** AD HOC & NORMAL THINGS

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
// policy Based ds
#include<ext/pb_ds/assoc_container.h
pp>
//#include<ext/pb_ds/tree_policy.hpp
>
using namespace __gnu_pbds;
typedef tree<long long , null_type,
greater_equal<long long>,
rb_tree_tag,tree_order_statistics_no
de_update> ordered_set;
//
cout<<*os.find_by_order(val)<<endl;
// k-th element it
// less_equal = multiset, less =
set, greater_equal = multiset
decreasing, greater = set
decreaseing
//
cout<<os.order_of_key(val)<<endl;  /
/ strictly smaller or greater
#define order(s, x)
s.order_of_key(x)
#define elemat(s ,x)
s.find_by_order(x)
#define
boost   ios_base::sync_with_stdio(0)
;cin.tie(0);cout.tie(0);
#define endl    "\n"
#define all(x)  x.begin(), x.end()
#define
read    freopen("input.txt","r",stdi
n)
#define
write   freopen("output.txt","w",std
out)
#define pb      push_back
#define ff      first
#define ss      second
#define GCD(a, b) __gcd(a, b)
#define PI 2.0124 * acos(0.0)125
#define LCM(a, b) (a * b) / GCD(a,
b)126
#define mem(a, b) memset(a, b,
sizeof(a))
#define popcountL
__builtin_popcountll
#define popcount __builtin_popcount
inline bool checkBit(int N, int
pos){return(bool)(N & (1 << pos));}
inline int setBit(int N, int pos) {
return N = N | (1 << pos); }
inline int unsetBit(int N, int pos)
{return N = (N & (~(1 << pos))); }
inline int toggleBit(int N, int pos)
{return N = (N ^ (1 << pos)); }
const ll sz=2e7+123;
#define INF 1000000000000000007
const ll MOD=1000000007;
const ll base = 31;
#define stringLen
18446744073709551620
inline void normal(ll &a) { a %=
MOD; (a < 0) && (a += MOD); }
inline ll modMul(ll a, ll b) { a %=
MOD, b %= MOD; normal(a), normal(b);
return (a*b)%MOD; }
inline ll modAdd(ll a, ll b) { a %=
MOD, b %= MOD; normal(a), normal(b);
return (a+b)%MOD; }
inline ll modSub(ll a, ll b) { a %=
MOD, b %= MOD; normal(a), normal(b);
a -= b; normal(a); return a; }
inline ll modPow(ll b, ll p) { ll r
= 1; while(p) { if(p&1) r =
modMul(r, b); b = modMul(b, b); p
>>= 1; } return r; }
inline ll modInverse(ll a) { return
modPow(a, MOD-2); }
inline ll modDiv(ll a, ll b) {
return modMul(a, modInverse(b)); }
//Graph direction array[8]
//dx[]={0,0,1,-1,-1,1,1,-1};
//dy[]={1,-1,0,0,-1,-1,1,1};
//Bishop direction array[8]
//dx[]={0,0,1,-1,-1,1,1,-1};
//dy[]={1,-1,0,0,-1,-1,1,1};
//Knight Direction array[8]
//dx[]={1,1,2,2,-1,-1,-2,-2};
//dy[]={2,-2,1,-1,2,-2,1,-1};
```

** NUMBER THEORY

FAISAL AMIN ABIR

PRIME GENERATION

```
bitset<sz>is_prime;
vi prime;
void primeGen(int n){
for(int i=3; i<=n;
i+=2)is_prime[i]=1;
int nn = sqrt(n)+1;
for(ll i=3; i<nn; i+=2){
if(is_prime[i]==0)continue;
for(int j=i*i; j<=n; j+=(i+i)){
is_prime[j]=0;}}
is_prime[2]=1;
prime.pb(2);
for(int i=3; i<=n; i+=2){
if(is_prime[i])prime.pb(i);}}
```

PRIME FACTORIZATION

```
vector<long long>factorization(long
long n){//O(sqrt(n)/ln(sqrt(n)) +
log2 n)
vector<long long>factors;
for(auto u:prime){
if(1LL*u*u > n) break;
if(n%u==0){//  factors.push_back(u);
//for generating unique factors keep
this line here
while(n%(u)==0){
factors.push_back(u);//for
generating all factors keep this
line here
n/=(u);}}}
if(n>1)factors.push_back(n);
return factors;}
```

SEGMENTED SIEVE

```
vector<char> segmentedSieve(long
long L,long long R){
// generate all primes up to sqrt(R)
long long lim = sqrt(R);
vector<char> mark(lim + 1, false);
vector<long long> primes;
for (long long i = 2; i <= lim; ++i)
{
if (!mark[i]) {
primes.emplace_back(i);
for (long long j=i*i;j<=lim;j+=i)
mark[j]=true;}}
vector<char> isPrime(R - L + 1,
true);
for (long long i:primes)
for (long long j=max(i*i, (L+i-
1)/i*i);j<=R;j+=i)
isPrime[j-L]=false;
if (L==1) isPrime[0] = false;
return isPrime;}
```

SUM OF DIVS

```
long long SOD(long long n){
long long res=1;
for(auto u:prime){
if(1LL*u*u > n)break;
if(n%u==0){
long long sum=1;
long long power = 1;
while(n%u==0){
n/=u;
power *= u;
sum += power;}
res *= sum;}}
if(n>1)res*=(1+n);
return res;}
```

SUM OF NUMBER OF DIVS

```
int SNOD(int n){
int sq = sqrt(n), res=0;
for(int i=1; i<=sq; i++){
res += (n/i) - i;}
res *= 2;
res += sq;
return res;}
```

NUMBER OF DIVS

```
ll NOD(ll n){
ll ans=1;
if(n>1000000000000){
for(ll i=0;;++i){
if(prime[i]*prime[i]*prime[i]>n){
break;
}
if(n%prime[i]==0){
ll cnt = 0;
while (n%prime[i]== 0){
n /= prime[i];
cnt++;}
ans*=(cnt+1);}}
if(isprime(n)){ans*=2;}
else if(issquareprime(n)){ans*=3;}
else if(n!=1){ans*=4;}}
else{
ll limit=sqrt(n);
for (ll i=0;prime[i]<=limit;++i){
if(n%prime[i]== 0){
ll cnt = 0;
while (n% prime[i]== 0){
n /= prime[i];
cnt++;
}
ans*=(cnt+1);
limit=sqrt(n);
}
}
if(n!=1)ans*=2;}
return ans;
}
//for NOD count each prime factors+1
and multiply all of them 2,2,3,3,3
(3)*(4)
```

GEOEMTRIC SUM

```
//**Give a,n will give
a^1+a^2+a^3+...+a^n**
const ll MOD=1e9+7;
ll GeoSum(ll a, ll n){
ll sz = 0;ll ret = 0;ll mul = 1;
int MSB = 63 - __builtin_clzll(n);
while(MSB >= 0){
ret = ret * (1 + mul); mul = (mul
*mul) % MOD; sz <<= 1;
if( (n >> MSB) & 1) {
mul = (mul *a) % MOD; ret += mul;
sz++;}
ret %= MOD; MSB--;}
return ret;}
```

NCR USING RECURRENCE

```
**ncr using recurrence{nCr = (n-1)Cr
+ (n-1)C(r-1)}**
void fncr(){
for(int i = 0; i < N; ++i) {
for(int j = 0; j < N; ++j) {
if (j == 0 || j == i) ncr[i][j] = 1;
else ncr[i][j] = ncr[i-1][j-1]
+ncr[i-1][j];
}}}
```

NCR%M USING MODULAR MULTIPLICATIVEINVERSE

```
void precal(){
fact[0]=invFact[0]=1;
for(int i=1; i<=N; i++){
fact[i]=((fact[i-
1]%MOD)*(i%MOD))%MOD;}
invFact[N]=bigmod(fact[N],MOD-2);
for(int i=N-1; i>=1; i--){
invFact[i]=((invFact[i+1]%MOD)*((i+1
)%MOD))%MOD;}}
cout<<((fact[n]*invFact[r])%MOD*invF
act[n-r])%MOD<<endl;
//**ncr if it stays in long long**
ll n,r,ans=1;
cin>>n>>r;
for(int i=1; i<=r; i++){
ans=ans*(n-i+1);
ans/=i;}
```

LUCAS THEOREM(N AND R IS VERY BIG BUT MOD IS SMALL)

```
precal(){factorial and inverse
factorialmod}
ll Lucas(ll n,ll r){
if(r<0||r>n){
return 0;}
if(r==0||r==n){
return 1;}
if(n>=MOD){
return(Lucas(n/MOD,r/MOD)%MOD*Lucas(
n%MOD,r%MOD)%MOD)%MOD;}
return(((fact[n]*invFact[r])%MOD)*in
vFact[n-r])%MOD;}
```

EXTENDED GCD AND LINEAR DIOPHANTINE EQUATION

```
int gcd(int a, int b, int& x, int&
y) {
if (b == 0) {
x = 1; y = 0;
return a;}
int x1, y1;
int d = gcd(b, a % b, x1, y1);
x = y1;
y = x1 - y1 * (a / b);
return d;}
```

LINEAR DIOPHANTINE

```
bool find_any_solution(int a, int b,
int c, int &x0, int &y0, int &g) {
g = gcd(abs(a), abs(b), x0, y0);
if (c % g) {
return false;}
x0 *= c / g;
y0 *= c / g;
if (a < 0) x0 = -x0;
if (b < 0) y0 = -y0;
return true;}
void shift_solution(int & x, int &
y, int a, int b, int cnt) {
x += cnt * b;
y -= cnt * a;}
int find_all_solutions(int a, int b,
int c, int minx, int maxx, int miny,
int maxy) {
int x, y, g;
if (!find_any_solution(a, b, c, x,
y, g))
return 0;
a /= g;
b /= g;
int sign_a = a > 0 ? +1 : -1;
int sign_b = b > 0 ? +1 : -1;
shift_solution(x, y, a, b, (minx -
x) / b);
if (x < minx)
shift_solution(x, y, a, b, sign_b);
if (x > maxx)
return 0;
int lx1 = x;
shift_solution(x, y, a, b, (maxx -
x) / b);
if (x > maxx)
shift_solution(x, y, a, b, -sign_b);
int rx1 = x;
shift_solution(x, y, a, b, -(miny -
y) / a);
if (y < miny)
shift_solution(x, y, a, b, -sign_a);
if (y > maxy)
return 0;
int lx2 = x;
shift_solution(x, y, a, b, -(maxy -
y) / a);
if (y > maxy)
shift_solution(x, y, a, b, sign_a);
int rx2 = x;
if (lx2 > rx2)
swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx)
return 0;
return (rx - lx) / abs(b) + 1;}
```

FAYSAL AHAMMED CHOWDHURY

LINEAR SIEVE in O(n) upto 10^7 but memory takes 32 times more:

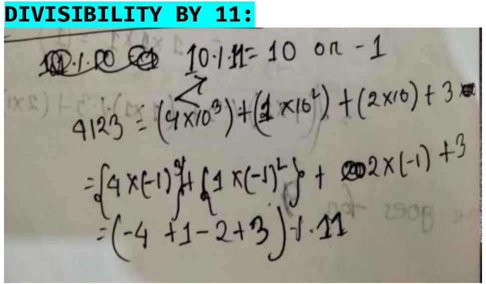
```
const int N = 10000000;
vector<int> spf(N + 1);
vector<int> primes;
void linear_sieve(){ // O(n)
for (int i = 2; i <= N; ++i){
if (spf[i] == 0){
spf[i] = i;
primes.push_back(i);}
for (int j = 0; i * primes[j] <= N;
++j){
spf[i * primes[j]] = primes[j];
if (primes[j] == spf[i])
{break;}}}}
```

SPF SIEVE

```
const int N = 1e6 + 9;
int spf[N];
void spf_sieve() {
for (int i = 2; i < N; i++) {
spf[i] = i;}
for (int i = 2; i < N; i++) {
if (spf[i] == i) {
for (int j = i; j < N; j += i) {
spf[j] = min(spf[j], i);}}}}
```

SOD: $((p1^{e1+1} - 1) / p1 - 1) * ((p2^{e2+1} - 1) / p2 - 1) * \dots$

DIVISIBILITY BY 11:



$10 \,\%\, 11 = 10$ or -1

$4123 = (4\times10^3)+(1\times10^2)+(2\times10)+3$

$= \{4\times(-1)^3\}+\{1\times(-1)^2\}+2\times(-1)+3$

$= (-4+1-2+3)\,\%\,11$

Legendre's Formula:

(N! / p^x) max value of x (p must be prime)

```
int legendre(int n, int p){
int ex = 0;
while(n) {
ex += (n / p);
n /= p;}
return ex;}
```

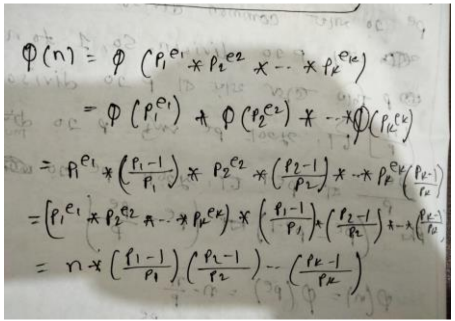
Digit Count of a number:

log10(n) + 1 (log10 for 10 base number)

How many Trailing zeros of n? –

Max power of base which divides n.

Phi function:



$\phi(n) = \phi(p_1^{e_1} * p_2^{e_2} * \cdots * p_k^{e_k})$

$= \phi(p_1^{e_1}) * \phi(p_2^{e_2}) * \cdots * \phi(p_k^{e_k})$

$= p_1^{e_1} * \left(\frac{p_1-1}{p_1}\right) * p_2^{e_2} * \left(\frac{p_2-1}{p_2}\right) * \cdots * p_k^{e_k}\left(\frac{p_k-1}{p_k}\right)$

$= (p_1^{e_1} * p_2^{e_2} * \cdots * p_k^{e_k}) * \left(\frac{p_1-1}{p_1}\right) * \left(\frac{p_2-1}{p_2}\right) * \cdots * \left(\frac{p_k-1}{p_k}\right)$

$= n * \left(\frac{p_1-1}{p_1}\right)\left(\frac{p_2-1}{p_2}\right)\cdots\left(\frac{p_k-1}{p_k}\right)$

* for n > 2, phi(n) is always even
* sum of all phi(d) - divisors of n, is n.

Euler Totient Function:

```
// if we need phi(x) multiple times,
then memoize it
map<int, int> dp;
int phi(int n) {
if (dp.count(n)) return dp[n];
int ans = n, m = n;
for (int i = 2; i * i <= m; i++) {
if (m % i == 0) {
while (m % i == 0) m /= i;
ans = ans / i * (i - 1);}}
if (m > 1) ans = ans / m * (m - 1);
return dp[n] = ans;}
```

Phi from 1 to n in O(nlog log n):

```
void phi_1_to_n(int n) {
vector<int> phi(n + 1);
for (int i = 0; i <= n; i++)
phi[i] = i;
for (int i = 2; i <= n; i++) {
if (phi[i] == i) {
for (int j = i; j <= n; j += i)
phi[j] -= phi[j] / i;}}}
```

Logarithm:

```
log(ab) = log(a) + log(b)
log(a^x) = xlog(a)
```

First K digit of n^k:

```
int firstk(int n, int k) {
double a = k * log10(n);
double b = a - floor(a);
double c = pow(10, b);
return floor(c * 100);}
```

Series:

Arithmetic progression:

S(n): (n/2)*(a+p) p is last element

Or, S(n) = (n/2) * (2a + (n-1) d) d is common difference

Geometric Progression:

S(n) = (a * (1 - r^n)) / (1-r) r < 1

S(n) = (a * (r^n - 1)) / (r-1) r > 1

* 1^2 + 2^2 +..+ n^2 = (n * (n+1) * (2n+1)) / 6

* 1^3 + 2^3 + … + n^3 = (n^2 * (n+1)^2) / 4

Problems:

* Minimal natural number N, so that N! contains exactly Q zeroes on the trail.
- Binary search over N and find how many trailing zeros N has by Legendre's formula.
* Given a number N^M, find out the number of integer bases in which it has exactly T trailing zeroes:

```
int solve_greater_or_equal(vector<int> e, int t) {
int ans = 1;
for(auto i: e) {
ans = 1LL * ans * (i / t + 1) % mod;
}
return ans;
}

int solve_equal(vector<int> e, int t) {
return (solve_greater_or_equal(e, t) - solve_greater_or_equal(e, t + 1) + mod) % mod;
}

int main() {
ios_base::sync_with_stdio(0);
cin.tie(0);

int n, m, t, cs = 0;
while(cin >> n >> m >> t and n != 0)
{
vector<int> e;
for(int i = 2; i * i <= n; i++) {
if(n % i == 0) {
int ex = 0;
while(n % i == 0) {
ex++;
n /= i;
}
e.push_back(ex * m);
}
}
if(n > 1) e.push_back(m);
cout << "Case " << ++cs << ": " << solve_equal(e, t) << '\n';
}

return 0;
}
```

* n! (factorial n) has at least t trailing zeroes in b based number system. Given the value of n and t, what is the maximum possible value of b?

```
const int N = 1e5 + 9, mod = 10000019;
vector<bool> is_prime(N, true);
vector<int> primes;

int main() {
ios_base::sync_with_stdio(0);
cin.tie(0);
sieve();

int t, cs = 0; cin >> t;
while(t--) {
int n, zeroes; cin >> n >> zeroes;
vector<pair<int, int>> ans;
for(auto p: primes) {
if(p > n) break;
int pw = legendre(n, p);
if(pw / zeroes != 0) {
ans.push_back({p, pw/ zeroes});
}
}
int max_base = 1;
for(auto i: ans) {
max_base = 1LL * max_base * power(i.first, i.second) % mod;
}
cout << "Case " << ++cs << ": ";
cout << (ans.size() == 0 ? -1 : max_base) << '\n';
}

return 0;
}
```

Combinatorics:

* nCr, nPr using recurrence -

```
const int N = 2005, mod = 1e9 + 7;
int C[N][N], fact[N];

void prec() { // O(n^2)
// nCr = (n-1)C(r-1) + (n-1)Cr
for (int i = 0; i < N; i++) {
C[i][0] = C[i][i] = 1;
for (int j = 1; j < N; j++) {
C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
}
}

fact[0] = 1;
for (int i = 1; i < N; i++) {
fact[i] = 1ll * fact[i - 1] * i % mod;
}
}

int nCr(int n, int r) {
if (n < r) return 0;
return C[n][r];
}

int nPr(int n, int r) {
if (n < r) return 0;
return 1ll * C[n][r] * fact[r] % mod;
}
```

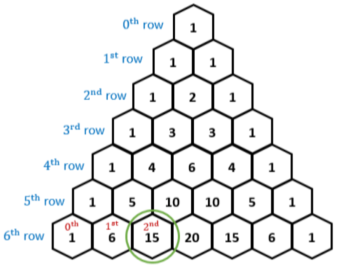
Combinations (nCr, nPr):

```
const int N = 1e6 + 1, mod = 1000003;
int fact[N], ifact[N];

int inverse(int x) {
return power(x, mod - 2, mod);
}

void prec() {
fact[0] = 1;
for (int i = 1; i < N; i++) {
fact[i] = 1ll * fact[i - 1] * i % mod;
}
ifact[N - 1] = inverse(fact[N - 1]);
for (int i = N - 2; i >= 0; i--) {
ifact[i] = 1ll * ifact[i + 1] * (i + 1) % mod;
}
}

int nCr(int n, int r) {
return 1ll * fact[n] * ifact[r] % mod * ifact[n - r] % mod;
}

int nPr(int n, int r) {
return 1ll * fact[n] * ifact[n - r] % mod;
}
```

Stars and Bars:

* Find the number of k-tuples of non-negative integers whose sum is n.
- (n + k - 1)! / (n! * (k-1)!) or C(n+k-1, n)
* Find the number of k-tuples of non-negative integers whose sum is <= n.
- (n + k)! / (n! * k!) or C(n+k, n)
* Combinations with repetitions
- C(n+k-1, k)
* How many ways to go from (0,0) to (n,m)
- C(n+m, n)

Pascals Triangle is equivalent to nCr



Multinomial Coefficient

(a1+a2+..+ak)^n

Powers are given of k numbers. coefficient?

```
int n, k; cin >> n >> k;
int ans = fact[n];
for (int i = 1; i <= k; i++) {
int x; cin >> x;
ans = 1ll * ans * ifact[x] % mod;
}
```

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Properties of Pascal's Triangle:

i = 0 to n ∑C(n,i) = 2^n

i = 0 to n where i is even ∑C(n,i) = 2^(n-1)

i = 0 to n where i is odd ∑C(n,i) = 2^(n-1)

Built-in Functions:

set(), reset(), flip(), count(), test(), any(), none(), all(), to_ullong()

__builtin_popcount(x), __builtin_clz(x), __builtin_ctz(x)

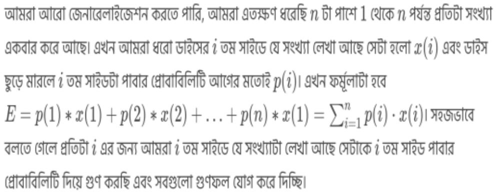
*** Power Tower:**

$$n^x \mod m = n^{\varphi(m)+x \mod \varphi(m)} \mod m$$

```
const int N = 1e5 + 9;
using ll = long long;

map<ll, ll> mp;
ll phi(ll n) {
if (mp.count(n)) return mp[n];
ll ans = n, m = n;
for (ll i = 2; i * i <= m; i++) {
```

```
if (m % i == 0) {
while (m % i == 0) m /= i;
ans = ans / i * (i - 1);
}
}
if (m > 1) ans = ans / m * (m - 1);
return mp[n] = ans;
}
inline ll MOD(ll x, ll m) {
if (x < m) return x;
return x % m + m;
}
ll power(ll n, ll k, ll mod) {
ll ans = MOD(1, mod);
while (k) {
if (k & 1) ans = MOD(ans * n, mod);
n = MOD(n * n, mod);
k >>= 1;
}
return ans;
}
int a[N];
// if x >= log2(m), then a^x =
a^(MOD(x, phi(m))) % m
ll yo(ll l, ll r, ll m) {
if (l == r) return MOD(a[l], m);
if (m == 1) return 1;
return power(a[l], yo(l + 1, r,
phi(m)), m);
}
int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);
int n, m; cin >> n >> m;
for (int i = 1; i <= n; i++) {
cin >> a[i];
}
int q; cin >> q;

while (q--) {
int l, r; cin >> l >> r;
cout << yo(l, r, m) % m << '\n';
}
return 0;
}
```

Expected value:



আমরা আরো জেনারেলাইজেশন করতে পারি, আমরা এতক্ষণ ধরেছি n টা পাশে 1 থেকে n পর্যন্ত প্রতিটা সংখ্যা একবার করে আছে। এখন আমরা ধরো ডাইসের i তম সাইডে যে সংখ্যা লেখা আছে সেটা হলো $x(i)$ এবং ডাইস ছুড়ে মারলে i তম সাইডটা পাবার প্রোবাবিলিটি আগের মতোই $p(i)$। এখন ফর্মুলাটা হবে $E = p(1) * x(1) + p(2) * x(2) + \ldots + p(n) * x(1) = \sum_{i=1}^{n} p(i) \cdot x(i)$। সহজভাবে বলতে গেলে প্রতিটা i এর জন্য আমরা i তম সাইডে যে সংখ্যাটা লেখা আছে সেটাকে i তম সাইড পাবার প্রোবাবিলিটি দিয়ে গুণ করছি এবং সবগুলো গুণফল যোগ করে দিচ্ছি।

** DATA STRUCTURES

Faysal Ahammed

Segment Tree:

```
const int N = 1e5 + 9;
int a[N];

struct ST {
int tree[4 * N];
void build(int n, int b, int e) {
if(b == e) {
tree[n] = a[b]; // change here
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
build(l, b, mid);
build(r, mid + 1, e);
tree[n] = tree[l] + tree[r]; //
change here
}
void upd(int n, int b, int e, int i,
int x) {
if(b > i || e < i) return;
if(b == e && b == i) {
tree[n] = x; // change here
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
upd(l, b, mid, i, x);
upd(r, mid + 1, e, i, x);
tree[n] = tree[l] + tree[r]; //
change here
}
int query(int n, int b, int e, int
i, int j) {
if(b > j || e < i) return 0; //
return appropriate value
if(b >= i && e <= j) return tree[n];
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
int L = query(l, b, mid, i, j);
int R = query(r, mid + 1, e, i, j);
return (L + R); // change this
}
} st;
```

Segment Tree Lazy:

```
const int N = 1e5 + 9;
int a[N];

struct ST {
int tree[4 * N], lazy[4 * N];
ST() {
memset(tree, 0, sizeof(tree));
memset(lazy, 0, sizeof(lazy));
}
void push(int n, int b, int e) {
if(lazy[n] == 0) return;
tree[n] += lazy[n] * (e - b + 1); //
change here
if(b != e) {
int l = n << 1, r = l + 1;
lazy[l] += lazy[n]; // change here
lazy[r] += lazy[n]; // change here
}
lazy[n] = 0;
}
void build(int n, int  b, int e) {
lazy[n] = 0;
if(b == e) {
tree[n] = a[b]; // change here
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
build(l, b, mid);
build(r, mid + 1, e);
tree[n] = tree[l] + tree[r]; //
change here
}
void upd(int n, int b, int e, int i,
int j, int x) {
push(n, b, e);
if(b > j || e < i) return;
if(b >= i && e <= j) {
lazy[n] += x; // change here
push(n, b, e);
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
upd(l, b, mid, i, j, x);
upd(r, mid + 1, e, i, j, x);
tree[n] = tree[l] + tree[r]; //
change here
}
int query(int n, int b, int e, int
i, int j) {
push(n, b, e);
if(b > j || e < i) return 0; //
return appropriate value
if(b >= i && e <= j) return tree[n];
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
int L = query(l, b, mid, i, j);
int R = query(r, mid + 1, e, i, j);
return L + R; // change here
}
} st;
```

Strongest Community:

```
const int N = 1e5 + 9;
int a[N];

struct node {
int first_element,
first_element_cnt;
int last_element, last_element_cnt;
int max_cnt;
};

node merge(node l, node r) {
if(l.first_element == -1) return r;
if(r.first_element == -1) return l;
node ans;
ans.max_cnt = max(l.max_cnt,
r.max_cnt);
if(l.last_element ==
r.first_element) {
ans.max_cnt = max(ans.max_cnt,
l.last_element_cnt +
r.first_element_cnt);
}
ans.first_element = l.first_element;
ans.first_element_cnt =
l.first_element_cnt;
if(l.first_element ==
r.first_element) {
ans.first_element_cnt +=
r.first_element_cnt;
}
ans.last_element = r.last_element;
ans.last_element_cnt =
r.last_element_cnt;
if(r.last_element == l.last_element)
{
ans.last_element_cnt +=
l.last_element_cnt;
}
return ans;
}

struct ST {
node tree[4 * N];
void build(int n, int b, int e) {
if(b == e) {
tree[n].first_element = a[b];
tree[n].first_element_cnt = 1;
tree[n].last_element = a[b];
tree[n].last_element_cnt = 1;
tree[n].max_cnt = 1;
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
build(l, b, mid);
build(r, mid + 1, e);
tree[n] = merge(tree[l], tree[r]);
// change here
}
node query(int n, int b, int e, int
i, int j) {
if(b > j || e < i) return {-1, -1, -
1, -1, -1};
if(b >= i && e <= j) return tree[n];
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
return merge(query(l, b, mid, i, j),
query(r, mid + 1, e, i, j)); //
change this
}
} st;
```

Segment tree with arithmetic progression [update the range l to r with the arithmetic progression a+b*(i-l)]:

```
using ll = long long;
const int N = 2e5 + 9;
const ll inf = 1e18;

pair<ll, ll> merge(pair<ll, ll> l,
pair<ll, ll> r) {
l.first = min(l.first, r.first);
l.second = max(l.second, r.second);
return l;
}

struct ST {
pair<ll, ll> tree[4 * N];
ll lazy[4 * N];
void push(int n, int b, int e) {
if (lazy[n] == 0) return;
tree[n].first += lazy[n];
tree[n].second += lazy[n];
if (b != e) {
int l = n << 1, r = l + 1;
lazy[l] += lazy[n];
lazy[r] += lazy[n];
}
lazy[n] = 0;
}
void build(int n, int  b, int e) {
lazy[n] = 0;
if (b == e) {
tree[n] = {0, 0};
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
build(l, b, mid);
build(r, mid + 1, e);
tree[n] = merge(tree[l], tree[r]);
}
void upd(int n, int b, int e, int i,
int j, ll x) {
push(n, b, e);
if (b > j || e < i) return;
if (b >= i && e <= j) {
lazy[n] += x;
push(n, b, e);
return;
}
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
upd(l, b, mid, i, j, x);
upd(r, mid + 1, e, i, j, x);
tree[n] = merge(tree[l], tree[r]);
}
pair<ll, ll> query(int n, int b, int
e, int i, int j) {
push(n, b, e);
if (b > j || e < i) return {inf, -
inf};
if (b >= i && e <= j) return
tree[n];
int mid = (b + e) >> 1, l = n << 1,
r = l + 1;
pair<ll, ll> L = query(l, b, mid, i,
j);
pair<ll, ll> R = query(r, mid + 1,
e, i, j);
return merge(L, R);
}
} st;

int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);

int n, q; cin >> n >> q;
st.build(1, 1, n);
while (q--) {
int type; cin >> type;
if (type == 1) {
int l, r; cin >> l >> r;
if (l == r) {
cout << 1 << '\n';
continue;
}
pair<ll, ll> ans = st.query(1, 1, n,
l + 1, r);
if (ans.first == 0 and ans.second ==
0) {
cout << 1 << '\n';
}
else {
cout << 0 << '\n';
}
}
else {
int l, r; ll a, b; cin >> l >> r >>
a >> b;
st.upd(1, 1, n, l, l, a);
if (l != r) {
st.upd(1, 1, n, l + 1, r, b);
}
ll x = 1ll * b * (r - l) + a;
st.upd(1, 1, n, r + 1, r + 1, -x);
}
}

return 0;
}
```

Searching for the first element greater than a given amount:

```
int get_first(int v, int tl, int tr,
int l, int r, int x) {
if (tl > r || tr < l) return -1;
if (t[v] <= x) return -1;

if (tl == tr) return tl;

int tm = tl + (tr - tl) / 2;
int left = get_first(2 * v, tl, tm,
l, r, x);
if (left != -1) return left;
return get_first(2 * v + 1, tm + 1,
tr, l , r, x);
}
```

Counting the number of zeros, searching for the k-th zero:

```
int find_kth(int v, int tl, int tr,
int k) {
if (k > t[v])
return -1;
if (tl == tr)
return tl;
int tm = (tl + tr) / 2;
if (t[v * 2] >= k)
return find_kth(v * 2, tl, tm, k);
else
return find_kth(v * 2 + 1, tm + 1,
tr, k - t[v * 2]);
}
```

Faisal Amin Abir

Trie Prefix tree

```
struct trieNode{
int cnt;
trieNode *child[2];
trieNode(){
cnt=0;
for(int i=0; i<2; i++){
child[i]=NULL;
}}
};
trieNode *root;
void insert(int num) {
binary form
trieNode *cur=root;
for(int i=20; i>=0; i--){
int ind=(bool)(num&(1<<i));
if(!cur->child[ind]){
cur->child[ind]=new
trieNode();
}
cur=cur->child[ind];
cur->cnt++;
}}
```

Min XOR with value set

-1 X: Add x to set
-2 x:for each element set it to element^x
-3: print the minimum of the set

```
int X=0;
void solve(int test){
int type,x;
si(type);
if(type==1){
si(x);
insert(x^X);}
else if(type==2){
si(x);
X^=x;}
else{
int res=query(X);
printf("%d\n",res);
}}
```

** Segment Tree **
**LAZY Propagation **

```
const ll N = 2e5+123;
ll arr[N], t[4*N], lazy[4*N],
lazySet[4*N];
bool isSet[4*N];
ll n, m;
void build(ll v, ll l, ll r){
if(l==r){
t[v]=arr[l];
return;
}
ll mid = (l+r)>>1;
build(2*v, l, mid);
build(2*v+1, mid+1, r);
t[v] = t[2*v] + t[2*v+1];
}
void checkSet(ll v, ll L, ll R){
if(isSet[v]==1){
isSet[v]=0;
ll x = lazySet[v];
lazySet[v]=0;
t[v] = (R-L+1) * x;
if(L!=R){
isSet[2*v] = isSet[2*v+1] = 1;
lazy[2*v] = lazy[2*v+1] = 0;
lazySet[2*v] = lazySet[2*v+1] = x;
}
}
}
void checkAdd(ll v, ll L, ll R){
if(lazy[v]!=0){
ll x = lazy[v];
lazy[v]=0;
t[v] += (R-L+1) * x;
if(L!=R){
lazy[2*v] += x;
lazy[2*v+1] += x;
}
}
}
void add(ll v, ll L, ll R, ll l, ll r, ll value){
checkSet(v, L, R);
checkAdd(v, L, R);
if(L > r or R < l) return;
if( L >= l and R <= r){
t[v] += (R-L+1) * value;
if(L!=R){
lazy[2*v] += value;
lazy[2*v+1] += value;
}
return;
}
ll mid = (L+R)>>1;
add(2*v, L, mid, l, r, value);
add(2*v+1, mid+1, R, l, r, value);
t[v] = t[2*v] + t[2*v+1];
}

void set_value(ll v, ll L, ll R, ll l, ll r, ll value){
checkSet(v, L, R);
checkAdd(v, L, R);
if(L>r or R < l)return;
if(L>=l and R<=r){
t[v] = (R-L+1) * value;
if(L!=R){
isSet[2*v] = isSet[2*v+1] = 1;
lazy[2*v] = lazy[2*v+1] = 0;
lazySet[2*v] = lazySet[2*v+1] = value;
}
return;
}
ll mid = (L+R)>>1;
set_value(2*v, L, mid, l, r, value);
set_value(2*v+1, mid+1, R, l, r , value);
t[v] = t[2*v] + t[2*v+1];
}

ll query(ll v, ll L, ll R, ll l, ll r){
checkSet(v, L, R);
checkAdd(v, L, R);
if(L>r or R<l)return 0;
if(L>=l and R <= r) return t[v];
ll mid = (L+R)>>1;
return query(2*v, L, mid, l, r) + query(2*v+1, mid+1, R, l, r);
}
void solve(){
cin>>n>>m;
for(ll i=1; i<=n; i++){
cin>>arr[i];
}
build(1, 1, n);
for(ll i=0; i<m; i++){
ll c;
cin>>c;
if(c==1){
ll a, b, x;
cin>>a>>b>>x;
add(1, 1, n, a, b, x);
}
else if(c==2){
ll a, b, x;
cin>>a>>b>>x;
set_value(1, 1, n, a, b, x);
}
else{
ll a, b;
cin>>a>>b;
cout << query(1, 1, n, a, b) << endl;
}}}
```

Max subarray sum in L to R with update

```
const int mx=50005;
LL num[mx];
struct node{
LL left,right,sum,answer;
};
node tree[4*mx];
node merge(node a,node b){
node x;
x.left=max(a.left,a.sum+b.left);
x.right=max(b.right,a.right+b.sum);
x.answer=max(max(a.answer,b.answer), a.right+b.left);
x.sum=a.sum+b.sum;
return x;
}
void buildSegmentTree(int at,int L,int R){
if(L==R){
tree[at].left=num[L];
tree[at].right=num[L];
tree[at].sum=num[L];
tree[at].answer=num[L];
return;
}
int mid=(L+R)/2;
buildSegmentTree(at*2,L,mid);
buildSegmentTree(at*2+1,mid+1,R);
tree[at]=merge(tree[at*2],tree[at*2+1]);
}
void updateSegemntTreeWithNew(int at,int L,int R,int pos,int value){
if(pos>R || pos<L){
return;
}
if(L>=pos && R<=pos){
tree[at].left=value;
tree[at].right=value;
tree[at].sum=value;
tree[at].answer=value;
return;
}
int mid=(L+R)/2;
updateSegemntTreeWithNew(at*2,L,mid, pos,value);
updateSegemntTreeWithNew(at*2+1,mid+1,R,pos,value);
tree[at]=merge(tree[at*2],tree[at*2+1]);
}
node def;
node querySegmentTree(int at,int L,int R,int l,int r){
if(r<L || R<l)return def;
if(l<=L && R<=r) return tree[at];
int mid=(L+R)/2;
node x=querySegmentTree(at*2,L,mid,l,r);
node y=querySegmentTree(at*2+1,mid+1,R,l,r);
return merge(x,y);
}
int main(){
def.answer=-100000000;
def.left=-100000000;
def.right-100000000;
def.sum=-100000000;
int n;
si(n);
Rep(i,1,n){
sl(num[i]);}
buildSegmentTree(1,1,n);
int q,l,r,x;
si(q);
while(q--){
siii(x,l,r);
if(x==0){
updateSegemntTreeWithNew(1,1,n,l,r);
}
else{
node ans=querySegmentTree(1,1,n,l,r);
printf("%lld\n",ans.answer);
}
}
return 0;
}
```

Longest increasing subsequence

```
for(int i=n; i>=1; i--){
int qr=query(1,1,m,arr[i]+1,m);
int now=query(1,1,m,arr[i],arr[i]);
now=max(now,1+qr);
update(1,1,m,arr[i],now);
}
```

All possible longest increasing subsequence

```
for(int i=1; i<=n; i++){
scl(arr[i].x)
arr[i].y=i+1;
}
sort(arr+1,arr+n+1,cmp);
for(int i=1; i<=n; i++){
lln d=arr[i].y;
lln q1=query(1,1,n+1,1,d-1);
update(1,1,n+1,d,d,q1+1);
}
lln ans=query(1,1,n+1,1,n+1);
printf("Case %d: %lld\n",xx,ans);
```

** Merge sort tree without update **

```
vector<int>:: iterator child;
vector<int>vi2[4*maxii];
int arr3[maxii];
int n,m;
int cc=1;
void segment(int node,int b,int e){
if(b==e){
vi2[node].pb(arr3[b]);
return;
}
int left=2*node;
int right=left+1;
int mid=(b+e)/2;
segment(left,b,mid);
segment(right,mid+1,e);
int ff=0,hh=0;
while(ff<vi2[left].size() &&hh<vi2[right].size()){
if(vi2[left][ff]<=vi2[right][hh]){
vi2[node].pb(vi2[left][ff]);
ff++;
}
else{
vi2[node].pb(vi2[right][hh]);
hh++;
}
}
while(ff<vi2[left].size()){
vi2[node].pb(vi2[left][ff]);
ff++;
}
while(hh<vi2[right].size()){
vi2[node].pb(vi2[right][hh]);
hh++;
}
}
int w;
int query(int node,int b,int e,int
st,int en){
if(st>e || en<b)return 0;
if(b>=st && e<=en){
int di=vi2[node].size();
int
p=lower_bound(vi2[node].begin(),vi2[
node].end(),w)-vi2[node].begin();
return di-p;
}
int mid=(b+e)/2;
int left=2*node;
int right=left+1;
int q1=query(left,b,mid,st,en);
int q2=query(right,mid+1,e,st,en);
return q1+q2;
}
```

Merge sort tree with update

```
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.h
pp>
#include<ext/pb_ds/tree_policy.hpp>
#define pii pair<int,int>
using namespace __gnu_pbds;
using namespace std;
typedef tree<pii, null_type,
less<pii>,rb_tree_tag,
tree_order_statistics_node_update>or
dered_set;
const int maxn=3e5+5;
int arr[maxn];
ordered_set segtree[4*maxn];
void build(int at,int L,int R){
if(L==R){
segtree[at].insert({arr[L],L});
return;
}
int mid=(L+R)>>1;
int left=at*2;
int right=at*2+1;
build(left,L,mid);
build(right,mid+1,R);
for(pii
i:segtree[left])segtree[at].insert(i
);
for(pii
i:segtree[right])segtree[at].insert(
i);
}
void update(int at,int L,int R,int
pos,pii rp,pii pt){
if(pos>R || pos<L){
return;
}
segtree[at].erase(rp);
segtree[at].insert(pt);
if(L==R){
return;
}
int mid=(L+R)>>1;
if(pos<=mid)update(at*2,L,mid,pos,rp
,pt);
else
update(at*2+1,mid+1,R,pos,rp,pt);
}
int query(int at,int L,int R,int
l,int r,int val){
if(r<L || R<l)return 0;
if(l<=L && R<=r)return
segtree[at].order_of_key({val,0});
int mid=(L+R)>>1;
return
query(at*2,L,mid,l,r,val)+query(at*2
+1,mid+1,R,l,r,val);
}
```

Total number of subarray whose sum less than t

```
void solve(int test){
LL n,t,res=0;
sll(n,t);
Rep(i,1,n){
sl(arr[i]);
arr[i]+=arr[i-1];
}
build(1,1,n);
Rep(i,1,n){
res+=query(1,1,n,i,n,arr[i-1]+t);
}
printf("%lld\n",res);
}
```

Mo's Algorithm distinct element in range

```
const int N = 2e5+2;
const int BLOCK = 450;
int arr[N], n, freq[N], cnt, ans[N];
struct query{
int l, r, index;
}q[200001];
void add(int value){
freq[value]++;
if(freq[value]==1)cnt++;
}
void remove(int value){
freq[value]--;
if(freq[value]==0)cnt--;
}
bool comp(query q1, query q2){
if((q1.l / BLOCK) == (q2.l /
BLOCK)){
return q1.r < q2.r;
}
return (q1.l / BLOCK ) < (q2.l /
BLOCK);
}
void solve(){
cin>>n;
int m;
cin>>m;
for(int i=0; i<n; i++) cin>>arr[i];
map<int, int>maps;
int c=0;
for(int i=0; i<n; i++){
if(maps[arr[i]]==0){
c++;
maps[arr[i]]=c;
}}
for(int i=0; i<n; i++) arr[i] =
maps[arr[i]];
for(int i=0; i<m; i++){
int x, y;
cin>>x>>y;
x--, y--;
q[i].index = i;
q[i].l = x;
q[i].r = y;
}
sort(q, q + m, comp);
int L=0, R=-1;
for(int i=0; i<m; i++){
int currL = q[i].l;
int currR = q[i].r;
int indx = q[i].index;
while(L-1 >= currL){//adding value
L--;
add(arr[L]);
}
while(R+1<=currR){
R++;
add(arr[R]);
}
while(L<currL){//removing value
remove(arr[L]);
L++;
}
while(R > currR){
remove(arr[R]);
R--;
}
ans[indx] = cnt;
}
for(int i=0; i<m; i++) cout <<
ans[i] << endl;
}
```

** GRAPH THEORY

Faysal Ahammed

BFS:

```
const int N = 1e5 + 9;
vector<int> g[N];
vector<bool> vis(N, false);

void bfs(int u) {
queue<int> q;
q.push(u);
vis[u] = true;
while(!q.empty()) {
int top = q.front(); q.pop();
for(auto v: g[top]) {
if(!vis[v]) {
q.push(v);
vis[v] = true;}}}}
```

Find Cycle:

```
const int N = 2e5 + 9;
vector<int> g[N];
vector<int> par(N, -1);
vector<int> col(N, 0);
int cycle_start, cycle_end;
set<int> cycle;

void find_cycle(int u) {
col[u] = 1;
for (auto v: g[u]) {
if (col[v] == 0) {
par[v] = u;
find_cycle(v);
}
else if(col[v] == 1 and v != par[u])
{
cycle_end = u;
cycle_start = v;
}
}
col[u] = 2;
}

int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);

int n, m; cin >> n >> m;
for(int i = 1; i <= m; i++) {
int u, v; cin >> u >> v;
g[u].push_back(v);
g[v].push_back(u);
}

find_cycle(1);
int cur = cycle_end;
while (cur != cycle_start) {
cycle.insert(cur);
cur = par[cur];
}
cycle.insert(cur);

return 0;
}
```

Topological Sort:

```
const int N = 1e5 + 9;
vector<int> g[N];
vector<int> indeg(N, 0);

int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);

int n, m; cin >> n >> m;
while(m--) {
int u, v; cin >> u >> v;
g[u].push_back(v);
indeg[v]++;
}

queue<int> q;
for(int i = 1; i <= n; i++) {
if(indeg[i] == 0) {
q.push(i);
}}

vector<int> ans;
while(!q.empty()) {
int top = q.front();
q.pop();
ans.push_back(top);
for(auto v: g[top]) {
indeg[v]--;
if(indeg[v] == 0) {
q.push(v);}}}

if(ans.size() == n) {
for(auto i: ans) {
cout << i << ' ';}
cout << '\n';}
else {
cout << "IMPOSSIBLE\n";}

return 0;
}
```

Bellman Ford:

```
using ll = long long;
const int N = 1005;
const ll inf = 1e18;
vector<pair<int, int>> g[N];
vector<ll> dis(N, inf);
int n, m;
bool cycle;

void bellman_ford(int src) {
dis[src] = 0;
for (int i = 1; i <= n; i++) {
for (int u = 1; u <= n; u++) {
for (auto [v, w] : g[u]) {
if (dis[v] > dis[u] + w and dis[u]
!= inf) {
if (i == n) cycle = true;
dis[v] = dis[u] + w;}}}}}

int main() {
cin >> n >> m;
while (m--) {
int u, v, w; cin >> u >> v >> w;
g[u].push_back({ v,w });
g[v].push_back({ u,w });
}

cycle = false;
bellman_ford(1);

for (int u = 1; u <= n; u++) {
cout << dis[u] << ' ';
}
cout << '\n';

return 0;
}
```

Dijkstra:

```
using ll = long long;
const int N = 1e5 + 9;
const ll inf = 1e18;
vector<pair<int, int>> g[N];
vector<bool> vis(N, false);
vector<ll> dis(N, inf);
int n, m;

void dijkstra(int u) {
dis[u] = 0;
priority_queue<pair<ll, int>,
vector<pair<ll, int>>,
greater<pair<ll, int>>> pq;
pq.push({ 0, u });
while (!pq.empty()) {
int selected_node = pq.top().second;
ll d = pq.top().first;
pq.pop();
if (vis[selected_node]) continue;
vis[selected_node] = true;
for (auto [v, w] : g[selected_node])
{
if (dis[v] > (1ll * d + w)) {
dis[v] = 1ll * d + w;
pq.push({ dis[v], v });}}}}
```

Floyd Warshall:

```
using ll = long long;
const int N = 505;
const ll inf = 1e18;
int g[N][N];
ll dis[N][N];
int n, m;

void floyd_warshall() {
for (int i = 1; i <= n; i++) {
for (int j = 1; j <= n; j++) {
if (i == j) dis[i][j] = 0;
else if (g[i][j] == 0) dis[i][j] =
inf;
else dis[i][j] = g[i][j];
}
}

for (int k = 1; k <= n; ++k) {
for (int i = 1; i <= n; ++i) {
for (int j = 1; j <= n; ++j) {
if (dis[i][k] < inf and dis[k][j] <
inf)
dis[i][j] = min(dis[i][j], dis[i][k]
+ dis[k][j]);
}}}}
```

Krushkal's MST:

```
const int N = 3e5 + 9, mod = 1e9;

struct dsu {
vector<int> par, rnk, size; int c;
dsu(int n) : par(n+1), rnk(n+1,0),
size(n+1,1), c(n) {
for (int i = 1; i <= n; ++i) par[i]
= i;
}
int find(int i) { return (par[i] ==
i ? i : (par[i] = find(par[i]))); }
bool same(int i, int j) { return
find(i) == find(j); }
int get_size(int i) { return
size[find(i)]; }
int count() { return c; }
//connected components
int merge(int i, int j) {
if ((i = find(i)) == (j = find(j)))
return -1; else --c;
if (rnk[i] > rnk[j]) swap(i, j);
par[i] = j; size[j] += size[i];
if (rnk[i] == rnk[j]) rnk[j]++;
return j;
}
};

int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);
int n, m; cin >> n >> m;
vector<array<int, 3>> ed;
for(int i = 1; i <= m; i++){
int u, v, w; cin >> u >> v >> w;
ed.push_back({w, u , v});
}
sort(ed.begin(), ed.end());
long long ans = 0;
dsu d(n);
for (auto e: ed){
int u = e[1], v = e[2], w = e[0];
if (d.same(u, v)) continue;
ans += w;
d.merge(u, v);
}
cout << ans << '\n';
return 0;
}
```

Tree Diameter: max cost(distance) between 2 nodes. Dfs from any node and get 1 of the 2 nodes. Then dfs again from this node and get another 1. From every node, 1 of these two nodes is the max cost.

Faisal Amin Abir

Bellman Ford with negative cycle

```
ll INF = 1e16+7; int n; int m;
struct edge{ ll a, b, cost;};
ll d[5005]; edge e[5005];
ll p[5005];
void solve(int v){
ll x=-1; d[v]=0;
for(int i=0; i<n; i++){
x=-1;
for(ll j=0; j<m; j++)
if(d[e[j].a] < INF)
if(d[e[j]].b > d[e[j].a] +
e[j].cost){
d[e[j].b] = max(-INF,d[e[j].a] +
e[j].cost);
p[e[j].b] = e[j].a; x = e[j].b;
}
}
if(x==-1) cout << "NO" << endl;
else{
```

```
ll y = x;
for(ll i = 0; i<n; i++) y = p[y];
vector<ll>path;
for(ll curr=y;;curr=p[curr]){
path.push_back(curr);
if(curr==y && path.size()>1){
break;
}
}
reverse(path.begin(), path.end());
cout << "YES" << endl;
for(auto u:path) cout << u << " ";
return ;
}
}
int main(){
cin>>n>>m;
for(int i=0; i<m; i++){
edge ee; cin>> ee[i].a;
cin>>ee[i].b>>ee[i].cost;
}
solve(1); return 0;
}
```

** FLOYD_Warshall**

```
void Floyd_Warshall(){
for(int i=0; i<=n; i++){
for(int j=0; j<=n; j++){
dis[i][j] = 100000000;}
dis[i][i]=0;}
for(int k=1; k<=n; k++){
for(int i=1; i<=n; i++){
for(int j=1; j<=n; j++){
dis[i][j] = min(dis[i][j], dis[i][k]
+ dis[k][j]);}}}}
```

Lexicographically minimum topological sort

```
vector<int> vi[maxii];
vector<int>:: iterator child;
int check[maxii];
int in[maxii];
int n,m;
int cycle =0;
void dfs(int node){
if(cycle){
return;}
check[node]=1;
for(auto child: vi[node]){
if(check[child]==0){
dfs(child);
}
else if(check[child]==1){
cycle=1;return ;
}}check[node]=2;
}
int main(){
int test=1;
for(int xx=1; xx<=test; xx++){
cin>>n>>m;
int a,b;
for(int i=1; i<=m; i++){
cin>>a>>b;
vi[a].push_back(b);
in[b]++;
}
for(int i=1; i<=n; i++){
if(check[i]==0){
dfs(i);
}}
if(cycle){
cout<<-1;
return 0;}
priority_queue<pair<int,int>,vector<
pair<int,int>>,greater<pair<int,int>
> > que;
for(int i=1; i<=n; i++){
if(in[i]==0){
que.push({i,0});
}}
while(!que.empty()){
int node=que.top().first;
int das=que.top().second;
que.pop();
if(das==0){
cout<<node<<" ";
for(int child:vi[node])
{
in[child]--;
que.push({child,in[child]});
}}}}}
```

** HLD to find max between u and v, with euler path, LCA and segtree **

```
// dfs for subtree cal, dfs2 for
euler
const int N = 1e4+1;;
const int K = __lg(N) + 2;
int LOG[N], parent[N], value[N],
level[N], subtree[N], first[N],
chain[N], in[N], out[N], lca[N][K],
head[N], t[4*N], n;
vector<pair<int, int>>g[N];
vector<int>euler;
void dfs(int v, int par, int val){
parent[v]=par;
value[v]=val;
lca[v][0] = par;
int sub = 1;
for(auto u:g[v]){
if(u.ff==par)continue;
level[u.ff] = level[v] + 1;
dfs(u.ff, v, u.ss);
sub += subtree[u.ff];
}
subtree[v]=sub;
}
void dfs2(int v, int par, int time,
int matha){
euler.pb(v);
head[v] = matha;
int sub=0, node=0;
in[v] = time;
for(auto u:g[v]){
if(u.ff==par) continue;
if(subtree[u.ff]>sub){
sub = subtree[u.ff];
node = u.ff;
}}
if(sub!=0){
dfs2(node, v, time+1, matha);
time = out[node];}
for(auto u:g[v]){
if(u.ff == par or u.ff == node)
continue;
dfs2(u.ff, v, time+1, u.ff);
time=out[u.ff];
}
out[v] = time;
}
void build(int v, int l, int r){
if(l==r){
t[v] = value[euler[l]];
return;
}
int mid = (l+r) >> 1;
build(2*v, l, mid);
build(2*v+1, mid+1, r);
t[v] = max(t[2*v] , t[2*v+1]);
}
int max_query(int v, int L, int R,
int l, int r){
if(L>r or R<l) return 0;
if(L>=l and R<=r) return t[v];
int mid = (L+R) >> 1;
int left = max_query(2*v, L, mid, l,
r);
int right = max_query(2*v+1, mid+1,
R, l, r);
return max(left, right);
}
void update(int v, int l, int r, int
index, int value){
if(l==r){
t[v] = value;
return;
}
int mid = (l+r) >> 1;
if(index <= mid) update(2*v, l, mid,
index, value);
else update(2*v+1, mid+1, r, index,
value);
t[v] = max(t[2*v], t[2*v+1]);
}
void buildLCA(){
for(int i=1; i<K; i++){
for(int j=1; j<=n; j++){
int x = lca[j][i-1];
if(x != -1){
lca[j][i] = lca[x][i-1];
}}}}
int find_Lca(int a, int b){
if(level[a] < level[b]) swap(a, b);
int z = level[a] - level[b];
while(z>0){
int i = LOG[z];
a = lca[a][i];
z-= (1<<i);
}
if(a==b) return a;
for(int i=K-1; i>=0; i--){
if(lca[a][i]!=-1 and
lca[a][i]!=lca[b][i]){
a = lca[a][i];
b = lca[b][i];
}
}
return lca[a][0];
}
void init(){
for(int i=1; i<=n; i++) {
g[i].clear();
for(int j=0; j<K; j++){
lca[i][j]=-1;}}
euler.clear();
}
int find_max(int a, int root){
if(chain[a] == chain[root]){
return max_query(1, 0, n-1,
in[root], in[a]);
}
else{
int x = max_query(1, 0, n-1,
in[head[a]], in[a]);
x = max(x, find_max(parent[head[a]],
root));
return x;
}}
void solve(){
cin>>n;
init();
vector<pair<int, int>>edge;
for(int i=1; i<n; i++){
int x,y, cost;
cin>>x>>y>>cost;
g[x].push_back({y, cost});
g[y].push_back({x, cost});
edge.pb({x, y});
}
dfs(1, -1, -1);
dfs2(1, -1, 0, 1);
int currChain=1;
for(int i=0; i<n; i++){
int x = euler[i];
chain[x] = currChain;
if(in[x]==out[x]) currChain++;
}
build(1, 0, n-1);
buildLCA();
while(1){
string s;
cin>>s;
if(s=="DONE")break;
if(s=="QUERY"){
int a, b;
cin>>a>>b;
int y = find_Lca(a, b);
update(1, 0, n-1, in[y], 0);
int ans = find_max(a, y);
ans = max(ans, find_max(b, y));
cout << ans << endl;
update(1,0, n-1, in[y], value[y]);
}
else{
int index, val;
cin>>index>>val;
index--;
int a, b;
a = edge[index].ff;
b = edge[index].ss;
if(parent[a]==b) update(1, 0, n-1,
in[a], val), value[a]=val;
else{
update(1, 0, n-1, in[b], val);
value[b]=val;
}}}
}
int main(){
LOG[1] = 0;
for(int i=2; i<N; i++){
LOG[i] = LOG[i/2] + 1;
}
solve();
return 0;}
```

SSC find toposort, component

->given a forest, tell which node to forward mail to reach max people, lexicographically smallest one;

```
const int N = 1e5+123;
vector<int>g1[N], g2[N], g3[N],
topo;
int totalCom, com[N], got,
countCom[N], minCom[N], dis[N],
indeg[N];
bool vis1[N], vis2[N], vis3[N];
int n, m;
void reset(){
for(int i=0; i<=n; i++){
g1[i].clear();
g2[i].clear();
g3[i].clear();
indeg[i]=dis[i]=countCom[i]=com[i]=v
is1[i]=vis2[i]=vis3[i]=0;
minCom[i]=INT_MAX;
}
totalCom=0;
topo.clear();
got=0;
}
void dfs1(int v){
if(vis1[v])return;
vis1[v]=1;
for(auto u:g1[v]){
dfs1(u);
}
topo.pb(v);
}
void dfs2(int v){
if(vis2[v])return;
vis2[v]=1;
com[v]=totalCom;
countCom[totalCom]++;
minCom[totalCom] =
min(minCom[totalCom], v);
for(auto u:g2[v]){
dfs2(u);
}
}
void solve(){
cin>>n;
reset();
vector<pair<int, int>>v;
for(int i=0; i<n; i++){
int x, y;
cin>>x>>y;
g1[x].pb(y);
g2[y].pb(x);
v.pb({x, y});
}
for(int i=1; i<=n; i++){
if(vis1[i]==0)
dfs1(i);
}
reverse(all(topo));
for(auto u:topo){
if(vis2[u]==0){
totalCom++;
dfs2(u);
}
}
for(auto u:v){
int f = com[u.ff];
int t = com[u.ss];
if(f!=t){ // from B to A, we find
the node in reverse direction to get
highest count
g3[t].pb(f);
indeg[f]++;
}
}
using pii = pair<int, int>;
priority_queue<pii, vector<pii>,
greater<pii>>q;
for(int i=1; i<=totalCom; i++){
if(indeg[i]==0){
q.push({countCom[i], i});
dis[i]=countCom[i];
}
}
while(!q.empty()){
int from = q.top().ss;
q.pop();
for(auto u:g3[from]){
if(dis[u]<dis[from]+countCom[u]){
dis[u] = dis[from] + countCom[u];
q.push({dis[u], u});
}
}
}
int tot=0;
int ans=INT_MAX;
for(int i=1; i<=totalCom; i++){
if(dis[i]>tot){
tot = dis[i];
ans = minCom[i];}
else if(tot == dis[i]){
ans = min(ans, minCom[i]);}
}
cout << "Case " << ++test << ": " <<
ans << endl;
}
```

SSC to find component of component

```
const int N = 1e5+123;
vector<int>g1[N], g2[N], g3[N],
topo, topoCom;
int com[N], totalCom;
bool vis1[N], vis2[N], vis3[N],
vis4[N];
void dfs1(int v){// find the
toposort of the given graph
if(vis1[v])return;
vis1[v]=1;
for(auto u:g1[v]){
dfs1(u);}
topo.pb(v);
}
void dfs2(int v){ // find the scc
from the toposort using the
transpose of given graph
if(vis2[v])return;
vis2[v]=1;
com[v]=totalCom;
for(auto u:g2[v]){
dfs2(u);}
}
void dfs3(int v){ // find the
toposort of the component graph
if(vis3[v])return;
vis3[v]=1;
for(auto u:g3[v]){
dfs3(u);}
topoCom.pb(v);
}
void dfs4(int v){// runs by the
toposort of the component graph
if(vis4[v])return;
vis4[v]=1;
for(auto u:g3[v]){
dfs4(u);}
}
void solve(){
int n,m;
cin>>n>>m;
vector<pair<int, int>>v;
for(int i=0; i<m; i++){
int x, y;
cin>>x>>y;
g1[x].pb(y);
g2[y].pb(x);
v.pb({x, y});
}
for(int i=1; i<=n; i++){
if(vis1[i]==0){
dfs1(i);}
}
reverse(all(topo));
for(auto u:topo){
if(vis2[u]==0){
totalCom++;
dfs2(u);}
}
for(auto u:v){
int from = com[u.ff];
int to = com[u.ss];
if(from!=to){
g3[to].pb(from);}
}
for(int i=1; i<=totalCom; i++){
if(vis3[i]==0){
dfs3(i);}
}
reverse(all(topoCom));
int start = topoCom[0];
dfs4(start);
for(int i=1; i<=totalCom; i++){
if(vis4[i]==0){
cout << 0 << endl;
return;}
}
vector<int>nodes;
for(int i=1; i<=n; i++){
if(com[i]==start){
nodes.pb(i);}
}
cout << nodes.size() << endl;
for(auto u:nodes)cout<<u<<" "; cout
<< endl;
}
```

Kruskals

```
#define MX 100005
int parent[MX], R[MX];
struct kruskalStruct{
int u,v,w;
};
static bool cmp(kruskalStruct &a,
kruskalStruct &b){
return a.w < b.w;
}
void init(int v){
for(int i = 0; i <= v; i++){
parent[i] = i;
R[i] = 1;
}
}
int Find(int p){
if(p == parent[p]) return p;
return parent[p] = Find(parent[p]);
}
bool Union(int u,int v){
int p = Find(u);
int q = Find(v);
if(p != q) {
if(R[p] >= R[q]){
parent[q] = p;
R[p] += R[q];
}
else{
parent[p] = q;
R[q] += R[p];
}
return true;
}
return false;
}
vector<kruskalStruct> store;
void kruskalsMST(){
int vertex,edge;
cin >> vertex >> edge;
init(vertex);
for(int i = 0; i < edge; i++) {
int u,v,w;
cin >> u >> v >> w;
kruskalStruct ks;
ks.u = u;
ks.v = v;
ks.w = w;
store.push_back(ks);
}
sort(store.begin(),store.end(),cmp);
int totalWeight = 0;
for(int i = 0; i < store.size();
i++){
if(Union(store[i].u,store[i].v))
totalWeight += store[i].w;
}
cout << "Kruskal's MST : " <<
totalWeight << endl;
}
```

** STRING

Faisal Amin Abir

String Multiply

```
string multiply(string num1, string
num2){
int len1 = num1.size();
int len2 = num2.size();
if (len1 == 0 || len2 == 0)return
"0";
vector<int> result(len1 + len2, 0);
int i_n1 = 0;
int i_n2 = 0;
for (int i=len1-1; i>=0; i--){
int carry = 0;
int n1 = num1[i] - '0';
i_n2 = 0;
for (int j=len2-1; j>=0; j--){
int n2 = num2[j] - '0';
int sum = n1*n2 + result[i_n1 +i_n2]
+ carry;
carry = sum/10;
result[i_n1 + i_n2] = sum % 10;
i_n2++;
}
if (carry > 0)result[i_n1 + i_n2]+=
carry;
i_n1++;
}
int i = result.size() - 1;
while (i>=0 && result[i] == 0)i--;
if (i == -1)return "0";
string s = "";
while (i >= 0)s
+=std::to_string(result[i--]);
return s;}
```

String division

```
string longDivision(string number,
int divisor){
string ans;
int idx = 0;
int temp = number[idx] - '0';
while (temp < divisor)
temp = temp * 10 + (number[++idx] -
'0');
while (number.size() > idx){
ans += (temp / divisor) + '0';
temp = (temp % divisor) * 10 +
number[++idx] - '0';
}
if (ans.length() == 0)return "0";
return ans;
}
```

String Double Hashing

```
const int MAXN=1000006;
namespace DoubleHash{
long long P[2][MAXN];
long long H[2][MAXN];
long long R[2][MAXN];
long long base[2];
long long mod[2];
void gen(){
base[0] = 1949313259ll;
base[1] = 1997293877ll;
mod[0]  = 2091573227ll;
mod[1]  = 2117566807ll;
for(int j=0;j<2;j++){
for(int i=0;i<MAXN;i++){
H[j][i]=R[j][i] = 0ll;
P[j][i] = 1ll;
}
}
for(int j=0;j<2;j++){
for(int i=1;i<MAXN;i++){
P[j][i] = (P[j][i-1] *
base[j])%mod[j];
}
}
}
void make_hash(string arr){
int len = arr.size();
for(int j=0;j<2;j++){
for (int i = 1; i <= len;
i++)H[j][i] = (H[j][i - 1] * base[j]
+ arr[i - 1] + 1007) % mod[j];
//            for (int i = len; i >=
1; i--)R[j][i] = (R[j][i + 1] *
base[j] + arr[i - 1] + 1007) %
mod[j];
}
}
inline long long range_hash(int
l,int r,int idx){
long long hashval = H[idx][r + 1] -
((long long)P[idx][r - l + 1] *
H[idx][l] % mod[idx]);
return (hashval < 0 ? hashval +
mod[idx] : hashval);
}
inline long long reverse_hash(int
l,int r,int idx){
long long hashval = R[idx][l + 1] -
((long long)P[idx][r - l + 1] *
R[idx][r + 2] % mod[idx]);
return (hashval < 0 ? hashval +
mod[idx] : hashval);
}
inline long long range_dhash(int
l,int r){
long long x = range_hash(l,r,0);
return (x<<32)^range_hash(l,r,1);
}
inline long long reverse_dhash(int
l,int r){
long long x = reverse_hash(l,r,0);
return (x<<32)^reverse_hash(l,r,1);
}
}
char str1[MAXN];
using namespace DoubleHash;
```

**Reverse Hashing to Find longest
palindromic substring***

```
int f1(int index){
if(index == 0 or index==n-1) return
1;
int ans = 1;
int l = 1, r = min(index, n-1-
index);
while( l <= r){
int mid = (l+r) >> 1;
ll h1 = range_hash(index+1, index +
mid);
ll h2 = reverse_range_hash(index-
mid, index-1);
if(h1==h2){
ans = 1 +  ( 2 * mid);
l = mid+1;
}
else r = mid-1;
}
return ans;
}
int f2(int index){
if(index==0 or index==n-2) return 2;
int ans=2;
int l = 1, r = min(index+1, n-1-
index);
while( l <= r){
int mid = (l+r) >> 1;
ll h1 = range_hash(index-mid+1,
index);
ll h2 = reverse_range_hash(index +
1, index+mid);
if(h1==h2){
ans = 2 * mid;
l = mid+1;
}
else r = mid-1;
}
return ans;
}
void solve(){
cin>>s;
n = s.size();
ll ans=1;
ll l = 0, r=0;
for(int i=0; i<n; i++){
ll x = f1(i);//for odd length i is
mid elem
if(x > ans){
ans = x;
l = i - (x/2);
r = i + (x/2);
}
if(i + 1 < n and s[i] == s[i+1]){
ll y = f2(i);// for even length
if( y > ans){
ans = y;
l = i - (y/2) + 1;
r = i + (y/2);
}}}
for(int i=l ; i<=r ; i++)
cout<<s[i];
}
```

**Manacher to Find the Longest
Palindromic Substring**

```
void solve(){
string s;
cin>>s;
string temp = "";
temp.pb('#');
for(auto u:s){
temp.pb(u);
temp.pb('#');
}
int n = temp.length();
int l=0, r=-1;
int pi[n]={0};
for(int i=0; i<n; i++){
int k=0;
if( i > r){
k = 0;
}
else{
k = min(r-i, pi[l + r - i]);
}
while( i + k + 1 < n && i - k - 1
>=0 && temp[i + k + 1] == temp[i - k
- 1]){
k++;
}pi[i]=k;
if( i + k > r){
r = i + k;
l = i - k;
}
}
int even=0, odd=0, index=0,
index2=0, ans=0;
for(int i=0; i<n; i++){
if(i%2==1){
index++;
}
if(pi[i]>ans){
ans = pi[i];
if(i%2){
odd=1;
even=0;
index2 = index;
}
else{
even=1;
odd=0;
index2=index;
}}}
index2--;
if(odd){
int l = index2, r = index2;
int k = 1;
while(k<ans){
l--, r++;
k+=2;
}for(int i=l; i<=r; i++)cout<<s[i];
}
else {
int l = index2, r = index2+1;int k =
2;
while( k < ans){
l--, r++;
k+=2;
}for(int i=l; i<=r; i++)cout<<s[i];
}}
```

->Given a string with m <= 2e5
operations:

a. Change the char at index to x
b. find if substring between given
l, r is palindrome?

```
const int N = 2e5+12;
ll t[4*N], reverse_t[4*N], power[N],
inv[N];
string s;
int n, q;
void build(int v, int l, int r){
if(l==r){
t[v] = (power[l] * (s[l]-'a'+1)) %
MOD;
reverse_t[v] = (power[n-1-l] *
(s[l]-'a'+1)) % MOD;
return;
}
int mid = (l+r) >> 1;
build(2*v, l, mid);
build(2*v+1, mid+1, r);
t[v] = (t[2*v] + t[2*v+1]) % MOD;
reverse_t[v] = (reverse_t[2*v] +
reverse_t[2*v+1]) % MOD;
}
void update(int v, int l, int r, int
index, char ch){
if(l==r){
s[l]=ch;
t[v] = (power[l] * (s[l]-'a'+1)) %
MOD;
reverse_t[v] = (power[n-1-l] *
(s[l]-'a'+1)) % MOD;
return;
}
int mid = (l+r) >> 1;
if(index <= mid) update(2*v, l, mid,
index, ch);
else update(2*v+1, mid+1, r, index,
ch);
t[v] = (t[2*v] + t[2*v+1]) % MOD;
reverse_t[v] = (reverse_t[2*v] +
reverse_t[2*v+1]) % MOD;
}

pair<ll, ll> query(int v, int L, int
R, int l, int r){
if(L>r or R<l ) return {0, 0};
if(L>=l and R<=r) return {t[v],
reverse_t[v]};
int mid = (L+R) >> 1;
pair<ll, ll>p1 = query(2*v, L, mid,
l, r);
pair<ll, ll>p2 = query(2*v+1, mid+1,
R, l, r);
return {(p1.ff + p2.ff) %
MOD,  (p1.ss + p2.ss) % MOD};
}
void solve(){
cin>>n>>q;
cin>>s;
power[0]=1;
power[1]=base;
inv[0]=1;
inv[1]=modPow(base, MOD-2);
for(int i=2; i<n; i++){
power[i] = (1LL * power[i-1] * base)
% MOD;
inv[i] = ( 1LL * inv[i-1] *
inv[1])  % MOD;
}
build(1, 0, n-1);
while(q--){
int x;
cin>>x;
if(x==1){
int index; char ch;
cin>>index >> ch;
update(1, 0, n-1, index-1, ch);
}
else{
int l, r;
cin>>l>>r;
l--, r--;
pair<ll, ll> p = query(1, 0, n-1, l,
r);
ll h1 = (inv[l] * p.ff) % MOD;
ll h2 = (inv[n-1-r] * p.ss) % MOD;
if(h1==h2){
cout << "YES" << endl;
}
else cout << "NO" << endl;
}}}
```

FAYSAL AHAMMED

Hashing:

*** Double Hashing with Reverse:**

```
const int MOD1 = 127657753, MOD2 =
987654319;
const int p1 = 137, p2 = 277; //
change here
int ip1, ip2;
pair<int, int> pw[N], ipw[N];
void prec() {
pw[0] = {1, 1};
for (int i = 1; i < N; i++) {
pw[i].first = 1ll * pw[i - 1].first
* p1 % MOD1;
pw[i].second = 1ll * pw[i -
1].second * p2 % MOD2;
}
ip1 = power(p1, MOD1 - 2, MOD1);
ip2 = power(p2, MOD2 - 2, MOD2);
ipw[0] = {1, 1};
for (int i = 1; i < N; i++) {
ipw[i].first = 1ll * ipw[i -
1].first * ip1 % MOD1;
ipw[i].second = 1ll * ipw[i -
1].second * ip2 % MOD2;
}
}
struct Hashing {
int n;
string s;
vector<pair<int, int>> hash_val;
vector<pair<int, int>> rev_hash_val;
Hashing() {}
Hashing(string _s) {
s = _s;
n = s.size();
hash_val.emplace_back(0, 0);
for (int i = 0; i < n; i++) {
pair<int, int> p;
p.first = (hash_val[i].first + 1ll *
s[i] * pw[i].first % MOD1) % MOD1;
p.second = (hash_val[i].second + 1ll
* s[i] * pw[i].second % MOD2) %
MOD2;
hash_val.push_back(p);
}
rev_hash_val.emplace_back(0, 0);
for (int i = 0, j = n - 1; i < n;
i++, j--) {
pair<int, int> p;
p.first = (rev_hash_val[i].first +
1ll * s[i] * pw[j].first % MOD1) %
MOD1;
p.second = (rev_hash_val[i].second +
1ll * s[i] * pw[j].second % MOD2) %
MOD2;
rev_hash_val.push_back(p);
}
}
pair<int, int> get_hash(int l, int
r) { // 1 indexed
pair<int, int> ans;
ans.first = (hash_val[r].first -
hash_val[l - 1].first + MOD1) * 1ll
* ipw[l - 1].first % MOD1;
ans.second = (hash_val[r].second -
hash_val[l - 1].second + MOD2) * 1ll
* ipw[l - 1].second % MOD2;
return ans;
}
pair<int, int> rev_hash(int l, int
r) { // 1 indexed
pair<int, int> ans;
ans.first = (rev_hash_val[r].first -
rev_hash_val[l - 1].first + MOD1) *
1ll * ipw[n - r].first % MOD1;
ans.second = (rev_hash_val[r].second
- rev_hash_val[l - 1].second + MOD2)
* 1ll * ipw[n - r].second % MOD2;
return ans;
}
pair<int, int> get_hash() { // 1
indexed
return get_hash(1, n);
}
bool is_palindrome(int l, int r) {
return get_hash(l, r) == rev_hash(l,
r);
}
};
```

Hashing with Updates and Reverse:

```
using T = array<int, 2>;
const T MOD = {127657753,
987654319};
const T p = {137, 277}; // change
here

T operator + (T a, int x) {return
{(a[0] + x) % MOD[0], (a[1] + x) %
MOD[1]};}
T operator - (T a, int x) {return
{(a[0] - x + MOD[0]) % MOD[0], (a[1]
- x + MOD[1]) % MOD[1]};}
T operator * (T a, int x) {return
{(int)((long long) a[0] * x %
MOD[0]), (int)((long long) a[1] * x
% MOD[1])};}
T operator + (T a, T x) {return
{(a[0] + x[0]) % MOD[0], (a[1] +
x[1]) % MOD[1]};}
T operator - (T a, T x) {return
{(a[0] - x[0] + MOD[0]) % MOD[0],
(a[1] - x[1] + MOD[1]) % MOD[1]};}
T operator * (T a, T x) {return
{(int)((long long) a[0] * x[0] %
MOD[0]), (int)((long long) a[1] *
x[1] % MOD[1])};}
ostream& operator << (ostream& os, T
hash) {return os << "(" << hash[0]
<< ", " << hash[1] << ")";}
T pw[N], ipw[N];
void prec() {
pw[0] =  {1, 1};
for (int i = 1; i < N; i++) {
pw[i] = pw[i - 1] * p;
}
ipw[0] =  {1, 1};
T ip = {power(p[0], MOD[0] - 2,
MOD[0]), power(p[1], MOD[1] - 2,
MOD[1])};
for (int i = 1; i < N; i++) {
ipw[i] = ipw[i - 1] * ip;
}
}
struct Hashing {
int n;
string s; // 1 - indexed
vector<array<T, 2>> t; // (normal,
rev) hash
array<T, 2> merge(array<T, 2> l,
array<T, 2> r) {
l[0] = l[0] + r[0];
l[1] = l[1] + r[1];
return l;
}
void build(int node, int b, int e) {
```

```
if (b == e) {
t[node][0] = pw[b] * s[b];
t[node][1] = pw[n - b + 1] * s[b];
return;
}
int mid = (b + e) >> 1, l = node <<
1, r = l | 1;
build(l, b, mid);
build(r, mid + 1, e);
t[node] = merge(t[l], t[r]);
}
void upd(int node, int b, int e, int
i, char x) {
if (b > i || e < i) return;
if (b == e && b == i) {
t[node][0] = pw[b] * x;
t[node][1] = pw[n - b + 1] * x;
return;
}
int mid = (b + e) >> 1, l = node <<
1, r = l | 1;
upd(l, b, mid, i, x);
upd(r, mid + 1, e, i, x);
t[node] = merge(t[l], t[r]);
}
array<T, 2> query(int node, int b,
int e, int i, int j) {
if (b > j || e < i) return {T({0,
0}), T({0, 0})};
if (b >= i && e <= j) return
t[node];
int mid = (b + e) >> 1, l = node <<
1, r = l | 1;
return merge(query(l, b, mid, i, j),
query(r, mid + 1, e, i, j));
}
Hashing() {}
Hashing(string _s) {
n = _s.size();
s = "." + _s;
t.resize(4 * n + 1);
build(1, 1, n);
}
void upd(int i, char c) {
upd(1, 1, n, i, c);
s[i] = c;
}
T get_hash(int l, int r) { // 1 -
indexed
return query(1, 1, n, l, r)[0] *
ipw[l - 1];
}
T rev_hash(int l, int r) { // 1 -
indexed
return query(1, 1, n, l, r)[1] *
ipw[n - r];
}
T get_hash() {
return get_hash(1, n);
}
bool is_palindrome(int l, int r) {
return get_hash(l, r) == rev_hash(l,
r);
}
};
```

Longest Common Prefix:

Binary Search over len and get true/false by hashing in O(1).

Lexicographically Min Cyclic Shift:

```
// return 0 if both equal
// return 1 if first substring
greater
// return -1 if second substring
greater
int compare(int i, int j, int x, int
y) {
int common_prefix = lcp(i, j, x, y);
int len1 = j - i + 1, len2 = y - x +
1;
if (common_prefix == len1 and len1
== len2) return 0;
else if (common_prefix == len1)
return -1;
else if (common_prefix == len2)
return 1;
else return (s[i + common_prefix -
1] < s[x + common_prefix - 1] ? -1 :
1);
}

int start = 1, end = k;
for (int i = 1; i + k - 1 <= n; i++)
{
int x = compare(start, end, i, i + k
- 1);
if (x == 1) {
start = i, end = i + k - 1;
}
}
cout << s.substr(start - 1, k) <<
'\n';
```

Cyclic Shift trick: s += s

Number of Palindromic Substring between l to r in O(n ^2):

```
int is_palindrome(int i, int j) { //
O(n^2)
if (i > j) return 1;
int &ans = dp2[i][j];
if (ans != -1) return ans;
ans = 1;
if (s[i] == s[j]) {
ans &= is_palindrome(i + 1, j - 1);
}
else {
ans = 0;
}
return ans;
}

int fun(int i, int j) { // O(n^2)
if (i > j) return 0;
int &ans = dp[i][j];
if (ans != -1) return ans;
ans = is_palindrome(i, j) + fun(i +
1, j) + fun(i, j - 1) - fun(i + 1, j
- 1);
return ans;
}
```

Number of Palindromic Substrings of a String in O(n logn):

```
string s;
Hashing hash_s;
int n;

bool ok(int l, int r) {
return hash_s.is_palindrome(l, r);
}

int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);
prec(); // must include

cin >> s;
n = s.size();
hash_s = Hashing(s);

long long ans = 0;
for (int i = 1; i <= n; i++) {
int l = 0, r = min(n - i, i - 1),
cnt = 1;
while (l <= r) {
int mid = (l + r) >> 1;
if (ok(i - mid, i + mid)) {
cnt = mid;
l = mid + 1;
}
else {
r = mid - 1;
}
}
ans += cnt + 1;
}
for (int i = 2; i <= n; i++) {
if (s[i - 1] == s[i - 2]) {
int l = 0, r = min(n - i, i - 1),
cnt = 2;
while (l <= r) {
int mid = (l + r) >> 1;
if (ok(i - 1 - mid, i + mid)) {
cnt = mid;
l = mid + 1;
}
else {
r = mid - 1;
}
}
ans += cnt + 1;
}
}
cout << ans << '\n';
return 0;
}
```

** DYNAMIC PROGRAMMING

FAYSAL AHAMMED

Knapsack-2 (constraints- n <= 100, w <= 10^9, value <= 1000):

```
int max_val = 0;
for (int i = n + 1; i >= 1; i--) {
for (int current_value = 0;
current_value <= n * 1000;
current_value++) {
if (i == n + 1) {
if (current_value == 0)
dp[i][current_value] = 0;
else dp[i][current_value] = inf;
}
else {
ll &ans = dp[i][current_value];
ans = dp[i + 1][current_value];
if (current_value - value[i] >= 0) {
ans = min(ans, weight[i] + dp[i +
1][current_value - value[i]]);
}
if (ans <= w) {
max_val = max(max_val,
current_value);
}}}}
cout << max_val << '\n';
```

LCS:

```
int fun(int i, int j) {
if (i >= n or j >= m) return 0;
int &ans = dp[i][j];
if (ans != -1) return ans;
ans = fun(i + 1, j);
ans = max(ans, fun(i, j + 1));
if (s1[i] == s2[j]) {
ans = max(ans, 1 + fun(i + 1, j +
1));
}
return ans;
}
void print(int i, int j) {
if (i >= n or j >= m) return;
if (s1[i] == s2[j]) {
cout << s1[i];
path(i + 1, j + 1);
return;
}
int ans1 = fun(i + 1, j);
int ans2 = fun(i, j + 1);
if (ans1 > ans2) {
path(i + 1, j);
}
else {
path(i, j + 1);
}
}
```

LIS:

```
st.build(1, 1, M);
for (int i = 1; i <= n; i++) {
dp[i] = 1;
if (a[i] != 1) {
int mx = st.query(1, 1, M, 1, a[i] -
1);
mx++;
dp[i] = max(dp[i], mx);
}
st.upd(1, 1, M, a[i], dp[i]);
}

int ans = 0;
for (int i = 1; i <= n; i++) {
ans = max(ans, dp[i]);
}
cout << ans << '\n';
```

Edit Distance (make two string same by add, remove, replace any index with min cost):

```
int fun(int i, int j) {
if (i == n) {
if (j == m) return 0;
return m - j;
}
if (j == m) return n - i;
int &ans = dp[i][j];
if (ans != -1) return ans;
ans = inf;
if (s1[i] == s2[j]) {
ans = min(ans, fun(i + 1, j + 1));
}
ans = min(ans, 1 + fun(i + 1, j +
1));
ans = min(ans, 1 + fun(i + 1, j));
ans = min(ans, 1 + fun(i, j + 1));
return ans;
}
```

Longest Palindromic Subsequence:

```
int fun(int i, int j) {
if (i > j) return 0;
if (i == j) return 1;
int &ans = dp[i][j];
if (ans != -1) return ans;
ans = 0;
if (s[i] == s[j]) {
ans = max(ans, 2 + fun(i + 1, j -
1));
}
else {
ans = max(ans, fun(i + 1, j));
ans = max(ans, fun(i, j - 1));
}
return ans;
}
```

Coin Change:

Repetition allowed (or max k times for k)

Repetition not allowed - fun(int i, int current_amount)

Repetition fixed - loop inside fun()

Order doesn’t matter - fun(amount) and loop inside fun()

Project (start time, ending time, profit are given, what's the max profit?):

-Sort by endtime, then dp[i] = max profit if i is the last index taken. Like LIS. Code-

```
sort(a + 1, a + 1 + n);
st.build(1, 1, M);
for (int i = 1; i <= n; i++) {
dp[i] = a[i][2];
ll mx = st.query(1, 1, M, 1, a[i][1]
- 1);
mx += a[i][2];
dp[i] = max(dp[i], mx);
st.upd(1, 1, M, a[i][0], dp[i]);
}
ll ans = 0;
for (int i = 1; i <= n; i++) {
ans = max(ans, dp[i]);
}
cout << ans << '\n';
```

Slime (n numbers are given, everytime merge 2 consecutive element until there is just 1 element such that cost is min, the cost of merge two element = sum of them):

- Think reverse: we are given the final sum, from i to j. Now we will cut any point between i to j and calculate the cost

```
ll fun(int i, int j) {
if (i == j) return 0;
ll &ans = dp[i][j];
if (ans != -1) return ans;
ll cur = 0;
for (int x = i; x <= j; x++) {
cur += a[x];
}
ans = inf;
for (int x = i; x < j; x++) {
ans = min(ans, cur + fun(i, x) +
fun(x + 1, j));
}
return ans;
}
```

Sub-Palindromic Tree (Given a tree, each node has a character. Now tell us the maximum path which has longest palindromic subsequence):

```
const int N = 2005, inf = 1e9;
vector<int> g[N];
int n;
string s;
int nxt[N][N];
vector<int> vec;
int dp[N][N];

void dfs(int u, int p) {
vec.push_back(u);
for (auto v: g[u]) {
if (v != p) {
dfs(v, u);
}}}
int fun(int u, int v) {
if (v == u) return 1;
int &ans = dp[u][v];
if (ans != -1) return ans;
ans = 0;
if (s[u] == s[v]) {
ans = 2 + (nxt[u][v] == v ? 0 :
fun(nxt[u][v], nxt[v][u]));}
else {
ans = max(fun(nxt[u][v], v), fun(u,
nxt[v][u]));
}
return ans;
}
void solve() {
cin >> n >> s;
s = '.' + s;
for (int i = 1; i < n; i++) {
int u, v; cin >> u >> v;
g[u].push_back(v);
g[v].push_back(u);
}
for (int u = 1; u <= n; u++) {
for (auto x : g[u]) {
vec.clear();
dfs(x, u);
for (auto v : vec) {
nxt[u][v] = x;
}}}
for (int u = 1; u <= n; u++) {
for (int v = 1; v <= n; v++) {
dp[u][v] = -1;
}}
int ans = 0;
for (int u = 1; u <= n; u++) {
for (int v = 1; v <= n; v++) {
ans = max(ans, fun(u, v));
}}
cout << ans << '\n';
for (int i = 1; i <= n; i++) {
g[i].clear();
}}
```

Dice-1 (choose N numbers from 1 to k, how many ways the sum of them is S?) [N, K <= 1000, S <= 15000]:

```
for (int i = n + 1; i >= 1; i--) {
for (int cur_sum = 0; cur_sum <= s;
cur_sum++) {
if (i == n + 1) {
dp[0][cur_sum] = (cur_sum == 0);
}
else {
int &ans = dp[0][cur_sum];
int mn = min(k, cur_sum);
ans = 0;
// for (int x = 1; x <= mn; x++) {
//   ans += dp[1][cur_sum - x];
//   ans %= mod;
// }
ans = (dp[1][cur_sum - 1] - (cur_sum
- mn - 1 < 0 ? 0 : dp[1][cur_sum -
mn - 1]) + mod) % mod;
}}
for (int cur_sum = 0; cur_sum <= s;
cur_sum++) {
dp[1][cur_sum] = dp[0][cur_sum];
}
for (int cur_sum = 1; i != 1 and
cur_sum <= s; cur_sum++) {
dp[1][cur_sum] += dp[1][cur_sum -
1];
dp[1][cur_sum] %= mod;
}}
cout << dp[0][s] << '\n';
```

Dice-2 (choose N numbers from 1 to k,if sum of them = s, then score = multiply of the n numbers, sum of scores?) [N, K <= 1000, S <= 15000]:

```
for (int i = n + 1; i >= 1; i--) {
for (int cur_sum = 0; cur_sum <= s;
cur_sum++) {
if (i == n + 1) {
dp[0][cur_sum] = cur_sum == 0;
}
else {
int &ans = dp[0][cur_sum];
ans = 0;
int mn = min(k, cur_sum);
// for (int x = 1; x <= mn; x++) {
//   ans += 1ll * x * dp[1][cur_sum
- x] % mod;
//   ans %= mod;
// }
int l = cur_sum - mn, r = cur_sum -
1;
ans = (pref[l] - pref[r + 1] + mod)
% mod;
ans -= 1ll * (s - r) * ((dp[1][r] -
(l ? dp[1][l - 1] : 0) + mod) % mod)
% mod;
ans += mod;
ans %= mod;
}}
for (int cur_sum = 0; cur_sum <= s;
cur_sum++) {
dp[1][cur_sum] = dp[0][cur_sum];
}
pref[0] = dp[1][0];
for (int cur_sum = 1; i != 1 and
cur_sum <= s; cur_sum++) {
pref[cur_sum] = dp[1][cur_sum];
dp[1][cur_sum] += dp[1][cur_sum -
1];
dp[1][cur_sum] %= mod;
}
for (int cur_sum = s - 1, j = 2; i
!= 1 and cur_sum >= 0; cur_sum--,
j++) {
pref[cur_sum] = (1ll * pref[cur_sum]
* j % mod) + pref[cur_sum + 1];
pref[cur_sum] %= mod;
}}
cout << dp[0][s] << '\n';
```

* If dp stores only true/false use bitset.

** MICELLANEOUS

FAYSAL AHAMMED

Ternary Search (Pyramid):

```
double suface_area;
double fun(double square_area) {
double base = sqrt(square_area);
double triangle_area = suface_area -
square_area;
double per_triangle_area =
triangle_area / 4;
double triangle_height =
(per_triangle_area * 2) / base;
double x = base / 2;
double height =
sqrt((triangle_height *
triangle_height) - (x * x));
double volume = (base * base *
height) / 3;
if (x > triangle_height) volume = 0;
return volume;
}
int cs = 0;
int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);
int t; cin >> t;
while (t--) {
cin >> suface_area;
cout << fixed << setprecision(4);
double l = 0, r = suface_area, ans =
-1;
int it = 100;
while (it--) {
double mid1 = l + (r - l) / 3;
double mid2 = r - (r - l) / 3;
double x = fun(mid1);
double y = fun(mid2);
if (x > y) {
ans = x;
r = mid2;
}
else {
l = mid1;
}}
cout << "Case " << ++cs << ": ";
cout << ans << '\n';
}
return 0;
}
```

2D Prefix Sum:

```cpp
int n, m; cin >> n >> m;
for (int i = 1; i <= n; i++) {
for (int j = 1; j <= m; j++) {
cin >> a[i][j];
}}
for (int i = 1; i <= n; i++) {
for (int j = 1; j <= m; j++) {
prefix[i][j] = prefix[i - 1][j] + prefix[i][j - 1] - prefix[i - 1][j - 1] + a[i][j];
}}
int q; cin >> q;
while (q--) {
int x1, y1, x2, y2; cin >> x1 >> y1 >> x2 >> y2;
cout << prefix[x2][y2] - prefix[x1 - 1][y2] - prefix[x2][y1 - 1] + prefix[x1 - 1][y1 - 1] << '\n';
}
```

2D Difference Array:

```cpp
int n, m; cin >> n >> m;
for (int i = 1; i <= n; i++) {
for (int j = 1; j <= m; j++) {
char c; cin >> c;
a[i][j] = c - '0';
}}

int q; cin >> q;
while (q--) {
int x1, y1, x2, y2, x; cin >> x1 >> y1 >> x2 >> y2;
x = 1;
d[x1][y1] += x;
d[x1][y2 + 1] -= x;
d[x2 + 1][y1] -= x;
d[x2 + 1][y2 + 1] += x;
}

for (int i = 1; i <= n; i++) {
for (int j = 1; j <= m; j++) {
d[i][j] += d[i - 1][j] + d[i][j - 1] - d[i - 1][j - 1];
}}

// new updated array
for (int i = 1; i <= n; i++) {
for (int j = 1; j <= m; j++) {
cout << (d[i][j] + a[i][j]) % 2;
}
cout << '\n';
}
```

PBDS:

```cpp
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T> using o_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
Custom Hash for GP HashTable:
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count();
struct chash {
int operator()(int x) const { return x ^ RANDOM; }
};
```

Direction Array:

```cpp
int dx[] = {+0, +0, +1, -1, -1, +1, -1, +1};
int dy[] = {-1, +1, +0, +0, +1, +1, -1, -1};
Custom Comparator:
bool cmp(pair<int, int> a, pair<int, int> b) {
if (a.first != b.first) return a.first > b.first;
return a.second < b.second; // comparator function must return false for equal elements
}
Custom Comparator for map,set, multiset, pq:
struct cmp {
bool operator()(const int& a, const int& b) const {
return a > b;
}
};
```

Mex with Array Updates:

```cpp
int n, q; cin >> n >> q;
set<int> missing_numbers;
for (int i = 0; i <= n + 200; i++) {
missing_numbers.insert(i);
}

int a[n + 1];
map<int, int> freq;
for (int i = 1; i <= n; i++) {
cin >> a[i];
freq[a[i]]++;
missing_numbers.erase(a[i]);
}

while (q--) {
int i, x; cin >> i >> x;
int y = a[i];
a[i] = x;
missing_numbers.erase(x);
freq[y]--;
freq[x]++;
if (freq[y] == 0) {
freq.erase(y);
missing_numbers.insert(y);
}
cout << *missing_numbers.begin() << '\n';
}
```

Coordinate Compression (faster):

```cpp
vector<int> a({100, 9, 10, 10, 9});
vector<int> v = a;
sort(v.begin(), v.end());
v.resize(unique(v.begin(), v.end()) - v.begin());

for (int i = 0; i < a.size(); i++) {
a[i] = lower_bound(v.begin(), v.end(), a[i]) - v.begin() + 1;
cout << a[i] << ' ';
}
```

Pigeonhole Principle:

-At least 1 subarray of an array of length N must be divisible by N.
-Build all possible sequences of length 10 whose value is between 1 to 100. At least any two sequences will be same.

* Given an array of length N (N <= 10^6) and M (M <= 10^3) check if there is any subsequence of the array whose sum is divisible by k? According to the pigeonhole principle if N >= M then it must be “YES”. Else we can do DP. where N < M <= 1000.

Contribution Technique (Calculate the contribution of each element separately):

* Sum of pair sums (i=1 to n Σ j= 1 to n Σ(ai+a):
=> Every element will be added 2n times.

$$\sum_{i=1}^{n} (2 \times n \times a_i) = 2 \times n \times \sum_{i=1}^{n} a_i.$$

* Sum of subarray sums:

$$\sum_{i=1}^{n} (a_i \times i \times (n - i + 1)).$$

* Sum of subset sums:

$$\sum_{i=1}^{n} (2^{n-1} \times a_i).$$

* Product of pair product:

$$\prod_{i=1}^{n} (a_i^{2\times n}).$$

* XOR of subarray XORS:
=> How many subarrays does an element have? (i* (n-i+1) times. If subarray length is odd then this element can contribute in total XORs.
* Sum of max-min over all subset:
=> Sort the array. Min = 2^(n-i), Max = 2^(i-1)
    i=1 to n Σ(ai * 2^(i-1))-(ai*2^(n-i))
* Sum using Bit:

$$\sum_{k=0}^{30} (cnt_k[1] \times 2^k).$$

* Sum of Pair XORs:
=> XOR = 1 if two bits are different

$$\sum_{k=0}^{30} (cnt_k[0] \times cnt_k[1] \times 2^k).$$

* Sum of pair ANDs:

$$\sum_{k=0}^{30} (cnt_k[1]^2 \times 2^k).$$

* Sum of pair ORs:

$$\sum_{k=0}^{30} ((cnt_k[1]^2 + 2 \times cnt_k[1] \times cnt_k[0]) \times 2^k).$$

* Sum of Subset XORs:

$$\sum_{k=0}^{30}(2^{cnt_k[1]+cnt_k[0]-1}\times 2^k).$$

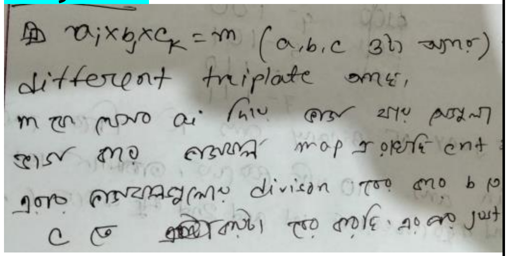
```
[where cnt0 != 0)
* Sum of Subset ANDs:
```

$$\sum_{k=0}^{30}((2^{cnt_k[1]}-1)\times 2^k).$$

```
* Sub of Subset ORs:
```

$$\sum_{k=0}^{30}((2^n-2^{cnt_k[0]})\times 2^k).$$

```
* Sum of subarray XORs:
=> Convert to prefix xor, then solve
for pairs.
Nafis and MEX:
void solve() {
int n, k; cin >> n >> k;
vector<int> a(n);
map<int, int> mp;
for (auto &x : a) {
cin >> x;
mp[x]++;
}
sort(a.begin(), a.end());
int max_mex = 0;
for (auto x : a) {
if (max_mex == x) max_mex++;
}
int ways[max_mex];
for (int i = 0; i < max_mex; i++) {
int mn = min(30ll, mp[i]);
ways[i] = (1 << mn) - 1;
}
for (int i = 1; i < max_mex; i++) {
int mn = (int)(1ll * ways[i] *
ways[i - 1]);
ways[i] = min(MIN, mn);
}
int cur = max_mex;
int add = (k + 1) / 2;
int minus = k - add;
long long ans = 0;
while (minus > 0) {
int right = n -
(upper_bound(a.begin(), a.end(),
cur) - a.begin());
int mn = min(30ll, right);
int possible = (1 << mn);
if (cur == 0) {
possible--;
int apply = min(minus, possible);
minus -= apply;
ans -= (1ll * cur * apply);
}
else {
possible = min(MIN, (int) (1ll *
ways[cur - 1] * possible));
int apply = min(minus, possible);
minus -= apply;
ans -= (1ll * cur * apply);
}
cur--;
}
cur = 0;
while (add > 0) {
int right = n -
(upper_bound(a.begin(), a.end(),
cur) - a.begin());
int mn = min(30ll, right);
int possible = (1 << mn);
if (cur == 0) {
possible--;
int apply = min(add, possible);
add -= apply;
ans += (1ll * cur * apply);
}
else {
possible = min(MIN, (int) (1ll *
ways[cur - 1] * possible));
int apply = min(add, possible);
add -= apply;
ans += (1ll * cur * apply);
}
cur++;
}
cout << ans << '\n';
}
How many triplets such that
ai*bj*ck=m:
```



ⓐ $a_i \times b_j \times c_k = m$ (a,b,c 3টা অ্যারে)
different triplate কত?,
m কে প্রথমে ai দিয়ে [illegible] করে [illegible] [illegible]
[illegible] [illegible] [illegible] map এ [illegible] cnt
[illegible] [illegible] division করে [illegible] b [illegible]
c তে [illegible] করে দেখি। [illegible] just

```
Formulas:
```

Sum of squares: 1^2 + 2^2 + 3^2 + … + n^2 = n(n+1) (n+2)/6

Sum of cubes: 1^3 + 2^3 + 3^3+ ... + n^3 = (n^2 * (n+1)^2)/4

Geometric Series: 1+x + x^2+x^3...+x^n = (x^(n+1)-1)/(x-1)

when |x|<1 then the sum = 1 / (1-x)

Harmonic Series: 1 + ½ + 1/3 + ¼ + … + 1/n = ln (n) + O(1)

> $$\sum_{k=0}^{n-1}(a_k-a_{k+1})=a_0-a_n .$$

> $$\sum_{k=1}^{n}(a_k-a_{k-1})=a_n-a_0 ,$$

>

$$\sum_{k=1}^{n-1}\frac{1}{k(k+1)} = \sum_{k=1}^{n-1}\left(\frac{1}{k}-\frac{1}{k+1}\right) = 1-\frac{1}{n} .$$

$$\ln n = \log_e n \quad \text{(natural logarithm)} ,$$

For all real $a>0$, $b>0$, $c>0$, and n,

$$a = b^{\log_b a}$$

$$\log_c(ab) = \log_c a + \log_c b ,$$

$$\log_b a = \frac{\log_c a}{\log_c b} ,$$

$$\log_b a = \frac{1}{\log_a b} ,$$

$$\log_b(1/a) = -\log_b a ,$$

$$a^{\log_b c} = c^{\log_b a} ,$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$a^2+b^2 = (a+b)^2-2ab$$

$$a^2+b^2 = (a-b)^2+2ab$$

$$(a+b)^2 = (a-b)^2+4ab$$

$$(a-b)^2 = (a+b)^2-4ab$$

$$a^2+b^2 = \frac{(a+b)^2+(a-b)^2}{2}$$

$$ab = \left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2$$

$$(x+a)(x+b) = x^2+(a+b)x+ab$$

$$2(ab+bc+ac) = (a+b+c)^2-(a^2+b^2+c^2)$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3 = a^3+b^3+3ab(a+b)$$

$$a^3+b^3 = (a+b)^3-3ab(a+b)$$

$$(a-b)^3 = a^3-3a^2b+3ab^2-b^3 = a^3-b^3-3ab(a-b)$$

$$a^3-b^3 = (a-b)^3+3ab(a-b)$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2},$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Geometric series:

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1}, \quad c \neq 1$$

$$\sum_{i=0}^{\infty} c^i = \frac{1}{1-c},$$

$$\sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, \quad |c|<1,$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!},$$

$$\sum_{k=0}^{n}\binom{n}{k} = 2^n$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$$

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$$

$$\sum_{k=0}^{n}\binom{r+k}{k} = \binom{r+n+1}{n},$$

$$\sum_{k=0}^{n}\binom{k}{m} = \binom{n+1}{m+1}$$

$$\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$$

$$\binom{n}{k} = (-1)^k\binom{k-n-1}{k}$$

$$\left\{{n \atop 1}\right\} = \left\{{n \atop n}\right\} = 1,$$

$$\left\{{n \atop 2}\right\} = 2^{n-1}-1,$$

$$\left\{{n \atop n-1}\right\} = \left[{n \atop n-1}\right] = \binom{n}{2}$$

$$\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} = n!,$$

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$\log_b x = \frac{\log_a x}{\log_a b}, \qquad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Euler's number e:

$$e = 1 + \tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{24} + \tfrac{1}{120} + \cdots$$

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x.$$

$$\left(1+\tfrac{1}{n}\right)^n < e < \left(1+\tfrac{1}{n}\right)^{n+1}.$$

$$\left(1+\tfrac{1}{n}\right)^n = e - \frac{e}{2n} + \frac{11e}{24n^2} - O\left(\frac{1}{n^3}\right).$$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Binomial distribution:

$$\Pr[X=k] = \binom{n}{k} p^k q^{n-k}, \qquad q = 1-p,$$

$$\mathrm{E}[X] = \sum_{k=1}^{n} k\binom{n}{k} p^k q^{n-k} = np.$$

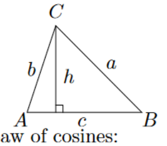
Euler's equation:

$$e^{ix} = \cos x + i \sin x, \qquad e^{i\pi} = -1.$$

Area:

$$A = \tfrac{1}{2}hc,$$
$$= \tfrac{1}{2}ab\sin C,$$
$$= \frac{c^2 \sin A \sin B}{2\sin C}.$$

Heron's formula:



Law of cosines:

$$c^2 = a^2+b^2-2ab\cos C.$$

$$A = \sqrt{s \cdot s_a \cdot s_b \cdot s_c},$$
$$s = \tfrac{1}{2}(a+b+c),$$
$$s_a = s-a,$$
$$s_b = s-b,$$
$$s_c = s-c.$$

$$\sin\tfrac{x}{2} = \sqrt{\frac{1-\cos x}{2}},$$

$$\cos\tfrac{x}{2} = \sqrt{\frac{1+\cos x}{2}},$$

$$\tan\tfrac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}, = \frac{1-\cos x}{\sin x}, = \frac{\sin x}{1+\cos x},$$

$$\cot\tfrac{x}{2} = \sqrt{\frac{1+\cos x}{1-\cos x}}, = \frac{1+\cos x}{\sin x}, = \frac{\sin x}{1-\cos x},$$

Euler's function: $\phi(x)$ is the number of positive integers less than x relatively prime to x. If $\prod_{i=1}^{n} p_i^{e_i}$ is the prime factorization of x then

$$\phi(x) = \prod_{i=1}^{n} p_i^{e_i-1}(p_i-1).$$

Euler's theorem: If a and b are relatively prime then

$$1 \equiv a^{\phi(b)} \bmod b.$$

Fermat's theorem:

$$1 \equiv a^{p-1} \bmod p.$$

The Euclidean algorithm: if $a > b$ are integers then

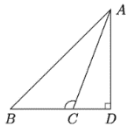
$$\gcd(a,b) = \gcd(a \bmod b, b).$$

If $\prod_{i=1}^{n} p_i^{e_i}$ is the prime factorization of x then

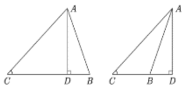
$$S(x) = \sum_{d|x} d = \prod_{i=1}^{n} \frac{p_i^{e_i+1}-1}{p_i-1}.$$

Perfect Numbers: x is an even perfect number iff $x = 2^{n-1}(2^n-1)$ and 2^n-1 is prime.
Wilson's theorem: n is a prime iff

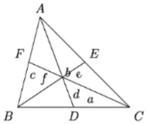
$$(n-1)! \equiv -1 \bmod n.$$



$$AB^2 = AC^2 + BC^2 + 2 \cdot BC \cdot CD$$



$$AB^2 = AC^2 + BC^2 - 2 \cdot BC \cdot CD$$



$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

$$d^2 = \frac{2(b^2+c^2)-a^2}{4}$$

$$e^2 = \frac{2(c^2+a^2)-b^2}{4}$$

$$f^2 = \frac{2(a^2+b^2)-c^2}{4}$$

$$, x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\log_a M = \log_b M \times \log_a b$$

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots + y^n$$

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

$$\binom{n}{r} = {}^nC_r, \quad {}^nC_n = 1$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}, \quad \binom{n}{0} = {}^nC_0 = 1$$

$$\binom{n}{n} = {}^nC_n = 1, \ 0! = 1$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!},$$

$$\sum_{k=0}^{n}\binom{n}{k} = 2^n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2},$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Triangle:

area = $\sqrt{s(s-a)(s-b)(s-c)}$

area = $\frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$

Cube:

area = abc

diag = $\sqrt{a^2+b^2+c^2}$

Circle:

circumference : $2\pi r$

area : πr^2

eqn : $(x-h)^2 + (y-k)^2 = r^2$ ... (i)

$(x-h)^2 + (y-k)^2 = r^2$ বা, $x^2 + y^2 - 2hx - 2ky + (h^2 +$

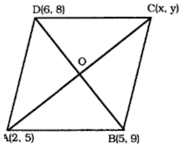
$h^2 + k^2 - r^2 = c$

বা, $g^2 + f^2 - c = r^2$

অতএব ব্যাসার্ধ, $r = \sqrt{g^2 + f^2 - c}.$

Cone:

volume: $\frac{1}{3}\pi r^2 h$



area = $= \frac{1}{2}(BD \times AC)$

$\therefore P(x_1,y_1)$ বিন্দু হতে $ax + by + c = 0$ রেখার লম্ব দৈর্ঘ্য = $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$\therefore$ সমান্তরাল রেখা দুইটির মধ্যবর্তী দূরত্ব $MN = ON - OM$

$$= \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$$

$x^2 + y^2 = r^2$ এবং $y = mx + c$

tangent condition: $c = \pm r\sqrt{1+m^2}$

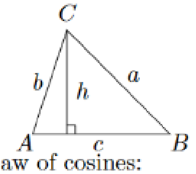
বহিঃস্থ কোন বিন্দু (x_1, y_1) থেকে $x^2 + y^2 + 2gx + 2fy + c = 0$ বৃত্তের অঙ্কিত স্পর্শক দুইটির সমীকরণ

$$(x^2 + y^2 + 2gx + 2fy + c)(x_1{}^2 + y_1{}^2 + 2gx_1 + 2fy_1 + c)$$
$$= \{x x_1 + yy_1 + g(x + x_1) + f(y + y_1) + c\}^2$$

Area:

$$A = \tfrac{1}{2}hc,$$
$$= \tfrac{1}{2}ab\sin C,$$
$$= \frac{c^2 \sin A \sin B}{2\sin C}.$$

Heron's formula:



Law of cosines:

$$c^2 = a^2+b^2-2ab\cos C.$$

$$A = \sqrt{s \cdot s_a \cdot s_b \cdot s_c},$$
$$s = \tfrac{1}{2}(a+b+c),$$
$$s_a = s-a,$$
$$s_b = s-b,$$
$$s_c = s-c.$$

Euler's function: $\phi(x)$ is the number of positive integers less than x relatively prime to x. If $\prod_{i=1}^{n} p_i^{e_i}$ is the prime factorization of x then

$$\phi(x) = \prod_{i=1}^{n} p_i^{e_i-1}(p_i-1).$$

Euler's theorem: If a and b are relatively prime then

$$1 \equiv a^{\phi(b)} \bmod b.$$

Fermat's theorem:

$$1 \equiv a^{p-1} \bmod p.$$