linear_sir_vs_seir

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[1]:
```
from sympy import *
from sympy.vector import *
from sympy.matrices import Matrix

init_printing()
from IPython.display import display, Math
```

0.1 SIR model

$$\frac{dS}{dt} = -\frac{\beta IS}{N}, \tag{1}$$
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I, \tag{2}$$
$$\frac{dR}{dt} = \gamma I, \tag{3}$$

Linearized SIR model

$$\frac{dS}{dt} = -\beta I, \tag{4}$$
$$\frac{dI}{dt} = (\beta - \gamma) I, \tag{5}$$
$$\frac{dR}{dt} = \gamma I, \tag{6}$$

Remove equations not relevant for I

$$\frac{dI}{dt} = (\beta - \gamma) I \tag{7}$$

Solution

$$I = I_0 e^{(\beta - \gamma)t} \tag{8}$$

Growth rate of $I(t)$

$$\beta - \gamma = \gamma(R_0 - 1) \tag{9}$$

0.2 SEIR model

$$\frac{dS}{dt} = -\frac{\beta IS}{N}, \tag{10}$$
$$\frac{dE}{dt} = \frac{\beta IS}{N} - \alpha E, \tag{11}$$
$$\frac{dI}{dt} = \alpha E - \gamma I, \tag{12}$$
$$\frac{dR}{dt} = \gamma I, \tag{13}$$

Linearized SEIR model

$$\frac{dS}{dt} = -\beta I, \tag{14}$$
$$\frac{dE}{dt} = \beta I - \alpha E, \tag{15}$$
$$\frac{dI}{dt} = \alpha E - \gamma I, \tag{16}$$
$$\frac{dR}{dt} = \gamma I, \tag{17}$$

Remove equations not relevant for I

$$\frac{dE}{dt} = \beta I - \alpha E, \tag{18}$$
$$\frac{dI}{dt} = \alpha E - \gamma I, \tag{19}$$
$$\tag{20}$$

Matrix form

$$\frac{d}{dt}\begin{pmatrix} E \\ I \end{pmatrix} = \begin{pmatrix} -\alpha & \beta \\ \alpha & -\gamma \end{pmatrix}\begin{pmatrix} E \\ I \end{pmatrix} \tag{21}$$

[2]:

```
alpha = Symbol('alpha', positive = True, real = True)
R0 = Symbol('R_0', positive = True, real = True)
gamma = Symbol('gamma', positive = True, real = True)
#beta = Symbol('beta')
beta = R0 * gamma

M = Matrix([[-alpha, beta],[alpha, -gamma]])
ll = list(M.eigenvals().keys())
[simplify(l) for l in ll]
```

[2]:

$$\left[-\frac{\alpha}{2} - \frac{\gamma}{2} - \frac{\sqrt{4R_0\alpha\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2}}{2}, \ -\frac{\alpha}{2} - \frac{\gamma}{2} + \frac{\sqrt{4R_0\alpha\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2}}{2} \right]$$

[3]:

```
[simplify(l.subs(alpha, gamma)) for l in ll]
```

[3]: $\left[-\gamma\left(\sqrt{R_0}+1\right),\ \gamma\left(\sqrt{R_0}-1\right)\right]$

Eigenvalues

$$\left[-\frac{\alpha}{2}-\frac{\gamma}{2}-\frac{\sqrt{4R_0\alpha\gamma+\alpha^2-2\alpha\gamma+\gamma^2}}{2},\ -\frac{\alpha}{2}-\frac{\gamma}{2}+\frac{\sqrt{4R_0\alpha\gamma+\alpha^2-2\alpha\gamma+\gamma^2}}{2}\right] \tag{22}$$

Eigenvalues in case $\alpha=\gamma$

$$\left[-\gamma\left(\sqrt{R_0}+1\right),\ \gamma\left(\sqrt{R_0}-1\right)\right] \tag{23}$$

Growth rate of $I(t)$ in case $\alpha=\gamma$

$$\gamma\left(\sqrt{R_0}-1\right) \tag{24}$$

[4]:
```
growth = ll[1]
R0sir = Symbol('R_0^\mathrm{sir}', positive = True, real = True)
s = solve(growth -(gamma * (R0sir - 1)) , R0)[0]
s = expand(factor(s, gamma))
display(Math('R_0^\mathrm{seir}=' + latex(s)))
```

$R_0^{\mathrm{seir}}=\frac{\left(R_0^{\mathrm{sir}}\right)^2\gamma}{\alpha}+R_0^{\mathrm{sir}}-\frac{R_0^{\mathrm{sir}}\gamma}{\alpha}$

Relationship between R_0 from SIR and SEIR

$$R_0^{\mathrm{seir}}=R_0^{\mathrm{sir}}+\frac{R_0^{\mathrm{sir}}(R_0^{\mathrm{sir}}-1)\gamma}{\alpha} \tag{25}$$