Normalisation in Deep Inference via Atomic Flows

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PROOF IDENTITY (HILBERT'S 24TH PROBLEM):

GIVEN TWO MATHEMATICAL PROOFS,
ARE THEY THE SAME?

... AND TWO ALGORITHMS?

NO GOOD ANSWERS SO FAR

PROOF COMPLEXITY:

HOW COMPLEX NEED PROOFS BE?

WE KNOW THAT:

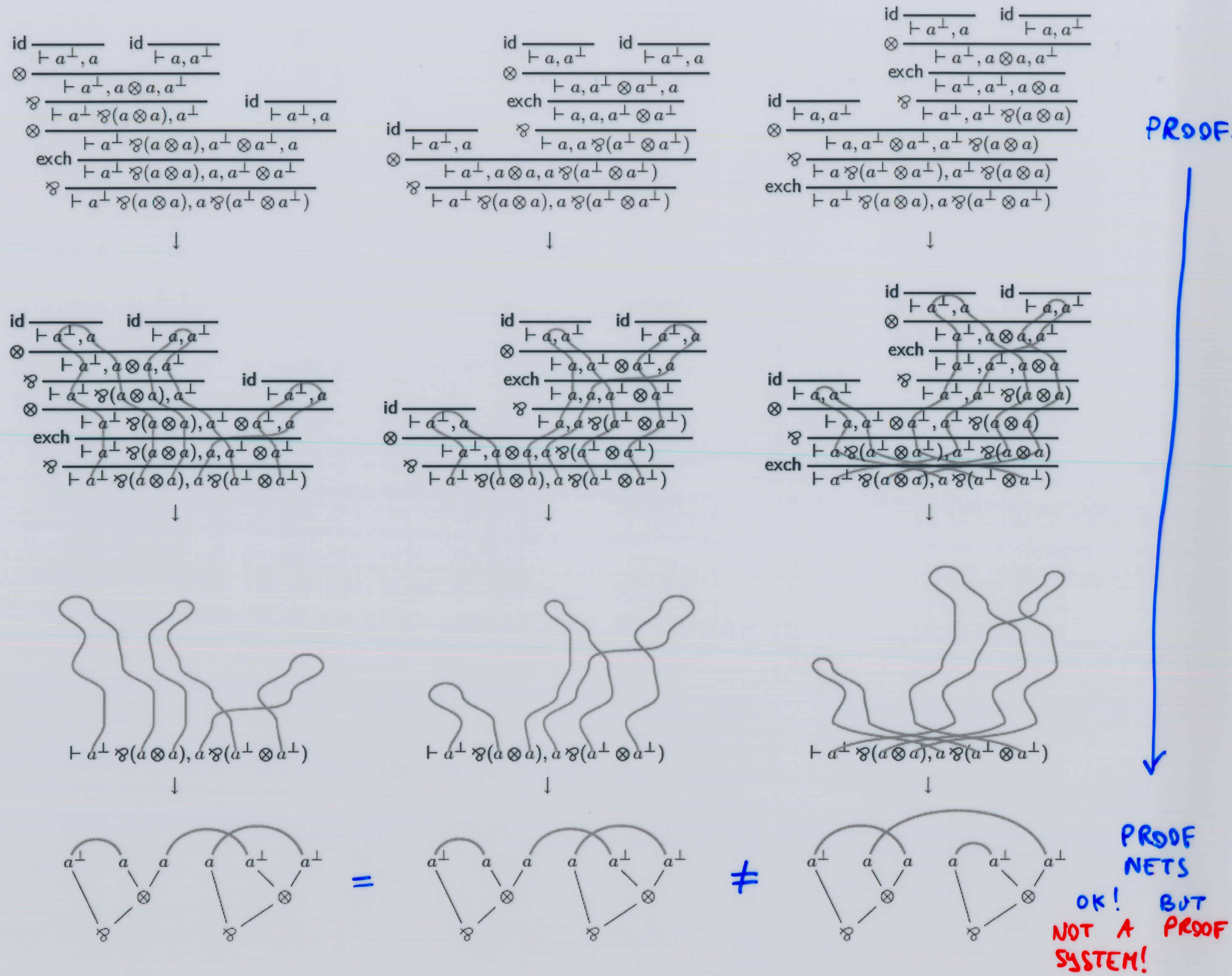
THERE EXISTS A PROOF SYSTEM
THAT PROVES POLYNOMIALLY ANY
PROPOSITIONAL TAUTOLOGY

IFF

$$coNP = NP$$

AND:

$$coNP \neq NP \Rightarrow P \neq NP$$


PROOFS
PROOF NETS
OK! BUT
NOT A PROOF SYSTEM!
=
≠

FOR DOING BETTER AT PROOF IDENTITY AND PROOF COMPLEXITY, WE NEED

- MORE FLEXIBILITY IN MANIPULATING PROOFS
- MORE FREEDOM IN BUILDING PROOFS

HOW CAN WE GET FLEXIBILITY AND FREEDOM?

IDEA: LET'S MAKE <u>INFERENCE RULES</u> SMALLER

E.G.: CONTRACTION

$$\frac{F \vee F}{F} \quad F\text{: FORMULA} \qquad \Bigg| \qquad \frac{a \vee a}{a} \quad a\text{: ATOM}$$

UNBOUNDED COMPLEXITY: BAD | CONSTANT COMPLEXITY: GOOD

SO, WE WANT LOCALITY

$$\boxed{\text{LOCALITY} \longrightarrow \text{GEOMETRY} \longrightarrow \text{SEMANTICS}}$$

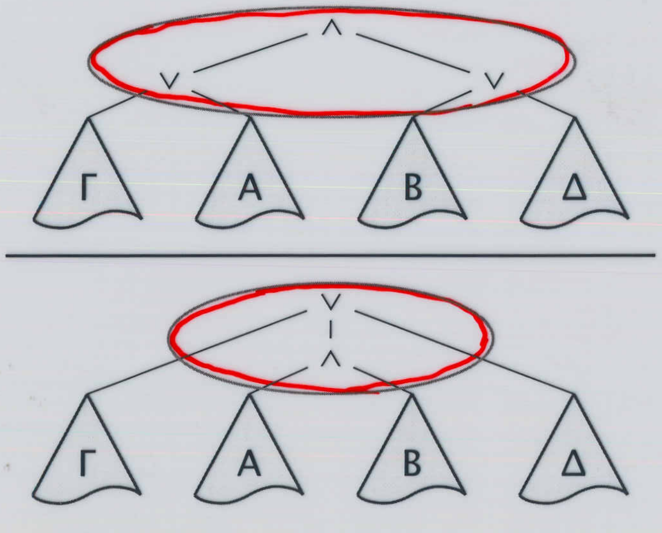
DEEP INFERENCE IS A QUEST FOR LOCALITY

Shallow Inference

Sequent Calculus

$$\frac{\Gamma, A \quad B, \Delta}{\Gamma, A \wedge B, \Delta}$$

$$\wr$$

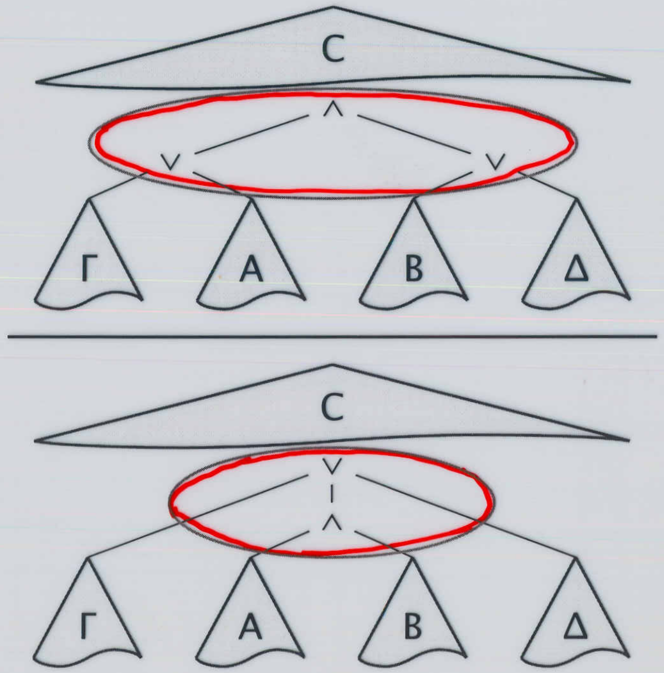


Deep Inference

Calculus of Structures

$$\frac{C\{(\Gamma \vee A) \wedge (B \vee \Delta)\}}{C\{\Gamma \vee (A \wedge B) \vee \Delta\}}$$

$$\wr$$



	Atomic structural rules			*Logical* rules	
SKS	$\mathsf{ai}\!\uparrow \dfrac{a \wedge \bar{a}}{\mathsf{f}}$ *cointeraction* or *cut*	$\mathsf{aw}\!\uparrow \dfrac{a}{\mathsf{t}}$ *coweakening*	$\mathsf{ac}\!\uparrow \dfrac{a}{a \wedge a}$ *cocontraction*		
	$\mathsf{ai}\!\downarrow \dfrac{\mathsf{t}}{a \vee \bar{a}}$ *interaction* or *identity*	$\mathsf{aw}\!\downarrow \dfrac{\mathsf{f}}{a}$ *weakening*	$\mathsf{ac}\!\downarrow \dfrac{a \vee a}{a}$ *contraction*	$\mathsf{s}\, \dfrac{A \wedge [B \vee C]}{(A \wedge B) \vee C}$ *switch*	$\mathsf{m}\, \dfrac{(A \wedge B) \vee (C \wedge D)}{[A \vee C] \wedge [B \vee D]}$ *medial*	KS

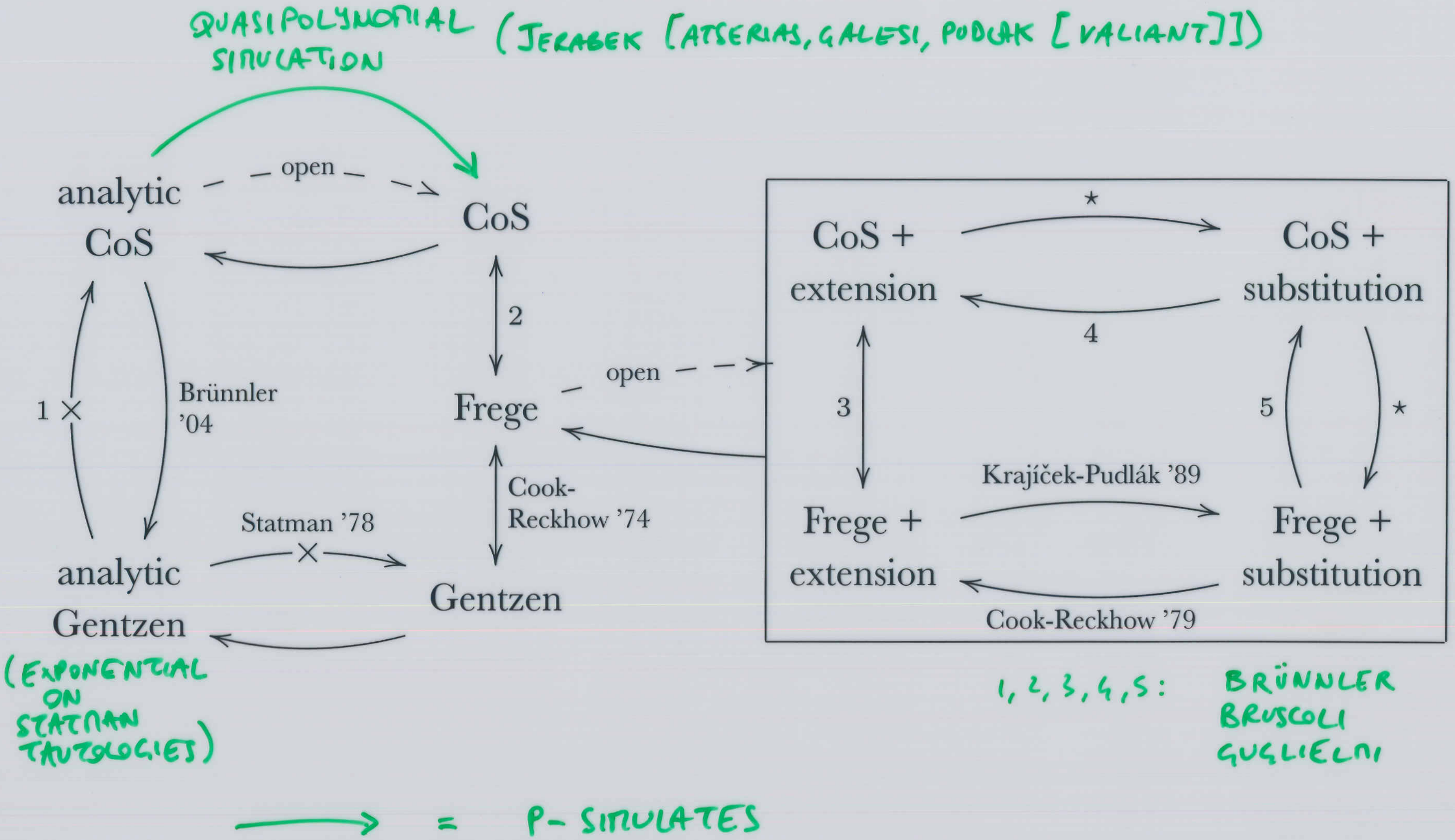
→ ANALYTIC

SELF-DUAL

Figure 3: Systems SKS and KS. (KAI BRÜNNLER & ALWEN TIU)

NOTICE THE TOP-DOWN SYMMETRY ← ONLY IN DI!

ANALYTICITY IS AN ASYMMETRIC NOTION!


QUASIPOLYNOMIAL SIMULATION
(JERABEK [ATSERIAS, GALESI, PUDLAK [VALIANT]])
analytic CoS
open
CoS
1
Brünnler '04
2
Frege
open
Cook-Reckhow '74
analytic Gentzen
Statman '78
Gentzen
(EXPONENTIAL ON STATMAN TAUTOLOGIES)
CoS + extension
⋆
CoS + substitution
4
3
5
⋆
Frege + extension
Krajíček-Pudlák '89
Frege + substitution
Cook-Reckhow '79
1, 2, 3, 4, 5: BRÜNNLER BRUSCOLI GUGLIELMI
= P-SIMULATES

FINAL OBJECTIVE

(JOINT WORK WITH TOM GUNDERSEN
MICHEL PARIGOT):

BUREAUCRACY-FREE FORMALISM

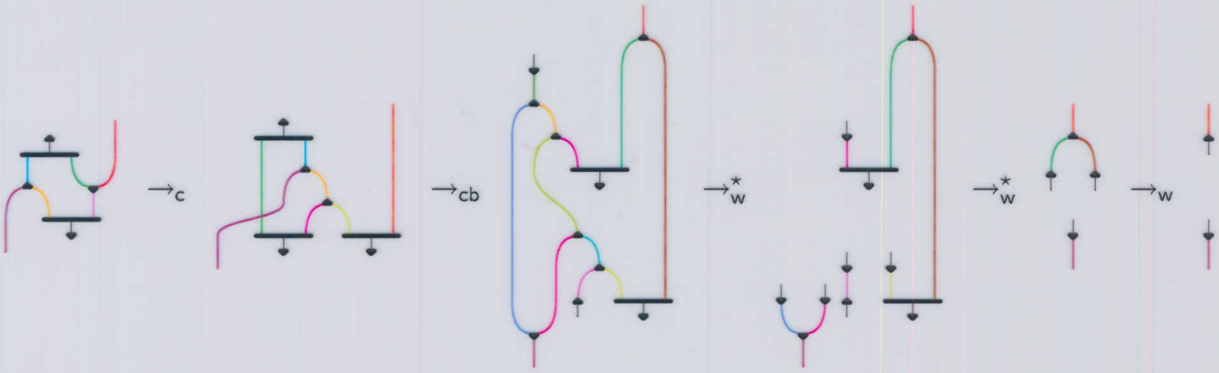
→ GOOD FOR
- PROOF SEMANTICS
- IDENTITY OF PROOFS

INTERMEDIATE STEP

(JOINT WORK WITH TOM GUNDERSEN):
TO APPEAR ON LMCS

BUREAUCRACY-FREE NORMALISATION

ATOMIC FLOWS



$$
\begin{array}{rc}
 & \bar{a} \\ \cline{2-2}
\mathsf{ai}\downarrow & [a \vee \bar{a}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & \bar{a} \vee (a \wedge \bar{a}) \\ \cline{2-2}
\mathsf{m} & [\bar{a} \vee \bar{a}] \wedge [\mathsf{t} \vee a] \\ \cline{2-2}
\mathsf{ac}\downarrow & \bar{a} \wedge [a \vee \mathsf{t}] \\ \cline{2-2}
\mathsf{ac}\uparrow & \bar{a} \wedge [(a \wedge a) \vee \mathsf{t}] \\ \cline{2-2}
\mathsf{s} & (\bar{a} \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ai}\uparrow & (\mathsf{f} \wedge a) \vee \mathsf{t}
\end{array}
$$

$$\downarrow_{\mathsf{c}}$$

$$
\begin{array}{rc}
 & \bar{a} \\ \cline{2-2}
\mathsf{ai}\downarrow & [\bar{a} \vee a] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & \bar{a} \vee (a \wedge \bar{a}) \\ \cline{2-2}
\mathsf{m} & [\bar{a} \vee \bar{a}] \wedge [\mathsf{t} \vee a] \\ \cline{2-2}
\mathsf{ac}\uparrow & [\bar{a} \vee \bar{a}] \wedge [(a \wedge a) \vee \mathsf{t}] \\ \cline{2-2}
\mathsf{s} & ([\bar{a} \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ac}\uparrow & ([\bar{a} \vee \bar{a}] \wedge a \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{s} & ([(a \wedge \bar{a}) \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ai}\uparrow & (\bar{a} \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ai}\uparrow & (\mathsf{f} \wedge a) \vee \mathsf{t}
\end{array}
$$

$$\to_{\mathsf{cb}}$$

$$
\begin{array}{rc}
 & \bar{a} \\ \cline{2-2}
\mathsf{c}\uparrow & \bar{a} \wedge \bar{a} \\ \cline{2-2}
\mathsf{aw}\downarrow & [\mathsf{t} \vee a] \wedge \bar{a} \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [\mathsf{t} \vee (a \wedge \bar{a})] \wedge \bar{a} \\ \cline{2-2}
\mathsf{m} & [\mathsf{t} \vee \bar{a}] \wedge [\mathsf{t} \vee a] \wedge \bar{a} \\ \cline{2-2}
\mathsf{ac}\uparrow & [\mathsf{t} \vee \bar{a}] \wedge [\mathsf{t} \vee (a \wedge a)] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [([\mathsf{t} \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{ac}\uparrow & [([\mathsf{t} \vee \bar{a}] \wedge a \wedge a \wedge a) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [([a \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [([(a \wedge \bar{a}) \vee a] \wedge a) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{ai}\uparrow & [(a \wedge a) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{ss}\downarrow & [(\mathsf{f} \wedge a) \vee \mathsf{t} \vee a] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & (\mathsf{f} \wedge a) \vee \mathsf{t} \vee (a \wedge \bar{a}) \\ \cline{2-2}
\mathsf{m} & [(\mathsf{f} \wedge a) \vee \mathsf{t} \vee \bar{a}] \wedge [\mathsf{t} \vee a] \\ \cline{2-2}
\mathsf{ac}\uparrow & [(\mathsf{f} \wedge a) \vee \mathsf{t} \vee \bar{a}] \wedge [\mathsf{t} \vee (a \wedge a)] \\ \cline{2-2}
\mathsf{s} & ([(\mathsf{f} \wedge a) \vee \mathsf{t} \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ac}\uparrow & ([(\mathsf{f} \wedge a) \vee \mathsf{t} \vee \bar{a}] \wedge a \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{s} & ([(a \wedge [(\mathsf{f} \wedge a) \vee \mathsf{t}]) \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\uparrow & ([(\mathsf{f} \wedge a) \vee \mathsf{t} \vee \bar{a}] \wedge a \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{s} & ([(a \wedge \bar{a}) \vee (\mathsf{f} \wedge a) \vee \mathsf{t}] \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ai}\uparrow & ([\mathsf{f} \vee (\mathsf{f} \wedge a) \vee \mathsf{t}] \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ss}\downarrow & (\mathsf{f} \wedge a) \vee (\mathsf{f} \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{c}\downarrow & (\mathsf{f} \wedge a) \vee \mathsf{t}
\end{array}
$$

$$\to^{\star}_{\mathsf{w}}$$

$$
\begin{array}{rc}
 & \bar{a} \\ \cline{2-2}
\mathsf{c}\uparrow & \bar{a} \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [\mathsf{t} \vee (\mathsf{f} \wedge \bar{a})] \wedge \bar{a} \\ \cline{2-2}
\mathsf{m} & [\mathsf{t} \vee \bar{a}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [([\mathsf{t} \vee \bar{a}] \wedge \mathsf{f}) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{s} & [(\bar{a} \wedge \mathsf{f}) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{aw}\downarrow & [(a \wedge \bar{a} \wedge \mathsf{f}) \vee \mathsf{t}] \wedge \bar{a} \\ \cline{2-2}
\mathsf{ai}\uparrow & \bar{a} \\ \cline{2-2}
\mathsf{s} & \mathsf{t} \vee (\mathsf{f} \wedge \bar{a}) \\ \cline{2-2}
\mathsf{m} & \mathsf{t} \vee \bar{a} \\ \cline{2-2}
\mathsf{s} & ([\mathsf{t} \vee \bar{a}] \wedge \mathsf{f}) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{s} & (\bar{a} \wedge \mathsf{f}) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\downarrow & ([a \vee \bar{a}] \wedge \mathsf{f}) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\uparrow & ([\mathsf{t} \vee \bar{a}] \wedge \mathsf{f}) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{s} & ([(\mathsf{f} \wedge \bar{a}) \vee \mathsf{t}] \wedge \mathsf{f}) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\downarrow & ([(a \wedge \bar{a}) \vee \mathsf{t}] \wedge \mathsf{f}) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{ai}\uparrow & \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\downarrow & (\mathsf{f} \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\downarrow & (\mathsf{f} \wedge a) \vee (\mathsf{f} \wedge a) \vee \mathsf{t} \\ \cline{2-2}
\mathsf{c}\downarrow & (\mathsf{f} \wedge a) \vee \mathsf{t}
\end{array}
$$

$$\to^{\star}_{\mathsf{w}}$$

$$
\begin{array}{rc}
 & \bar{a} \\ \cline{2-2}
\mathsf{c}\uparrow & \bar{a} \wedge \bar{a} \\ \cline{2-2}
\mathsf{aw}\uparrow & \bar{a} \\ \cline{2-2}
\mathsf{aw}\uparrow & \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\downarrow & (\mathsf{f} \wedge a) \vee \mathsf{t}
\end{array}
$$

$$\downarrow_{\mathsf{w}}$$

$$
\begin{array}{rc}
 & \bar{a} \\ \cline{2-2}
\mathsf{aw}\downarrow & \mathsf{t} \\ \cline{2-2}
\mathsf{aw}\uparrow & (\mathsf{f} \wedge a) \vee \mathsf{t}
\end{array}
$$

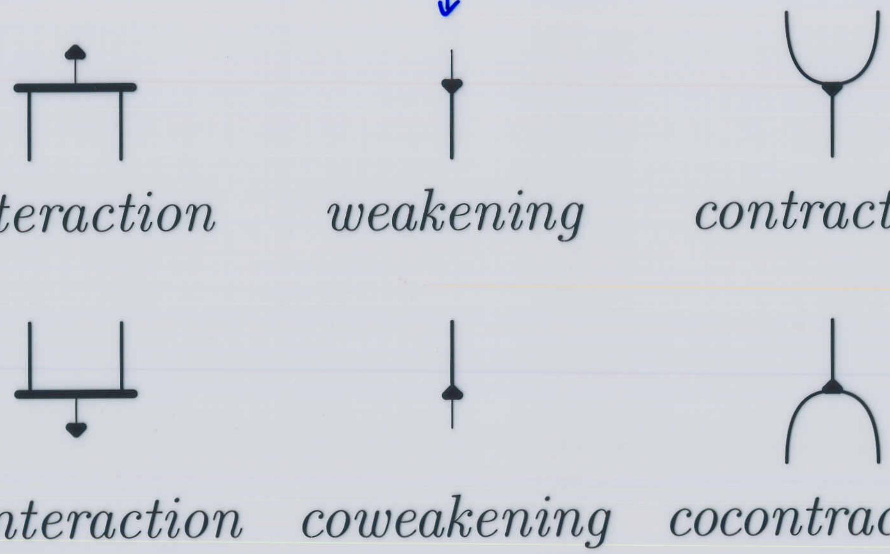
Figure 7: Example of application of procedure Str to an atomic flow.

ATOMIC FLOWS CONTROL NORMALISATION

ATOMIC FLOWS

$\frac{f}{a}$ $\downarrow$

$\frac{t}{[a \vee \bar{a}]} \rightarrow$ $\leftarrow \frac{[a \vee a]}{a}$



$\frac{(a \wedge \bar{a})}{f} \rightarrow$ $\leftarrow \frac{a}{(a \wedge a)}$

$\uparrow$ $\frac{a}{t}$

PROOF $\underset{!\;*}{\rightleftarrows}$ ATOMIC FLOW

$* \neq$ PROOF NETS!

$$
\begin{array}{rc}
 & \mathsf{t} \\ \hline
\mathsf{ai}\!\downarrow & a \vee \bar{a} \\ \hline
= & (a \wedge \mathsf{t}) \vee (\mathsf{t} \wedge \bar{a}) \\ \hline
\mathsf{m} & [a \vee \mathsf{t}] \wedge [\mathsf{t} \vee \bar{a}] \\ \hline
= & [a \vee \mathsf{t}] \wedge [\bar{a} \vee \mathsf{t}] \\ \hline
\mathsf{s} & ([a \vee \mathsf{t}] \wedge \bar{a}) \vee \mathsf{t} \\ \hline
= & (\bar{a} \wedge [a \vee \mathsf{t}]) \vee \mathsf{t} \\ \hline
\mathsf{s} & [(\bar{a} \wedge a) \vee \mathsf{t}] \vee \mathsf{t} \\ \hline
= & (a \wedge \bar{a}) \vee \mathsf{t} \\ \hline
\mathsf{ai}\!\uparrow & \mathsf{f} \vee \mathsf{t} \\ \hline
= & \mathsf{t}
\end{array}
\qquad
\begin{array}{rc}
 & a \wedge [\bar{a} \vee \mathsf{t}] \wedge \bar{a} \\ \hline
\mathsf{ai}\!\downarrow & a \wedge [\bar{a} \vee [\bar{a} \vee a]] \wedge \bar{a} \\ \hline
= & (a \wedge [[\bar{a} \vee \bar{a}] \vee a]) \wedge \bar{a} \\ \hline
\mathsf{s} & [(a \wedge [\bar{a} \vee \bar{a}]) \vee a] \wedge \bar{a} \\ \hline
\mathsf{ac}\!\downarrow & [(a \wedge \bar{a}) \vee a] \wedge \bar{a} \\ \hline
\mathsf{ai}\!\uparrow & [\mathsf{f} \vee a] \wedge \bar{a} \\ \hline
= & a \wedge \bar{a} \\ \hline
\mathsf{ac}\!\uparrow & (a \wedge a) \wedge \bar{a} \\ \hline
= & a \wedge (a \wedge \bar{a}) \\ \hline
\mathsf{ai}\!\uparrow & a \wedge \mathsf{f}
\end{array}
\qquad
\begin{array}{rc}
 & [a \vee b] \wedge c \\ \hline
\mathsf{ac}\!\uparrow & [(a \wedge a) \vee b] \wedge c \\ \hline
\mathsf{ac}\!\uparrow & [(a \wedge a) \vee (b \wedge b)] \wedge c \\ \hline
\mathsf{ac}\!\uparrow & [(a \wedge a) \vee (b \wedge b)] \wedge (c \wedge c) \\ \hline
\mathsf{m} & ([a \vee b] \wedge [a \vee b]) \wedge (c \wedge c) \\ \hline
= & ([a \vee b] \wedge c) \wedge ([a \vee b] \wedge c)
\end{array}
$$

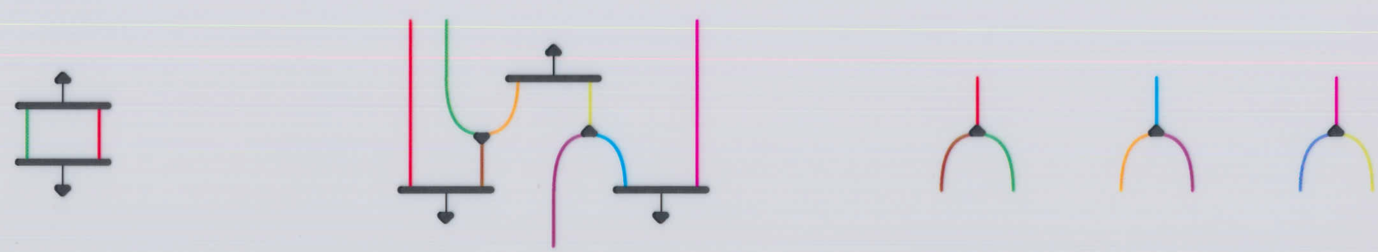
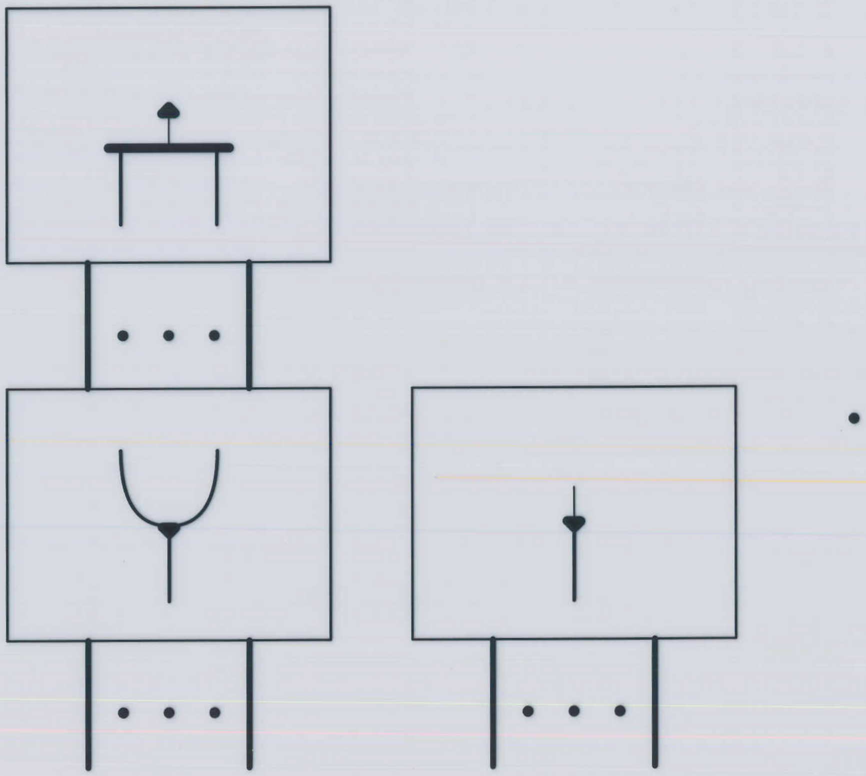


Figure 1: Examples of atomic flows associated with derivations.

SHAPE OF A NORMALISED PROOF

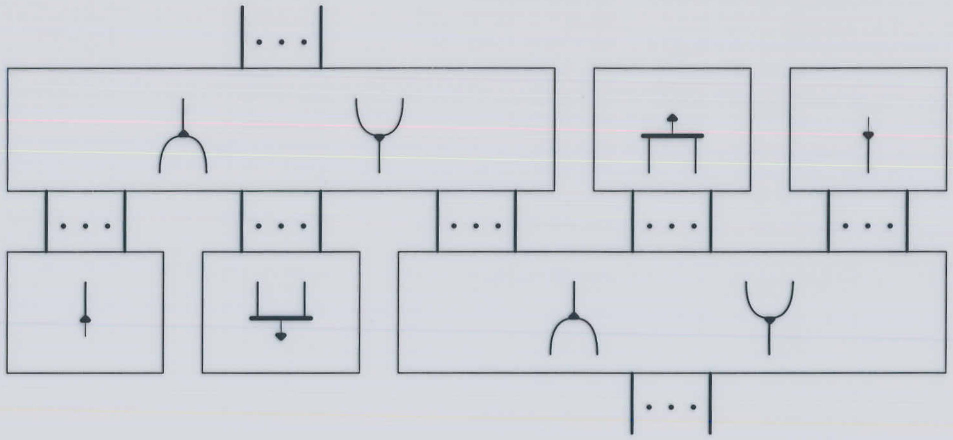


FUTURE: A SYNTAX AS CLEAN AS ATOMIC FLOWS: NO BUREAUCRACY

PRESENT: THERE IS A USEFUL GEOMETRIC NATURE OF PROOFS

STREAMLINED DERIVATIONS
(NEW NORMAL FORM)

3.13 Remark. The following diagram shows the shape of a streamlined derivation; the boxes stand for flows obtained by freely composing wires and the nodes indicated on them:



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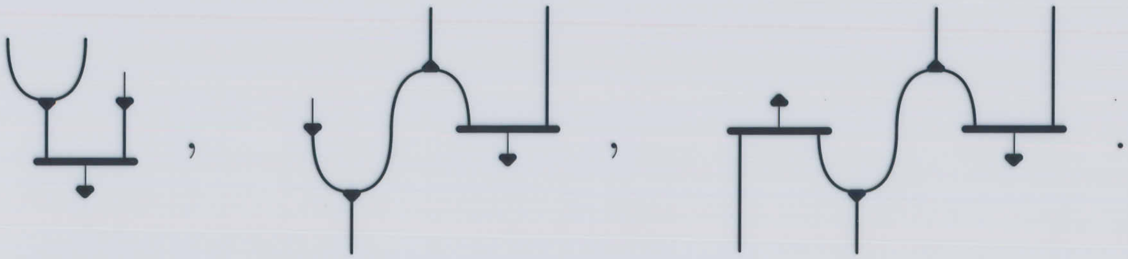
3.14 Example. The first flow is not streamlined, the other two are streamlined:



3.15 Remark. A streamlined SKS proof is cut-free.

FURTHER NORMALISATIONS:

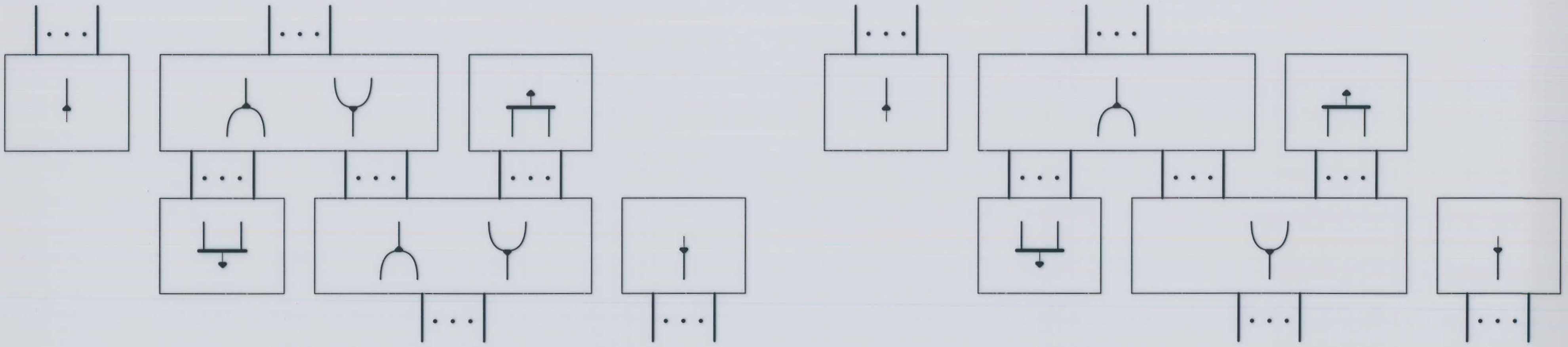


Figure 8: Super-streamlined (left) and hyper-streamlined (right) flows.

THEOREMS: ALL DERIVATIONS CAN BE SO STREAMLINED

COROLLARY: CUT-ELIMINATION

FLOWS ARE ENOUGH!

SOUND REDUCTIONS — I

FOR EVERY REDUCTION THERE'S A CORRESPONDING PROOF TRANSFORMATION

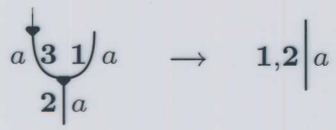
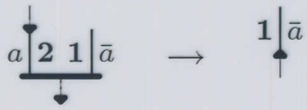
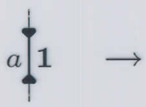
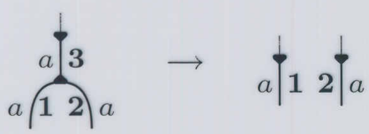
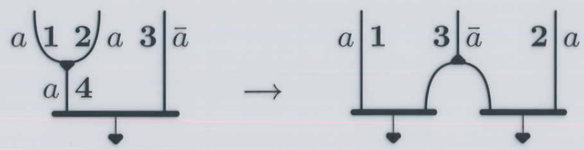
w↓-c↓: 	$\begin{array}{c} \mathsf{aw}{\downarrow}\,\dfrac{\xi\{\mathsf{f}\}}{\xi\{a^{\mathbf{3}}\}} \\ \Delta \Big\Vert \\ \mathsf{ac}{\downarrow}\,\dfrac{\zeta\{a^{\mathbf{3}} \vee a^{\mathbf{1}}\}}{\zeta\{a^{\mathbf{2}}\}} \end{array} \quad \to_{\mathsf{w}\downarrow\text{-}\mathsf{c}\downarrow} \quad \begin{array}{c} \xi\{\mathsf{f}\} \\ \Delta\{a^{\mathbf{3}} \leftarrow \mathsf{f}\} \Big\Vert \\ =\dfrac{\zeta\{\mathsf{f} \vee a^{\mathbf{1,2}}\}}{\zeta\{a^{\mathbf{1,2}}\}} \end{array}$
w↓-i↑: 	$\begin{array}{c} \mathsf{aw}{\downarrow}\,\dfrac{\xi\{\mathsf{f}\}}{\xi\{a^{\mathbf{2}}\}} \\ \Delta \Big\Vert \\ \mathsf{ai}{\uparrow}\,\dfrac{\zeta\{a^{\mathbf{2}} \wedge \bar{a}^{\mathbf{1}}\}}{\zeta\{\mathsf{f}\}} \end{array} \quad \to_{\mathsf{w}\downarrow\text{-}\mathsf{i}\uparrow} \quad \begin{array}{c} \xi\{\mathsf{f}\} \\ \Delta\{a^{\mathbf{2}} \leftarrow \mathsf{f}\} \Big\Vert \\ \mathsf{aw}{\uparrow}\,\dfrac{\zeta\{\mathsf{f} \wedge \bar{a}^{\mathbf{1}}\}}{=\dfrac{\zeta\{\mathsf{f} \wedge \mathsf{t}\}}{\zeta\{\mathsf{f}\}}} \end{array}$
w↓-w↑: 	$\begin{array}{c} \mathsf{aw}{\downarrow}\,\dfrac{\xi\{\mathsf{f}\}}{\xi\{a^{\mathbf{1}}\}} \\ \Delta \Big\Vert \\ \mathsf{aw}{\uparrow}\,\dfrac{\zeta\{a^{\mathbf{1}}\}}{\zeta\{\mathsf{t}\}} \end{array} \quad \to_{\mathsf{w}\downarrow\text{-}\mathsf{w}\uparrow} \quad \begin{array}{c} \xi\{\mathsf{f}\} \\ \Delta\{a^{\mathbf{1}} \leftarrow \mathsf{f}\} \Big\Vert \\ =\dfrac{\zeta\{\mathsf{f}\}}{\mathsf{s}\,\dfrac{\zeta\{\mathsf{f} \wedge [\mathsf{f} \vee \mathsf{t}]\}}{=\dfrac{\zeta\{(\mathsf{f} \wedge \mathsf{f}) \vee \mathsf{t}\}}{\zeta\{\mathsf{t}\}}}} \end{array}$
w↓-c↑: 	$\begin{array}{c} \mathsf{aw}{\downarrow}\,\dfrac{\xi\{\mathsf{f}\}}{\xi\{a^{\mathbf{3}}\}} \\ \Delta \Big\Vert \\ \mathsf{ac}{\uparrow}\,\dfrac{\zeta\{a^{\mathbf{3}}\}}{\zeta\{a^{\mathbf{1}} \wedge a^{\mathbf{2}}\}} \end{array} \quad \to_{\mathsf{w}\downarrow\text{-}\mathsf{c}\uparrow} \quad \begin{array}{c} \xi\{\mathsf{f}\} \\ \Delta\{a^{\mathbf{3}} \leftarrow \mathsf{f}\} \Big\Vert \\ =\dfrac{\zeta\{\mathsf{f}\}}{\mathsf{aw}{\downarrow}\,\dfrac{\zeta\{\mathsf{f} \wedge \mathsf{f}\}}{\mathsf{aw}{\downarrow}\,\dfrac{\zeta\{a^{\mathbf{1}} \wedge \mathsf{f}\}}{\zeta\{a^{\mathbf{1}} \wedge a^{\mathbf{2}}\}}}} \end{array}$

Figure 2: 'Downwards' reductions for weakening.

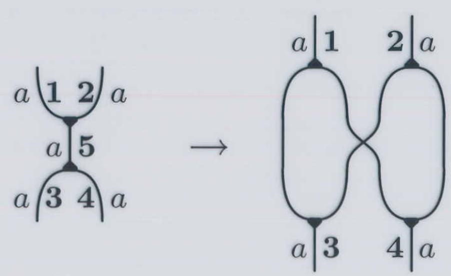
SOUND REDUCTIONS – II

c↓-i↑ :



$$\begin{array}{rc}
 & \xi\{a^{\mathbf{1}} \vee a^{\mathbf{2}}\} \\ \cline{2-2}
\mathsf{ac}\!\downarrow & \xi\{a^{\mathbf{4}}\} \\
 & \Delta \,\| \\
 & \zeta\{a^{\mathbf{4}} \wedge \bar{a}^{\mathbf{3}}\} \\ \cline{2-2}
\mathsf{ai}\!\uparrow & \zeta\{\mathsf{f}\}
\end{array}
\quad \xrightarrow{}_{\mathsf{c}\downarrow\text{-}\mathsf{i}\uparrow} \quad
\begin{array}{rc}
 & \xi\{a^{\mathbf{1}} \vee a^{\mathbf{2}}\} \\
 & \Delta\{a^{\mathbf{4}} \leftarrow a \vee a\} \,\| \\
 & \zeta\{[a^{\mathbf{1}} \vee a^{\mathbf{2}}] \wedge \bar{a}^{\mathbf{3}}\} \\ \cline{2-2}
\mathsf{ac}\!\uparrow & \zeta\{[a^{\mathbf{1}} \vee a^{\mathbf{2}}] \wedge (\bar{a} \wedge \bar{a})\} \\ \cline{2-2}
= & \zeta\{(\bar{a} \wedge [a^{\mathbf{1}} \vee a^{\mathbf{2}}]) \wedge \bar{a}\} \\ \cline{2-2}
\mathsf{s} & \zeta\{[(\bar{a} \wedge a^{\mathbf{1}}) \vee a^{\mathbf{2}}] \wedge \bar{a}\} \\ \cline{2-2}
\mathsf{ai}\!\uparrow & \zeta\{[\mathsf{f} \vee a^{\mathbf{2}}] \wedge \bar{a}\} \\ \cline{2-2}
= & \zeta\{a^{\mathbf{2}} \wedge \bar{a}\} \\ \cline{2-2}
\mathsf{ai}\!\uparrow & \zeta\{\mathsf{f}\}
\end{array}$$

c↓-c↑ :



$$\begin{array}{rc}
 & \xi\{a^{\mathbf{1}} \vee a^{\mathbf{2}}\} \\ \cline{2-2}
\mathsf{ac}\!\downarrow & \xi\{a^{\mathbf{5}}\} \\
 & \Delta \,\| \\
 & \zeta\{a^{\mathbf{5}}\} \\ \cline{2-2}
\mathsf{ac}\!\uparrow & \zeta\{a^{\mathbf{3}} \wedge a^{\mathbf{4}}\}
\end{array}
\quad \xrightarrow{}_{\mathsf{c}\downarrow\text{-}\mathsf{c}\uparrow} \quad
\begin{array}{rc}
 & \xi\{a^{\mathbf{1}} \vee a^{\mathbf{2}}\} \\
 & \Delta\{a^{\mathbf{5}} \leftarrow a \vee a\} \,\| \\
 & \zeta\{a^{\mathbf{1}} \vee a^{\mathbf{2}}\} \\ \cline{2-2}
\mathsf{ac}\!\uparrow & \zeta\{a^{\mathbf{1}} \vee (a \wedge a)\} \\ \cline{2-2}
\mathsf{ac}\!\uparrow & \zeta\{(a \wedge a) \vee (a \wedge a)\} \\ \cline{2-2}
\mathsf{m} & \zeta\{[a \vee a] \wedge [a \vee a]\} \\ \cline{2-2}
\mathsf{ac}\!\downarrow & \zeta\{a^{\mathbf{3}} \wedge [a \vee a]\} \\ \cline{2-2}
\mathsf{ac}\!\downarrow & \zeta\{a^{\mathbf{3}} \wedge a^{\mathbf{4}}\}
\end{array}$$

Figure 3: 'Downwards' reductions for contraction.

(THESE CREATE COMPLEXITY)

ONE SOURCE OF COMPLEXITY AND NON-TERMINATION ...

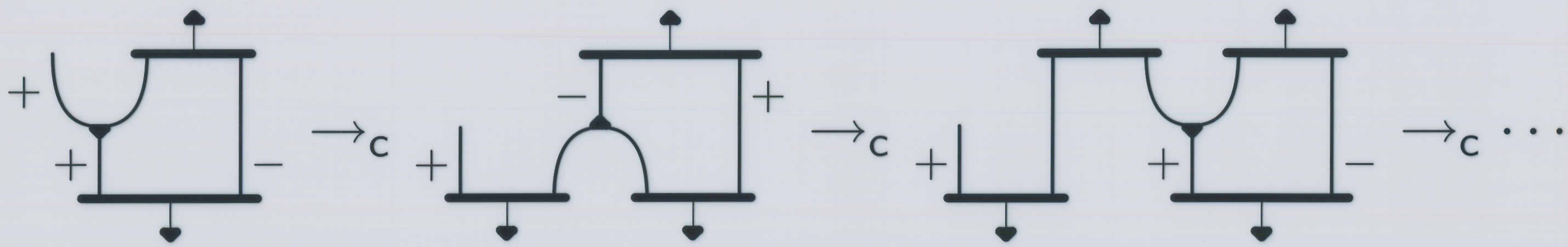
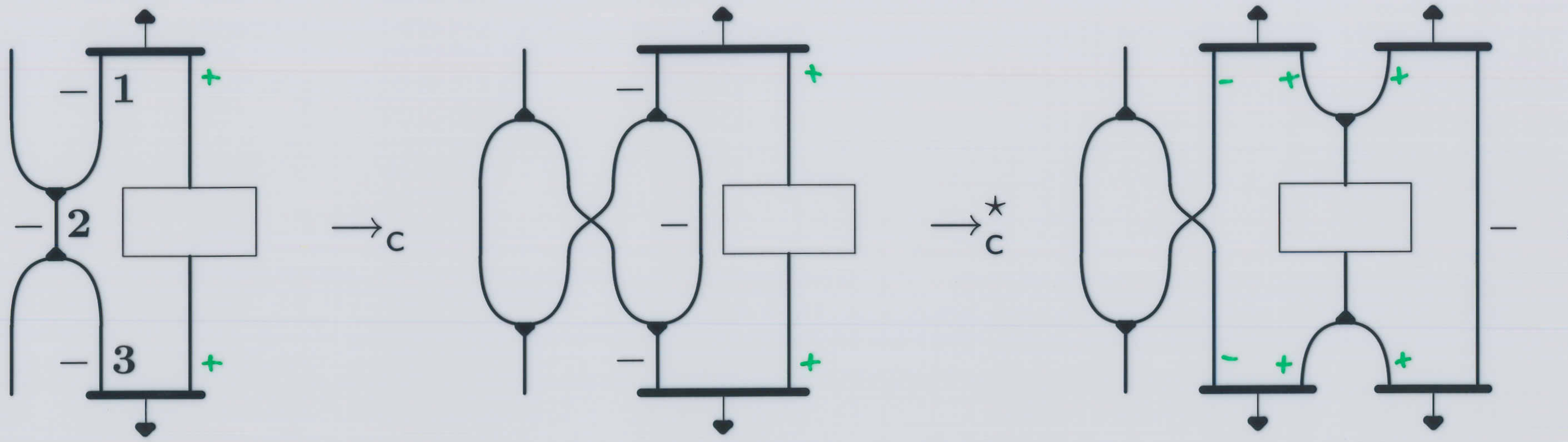


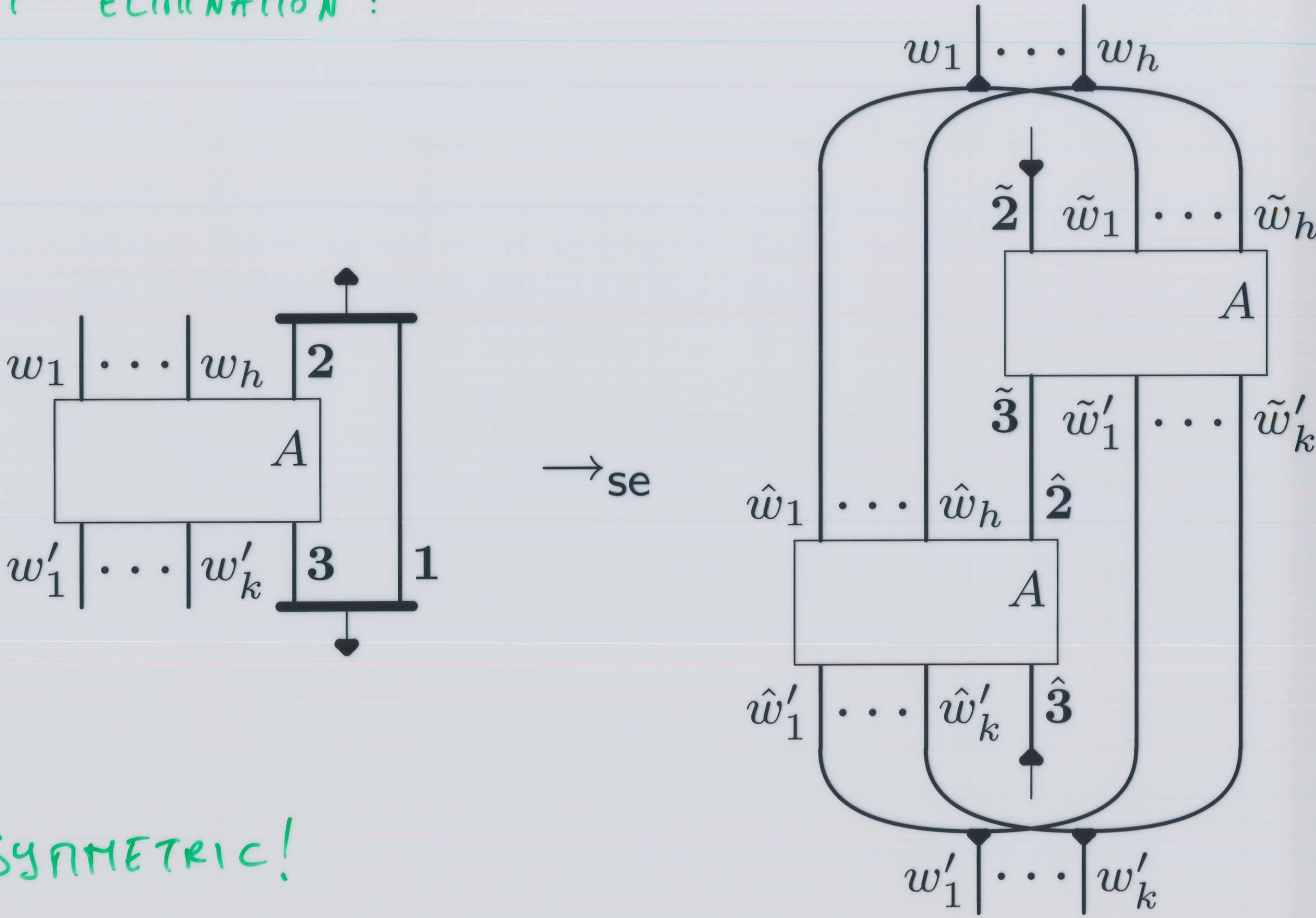
Figure 4: Infinite chain generated by flow rewriting system c.

... BUT WE CAN DEAL WITH IT!

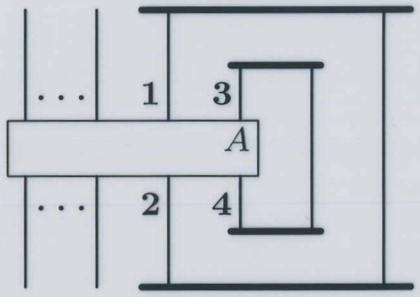


WE CAN MOVE ALL CONTRACTIONS AND COCONTRATIONS
TO A POLARITY OF CHOICE! (AND THEN GET RID OF THEM)

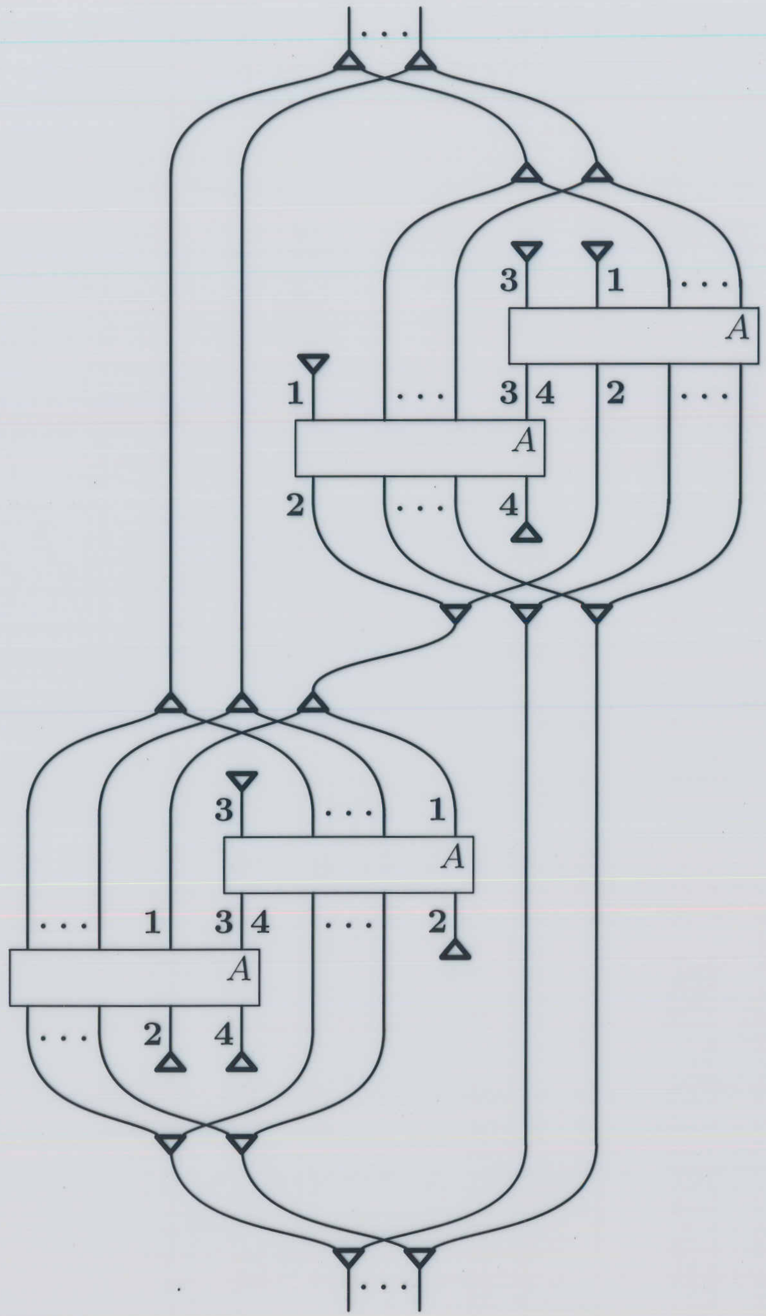
CUT ELIMINATION:



SYMMETRIC!



$\longrightarrow_{\mathsf{bc}}$



CONCLUSIONS

- WE ARE GETTING RID OF BUREAUCRACY 'VIA GEOMETRY'
- GOOD PROGRESS TOWARDS SOLVING THE PROOF IDENTITY PROBLEM
- WE TAKE INPUTS ALSO FROM
  - LAMARCHE / STRAßBURGER PROOF NETS
  - HUGHES' COMBINATORIAL PROOFS

ALSO IN THE FUTURE

- COMPUTATIONAL INTERPRETATIONS (WITH MICHEL PARIGOT)
- QUASIPOLYNOMIAL STREAMLINING (VIA THRESHOLD FUNCTIONS)