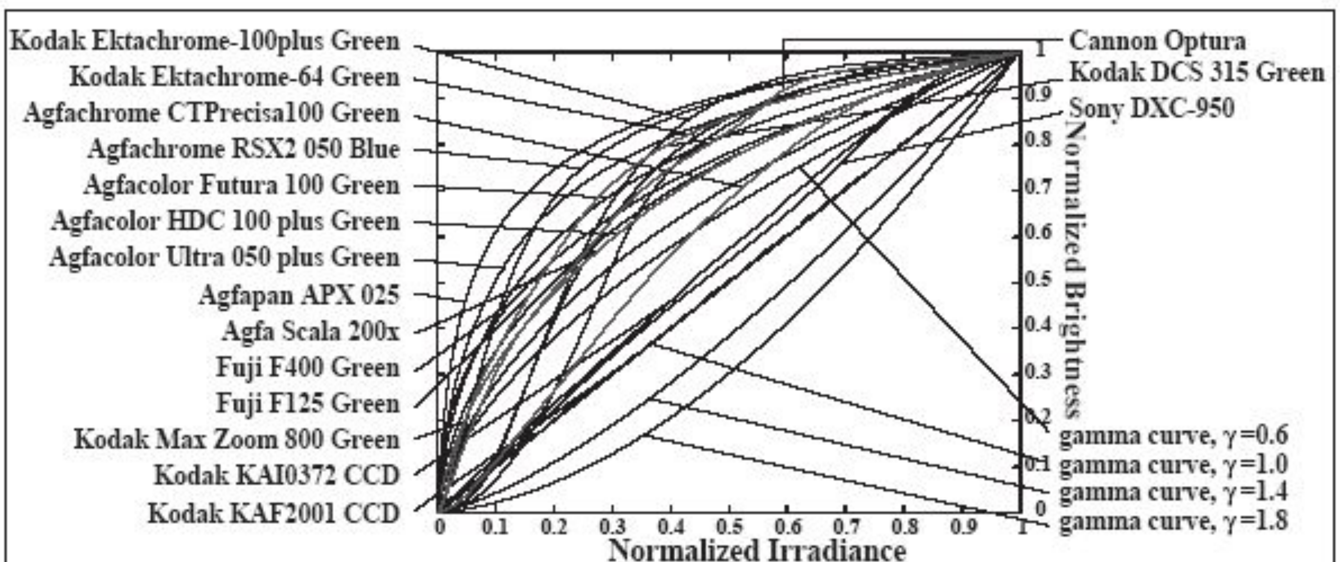
Goal: Measured $I(x,y)$ should be “what is out there?”

Pixel Value & Irradiance: Relation

Image Irradiance E → Camera Electronics → Measured Pixel Values, I

$$g : E \rightarrow I$$

- After light hits the image plane
  - The camera response function

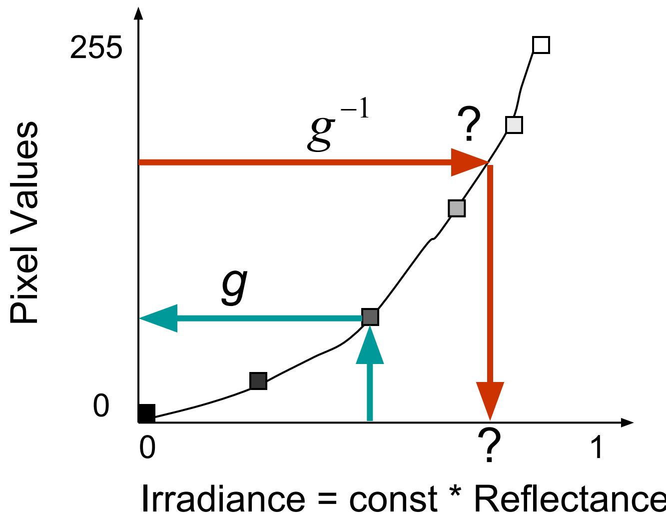


(Grossberg and Nayar)

Need to invert

- Important preprocessing step for many vision and image processing algorithm
- Usual practice to use a color chart with precisely known reflectances
  - Method assumes constant lighting on all patches and works best when the source is far away
  - Unique inverse exists and smooth for all cameras

$$g^{-1} : I \rightarrow E$$



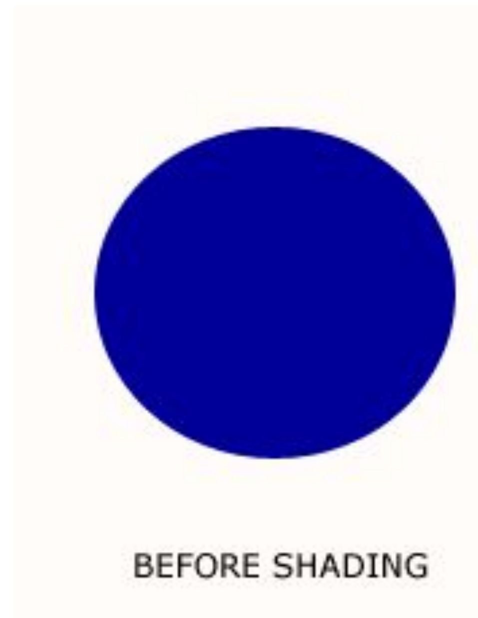
Contents

Problem: Shape from shading

- Given an image $I(x,y)$ of a surface, determine the depth $Z(x,y)$ at each (visible) point on the surface.

Problem: Shape from shading

- With the assumption that scene radiance equals image irradiance, given an image *I(x,y)* of a surface, determine the depth at each point on the surface.
  - What is shading, and why would shading reveal the depth?
  - Depth only for **visible** points

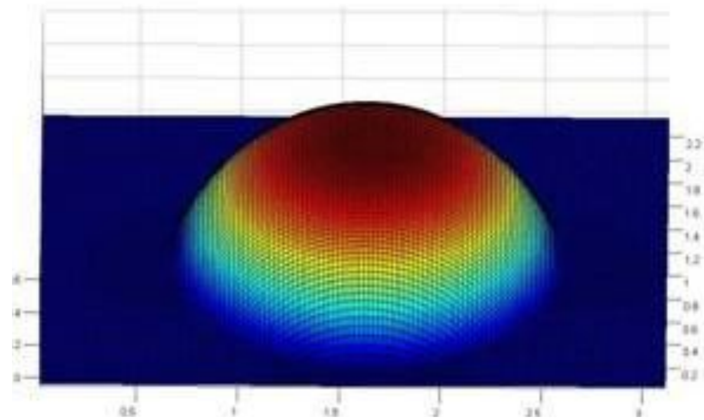


Recall Example

Depth from Shading: Qualitative

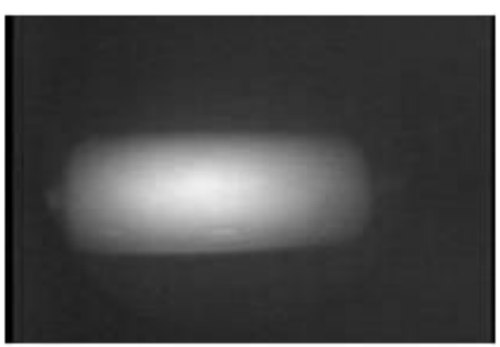
- Depth is shown as a function *Z(x,y)* (not as distance from the camera)

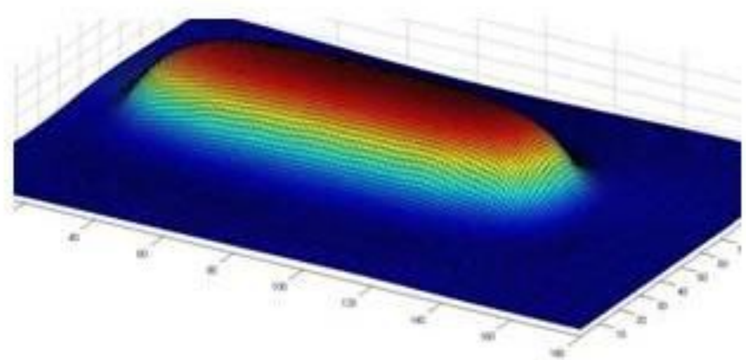
I(x,y)



Z(x,y)

I(x,y)





Z(x,y)

Depth from Shading: Quantitative

- The depth of an object reveals itself from its surface normal …
- The surface normal reveals itself from the shading because
- scene radiance is a function of
  - scene illumination
  - material properties
  - local geometry (depth)
  - viewer position
- We have to disentangle these last four things

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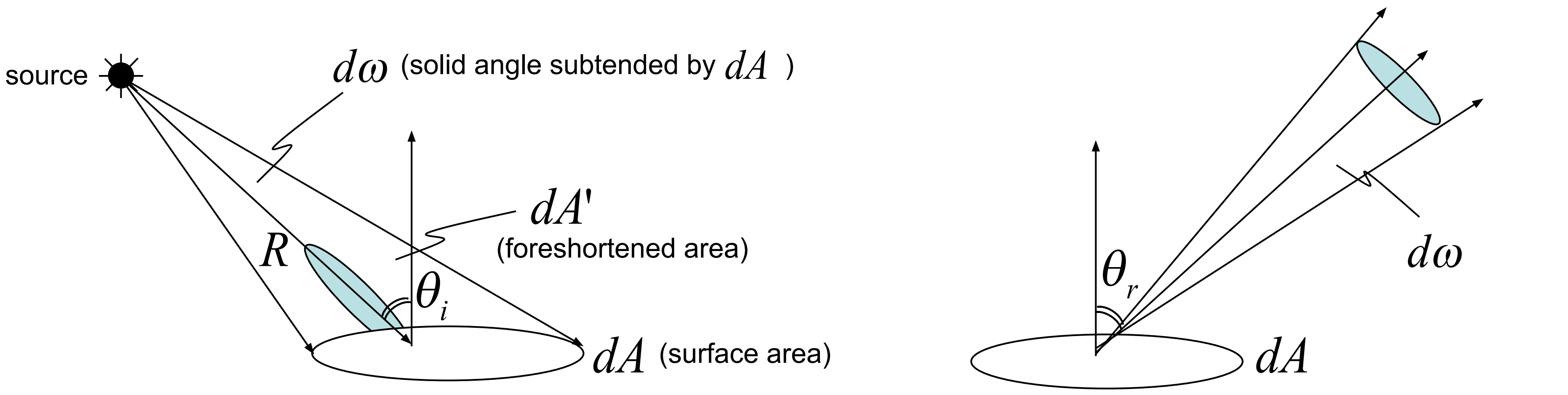
Reflectance Models

Radiometric concepts – boring…but, important!



(1) Solid Angle : $d\omega = \dfrac{dA'}{R^2} = \dfrac{dA\cos\theta_i}{R^2}$ (steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source : $J = \dfrac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \dfrac{d\Phi}{dA}$ (watts / m)

Light Flux (power) incident per unit surface area.

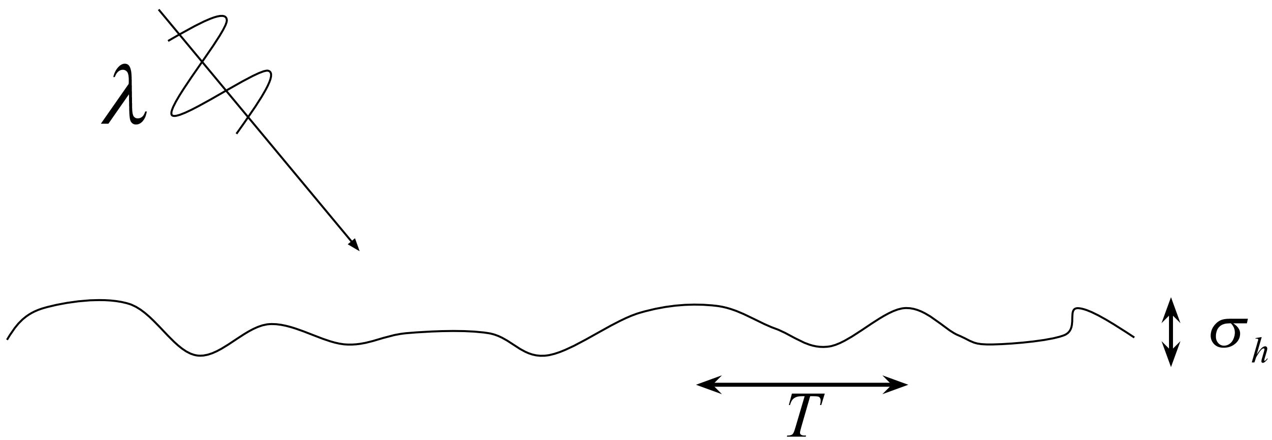
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- *L* depends on direction θ_r
- Surface can radiate into whole hemisphere.
- *L* depends on reflectance properties of surface.

Reflectance Models



- Reflection is an electromagnetic phenomenon
- So to determine the reflectance model
  - Wave optics and Ray Optics
- Geometric models are easier
  - Sometimes we need wave optics

Scene and image irradiance

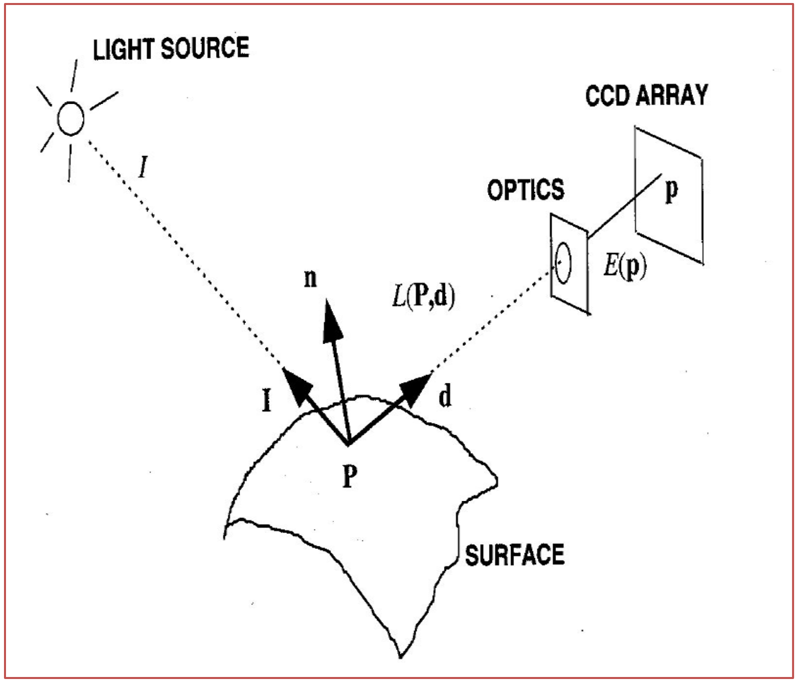


Image irradiance: Power of light per unit area incident around a point **p** on the image plane

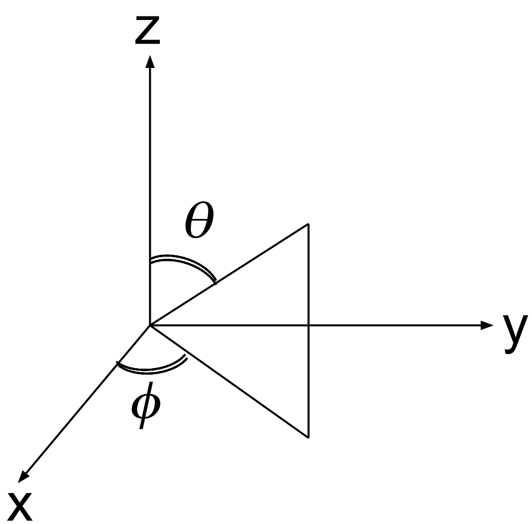
Scene radiance: Power of light per unit area reflected from a small area around a point **P** on the surface of the 3D object in a direction **d**.

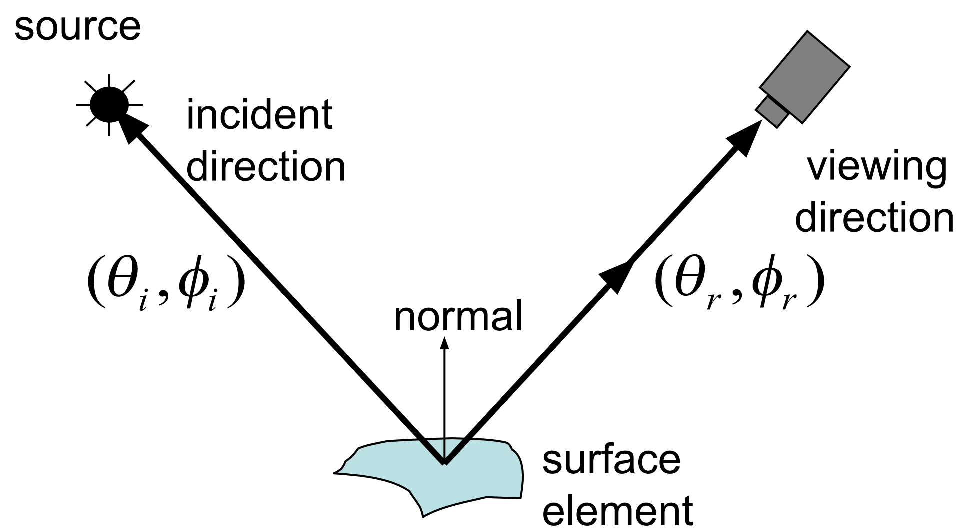
Scene radiance depends on the following factors:

- Light: intensity & direction (w.r.t. normal)
- Material properties
- Surface (surface normal) at point **P**
- Viewing direction (w.r.t. normal)

The model in which the surface reflects light is called as the **surface reflectance model**.

BRDF





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

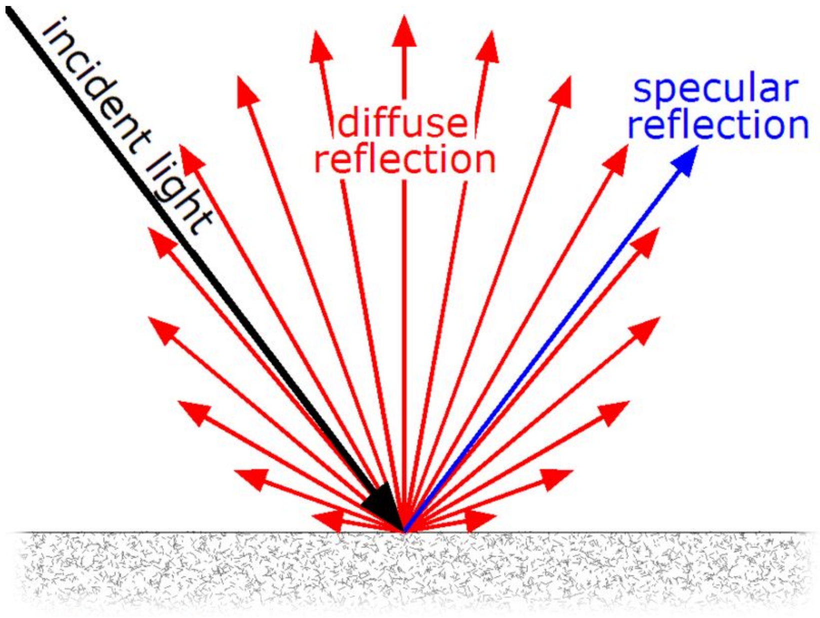
BRDF : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \dfrac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$

Problem: Shape from shading

- Given an image *I(x,y)* of a surface **with a known reflectance model**, determine the depth at each (visible) point on the surface.

Lambertian Model

- It is one particular type of reflectance model, for a diffusely reflecting surface
- Surface **appears equally bright** when viewed from all directions from which the surface is visible



en.wikipedia.org/wiki/Diffuse_reflection

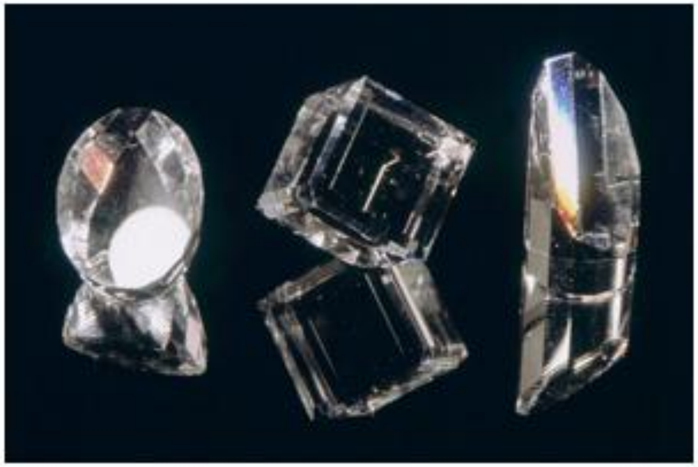
- Many surfaces are **not** Lambertian

Diffuse and Specular Reflection

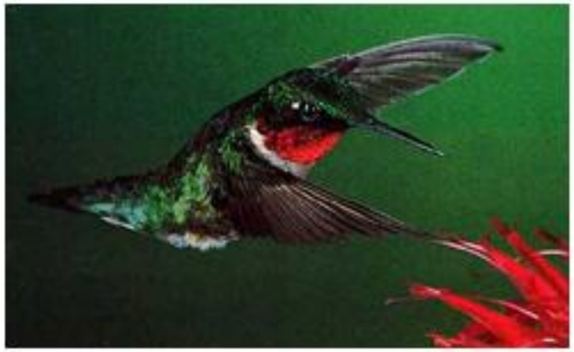
diffuse specular diffuse+specular

Recall Wave Optics

Life can get very complicated so ...

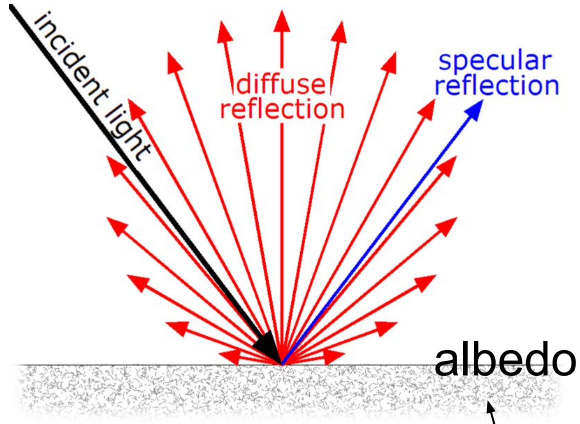




Back to Lambertian Model

- Lots of surfaces are Lambertian
  - Or even if not, it is a significant component
- The BRDF is constant
- Very common in vision and graphics



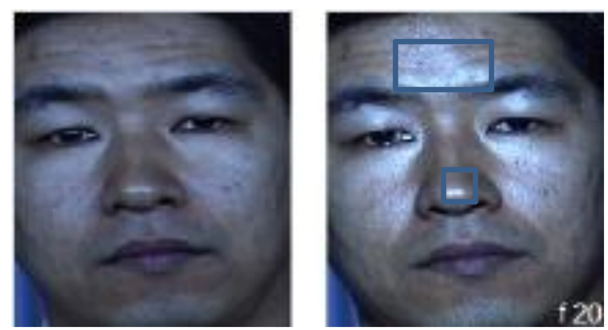
$$f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$$

Albedo

- The albedo ρ is the fraction of the reflected light over the incident light. It is a quantity between 0 and 1.
- It is a property of the material. Different materials have different albedos.
- Example: charcoal has an albedo of 0.04, snow has an albedo of 0.9.

Examples

- Matte paint, paper are Lambertian.
- Human face is (generally) Lambertian, except for some "shiny" spots.
- Mirrors, oil-polished surfaces are **not** Lambertian.



Lambertian Model

- The scene radiance is given by the equation:

$$I = L\rho \boldsymbol{l}^T \mathbf{N} = L\rho \cos(\theta),$$

The term in the circle(s) is the effective surface area as seen from the light source

I = scene radiance = image irradiance at the appropriate image point

L = lighting intensity

$\boldsymbol{l}$ = lighting direction (unit vector), assuming distant point light source

$\mathbf{N}$ = unit surface normal at the point under consideration

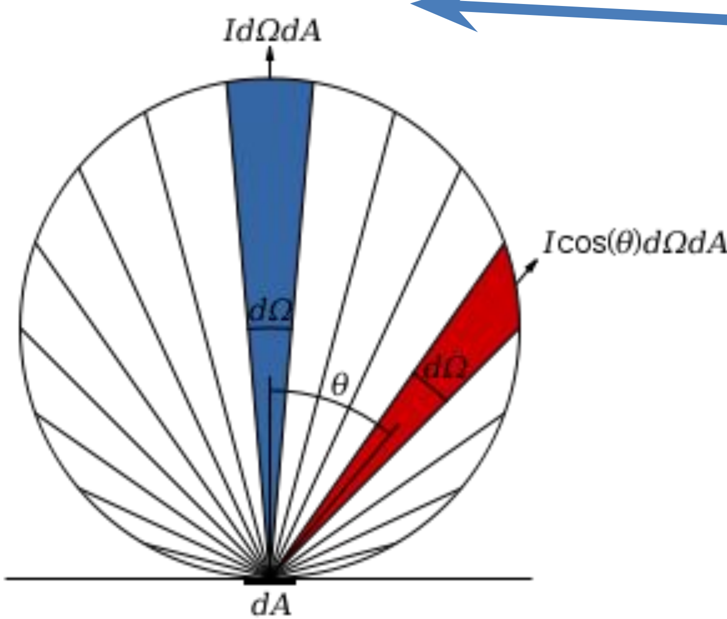
ρ = surface reflectivity (albedo) at the point under consideration

- What if cos(ϴ) < 0?
  - surface does not face the light source
  - set irradiance to 0.

$$I = \max(0, L\rho \cos(\theta))$$

Lambert's cosine law

- Intensity reflected from a diffusely reflecting surface is proportional to the cosine of the angle between the surface normal and the light direction and **independent** of the viewing direction

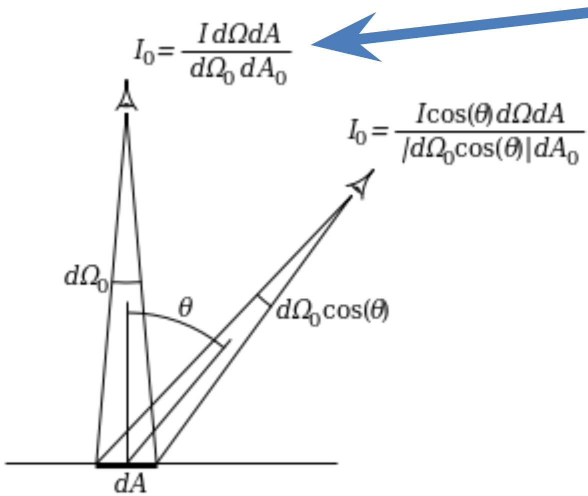


Observed photons being emitted

http://en.wikipedia.org/wiki/Lambert%27s_cosine_law

Lambertian BRDF

- The Lambertian model does **not** mean that equal amounts of light are **reflected** in all viewing directions. The amount of light reflected gets multiplied by a cosine term, but the perceived area also gets multiplied. Hence **irradiance** is the same.



Irradiance Observed =
Power per unit area

http://en.wikipedia.org/wiki/Lambert%27s_cosine_law

Depth from Shading: Quantitative

- Scene radiance is a function of
  a. scene illumination (point source, no interreflections)
  b. material properties (Lambertian)
  c. local geometry (depth)
  d. viewer position (far away from object)
- We will use (d) to suitable parametrize (c)