Bansal Maths Calsses

B.Sc.- III (Complex Analysis –Analytic Function)

Assignment

1. Cauchy –Riemann equation for $w = u + iv = f(z)$ are :

   a. $u_x = v_x, u_y = v_y$ b. $u_x = v_y, u_y = v_x$ c. $u_x = v_y, u_y = -v_x$ d. none of these

2. An analytic function with constant modulo is :

   a. variable   b. may be variable or constant   c. constant   d. none of these

3. If a function is analytic at all points of a bounded domain except finitely many points , then these points are called :

   a. singular points   b. simple points   c. continuous points   d. none of these

4. $Exp(2 \pm 3i\pi) =$

   a. e^{-2} b. $e^{\pm 3i}$ c. $e^{\pm 3}$ d. $-e^{2}$

5. At z=0 , the function $f(z) = \overline{z}$ is :

   a. not analytic   b. not differentiable   c. not continuous   d. none of these

6. The polar form of the complex number $-5+5i$ is :

   a. $5\sqrt{2}e^{i\pi/4}$ b. $5\sqrt{2}e^{-i3\pi/4}$ c. $5\sqrt{2}e^{i3\pi/4}$ d. none of these

7. If $f(z) = u + iv$ is analytic function in a finite region and $u = x^3 - 3xy^2$, then $v =$

   a. $3x^2y - y^3 + c$ b. $3x^2y^2 - y^3$ c. $3x^2y - y^2 + c$ d. none of these

8. For any two complex numbers $z_1, z_2, |z_1 - z_2|$ is :

   a. $\geq |z_1| - |z_2|$ b. $\geq |z_1| + |z_2|$ c. $= |z_1| + |z_2|$ d. none of these

9. A conjugate harmonic of $u(x, y) = e^x \sin y$ is :

   a. $e^y \cos x$ b. $e^x \cos y$ c. $-e^x \cos y + 1$ d. $e^y / \sin x$

10. Which of the function $f(z)$ is analytic in complex plane where $f(z) =$

    a. Re z   b. Im z   c. cot z   d. e^z

11. Cauchy –Riemann equations (in usual notations) are :

    a. $u_x = v_x, u_y = v_y$ b. $u_x = v_y, u_y = v_x$ c. $u_x = v_x, u_y = -v_y$ d. $u_x = v_y, u_y = -v_x$

12. Which of the following is correct for $w = f(z)$?

    a. $\frac{dw}{dz} = \frac{\partial w}{\partial x}$ b. $\frac{dw}{dz} = -\frac{\partial w}{\partial x}$ c. $\frac{dw}{dz} = \frac{\partial w}{\partial y}$ d. $\frac{dw}{dz} = -\frac{\partial w}{\partial y}$

13. Cauchy –Riemann equations in polar form are :

    a. $\frac{\partial u}{\partial r} = \frac{1}{r}.\frac{\partial v}{\partial \theta}$ and $\frac{1}{r}.\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$ b. $\frac{\partial u}{\partial r} = r.\frac{\partial v}{\partial \theta}$ and $r.\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$

    c. $\frac{\partial u}{\partial r} = -\frac{1}{r}.\frac{\partial v}{\partial \theta}$ and $\frac{1}{r}.\frac{\partial u}{\partial \theta} = \frac{\partial v}{\partial r}$ d. $\frac{\partial u}{\partial r} = -r.\frac{\partial v}{\partial \theta}$ and $r.\frac{\partial u}{\partial \theta} = \frac{\partial v}{\partial r}$

14. The derivative of a function $w = f(z)$ in polar form is given by :

a. $\frac{dw}{dz} = \frac{\partial w}{\partial r} e^{i\theta}$ b. $\frac{dw}{dz} = -\frac{\partial w}{\partial \theta} e^{i\theta}$ c. $\frac{dw}{dz} = \frac{\partial w}{\partial r} e^{-i\theta}$ d. $\frac{dw}{dz} = -\frac{\partial w}{\partial \theta} e^{-i\theta}$

15. If $f(z) = u(x,y) + iv(x,y)$ is an analytic and $u = \log(x^2 + y^2)$ then v is :

a. $2\arctan\frac{y}{x} + c$ b. $\frac{1}{2}\arctan\frac{y}{x} + c$ c. $\tan^{-1}\frac{y}{x} + c$ d. none of these

16. If the function $f(z) = e^x(\cos ky + i\sin ky), z = x + iy$, is analytic iff $k =$

a. 1 b. 0 c. 2 d. none of these

17. The function conjugate harmonic to $u = e^{xy}\cos\frac{1}{2}(x^2 - y^2)$ is :

a. $-e^{xy}\sin\frac{1}{2}(x^2 - y^2) + c$ b. $e^{xy}\sin\frac{1}{2}(x^2 - y^2) + c$

c. $\frac{1}{2}e^{xy}\sin\frac{1}{2}(x^2 - y^2)$ d. none of these

18. The analytic function whose real part is $e^x \cos y$ is :

a. $e^z + ci$ b. e^{2z} c. xe^z d. none of these

19. The analytic function whose real part is $e^x(x\cos y - y\sin y)$ is :

a. $ze^z + ci$ b. z^2e^z c. ze^{x^2+iy} d. none of these

20. If $f(z)$ is analytic in a simply connected domain D enclosed by a rectifiable Jordan curve C and $f(z)$ is continuous on C, then for any point z_0 in D , we have $f(z_0) =$

a. $\frac{1}{2\pi}\int_C \frac{f(z)}{z - z_0}dz$ b. $\frac{1}{2\pi i}\int_C \frac{f(z)}{z - z_0}dz$ c. $2\pi\int_C \frac{f(z)}{z - z_0}dz$ d. $2\pi i\int_C \frac{f(z)}{z - z_0}dz$

21. Which of the following is true :

a. $|\int_L f(z)dz| = \int_L |f(z)||dz|$ b. $|\int_L f(z)dz| \le \int_L |f(z)||dz|$

c. $|\int_L f(z)dz| < \int_L |f(z)||dz|$ d. $|\int_L f(z)dz| > \int_L |f(z)||dz|$

22. If L is circle $|z - a| = r$, then $\int_L \frac{dz}{z - a}$ is :

a. $2\pi i$ b. πi c. 0 d. - $2\pi i$

23. If L is circle $|z| = r$, then $\int_L \bar{z}dz$ is :

a. πi b. 2 πi c. 0 d. none of these

24. Let $f(z)$ be continuous in a simply connected domain G enclosed by a rectifiable Jordan curve L and let $f(z)$ be continuous on L . Then $\int_L \frac{f(z)}{z - z_0}dz$ is :

a. $2\pi i f(z_0)$ b. $2\pi i f'(z_0)$ c. $2\pi f(z_0)$ d. none of these

25. When 0<|z|<4 , the expansion of $\frac{1}{4z-z^2}$ is :

a. $\sum_{n=0}^{\infty}\frac{z^{n+1}}{4^{n+1}}$ b. $\sum_{n=0}^{\infty}\frac{(-1)^n z^{n+1}}{4^{n+1}}$ c. $\sum_{n=0}^{\infty}\frac{z^{n-1}}{4^{n+1}}$ d. none of these

26. The value of $\frac{1}{2\pi i}\int_{|z|=4}\frac{e^z}{(z+2)^2}dz$ is :

a. e^3 b. e^2 c. 0 d. none of these

27. The value of $\frac{2!}{2\pi i}\int_{|z|=3}\frac{z^2+3z+4}{(z-1)^3}dz$ is :

a. 2 b. 0 c. πi d. none of these

28. The value of $\frac{n!}{2\pi i}\int_{|z|=3}\frac{z^n+az^{n-2}+bz}{(z-3)^{n+1}}dz$ is :

a. $n!$ b. $(n+1)!$ c. $2n\pi i$ d. 0

29. If L is a closed curve and $z=a$ is out side L, then $\int_L \frac{1}{z-a}dz$ is :

a. 0 b. πi c. 2 πi d. ∞

30. If L is the circle , the value of then $\int_L \frac{1}{z-2}dz$ is :

a. 0 b. 2 πi c. πi d. ∞

31. The function $f(z)=\overline{z}$ is :

a. not analytic b. analytic c. not continuous d. none of these

32. The function $f(z)=z^n$ is :

a. analytic b. differentiable c. both (a) and (b) d. none of these

33. A function which satisfying the Laplace equation is called :

a. analytic function b. harmonic function c differentiable d. none of these

34. For $w=f(z)$, which of the following is correct :

a. $\frac{dw}{dz}=\frac{-\partial w}{\partial x}$ b. $\frac{dw}{dz}=\frac{\partial w}{\partial x}$ c. $\frac{dw}{dz}=\frac{\partial w}{\partial y}$ d. none of these

35. If $w=u+iv$ be an analytic function $z=x+iy$ then the families of the curve $u(x,y)=c_1$ and $v(x,y)=c_2$ form :

a. orthonormal system b. analytic system c. harmonic system d. none of these

36. If u is a harmonic function . then a function is said to be harmonic conjugate of u if :

a. v is harmonic b. $u\,\&\,v$ satires C-R equation
c. both (a) and (b) are true d. none of these

37. Two function $u\,\&\,v$ are harmonic conjugate to each other if :

a. u is constant b. v is constant c. both u and v are constant d. none of these

38. If $u = x^2 - y^2$ is harmonic , then its conjugate harmonic is :

a. 2xy b. 2xy+c c. x^2y^2 d. xy+c

39. Harmonic conjugate of the function $e^x \cos y$ is :

a. $e^x \sin y + c$ b. $e^{-x} \cos y + c$ c. $e^x \sin y + e^x \cos y$ d. none of these

40. Any function of x and y possessing continuous partial derivatives of the first and second order is said to be harmonic function if it satisfying :

a. Laplace equation b. Euler's equation c. C-R equation d. none of these

41. If harmonic function u and v satisfying C-R equation then $u + iv$ is :

a. not analytic b. analytic c. can't anything d. none of these

42. Analytic function with its derivatives zero for all points in the domain is :

a. z b. ze^z +c c. constant d. none of these

43. If $f(z) = u + iv$ and $u - v = e^x(\cos y - \sin y)$ then $f(z)$ in terms of z is :

a. $e^z + c$ b. $e^z + z + c$ c. $z^2 + c$ d. none of these

44. The function $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is :

a. harmonic b. non-harmonic c. can't say d. none of these

46. value of $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ is :

a. $4\frac{\partial^2}{\partial z.\partial \overline{z}}$ b. $2\frac{\partial^2}{\partial z.\partial z}$ c. $-4\frac{\partial^2}{\partial z.\partial z}$ d. none of these

47. Which of the functions $f(z)$ and $g(z)$ in a region R :

a. $f(z) + g(z)$ is analytic in R b. $f(z) - g(z)$ is analytic in R

c. $f(z).g(z)$ is analytic in R d. $f(z)/g(z)$ is analytic in R

48. Which of the function is not analytic :

a. $f(z) = z^6$ b. $f(z) = 1/z^4 \, (z \neq 0)$

c. $f(z) = \log r + i\theta$ d. $f(z) = 1/(z-1)^3$

e. $f(z) = 1/(z-1)^3, z \neq 1$

49. Which of the followings functions $f(z)$ satisfies C-R equation :

a. $f(z) = \overline{z} = x - iy$ at $z = 1 + i$ b. $f(z) = |z|^2$ at $z (z \neq 0)$

c. $f(z) = \sqrt{|xy|}$ at $z = 0$ d. $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ at $z \neq 0, f(0) = 0$

50. The analytic function $w = u + iv$ where $u = e^{-x}\left\{(x^2 - y^2)\cos y + 2xy \sin y\right\}$ is :

a. $e^{-x}(x - iy)^2\{\cos y - i \sin y\}$ b. $e^{-x}(x - iy)^2\{\cos y + i \sin y\}$

c. $z^2 e^{-x}\{\cos y - i \sin y\}$ d. none of these