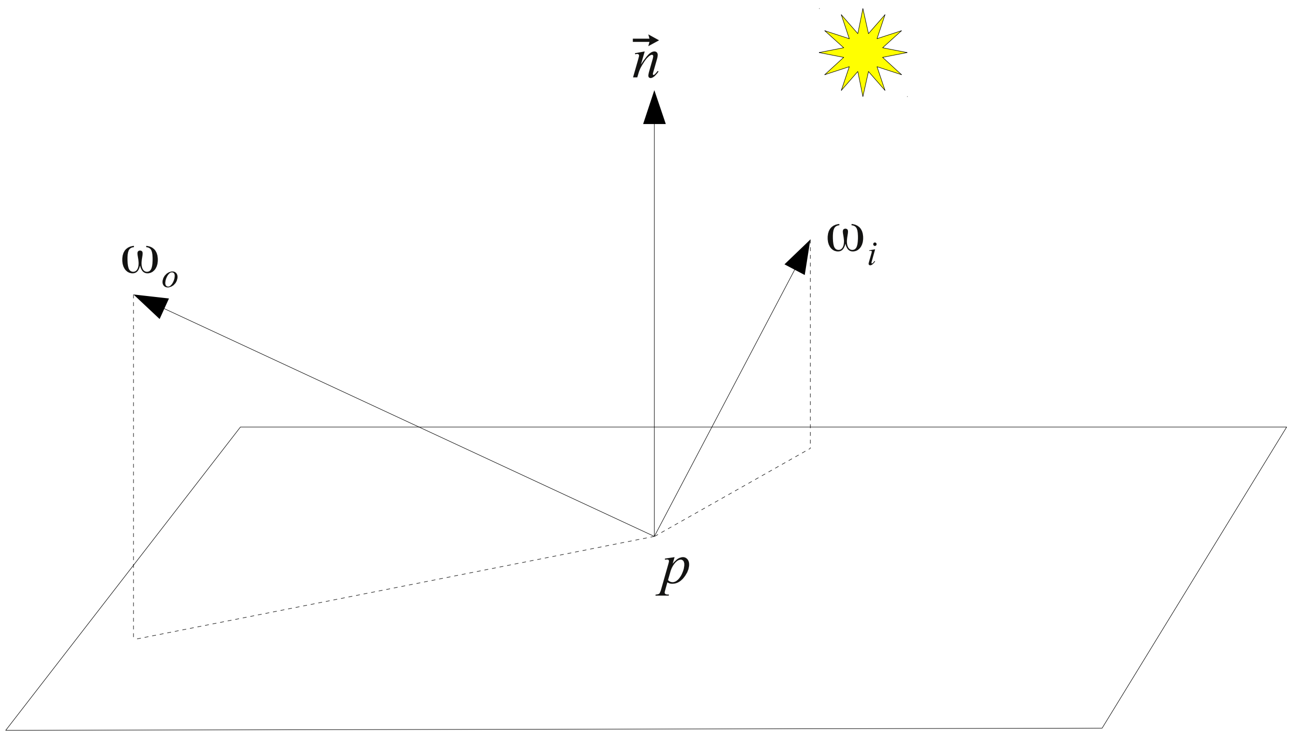
CS 775: Advanced Computer Graphics

Lecture 2 : The BRDF

The BRDF

Differential Irradiance at p is $dE(p,\omega_i) = L_i(p,\omega_i)\cos\theta_i \, d\omega_i$



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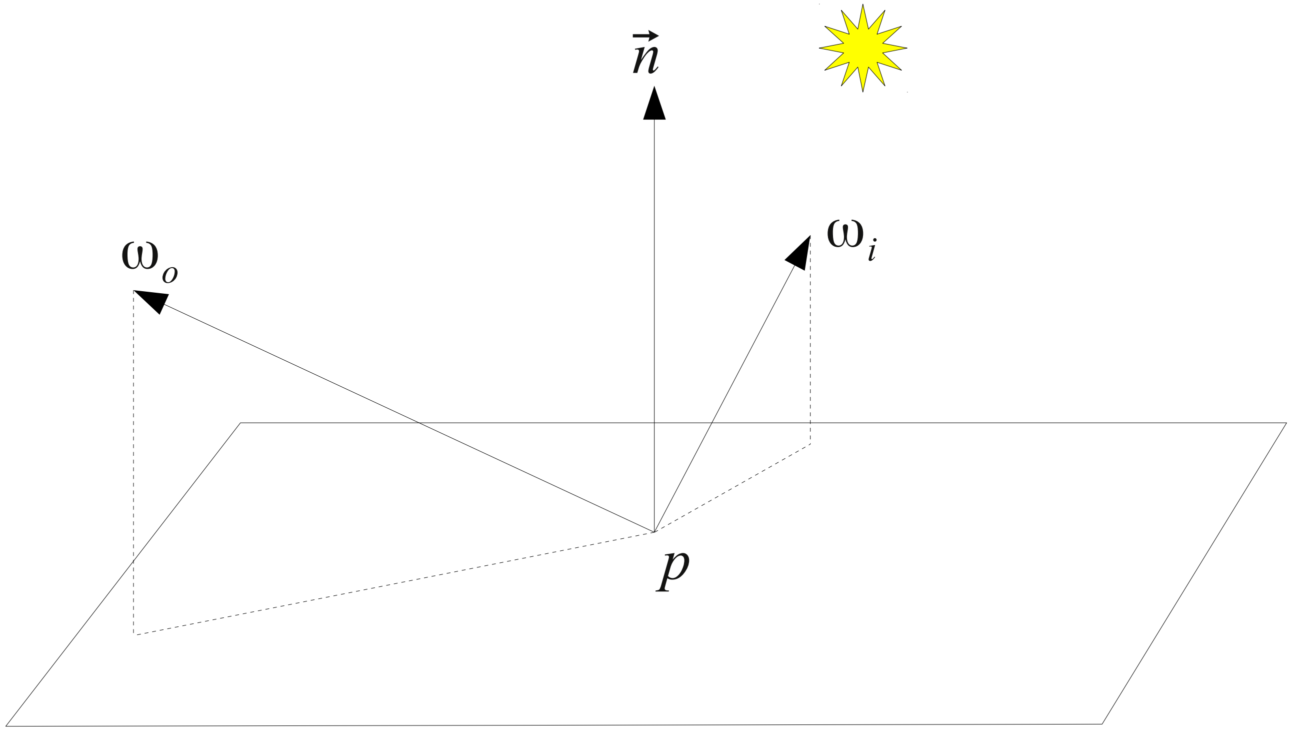
Differential Irradiance at p is $dE(p,\omega_i) = L_i(p,\omega_i)\cos\theta_i d\omega_i$

Reflected differential radiance is then given by $dL_o(p,\omega_o) \propto dE(p,\omega_i)$



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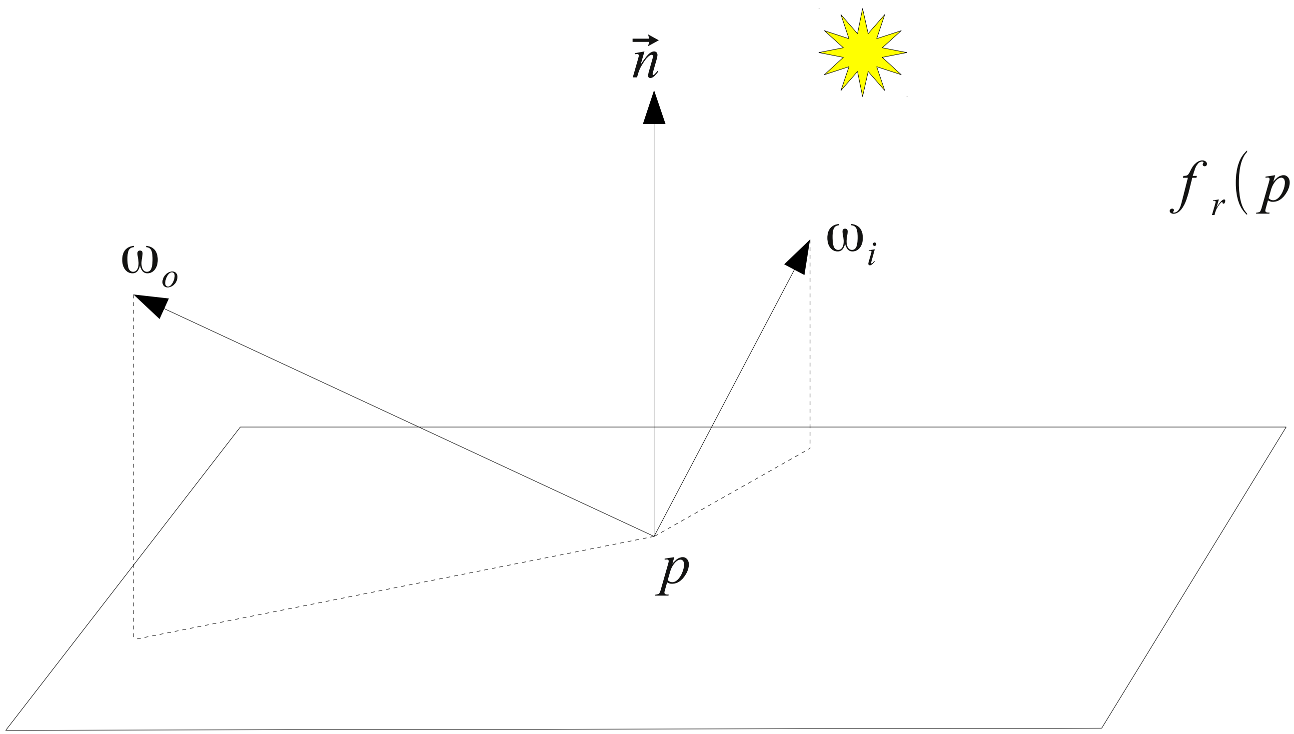


$$f_r(p,\omega_o,\omega_i) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)}$$

The BRDF

$$f_r(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)}$$

- Reciprocity

$$f_r(p, \omega_o, \omega_i) = f_r(p. \omega_i, \omega_o)$$

- Energy Conservation

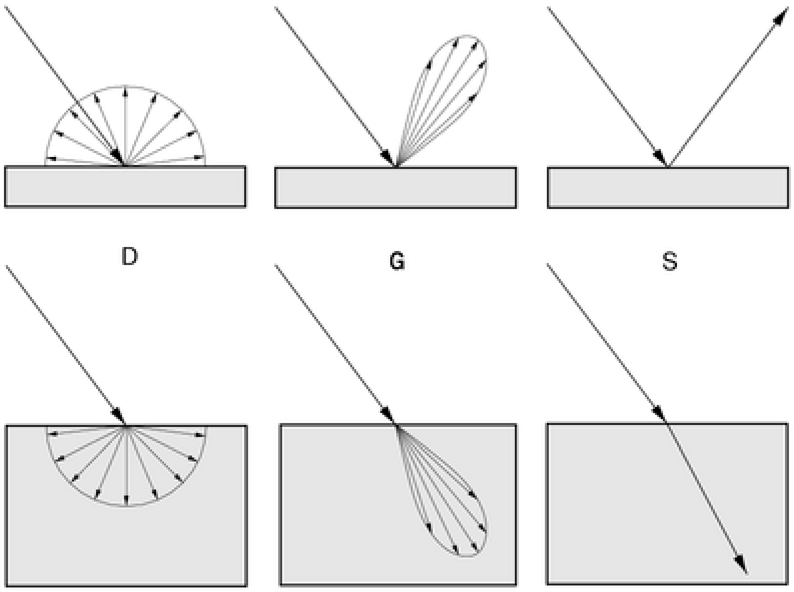
$$\int_{H^2(\vec{n})} f_r(p, \omega_o, \omega')\cos\theta' \, d\omega' \leq 1$$

- Consider a general $f()$ over the sphere of all directions and we get the BSDF (=BRDF+BTDF)

$$L_o(p, \omega_o) = \int_{S^2} f(p, \omega_o, \omega_i) L_i(p, \omega_i) |\cos\theta_i| \, d\omega_i$$

The BRDF

- Sources
  - Measured Data
  - Phenomenological Models
  - Simulation
  - Physical (Wave) Optics
  - Geometric optics
- Types of Surfaces
  - Diffuse
  - Glossy Specular
  - Perfect Specular
  - Retro-reflective



http://www.lamrug.org/resources/indirectips.html

The Ideal Diffuse BRDF

$$Let\ f_r(p, \omega_o, \omega_i) = k_d$$

- Assume BRDF reflects a fraction ρ of the incoming light

$$\int_{H^2(\vec{n})} f(p, \omega_o, \omega_i) \cos\theta_i \, d\omega_i = \rho$$

$$k_d \int_0^{2\pi} \int_0^{\pi/2} \cos\theta_i \sin\theta_i \, d\theta_i \, d\phi_i = \rho$$

$$2\pi k_d \int_0^{\pi/2} \cos\theta_i \sin\theta_i \, d\theta_i = \rho$$

$$k_d = \frac{\rho}{\pi}$$

- The quantity ρ is known as the albedo of the surface.

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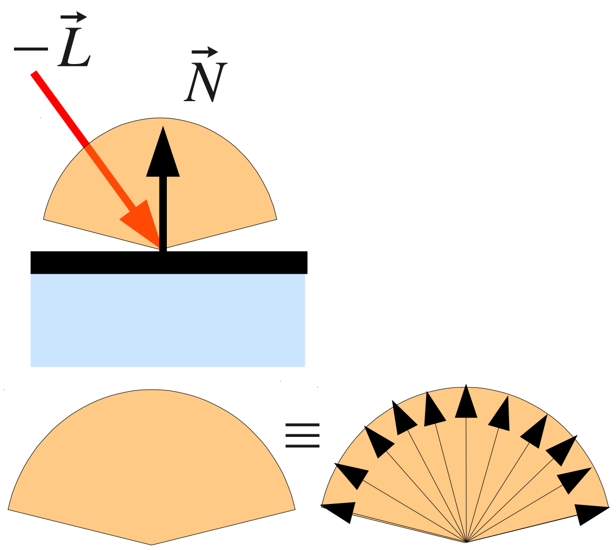
Remember this?



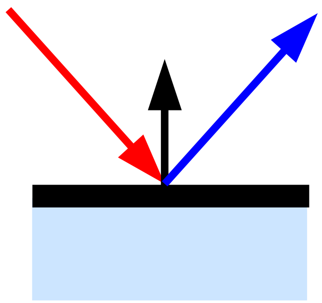
- The quantity ρ is known as the albedo of the surface.

The Ideal Mirror BRDF

- BRDF is zero everywhere except where

$$\theta_o = \theta_i$$

$$\phi_o = \phi_i + \pi$$



Assume $f_r(p, \omega_o, \omega_i) = F_r(\omega_i) \dfrac{\delta(\omega_i - R(\omega_o, \vec{n}))}{\cos\theta_i}$

and we know $\int f(x)\delta(x - x_o)dx = f(x_o)$

gives us $L_o(p, \omega_o) = \int_{H^2(\vec{n})} f_r(p, \omega_o, \omega_i) L_i(p, \omega_i) \cos\theta_i \, d\omega_i$

$$= F_r(R(\omega_o, \vec{n})) L_i(p, R(\omega_o, \vec{n})) = F_r(\omega_r) L_i(p, \omega_r)$$

Fresnel Reflectance

- For conducting specular surfaces, the amount of reflected light is given by

$$r_{\parallel} = \left| \frac{(\eta^2 + k^2)\cos\theta_i^2 - 2\eta\cos\theta_i + 1}{(\eta^2 + k^2)\cos\theta_i^2 + 2\eta\cos\theta_i + 1} \right|^2 \quad \text{for parallel polarized light}$$

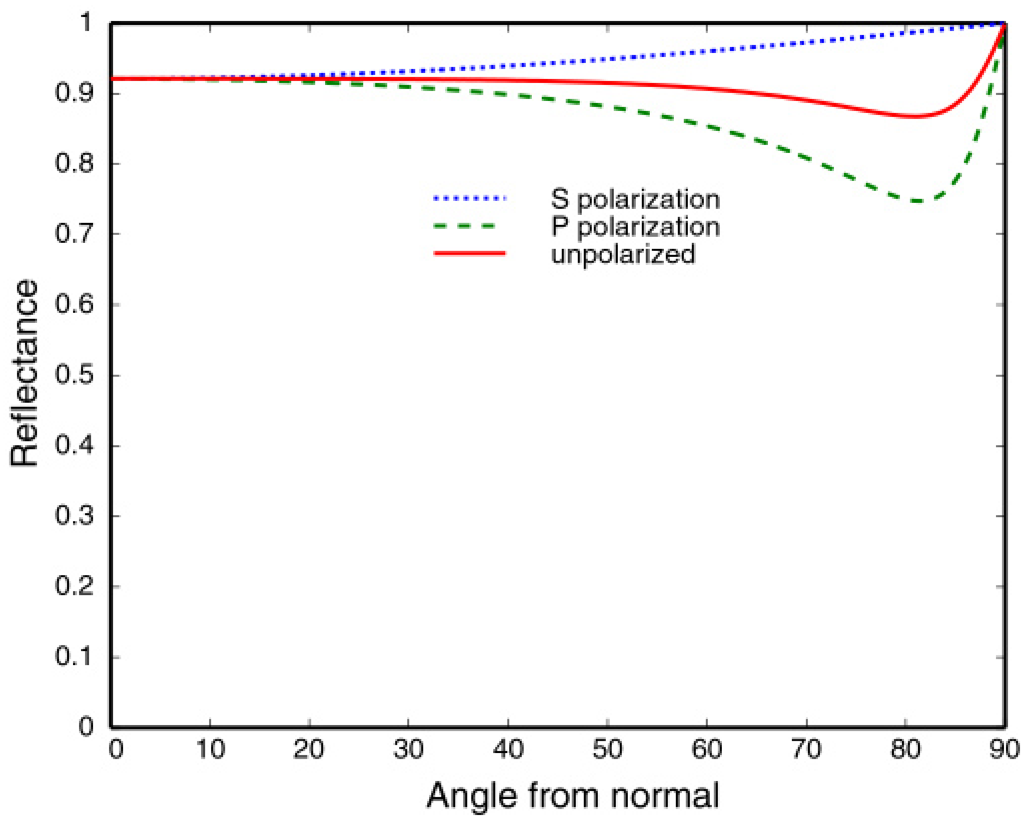
$$r_{\perp} = \left| \frac{(\eta^2 + k^2)\cos\theta_i^2 - 2\eta\cos\theta_i + \cos\theta_i^2}{(\eta^2 + k^2)\cos\theta_i^2 + 2\eta\cos\theta_i + \cos\theta_i^2} \right|^2 \quad \text{for perpendicular polarized light}$$

$$F_r = \frac{1}{2}(r_{\parallel} + r_{\perp}) \quad \text{for unpolarized light}$$

- Obtained from solution to Maxwell's equation for reflection off smooth surfaces.

Fresnel Reflectance

- Fresnel Reflectance for pure aluminum at 550 nanometers

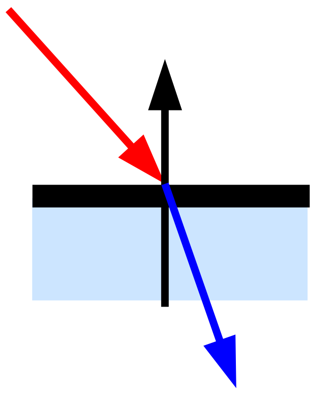


http://www.graphics.cornell.edu/~westin/misc/fresnel.html

Specular Transmittance

- BTDF is zero everywhere except where

$$\frac{\cos\theta_o \, d\theta_o}{\cos\theta_i \, d\theta_i} = \frac{\eta_i}{\eta_o}$$



$$f_t(p, \omega_o, \omega_i) = \frac{\eta_o^2}{\eta_i^2}(1 - F_r(\omega_i)) \frac{\delta(\omega_i - T(\omega_o, \vec{n}))}{|\cos\theta_i|}$$

Fresnel Reflectance

- For dielectric specular surfaces, the amount of reflected light is given by

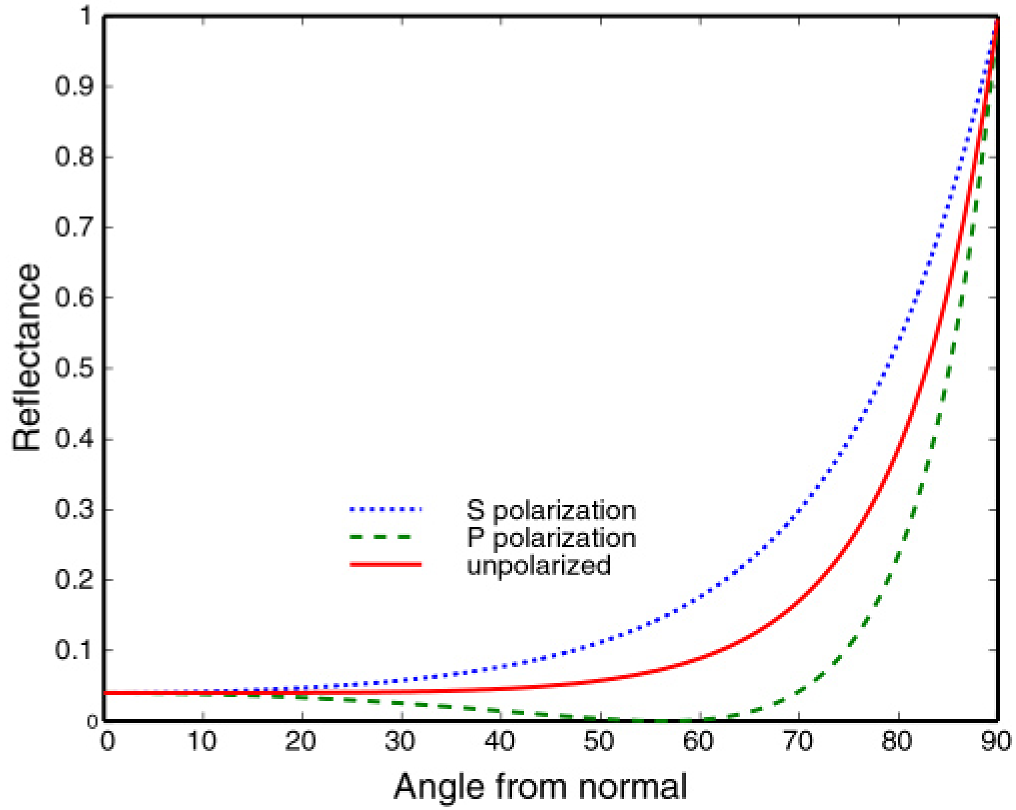
$$r_{\parallel} = \left| \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t} \right|^2$$ for parallel polarized light

$$r_{\perp} = \left| \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t} \right|^2$$ for perpendicular polarized light

$$F_r = \frac{1}{2}\left(r_{\parallel} + r_{\perp}\right)$$ for unpolarized light

Fresnel Reflectance

- For a typical dielectric $\eta = 1.5$



http://www.graphics.cornell.edu/~westin/misc/fresnel.html

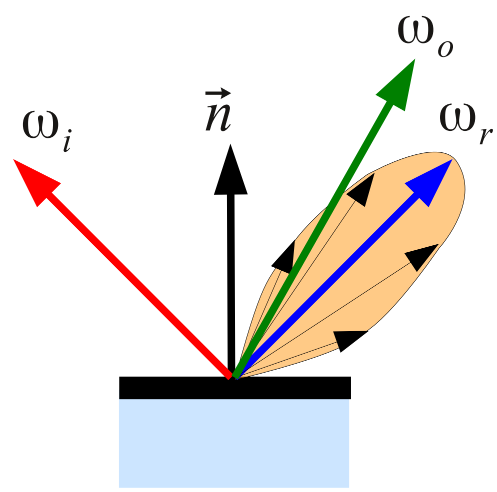
Glossy BRDF

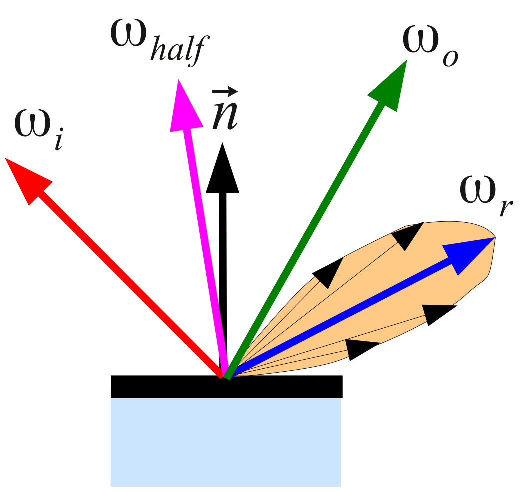
$$f_{Phong} = k_s(\omega_o . R(\omega_i, \hat{n}))^g$$

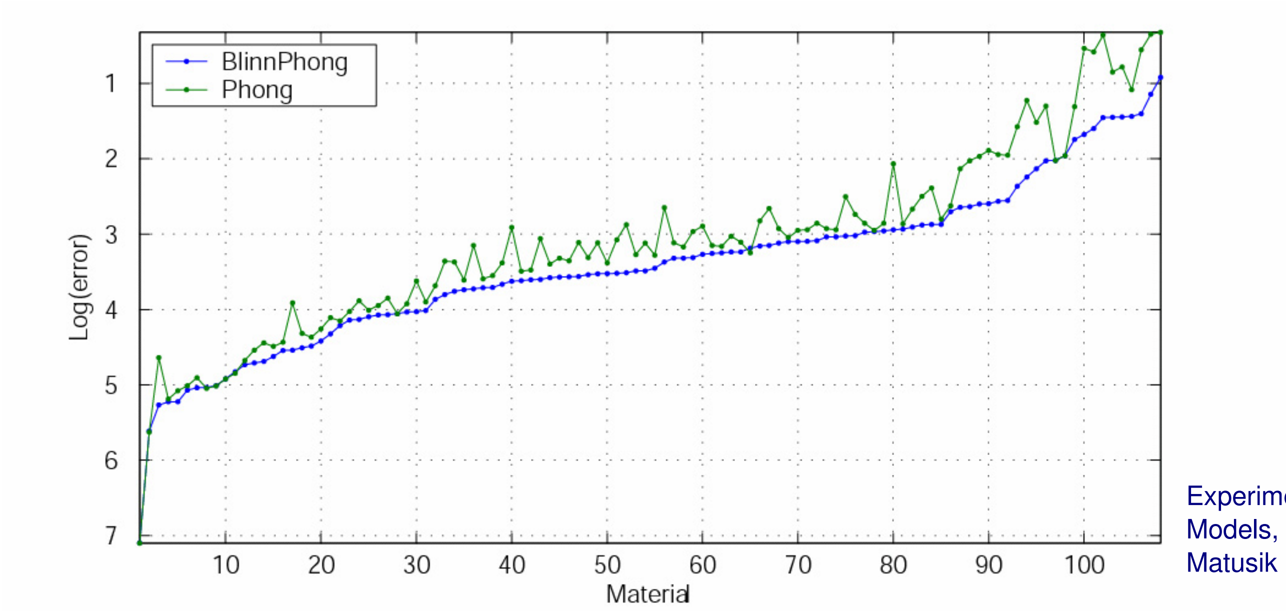
Phenomenological

$$f_{Blinn-Phong} = k_s'(\omega_{half} . \hat{n})^h$$

Also, phenomenological but gives less error

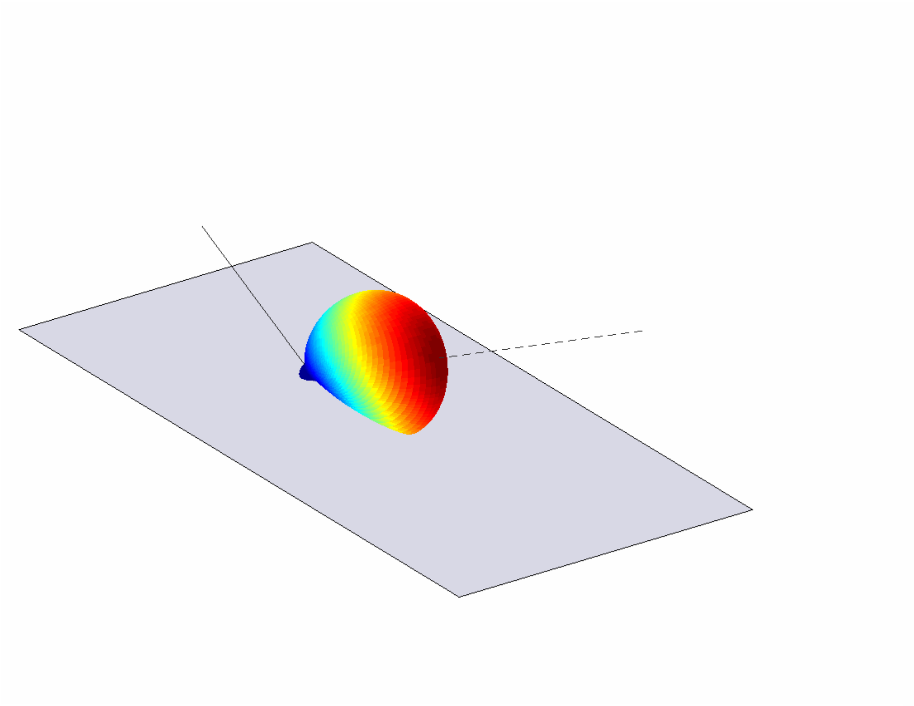




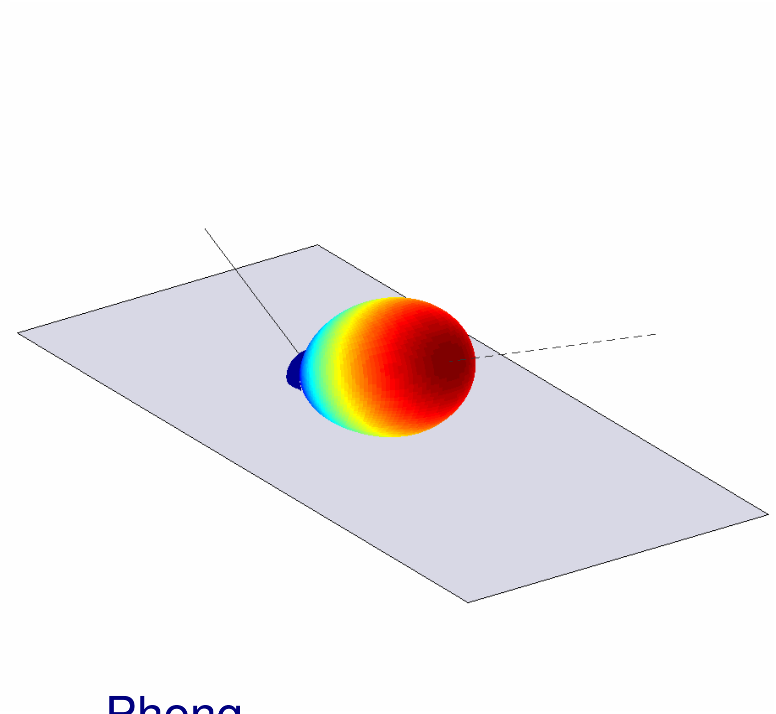


Experimental Validation of Analytical BRDF Models, Addy Ngan Fredo Durand, Wojciech Matusik , Technical Sketch, SIGGRAPH2004

Glossy BRDFs



Blinn-Phong



Phong

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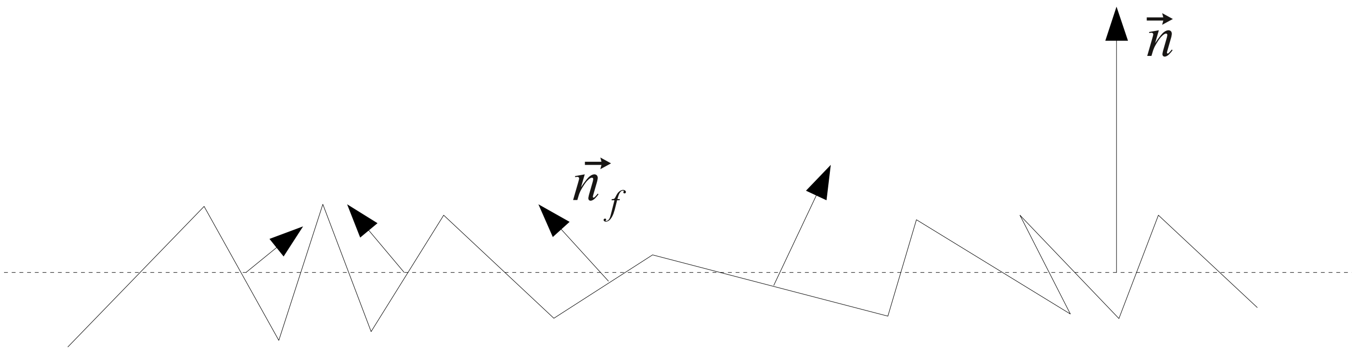
Microfacet BRDFs

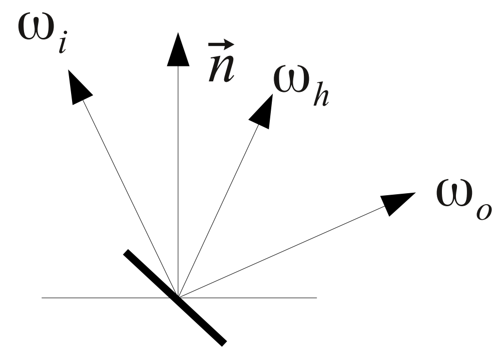


Described by a function giving the distribution of microfacet normals, $\vec{n}_f$ with respect to the surface normal $\vec{n}$. Greater variation indicates a rougher surface.

Also, necessary to describe the BRDF for the individual facets.

First described by Torrance-Sparrow (1967).

Microfacet BRDFs



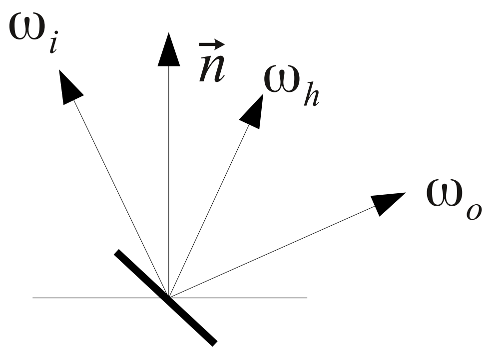
In Blinn-Phong, the distribution of microfacet normals is approximated by an exponential falloff [Blinn 1977].

$$D(\omega_h) \propto (\omega_h \cdot \hat{n})^h$$

$$D(\omega_h) = \frac{h+2}{2\pi}(\omega_h \cdot \hat{n})^h$$

The most likely orientation in this model is in the direction of the surface normal direction, falling-off to no microfacets oriented perpendicular to the normal. For smooth surfaces the fall-off is fast compared to a slow fall-off for rough surfaces.

Microfacet BRDFs



The complete Torrence-Sparrow BRDF is

$$f(p, \omega_o, \omega_i) = \frac{D(\omega_h)\, G(\omega_o, \omega_i)\, F_r(\omega_i)}{4 \cos\theta_o \cos\theta_i}$$

$D(\omega_h)$ gives microfacet distribution

$G(\omega_o, \omega_i)$ resolves visibility between a given pair of directions

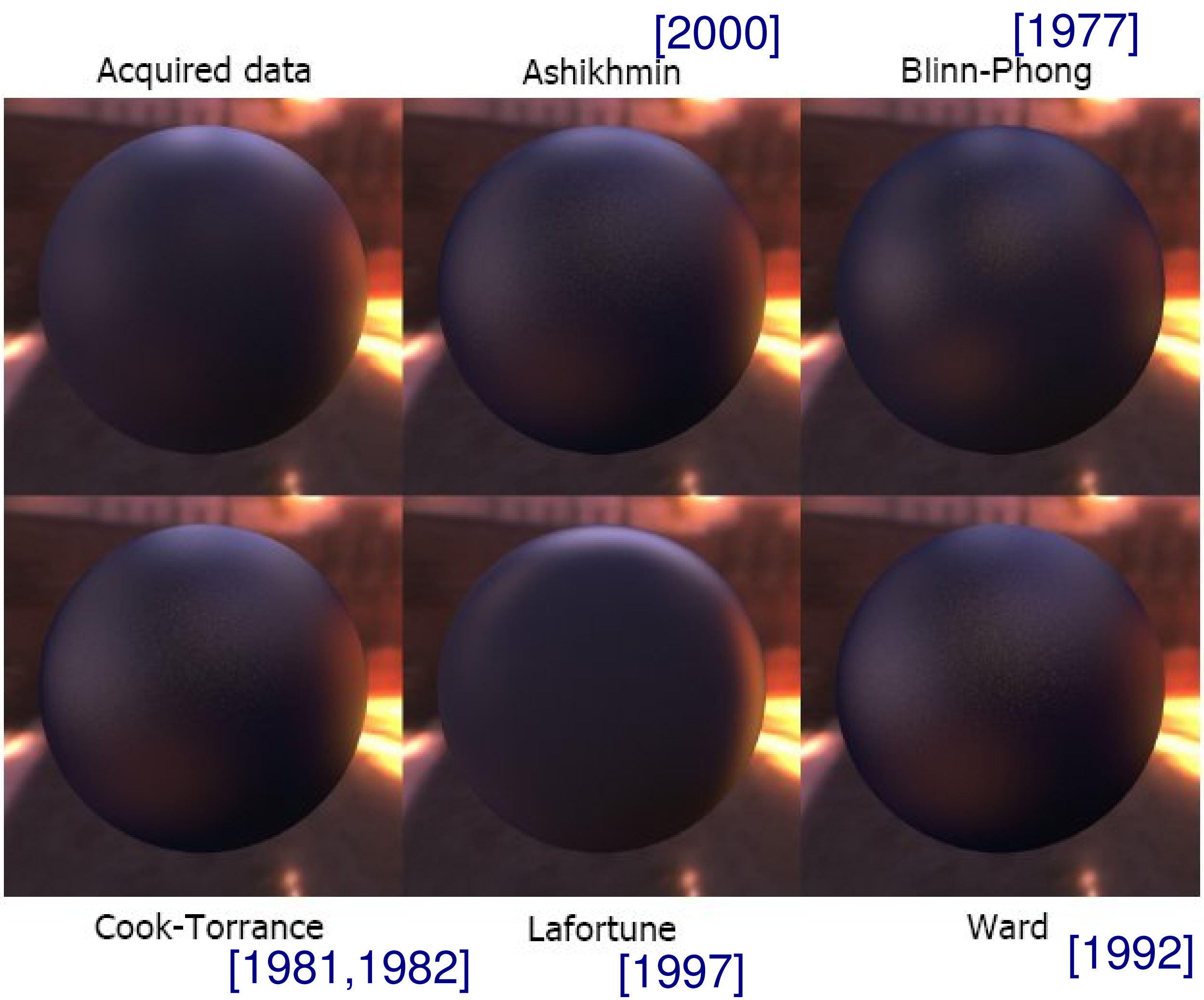
$F_r(\omega_i)$ is the Fresnel Term

The most likely orientation in this model is in the direction of the surface normal direction, falling off to no microfacets oriented perpendicular to the normal. For smooth surfaces the fall of is fast compared to a slow fall of for rough surfaces.

The model also assumes facets are along infinitely long V-shaped grooves.

Parag Chaudhuri

Microfacet BRDFs



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Microfacet BRDFs

Anisotropic BRDFs

http://www.evermotion.org/tutorials/show/7875/anisotropic-shader-tutorial-using-vray-1-5-final-sp-1

Microfacet BRDFs

http://en.wikipedia.org/wiki/Oren–Nayar_reflectance_model

- Oren-Nayar Diffuse BRDF [1994]
  - Symmetric V-shaped grooves
  - Gaussian Distribution of Microfacets
  - Each facet is perfectly Lambertian

http://www.larrygritz.com/arman/materials.html

Oren-Nayar

Ideal Diffuse

The Rendering Equation

- Now that we know the 4D-5D BRDFs, we can write the rendering equation as:

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{\Omega} f_r(p,\omega_o,\omega_i)\, L_i(p,\omega_i) \cos\theta_i \, d\omega_i$$

- Next question: How to solve the rendering equation for all points in the environment.