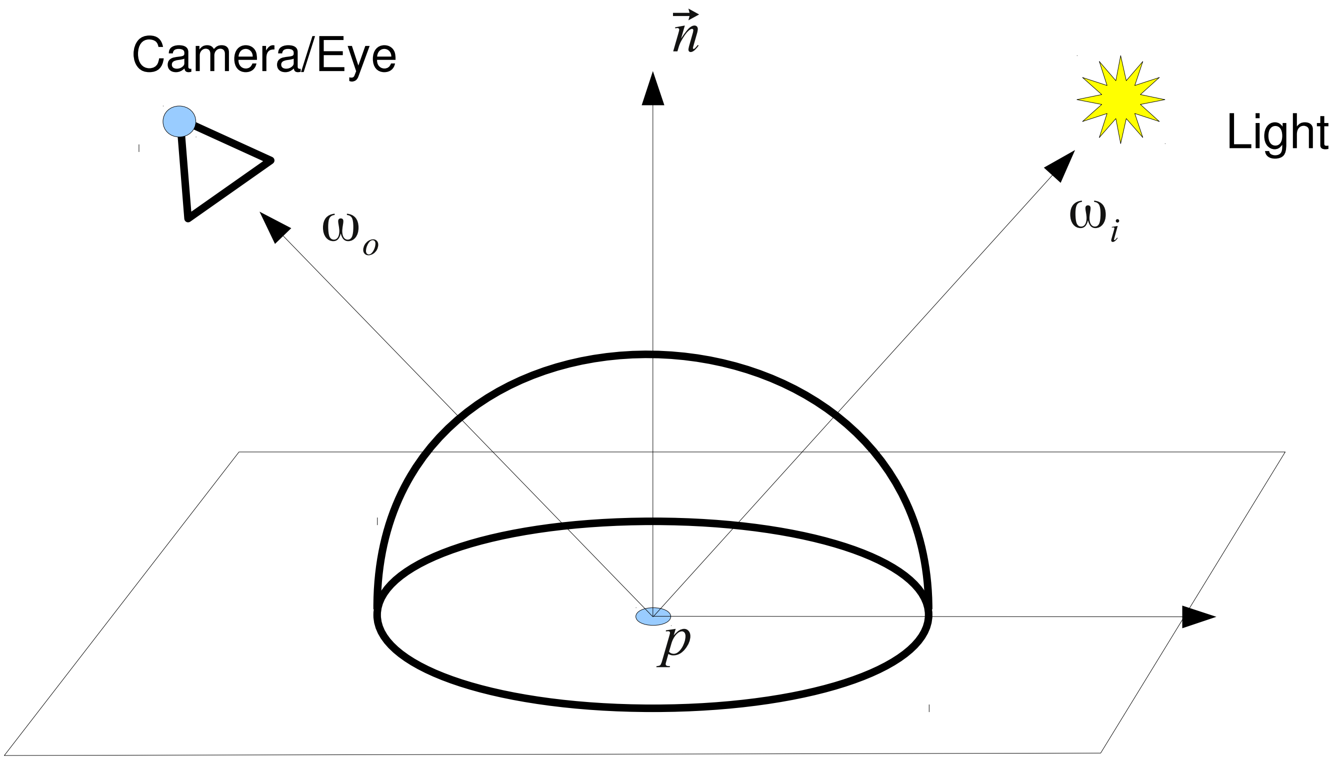
CS 775: Advanced Computer Graphics

Lecture 3 : Radiosity

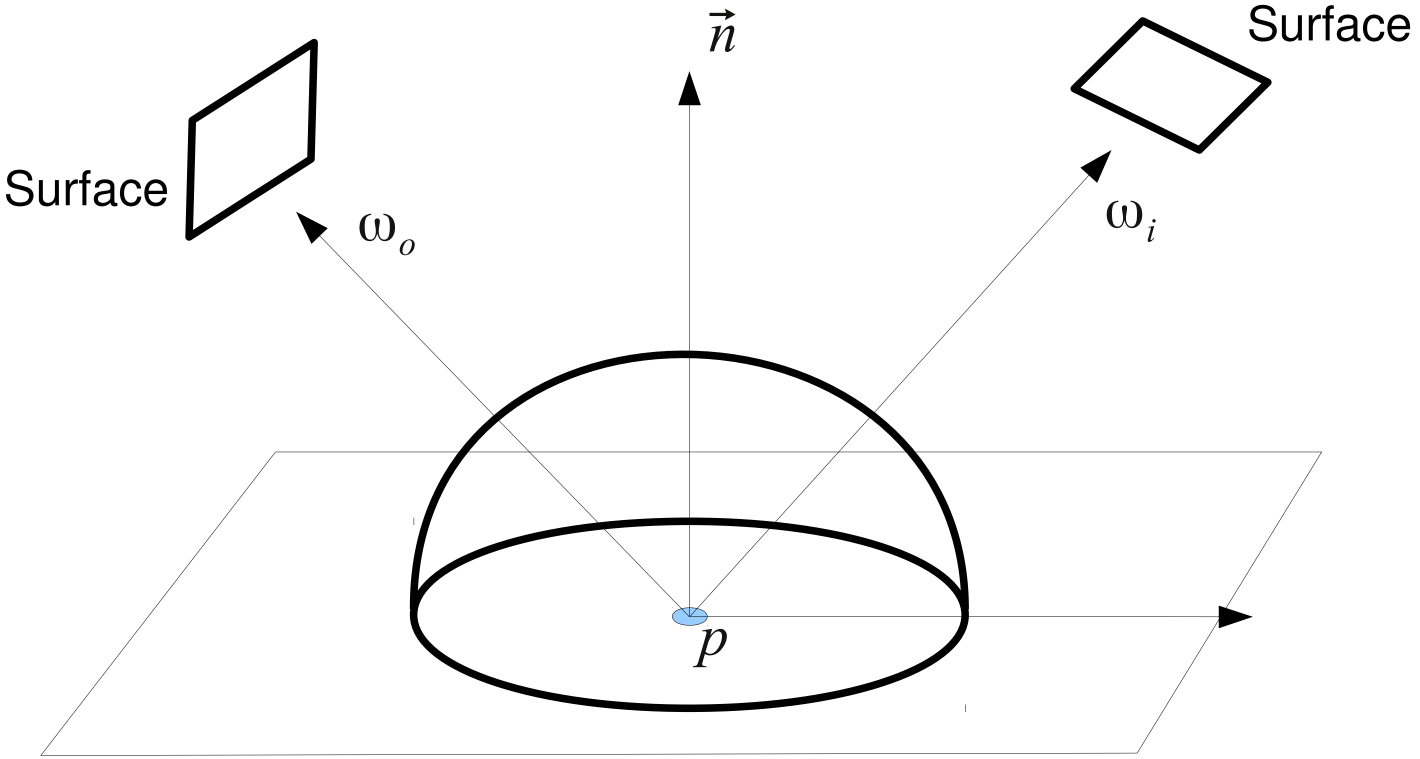
The Rendering Equation

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{\Omega} f_r(p,\omega_o,\omega_i) L_i(p,\omega_i) \cos\theta_i \, d\omega_i$$



The Rendering Equation

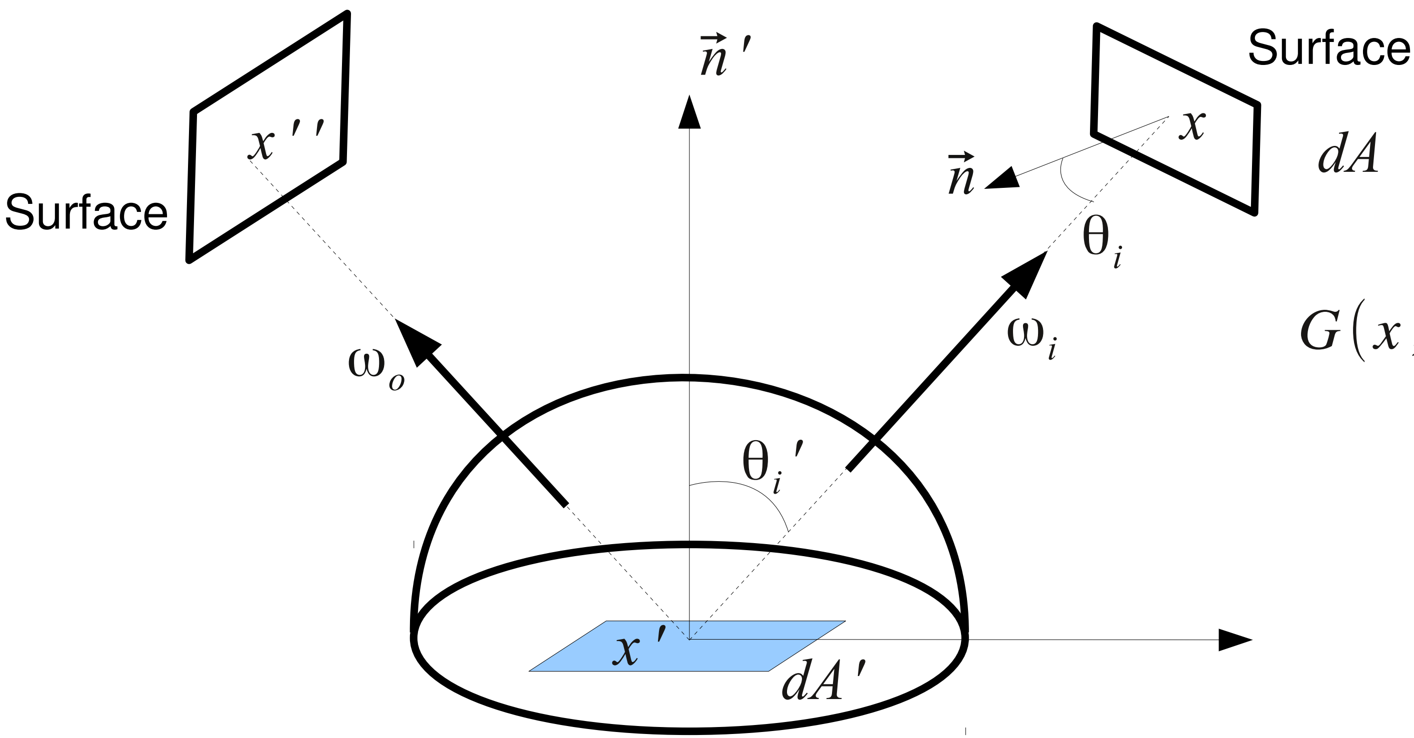
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{\Omega} f_r(p,\omega_o,\omega_i)\, L_i(p,\omega_i) \cos\theta_i \, d\omega_i$$



The Rendering Equation

$$L_o(x' \rightarrow x'') = L_e(x' \rightarrow x'') + \int_S f_r(x \rightarrow x' \rightarrow x'') L_i(x \rightarrow x') V(x, x') G(x, x') \, dA$$

The Rendering Equation, J. T. Kajiya, SIGGRAPH 1986.



$$G(x, x') = \frac{\cos\theta_i \cos\theta_i'}{\|x - x'\|^2}$$

Solutions to the Rendering Eqn.

- OpenGL
- Ray Tracing
- Radiosity
- Distribution Ray Tracing and Path Tracing
- Photon Mapping

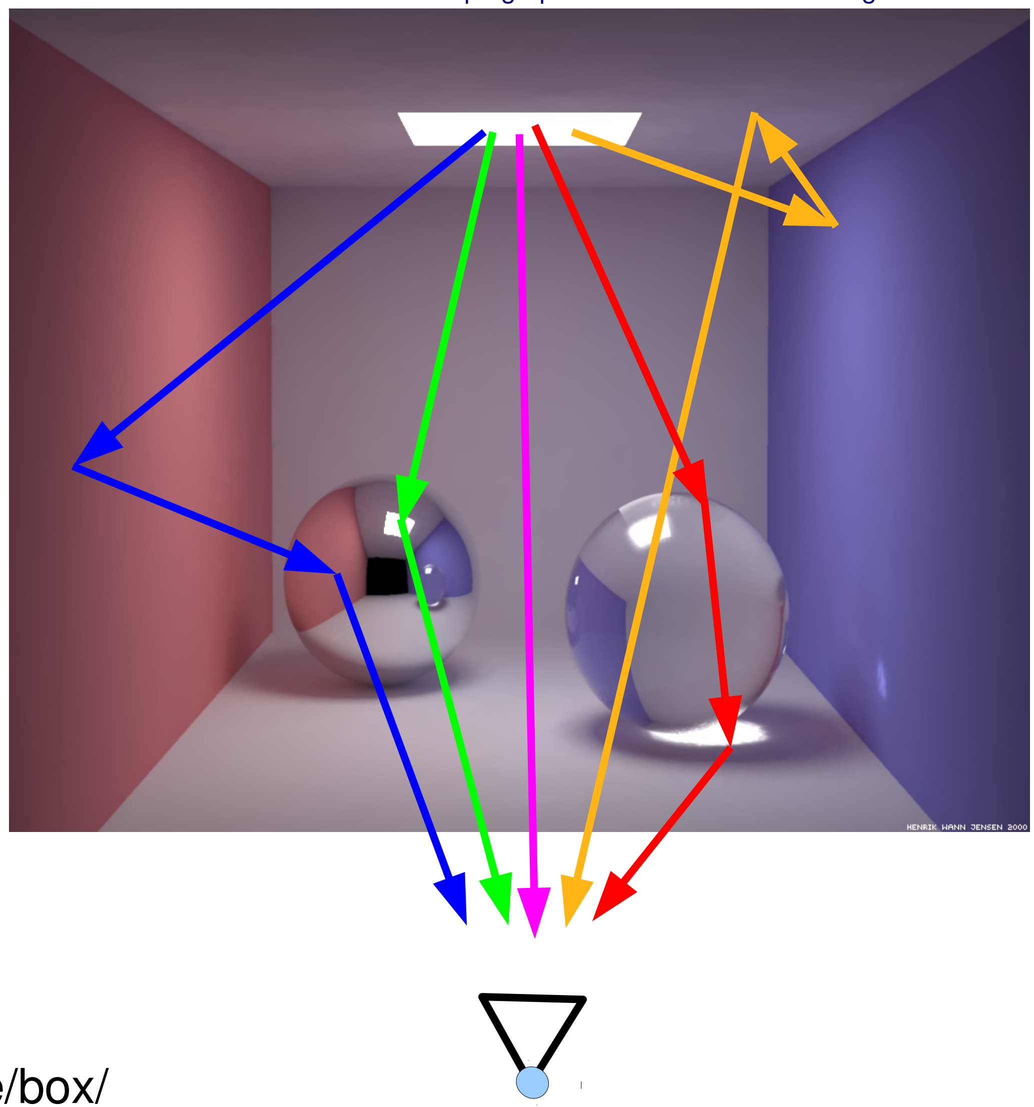
Light Paths

- A grammar for light paths
- Alphabet
  - L : Point on the light source
  - D : Point on a diffuse surface
  - S : Point on a specular surface
  - E : Point on the eye/camera
- Regex notation
  - ab – concatanate a AND b
  - a|b – either a OR b
  - a^* – Zero or more repetition of a
  - a^+ – One or more repetition of a

Light Paths

- All light paths: L(D|S)*E

http://graphics.ucsd.edu/~henrik/images/cbox.html



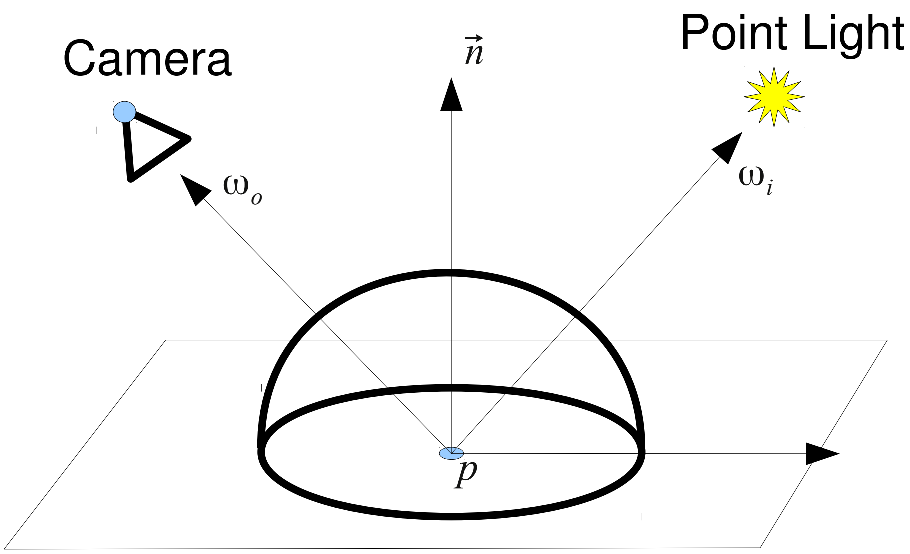
Rendering Trivia: The Cornell Box

http://www.graphics.cornell.edu/online/box/

OpenGL

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega f_r(p,\omega_o,\omega_i) L_i(p,\omega_i) \cos\theta_i \, d\omega_i$$

- LD|SE
- Point Lights
- Only Direct Illumination
- Lambertian/Phong BRDF
- Visibility ignored

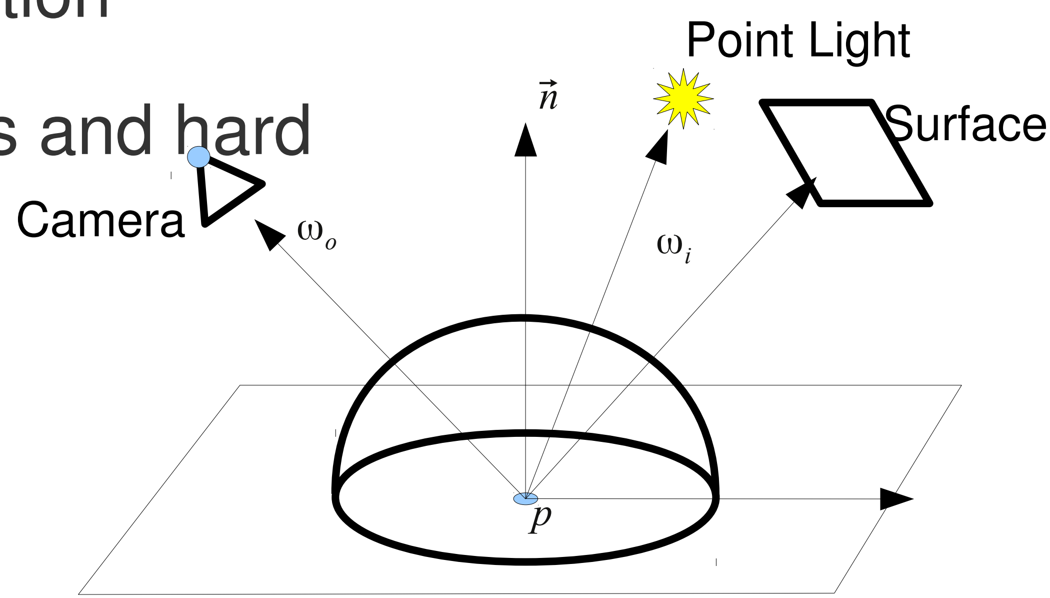


$$L_o(p,\omega_o) = L_a + L_e(p,\omega_o) + \sum_{1}^{nLights} f_r(p,\omega_o,\omega_i) L_i(p,\omega_i) \cos\theta_i$$

Ray Tracing

$$L_o(p,\omega_o)=L_e(p,\omega_o)+\int_{\Omega} f_r(p,\omega_o,\omega_i)L_i(p,\omega_i)\cos\theta_i\, d\omega_i$$

- LDS*E
- Specular Reflection and Transmission only
- No other Indirect Illumination
- Whitted – only point lights and hard shadows.

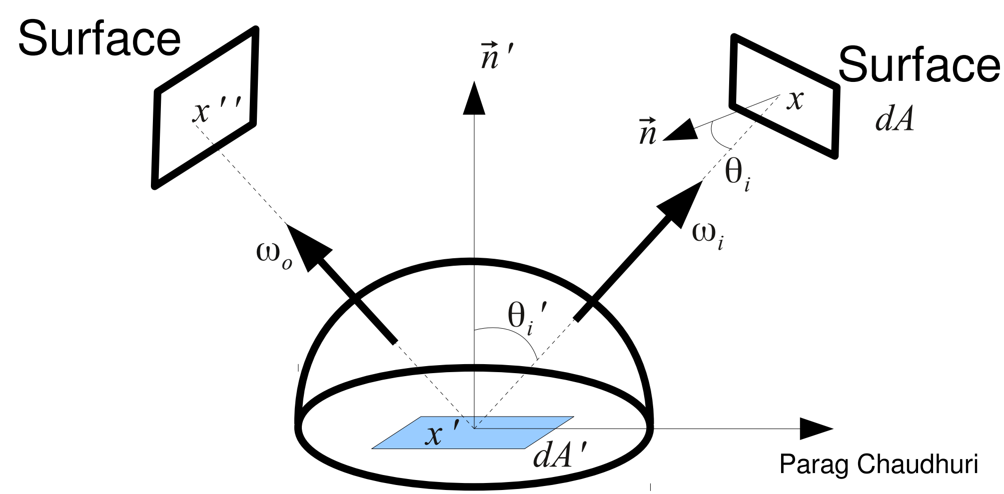


$$L_o(p,\omega_o)=L_a+L_e(p,\omega_o)+\sum_{1}^{nLights} f_r(p,\omega_o,\omega_i)L_i(p,\omega_i)V(p,\omega_i)\cos\theta_i+\textit{indirect specular}$$

Radiosity

$$L_o(x' \rightarrow x'') = L_e(x' \rightarrow x'') + \int_S f_r(x \rightarrow x' \rightarrow x'') L_i(x \rightarrow x') V(x, x') G(x, x') dA$$

- LD*E
- Assume all surfaces are Lambertian
- The term has its origin in Thermodynamics
- Has the same dimensions/units as Irradiance/Radiant Emittance



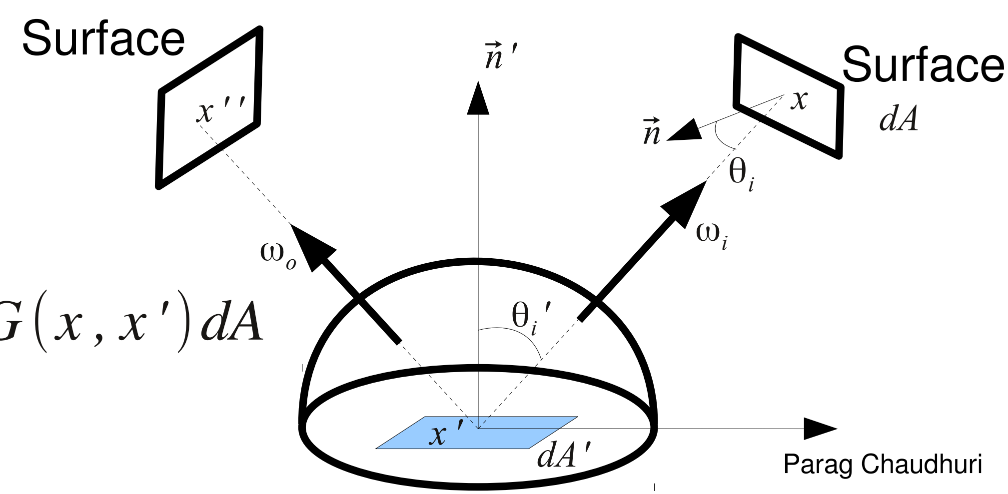
Radiosity

$$L_o(x' \rightarrow x'') = L_e(x' \rightarrow x'') + \int_S f_r(x \rightarrow x' \rightarrow x'') L_i(x \rightarrow x') V(x, x') G(x, x') \, dA$$

The All-Lambertian Assumption $\quad f_r(x \rightarrow x' \rightarrow x'') = k_d = \frac{\rho}{\pi}$

$$L_o(x') = L_e(x') + \frac{\rho}{\pi} \int_S L_i(x) V(x, x') G(x, x') \, dA$$

Convert to Radiosities $\quad B = \int_\Omega L \cos\theta \, d\omega \quad \text{gives} \quad L = \frac{B}{\pi}$



$$B_o(x') = B_e(x') + \frac{\rho}{\pi} \int_S B_i(x) V(x, x') G(x, x') \, dA$$

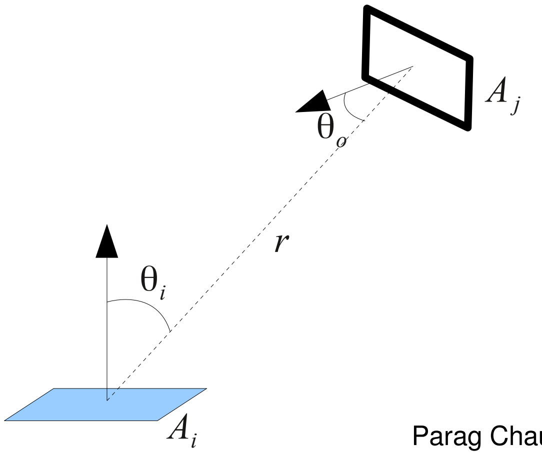
Radiosity

$$B_o(x')=B_e(x')+\frac{\rho}{\pi}\int_S B_i(x)V(x,x')G(x,x')\,dA$$

Radiosity Approximation: Discretize the surface into smaller elements.

$$B_i=E_i+\rho_i\sum_{j=1}^{N}B_jF_{ij}$$

$$\text{where } F_{ij}=\frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{V_{ij}\cos\theta_i\cos\theta_o}{\pi r^2}\,dA_j\,dA_i$$



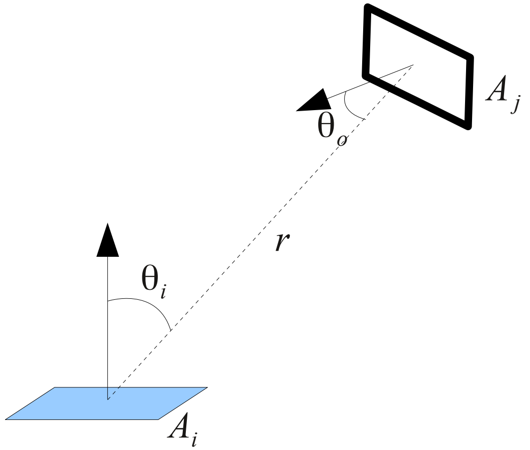
Radiosity

Form a system of Equations and Solve

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij} = E_i$$

$$(1 - \rho_i F_{ii}) B_i - \rho_i \sum_{j=1, j \neq i}^{N} F_{ij} B_j = E_i$$

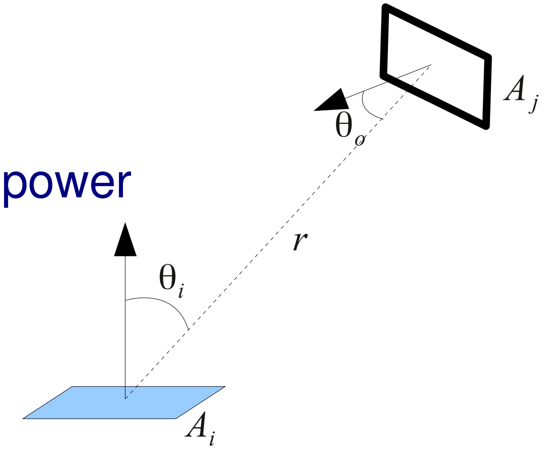


$$\begin{bmatrix} 1-\rho_1 F_{1,1} & \dots & \dots & \dots & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & \dots & \dots & -\rho_2 F_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\rho_{n-1} F_{n-1,1} & \dots & \dots & 1-\rho_{n-1} F_{n-1,1} & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & \dots & \dots & \dots & 1-\rho_n F_{n,1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{n-1} \\ E_n \end{bmatrix}$$

Radiosity

Intuition behind Radiosity is conservation of energy – How?

$$B_i dA_i = E_i dA_i + \rho_i \int_{\Omega} B_j F_{ji} dA_j$$



F_{ji} or the **Form Factor from *j* to *i*** is the proportion of total power leaving patch j that is received by patch i.

Since there is no loss of power in between,

$$F_{ij} A_i = F_{ji} A_j$$

$$\underbrace{B_i dA_i}_{\text{outgoing power}} = \underbrace{E_i dA_i}_{\text{emitted power}} + \underbrace{\rho_i}_{\text{reflectance}} \overbrace{\int_{\Omega} B_j F_{ji} dA_j}^{\text{power received from } j}$$

$$\text{or } B_i dA_i = E_i dA_i + \rho_i \int_{\Omega} B_j F_{ij} dA_i$$

$$\text{or } B_i = E_i + \rho_i \int_{\Omega} B_j F_{ij}$$

Radiosity

Deriving the Form Factor

F_{ij} or the **Form Factor** is the proportion of total power leaving patch i that is received by patch j.

Solid angle subtended by dA_j as seen from dA_i is $d\omega = \frac{dA_j \cos\theta_j}{r^2}$

If exitant radiance leaving the surface at dA_i in the direction θ_i is dM_i in the solid angle $d\omega$

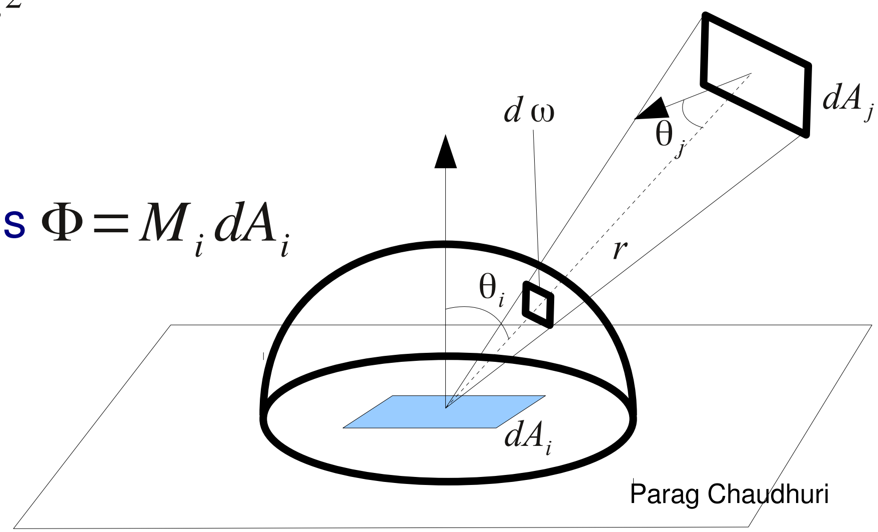
Flux leaving dA_i in the direction of dA_j is given by

$$d\Phi = dM_i\, dA_i = L_i \cos\theta_i\, d\omega\, dA_i = \frac{L_i \cos\theta_i \cos\theta_j\, dA_j\, dA_i}{r^2}$$

$$= \frac{M_i \cos\theta_i \cos\theta_j\, dA_j\, dA_i}{\pi r^2}$$

Total flux leaving dA_i over the entire hemisphere is $\Phi = M_i\, dA_i$

$$\Rightarrow F_{dA_i \to dA_j} = \frac{d\Phi}{\Phi} = \frac{\cos\theta_i \cos\theta_j\, dA_j}{\pi r^2}$$



Radiosity

F_{ij} or the **Form Factor** is the proportion of total power leaving patch i that is received by patch j.

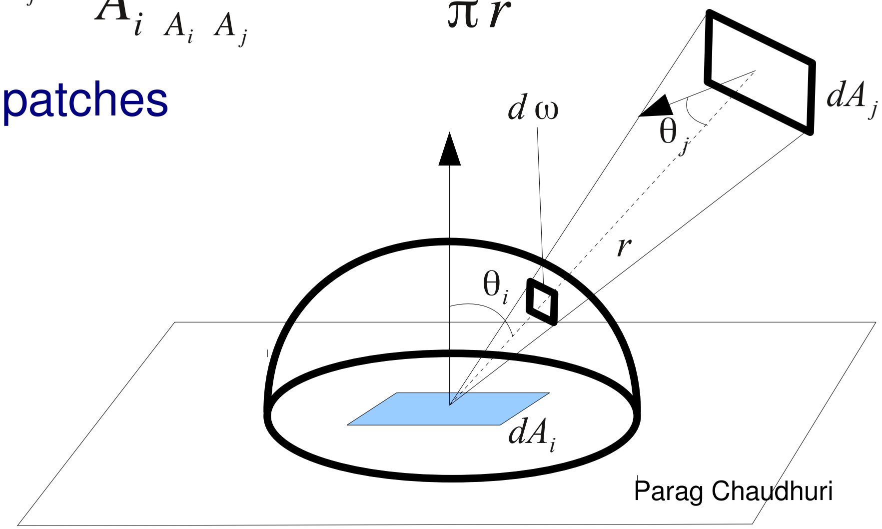
Form factor from dA_i to dA_j is given by $F_{dA_i \rightarrow dA_j} = \frac{\cos\theta_i \cos\theta_j \, dA_j}{\pi r^2}$

Form factor from dA_i to A_j is given by $F_{dA_i \rightarrow A_j} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j \, dA_j}{\pi r^2}$

Form factor from A_i to A_j is given by $F_{ij} = F_{A_i \rightarrow A_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j \, dA_j \, dA_i}{\pi r^2}$

If distance r is large compared to area of the two patches then

$$F_{ij} \approx F_{dA_i \rightarrow A_j} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j \, dA_j}{\pi r^2}$$



Radiosity

Algorithm

1. Discretize the environment into patches.

2. **Calculate the form-factor for every patch.**

3. Solve the system of equations.

4. Render the scene with the computed radiosity.

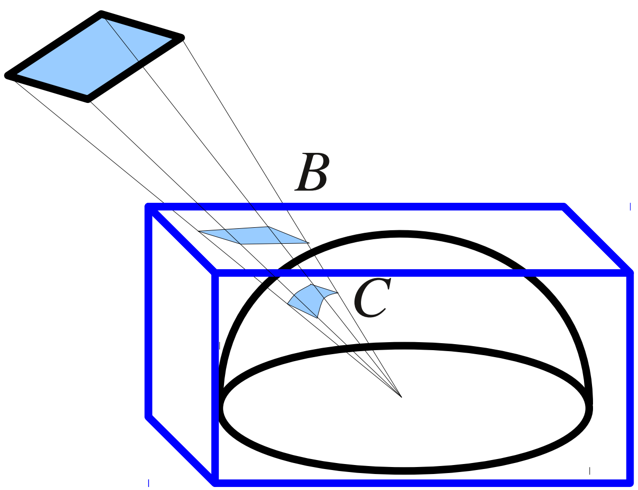
Radiosity

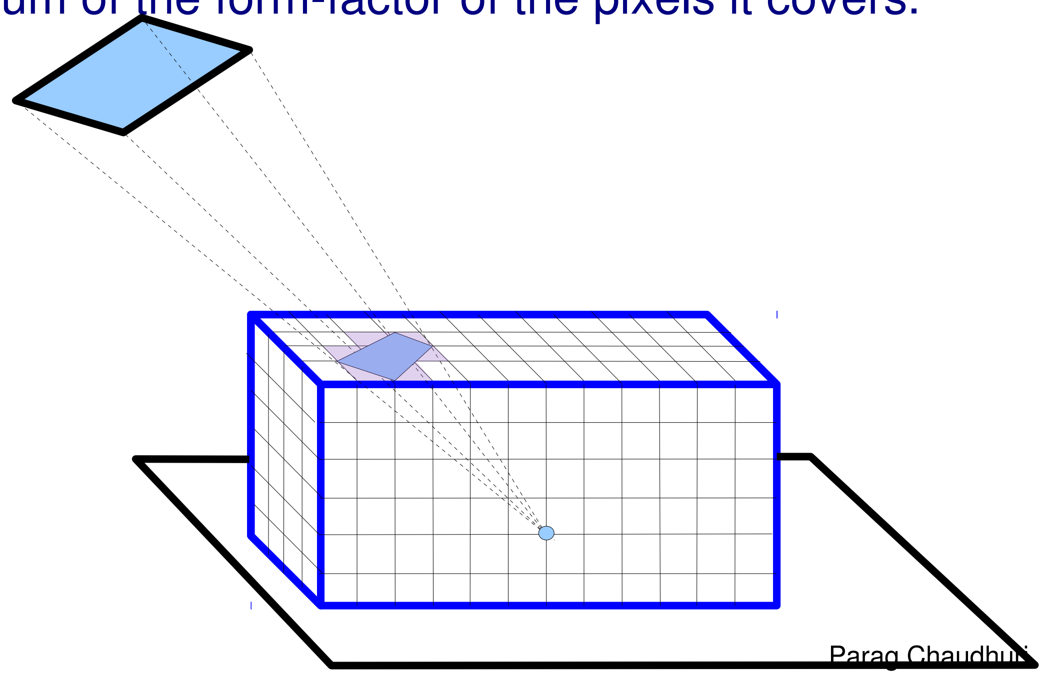
Hemicube Form-factors

- A **hemicube** is a half cube centered at the patch.

- The **Nusselt Analogue** Justification – Form factor of a patch is equivalent to the fraction of the unit circle that is formed by the projection of the patch.

- Form-factor of a patch is equal to sum of the form-factor of the pixels it covers.



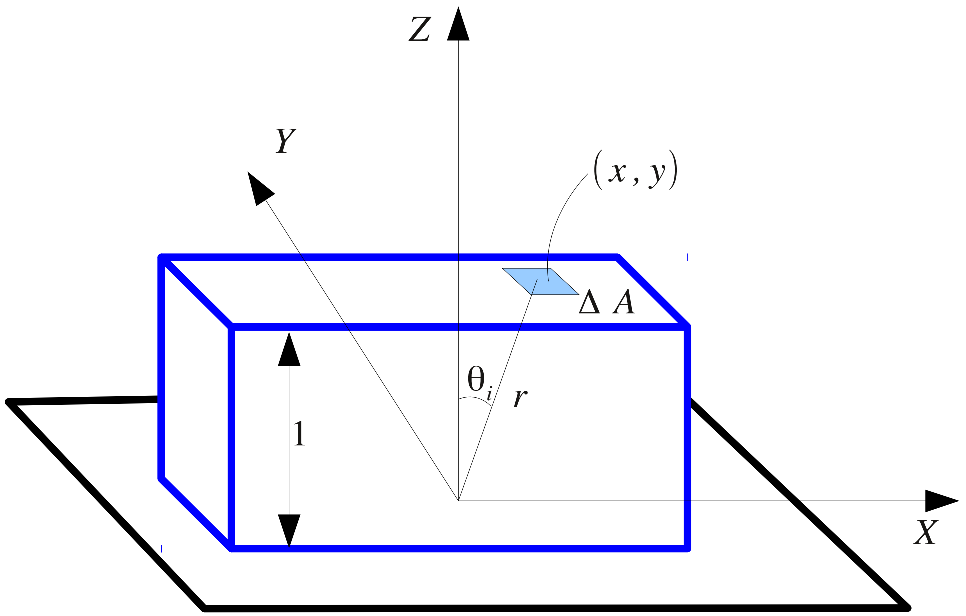


Radiosity

Hemicube Form-factors

- Advantages
  - Pre-compute pixel form factors
  - Which pixels are covered can be obtained from projection on the corresponding hemicube faces planes.

For a pixel q on the top surface of the hemicube,



$$\Delta F_q = \frac{1}{\pi(x^2 + y^2 + 1)^2} \Delta A$$

$$r = (x^2 + y^2 + 1)^{1/2}$$

$$\cos\theta_i = \cos\theta_j = \frac{1}{(x^2 + y^2 + 1)^{1/2}}$$

Radiosity

Algorithm

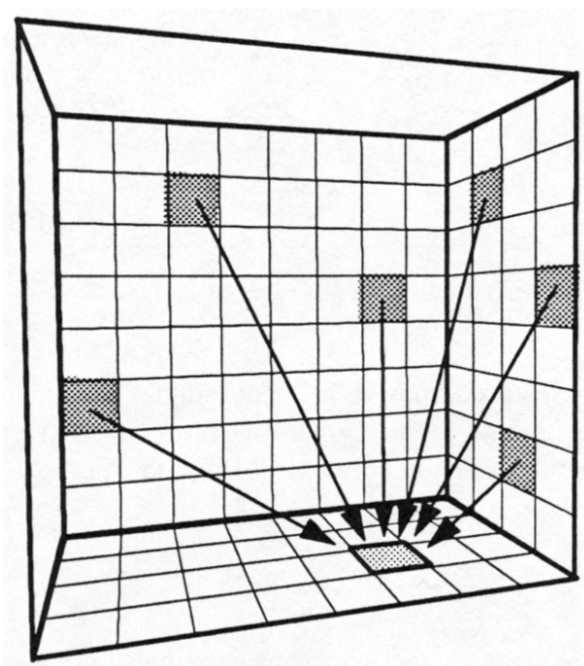
1. Discretize the environment into patches.
2. Calculate the form-factor for every patch.
3. **Solve the system of equations.**
4. Render the scene with the computed radiosity.

Radiosity

Gauss-Seidel Solution

$$
\begin{bmatrix}
1-\rho_1 F_{1,1} & \cdots & \cdots & \cdots & -\rho_1 F_{1,n} \\
-\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & \cdots & \cdots & -\rho_2 F_{2,n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
-\rho_{n-1} F_{n-1,1} & \cdots & \cdots & 1-\rho_{n-1} F_{n-1,n-1} & -\rho_{n-1} F_{n-1,n} \\
-\rho_n F_{n,1} & \cdots & \cdots & \cdots & 1-\rho_n F_{n,n}
\end{bmatrix}
\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix}
=
\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{n-1} \\ E_n \end{bmatrix}
$$

$$\boldsymbol{K}\,\boldsymbol{B} = \boldsymbol{E}$$



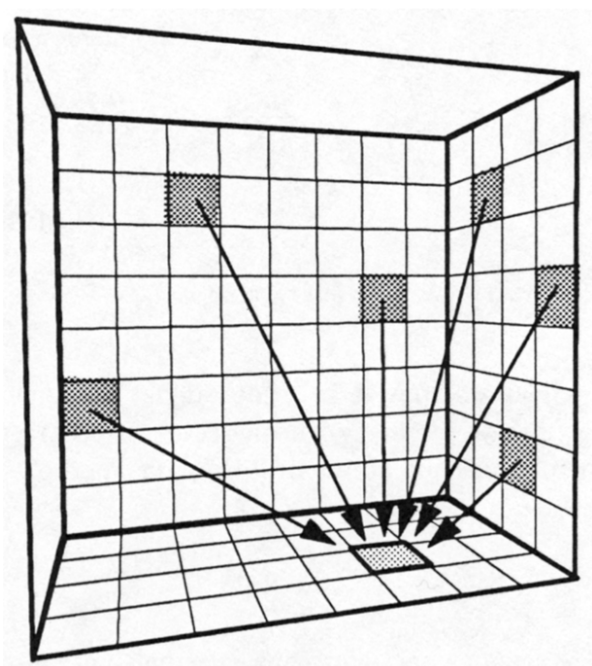
A Gathering Solution

```
Compute Form-factors for all patches
for all i  do { Bi = Ei }
while (not converged) do
{
    for each i do
    {
        sum = 0;
        for all j except i do
            sum += Kij*Bj;
        Bi = Ei – sum;
    }
}
```

Radiosity

Gauss-Seidel Solution

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{n-1} \\ E_n \end{bmatrix} + \begin{bmatrix} \rho_1 F_{1,1} & \cdots & \cdots & \cdots & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & \rho_2 F_{2,2} & \cdots & \cdots & -\rho_2 F_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\rho_{n-1} F_{n-1,1} & \cdots & \cdots & rho_{n-1} F_{n-1,n-1} & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & \cdots & \cdots & \cdots & \rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix}$$

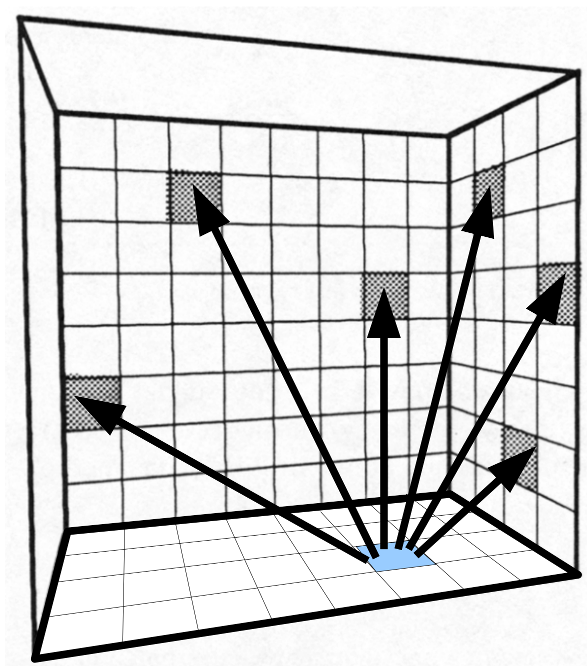


Relaxing one row at a time

A *Gathering* Solution

Radiosity

Progressive Solution



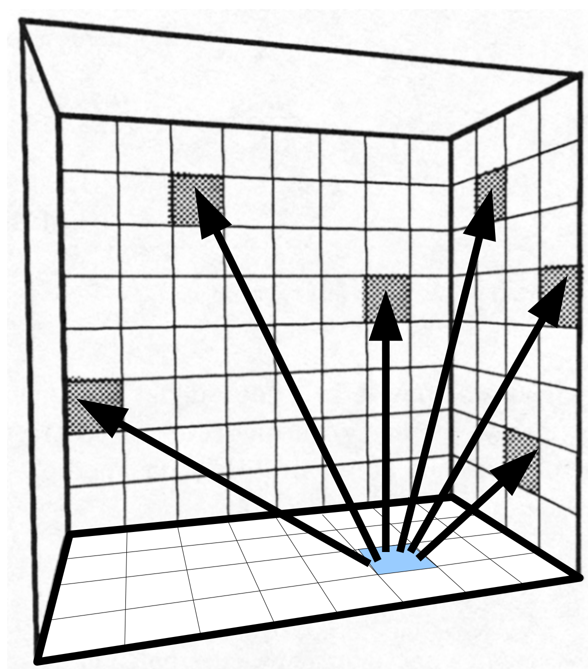
A *Shooting* Solution

```
for all i do
{
  B_i = E_i, ΔB_i = E_i
  Compute Form-factors F_ij if not done
}
while not converged do
{
   Pick i such that ΔB_i*A_i is largest
    for every patch j except i
     {
       Δrad = ρ_j ΔB_i F_ij A_i/A_j
      ΔB_j = ΔB_j + Δrad
      B_j = B_j + Δrad
    }
   ΔB_i = 0
}
```

Radiosity

Progressive Solution

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} + \begin{bmatrix} B_1 \\ \boxed{B_2} \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} \begin{bmatrix} \rho_1 F_{1,1} & \cdots & \cdots & \cdots & \rho_1 F_{1,n} \\ \rho_2 F_{2,1} & \rho_2 F_{2,2} & \cdots & \cdots & \rho_2 F_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{n-1} F_{n-1,1} & \cdots & \cdots & \rho_{n-1} F_{n-1,n-1} & \rho_{n-1} F_{n-1,n} \\ \rho_n F_{n,1} & \cdots & \cdots & \cdots & \rho_n F_{n,n} \end{bmatrix}$$



Evaluating a column at a time

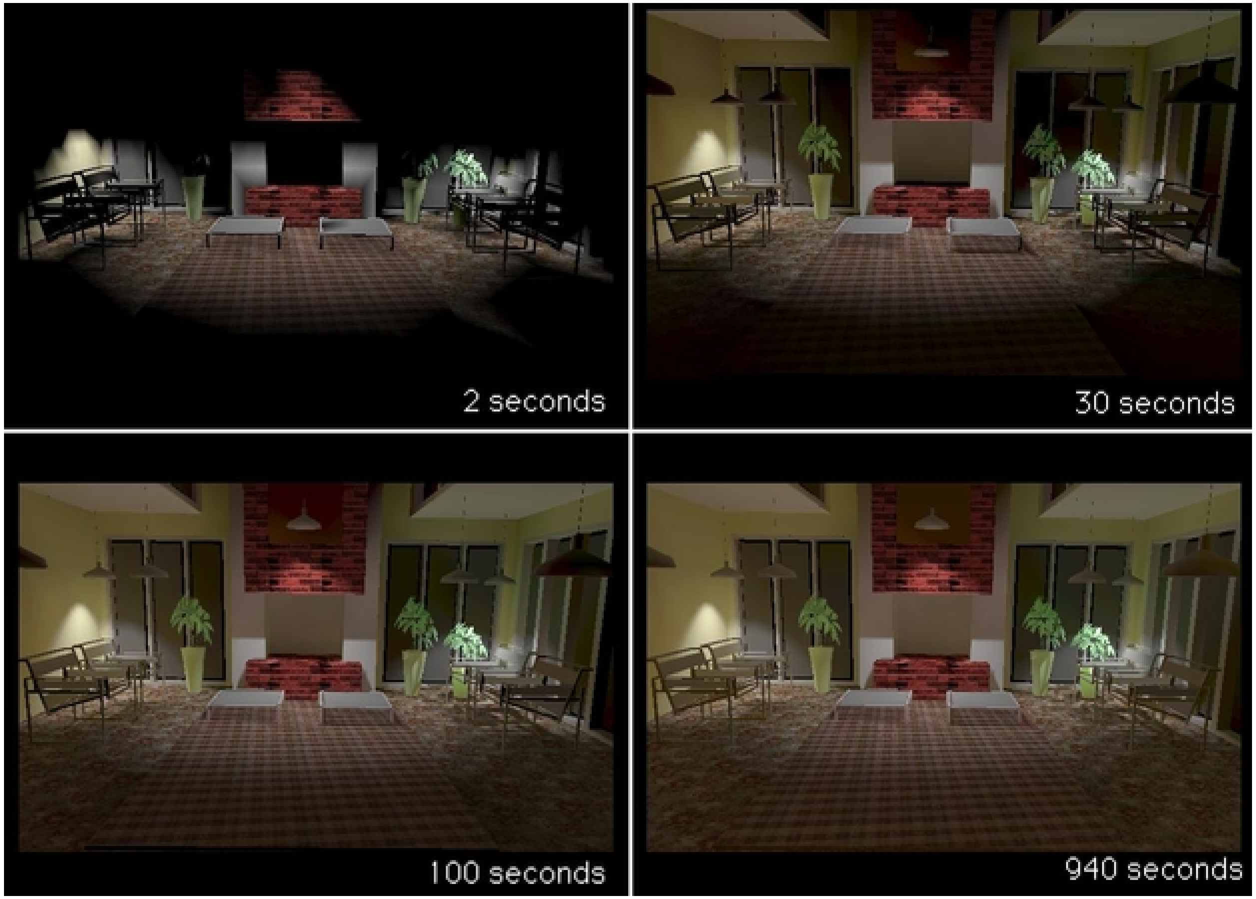
A *Shooting* Solution

Radiosity

Algorithm

1. Discretize the environment into patches.
2. Calculate the form-factor for every patch.
3. Solve the system of equations.
4. **Render the scene with the computed radiosity.**

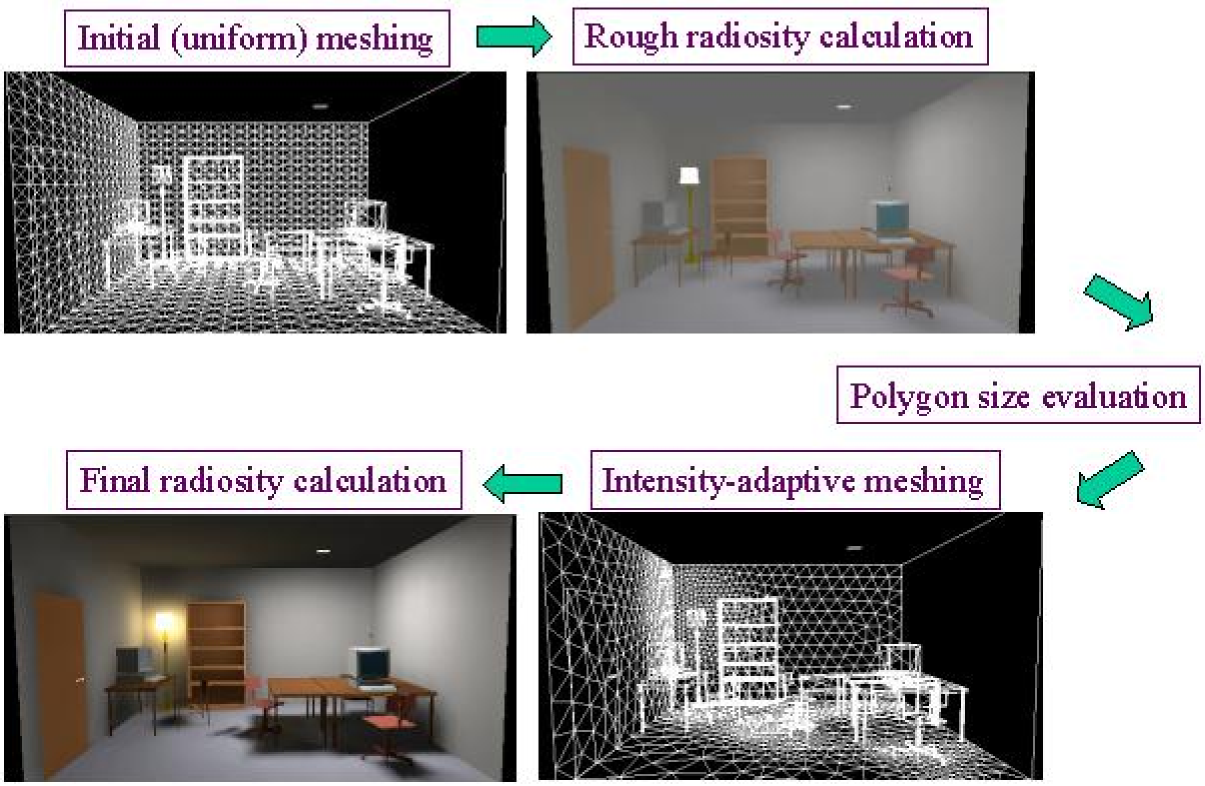
Radiosity



http://www.mpi-inf.mpg.de/resources/atrium/hab/chapter_3/chapter_3.html

Radiosity

- Adaptive meshing



http://hawk.ise.chuo-u.ac.jp/makino-lab/person/itot/mytopics/radios.html

Radiosity

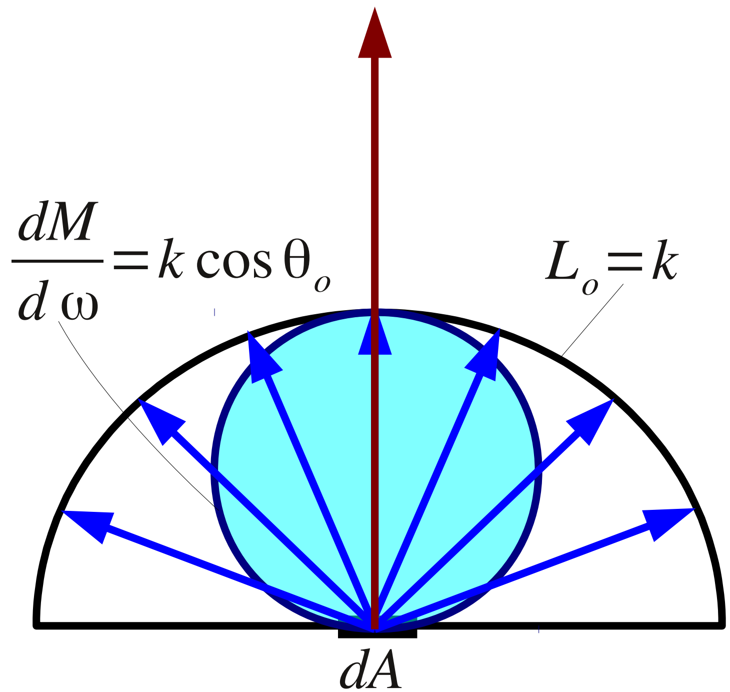
- Extentions
  - Discontinuity Meshing
  - Hierarchical Radiosity
  - Add Participating Media
  - Combine with RayTracing

Lambertian Surfaces

A **Lambertian** surface is one that follows **Lambert's** law : Illumination emitted by a surface in a particular direction varies as the cosine of the angle between the said direction and the normal to the surface.

$$L_o = \frac{dM}{\cos\theta_o \, d\omega} = k$$

$$M = \int_{H^2(\vec{n})} L_o \cos\theta_o \, d\omega = \pi L_o$$

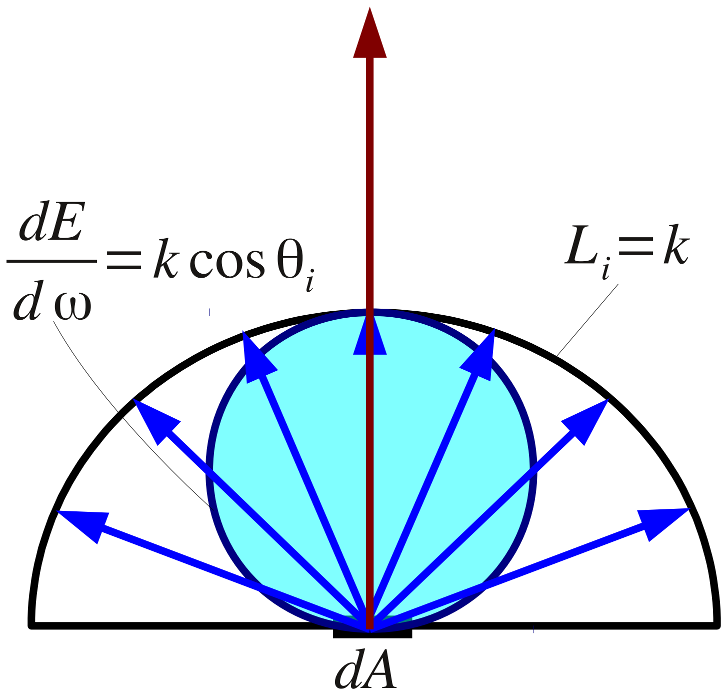


Lambertian Surfaces

A **Lambertian** surface is one that follows **Lambert's** law : Illumination received by a surface in a particular direction varies as the cosine of the angle between the said direction and the normal to the surface.

$$L_i = \frac{dE}{\cos\theta_i \, d\omega} = k$$

$$E = \int_{H^2(\vec{n})} L_i \cos d\,\theta_i = \pi L_i$$



Lambertian Surfaces

A **Lambertian** surface is one that follows **Lambert's** law : Illumination reflected by a surface in a particular direction varies as the cosine of the angle between the said direction and the normal to the surface.

$$L_o(p,\omega_o) = \int_{H^2(\vec{n})} f_r(p,\omega_o,\omega_i)\, dE(p,\omega_i)\, d\omega_i$$

$$= \int_{H^2(\vec{n})} f_r(p,\omega_o,\omega_i)\, L_i(p,\omega_i) \cos\theta_i\, d\omega_i = \rho L_i = \pi k_d L_i$$