Computer Vision

Triangulation and Absolute Orientation

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Motivation

Given the relative orientation of two images, compute the points in 3D

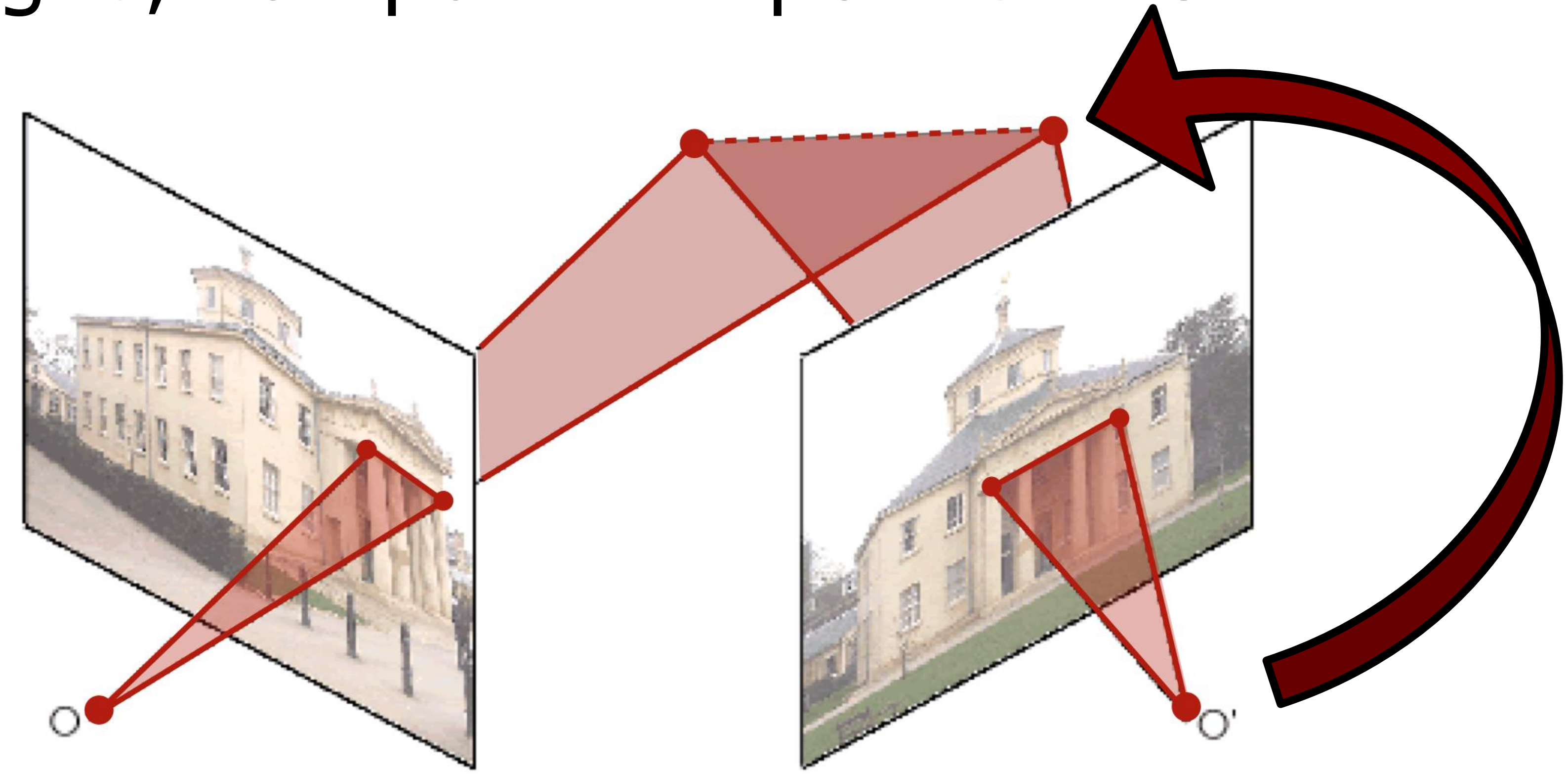

Image courtesy: Schindler

Topic

Last lectures

Computing the relative orientation of two images

Today

Given the relative orientation of the images, compute the **3D location of corresponding points**

Triangulation / forward intersection

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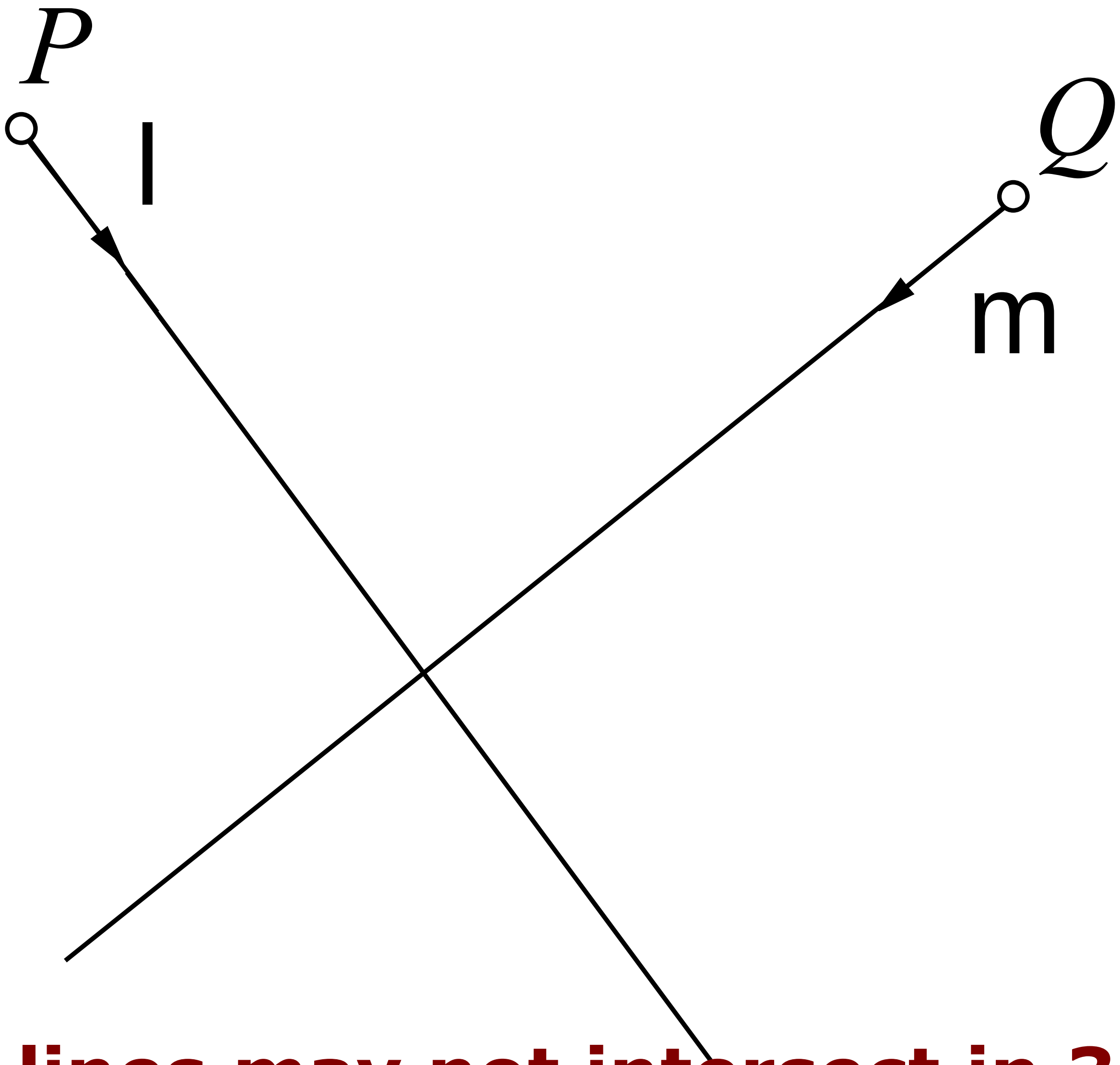
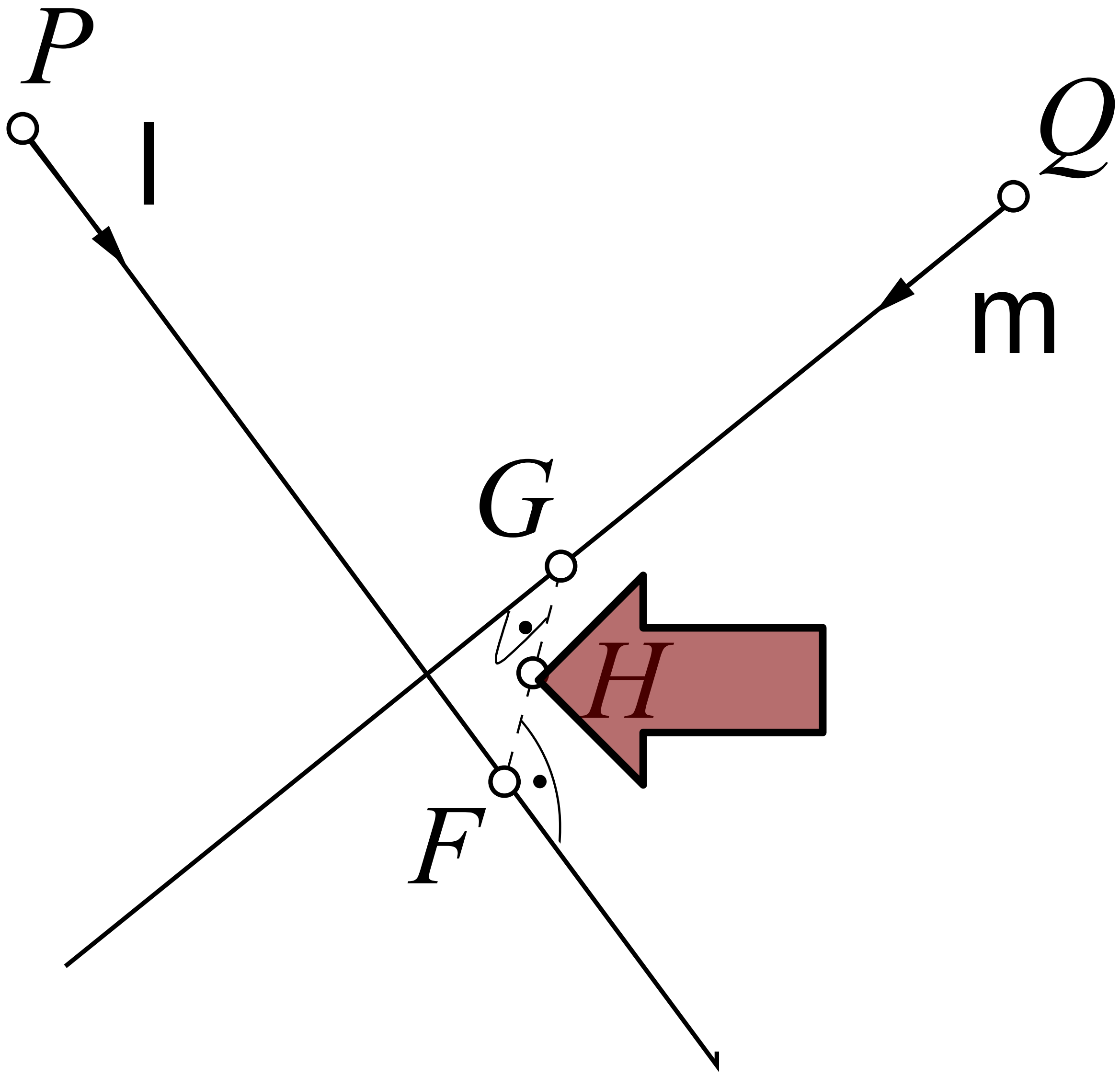
Geometric Approach

The Problem



The lines may not intersect in 3D!

Find the Point H



Refresher from Camera Geometry

- Starting from $\lambda \mathbf{x} = \mathsf{K}R\boldsymbol{X} - \mathsf{K}R\boldsymbol{X}_O$
- we obtain

$$
\begin{aligned}
\boldsymbol{X} &= (\mathsf{K}R)^{-1}\mathsf{K}R\boldsymbol{X}_O + \lambda(\mathsf{K}R)^{-1}\mathbf{x} \\
&= \boldsymbol{X}_O + \lambda(\mathsf{K}R)^{-1}\mathbf{x}
\end{aligned}
$$

3x1 Euclidean

- The term $(\mathsf{K}R)^{-1}\mathbf{x}$ describes the direction of the ray from the camera origin $\boldsymbol{X}_O$ to the 3D point $\boldsymbol{X}$

Geometric Solution

- Equation for two lines in 3D

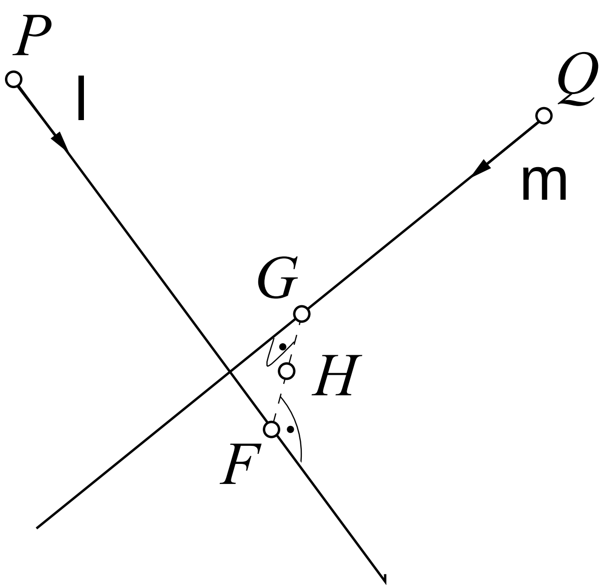
$$\boldsymbol{f} = \boldsymbol{p} + \lambda \boldsymbol{r} \qquad \boldsymbol{g} = \boldsymbol{q} + \mu \boldsymbol{s}$$

- with the points $\boldsymbol{p} = \boldsymbol{X}_{O'} \quad \boldsymbol{q} = \boldsymbol{X}_{O''}$
- and the directions (calibrated camera)

$$\boldsymbol{r} = R'^{\mathsf{T}}\, {}^{k}\mathbf{x}' \qquad \boldsymbol{s} = R''^{\mathsf{T}}\, {}^{k}\mathbf{x}''$$

- with ${}^{k}\mathbf{x}' = (x', y', c)^{\mathsf{T}} \quad {}^{k}\mathbf{x}'' = (x'', y'', c)^{\mathsf{T}}$

Geometric Solution



- The shortest connection requires that FG is orthogonal to both lines
- This leads to the constraint

$$(\boldsymbol{f}-\boldsymbol{g})\cdot\boldsymbol{r}=0 \qquad (\boldsymbol{f}-\boldsymbol{g})\cdot\boldsymbol{s}=0$$

which directly leads to

$$\begin{aligned}(\boldsymbol{q}+\lambda\boldsymbol{s}-\boldsymbol{p}-\mu\boldsymbol{r})\cdot\boldsymbol{s} &= 0\\ (\boldsymbol{q}+\lambda\boldsymbol{s}-\boldsymbol{p}-\mu\boldsymbol{r})\cdot\boldsymbol{r} &= 0\end{aligned}$$

- Two equations, two unknowns

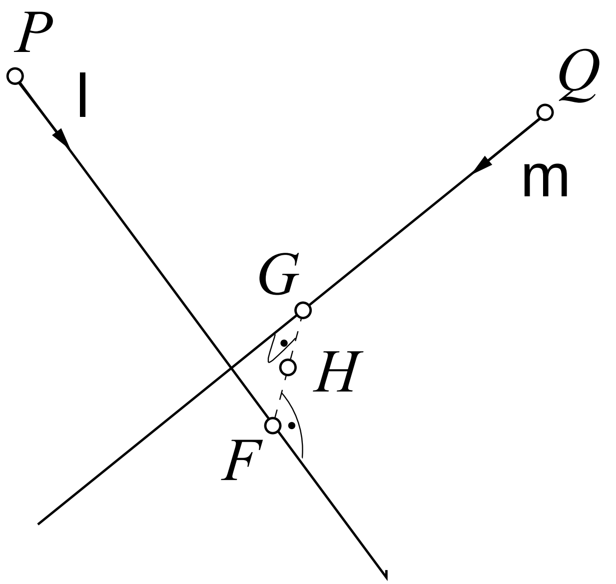
Geometric Solution

- By solving the equations

$$
\begin{aligned}
(\boldsymbol{q} + \lambda \boldsymbol{s} - \boldsymbol{p} - \mu \boldsymbol{r}) \cdot \boldsymbol{s} &= 0 \\
(\boldsymbol{q} + \lambda \boldsymbol{s} - \boldsymbol{p} - \mu \boldsymbol{r}) \cdot \boldsymbol{r} &= 0
\end{aligned}
$$

  we obtain λ, μ

- dirλ, μy yield F and G
- We compute H as the middle of the line connecting F and G



2.
For the Stereo Normal Case

Stereo Normal Case

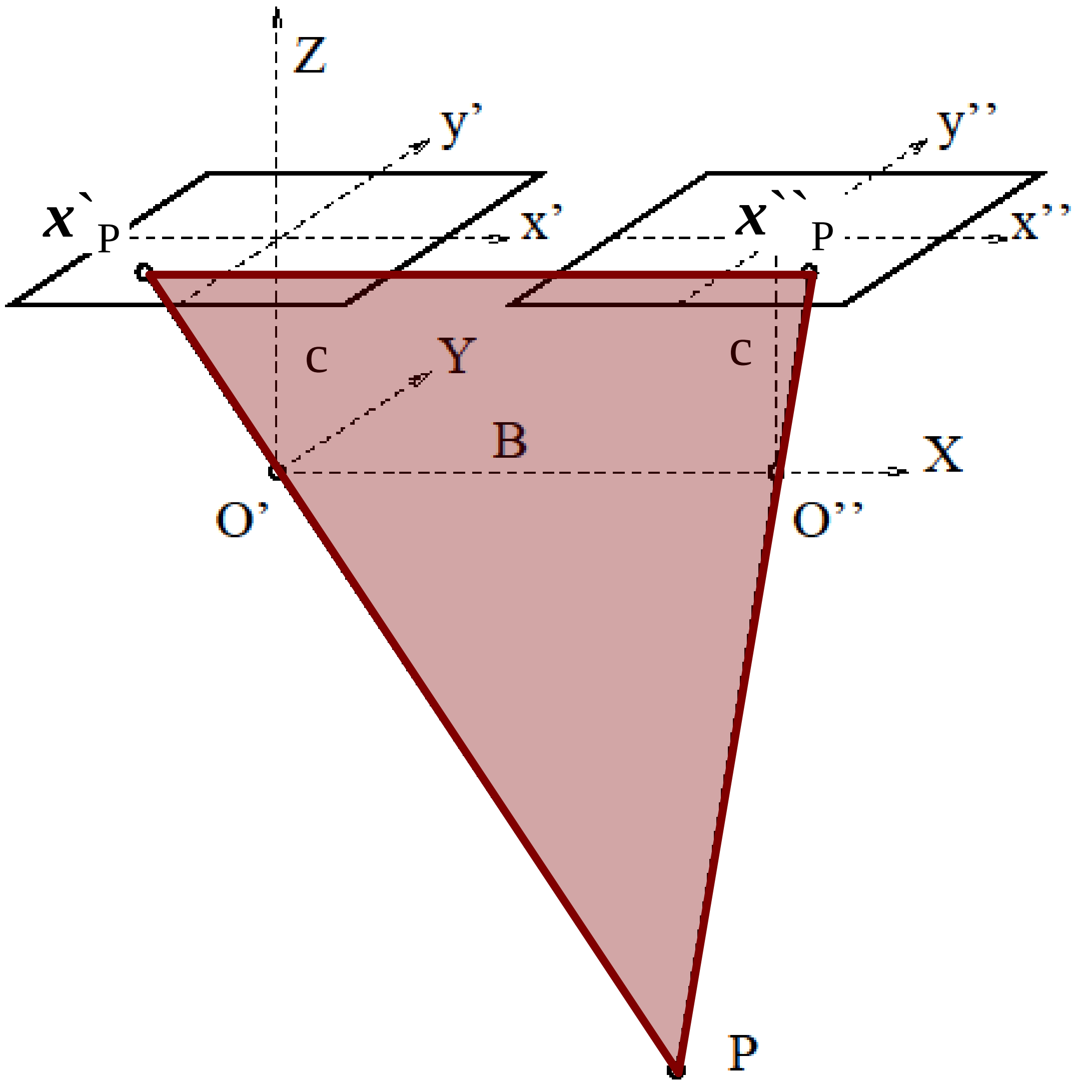


Image courtesy: Förstner

Stereo Normal: Intersection

1. Z-coordinate from intercept theorem

$$\frac{Z}{c} = \frac{B}{-\underbrace{(x'' - x')}_{p_x}}$$

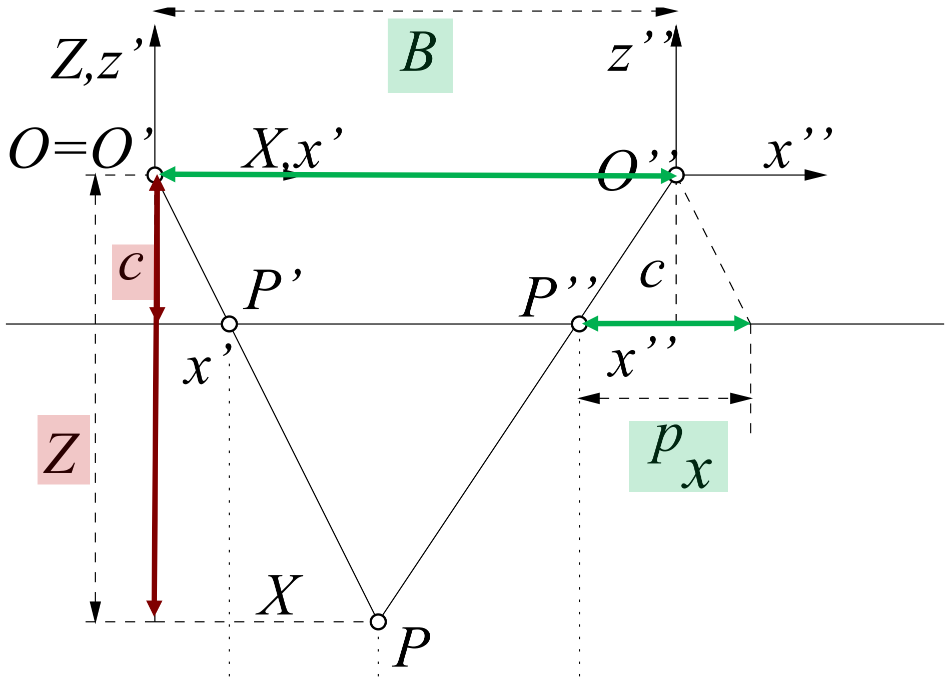


Image courtesy: Förstner

Stereo Normal: Intersection

1. Z-coordinate from intercept theorem

$$\frac{Z}{c} = \frac{B}{-\underbrace{(x'' - x')}_{p_x}}$$

$$\Rightarrow \quad Z = c\frac{B}{-(x'' - x')}$$

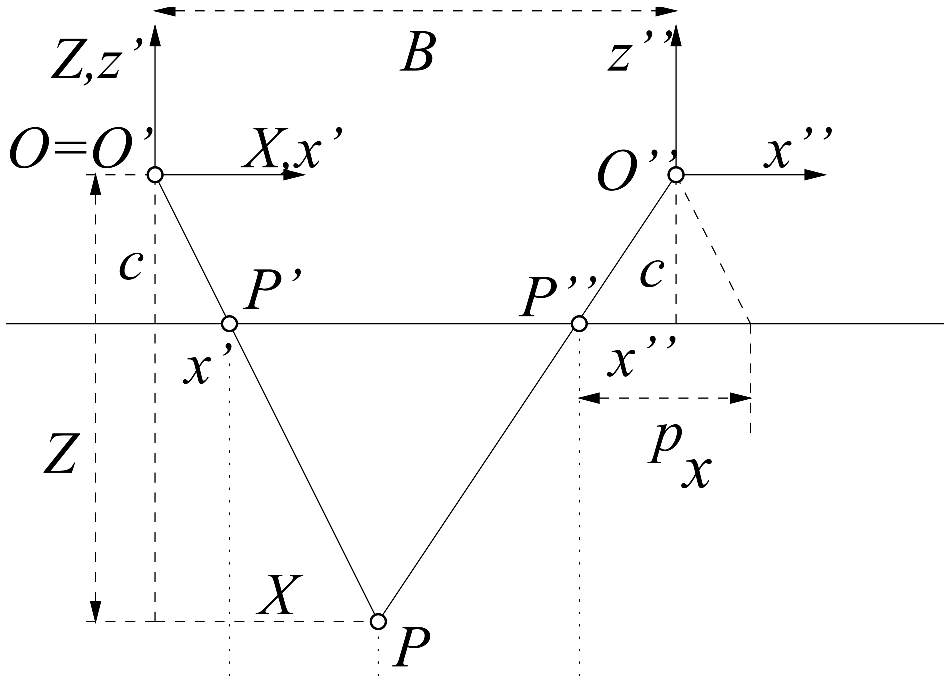


Image courtesy: Förstner

Stereo Normal: Intersection

1. Z-coordinate from intercept theorem

$$\frac{Z}{c} = \frac{B}{-(x'' - x')}$$

2. X-coordinate

$$\frac{X}{x'} = \frac{Z}{c}$$

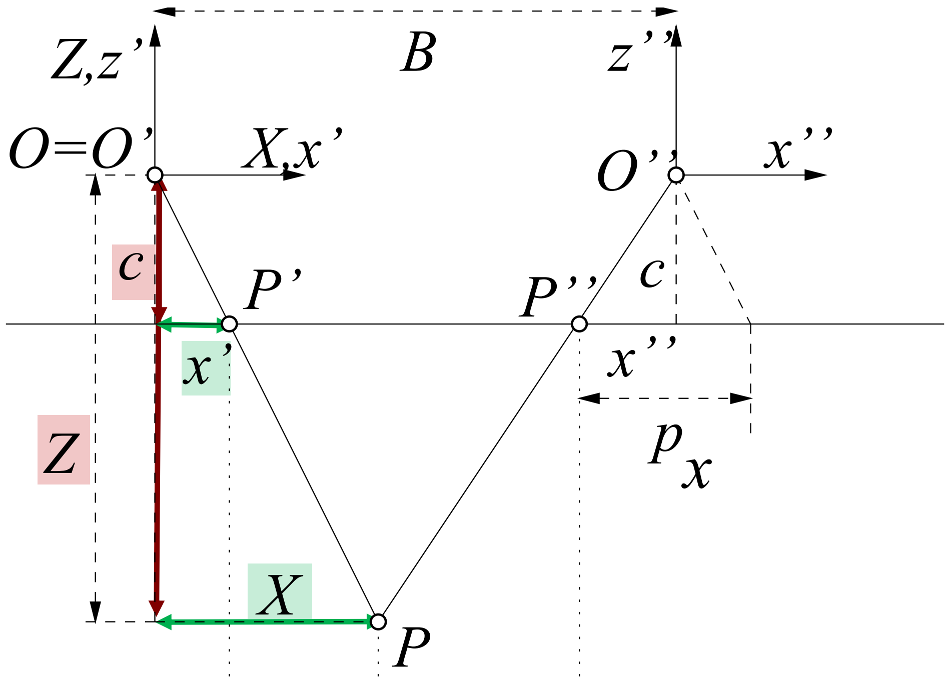


Image courtesy: Förstner

Stereo Normal: Intersection

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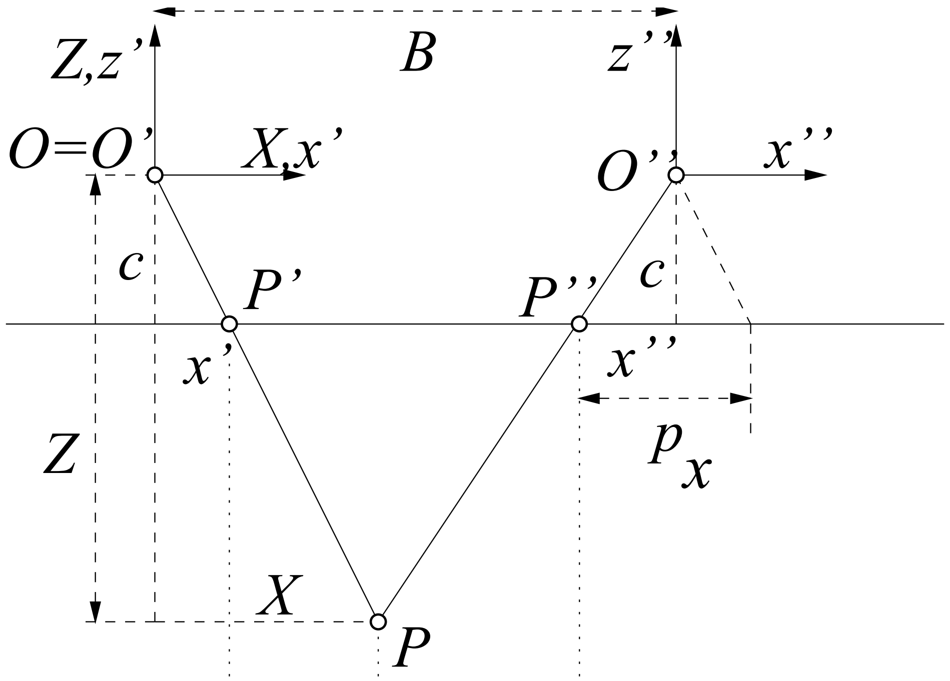

$$\Rightarrow \quad X = x' \frac{B}{-(x'' - x')}$$

Image courtesy: Förstner

Stereo Normal: Intersection

1. Z-coordinate from intercept theorem

$$\frac{Z}{c} = \frac{B}{-(x'' - x')}$$

2. X-coordinate

$$\frac{X}{x'} = \frac{Z}{c}$$

3. Y-coordinate by mean

$$\frac{Y}{X} = \frac{\frac{y'+y''}{2}}{x'}$$

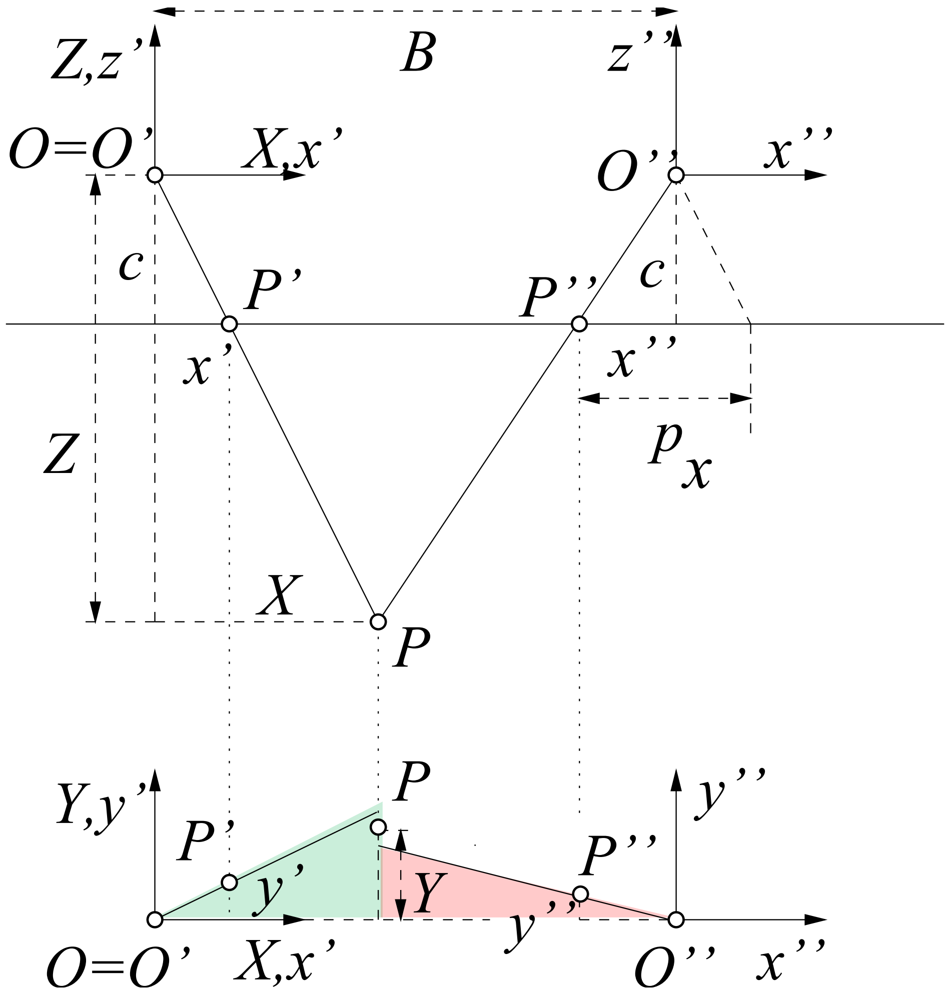

Image courtesy: Förstner

Stereo Normal: Intersection

1. Z-coordinate from intercept theorem

$$\frac{Z}{c} = \frac{B}{-(x'' - x')}$$

2. X-coordinate

$$\frac{X}{x'} = \frac{Z}{c}$$

3. Y-coordinate by mean

$$\frac{Y}{X} = \frac{\frac{y'+y''}{2}}{x'}$$

$$\Rightarrow \quad Y = \frac{y' + y''}{2} \frac{B}{-(x'' - x')}$$

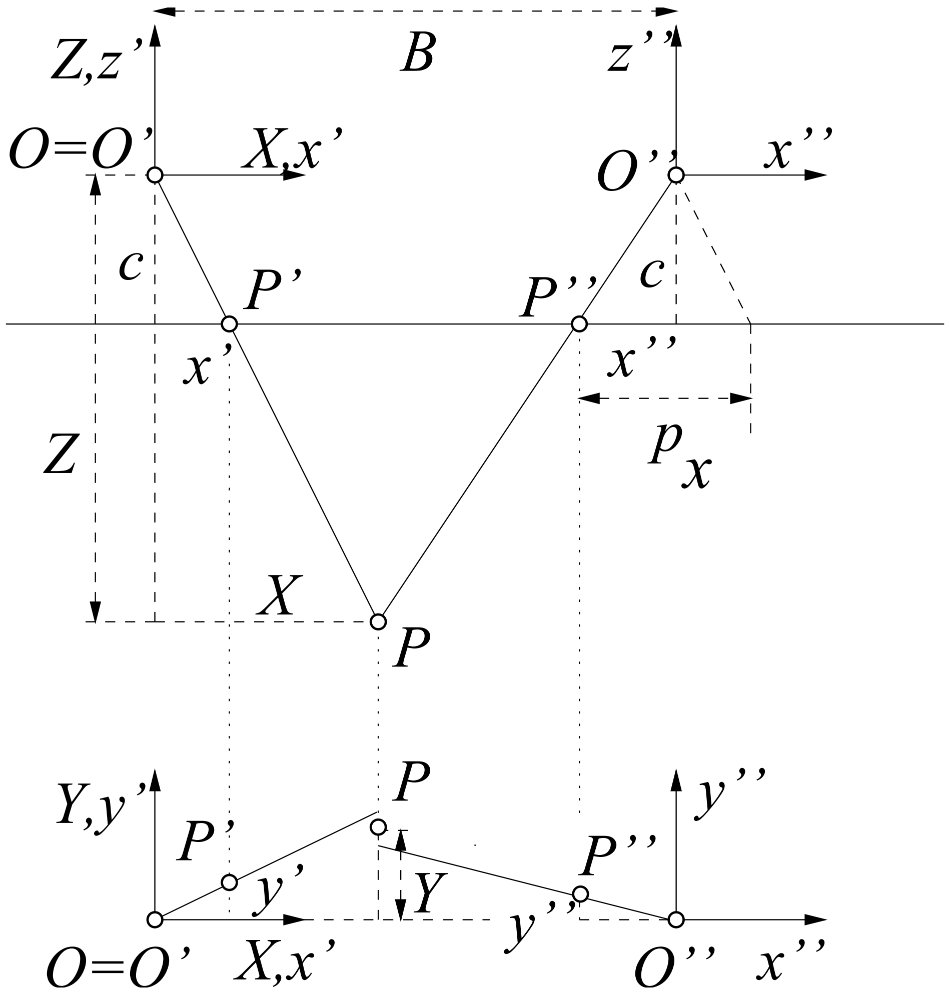

Image courtesy: Förstner

Intersection of Two Rays for the Stereo Normal Case

$$X = x' \frac{B}{-(x'' - x')} \qquad Y = \frac{y' + y''}{2} \frac{B}{-(x'' - x')} \qquad Z = c \frac{B}{-(x'' - x')}$$

- x-parallax $p_x = x'' - x'$ corresponds to depth Z
- y-parallax $p_y = y'' - y'$ corresponds to the consistency of image points and should be small (due to stereo normal case)
- The parallax is also called disparity

Literature

- Förstner, Wrobel: Photogrammetric Computer Vision, Ch. 12.4 - 12.6

Slide Information

- The slides have been created by Cyrill Stachniss as part of the photogrammetry and robotics courses.
- I tried to acknowledge all people from whom I used images or videos. In case I made a mistake or missed someone, please let me know.
- The photogrammetry material heavily relies on the very well written lecture notes by Wolfgang Förstner and the Photogrammetric Computer Vision book by Förstner & Wrobel.
- Parts of the robotics material stems from the great Probabilistic Robotics book by Thrun, Burgard and Fox.

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