

Computer Vision (CS763)

Teaching cameras to “see”

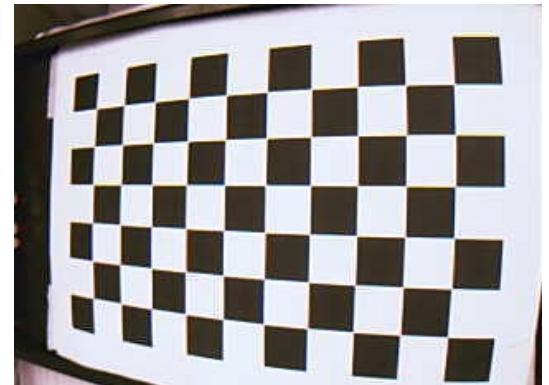
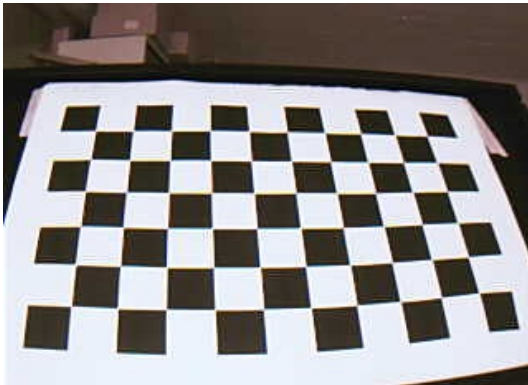
Camera Extrinsics and Intrinsics

Arjun Jain

Camera Calibration Using a 2D Checkerboard (Zhang)

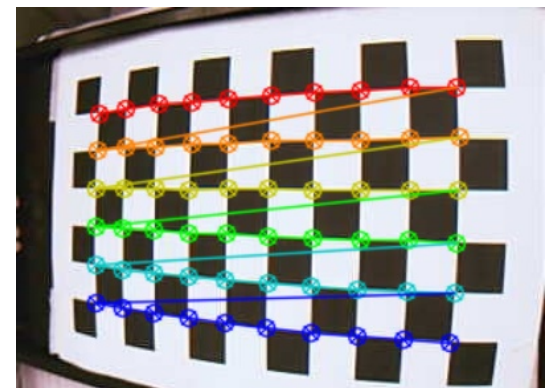
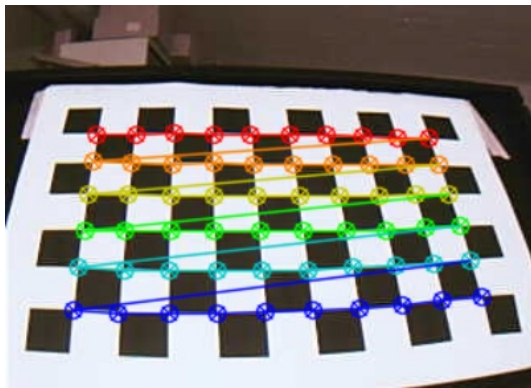
Camera Calibration Using a 2D Checkerboard (Zhang)

- Observed 2D pattern (checkerboard)
- **Known size and structure**



Trick for Checkerboard Calibration

- **Set the world coordinate system to the corner of the checkerboard**
- All points on the checkerboard lie in the X/Y plane, i.e., $Z=0$



Simplification

- The Z coordinate of all points on the checkerboard is equal to zero

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ \boxed{Z} \\ 1 \end{bmatrix}$$

Simplification

- The Z coordinate of all points on the checkerboard is equal to zero

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Simplification

- The Z coordinate of all points on the checkerboard is equal to zero

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

The diagram shows the extrinsic parameter matrix with red boxes and X marks indicating the simplification process. The third column of the extrinsic matrix (containing r_{13} , r_{23} , and r_{33}) is boxed and crossed out with a red X. A red line connects this box to the Z coordinate in the point vector, which is also boxed and crossed out with a red X.

- We can delete the 3rd column of the extrinsic parameter matrix

Simplification

- The Z coordinate of all points on the checkerboard is equal to zero
- Deleting the 3rd column of the extrinsic parameter matrix leads to

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Setting Up the Equations for Determining the Parameter

$$H = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_K \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[r_1, r_2, t]}$$

Setting Up the Equations for Determining the Parameter

$$H = [h_1, h_2, h_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_K \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[r_1, r_2, t]}$$

One point generates the equation:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = K[r_1, r_2, t] \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Setting Up the Equations for Determining the Parameter

- For multiple points, we obtain

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \underset{3 \times 3}{\mathbf{H}} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix} \quad i = 1, \dots, I$$

How to proceed?

Setting Up the Equations for Determining the Parameter

- For multiple points, we obtain

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \underset{3 \times 3}{\mathbf{H}} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix} \quad i = 1, \dots, I$$

- Analogous to **steps 1-3 of the DLT**

DLT-Like Estimation

- We estimate a 3x3 homography instead of a 3x4 projection matrix
- Rest is identical (instead of Z coord.)

- We use $a_{x_i}^\top h = 0$
 $a_{y_i}^\top h = 0$

- with

$$h = (h_k) = \text{vec}(H^\top)$$

$$a_{x_i}^\top = (-X_i, -Y_i, -\cancel{Z_i}, -1, 0, 0, \cancel{0}, 0, x_i X_i, x_i Y_i, x_i \cancel{Z_i}, x_i)$$

$$a_{y_i}^\top = (0, 0, \cancel{0}, 0, -X_i, -Y_i, -\cancel{Z_i}, -1, y_i X_i, y_i Y_i, y_i \cancel{Z_i}, y_i)$$

DLT-Like Estimation

- We estimate a 3x3 homography instead of a 3x4 projection matrix
- Rest is identical (instead of Z coord.)
- We use $a_{x_i}^\top h = 0$
 $a_{y_i}^\top h = 0$
- with

$$\begin{aligned} h &= (h_k) = \text{vec}(H^\top) \\ a_{x_i}^\top &= (-X_i, -Y_i, -1, 0, 0, 0, x_i X_i, x_i Y_i, x_i) \\ a_{y_i}^\top &= (0, 0, 0, -X_i, -Y_i, -1, y_i X_i, y_i Y_i, y_i) \end{aligned}$$

DLT-Like Estimation

- Solving a system of linear equation leads to an estimate of H
- We need to identify at least 4 points as H has 8 DoF and each point consists of two observations

**We estimated H and
now we need to compute K from H**

Computing K Given H

$$H = [h_1, h_2, h_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_K \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[r_1, r_2, t]}$$



**no rotation matrix, thus
QR decomposition is not
applicable as for DLT**

Computing K Given H is Different From the DLT Solution

- Homography H has only 8 DoF
- No direct decomposition as in DLT
- Exploit constraints on the intrinsic parameters

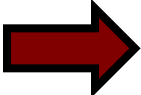
$$H = [h_1, h_2, h_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_K \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[r_1, r_2, t]}$$

$$[h_1, h_2, h_3] = K[r_1, r_2, t]$$

Exploiting Constraints for Determining the Parameter

$$H = [h_1, h_2, h_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_K \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[r_1, r_2, t]}$$

$$[h_1, h_2, h_3] = K[r_1, r_2, t]$$

 $r_1 = K^{-1}h_1 \quad r_2 = K^{-1}h_2$

Exploiting Constraints for Determining the Parameter

$$H = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_K \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]}$$

$$[\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3] = K[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]$$

$$\mathbf{r}_1 = K^{-1}\mathbf{h}_1 \quad \mathbf{r}_2 = K^{-1}\mathbf{h}_2$$

As $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ form an orthonormal basis

$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = \|\mathbf{r}_2\| = 1$$

Exploiting Constraints

$$r_1 = K^{-1}h_1 \quad r_2 = K^{-1}h_2$$

$$r_1^T r_2 = 0$$



$$h_1^T K^{-T} K^{-1} h_2 = 0$$

Exploiting Constraints

$$r_1 = K^{-1}h_1 \quad r_2 = K^{-1}h_2$$

$$r_1^T r_2 = 0$$



$$h_1^T K^{-T} K^{-1} h_2 = 0$$

$$\|r_1\| = \|r_2\| = 1$$



$$h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$$

Exploiting Constraints

$$r_1 = K^{-1}h_1 \quad r_2 = K^{-1}h_2$$

$$r_1^T r_2 = 0$$



$$h_1^T K^{-T} K^{-1} h_2 = 0$$

$$\|r_1\| = \|r_2\| = 1$$



$$h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$$

$$h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0$$

Exploiting Constraints

$$r_1 = K^{-1}h_1 \quad r_2 = K^{-1}h_2$$

$$h_1^T K^{-T} K^{-1} h_2 = 0$$

$$h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$$

$$h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0$$

Exploiting Constraints

$$h_1^T \underline{K^{-T} K^{-1}} h_2 = 0$$

$$\underline{h_1^T K^{-T} K^{-1} h_1} - \underline{h_2^T K^{-T} K^{-1} h_2} = 0$$

- Define symmetric and positive definite matrix $B := K^{-T} K^{-1}$

Exploiting Constraints

$$h_1^T \underline{B} h_2 = 0$$

$$h_1^T \underline{B} h_1 - h_2^T \underline{B} h_2 = 0$$

- Define symmetric and positive definite matrix $B := K^{-T} K^{-1}$

Exploiting Constraints

$$\underline{h_1^T B h_2} = 0$$

$$\underline{h_1^T B h_1} - \underline{h_2^T B h_2} = 0$$

- Define symmetric and positive definite matrix $B := K^{-T} K^{-1}$
- If B is known, the calibration matrix can be recovered through Cholesky decomp.

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$$

$$\text{chol}(B) = A A^T$$

$$A = K^{-T}$$

Recap: Cholesky Decomposition

- Cholesky decomposition or Cholesky factorization is a decomposition of a positive-definite matrix into the product of a lower triangular matrix and its transpose

$$\begin{pmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 6 & 1 & 0 \\ -8 & 5 & 3 \end{pmatrix} \begin{pmatrix} 2 & 6 & -8 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

Exploiting Constraints

- Define a vector $b = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33})$ of unknowns

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$$

Exploiting Constraints

- Define a vector $b = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33})$ of unknowns

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$$

- Construct a system of linear equations $Vb = 0$ using $h_i^T B h_j = v_{ij}^T b$ (v_{ij} see next slide) exploiting the constraints:

$$v_{12}^T b = 0$$

(first constraint)

$$r_1^T r_2 = 0$$

$$v_{11}^T b - v_{22}^T b = 0$$


(second constraint)

$$\|r_1\| = \|r_2\| = 1$$

The Matrix V

- The matrix V is given as

$$V = \begin{pmatrix} & v_{12}^T \\ v_{11}^T & -v_{22}^T \end{pmatrix} \quad \text{with} \quad v_{ij} = \begin{bmatrix} h_{1i}h_{1j} \\ h_{1i}h_{2j} + h_{2i}h_{1j} \\ h_{3i}h_{1j} + h_{1i}h_{3j} \\ h_{2i}h_{2j} \\ h_{3i}h_{2j} + h_{2i}h_{3j} \\ h_{3i}h_{3j} \end{bmatrix}$$


elements of H

- For one image, we obtain

$$\begin{pmatrix} & v_{12}^T \\ v_{11}^T & -v_{22}^T \end{pmatrix} b = 0$$

The Matrix V

- For multiple images, we stack the matrices to a $2n \times 6$ matrix

$$\begin{array}{l} \text{image 1} \longrightarrow \\ \text{image n} \longrightarrow \end{array} \begin{pmatrix} v_{11}^T & v_{12}^T & -v_{22}^T \\ \vdots & \vdots & \vdots \\ v_{11}^T & v_{12}^T & -v_{22}^T \end{pmatrix} b = 0$$

- We need to solve the linear system $Vb = 0$ to obtain b and thus K

Solving the Linear System

- The system $Vb = 0$ has a trivial solution which (invalid matrix B)
- Impose additional constraint $\|b\| = 1$

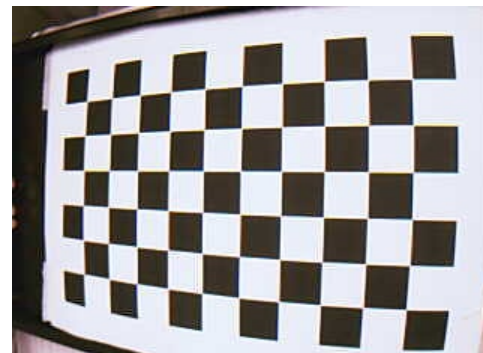
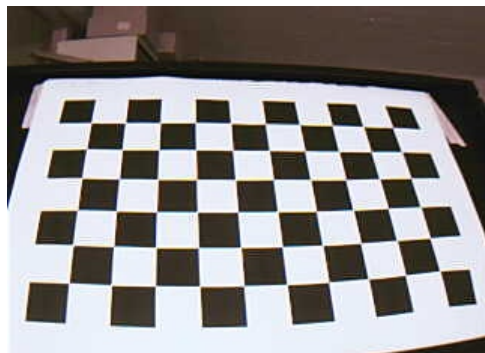
Solving the Linear System

- The system $Vb = 0$ has a trivial solution which (invalid matrix B)
- Impose additional constraint $\|b\| = 1$
- Real measurements are noisy
- Find the solution that minimizes the squares error

$$b^* = \arg \min_b \|Vb\| \text{ with } \|b\| = 1$$

- Eigenvector/Eigenvalue problem similar to the DLT computation

What is Needed?

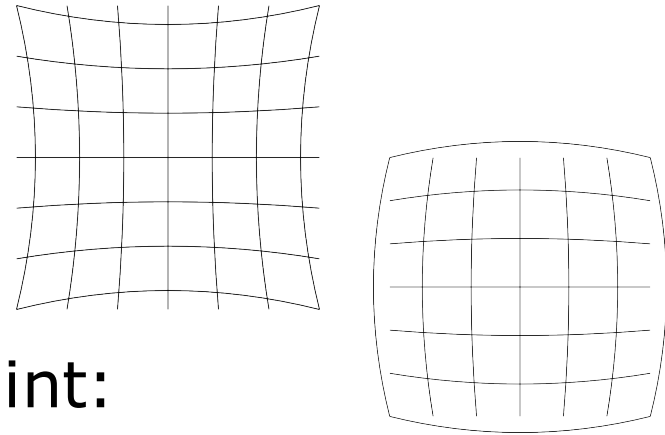


- We need at least **4 points per plane** to compute the matrix H
- Each **plane** gives us **two equations**
- Since B has 6 DoF, we need at least **3 different views of a plane**
- Solve $\forall b = 0$ to compute K

Example Lens Distortion Model

Non-linear effects:

- Radial distortion
- Tangential distortion



- Compute the corrected image point:

$$(1) \quad \begin{aligned} x' &= x/z \\ y' &= y/z \end{aligned}$$

$$(2) \quad \begin{aligned} x'' &= x'(1 + k_1 r^2 + k_2 r^4) + 2p_1 x' y' + p_2 (r^2 + 2x'^2) \\ y'' &= y'(1 + k_1 r^2 + k_2 r^4) + p_1 (r^2 + 2y'^2) + 2p_2 x' y' \end{aligned}$$

where $r^2 = x'^2 + y'^2$ k_1, k_2 : radial distortion coefficients

p_1, p_2 : tangential distortion coefficients

$$(3) \quad \begin{aligned} u &= f_x \cdot x'' + c_x \\ v &= f_y \cdot y'' + c_y \end{aligned}$$

Error Minimization

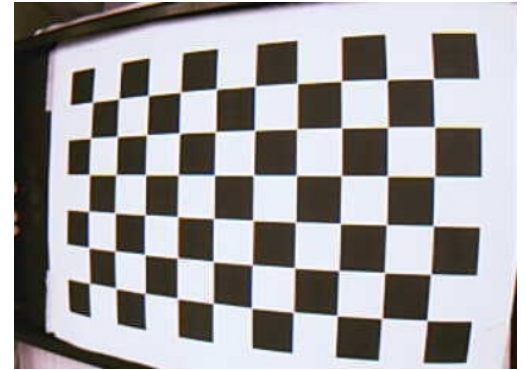
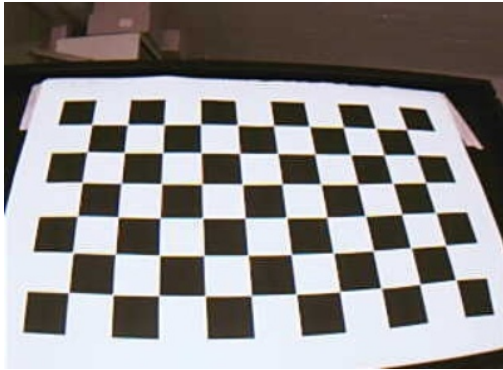
Lens distortion can be calculated by minimizing a non-linear error function

$$\min_{(K, \mathbf{q}, R_n, \mathbf{t}_n)} \sum_n \sum_i \|\mathbf{x}_{ni} - \hat{\mathbf{x}}(K, \mathbf{q}, R_n, \mathbf{t}_n, \mathbf{X}_{ni})\|^2$$

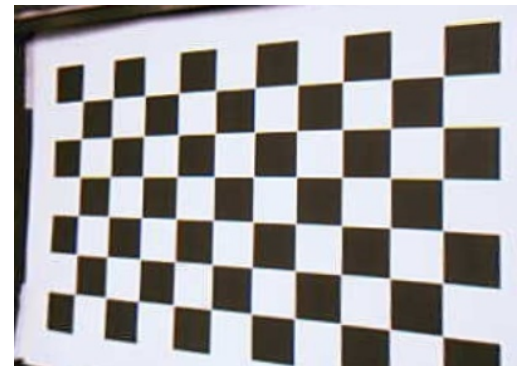
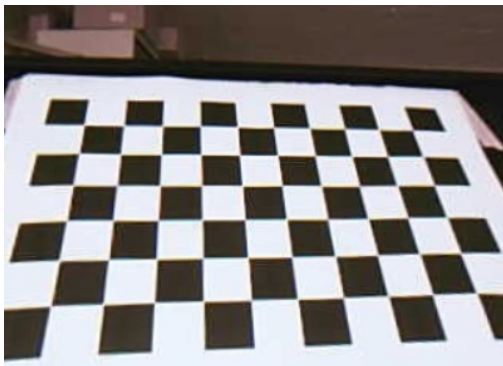
...linearize to obtain a quadratic function, compute derivative, set it to 0, solve linear system, iterate...
(solved using Levenberg-Marquardt)

Example Results

- Before calibration:



- After calibration:

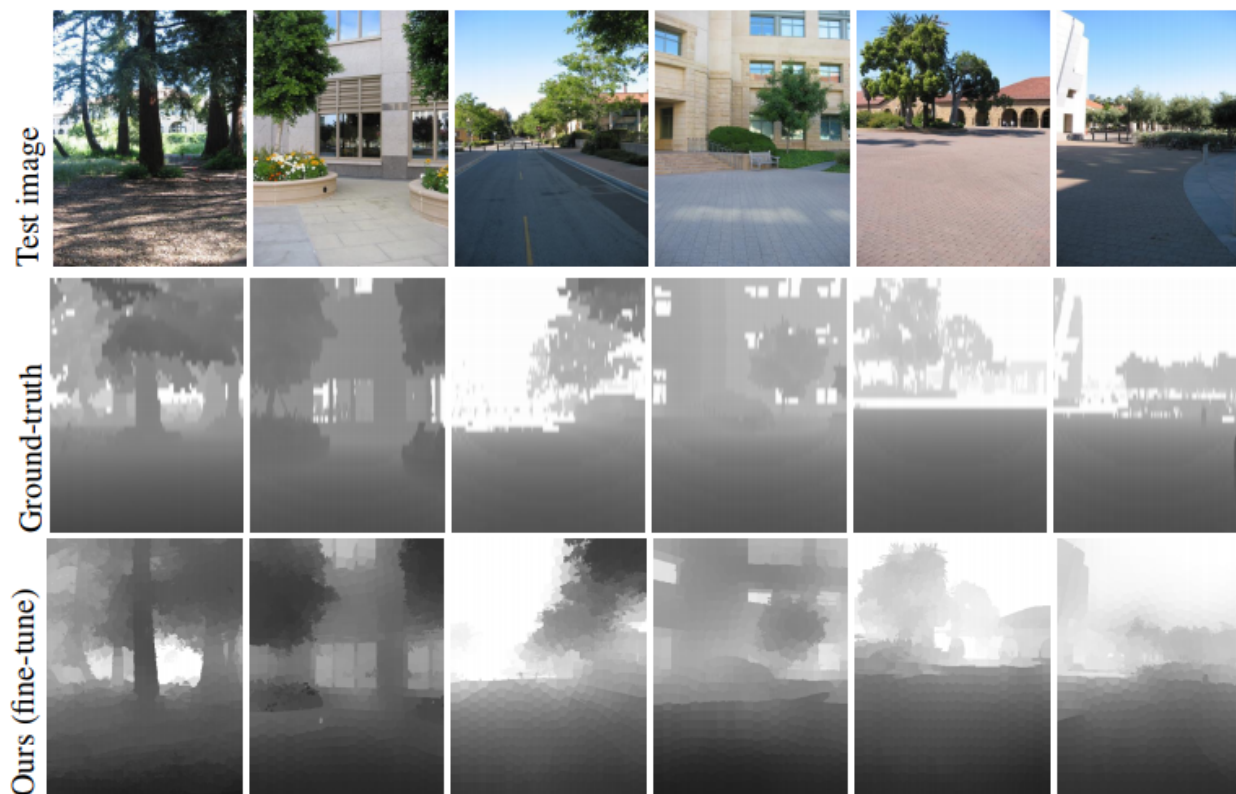


Summary on Camera Calibration Using a Checkerboard

- Pinhole camera model (first step)
- Non-linear model for lens distortion (second step)
- Approach to 2D camera calibration that
 - accurately determines the model parameters
 - is easy to realize

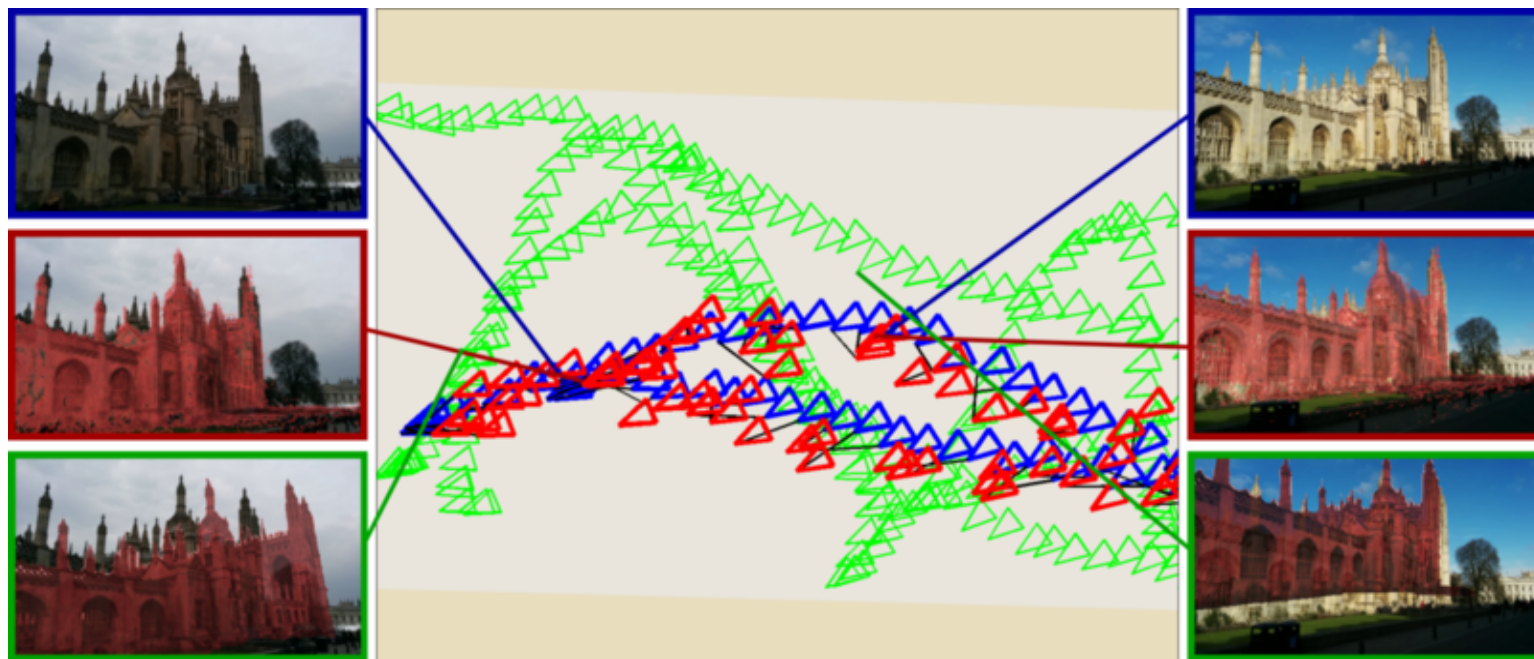
Depth from Single Image

- Using CNN to learn unary and pairwise potential of continuous CRF. Does not require geometric priors.

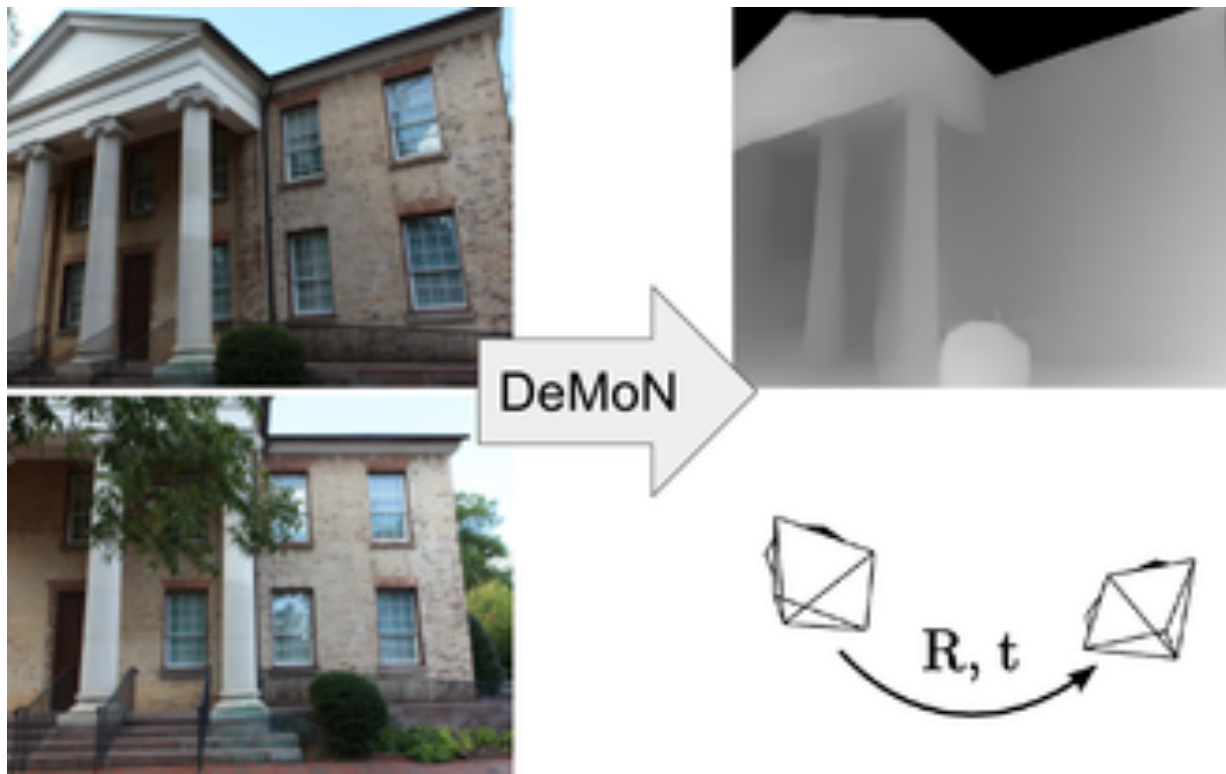


6-DOF Camera Relocalization

- Using CNN to regress a camera's 6-DOF relative to a scene using a RGB image.



Relative Motion Between Two Cameras



Slide Information

- The slides have been created by Cyrill Stachniss (cyrill.stachniss@igg.uni-bonn.de) as part of the photogrammetry and robotics courses.
- A lot of material from Ajit Rajwade's CS763 course
- Thanks to Parag for some slides
- Thanks to Rahul Mitra for DL slides
- **I tried to acknowledge all people from whom I used images or videos. In case I made a mistake or missed someone, please let me know.**
- The photogrammetry material heavily relies on the very well written lecture notes by Wolfgang Förstner and the Photogrammetric Computer Vision book by Förstner & Wrobel.
- Parts of the robotics material stems from the great Probabilistic Robotics book by Thrun, Burgard and Fox.
- If you are a university lecturer, feel free to use the course material. If you adapt the course material, please make sure that you keep the acknowledgements to others and please acknowledge me as well. To satisfy my own curiosity, please send me email notice if you use my slides.

Arjun Jain, ajain@cse.iitb.ac.in