1805098

Proof of NP-completeness of Clique problem

Soln: To prove it we have to prove i) clique problem is NP

ii) A NP problem can be reduced to clique problem

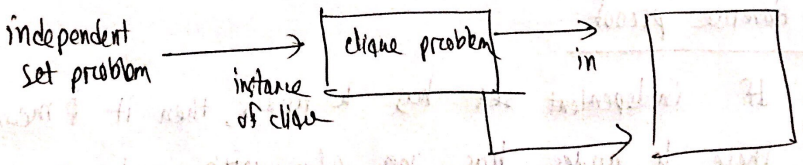
We know independent set problem is a NP problem

Decision Problem: Find if a graph is having clique of size k

i) Polynomial verification: If the ~~circuit~~ certificate of the problem gives us that ~~it~~ ~~k~~ 'yes' answer to question of decision problem and also gives the nodes of the clique. Then we can do the check in $O(k^2)$ time to see that every node in this clique is connected to other nodes of the clique.

So, clique problem is a NP problem.

ii)



If our graph is G. Then let G' is another graph have edges between two nodes where these two nodes had no edge in our G.

We want to know if independent set can have k nodes or not.

If the clique problem gives k nodes then we are sure that these nodes are connected to each others in G'. That means these nodes don't have with others in our original G. And so these nodes form independent set.

So it is proved that if clique has k nodes then independent set has those k nodes.

Reverse proof:

If independent set has k nodes, then it means these k nodes has no edge with each other. So in G', these k nodes has edges with every

other (k-1) nodes. So clique size will be k.

So, we can say clique problem is NP complete.