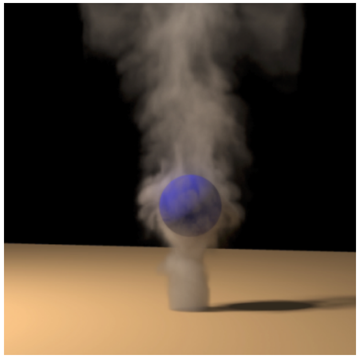
CS 775: Advanced Computer Graphics

Lecture 7: Particle Fluids

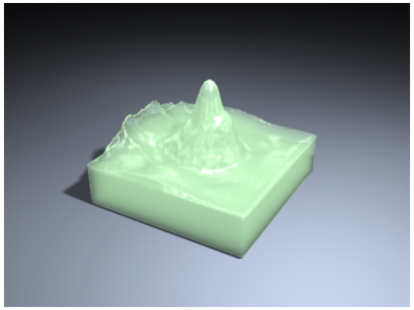
Fluid Simulation

- Why?
  - Games
  - Movies
  - Scientific Visualization
  - Medical Simulation
- What?
  - Smoke, Fire, Sand, Water,
  - Honey, Blood

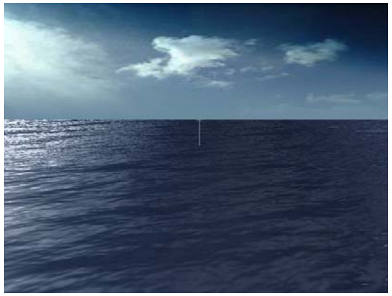


Visual Simulation of Smoke, Fedkiw, Stam, Jensen, SIGGRAPH 2001

Water Drops on Surfaces, Wang, Mucha, Turk, SIGGRAPH 2005



A Method for Animating Viscoelastic Fluids, Goktekin, Bargteil, O'Brien, SIGGRAPH 2004



Simulating Ocean Water, Tessendorf, SIGGRAPH 2001

Framestore VFX, Dawn Treader

Weta Digital, X-Men: First Class

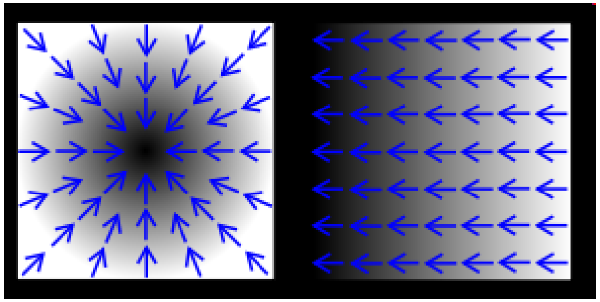
Fluid Simulation

- A **fluid** is a substance that continually deforms (flows) under an applied shear stress.
- Different types of fluids:

  - Incompressible (divergence-free) fluids: Fluid does not change volume (very much).
  - Compressible fluids: Fluids change their volume significantly.

  - Viscous fluids: Fluids tend to resist a certain degrees of deformation
  - Inviscid (Ideal) fluids: Fluids don’t have resistance to the shear stress
  - Newtonian fluids: Fluids in which stress is directly proportional to rate of strain
  - Non-Newtonian fluids: Fluids that have non-constant viscosity

Fluid Simulation



http://en.wikipedia.org/wiki/Gradient

- Calculus Review
  - Gradient (∇): A vector pointing in the direction of the greatest rate of increment

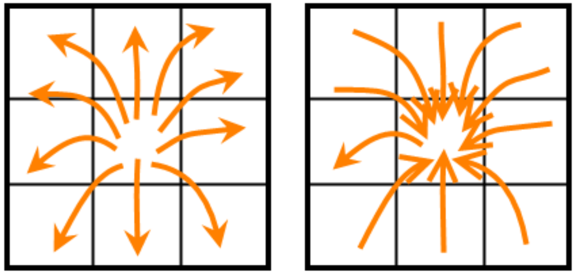
$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right)$$

u can be a scalar or a vector

  - Divergence ($\nabla \cdot$): Measure how the vectors are converging or diverging at a given location (volume density of outward flux)

$$\nabla \cdot u = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

u can only be a vector



http://www.cs.unc.edu/~lin/COMP768-S09/LEC/fluid.ppt

Fluid Simulation

- Calculus Review
  - Laplacian (∇^2): Divergence of the gradient

$$\nabla^2 u = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

u can be a scalar or a vector

Fluid Simulation

- Calculus Review

$$\text{If } F(x,y,z)=f(x,y,z)\hat{i}+g(x,y,z)\hat{j}+h(x,y,z)\hat{k}$$

$$\text{then, } \nabla F=\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial z} \end{bmatrix}$$

$$\nabla\cdot F=\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y}+\frac{\partial h}{\partial z}$$

$$\text{and, } \nabla^2 F=\nabla\cdot(\nabla F)=\begin{bmatrix} \frac{\partial}{\partial x}(\frac{\partial f}{\partial x})+\frac{\partial}{\partial y}(\frac{\partial f}{\partial y})+\frac{\partial}{\partial z}(\frac{\partial f}{\partial z}) \\ \frac{\partial}{\partial x}(\frac{\partial g}{\partial x})+\frac{\partial}{\partial y}(\frac{\partial g}{\partial y})+\frac{\partial}{\partial z}(\frac{\partial g}{\partial z}) \\ \frac{\partial}{\partial x}(\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(\frac{\partial h}{\partial z}) \end{bmatrix}$$

The Navier-Stokes

- Consider the fluid as a continuum
- $\boldsymbol{f} = m\,\boldsymbol{a}$ for fluids
  - For a small volume of fluid, $m = \rho v$
  - Acceleration = rate of change of velocity, $a = \frac{D\boldsymbol{u}}{Dt}$
  - Net force per unit volume, $\rho \frac{D\boldsymbol{u}}{Dt}$

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where $\frac{D\phi(x,t)}{Dt} = \frac{\partial \phi(x,t)}{\partial t} + \frac{\partial \phi(x,t)}{\partial x} \cdot \frac{\partial x}{\partial t}$

$= \frac{\partial \phi(x,t)}{\partial t} + \nabla \phi(x,t) \cdot \vec{u}$

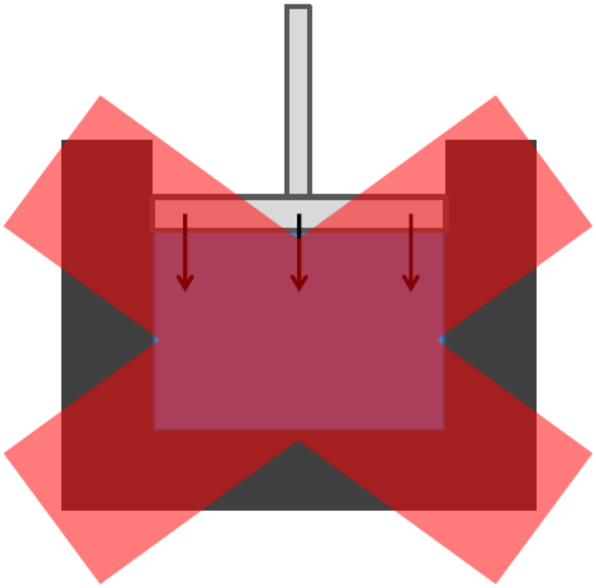
Material Derivative

The Navier-Stokes

- Forces on the fluid
- Gravity $\boldsymbol{f}_{gravity} = \rho\, \boldsymbol{g}$
  - Constant downward Force

The Navier-Stokes

- Forces on the fluid
- Pressure
  - Most fluids (liquids) are incompressible
  - This gives a constraint
    of the form $\nabla \cdot \boldsymbol{u} = 0$



The Navier-Stokes

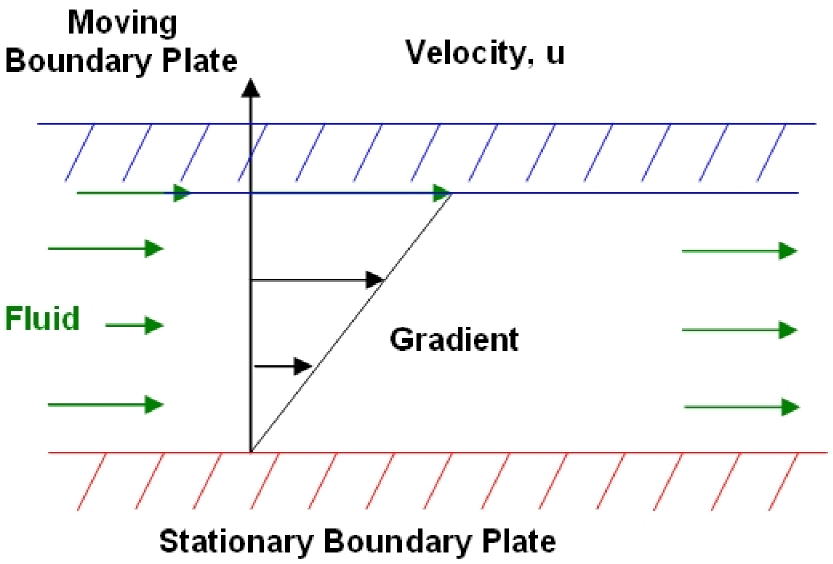
- Forces on the fluid
- Pressure
  - Pressure differences cause change in velocity, force is the gradient of pressure

$$\boldsymbol{f}_{\boldsymbol{pressure}} = -\nabla p$$

The Navier-Stokes

- Forces on the fluid
- Viscosity
  - Internal friction – Newtonian Fluids

$$\boldsymbol{f}_{viscosity} = \mu \nabla . \nabla \boldsymbol{u}$$



The Navier-Stokes

- $$\rho \frac{D\boldsymbol{u}}{Dt} = \boldsymbol{f}_{pressure} + \boldsymbol{f}_{viscosity} + \boldsymbol{f}_{gravity} + \boldsymbol{f}_{other}$$

  $$= -\nabla p + \mu \nabla . \nabla \boldsymbol{u} + \rho g + \boldsymbol{f}_{other}$$

Momentum Equation

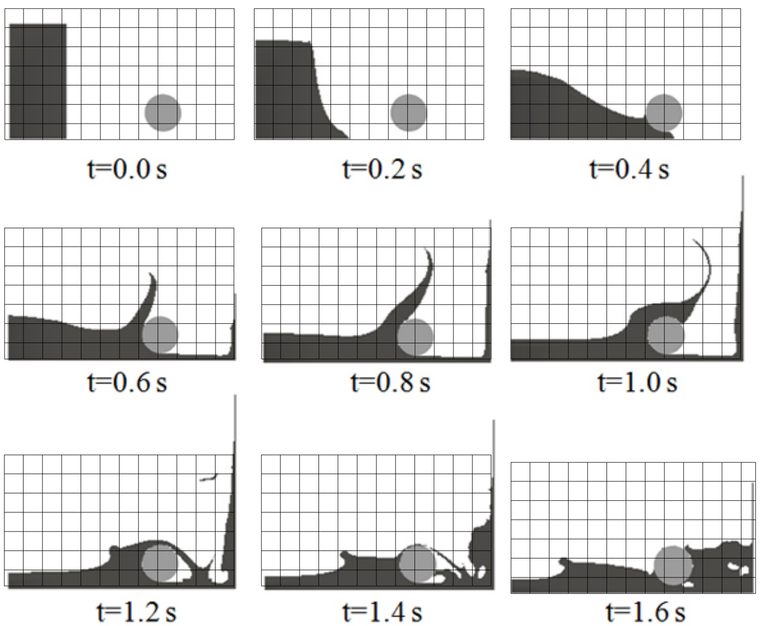
- $$\nabla \cdot \boldsymbol{u} = 0$$

Continuity Equation

Fluid Simulation

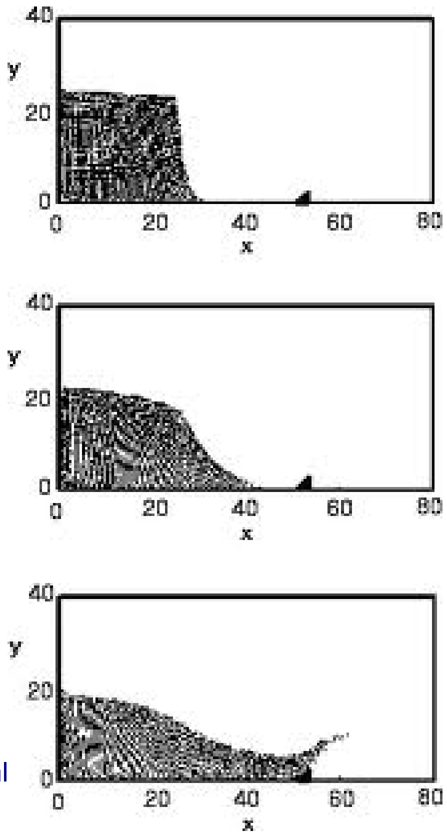
- Eulerian Viewpoint
  - Discretize the domain using *finite differences*
  - Define scalar & vector fields on the grid
  - Use the *operator splitting* technique to solve each term separately
  - Advantages
    - Derivative approximation
    - Adaptive time step/solver
  - Disadvantages
    - Memory usage & speed
    - Grid artifact/resolution limitation

http://www.eng.nus.edu.sg/EResnews/022014/images/rd02-fig1.jpg



Fluid Simulation

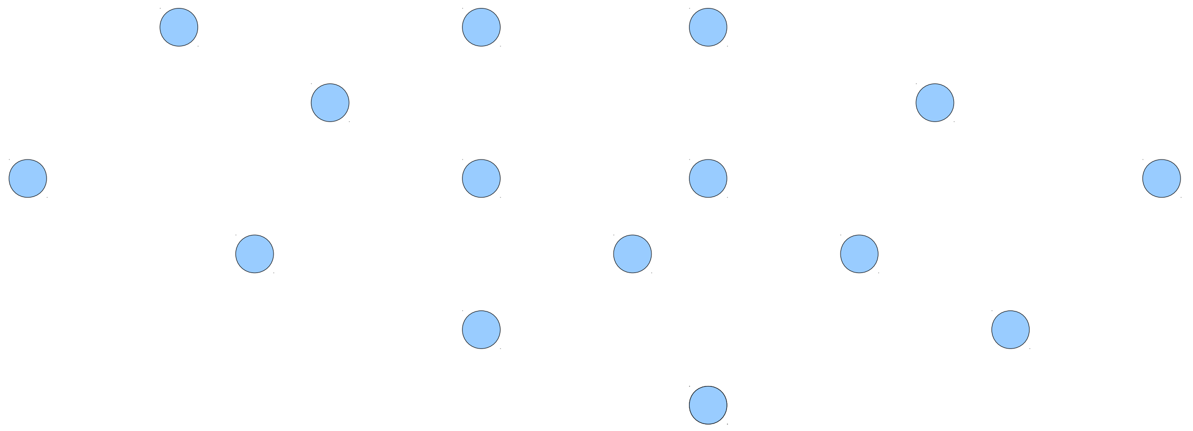
- Lagrangian Viewpoint
  - Discretize the domain using *particles*
  - Define interaction forces between neighbouring particles using smoothing kernels.
  - Advantages
    - Mass Conservation
    - Intuitive
  - Disadvantages
    - Incompressibility is indirectly enforced
    - Surface tracking



http://mec-mail.eng.monash.edu.au/~mct/mct/docs/sph/sph.html

Particle Fluids

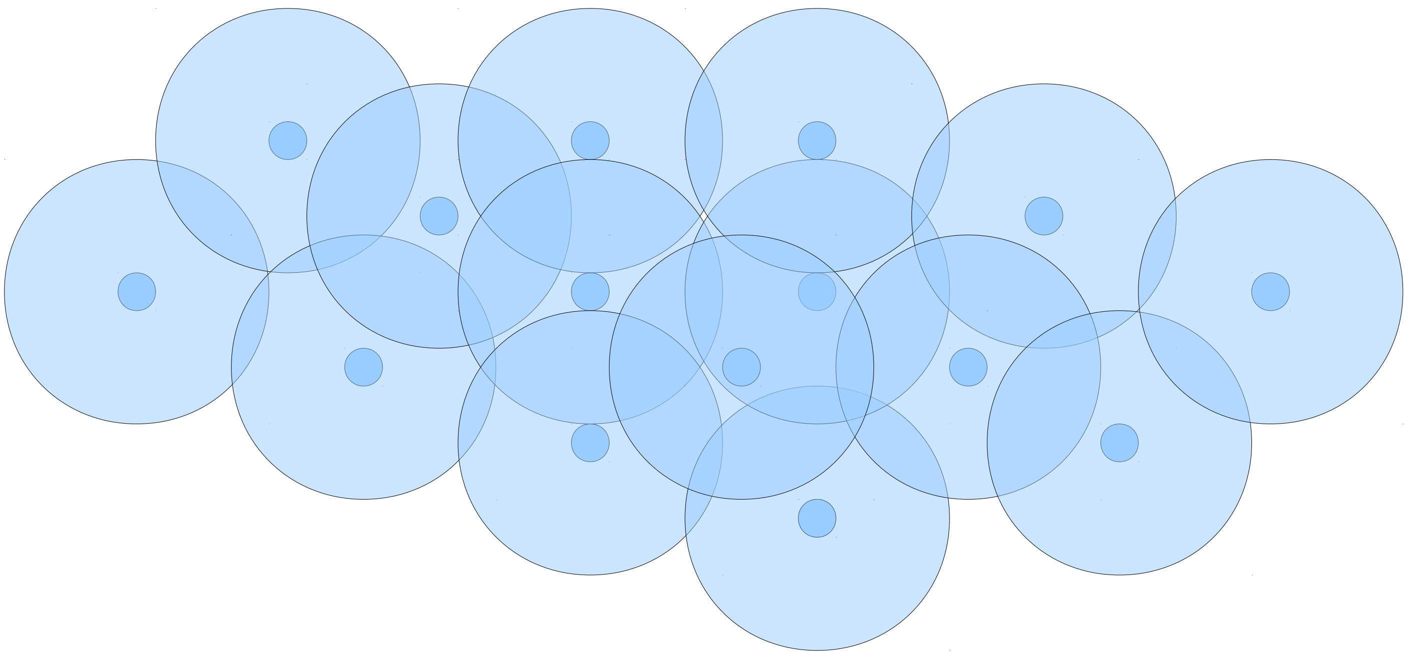
- Particles have mass, velocity and position



- But for a fluid we need to solve the NV everywhere in the fluid not just at the particles.

Particle Fluids

- "Smooth" particle information over an area

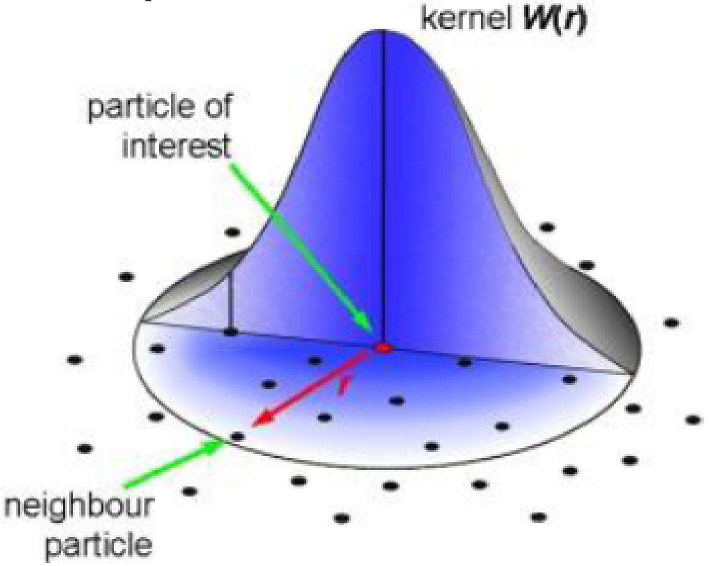


- Smoothed Particle Hydrodynamics (SPH)

Particle Fluids

- Use a smoothing kernel to spread out particle information

$$A_s(\boldsymbol{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\boldsymbol{r} - \boldsymbol{r}_j, h)$$



- Any field quantity $A_s(\boldsymbol{r})$ can be spread around the fluid particle by using the above form.

Particle Fluids

- Some Kernel Properties
  - Symmetric
  - Finite Support
  - Normalized
  - Smooth

$$\nabla A_s(\boldsymbol{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\boldsymbol{r} - \boldsymbol{r}_j, h)$$

$$W_{poly6}(\boldsymbol{r}, \boldsymbol{r}_j, h) = \frac{315}{64\pi h^9} \begin{cases} \left(h^2 - \|\boldsymbol{r} - \boldsymbol{r}_j\|^2\right)^3, & 0 \le \|\boldsymbol{r} - \boldsymbol{r}_j\| \le h \\ 0, & \text{otherwise} \end{cases}$$

Particle Fluid Simulation

- For each simulation timestep
  - Compute Density
  - Compute Pressure
  - Compute Pressure Force
  - Compute Viscocity Force
  - Compute Other Forces
  - Compute acceleration and velocity
  - Update particle position

Particle Fluid Simulation

- Compute Density $$\rho_i = \sum_j m_j W(\boldsymbol{r}_i - \boldsymbol{r}_j, h)$$
- Compute Pressure $$p_i = k(\rho_i - \rho_o)$$
- Compute Pressure Force

$$\boldsymbol{f}_i^{pressure} = -\frac{m_i}{\rho_i} \sum_j \frac{m_j}{\rho_j} \frac{p_i + p_j}{2} W(\boldsymbol{r}_i - \boldsymbol{r}_j, h)$$

- Compute Viscocity Force

$$\boldsymbol{f}_i^{viscosity} = \frac{m_i}{\rho_i} \sum_j \frac{m_j}{\rho_j} \frac{\mu_i + \mu_j}{2} (\boldsymbol{v}_i - \boldsymbol{v}_j) \nabla^2 W(\boldsymbol{r}_i - \boldsymbol{r}_j, h)$$

Particle Fluid Simulation

- Compute Surface Tension

$$\boldsymbol{f}_{\boldsymbol{i}}^{\boldsymbol{stension}} = -\gamma \sum_j m_j W(\boldsymbol{r}_i - \boldsymbol{r}_j, h)(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

Particle Fluid Simulation

- Incompressibility enforcement is indirect
  - Fluid is “mildly” compressible
- Enforced with an equation of state
  - Ideal gas law [Muller 2003] $p_i = k(\rho_i - \rho_o)$
  - Tait equation [Becker & Teschner 2007]

$$p_i = B\left(\left(\frac{\rho_i}{\rho_o}\right)^{\gamma} - 1\right)$$

Particle Fluid Simulation

- Update velocity and position

$$\boldsymbol{v}_{i+1/2} = \boldsymbol{v}_{i-1/2} + \boldsymbol{a}_i \delta t$$
$$\boldsymbol{x}_{i+1} = \boldsymbol{x}_i + \boldsymbol{v}_{i+1/2} \delta t$$

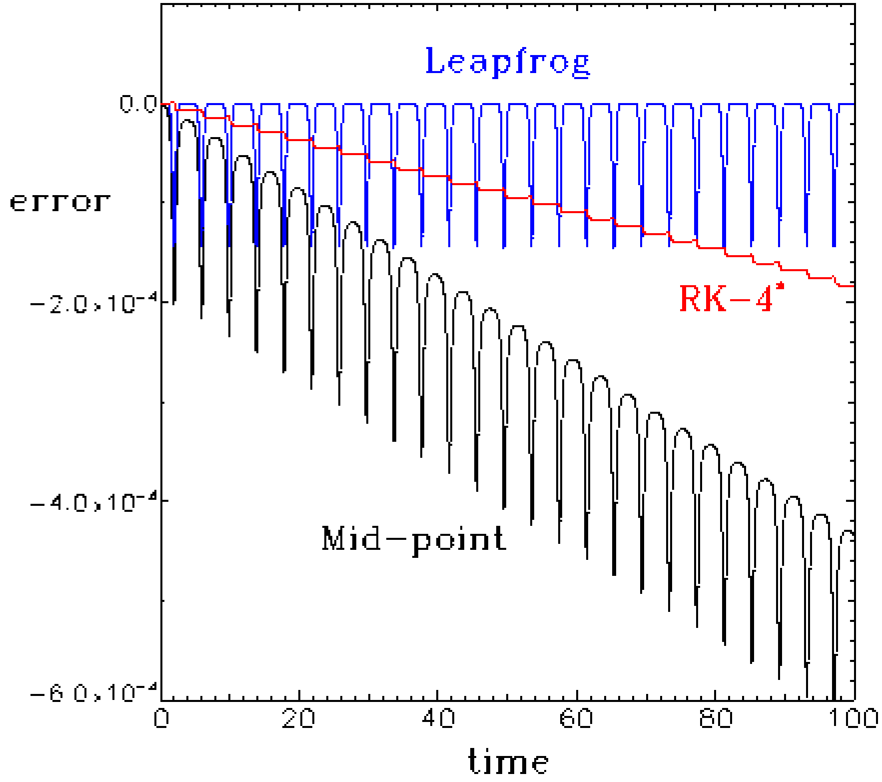
$$\boldsymbol{v}_{i+1} = \boldsymbol{v}_{i+1/2} + \boldsymbol{a}_i \delta t/2$$

Leapfrog Integrator
- Time reversible
- Symplectic

Müller et al., Particle-Based Fluid Simulation for Interactive Applications, SCA 2003

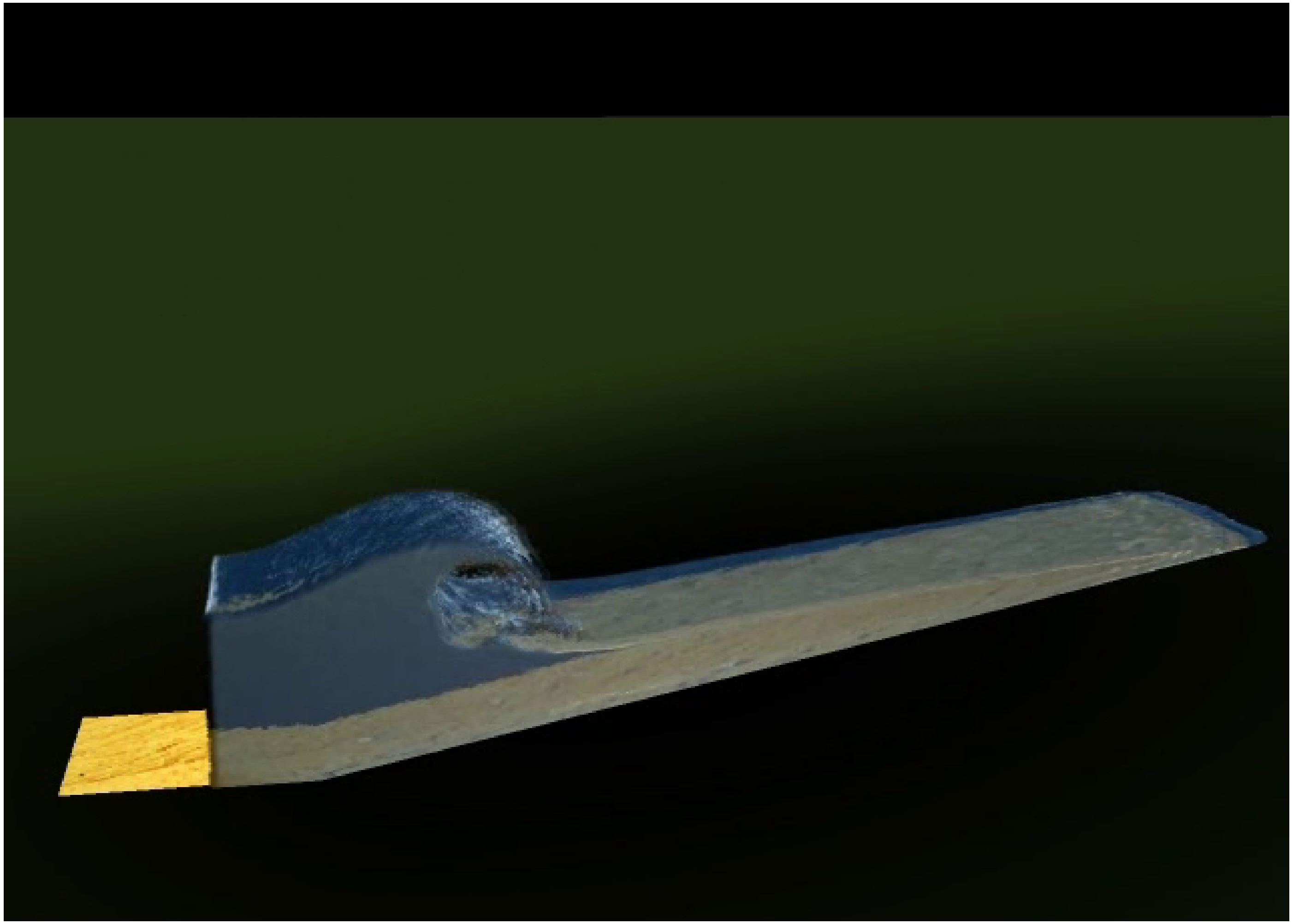
Leapfrog Integrator

- For a mass under an inverse square law, the error in orbital parameters is shown below.



https://www.physics.drexel.edu/~valliere/PHYS305/Diff_Eq_Integrators/time_reversal/

Weakly Compressible SPH

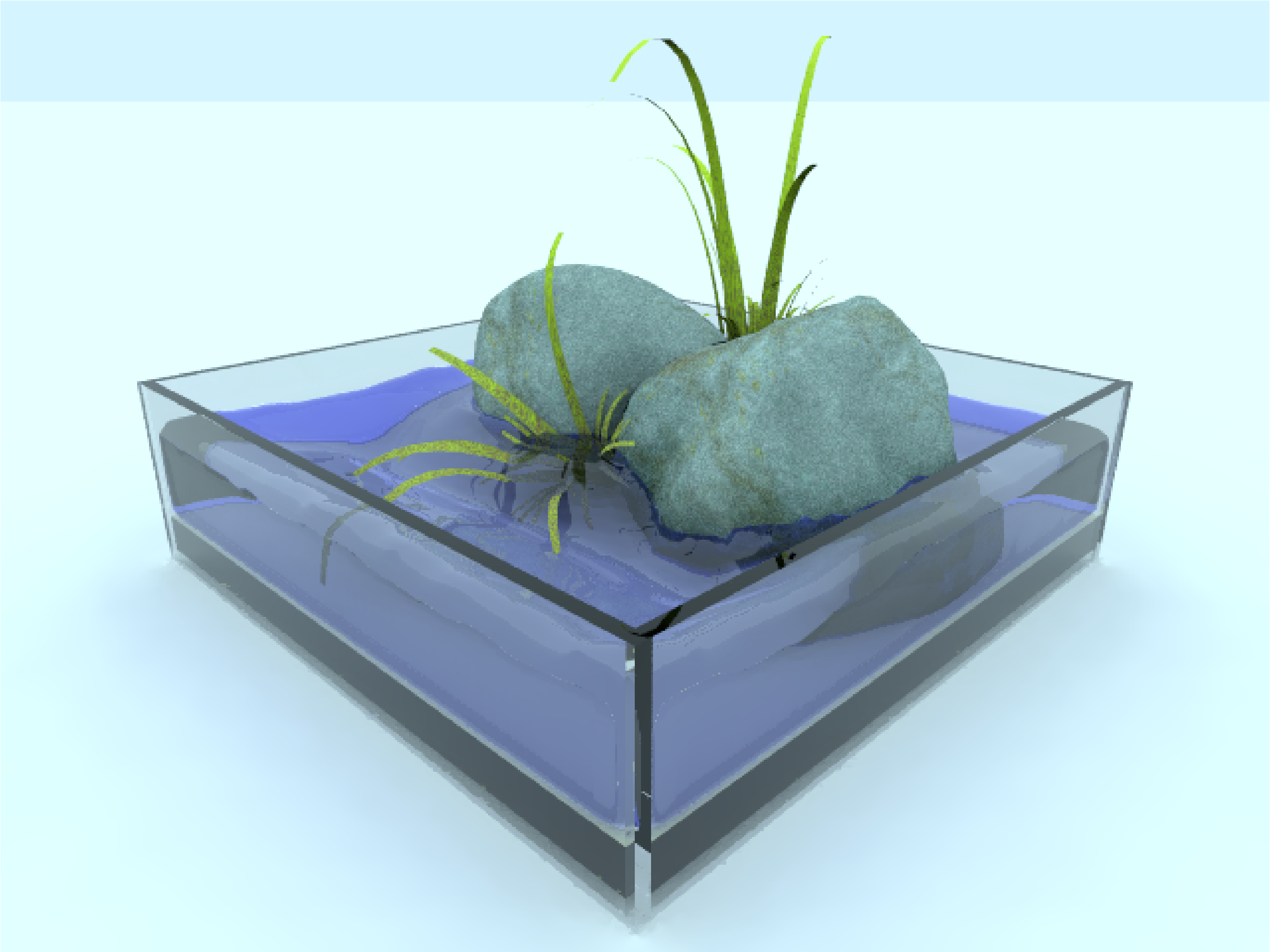


Weakly Compressible SPH for Free Surface Flows, Becker & Teschner, SCA 2007

More SPH

Toon Lenaerts, Bart Adams, and Philip Dutré. Porous Flow in Particle-Based Fluid Simulations. ACM Transactions on Graphics, 27(3), Proceedings of ACM SIGGRAPH 2008, pages 49:1-49:8, 2008.

Still more SPH



Saket Patkar, Parag Chaudhuri, Wetting of Porous Solids, IEEE TVCG 2013