G_MATH & H_MATH[9-10]

Symmetric difference of two sets: is denoted by $A \, \Delta \, B = (A - B) \cup (B - A)$

$= (A \cup B) - (A \cap B)$

De-Morgan's laws:

i. $(A \cup B)' = A' \cap B'$

ii. $(A \cap B)' = A' \cup B'$

iii. $A - (B \cap C) = (A - B) \cup (A - C)$

iv. $A - (B \cup C) = (A - B) \cap (A - C)$

If A, B and C are any three sets, then

i. $A \cap (B - C) = (A \cap B) - (A \cap C)$

ii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

iii. $P(A) \cap P(B) = P(A \cap B)$

iv. $P(A) \cup P(B) = P(A \cup B)$

v. If $P(A) = P(B) \Rightarrow A = B$

where, $P(A)$ is the power set of A.

$A \subseteq A \cup B, B \subseteq A \cup B, A \cup B \subseteq A,$

$A \cap B \subseteq B$

$A - B = A \cap B', B - A = B \cap A'$

$(A - B) \cap B = \phi$

$(A - B) \cup B = A \cup B$

$A \subseteq B \Leftrightarrow B' \subseteq A'$

$A - B = B' - A'$

$(A \cup B) \cap (A \cup B') = A$

$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

$A - (A - B) = A \cap B$

$A - B = B - A \Leftrightarrow A = B$ and

$A \cup B = A \cap B \Rightarrow A = B$

Results on cardinal number of some sets:

If A, B and C are finite sets and U be the universal set, then

i. $n(A \cup B) = n(A) + n(B)$ if A and B are disjoint sets.

ii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

iii. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

Algebraic Formulae

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 - 3abc = \tfrac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$$

$$(a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$$

Corollary 6. if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Corollary 7. if $a^3 + b^3 + c^3 = 3abc$, so $a + b + c = 0$ or $a = b = c$

Corollary 1. $a^2 + b^2 = (a + b)^2 - 2ab$

Corollary 2. $a^2 + b^2 = (a - b)^2 + 2ab$

Corollary 3. $(a + b)^2 = (a - b)^2 + 4ab$

Corollary 4. $(a - b)^2 = (a + b)^2 - 4ab$

Corollary 5. $a^2 + b^2 = \dfrac{(a + b)^2 + (a - b)^2}{2}$

Corollary 6. $ab = \left(\dfrac{a + b}{2}\right)^2 - \left(\dfrac{a - b}{2}\right)^2$

Formula 3. $a^2 - b^2 = (a + b)(a - b)$

Formula 4. $(x + a)(x + b) = x^2 + (a + b)x + ab$

Formula 5. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

Corollary 7. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ac)$

Corollary 8. $2(ab + bc + ac) = (a + b + c)^2 - (a^2 + b^2 + c^2)$

Formulae of Cubes

Formula 6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$

Corollary 9. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

Formula 7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a - b)$

Corollary 10. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

Formula 8. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Formula 9. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Binomial Expansion

Value of n		Pascal's Triangle	Number of Terms
$n = 0$	$(1 + y)^0 =$	1	1
$n = 1$	$(1 + y)^1 =$	$1 + y$	2
$n = 2$	$(1 + y)^2 =$	$1 + 2y + y^2$	3
$n = 3$	$(1 + y)^3 =$	$1 + 3y + 3y^2 + y^3$	4
$n = 4$	$(1 + y)^4 =$	$1 + 4y + 6y^2 + 4y^3 + y^4$	5
$n = 5$	$(1 + y)^5 =$	$1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$	6

$$(1 + y)^n = 1 + ny + \frac{n(n - 1)}{1 \cdot 2}y^2 + \frac{n(n - 1)(n - 2)}{1 \cdot 2 \cdot 3}y^3 + \cdots + y^n$$

$$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \cdots + y^n$$

Logarithms

1) $\log_a b = x$ if and only If $a^x = b$.

2) $\log_a(a^x) = x$

3) $a^{\log_a b} = b$

(i) If $x > 0,\ y > 0$ and $a \neq 1$ then $x = y$ if and only if $\log_a x = \log_a y$

(ii) If $a > 1$ and $x > 1$ then $\log_a x > 0$

(iii) If $0 < a < 1$ and $0 < x < 1$ then $\log_a x > 0$

(iv) If $a > 1$ and $0 < x < 1$ then $\log_a x < 0$

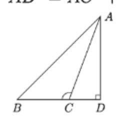
Formula 10 (change of base). $\log_a M = \log_b M \times \log_a b$

Corollary 1. $\log_a b = \dfrac{1}{\log_b a}$ or $\log_b a = \dfrac{1}{\log_a b}$

Lines, Angles and Triangles

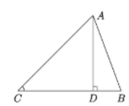
for obtuse angle C

$AB^2 = AC^2 + BC^2 + 2 \cdot BC \cdot CD$



For acute angle c

$AB^2 = AC^2 + BC^2 - 2 \cdot BC \cdot CD$



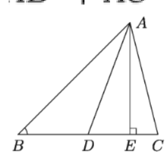
1) If $\angle ACB$ is an obtuse angle,, $AB^2 > AC^2 + BC^2$

2) If $\angle ACB$ is a right angle, $AB^2 = AC^2 + BC^2$

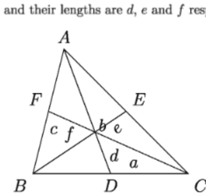
3) If $\angle ACB$ is an acute angle, $AB^2 < AC^2 + BC^2$

Theorem 5 (Theorem of Apollonius). The sum of the areas of the squares drawn on any two sides of a triangle is equal to twice the sum of area of the squares drawn on the median of the third side and on either half of that side.

$$AB^2 + AC^2 = 2(AD^2 + BD^2).$$



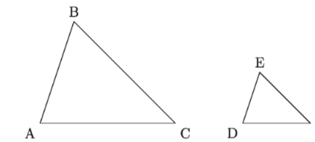
Let, length of the sides of BC, CA and AB of the $\triangle ABC$ are a, b and c respectively. AD, BE and CF are the medians drawn on sides BC, CA and AB and their lengths are d, e and f respectively.



Or, $d^2 = \dfrac{2(b^2 + c^2) - a^2}{4}$

Similarly we can get, $e^2 = \dfrac{2(c^2 + a^2) - b^2}{4}$ and $f^2 = \dfrac{2(a^2 + b^2) - c^2}{4}$

$\therefore 3(a^2 + b^2 + c^2) = 4(d^2 + e^2 + f^2)$



Theorem 9. The ratio of the areas of the two similar triangles is equal to the ratio of the areas of the squares drawn on their two corresponding sides.

$$\frac{\triangle ABC}{\triangle DEF} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

Circumcenter of a Triangle: The **circumcenter** of a triangle is the point of intersection of two perpendicular bisectors of that triangle. Noted that, the perpendicular bisector of the third side of the triangle would pass through the circumcenter too.

Centroid of a Triangle: The **centroid** of a triangle is the point of intersection of three medians of that triangle. The centroid of a triangle divides each median

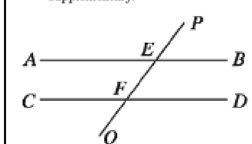
Orthocenter of a Triangle: The **orthocenter** of a triangle is the point of intersection of the perpendiculars drawn from each vertex to their respective opposite side.

Theorem 3. When a transversal cuts two parallel straight lines,

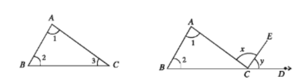
1) the pair of corresponding angles are equal.

2) the pair of alternate angles are equal.

3) that pair of interior angles on the same side of the transversal are supplementary.



Theorem 5. The sum of the three angles of a triangle is equal to two right angles.



$\therefore \angle ABC + \angle BAC = \angle ECD + \angle ACE = \angle ACD$

$\angle ABC + \angle BAC + \angle ACB = \angle ECD + \angle ACE + \angle ACB = \angle ACD + \angle ACB = 2$ right angles.

Corollary 2. If a side of a triangle is produced then exterior angle so formed is equal to the sum of the two opposite interior angles.

Corollary 3. If a side of a triangle is produced, the exterior angle so formed is greater than each of the two interior opposite angles.

Corollary 4. The acute angles of a right angled triangle are complementary to each other.

Theorem 12. If one side of a triangle is greater than another, the angle opposite the greater side is greater than the angle opposite the lesser sides.

Let, in triangle $\triangle ABC$, $AC > AB$. Therefore $\angle ABC > \angle ACB$

Corollary 5. The difference of the lengths of any two sides of a triangle is smaller than the third side.

Theorem 15. The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and in length it is half.

Circle

Theorem 17. The line segment drawn from the centre of a circle to bisect a chord other than diameter is perpendicular to the chord.

Theorem 19. Chords equidistant from the centre of a circle are equal.

Theorem 20. The angle subtended by the same arc at the centre is double of the angle subtended by it at any point on the remaining part of the circle.

Theorem 21. Angles in a circle standing on the same arc are equal.

Theorem 22. The angle inscribed in the semi- circle is a right angle.

Corollary 4. The circle drawn with hypotenuse of a right-angled triangle as diameter passes through the vertices of the triangle.

Corollary 5. The angle inscribed in the major arc of a circle is an acute angle.

Theorem 23. The sum of the two opposite angles of a quadrilateral inscribed in a circle is two right angles.

Corollary 6. If one side of a cyclic quadrilateral is extended, the exterior angle formed is equal to the opposite interior angle.

Corollary 7. A parallelogram inscribed in a circle is a rectangle.

Theorem 24. If two opposite angles of a quadrilateral are supplementary, the four vertices of the quadrilateral are concyclic.

Trigonometric Ratio

Proposition 5. Any arc of length s produces an angle θ in the centre of t circle of radius r then $s = r\theta$.

Proposition 6. $1^\circ = \left(\frac{\pi}{180}\right)^c$ and $1^c = \left(\frac{180}{\pi}\right)^\circ$

(i) $1^\circ = \left(\frac{\pi}{180}\right)^c$

(ii) $30^\circ = \left(30 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{6}\right)^c$

(iii) $45^\circ = \left(45 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{4}\right)^c$

(iv) $60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

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(v) $90^\circ = \left(90 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{2}\right)^c$

(vi) $180^\circ = \left(180 \times \frac{\pi}{180}\right)^c = \pi^c$

(vii) $360^\circ = \left(360 \times \frac{\pi}{180}\right)^c = (2\pi)^c$

$\therefore \sin(2\pi - \theta) = \sin(-\theta) = -\sin\theta,\ \cos(2\pi - \theta) = \cos(-\theta) = \cos\theta$

$\tan(2\pi - \theta) = \tan(-\theta) = -\tan\theta,\ \mathrm{cosec}(2\pi - \theta) = \mathrm{cosec}(-\theta) = -\mathrm{cosec}\theta$

$\sec(2\pi - \theta) = \sec(-\theta) = \sec\theta$ and $\cot(2\pi - \theta) = \cot(-\theta) = -\cot\theta$

$\therefore \sin(2\pi + \theta) = \sin\theta,\ \cos(2\pi + \theta) = \cos\theta$

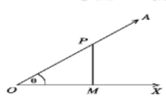
$\tan(2\pi + \theta) = \tan\theta,\ \mathrm{cosec}(2\pi + \theta) = \mathrm{cosec}\theta$

$\sec(2\pi + \theta) = \sec\theta,\ \cot(2\pi + \theta) = \cot\theta.$

$$\sin\theta = \frac{PM}{OP} = \frac{\text{opposite side}}{\text{Hypotenuse}} \text{ [sine of angle } \theta]$$

$$\cos\theta = \frac{OM}{OP} = \frac{\text{adjacent side}}{\text{Hypotenuse}} \text{ [cosine of angle } \theta]$$

$$\tan\theta = \frac{PM}{OM} = \frac{\text{opposite side}}{\text{adjacent side}} \text{ [tangent of angle } \theta]$$



$$\mathrm{cosec}\theta = \frac{1}{\sin\theta} \text{ [cosecant of angle } \theta]$$

$$\sec\theta = \frac{1}{\cos\theta} \text{ [secant of angle } \theta]$$

$$\cot\theta = \frac{1}{\tan\theta} \text{ [cotangent of angle } \theta]$$

$\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta}}$ $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$ $\boxed{\sec^2\theta - \tan^2\theta = 1}$

$\boxed{(\sin\theta)^2 + (\cos\theta)^2 = 1}$ $\boxed{\mathrm{cosec}^2\theta - \cot^2\theta = 1}$

Algebraic Ratio and Proportion

3. if $a : b = c : d$ then, $\frac{a+b}{b} = \frac{c+d}{d}$ [Componendo]

if $a : b = c : d$ then, $\frac{a-b}{b} = \frac{c-d}{d}$ [Dividendo]

if $a : b = c : d$, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ [Componendo-Dividendo]

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$ then each of the ratio $= \frac{a+c+e+g}{b+d+f+h}$

If $a : b = b : c$, prove that, $\left(\frac{a+b}{b+c}\right)^2 = \frac{a^2+b^2}{b^2+c^2}$

If $\frac{a}{b} = \frac{c}{d}$ show that, $\frac{a^2+b^2}{a^2-b^2} = \frac{ac+bd}{ac-bd}$

Ratio/Angle	0°	30°	45°	60°	90°
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cotangent	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cosecant	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

$\therefore \sin(90^\circ - \theta) = \frac{OM}{OP} = \cos\angle POM = \cos\theta$

$\cos(90^\circ - \theta) = \frac{PM}{OP} = \sin\angle POM = \sin\theta$

$\tan(90^0 - \theta) = \frac{OM}{PM} = \cot\angle POM = \cot\theta$

$\cot(90^0 - \theta) = \frac{PM}{OM} = \tan\angle POM = \tan\theta$

$\sec(90^0 - \theta) = \frac{OP}{PM} = \mathrm{cosec}\angle POM = \mathrm{cosec}\theta$

$\mathrm{cosec}(90^0 - \theta) = \frac{OP}{OM} = \sec\angle POM = \sec\theta$

Finite Series

Arithmetic Series

$\therefore n$ th term $= a + (n-1)d$

Sum of n terms of an arithmetic series

Let the first term of any arithmetic series be a, last term be p, common difference be d, number of terms be n and sum of n terms be S_n.

$$\therefore S_n = \frac{n}{2}(a+p) \ldots (3)$$

$$\text{i.e., } S_n = \frac{n}{2}\{2a + (n-1)d\} \ldots (4)$$

1. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
2. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

N.B: $1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$

Coordinate Geometry

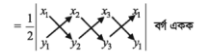
The distance of P from Q is, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ Area of the Triangle ABC

$= \frac{1}{2} \times (BE + AF) \times EF + \frac{1}{2} \times (CD + BE) \times DE - \frac{1}{2} \times (CD + AF) \times DF$

$= \frac{1}{2} \times (y_2 + y_1) \times (x_1 - x_2) + \frac{1}{2} \times (y_3 + y_2) \times (x_2 - x_3) - \frac{1}{2} \times (y_3 + y_1) \times (x_1 - x_3)$

$= \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$



Geometric Series

Let the first term of a geometric series be a and common ratio be r. Then, of the series

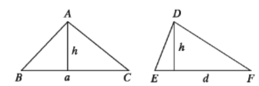
nth term $= ar^{n-1}$

Let the first term of the geometric series be a, common ratio r and number of terms n. If S_n is the sum of n terms,

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1}, \text{ when } r > 1$$

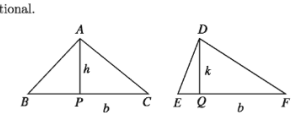
Ratio, Similarity and Symmetry

1. If the heights of two triangles are equal, their bases and areas are proportional.



$$= \frac{1}{2} \times a \times h : \frac{1}{2} \times d \times h = a : d = BC : EF$$

2. If the bases of two triangles are equal, their heights and areas are proportional.



$$\frac{1}{2} \times b \times h : \frac{1}{2} \times b \times k = h : k = AP : DQ$$

Theorem 28. A straight line drawn parallel to one side of a triangle intersects the other two sides or those sides produced proportionally.

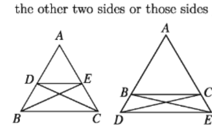


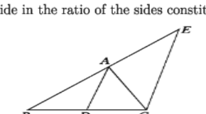
Figure 1 Figure 2

$AD : DB = AE : EC$

Corollary 1. If the line parallel to BC of the triangle ABC intersects the sides AB and AC at D and E respectively, then $\frac{AB}{AD} = \frac{AC}{AE}$ and $\frac{AB}{BD} = \frac{AC}{CE}$.

Corollary 2. The line through the mid point of a side of a triangle parallel to another side bisects the third line.

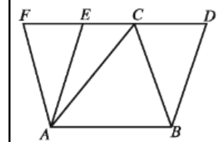
Theorem 30. The internal bisector of an angle of a triangle divides its opposite side in the ratio of the sides constituting to the angle.



$$\therefore \frac{BD}{DC} = \frac{BA}{AC}$$

Area Related Theorems and Constructions

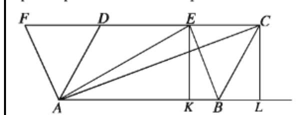
Theorem 36. Areas of all the triangular regions having same base and lying between the same pair of parallel lines are equal to one another.



$\triangle$ region $ABC = \triangle$ region DBC

Corollary 1. If a triangle and a parallelogram lie on bases with equal length and between same pair of parallel lines, the area of the triangle is equal to exactly half of the area of the parallelogram.

Theorem 38. Parallelograms lying on the same base and between the same pair of parallel lines are of equal area.



$\triangle ABC = \triangle ABE$

$\Rightarrow \frac{1}{2}$ area of the parallelogram $ABCD = \frac{1}{2}$ area of the parallelogram $ABEF$.

Area of the parallelogram $ABCD$ = area of the parallelogram $ABEF$. (proved)

Area of Triangular region

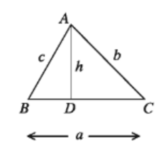
1. **Right angled triangle:** $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2}ab$
2. **Two sides of a triangular region and the angle included between them are given:**

Area of $\triangle ABC = \frac{1}{2} BC \times AD$

$= \frac{1}{2} a \times b \sin C = \frac{1}{2} ab \sin C$

Similarly, area of $\triangle ABC$

$= \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$



3. **Three sides of a triangle are given:**

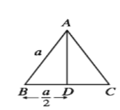
$2s = a + b + c.$

$\therefore$ Area of $\triangle ABC$

$= \frac{1}{2} BC \cdot AD = \frac{1}{2} \cdot a \cdot \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{s(s-a)(s-b)(s-c)}$

4. **Equilateral triangle:**

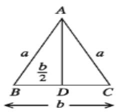
$$\triangle ABC = \frac{1}{2} \cdot BC \cdot AD = \frac{1}{2} \cdot a \cdot \frac{\sqrt{3}a}{2} = \frac{\sqrt{3}}{4}a^2$$



5. Isosceles triangle:

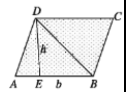
Area of isosceles $\triangle ABC = \frac{1}{2} \cdot BC \cdot AD$

$= \frac{1}{2} \cdot b \cdot \frac{\sqrt{4a^2 - b^2}}{2} = \frac{b}{4}\sqrt{4a^2 - b^2}$



Area of a parallelogram region:

a) Base and height are given: Let, the base $AB = b$ and height $DE = h$ of parallelogram $ABCD$. The diagonal BD divides the parallelogram into two equal triangular regions.

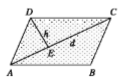


$\therefore$ The area of the parallelogram $ABCD$

$= 2\times$ area of $\triangle ABD = 2 \times \frac{1}{2} b \cdot h = bh$

b) The length of a diagonal and the length of a perpendicular drawn from the opposite angular point on that diagonal are given:

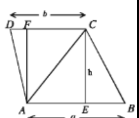
Let, in a parallelogram $ABCD$, the diagonal be $AC = d$ and the perpendicular from opposite angular point D on AC be $DE = h$. Diagonal AC divides the parallelogram into two equal triangular regions.



$\therefore$ area of the parallelogram $ABCD$

$= 2\times$ area of $\triangle ACD = 2 \times \frac{1}{2} d \cdot h = dh$

Area of trapezium region: Two parallel sides of trapezium region and the distance of perpendicular between them are given. Let $ABCD$ be a trapezium whose lengths of parallel sides are $AB = a$ unit, $CD = b$ unit and distance between them be $CE = AF = h$. Diagonal AC divides the trapezium region $ABCD$ into $\triangle ABC$ and $\triangle ACD$.



Area of trapezium region $ABCD$

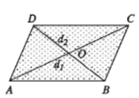
$=$ area of $\triangle ABC$ + area of $\triangle ACD$

$= \frac{1}{2} AB \times CE + \frac{1}{2} CD \times AF$

$= \frac{1}{2} ah + \frac{1}{2} bh = \frac{h(a+b)}{2}$

Area of Rhombus Region: Two diagonals of a rhombus region are given. Let the diagonals be $AC = d_1$, $BD = d_2$ of the rhombus $ABCD$ and the diagonals intersect each other at O.

Diagonal AC divides the rhombus region into two equal triangular regions. We know that the diagonals of a rhombus bisect each other at right angles.



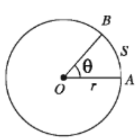
$\therefore$ height of $\triangle ACD = \frac{d_2}{2}$

$\therefore$ area of the rhombus $ABCD$

$= 2\times$ area of $\triangle ACD = 2 \times \frac{1}{2} d_1 \cdot \frac{d_2}{2} = \frac{1}{2} d_1 d_2$

Measurement regarding circle

$\therefore$ diameter of the circle $= 2r$ and circumference $= 2\pi r$



Length of arc of a circle

$$\therefore \frac{\theta}{360^\circ} = \frac{s}{2\pi r} \text{ or, } s = \frac{\pi r \theta}{180^\circ}$$

3. **Area of circular region and circular segment**

$\frac{\theta}{360^\circ} \times \pi r^2$

Solid Geometry

Rectangular solid

$\therefore$ the diagonal of the rectangular solid $= \sqrt{a^2 + b^2 + c^2}$

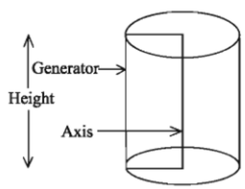
area of the whole surface: $2(ab + bc + ca)$

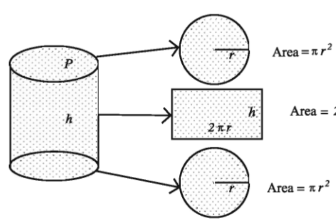
Volume of the rectangular solid = length $\times$ width $\times$ height $= abc$

Cube

1. The length of diagonal of the cube $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$
2. The area of the whole surface of the cube $= 2(a \cdot a + a \cdot a + a \cdot a) = 2(a^2 + a^2 + a^2) = 6a^2$
3. The volume of the cube $= a \cdot a \cdot a = a^3$

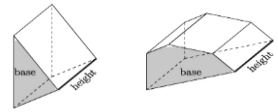
Cylinder





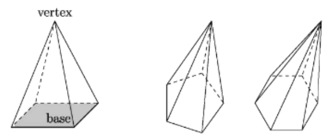
1. Area of the base $= \pi r^2$
2. Area of the curved surface = perimeter of the base $\times$ height $= 2\pi rh$
3. Area of the whole surface $= (\pi r^2 + 2\pi rh + \pi r^2) = 2\pi r(r + h)$
4. Volume = Area of the base $\times$ height $= \pi r^2 h$

3. Prism



1) The area of total surfaces of a prism
   = 2 (area of the base) + area of the lateral surfaces
   = 2 (area of the base) + perimeter of the base $\times$ height
2) volume = area of the base $\times$ height

4. Pyramid



The base of a pyramid is a any polygon and its lateral surfaces are of any triangular shape. But if the base is a regular polygon and the lateral faces are congruent triangles, the pyramid is called **regular pyramid**. The regular pyramids are eye-catching. The line joining the vertex and any corner of the base is called the **edge** of the pyramid. The length of the perpendicular from the vertex to the base is called the **height** of the pyramid. Usually, a solid with a square base and four congruent triangles meeting at a point is considered as a pyramid. These pyramids are in wide use.

A solid enclosed by four equilateral triangles is known as **regular tetrahedron** which is also a pyramid. This pyramid has $3 + 3 = 6$ edges and 4 vertices. The perpendicular from the vertex falls on the centroid of the base.

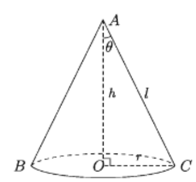
1) The area of all surfaces of pyramid = Area of the base + area of the lateral surfaces

   But if the lateral surfaces are congruent triangles,

   The area of all surafcs of the pyramid = Area of the base+$\frac{1}{2}$ (perimeter of the base $\times$ slant height)

   If the height of the perimid is h, radius of the inscribed circle of the base is r and l is its slant height, then $l = \sqrt{h^2 + r^2}$
2) volume $= \frac{1}{3}\times$ area of the base $\times$ height

5. Right circular cone



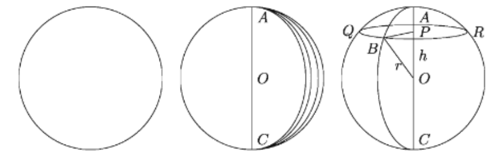
In the figure, the right circular cone ABC is formed by revolving the right-angled triangle OAC about OA. In this case, if θ is the vertical angle $\angle OAC$ of the triangle then it is called the Semi-vertical Angle of the cone.

If the circular cone has height $OA = h$, radius of the base $OC = r$ and slant height $AC = l$, then

1) Area of the curved surface $= \frac{1}{2}\times$ circumference of the base $\times$ slant height
   $= \frac{1}{2} \times 2\pi r \times l = \pi rl$ square units
2) Area of the whole surface = Area of the curved surface + area of base
   $= \pi rl + \pi r^2 = \pi r(r + l)$ square units
3) Volume $= \frac{1}{3}\times$ area of base $\times$ height
   $= \frac{1}{3}\pi r^2 h$ cubic units [You will learn the method of deduction of this formula in higher classes]

6. Sphere

The solid formed by a complete revolution of a semi-circle about its diameter as axis is called a sphere. The centre of the semi-circle is the centre of the sphere. The surface formed by the revolution of the semi-circle about its diameter is the surface of the sphere.



The centre of the sphere $CQAR$ is the point O, radius $OA = OB = OC$ and a plane perpendicular to OA and passing through a point at a distance h from the centre cuts the sphere and form the circle QBR. The centre of this circle is P and radius PB. Then PB and OP are perpendicular to each other.

$\therefore OB^2 = OP^2 + PB^2$

$\therefore PB^2 = OB^2 - OP^2 = r^2 - h^2$

If the radius of the sphere is r then

1) Area of the surface of the sphere $= 4\pi r^2$ sq units
2) Volume $= \frac{4}{3}\pi r^3$ cubic units
3) Radius of the circle formed by the section of a plane at a distance h from the centre $= \sqrt{r^2 - h^2}$ unit

So, volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ cubic units

Volume of the semi-sphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$ cubic units

Volume of the cylinder $= \pi r^2 h = \pi r^3$ cubic units

Example 8. If the volume of a right circular cone is V, the area of it curved surface is S, radius of the base is r, height is h and semi-vertica angle is α. Then show that,

1) $S = \frac{\pi h^2 \tan\alpha}{\cos\alpha} = \frac{\pi r^2}{\sin\alpha}$ square units.
2) $V = \frac{1}{3}\pi h^3 \tan^2\alpha = \frac{\pi r^3}{3\tan\alpha}$ cubic units.