CHOICE OF CAPITAL INTENSITY FURTHER CONSIDERED*

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I

Discussions on the choice of capital intensity of investment as a problem of development planning that have taken place in recent years,[1] have nearly all been based (explicity or implicitly) on a set of simplifying assumptions about the economic world. This is perhaps as it should be, for if one introduced all the complications at once the picture might be too confused to be studied and even the most elementary aspects of the problem (e.g., the choice of time horizon involved in the question) might be lost sight of. This article is meant as a sequel to my last one in this journal,[2] as an attempt to carry the argument a little further by relaxing some of the limiting assumptions of my last model. Since these assumptions are shared

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1. To mention some of the longer articles, A. E. Kahn, "Investment Criteria in Development Programs," *this* Journal, LXV (Feb. 1951), 38; H. B. Chenery, "The Application of Investment Criteria," *this* Journal, LXVII (Feb. 1953), 76; Maurice Dobb, "A Note on the So-called Degree of Capital-Intensity of Investment in Underdeveloped Countries," *On Economic Theory and Socialism* (London, 1955); *idem*, "Second Thoughts on Capital-Intensity of Investment," *Review of Economic Studies*, XXIV (1956–57), 33; W. A. Lewis, *The Theory of Economic Growth* (London and Homewood, Ill., 1955); *idem*, "Unlimited Labour: Further Notes," *Manchester School*, XXVI (Jan. 1958); W. Galenson and H. Leibenstein, "Investment Criteria, Productivity and Economic Development," *this* Journal, LXIX (Aug. 1955), 343; Otto Eckstein, "Investment Criteria for Economic Development and the Theory of Intertemporal Welfare Economies," *this* Journal, LXXVI (Feb. 1957); Francis M. Bator, "On Capital Productivity, Input Allocation and Growth," *this* Journal LXXI (Feb. 1957); A. K. Sen, "Some Notes on the Choice of Capital Intensity in Development Planning," *this* Journal, LXXI (Nov. 1957), 561.

2. *Op. cit.*

by nearly all the writings on the subject, the exercise is perhaps worth doing. It appears that the nature of the problem changes considerably with the release of these assumptions.

We shall try to *eliminate*, one by one, the following assumptions and see how they affect our choice:

(1) Capital goods needed for the production of consumer goods can be made by labor unassisted by capital.

(2) There is only one type of consumer good.

(3) When capital and labor are increased in the same proportion, given the technique, output too goes up in that proportion, i.e., there are constant returns to "scale" (capital and labor).

(4) There is no progress of technological knowledge.

II

When we release the assumption that capital goods necessary to make consumer goods can be produced by unassisted labor, we are confronted with two problems. First, how should we choose the capital intensity of the capital goods sector? Our last model dealt only with the choice for the consumer goods industries. Secondly, how does this consideration affect the choice of capital intensity for the consumer goods sector? We shall examine the former question first. We start with a rather simple case. The economy has two sectors — "backward" and "advanced." The first is a backward rural economy with an enormous volume of surplus labor. The second is just being started and we are interested in the techniques of production of this sector. Thanks to the presence of surplus labor in the backward sector, the advanced sector can get labor without any social opportunity cost. In the advanced sector there are three departments — C, I, and J. Department C is producing corn (the consumer good), department I is producing machinery for department C and department J machinery for I. We assume temporarily that in department J, machinery for department I can be made by unassisted labor. The problem is to choose the capital intensity for department I. We shall refer to the amounts of labor employed in departments J, I and C as Lj, Li and Lc respectively. Just as a unit of Li makes a stock-contribution to the flow of output in department C, a unit of Lj increases the production flow per year of department I. For example, just as one tractor made in department I increases the flow of annual output of corn (and also the annual employment opportunity in the corn sector), one tractor-making machine made in department J increases the flow of annual output of tractors (and also the annual employment opportunity in the tractor sector). The problem is essentially similar. But unlike the case of department C, in department I the product (tractors) is different in nature from the real cost of employing labor (corn consumed) in that department. Thus we cannot, as in the case of department C,[3] easily derive a "rate of surplus."

This does not mean, however, that the problem cannot be solved. We can still find out which of the various techniques of production applicable to department I will give us the maximum total consumption within our time-horizon U.[4] For this we have to study the interrelationship between the various departments. Let us assume temporarily that the capital intensity of department C is already given and is not affected by our choice of technique in department I.[5]

We shall use the following notation:

Lj = labor employed in sector J,
Li = labor employed in sector I,
Lc = labor employed in sector C,
Pc = productivity in terms of corn per man-year in department C,
a = number of people employed in department I for one year (working with one type of machinery or another) to produce the machinery required to employ one man in department C,
b = number of people employed in department J for one year (working *without* machinery) to produce the machinery required, to employ one man in department I,
w = wage rate per man-year,
$\overline{S}$ = annual surplus of corn extracted from the "backward" sector which starts the process off,[6]
S = total investible surplus in the "advanced" sector consisting of surplus produced in the "advanced" sector *plus* $\overline{S}$,
$\overline{Y}$ = annual income of the "backward" sector (unaffected by the movement of labor out of it),
and Y = total consumer goods output for the economy ("advanced" and "backward") as a whole.

The numerical suffixes will refer to the relevant periods of time.

3. Even in the case of department C there are some problems when raw material costs and depreciation are introduced or the assumption of one commodity is released. We shall discuss this later in this paper.

4. See the present writer's article, *op. cit*, pp. 568–71 and pp. 582–84.

5. This is not a good assumption and we shall release it soon.

6. In Marx's language this is the volume of "primitive accumulation," *Capital*, Vol. I, Part VIII.

Production periods of one year each are assumed for both departments J and I. We assume that all the machinery lasts for ever.

Total employment possibility in department I in period t depends upon the number of laborers employed in department J until period $(t - 1)$. Hence we have

$$Li_t = \left(Lj_{t-1} + Lj_{t-2} + \ldots\ldots\ldots\ldots + Lj_1\right) . \frac{1}{b}$$

$$= \frac{1}{b} \sum_{x=1}^{t-1} Lj_x \tag{1}$$

Similarly we have

$$Lc_t = \left(Li_{t-1} + Li_{t-2} + \ldots\ldots\ldots\ldots\ldots + Li_2\right)\frac{1}{a} = \frac{1}{a} \sum_{x=2}^{t-1} Li_x$$

We ignore Li_1 as it is equal to zero.
This can be written as

$$Lc_t = \left[Lj_{t-2} + 2 . Lj_{t-3} + 3 . Lj_{t-4} + \ldots\ldots + (t-2)Lj_1\right]\frac{1}{a . b}$$

$$= \frac{1}{a . b} \sum_{x=1}^{t-2} x . Lj_{t-x-1} \tag{2}$$

The total corn output of the economy in period t is given by

$$Y_t = Lc_t . \quad Pc + \overline{Y} \tag{3}$$

The total corn surplus in period t is expressed as

$$S_t = Lc_t(Pc - w) + \overline{S} \tag{4}$$

This must equal the total corn demanded by the laborers employed in department I and J in period t. This is a crucial consistency-equation.

$$S_t = (Lj_t + Li_t) . w \tag{5}$$

The aggregate consumption over U periods is

$$A = \sum_{t=1}^{U} Y_t \tag{6}$$

To use the same criterion as was employed in my last article, we assume that we wish to maximize this aggregate consumption over the given time period U. Thus we must choose that b which maximizes A. Assuming continuity we require

$$\frac{\delta A}{\delta b} = 0, \text{ (also satisfying } \frac{\delta^2 A}{\delta b^2} < 0) \tag{7}$$

As we raise b, a falls, (i.e., the amount of machinery for department C produced per person in department I *rises*) given the type of machinery used in department C (i.e., given Pc). Thus there is a functional (inverse) relationship between b and a. This is really the production function for department I.

$$a = f\,(b) \tag{8}$$

We have in our system 5 U unknowns in the shape of

U . . . Y's
U . . . S's
U . . . Lj's,
$(U - 1)$ . . . Li's, ignoring Li_1 $(= 0)$,
$(U - 2)$ . . . Lc's, ignoring $Lc_1 = Lc_2$ $(=0)$,
1 . . . A,
1 . . . a,
and 1 . . . b.

There are 5 U equations as well, consisting of

$(U - 1)$ equations (1), regarding Li_t,
$(U - 2)$ equations (2), regarding Lc_t,
U equations (3), regarding Y_t,
U equations (4), regarding S_t on the *supply* side,
U equations (5), regarding S_t on the *demand* side,
1 equation (6), regarding A,
1 equation (7), a maximizing condition,
1 equation (8), regarding a.

Hence we can find out the technique b that maximizes aggregate consumption over period U.[7] The problem can thus be solved with these simultaneous equations.

If the production function is discontinuous we cannot, of course, use $\delta A/\delta b$ for maximization. In this case we can take alternative values of b and find out the corresponding values of aggregate consumption over U periods. We choose the one that gives the maximum value of A.

7. Equality of the number of equations and that of unknowns is not always a sufficient condition for determinacy, but there is no particular reason to believe that the equations in the above system will fail to be consistent and independent.

Now we may release the assumption that the capital intensity of department C is given irrespective of the choice of technique for department I. Actually the capital intensities of the two sectors have to be chosen simultaneously. We take alternative combinations of techniques for departments I and C, i.e., alternative combinations of a, b and Pc, and choose the combination that maximizes aggregate consumption A over the given time period U. Pc can be raised either by increasing a (given b) or by raising b (given a). When we know how Pc is related to a (given b) and to b (given a), i.e., when the production function is completely known, the problem is again fully soluble.

When we introduce the question of capital goods needed for sector J, the problem becomes somewhat more complicated. In principle, however, we can go on moving backwards (to departments J, K, L, ...) until we arrive at labor being applied unaccompanied by capital (though perhaps accompanied by "land"). The problem remains soluble all the way. This flight of fancy is not, however, as absurd as it sounds, since had there been no foreign trade, an underdeveloped economy without any capital goods would really have had to start from land and labor. In practice, of course, even an underdeveloped economy does have some capital goods, which we may consider equivalent to "land," since in the initial situation they are fixed and given to us. More important is the fact that an underdeveloped economy can *import* machinery from abroad to be in a position to use modern technology. In these cases the value of capital is given by its price in the international market, and even if loans are not available from abroad, capital goods can still be bought by exporting a part of the product of the underdeveloped country (corn, for example). In this case the cost of capital to the economy can be equated to the export-equivalent of imports. This particular question was discussed in my previous article on the choice of capital intensity.[8]

One definite result that emerges from what has been said above is that while the problem of choice of capital intensity for the consumer goods department and that for the capital goods department can be solved (given our value judgments), in terms of a model of general equilibrium, the two choices cannot really be separated. That the choice for the capital goods department cannot be studied without bringing in the consumer goods sector may appear to be obvious enough, but perhaps one ought to emphasize that the reverse is also true. The value of a capital good cannot be unambiguously expressed

8. "The Second Model," pp. 577–82.

in terms of its wage cost unless we assume that unassisted labor can produce it in a unit period of time. Once capital goods to make capital goods are introduced, the problem can be solved only in terms of a general equilibrium of the different stages of production in the manner of the above model. The compartmental approach to the problem which is so much in vogue, does not seem to be at all satisfactory.

III

The general equilibrium solution worked out above is somewhat frustrating since one does not know what set of capital intensities will be optimal until one has worked out an elaborate system of simultaneous equations. To appreciate some aspects of the problem it may be useful to simplify the picture by making some restrictive assumptions. In this section a simplified model is presented to discuss some problems of optimization involving the two departments.

We assume that there is no time lag in making machinery and that the object of the exercise is to maximize the rate of growth of consumption.[9] With the former assumption it is possible to have an immediate circular flow in the investment goods sector. The latter provides the policy assumption for optimization.

We assume that to employ one *additional* man in the consumer goods sector ("corn" sector), a men have to be employed in the investment goods sector to make enough machinery for him. The men in the investment sector themselves work with other machinery. In order to employ one *additional* man in the investment goods sector, b men, we assume, have to be employed in the same sector. The relationship is one of stock-and-flow; a and b are once-for-all employment for the flow of an additional unit employment in the two sectors respectively. Choice of techniques of production consists, in this case in choosing between alternative combinations of a and b.

Li, the number of people employed in the investment goods sector is equal to a times the rate of growth of employment in the corn sector plus b times the rate of growth of employment in the investment sector.

$$Li = \frac{dLc}{dt} \cdot a + \frac{dLi}{dt} \cdot b \tag{9}$$

But this employment in the investment goods sector must equal the

9. This corresponds to the policy assumption of Mr. Maurice Dobb, *op. cit.;* Galenson and Leibenstein, *op. cit.;* and of my first model, *op. cit.*, pp. 571–77.

surplus of corn produced in the corn sector divided by the wage rate.

$$Li = \frac{Lc(Pc - w)}{w}, \tag{10}$$

Pc being the productivity per laborer in the corn sector and w the real wage rate.

From (9) and (10) we have (assuming unchanging techniques over time):

$$\frac{\frac{dLc}{dt}}{Lc} = \frac{\frac{dLi}{dt}}{Li} = \frac{Pc - w}{b(Pc - w) + a \,.\, w} \tag{11}$$

This also expresses the proportional rate of growth of corn production and is to be maximized according to our policy assumption.

The choice of technique involves choosing between different combinations of a, b and Pc. Given b, a rise in a, will raise Pc; if it does not, the technique involving the higher a is just inferior and should not be taken into account. Similarly, given a, a rise in b will raise Pc. The system is completely soluble once the relevant production function is obtained. There are four unknowns in the model: a, b, Pc and g, the rate of growth $\left(\frac{dLc}{dt}\Big/ Lc\right)$. There are also four equations:

(I) $Pc = f\,(a, b)$, the production function,

(II) $g = \frac{Pc - w}{b(Pc - w) + a \,.\, w}$, the growth rate equation i.e., equation (11) above.

(III) $\frac{\partial g}{\partial a} = 0$,

and (IV) $\frac{\partial g}{\partial b} = 0$, the two maximizing conditions.

When a cannot be varied without b, the maximizing condition changes. Provided we know the relationship between a and b, however, the system is soluble.

One special case of the above model is when $b = 0$, i.e., when no machinery is needed for the investment goods sector. The growth rate equation (11), in this case becomes:

$$g = \frac{Pc - w}{a \,.\, w} \tag{12}$$

This corresponds to Maurice Dobb's $\frac{Pi\,(Pc - w)}{w}$, Galenson and Leibenstein's $\frac{p - e\,.\,w}{c}$, and is the same as my relationship (2) of the "First Model" (which had corresponding assumptions).[1]

Another special case is when $a = b$, i.e., when the capital intensities of the two sectors must be equal. The growth rate, in this case, is:

$$g = \frac{Pc - w}{Pc\,.\,a} \tag{13}$$

The growth rate is maximized when $\frac{dg}{da} = 0$,

i.e., when $\frac{f'.f\,.\,a - (f - w)\,(f'.\,a + f)}{a^2\,.\,f^2} = 0$,

$$\text{i.e., when } f' = \frac{f\,(f - w)}{a\,.\,w}, \tag{14}$$

where f stands for Pc and f' for $\frac{dPc}{da}$.

Geometrically the optimum choice can be represented in the following manner. In Figure I the horizontal axis represents different

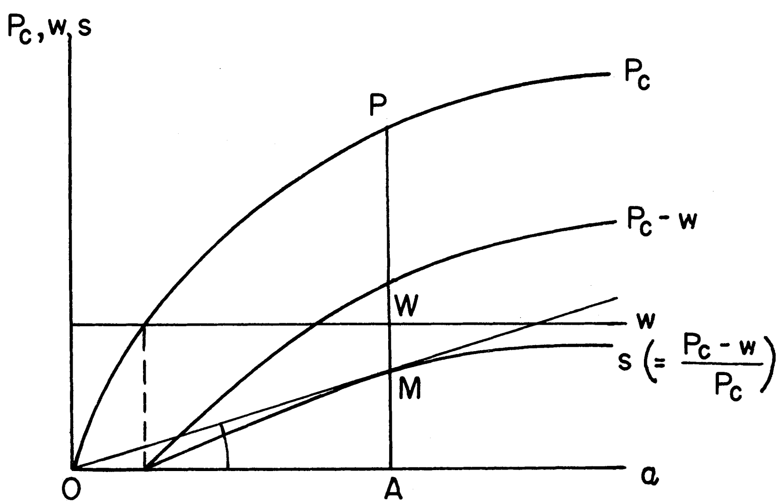

FIGURE I

1. See M. H. Dobb, *op. cit.*, pp. 35–37; Galenson and Leibenstein, *op. cit.*, p. 357; and the writer's previous article, *op. cit.*, p. 573.

values of a, and the vertical axis measures Pc and w in units of corn. As a is raised Pc also rises; otherwise raising a will be pointless. The wage rate w is given. From the lines Pc and w, we derive the curve $(Pc - w)$, which gives the absolute surplus of corn corresponding to each a. The surplus rate curve s is derived from the curves Pc and $(Pc - w)$ and represents rates of surplus per unit of corn output, i.e., $\frac{Pc - w}{Pc}$. For this the units of the y-axis are different as s is measured not in units of corn, but in terms of pure numerical ratios. Obviously s continues to rise indefinitely as long as Pc is rising. The ratio of s to a, however, reaches a peak at M and then declines. And this ratio is the rate of growth of consumption, as will be obvious from equation (13). To choose the maximum rate of growth, therefore, we choose capital intensity OA, productivity PA and the rate of growth represented by the angle MOA. It can be shown that the higher the real wage rate, the higher should be the degree of capital intensity chosen, provided (i) the production function f is continuous and (ii) a higher wage rate, given other things, does not raise productivity correspondingly.[2]

We should not attach too much importance to this rather neat picture. It has been obtained with many simplifying assumptions and the choice in the real world will look more like the one described in the last section than like this piece of convenient geometry. In fact the real choice will be a good deal *more* complicated than the one shown in the last section because of other problems, e.g., multiplicity of consumer goods, diminishing returns, technological progress. These difficulties are briefly discussed in the following section, not with the object of working out all their effects on the choice, but to indicate (i) the direction of their effects, and (ii) the complexity of the problem of choice of technology, which (for reasons that are not difficult to see) nearly every writer on the subject prefers to avoid.

IV

We may now examine the problems that arise when the assumption of one commodity is dropped and the choice of capital intensity for individual commodities is studied. The usual practice of extending the results of a single-commodity model to a many-commodity world would have been justified if we could have treated the problem

2. See M. H. Dobb, *op. cit.*, pp. 37–38, for the problem introduced by the connection between real wages and efficiency.

of relative prices as only an "index number problem." It is, however, more fundamental than that.

The difficulty is that while a worker produces only one or a few goods, he spends his income on a number of goods. Thus we cannot get an unambiguous measure of the "surplus" created in that industry without knowing the relative prices.[3] The relative prices, on the other hand, depend on the choice of capital intensity in the production of different goods and hence the problem can be solved only in terms of a set of simultaneous equations. The problem may become clearer if we first work out a solution for the economy as a whole and then try to see whether the partial approach can be used to get the same results. To simplify our exercise we shall take only two commodities and shall assume that machinery to produce consumer goods can be made by labor unaccompanied by any capital good, i.e., department J is redundant. We shall also assume that the workers consume the two commodities in fixed proportions.[4]

Let there be two commodities X and Y. Each wage earner consumes $\bar{x}$ units of X and $\bar{y}$ units of Y. We assume that Xo amount of commodity X and Yo amount of commodity Y are available in the initial year (once for all) from outside the "advanced" sector, i.e., from the "backward" sector (or from a foreign country). This provides us with a fund to initiate the "advanced" sector. It is also assumed that $\dfrac{Xo}{Yo} = \dfrac{\bar{x}}{\bar{y}}$, so that the whole of the surplus of both can be used, as the wage earners consume them in that proportion.

The number of people employed in year O in the investment department is:

$$Li_o = \frac{Xo}{\bar{x}} = \frac{Yo}{\bar{y}}$$

They can be employed in making machinery for commodity X or for commodity Y. The first thing we have to find out is how to distribute the total number of people between the two pursuits.

The notation of the original model is used with the difference that x or y is added after the symbols to represent which of the two

3. A similar problem is introduced by raw materials, replacements and repairs. Any *individual* consumer-good industry has something in common with the capital-goods industry, viz., its "inputs" are not (all) produced by it.

4. This is a very restrictive assumption and we get a simple result below because of this assumption. The effects of relaxing this assumption are discussed later.

sectors they refer to. For example *Lix* refers to labor employed in the investment sector to produce machinery for the production of X and *Liy* the same for Y, and *Pcx* refers to labor-productivity in X-production and *Pcy* that in Y-production.

Obviously,

$$Li_o = Lix_o + Liy_o\,.$$

Assuming one-year gestation period for investment in both fields, we have:

$$Lcx_1 = Lix_o \,.\, \frac{1}{ax}$$

and

$$Lcy_1 = Liy_o \,.\, \frac{1}{ay}$$

If we represent the output of X and Y as Px and Py and the total consumption of X and Y by people employed in the consumer goods industries as Cx and Cy, we have:

$$Px_1 = Lcx_1 \,.\, Pcx = Lix_o \,.\, \frac{1}{ax} \,.\, Pcx,$$

$$Py_1 = Lcy_1 \,.\, Pcy = Liy_o \,.\, \frac{1}{ay} \,.\, Pcy,$$

$$Cx_1 = (Lcx_1 + Lcy_1)\,\bar{x}\,, \text{ and}$$

$$Cy_1 = (Lcx_1 + Lcy_1)\,\bar{y}$$

Thus the surplus of X and Y produced in year 1 is given by

$$Sx_1 = Px_1 - Cx_1 = Lix_o \,.\, \frac{1}{ax} \,.\, Pcx - \left(Lix_o \,.\, \frac{1}{ax} + Liy_o \,.\, \frac{1}{ay}\right)\bar{x}$$

$$Sy_1 = Py_1 - Cy_1 = Liy_o \,.\, \frac{1}{ay} \,.\, Pcy - \left(Lix_o \,.\, \frac{1}{ax} + Liy_o \,.\, \frac{1}{ay}\right)\bar{y}$$

Now, in order to be fully utilized the surpluses of X and Y must be in the proportion $\bar{x}/\bar{y}$ since that is the ratio in which the workers consume them. Thus we require that

$$\frac{Sx_1}{Sy_1} = \frac{\bar{x}}{\bar{y}},$$

from which we have

$$\frac{Lix_o}{Liy_o} = \frac{Pcy \, . \, \bar{x} \, . \, ax}{Pcx \, . \, \bar{y} \, . \, ay} \tag{15}$$

This is the proportion in which we divide employment in department I. We have from (15),

$$Lix_o = \frac{Pcy \, . \, \bar{x} \, . \, ax}{Pcy \, . \, \bar{x} \, . \, ax + Pcx \, . \, \bar{y} \, . \, ay} \, . \, Li_o$$

$$Liy_o = \frac{Pcx \, . \, \bar{y} \, . \, ay}{Pcy \, . \, \bar{x} \, . \, ax + Pcx \, . \, \bar{y} \, . \, ay} \, . \, Li_o$$

Now we can express the output, the wages bill and the surplus equations in the following way:

$$\begin{aligned} Px_1 &= Lcx_1 \, . \, Pcx \\ &= \frac{Pcx \, . \, Pcy \, . \, x}{Pcy \, . \, \bar{x} \, . \, ax + Pcx \, . \, \bar{y} \, . \, ay} \, . \, Li_o \end{aligned} \tag{16}$$

$$\begin{aligned} Py_1 &= Lcy_1 \, . \, Pcy \\ &= \frac{Pcx \, . \, Pcy \, . \, \bar{y}}{Pcy \, . \, \bar{x} \, . \, ax + Pcx \, . \, \bar{y} \, . \, ay} \, . \, Li_o \end{aligned} \tag{17}$$

Therefore

$$\begin{aligned} Sx_1 &= Px_1 - (Lcx_1 + Lcy_1) \, . \, \bar{x} \\ &= Li_o \, . \, \frac{Pcx \, . \, Pcy \, . \, \bar{x}}{Pcy \, . \, \bar{x} \, . \, ax + Pcx \, . \, \bar{y} \, . \, ay} \left(1 - \frac{\bar{x}}{Pcx} - \frac{\bar{y}}{Pcy}\right) \\ &= Li_o \, . \, A \, . \, B \, . \, \bar{x} \end{aligned} \tag{18}$$

putting A for

$$\frac{Pcx \, . \, Pcy}{Pcy \, . \, \bar{x} \, . \, ax + Pcx \, . \, \bar{y} \, . \, ay}$$

and B for

$$\left(1 - \frac{\bar{x}}{Pcx} - \frac{\bar{y}}{Pcy}\right)$$

The number of people we can employ in the capital goods department in year 1 depends upon the surplus created in sector A.

$$\begin{aligned} Li_1 &= \frac{Sx_1}{\bar{x}} \\ &= Li_o \, . \, A \, . \, B \, . \end{aligned}$$

This is divided between the production of machines to make X and machines to make Y in the required proportion (15). Proceeding this way we find that

$$Px_2 = Xo \,.\, A(1 + A \,.\, B)$$
$$Px_3 = Xo \,.\, A(1 + A \,.\, B)^2$$
$$Px_n = Xo \,.\, A(1 + A \,.\, B)^{n-1} \tag{19}$$

Similarly

$$Py_n = Yo \,.\, A(1 + A \,.\, B)^{n-1} \tag{20}$$

Thus given the capital intensities (ax and ay), the time series of physical units of output of X and Y can be found out. We may take different values of ax and ay and find out the combination that gives us the time series we like best, according to our values. We may even use our "period of recovery" concept as applied to the output of X and of Y in choosing between two techniques.[5]

If we wish to apply the Growth Rate Criterion,[6] we should try to maximize AB, i.e., maximize

$$G = A \,.\, B = \frac{Pcx \,.\, Pcy}{Pcy \,.\, \bar{x} \,.\, ax + Pcx \,.\, \bar{y} \,.\, ay}\left(1 - \frac{\bar{x}}{Pcx} - \frac{\bar{y}}{Pcy}\right)$$
$$= \frac{Pcx \,.\, Pcy - Pcy \,.\, \bar{x} - Pcx \,.\, \bar{y}}{Pcy \,.\, \bar{x} \,.\, ax + Pcx \,.\, \bar{y} \,.\, ay} \tag{21}$$

We choose that combination of ax and ay that maximizes G.

It is clear from all this that one cannot choose the technique for one commodity irrespective of that for the others. The capital intensity ax that will give us the maximum rate of growth (or the "most desired" time series) depends on the values of ay and Pcy. And since the choice of ay similarly depends upon ax, the partial equilibrium approach to this question must necessarily be imperfect. And even if ay and Pcy remain unchanged, the usual type of approach in terms of given relative prices of X vis-à-vis Y would not work as the price-structure which would give us maximum growth would vary with the ratios ax/ay and Pcx/Pcy. Actually equation (21) can be rewritten as

$$G = \frac{Pcx - \bar{x} - \bar{y} \,.\, \dfrac{Pcx}{Pcy}}{ax\left(\bar{x} + \bar{y} \,.\, \dfrac{ay}{ax} \,.\, \dfrac{Pcx}{Pcy}\right)}$$
$$= \frac{Pcx - \bar{x} - \bar{y} \,.\, m}{ax\,(\bar{x} + \bar{y} \,.\, m \,.\, n)}, \text{ putting}$$
$$m = \frac{Pcx}{Pcy}, \quad n = \frac{ay}{ax}$$

5. Sen, *op. cit.*, pp. 568–71.
6. Dobb, *op. cit.;* Galenson — Leibenstein, *op. cit.*

Thus, only if a unit of Y is valued at m units of X in department C and at (mn) units in department I, would the maximum profit position in the production of X lead to the maximization of the rate of growth. But $m\left(=\frac{Pcx}{Pcy}\right)$ and $mn\left(=\frac{Pcx}{Pcy}\cdot\frac{ay}{ax}\right)$ change with ax and Pcx, given ay and Pcy. Thus if we wish to use the usual partial equilibrium approach we must assume variable price-ratios.

One way out may be to assume that when Pcx is changed so is Pcy, so that $\frac{Pcx}{Pcy}$ remains unchanged. If the commodities have *similar* production functions so that $\frac{ay}{ax}$ remains unchanged so long as $\frac{Pcx}{Pcy}$ is the same, this assumption is not illegitimate. But the assumption is so restrictive that it is not much better than the assumption of one commodity.

In criticizing this model it may be pointed out that no possibility of consumers' substitution between X and Y is allowed for.[7] The ratio $\bar{x}/\bar{y}$ may change with their price ratio, and if X and Y happen to be perfect substitutes for one another even a very small change in the price-ratio will bring about a complete substitution of one for the other. In this case the price-ratio must be given (one, for corresponding units), and we can ignore other commodities in discussing the production of one, since by an infinitesimal lowering of prices we can induce everybody to buy that commodity only and nothing else. This assumption is actually a way of getting out of the multi-commodity world and is not very helpful. The reason why we are discussing these rather strange assumptions is that something of this kind must be assumed in order to apply the results obtained in a single-commodity world to the case of one commodity in a world of many commodities.[8] For a satisfactory solution of the problem of choice of capital intensity in one sector we must bring the rest of the

7. In fact we have assumed that the consumption of each commodity by a worker is fixed in real terms irrespective of the relative prices.

8. In discussing, for example, the choice of techniques for the cotton textile industry, the common practice is to take the value of output (cloth) and to subtract from it (at current prices) the value of raw cotton, other materials and depreciation to get the "value added." Further subtraction (at current prices) of the consumption of the cotton textile workers from this gives the volume of "surplus." One must not forget that this practice involves many *implicit* assumptions about relative prices. Examples of the partial approach are seen in Galenson and Leibenstein, *op. cit.*, pp. 358–59, in Chenery, *op. cit.*, pp. 83–85, and in the controversy on Indian cotton textile techniques in the *Economic Weekly*, April to December, 1956.

economy into the picture. This is, of course, not to deny that in certain cases the partial equilibrium approach may bring us fairly close to the desired result (depending on the relative shapes of the production functions and their substitutability), and, as such, may be of great practical value. It does not, however, give us the precise result we want. The partial approach is indeed too partial to be acceptable.

V

The complications introduced in the two preceding sections are enormous and it is doubtful whether one can in actual situations have a complete solution of the problem. The removal of the assumptions of constant returns to "scale" (capital and labor) and of absence of growth of technological knowledge will make the problem even more complicated.

If a nonreproducible factor of production ("land") is introduced, the ratio of capital to labor, i.e., the capital intensity, is no longer the only determinant of productivity. As the stock of capital grows, the flow of output may not increase at the same rate even if the technique of production is the same. The introduction of the problem of diminishing returns (to capital and labor) thus destroys the convenient independence of the problem of capital intensity from the scale of production. Depending on the relative rates of diminishing returns, technique *A* can be chosen for one scale of production and technique *B* for another. We may even choose a combination of techniques rather than employ one technique to produce the entire output. Again, due to the interdependence of production in the different sectors shown in the last section, diminishing returns in one field will lead to that in the others. Thus even if a particular industry is enjoying constant returns to scale in physical terms, there may be *de facto* diminishing returns arising from the rising relative price of commodities produced under conditions of diminishing returns and consumed by the workers in this industry. Similarly diminishing returns in the production of raw materials and other intermediate products may affect the rates of surplus. Thus the diminishing returns in agriculture may have a significant effect on the choice of techniques for industries via relative prices of food and raw materials.

Once this complication is introduced we have to dispense with an assumption which is used by almost all economists writing on this question, viz., a technique is chosen once for all. In order to derive the alternative sets of time series of output open to the economy we have to look at alternative time-combinations of techniques. Thus the number of alternative cases that we have to examine for a complete solution of the problem becomes very much larger.

Again, once the possibility of variation of technique over time is introduced, we have to examine the problem of "terminal capital" at the end of our arbitrarily defined period U. We may try to maximize our consumption only over this period, but surely we cannot forget the future beyond that point completely. Or, in other words, we should not choose techniques with lower capital intensity giving higher output but leading to little, or no, or negative, growth as the margin is approached so that we end up with zero terminal capital. This problem did not appear explicitly earlier because we had assumed that a technique was chosen for good, i.e., till the end of the time horizon. In deriving the alternative sets of time series, therefore, we may see that all end up with the same quantity of "terminal capital" so that their contributions to the future are equal. The choice of the precise *amount* of terminal capital is, however, a problem that depends very much on our value judgments.[9] Even the *measurement* of the amount of terminal capital may not be simple.

When we bring technological progress into the picture the argument for not treating the choice of technology as an once-for-all choice becomes even stronger. The number of alternatives open to us are now increasing over time and we have to decide whether or not we should change over from the technique adopted earlier. If there are m techniques existing this year, n in the next, o the year after and p in the following year, the number of possible combinations over the four years is ($mnop$) and this does not consider the possibility of utilizing more than one technique at a time. When we take into account the number of alternative time possibilities open to us in the production of any one commodity and also remember that we cannot treat a choice of this sort partially, i.e., we have to study the choice for all the products simultaneously, we can realize the complexity of the task. Theoretically it is still possible to work out all the alternative sets of time series of the production of the various commodities corresponding to all the relevant time combinations of techniques for all commodities,[1] but it is, certainly, not a very practical approach.[2]

9. See J. De V. Graaff, *Theoretical Welfare Economics*, Chap. VI (Cambridge University Press, 1957).

1. If, however, our choice of techniques today itself affects the progress of technological knowledge in the future, the task will be a good deal more difficult.

2. It may be argued that for quite a few years in the initial stages of her growth an underdeveloped economy will have to use the techniques already developed in the West and new techniques of production may not be very useful

It will appear from what has been said in this article that an economist can hardly put forward a criterion or even a set of criteria that will, on the one hand, conform to the usual set of value judgments (however combined into a social welfare function) and, on the other, be applicable in practice.[3] We may, of course, apply simple criteria such as that of choosing the minimum "capital coefficient" or the maximum "rate of surplus" with given relative prices. These will not be fair to our value judgments in these fields which are usually a good deal more sophisticated. Alternatively, we may bear in mind the sort of criteria we have been discussing above and all the complications involved therein, trying to evolve some kind of short-hand practical method to get as close as possible to the desired result. This is really a matter of good practical judgment and economists are not the best people to advise on how to achieve this.

VI

Summary and Conclusions

The main points of this paper can be summarized as follows.

(1) Given the policy objectives, the optimum techniques for the consumer goods sector and for the investment goods sector can be worked out in terms of a system of general equilibrium. Neither choice can, however, be divorced from that of the other. Two models have been used in the paper to indicate the nature of the interdependence involved.

(2) Techniques for the production of individual commodities can be chosen with a general equilibrium system. The choice for any particular commodity cannot, however, be divorced from that of the others. The usual method of choice of techniques for an individual commodity with the assumption of constant relative prices is shown to be based on some very dubious assumptions.

for her as they are very capital intensive. This might make the progress of technological knowledge over time not very important for the underdeveloped countries. This argument is not entirely correct, for often the new techniques do not involve much higher capital-output ratios and often in spite of their higher capital coefficient they are of great significance to the underdeveloped countries. Further, when the time horizon to be taken into account is long, an underdeveloped country may "age" within our period of calculation.

3. If the complications already introduced are not enough to make us sufficiently sceptical of the possibility of applying the usual economic criteria to actual policymaking, we may point out that we have not taken quite a few others into account, e.g., the external economies of production and consumption, the possibility of variations in the *quality* of the product, and above all the non-economic (political, cultural, etc.) effects of the choice of techniques of production in a particular society.

(3) No technique of production may be distinctly superior to others, even when definite policy assumptions are made. The optimum choice is quite likely to be a combination of different techniques of production to produce the same good.

(4) The usual assumption of an *once-for-all* choice of techniques ignores some essential aspects of the problem.

(5) The approaches that are intellectually satisfactory, given the common value judgments, may be impossible to apply fully in practice. The planners will have to evolve short-hand practical methods of getting as close as possible to the desired results. This is yet another field where economic theory is no substitute for practical experience and skilled judgment.

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