CSE206

Digital Logic Design Sessional

Offline 1

Truth tables and simplification

using Boolean Algebra

Bangladesh University of Engineering & Technology

Group: 02

Section: B2

Md. Farhan Mahtab
1805096

Md. Sultanul Arifin
1805097

Md. Shahrukh Islam
1805098

Kamruj Jaman Sheam
1805099

Utchchhwas Singha
1805100

Level - 2, Term - 1

Department of Computer Science and Engineering

Submitted on March 13, 2021

Question No: 1

Problem Specification:

The given equation is -

$$F(A,B,C,D) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + ABCD + AB\overline{C}D + \overline{A}\,\overline{B}C\overline{D} + \overline{A}B\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}C\overline{D} + \overline{A}BCD$$

We have to simplify this equation using Boolean algebra and implement the simplified equation using logic gates.

Required Instruments:

1. NOT Gate (IC 7404) – Quantity: 1
2. AND Gate (IC 7408) – Quantity: 1
3. OR Gate (IC 7432) – Quantity: 1
4. Input pins – Quantity: 4
5. Output pins – Quantity: 1
6. Wires, power source etc.

Truth Table:

The truth table for the given equation is shown below -

Input				Output
A	**B**	**C**	**D**	**F**
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Now, the given equation is -

$$F(A,B,C,D) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}+ABCD+AB\overline{C}D+\overline{A}\,\overline{B}C\overline{D}+\overline{A}B\overline{C}D+A\overline{B}\,\overline{C}\,\overline{D}+A\overline{B}C\overline{D}+\overline{A}BCD$$

$$= (ABCD+AB\overline{C}D)+(\overline{A}\,\overline{B}C\overline{D}+\overline{A}\,\overline{B}\,\overline{C}\,\overline{D})+$$
$$(A\overline{B}C\overline{D}+A\overline{B}\,\overline{C}\,\overline{D})+(\overline{A}BCD+\overline{A}B\overline{C}D)\ [\textit{Commutative Law}]$$

$$= ABD(C+\overline{C})+\overline{A}\,\overline{B}\,\overline{D}(C+\overline{C})+A\overline{B}\,\overline{D}(C+\overline{C})+\overline{A}BD(C+\overline{C})\ [\textit{Distributive Law}]$$

$$= (ABD+\overline{A}\,\overline{B}\,\overline{D}+A\overline{B}\,\overline{D}+\overline{A}BD)(C+\overline{C})$$

$$= (ABD+\overline{A}\,\overline{B}\,\overline{D}+A\overline{B}\,\overline{D}+\overline{A}BD)\cdot 1\ [\textit{Complement Law}]$$

$$= ABD+\overline{A}\,\overline{B}\,\overline{D}+A\overline{B}\,\overline{D}+\overline{A}BD\ [\textit{Identity Law}]$$

$$= (ABD+\overline{A}BD)+(A\overline{B}\,\overline{D}+\overline{A}\,\overline{B}\,\overline{D})$$

$$= BD(A+\overline{A})+\overline{B}\,\overline{D}(A+\overline{A})$$

$$= (BD+\overline{B}\,\overline{D})(A+\overline{A})$$

$$= (BD+\overline{B}\,\overline{D})\cdot 1\ [\textit{Complement Law}]$$

$$= BD+\overline{B}\,\overline{D}\ [\textit{Identity Law}]$$

$$\textit{Therefore}, F=BD+\overline{B}\,\overline{D}$$

<u>Circuit Diagram</u>:

a) **<u>Logic Circuit</u>:**

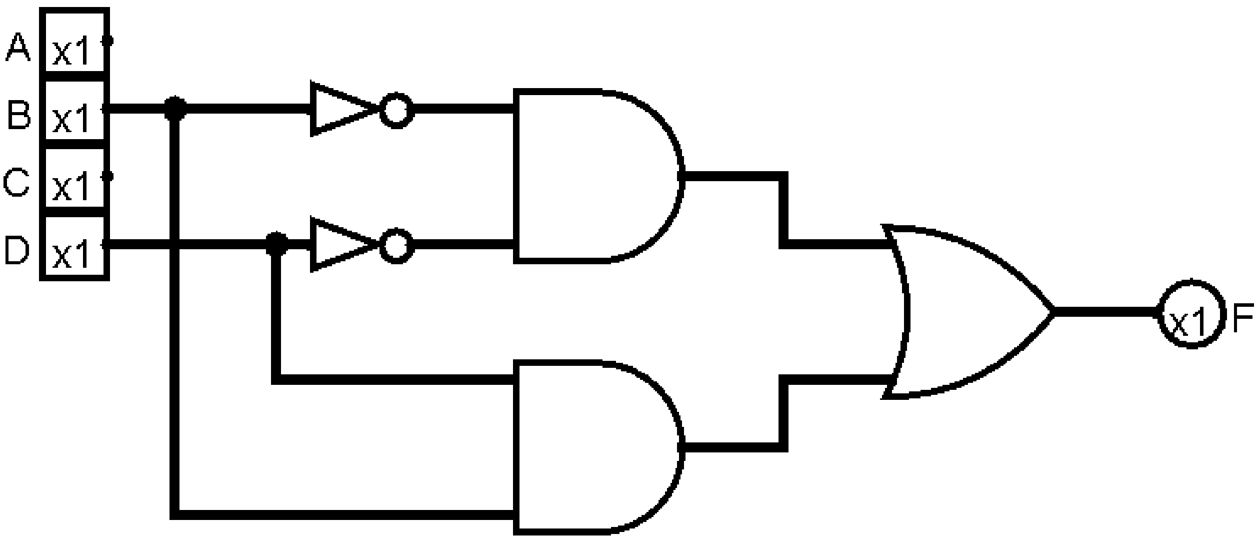

Figure 1: Implementation of simplified boolean expression using logic gates

b) Electronic Circuit:

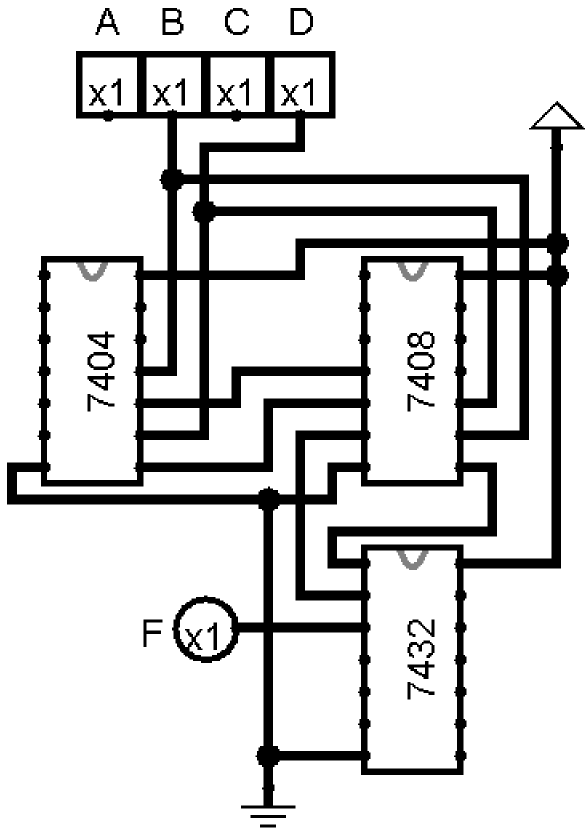

Figure 2: Implementation of simplified boolean expression using 7400 series ICs

Observations:

Here, the given Boolean formula can be simplified to $F = BD + \overline{B}\,\overline{D}$
This simplified formula represents the XNOR operation between B and D and can be written as B ⊙ D.
Also, since the simplified equation does not contain the boolean variables *A* and *C*, the output does not depend of the truth values of these two boolean variables.

Question No: 2

Problem Specification:

We have to derive the output equations for a 3-bit gray to binary converter from the truth table and then implement those equations with necessary logic gates.

Required Instruments:

1. NOT Gate (IC 7404) – Quantity: 1
2. AND Gate (IC 7408) – Quantity: 1
3. OR Gate (IC 7432) – Quantity: 1
4. Input pins – Quantity: 3
5. Output pins – Quantity: 3
6. Wires, power source etc.

Truth Table:

The truth table of a 3-bit gray to binary converter is shown below -

Input (3-bit Gray Code)			Output (Binary Code)		
G_2	G_1	G_0	B_2	B_1	B_0
0	0	0	0	0	0
0	0	1	0	0	1
0	1	1	0	1	0
0	1	0	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1
1	0	1	1	1	0
1	0	0	1	1	1

From the truth table,

$$B_0 = \sum(1,3,5,7)$$

$$= \overline{G_2}\,\overline{G_1}\,G_0 + \overline{G_2}\,G_1\,\overline{G_0} + G_2 G_1 G_0 + G_2\,\overline{G_1}\,\overline{G_0}$$

$$= G_2 G_1 G_0 + G_2\,\overline{G_1}\,\overline{G_0} + \overline{G_2}\,G_1\,\overline{G_0} + \overline{G_2}\,\overline{G_1}\,G_0$$

$$Therefore,\ B_0 = G_2 G_1 G_0 + G_2\,\overline{G_1}\,\overline{G_0} + \overline{G_2}\,G_1\,\overline{G_0} + \overline{G_2}\,\overline{G_1}\,G_0$$

$$B_1 = \sum(2,3,6,7)$$

$$= \overline{G_2}G_1G_0 + \overline{G_2}G_1\overline{G_0} + G_2\overline{G_1}G_0 + G_2\overline{G_1}\,\overline{G_0}$$

$$= \overline{G_2}G_1(G_0 + \overline{G_0}) + G_2\overline{G_1}(G_0 + \overline{G_0})\ [\mathit{Distributive\ Law}]$$

$$= \overline{G_2}G_1 \cdot 1 + G_2\overline{G_1} \cdot 1\ [\mathit{Complement\ Law}]$$

$$= \overline{G_2}G_1 + G_2\overline{G_1}\ [\mathit{Identity\ Law}]$$

$$\mathit{Therefore},\ B_1 = \overline{G_2}G_1 + G_2\overline{G_1}$$

$$B_2 = \sum(4,5,6,7)$$

$$= G_2G_1\overline{G_0} + G_2G_1G_0 + G_2\overline{G_1}G_0 + G_2\overline{G_1}\,\overline{G_0}$$

$$= G_2G_1(G_0 + \overline{G_0}) + G_2\overline{G1}(G_0 + \overline{G_0})\ [\mathit{Distributive\ Law}]$$

$$= G_2G_1 \cdot 1 + G_2\overline{G_1} \cdot 1\ [\mathit{Complement\ Law}]$$

$$= G_2G_1 + G_2\overline{G_1}\ [\mathit{Identity\ Law}]$$

$$= G_2(G_1 + \overline{G_1})\ [\mathit{Distributive\ Law}]$$

$$= G_2 \cdot 1\ [\mathit{Complement\ Law}]$$

$$= G_2\ [\mathit{Identity\ Law}]$$

$$\mathit{Therefore},\ B_2 = G_2$$

The following forms of the boolean equations are used for implementation -

$$B_0 = G_2 G_1 G_0 + G_2 \overline{G_1}\,\overline{G_0} + \overline{G_2} G_1 \overline{G_0} + \overline{G_2}\,\overline{G_1} G_0$$

$$= (G_2 G_1 + \overline{G_2}\,\overline{G_1}) G_0 + (\overline{G_2} G_1 + G_2 \overline{G_1}) \overline{G_0} \quad [\mathit{Distributive\ Law}]$$

$$= \overline{\overline{(G_2 G_1 + \overline{G_2}\,\overline{G_1})}} G_0 + (\overline{G_2} G_1 + G_2 \overline{G_1}) \overline{G_0} \quad [\mathit{Double\ Negation\ Law}]$$

$$= \overline{(\overline{G_2} G_1 + G_2 \overline{G_1})} G_0 + (\overline{G_2} G_1 + G_2 \overline{G_1}) \overline{G_0} \quad [\mathit{de\ Morgan's\ Law}]$$

$$B_1 = \overline{G_2} G_1 + G_2 \overline{G_1}$$

$$B_2 = G_2$$

Circuit Diagram:

a) **Logic Circuit**:

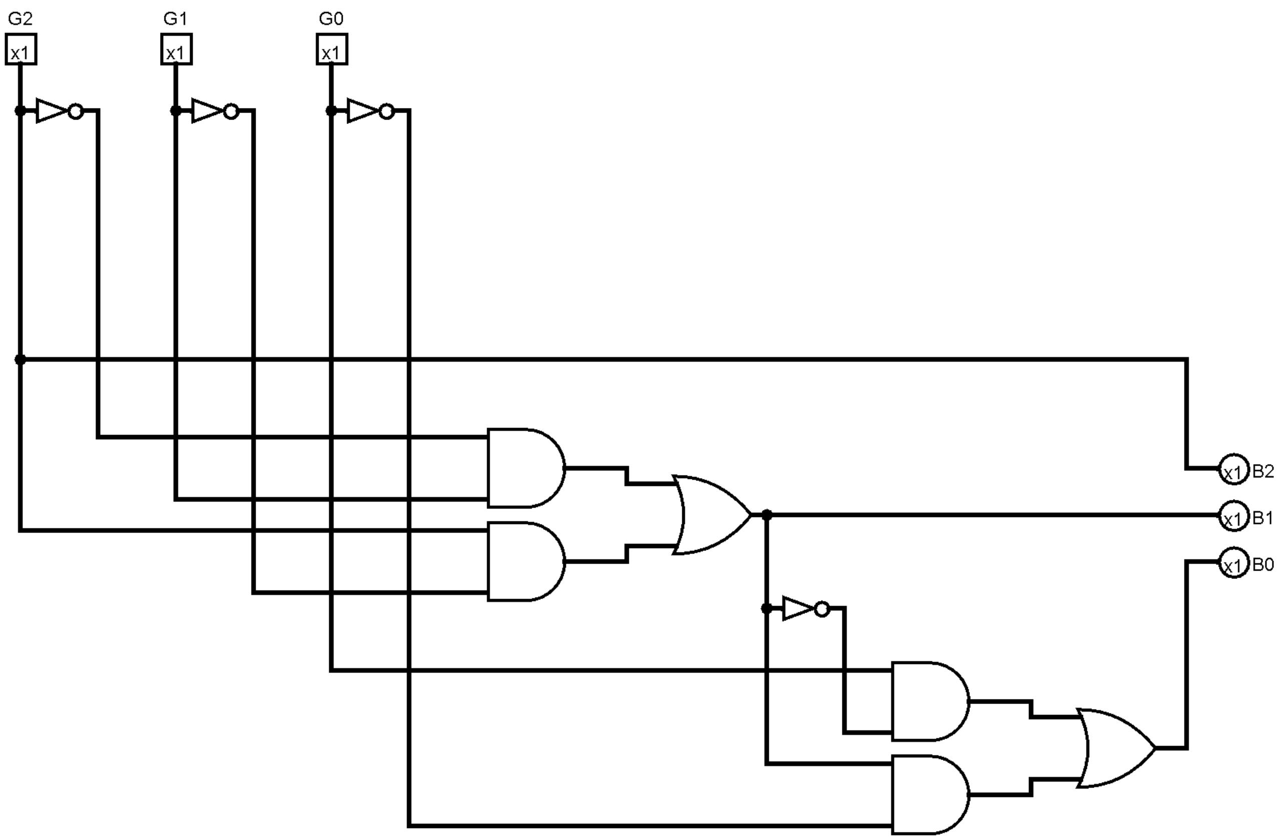

Figure 3: Implementation of Boolean equations using logic gates

b) Electronic Circuit:

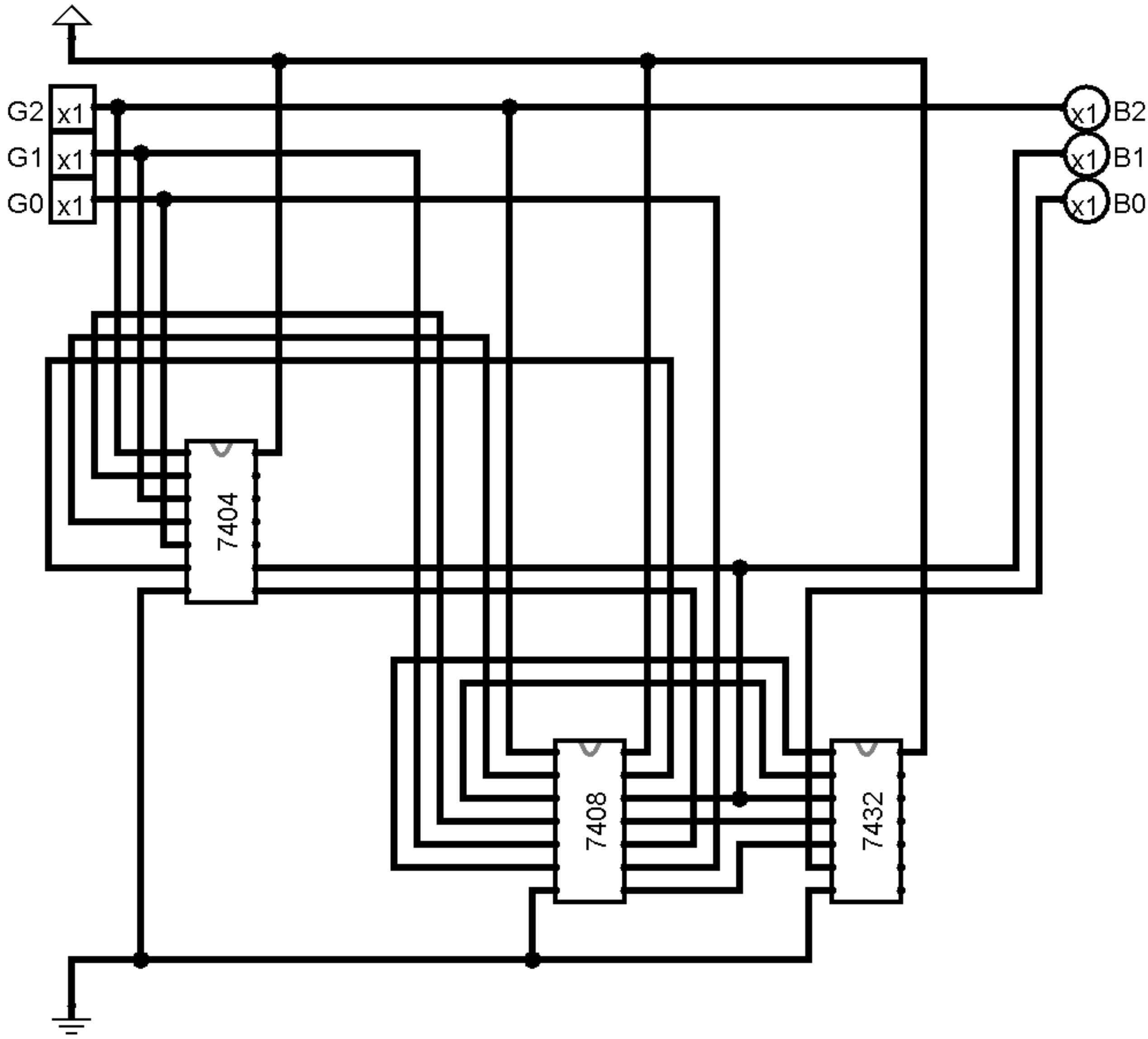

Figure 4: Implementation of Boolean equations using 7400 series ICs

Question No: 3

Problem Specification:

There are 3 inputs into a system. The system glows LED1 and LED0 in such away that the pattern represents the number of set bits in the input.

We have to derive the truth table and the corresponding output equations for the given condition and implement those equations using necessary logic gates.

Required Instruments:

1. NOT Gate (IC 7404) – Quantity: 1
2. AND Gate (IC 7408) – Quantity: 3
3. OR Gate (IC 7432) – Quantity: 2
4. Input pins – Quantity: 3
5. Output pins – Quantity: 2
6. Wires
7. Power source etc.

Truth Table:

The truth table for the given condition is shown below -

Input			**Output**	
A	**B**	**C**	**X (LED1)**	**Y (LED0)**
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

From the truth table,

$$X = \sum(3,5,6,7)$$

$$= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= ABC + \overline{A}BC + A\overline{B}C + AB\overline{C}$$

$$= BC(A+\overline{A}) + A\overline{B}C + AB\overline{C}$$

$$= BC\cdot 1 + A\overline{B}C + AB\overline{C} \ \ [\textit{Complement Law}]$$

$$= BC + A\overline{B}C + AB\overline{C} \ \ [\textit{Identity Law}]$$

$$= (B+\overline{B}A)C + AB\overline{C} \ \ [\textit{Distributive Law}]$$

$$= (B+A)C + AB\overline{C} \ \ [\textit{Absorption Law}]$$

$$= BC + AC + AB\overline{C}$$

$$= BC + A(C+\overline{C}B)$$

$$= BC + A(C+B) \ \ [\textit{Absorption Law}]$$

$$= BC + AC + AB$$

$$= AB + BC + CA$$

$$\textit{Therefore},\ X = AB + BC + CA$$

$$Y = \sum(1,2,4,7)$$

$$= \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$$

$$= ABC + A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C$$

$$\textit{Therefore},\ Y = ABC + A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C$$

Circuit Diagram:

a) Logic Circuit:

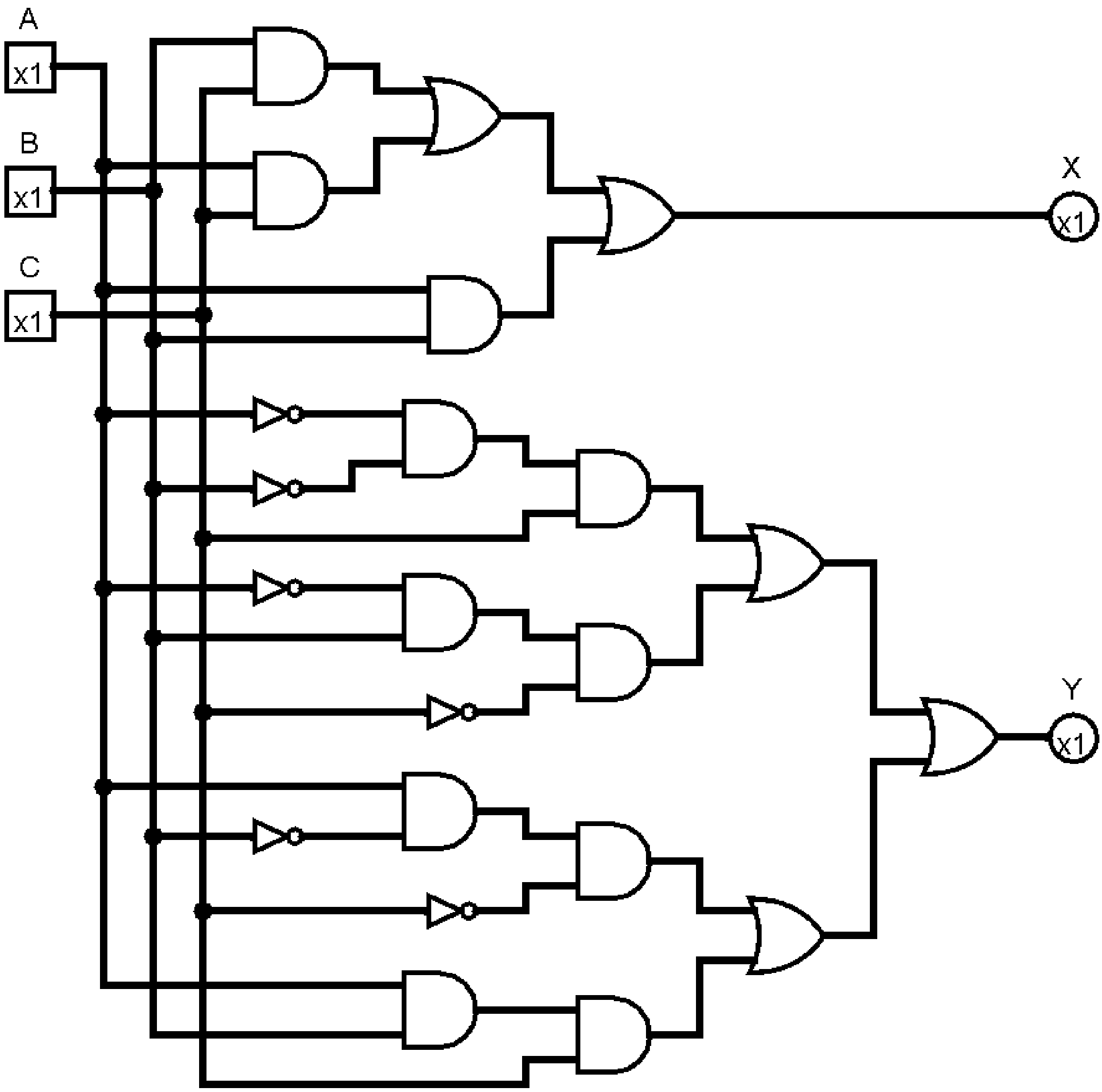

Figure 5: Implementation of simplified boolean equations using logic gates

b) Electronic Circuit:

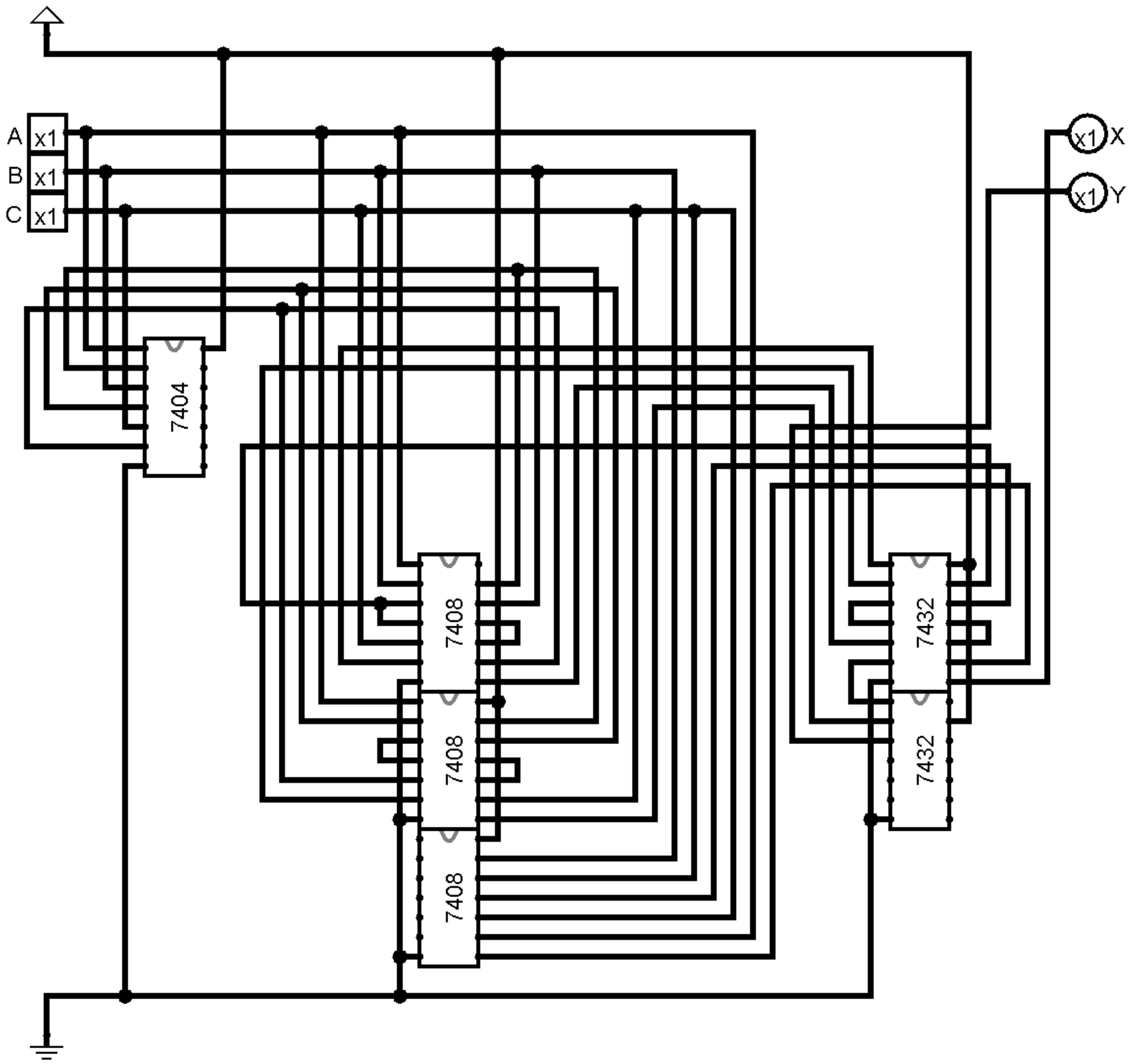

Figure 6: Implementation of the logical expressions using 7400 series ICs

Observations:

Here, the system works like a full adder. The three input A, B and C correspond to the three inputs of a full adder (two bits to sum and another carry bit). The output X and Y correspond to the carry and the sum of the full adder respectively.

<u>Question No</u>: 4

<u>Problem Specification</u>:

The given logic function is - $F(A,B,C,D) = \sum(6,9,12,15)$
We have to find the truth table and the logic expression of this function. Then we have to simplify it and implement it using logic gates.

<u>Required Instruments</u>:

1. NOT Gate (IC 7404) – Quantity: 1
2. AND Gate (IC 7408) – Quantity: 4
3. OR Gate (IC 7432) – Quantity: 1
4. Input pins – Quantity: 4
5. Output pins – Quantity: 1
6. Wires, power source etc.

<u>Truth Table</u>:

The truth table of the given function is shown below -

Input				**Output**
A	**B**	**C**	**D**	**F**
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

From the truth table,

$$F = \sum(6,9,12,15)$$

$$= \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}D + ABC\overline{D} + ABCD$$

This equation is already simplified and thus we cannot simplify it any further.

<u>Circuit Diagram</u>:

a) <u>Logic Circuit</u>:

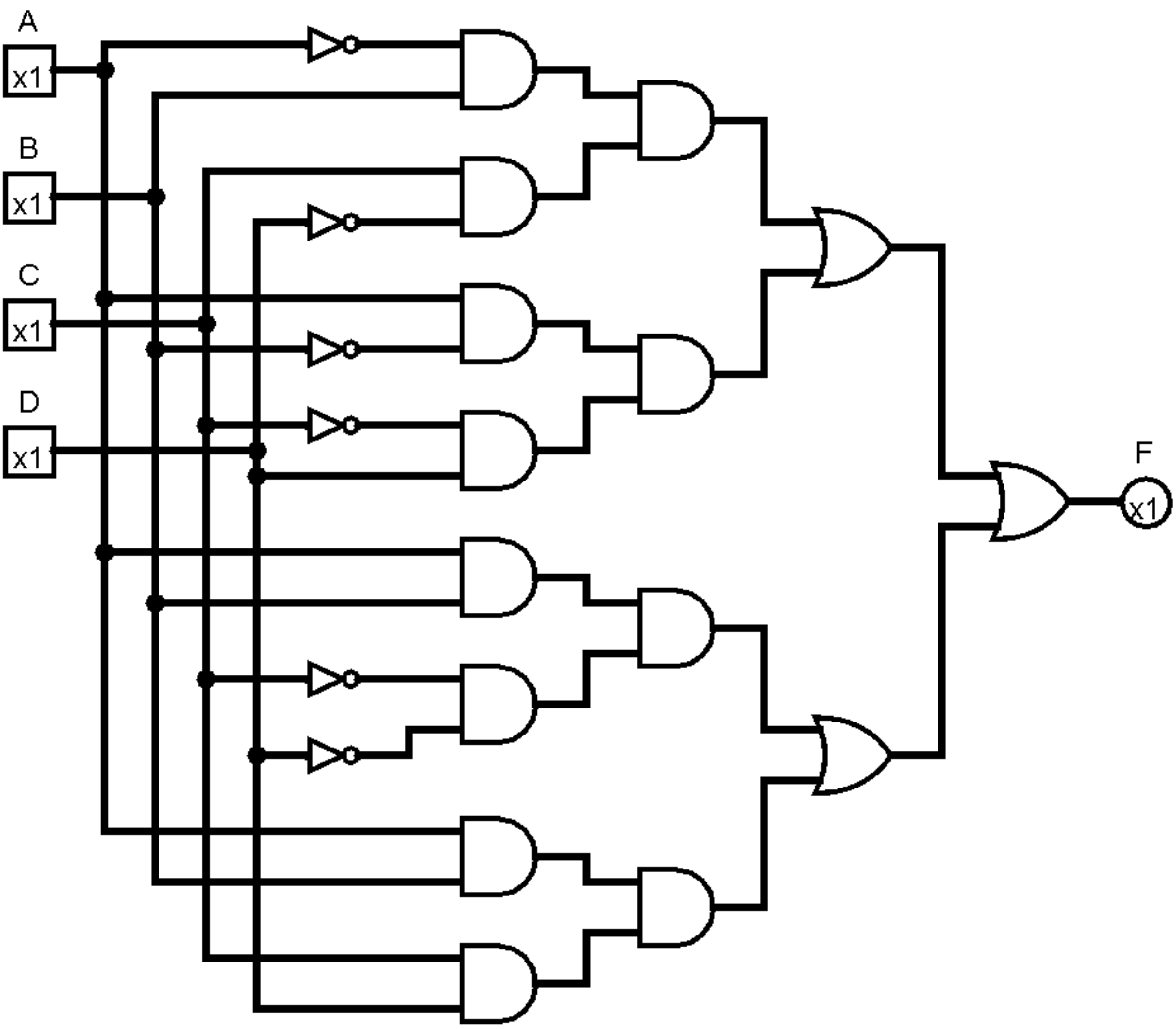

Figure 7: Implementation of simplified boolean equation using logic gates

b) <u>Electronic Circuit</u>:

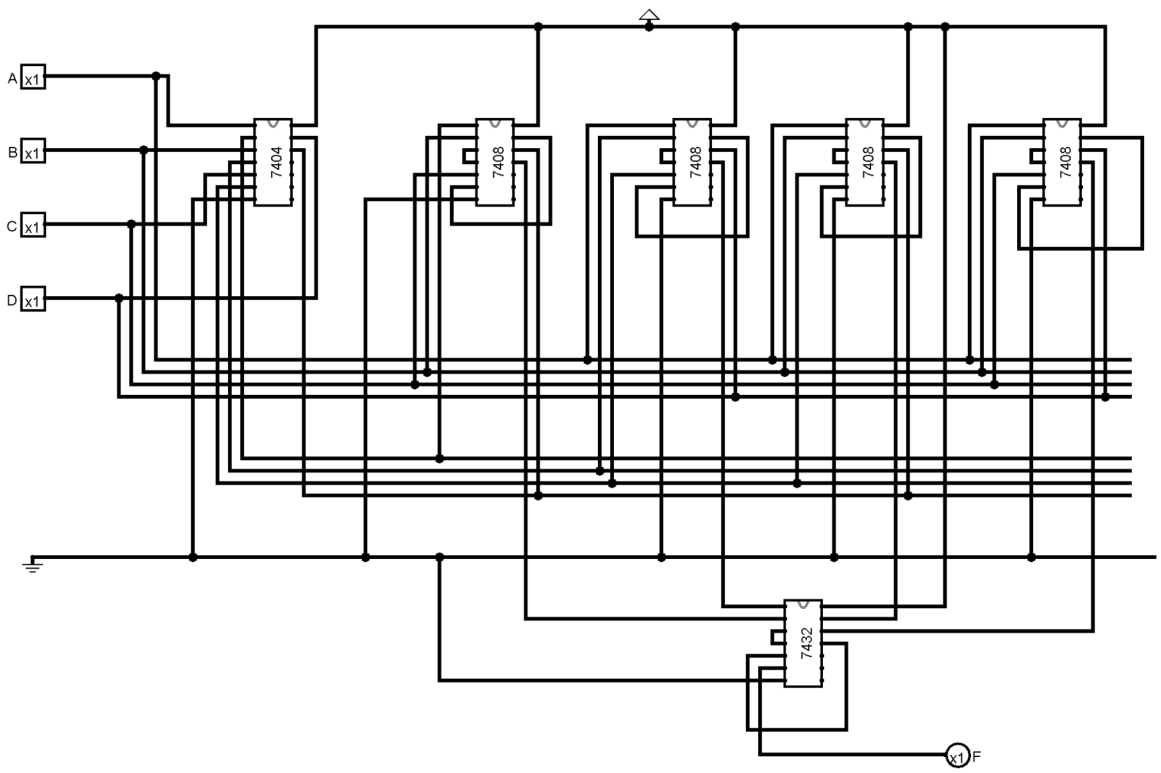

Figure 8: Implementation of simplified boolean equation using 7400 series ICs