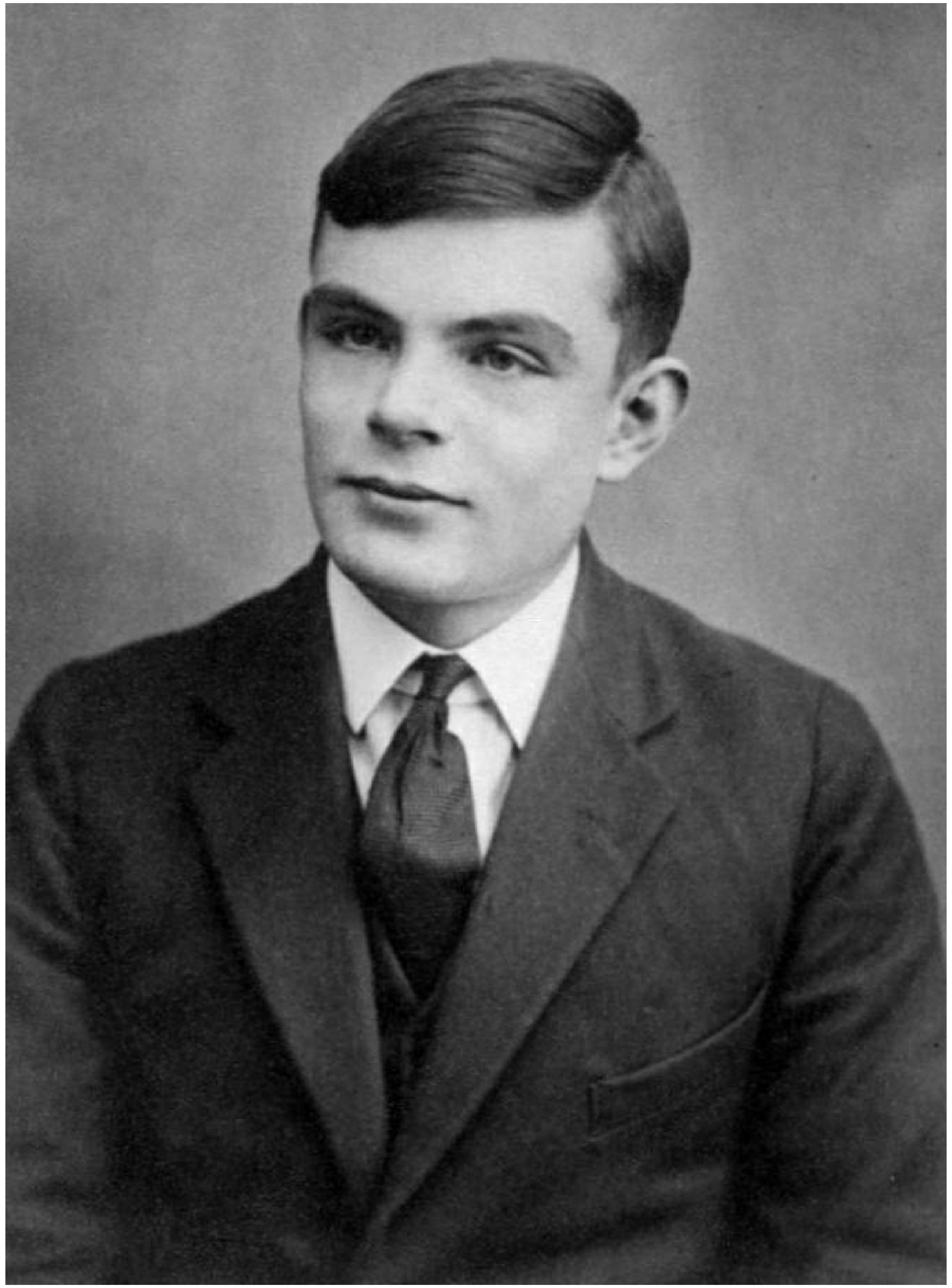
FUNDAMENTALS OF BAYESIAN DATA ANALYSIS IN R

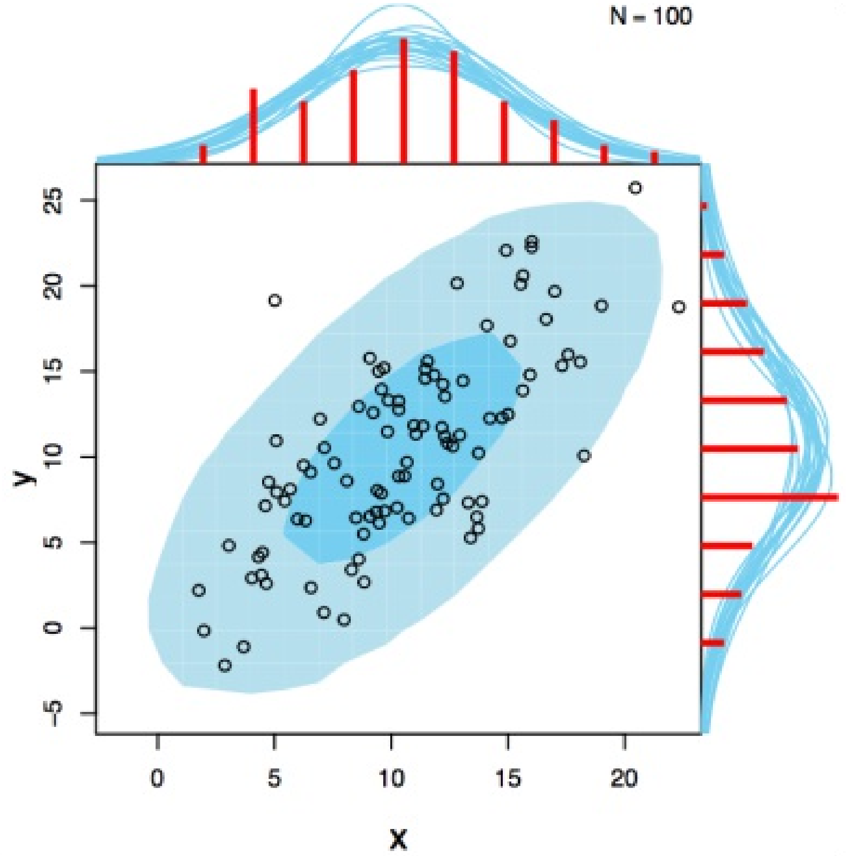
Welcome to the course!

Rasmus Bååth
Data Scientist

[1] https://commons.wikimedia.org/wiki/File:Enigma_08.jpg




A
B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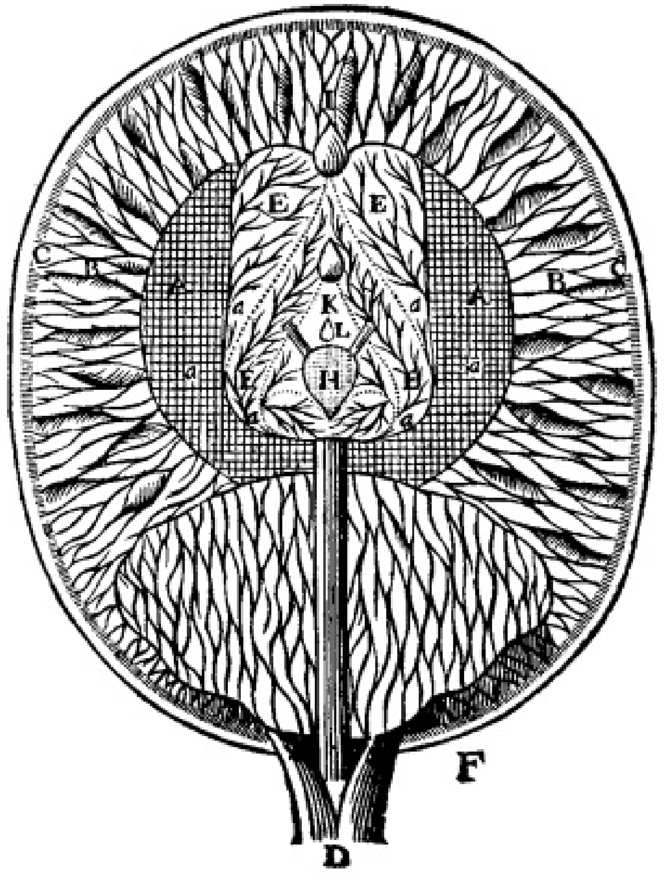
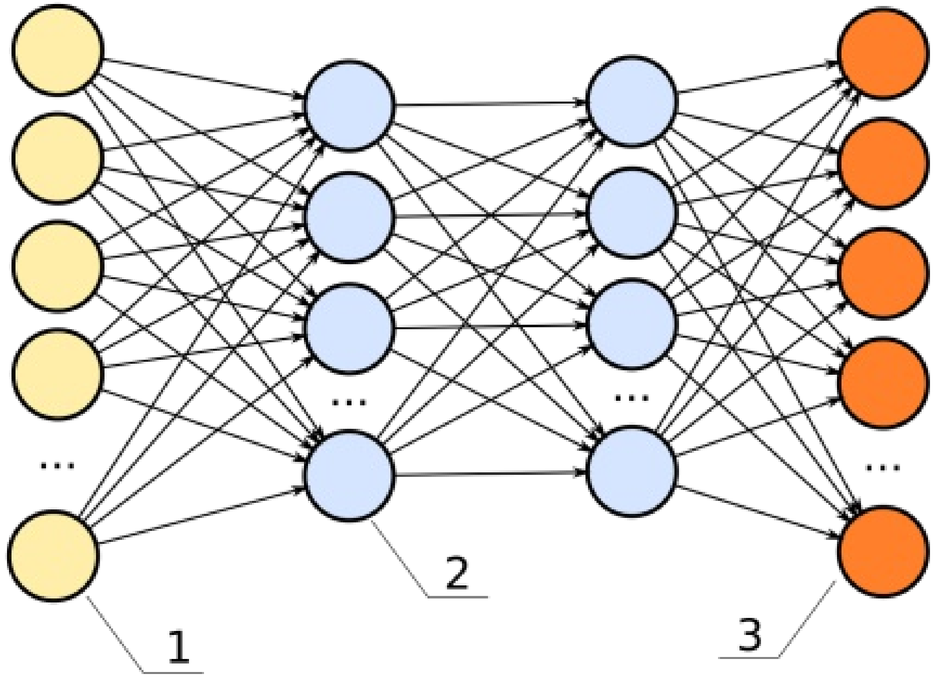
y
x
25
20
15
10
5
0
-5
0
5
10
15
20


E
E
K
L
H
B
C
F
D

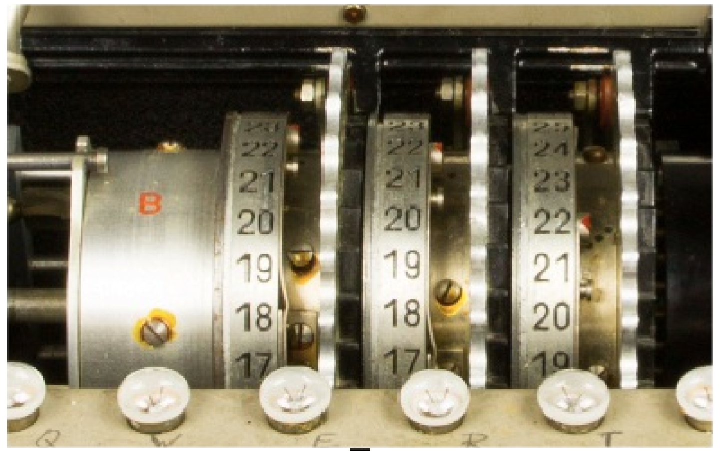

1
2
3

Bayesian inference in a nutshell

A method for figuring out unobservable quantities given known facts that uses probability to describe the uncertainty over what the values of the unknown quantities could be.

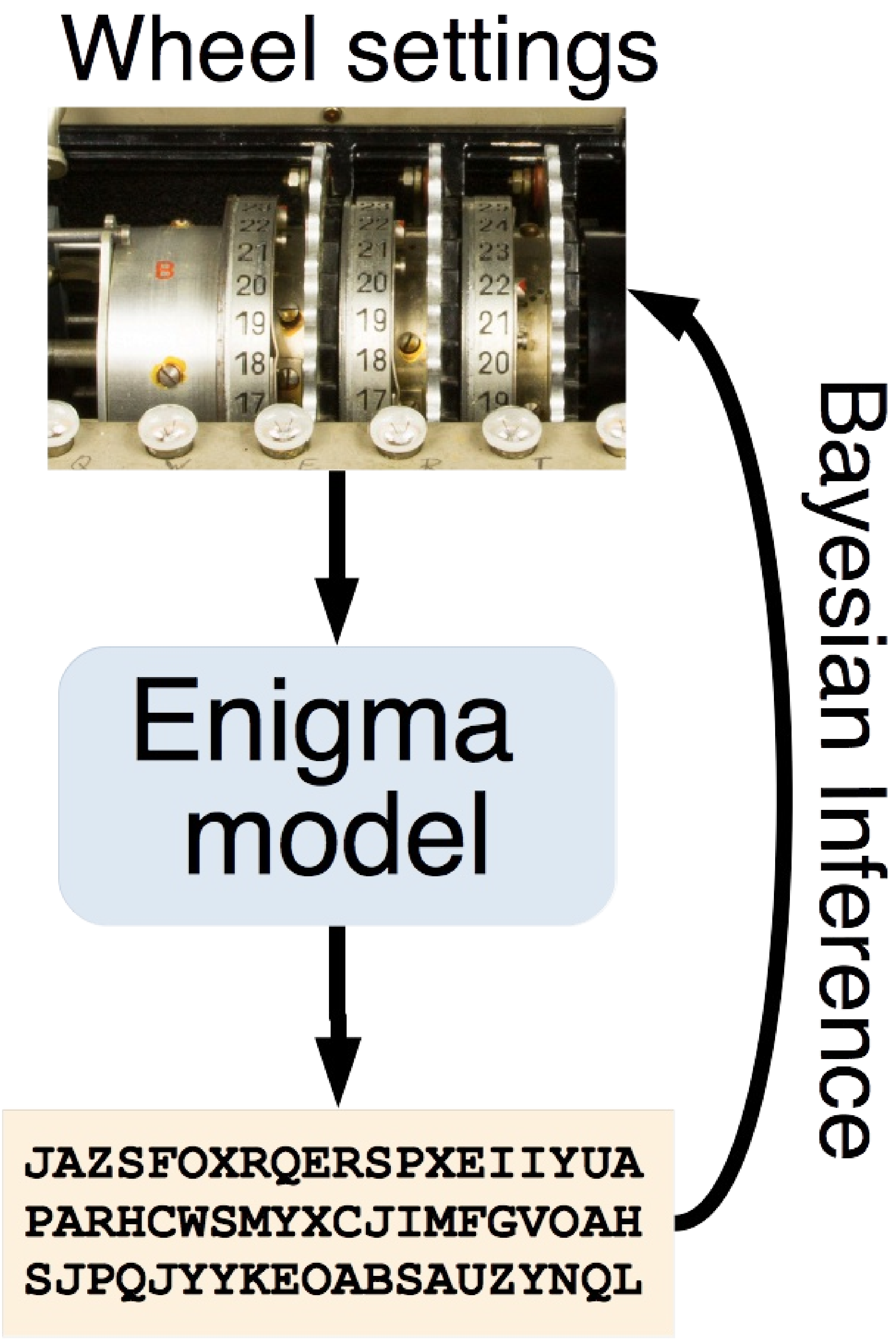

Wheels

Wheel settings



Enigma model

JAZSFOXRQERSPXEIIYUA
PARHCWSMYXCJIMFGVOAH
SJPQJYYKEOABSAUZYNQL


Wheel settings
Enigma model
JAZSFOXRQERSPXEIIYUA
PARHCWSMYXCJIMFGVOAH
SJPQJYYKEOABSAUZYNQL
Bayesian Inference

Bayesian data analysis

- The use of Bayesian inference to learn from data.
- Can be used for hypothesis testing, linear regression, etc.
- Is flexible and allows you to construct problem-specific models.

Course overview

- **Chapter 1**: A small Bayesian analysis.
- **Chapter 2**: How Bayesian inference works.
- **Chapter 3**: Why you would want to use Bayesian data analysis?
- **Chapter 4**: Bayesian inference with Bayes theorem.
- **Chapter 5**: Wrapping up + a practical tool for Bayesian Data Analysis in R.

FUNDAMENTALS OF BAYESIAN DATA ANALYSIS IN R

Bayesian Data analysis: a tool to make sense of your data.

FUNDAMENTALS OF BAYESIAN DATA ANALYSIS IN R

A little bit of background

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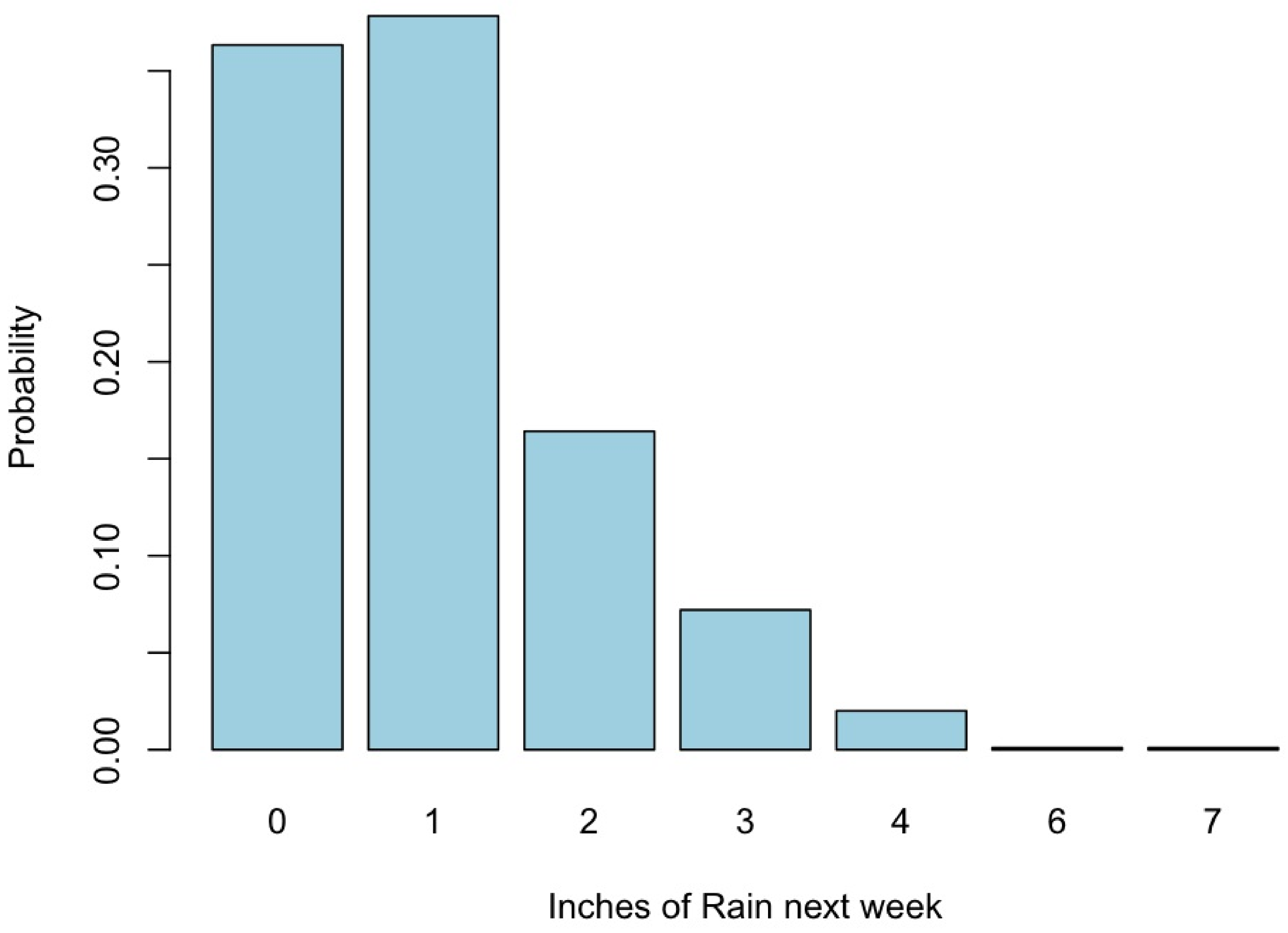
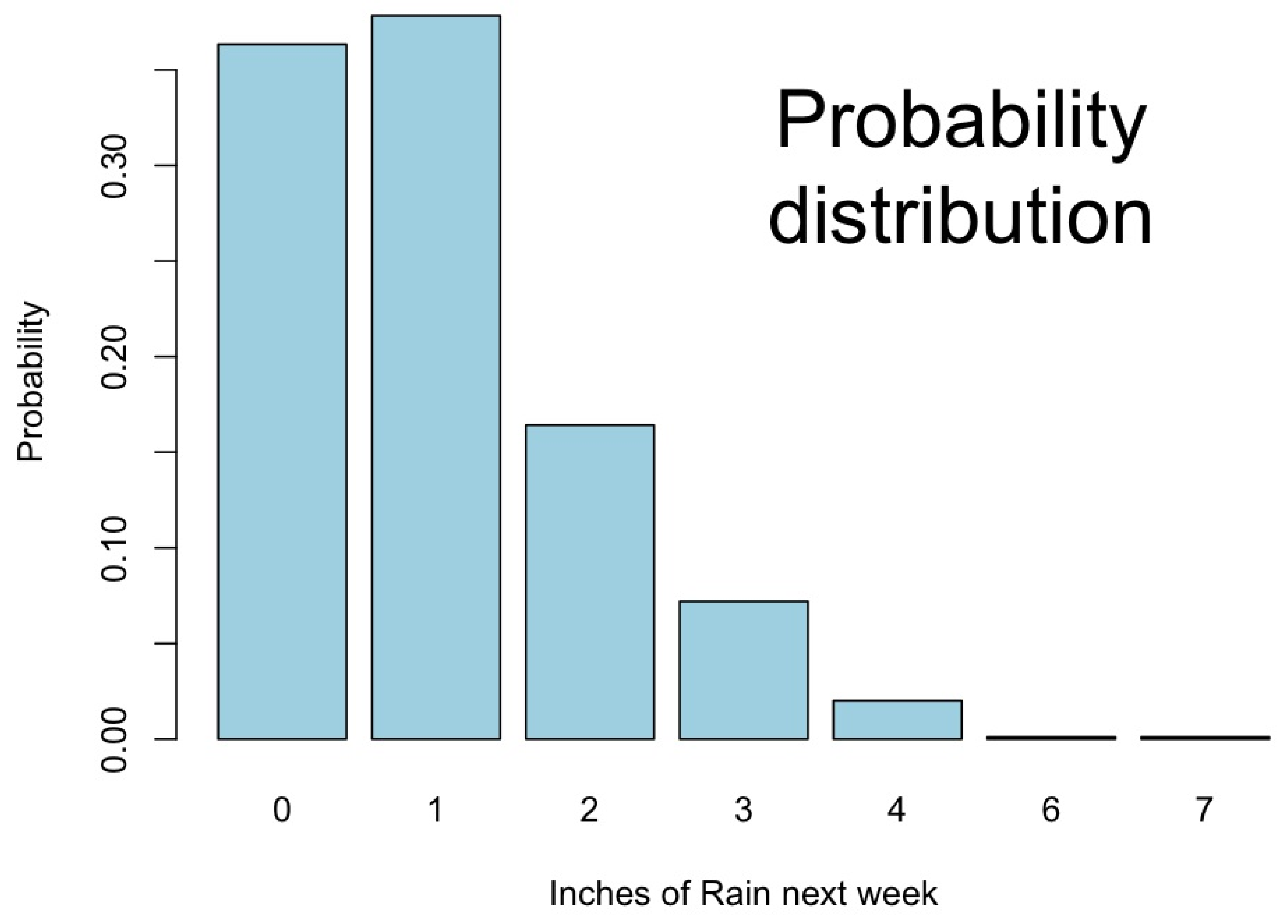
Data Scientist

Thomas Bayes (1702-1761)

Probability

- A number between 0 and 1.
- A statement about certainty / uncertainty.
- 1 is complete certainty something is the case.
- 0 is complete certainty something is *not* the case.
- Not only about yes/no events.


Probability
0.00
0.10
0.20
0.30
0
1
2
3
4
6
7
Inches of Rain next week


Probability distribution
Probability
0.00
0.10
0.20
0.30
0
1
2
3
4
6
7
Inches of Rain next week

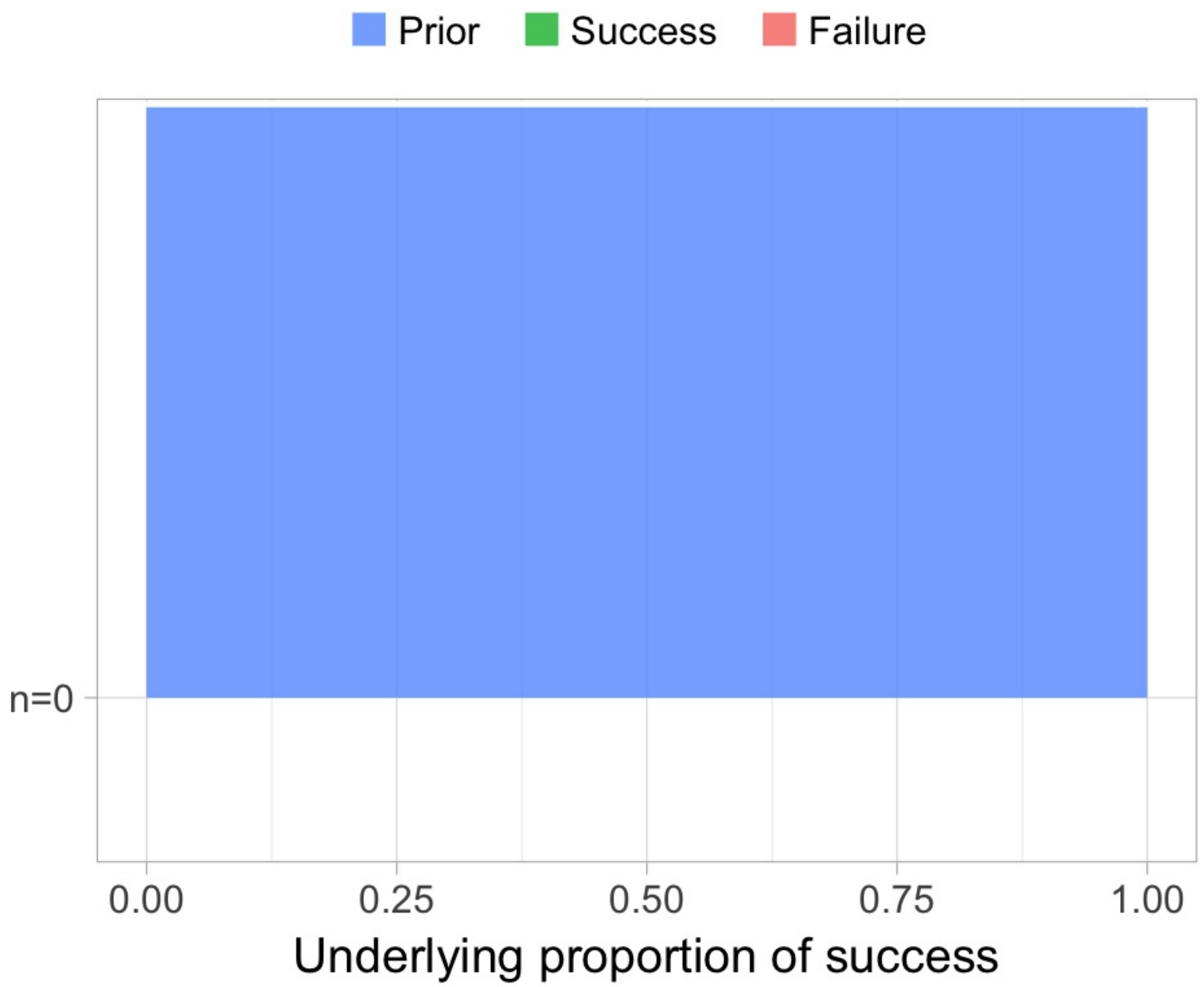
The role of probability distributions in Bayesian data analysis is to represent uncertainty, and the role of Bayesian inference is to update probability distributions to reflect what has been learned from data.

A Bayesian model for the proportion of success

- prop_model(data)
- The data is a vector of successes and failures represented by 1s and 0s.
- There is an unknown underlying *proportion of success*.
- If data point is a success is only affected by this proportion.
- Prior to seeing any data, any underlying proportion of success is equally likely.
- The result is a probability distribution that represents what the model knows about the underlying proportion of success.

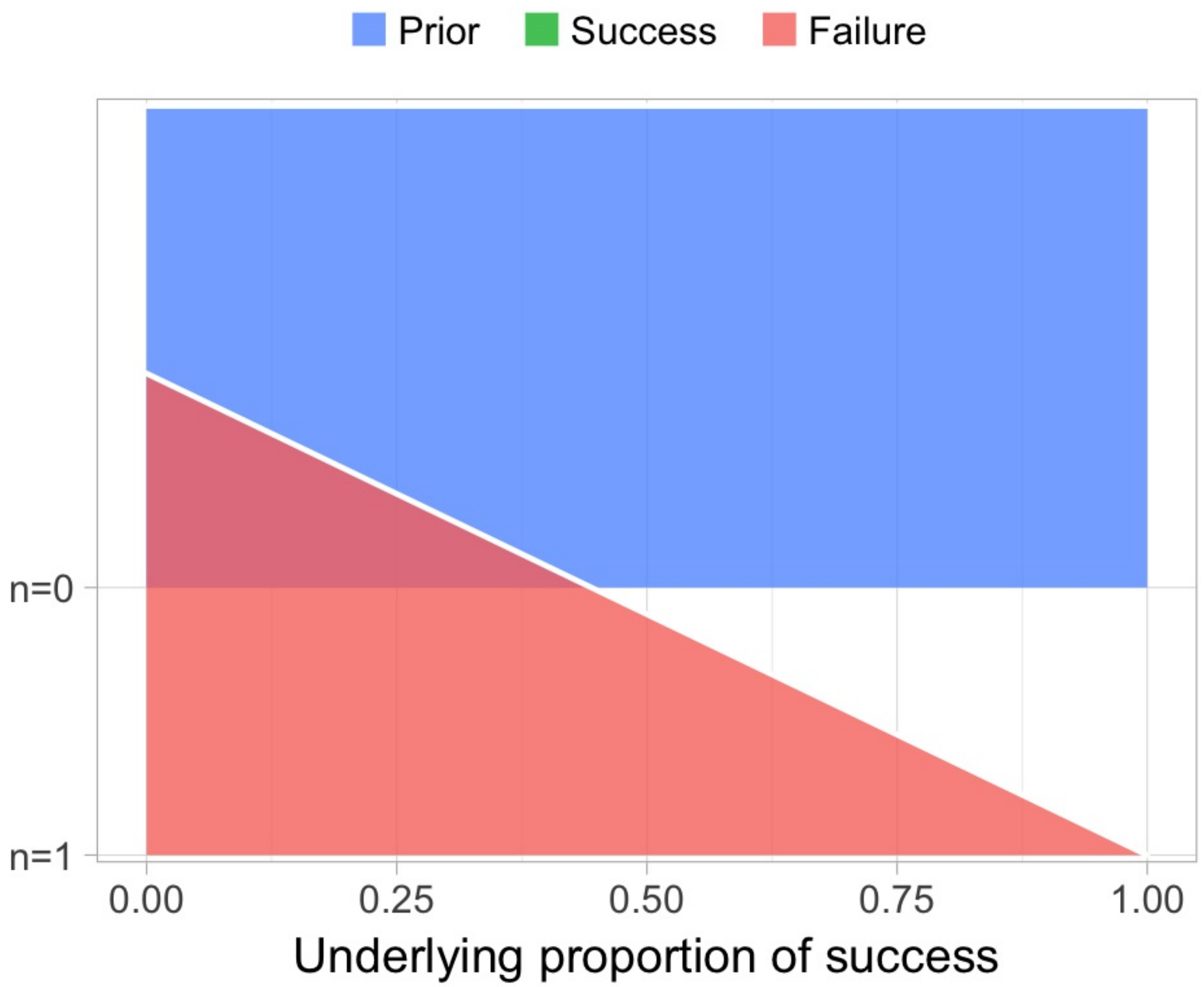
Trying out prop_model

```
data <- c()
prop_model(data)
```



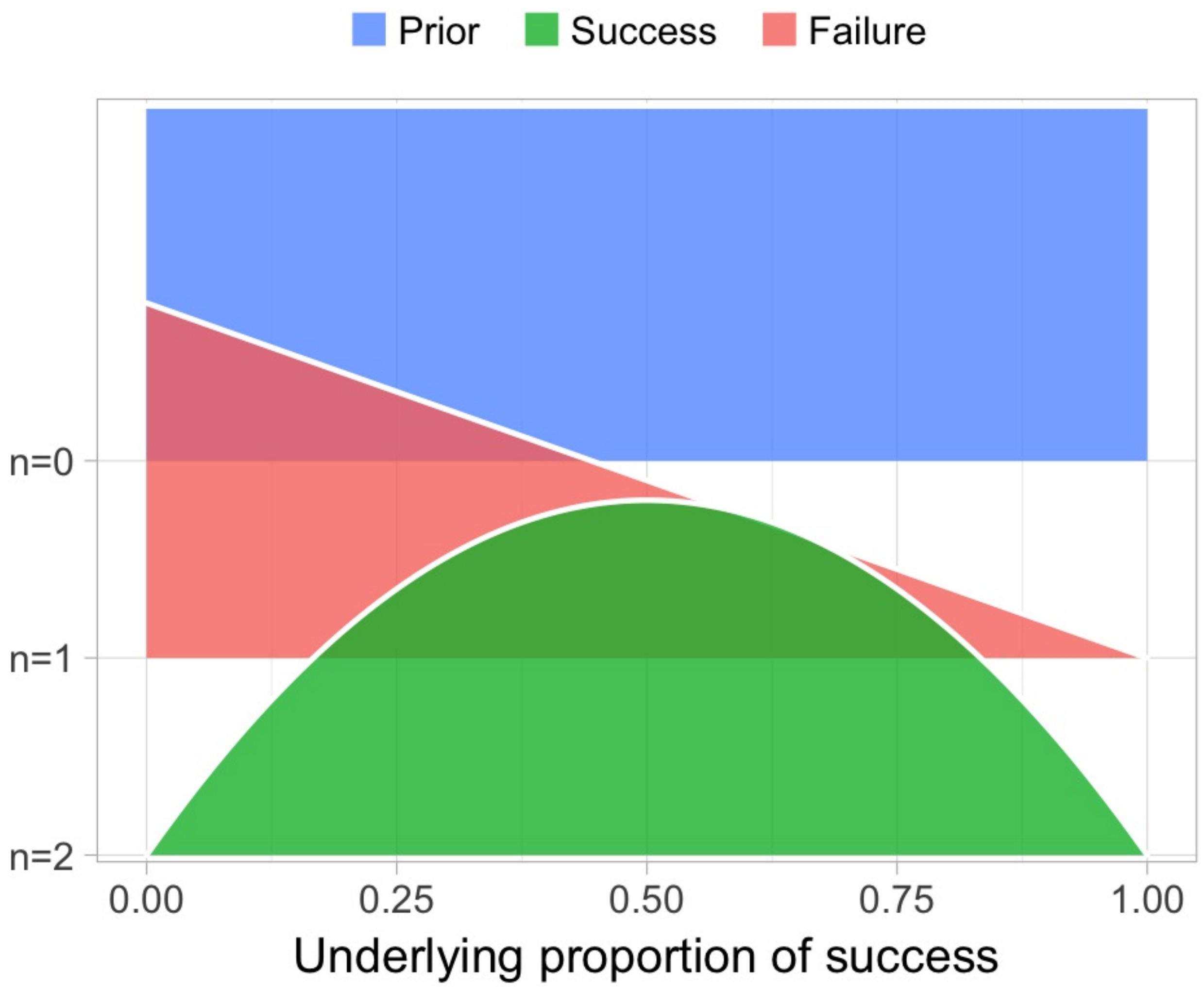
Trying out prop_model

```
data <- c(0)
prop_model(data)
```



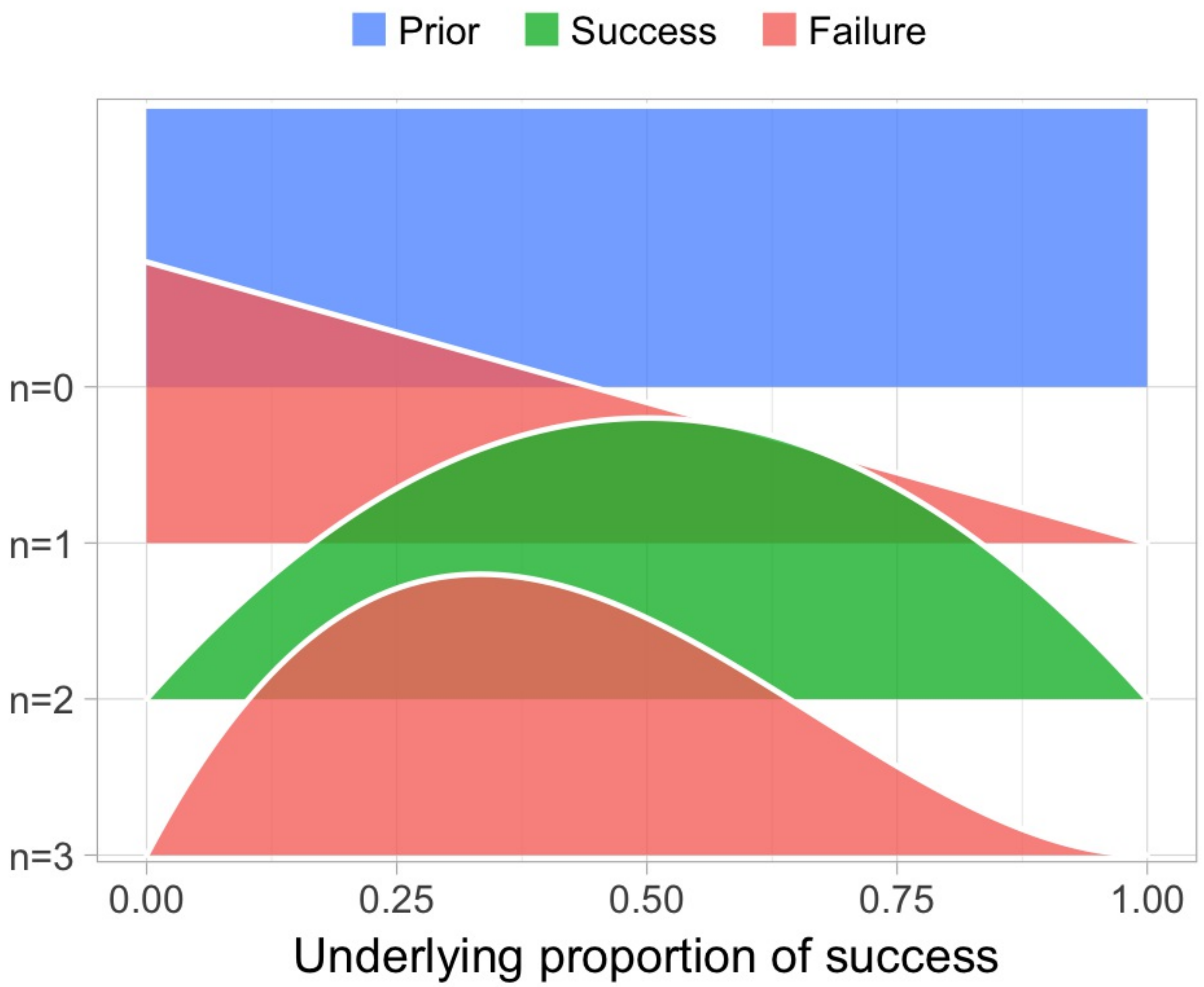
Trying out prop_model

```
data <- c(0, 1)
prop_model(data)
```



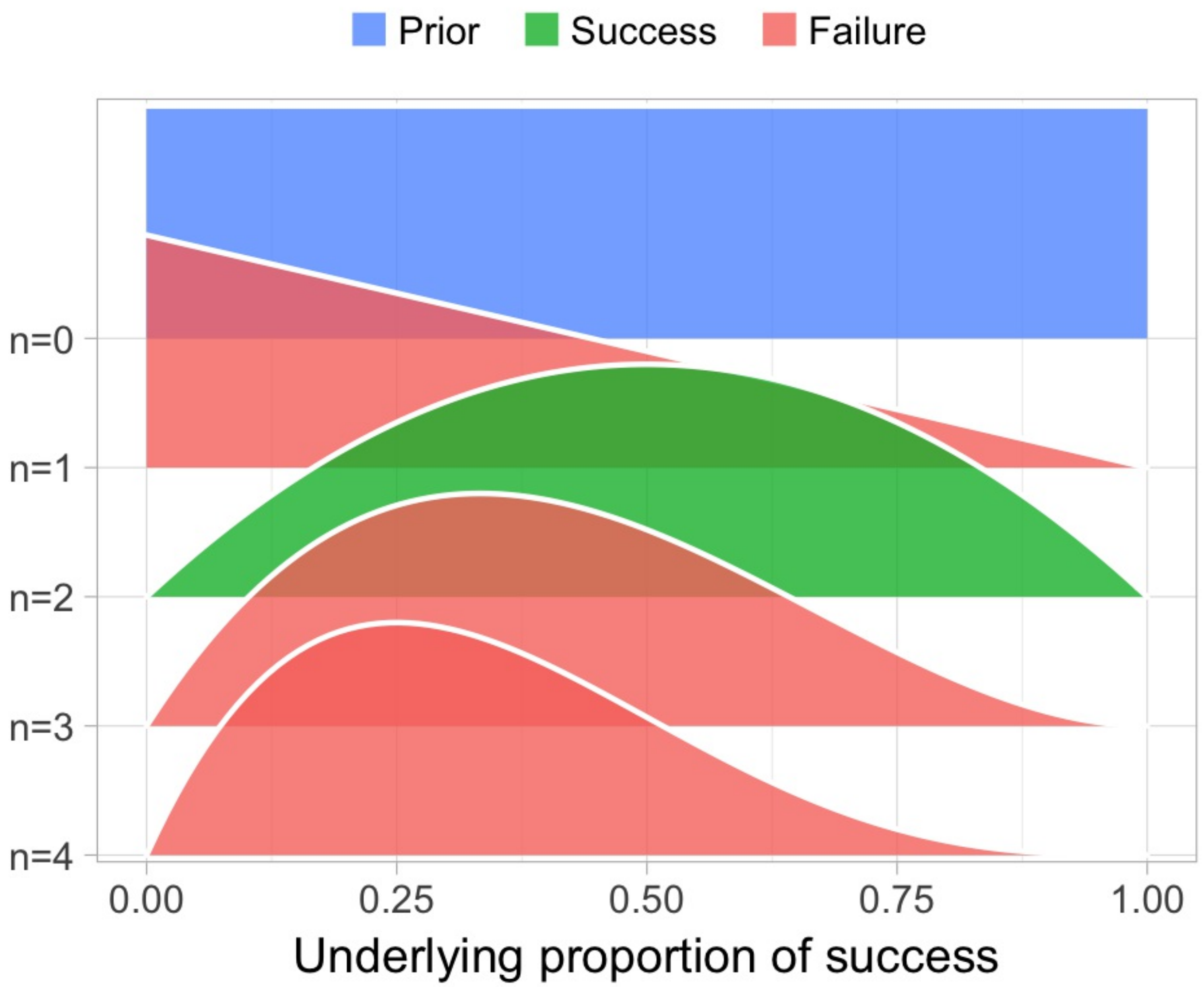
Trying out prop_model

```
data <- c(0, 1, 0)
prop_model(data)
```



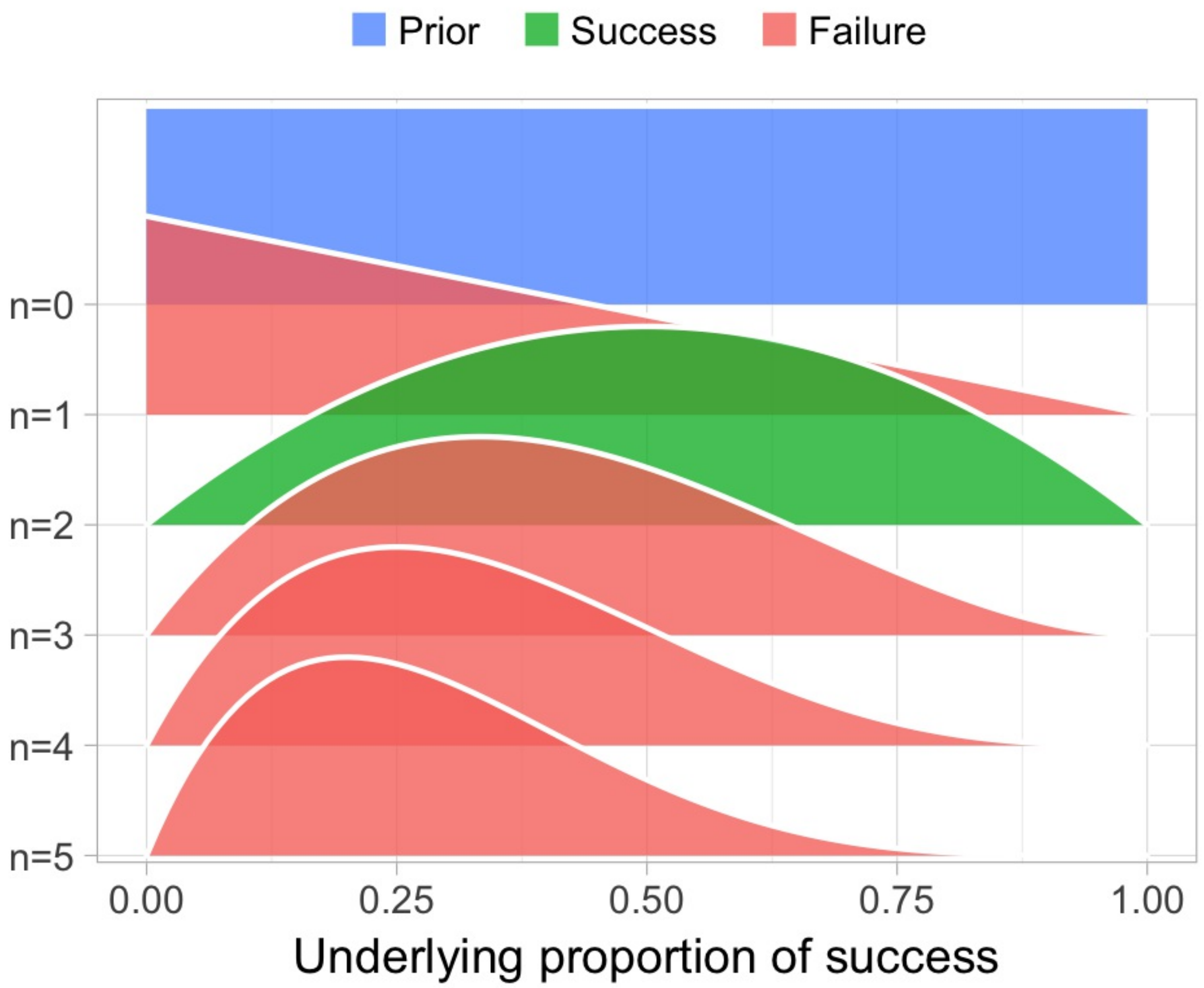
Trying out prop_model

```
data <- c(0, 1, 0, 0)
prop_model(data)
```



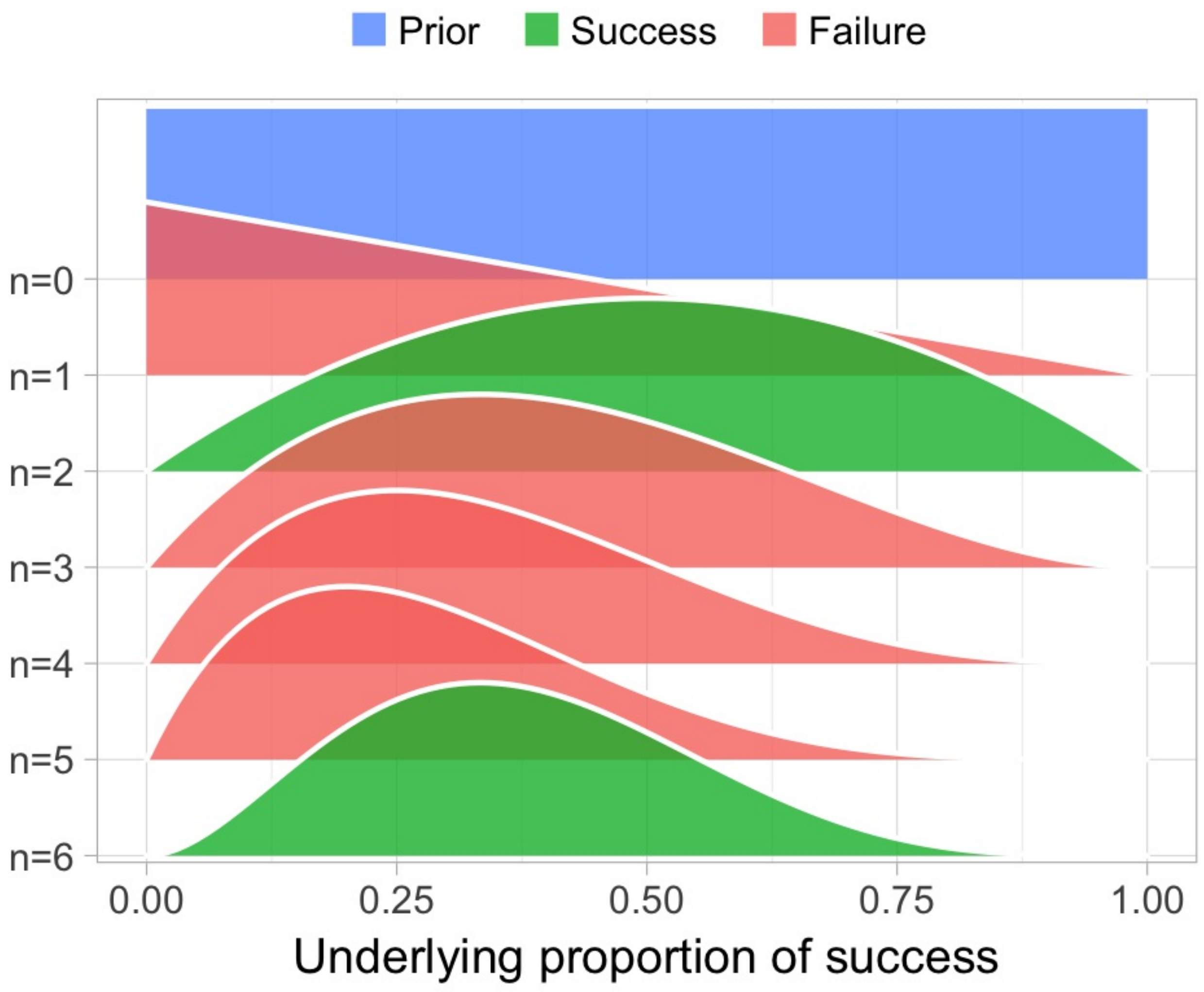
Trying out prop_model

```
data <- c(0, 1, 0, 0, 0)
prop_model(data)
```



Trying out prop_model

```
data <- c(0, 1, 0, 0, 0, 1)
prop_model(data)
```



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Now, you try out prop_model!

FUNDAMENTALS OF BAYESIAN DATA ANALYSIS IN R

You just did some Bayesian data analysis!

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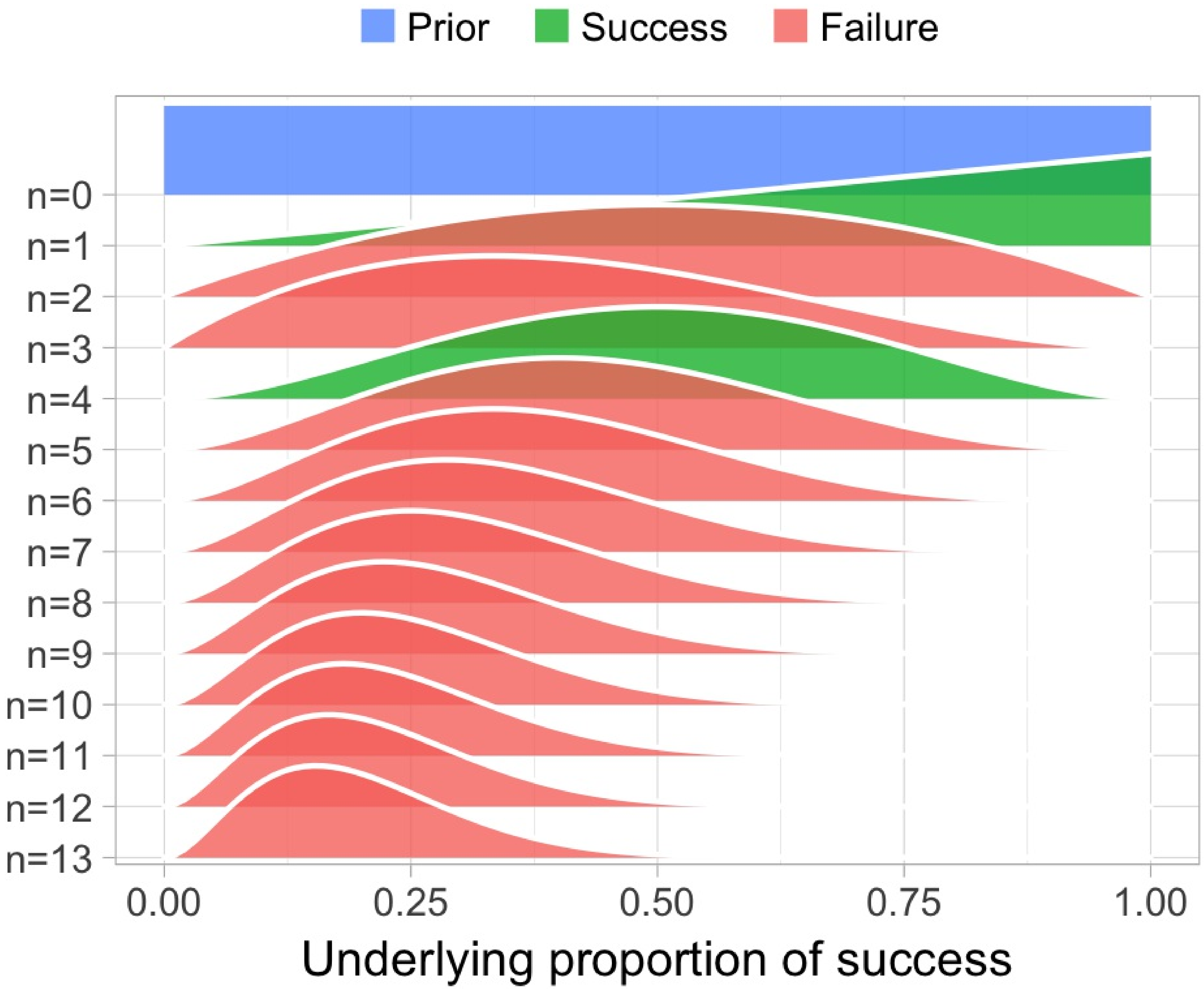
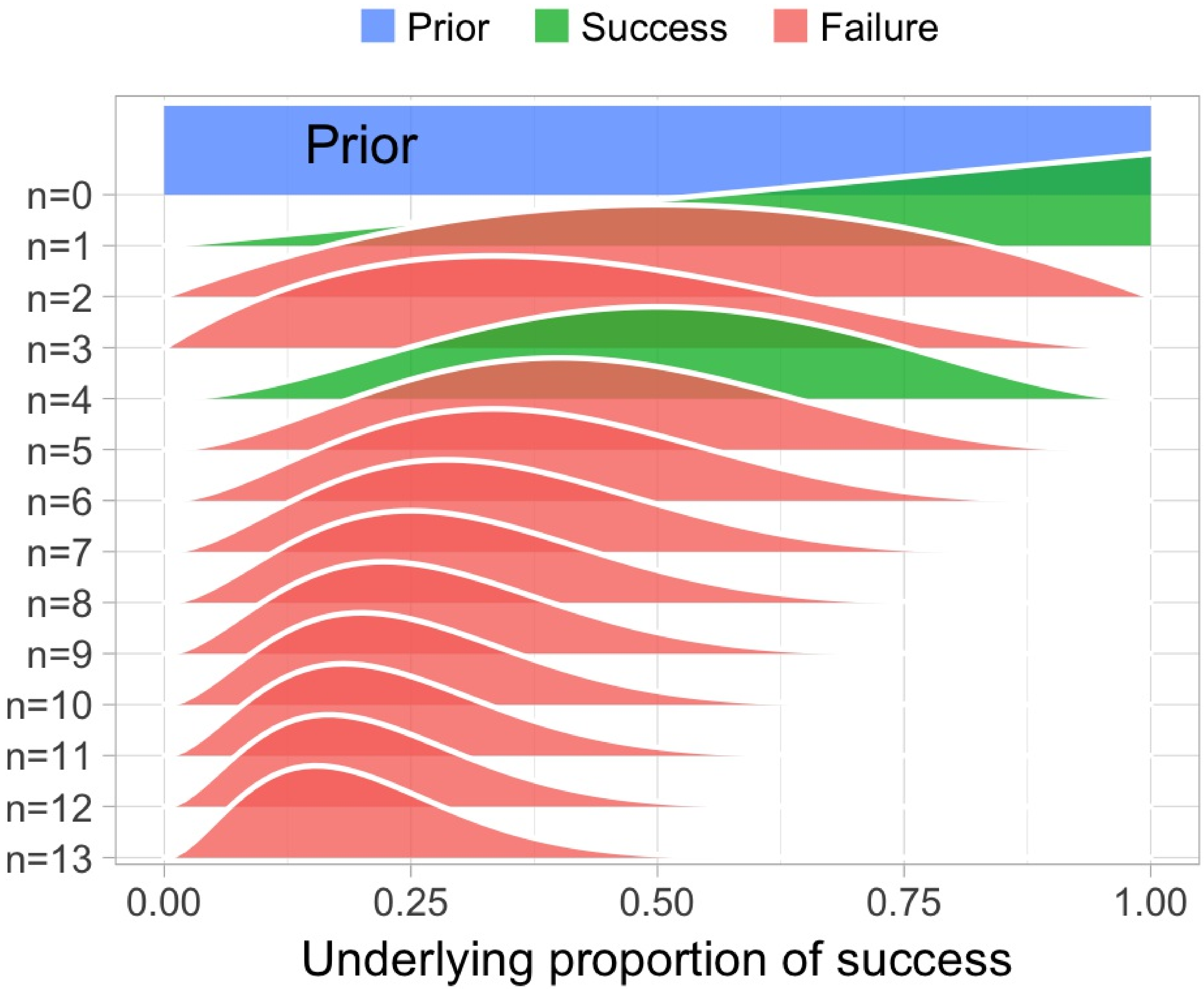
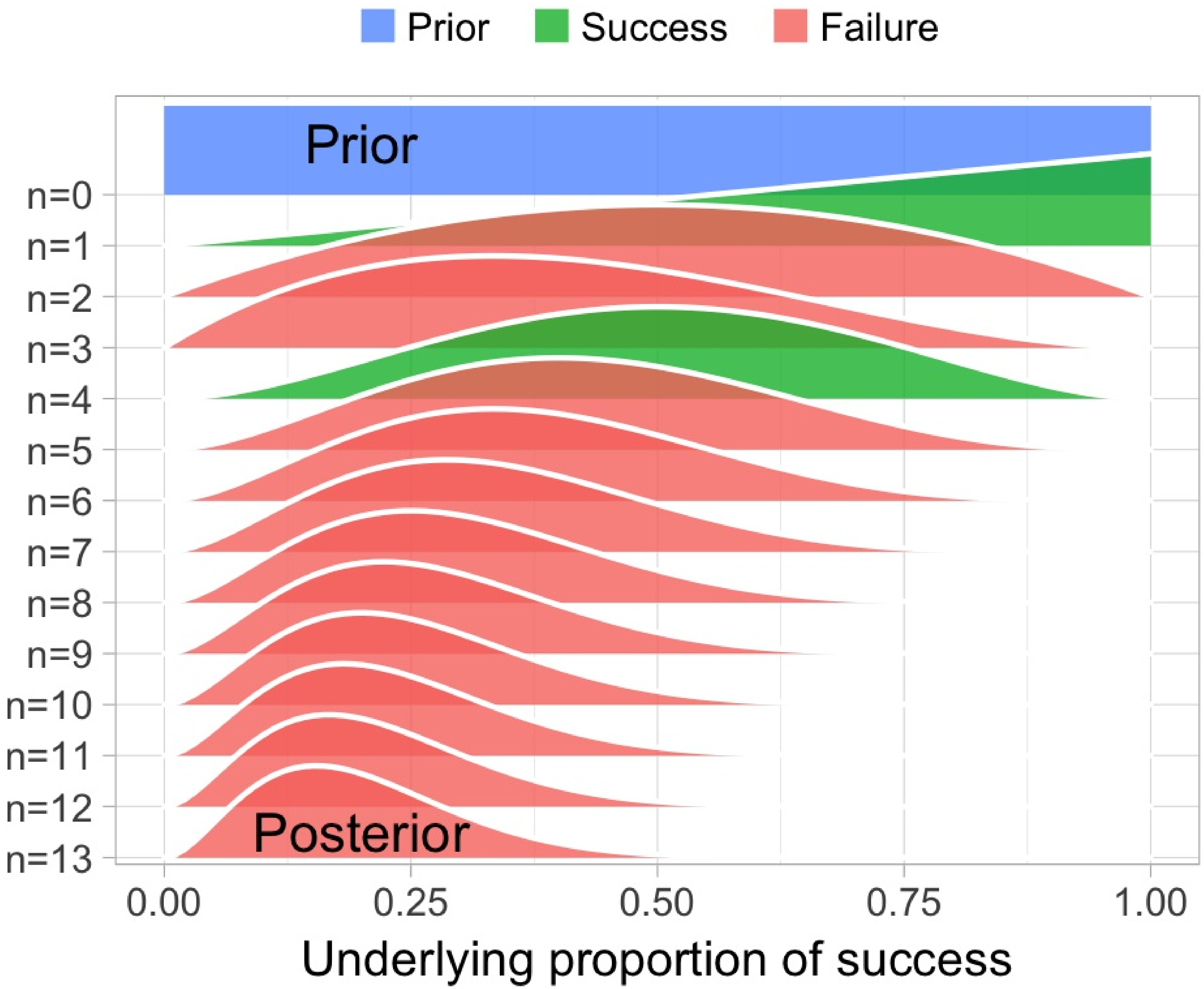
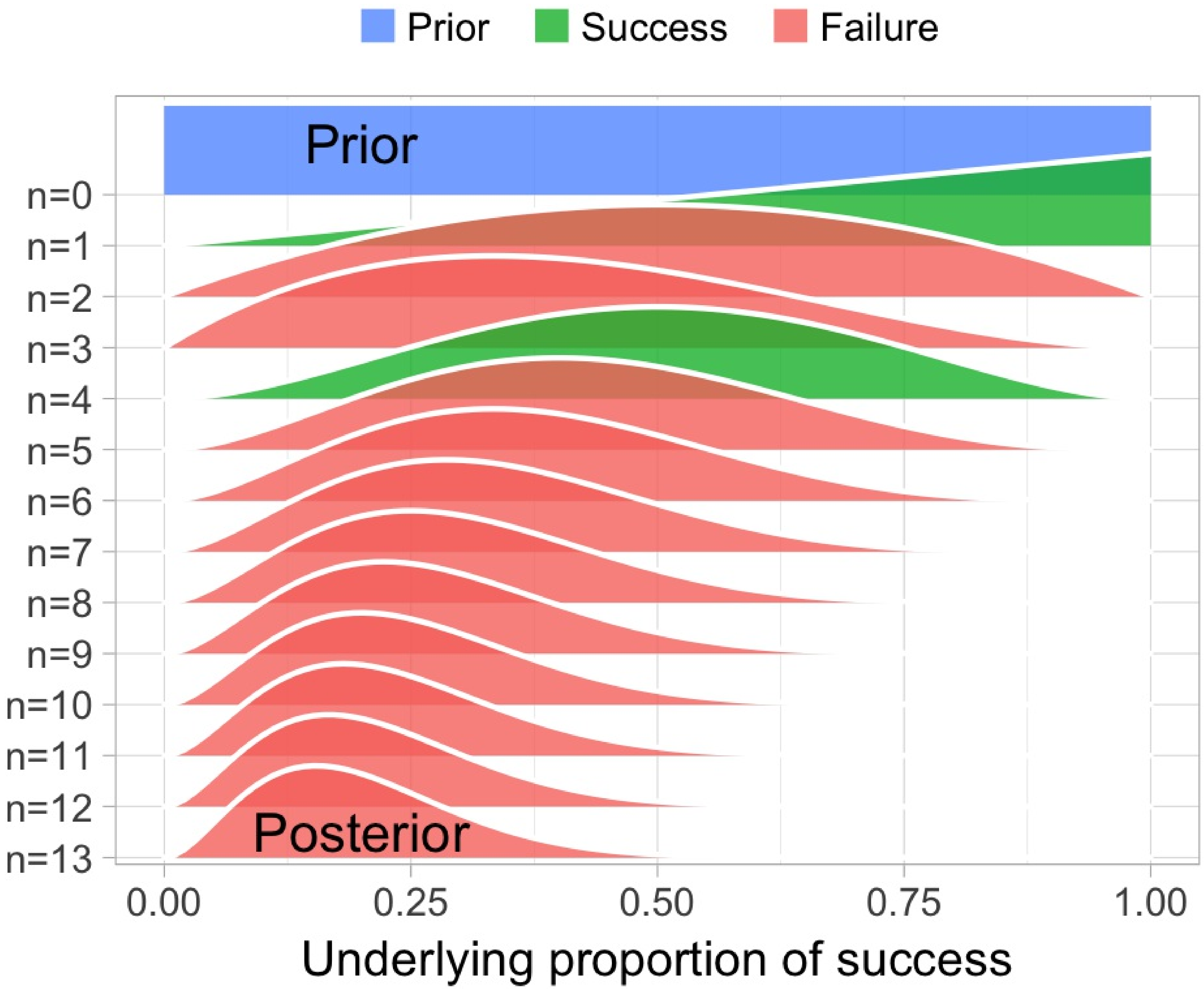

Prior
Success
Failure
n=0
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9
n=10
n=11
n=12
n=13
0.00
0.25
0.50
0.75
1.00
Underlying proportion of success


Prior
Success
Failure
Prior
n=0
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9
n=10
n=11
n=12
n=13
0.00
0.25
0.50
0.75
1.00
Underlying proportion of success

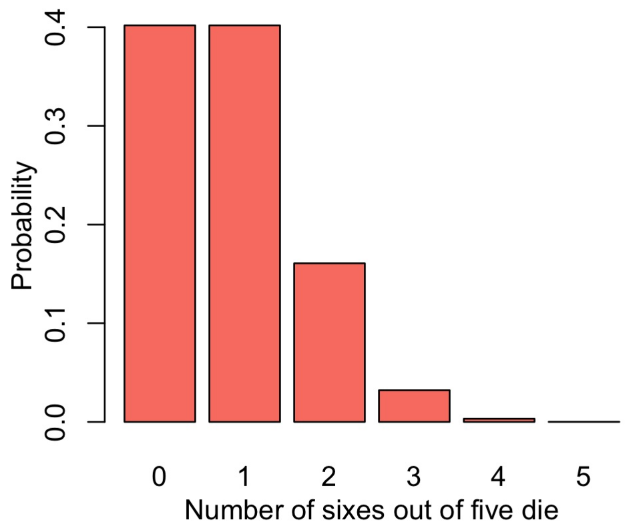

Prior
Success
Failure
Prior
Posterior
n=0
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9
n=10
n=11
n=12
n=13
0.00
0.25
0.50
0.75
1.00
Underlying proportion of success

Priors & Posteriors

- A **prior** is a probability distribution that represents what the model knows before seeing the data.
- A **posterior** is a probability distribution that represents what the model knows after having seen the data.


Prior
Success
Failure
Prior
Posterior
n=0
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9
n=10
n=11
n=12
n=13
0.00
0.25
0.50
0.75
1.00
Underlying proportion of success

The probability distribution over #? when rolling 5 dice



$$p(x) = \binom{5}{x}(\frac{1}{6})^x(1-(\frac{1}{6}))^{5-x}$$

```
> number_of_sixes
   [1] 1 1 0 0 1 0 1 2 0 1 0 0 1 0 0 0 0 0 1 1 1 0 1 1 0 0 2 0
  [29] 0 1 0 0 1 0 0 1 0 1 2 0 1 0 0 0 1 2 1 2 0 0 1 1 3 3 0 0
  [57] 1 1 1 1 1 0 0 1 2 0 1 3 1 1 1 0 1 0 1 2 0 1 1 0 1 1 1 0
  [85] 2 1 0 4 0 1 2 1 1 1 2 0 1 0 1 1 0 0 2 0 0 0 0 0 1 1 0 1
 [113] 0 0 0 0 2 0 0 0 0 0 1 1 0 0 2 1 1 1 0 2 1 1 1 0 0 1 1 1
  ...
```

```
> mean(number_of_sixes)
[1] 0.83
```

```
posterior <- prop_model(data)
print(posterior)

[1] 0.23 0.36 0.20 0.21 0.12
[6] 0.10 0.03 0.16 0.09 0.14
[11] 0.23 0.05 0.15 0.26 0.22
...
```

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Finish off the Zombie drug analysis!

FUNDAMENTALS OF BAYESIAN DATA ANALYSIS IN R

Wrapping up the zombie analysis

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Data Scientist

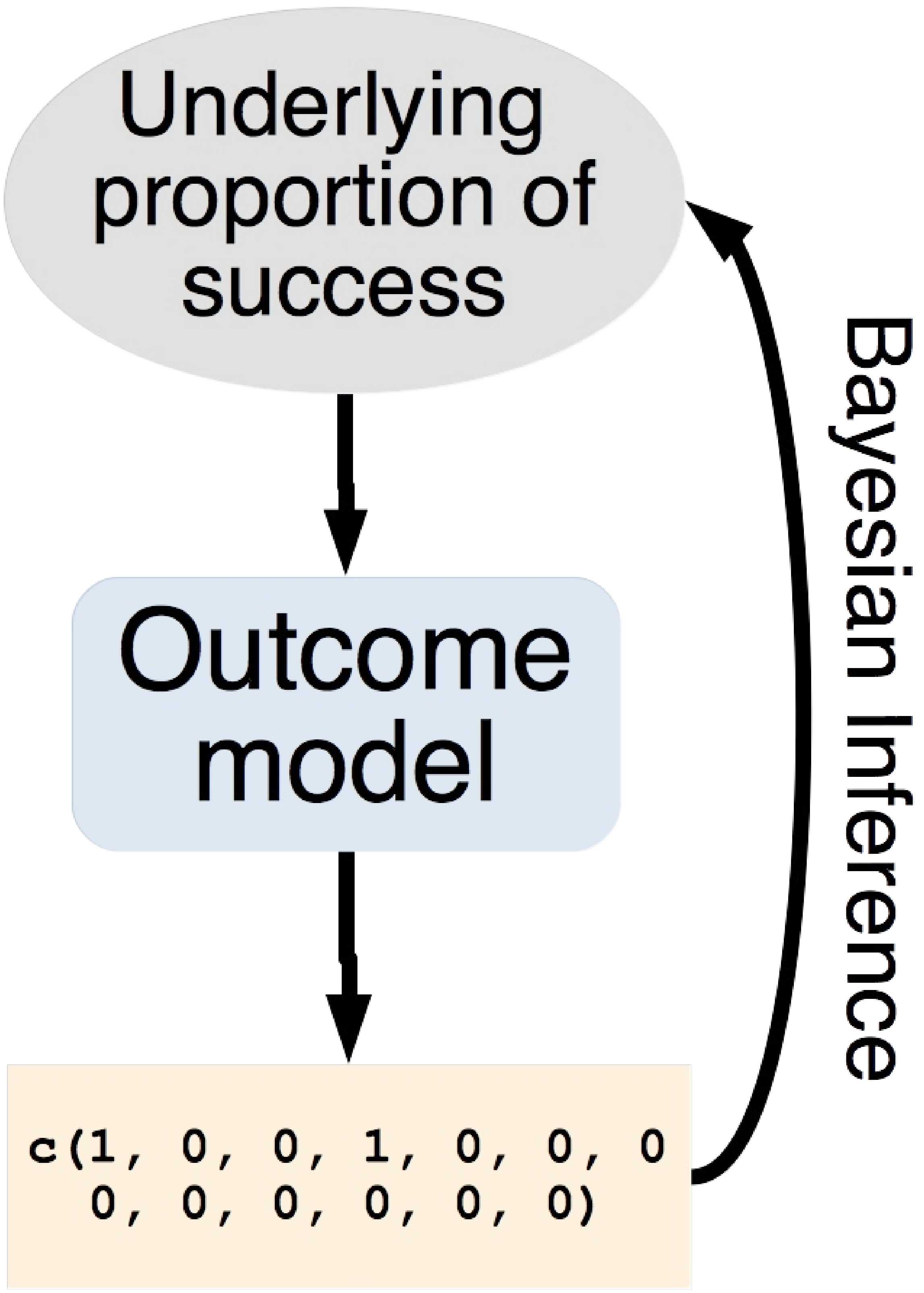
The result of the zombie analysis

```
data = c(1, 0, 0, 1, 0, 0, 0,
         0, 0, 0, 0, 0, 0)
posterior <- prop_model(data)

median(posterior)
## 0.19
quantile(posterior, c(0.05, 0.95))
##  5%    95%
## 0.06  0.39
sum(posterior > 0.07) / length(posterior)
## 0.93
```

The result in a journal

Given the data of two cured and 11 relapsed zombies, and using the Bayesian model described before, there is a 90% probability that our drug cures between 6% and 39% of treated zombies. Further, there is 93% probability that our drug cures zombies at a higher rate than the current state of the art drug.


Underlying proportion of success
Outcome model
c(1, 0, 0, 1, 0, 0, 0
0, 0, 0, 0, 0, 0)
Bayesian Inference

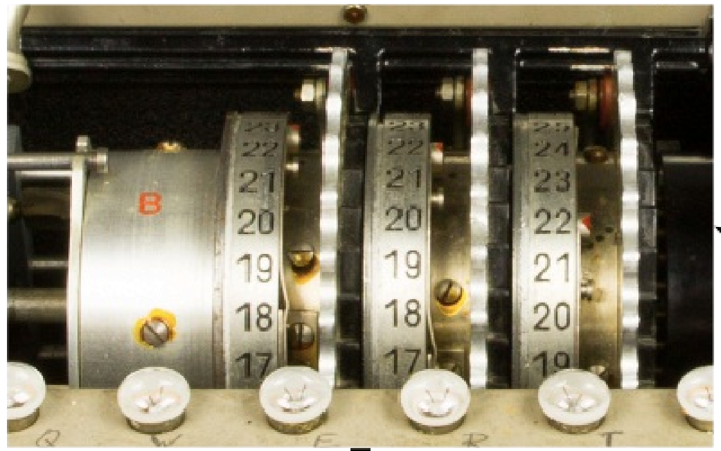
Underlying proportion of success

Outcome model

```
c(1, 0, 0, 1, 0, 0, 0
 0, 0, 0, 0, 0, 0)
```

Bayesian Inference

Wheel settings



Enigma model

```
JAZSFOXRQERSPXEIIYUA
PARHCWSMYXCJIMFGVOAH
SJPQJYYKEOABSAUZYNQL
```

Bayesian Inference

FUNDAMENTALS OF BAYESIAN DATA ANALYSIS IN R

Next up: How does Bayes work?