CS 775: Advanced Computer Graphics

Lecture 9 : Cloth Simulation

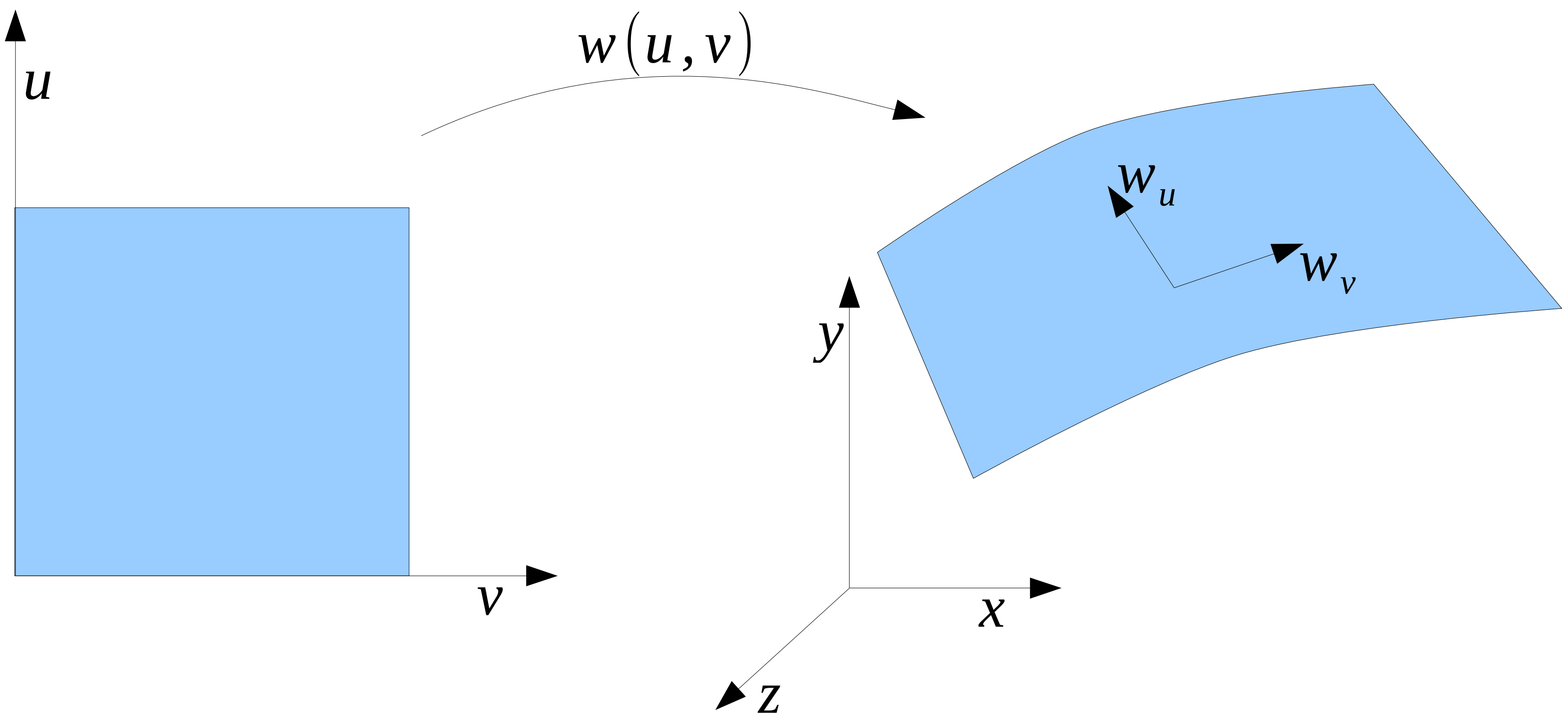
Simulating Cloth

- Cloth is thin and flexible
- Does not stretch easily – resistant to stretch
- Bends easily – folds and wrinkles
- Self Collision
- Woven vs Knit

Efficient Simulation of Inextensible Cloth, Goldenthal et al. SIGGRAPH 2007

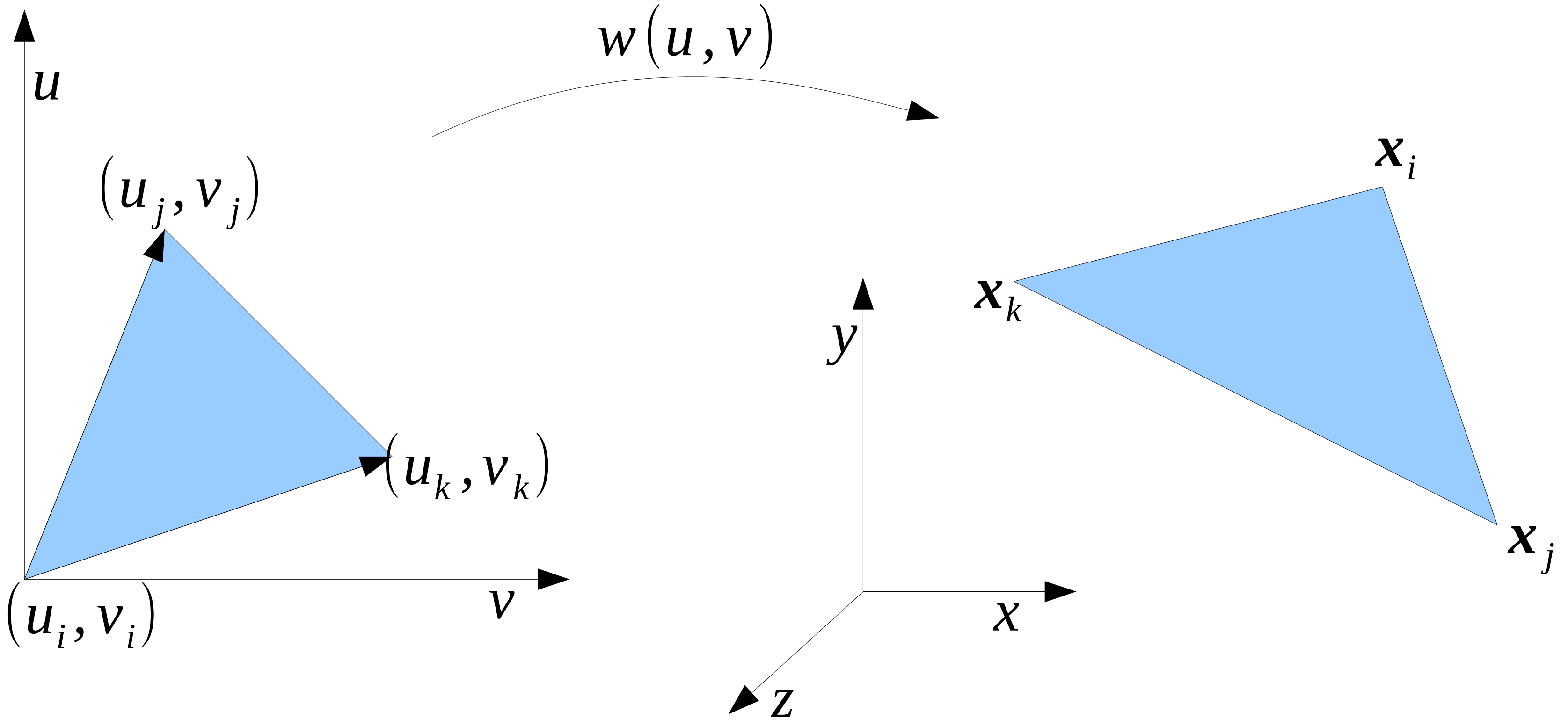
Modeling cloth

- A triangle mesh of particles.
- Rest state of cloth in *material* (u, v) space



Modeling cloth

- Assume $w(u,v)$ is linear over each triangle, and so its gradient wrt u and v is constant.



Modeling Cloth

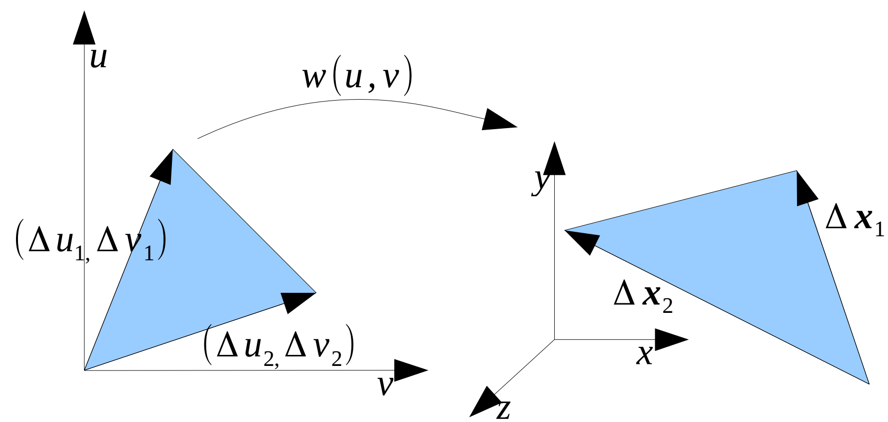
- Let $\Delta \boldsymbol{x}_1 = \boldsymbol{x}_j - \boldsymbol{x}_i \quad \Delta \boldsymbol{x}_2 = \boldsymbol{x}_k - \boldsymbol{x}_i$

$$\Delta u_1 = u_j - u_i \qquad \Delta v_1 = v_j - v_i$$
$$\Delta v_2 = v_k - v_i \qquad \Delta u_2 = u_k - u_i$$

$$\boldsymbol{w}_u = \frac{\partial \boldsymbol{w}}{\partial u} \qquad \boldsymbol{w}_v = \frac{\partial \boldsymbol{w}}{\partial v}$$

- then $\Delta \boldsymbol{x}_1 = \boldsymbol{w}_u \Delta u_1 + \boldsymbol{w}_v \Delta v_1 \qquad \Delta \boldsymbol{x}_2 = \boldsymbol{w}_u \Delta u_2 + \boldsymbol{w}_v \Delta v_2$
- Solving for $\boldsymbol{w}_u$ and $\boldsymbol{w}_v$,

$$\begin{pmatrix} \boldsymbol{w}_u & \boldsymbol{w}_v \end{pmatrix} = \begin{pmatrix} \Delta \boldsymbol{x}_1 & \Delta \boldsymbol{x}_2 \end{pmatrix} \begin{pmatrix} \Delta u_1 & \Delta u_2 \\ \Delta v_1 & \Delta v_2 \end{pmatrix}^{-1}$$



Equations of Motion

$$\boldsymbol{M}\,\ddot{\boldsymbol{x}} = \boldsymbol{f}_{\mathrm{int}} + \boldsymbol{f}_{ext}$$

$\boldsymbol{M}$	Mass matrix	$\boldsymbol{R}^{3n \times 3n}$
$\ddot{\boldsymbol{x}}$	Acceleration of particles	$\boldsymbol{R}^{3n}$
$\boldsymbol{f}_{\mathrm{int}}$	Cloth internal forces	$\boldsymbol{R}^{3n}$
$\boldsymbol{f}_{ext}$	External forces (Gravity, contact, wind)	$\boldsymbol{R}^{3n}$

We derive forces from potential energy function.

Potential Energy Functions

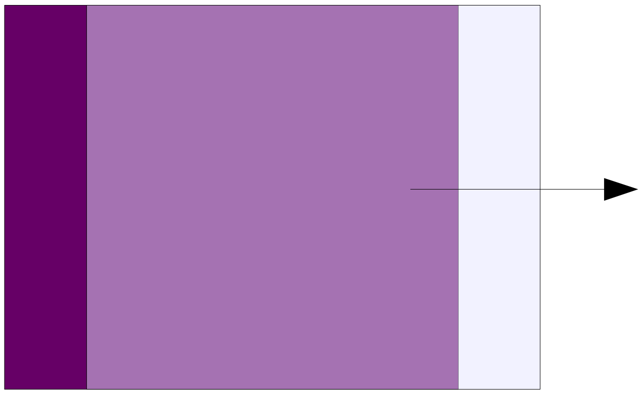
- If $\boldsymbol{C}(\boldsymbol{x})$ is a vector condition function we want to be zero
- Then the potential energy functions is defined as

$$E(\boldsymbol{x}) = \frac{k}{2}\boldsymbol{C}^T(\boldsymbol{x})\boldsymbol{C}(\boldsymbol{x})$$

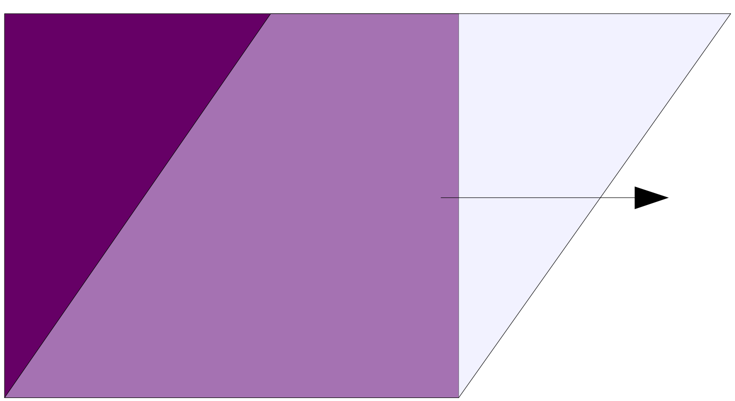
- Force is the negative gradient of potential energy, so

$$\boldsymbol{f} = \frac{-\partial E}{\partial \boldsymbol{x}} = -k\frac{\partial \boldsymbol{C}(\boldsymbol{x})^T}{\partial \boldsymbol{x}}\boldsymbol{C}(\boldsymbol{x})$$

Stretch and Shear Energy

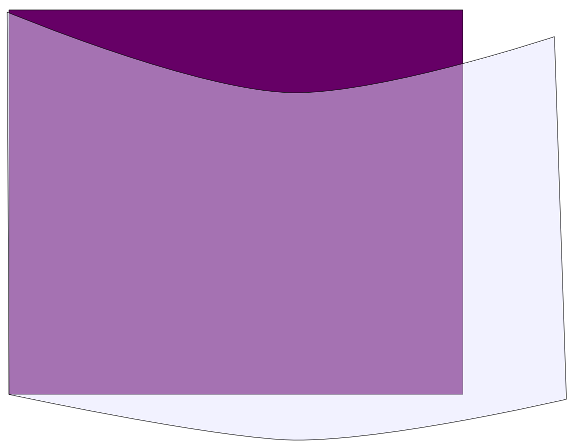


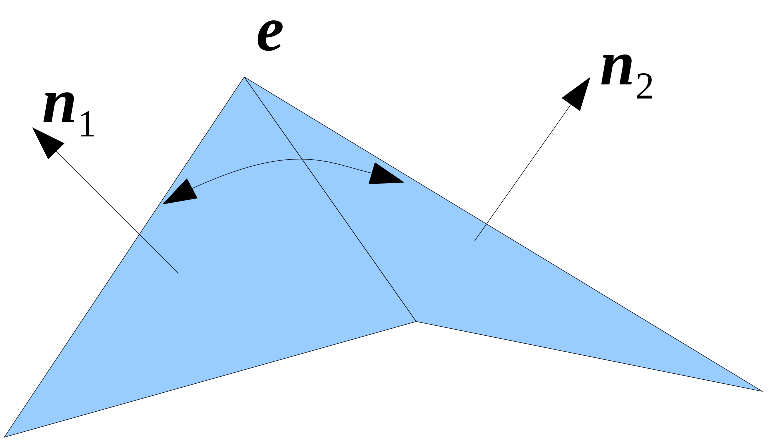
$$C(\boldsymbol{x}) = a\begin{pmatrix} \|\boldsymbol{w}_u(\boldsymbol{x})\| - b_u \\ \|\boldsymbol{w}_v(\boldsymbol{x})\| - b_v \end{pmatrix}$$



$$C(\boldsymbol{x}) = a\,\boldsymbol{w}_u(\boldsymbol{x})^T \boldsymbol{w}_v(\boldsymbol{x})$$

Bend Energy





$$\sin\theta = (\boldsymbol{n}_1 \times \boldsymbol{n}_2) \cdot \boldsymbol{e}$$

$$\cos\theta = (\boldsymbol{n}_1 \cdot \boldsymbol{n}_2)$$

$$\boldsymbol{C}(\boldsymbol{x}) = \theta$$

What solver to use?

- Cloth shows little in-plane stretch.
- So the stiffness (resistance) for the stretch forces is very high.
- Use implicit solvers.

Implicit Solver for Cloth

Re-write the second order ODE as two coupled first order ODE's

$$\frac{d}{dt}\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{v}\end{pmatrix}=\begin{pmatrix}\boldsymbol{v}\\ \boldsymbol{M}^{-1}\boldsymbol{f}(\boldsymbol{x},\boldsymbol{v})\end{pmatrix}$$

Compute derivative at next state to get implicit form

$$\Delta\boldsymbol{x}=\boldsymbol{x}(t_0+h)-\boldsymbol{x}_0 \qquad \Delta\boldsymbol{v}=\boldsymbol{v}(t_0+h)-\boldsymbol{v}_0$$

$$\begin{pmatrix}\Delta\boldsymbol{x}\\ \Delta\boldsymbol{v}\end{pmatrix}=h\begin{pmatrix}\boldsymbol{v}_0+\Delta\boldsymbol{v}\\ \boldsymbol{M}^{-1}\boldsymbol{f}(\boldsymbol{x}_0+\Delta\boldsymbol{x},\boldsymbol{v}_0+\Delta\boldsymbol{v})\end{pmatrix}$$

Implicit Solver for Cloth

Linearize forces about current state

$$\boldsymbol{f}(\boldsymbol{x}_0+\Delta\boldsymbol{x},\boldsymbol{v}_0+\Delta\boldsymbol{v})=\boldsymbol{f}_0+\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}\Delta\boldsymbol{x}+\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\Delta\boldsymbol{v}$$

Compute next time step

$$\begin{pmatrix}\Delta\boldsymbol{x}\\ \Delta\boldsymbol{v}\end{pmatrix}=h\begin{pmatrix}\boldsymbol{v}_0+\Delta\boldsymbol{v}\\ \boldsymbol{M}^{-1}\left(\boldsymbol{f}_0+\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}\Delta\boldsymbol{x}+\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\Delta\boldsymbol{v}\right)\end{pmatrix}$$

$$\Rightarrow\Delta\boldsymbol{v}=h\boldsymbol{M}^{-1}\left(\boldsymbol{f}_0+\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}\Delta\boldsymbol{x}+\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\Delta\boldsymbol{v}\right)$$

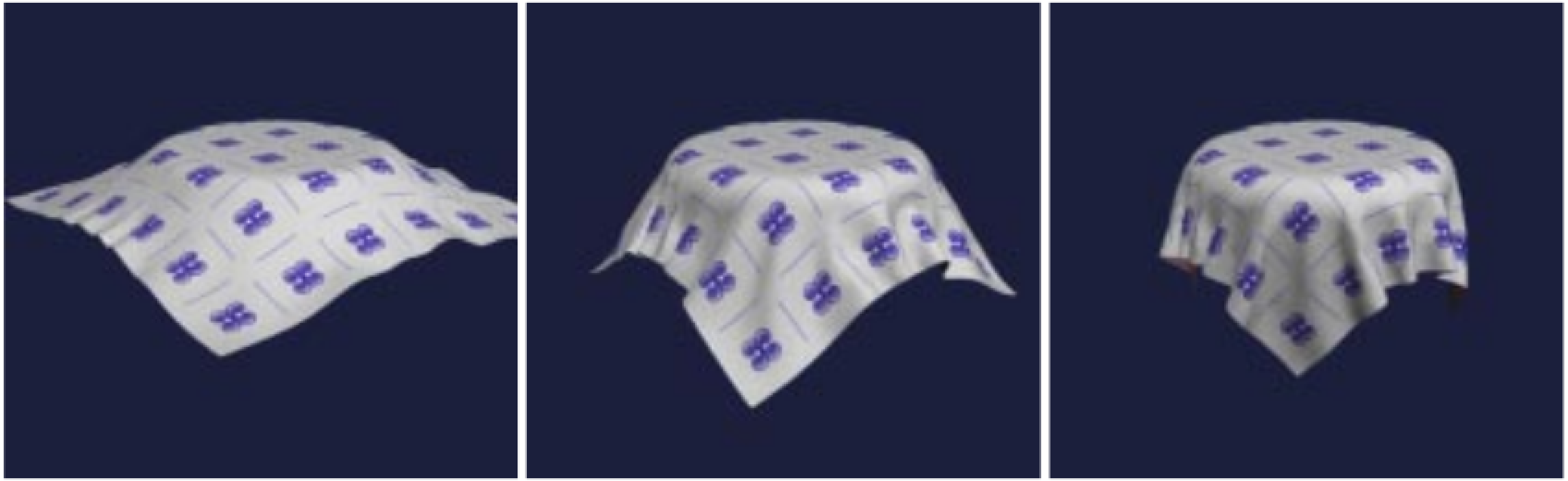
$$\Rightarrow\left(\boldsymbol{I}-h\boldsymbol{M}^{-1}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}-h^2\boldsymbol{M}^{-1}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}\right)\Delta\boldsymbol{v}=h\boldsymbol{M}^{-1}\left(\boldsymbol{f}_0+h\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}\Delta\boldsymbol{v}_0\right)$$

Cloth Simulation

Update position and velocity

$$\boldsymbol{v} = \boldsymbol{v}_0 + \Delta\,\boldsymbol{v}$$

$$\boldsymbol{x} = \boldsymbol{x}_0 + h\,\boldsymbol{v}$$



Large Steps in Cloth Simulation, Baraff and Witkin, SIGGRAPH 1998