BAYESIAN REGRESSION MODELING WITH RSTANARM

Using the R^2 Statistic

Jake Thompson
Psychometrician, ATLAS, University of Kansas

What is the R^2?

- Coefficient of determination

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}$$

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$$R^2 = 1 - \frac{\sum_i \left(\overset{\text{Observed value}}{\boxed{y_i}} - \overset{\text{Predicted value}}{\boxed{\hat{y}_i}}\right)^2}{\sum_i (y_i - \bar{y})^2}$$

What is the R^2?

- Coefficient of determination

$$R^2 = 1 - \frac{\sum_i (\overset{\text{Observed value}}{\boxed{y_i}} - \overset{\text{Predicted value}}{\boxed{\hat{y}_i}})^2}{\sum_i (\underset{\text{Observed value}}{\boxed{y_i}} - \underset{\text{Mean value}}{\boxed{\bar{y}}})^2}$$

Calculating the R^2

```r
lm_model <- lm(kid_score ~ mom_iq, data = kidiq)

lm_summary <- summary(lm_model)
lm_summary$r.squared
#> [1] 0.2009512
```

```r
ss_res <- var(residuals(lm_model))
ss_total <- var(residuals(lm_model)) + var(fitted(lm_model))
1 - (ss_res / ss_total)
#> [1] 0.2009512
```

The R^2 of a Bayesian Model

```
stan_model <- stan_glm(kid_score ~ mom_iq, data = kidiq)

ss_res <- var(residuals(stan_model))
ss_total <- var(fitted(stan_model)) + var(residuals(stan_model))
1 - (ss_res / ss_total)
#> [1] 0.2004996
```

```
lm_summary$r.squared
#> [1] 0.2009512
```

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Let's practice!

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Posterior Predictive Model Checks

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Using posterior distributions

```
stan_model <- stan_glm(kid_score ~ mom_iq, data = kidiq)
```

```
spread_draws(stan_model, `(Intercept)`, mom_iq) %>%
  select(-.draw)
#> # A tibble: 4,000 x 4
#>    .chain .iteration `(Intercept)` mom_iq
#>     <int>      <int>         <dbl>  <dbl>
#>  1      1          1          19.9  0.654
#>  2      1          2          20.7  0.643
#>  3      1          3          27.2  0.604
#>  4      1          4          24.9  0.613
#>  5      1          5          26.4  0.610
#>  6      1          6          25.2  0.619
#>  7      1          7          17.8  0.702
#>  8      1          8          35.5  0.502
#>  9      1          9          32.9  0.540
#> 10      1         10          27.3  0.599
#> # ... with 3,990 more rows
```

Posterior predictions

```
predictions <- posterior_linpred(stan_model)

predictions[1:10, 1:5]
#>
#> iterations         1        2        3        4        5
#>       [1,] 100.18694 79.04791 96.40964 85.76310 81.30045
#>       [2,] 100.24843 82.00786 96.98905 87.80231 83.95155
#>       [3,] 100.85608 81.13109 97.33146 87.39709 83.23295
#>       [4,] 102.31392 80.81881 98.47300 87.64712 83.10930
#>       [5,]  97.25617 81.18278 94.38404 86.28879 82.89553
#>       [6,] 100.86263 79.89830 97.11655 86.55800 82.13223
#>       [7,]  99.36166 81.10329 96.09910 86.90339 83.04887
#>       [8,] 101.13487 80.97878 97.53321 87.38173 83.12658
#>       [9,]  98.72686 79.97596 95.37629 85.93252 81.97403
#>      [10,] 100.22835 81.04603 96.80069 87.13964 83.09007
```

Comparing score distributions

```
predictions <- posterior_linpred(stan_model)

# First replication
iter1 <- predictions[1,]

# Second replication
iter2 <- predictions[2,]

# Data summaries
summary(kidiq$kid_score)
#>    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
#>    20.0    74.0    90.0    86.8   102.0   144.0

summary(iter1)
#>    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
#>   68.54   79.86   85.80   87.14   93.74  112.12

summary(iter2)
#>    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
#>   70.05   80.19   85.51   86.71   92.62  109.08
```

Comparing single scores

```
predictions <- posterior_linpred(stan_model)

kidiq$kid_score[24]
#> [1] 87
summary(predictions[, 24])
#>    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
#>   83.34   86.17   86.77   86.75   87.34   90.23
```

```
kidiq$kid_score[185]
#> [1] 111
summary(predictions[, 185])
#>    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
#>   82.81   85.65   86.25   86.24   86.83   89.69
```

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Model Fit With Posterior Predictive Model Checks

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R^2 Posterior Distribution

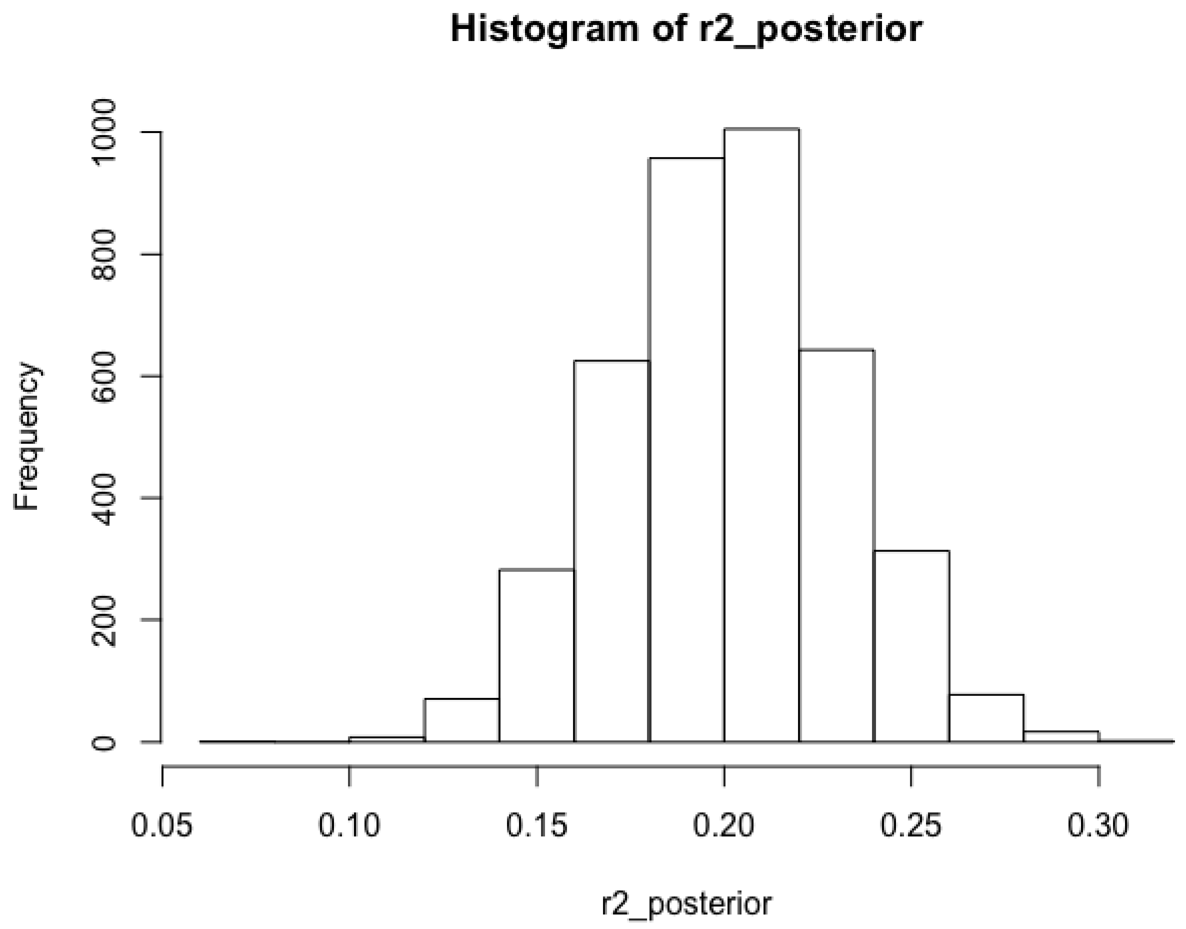
```
stan_model <- stan_glm(kid_score ~ mom_iq, data = kidiq)

r2_posterior <- bayes_R2(stan_model)
```

```
summary(r2_posterior)
#>    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
#> 0.09677 0.18034 0.20006 0.20042 0.22048 0.33414
```

```
quantile(r2_posterior, probs = c(0.025, 0.975))
#>      2.5%     97.5%
#> 0.1402846 0.2619605
```

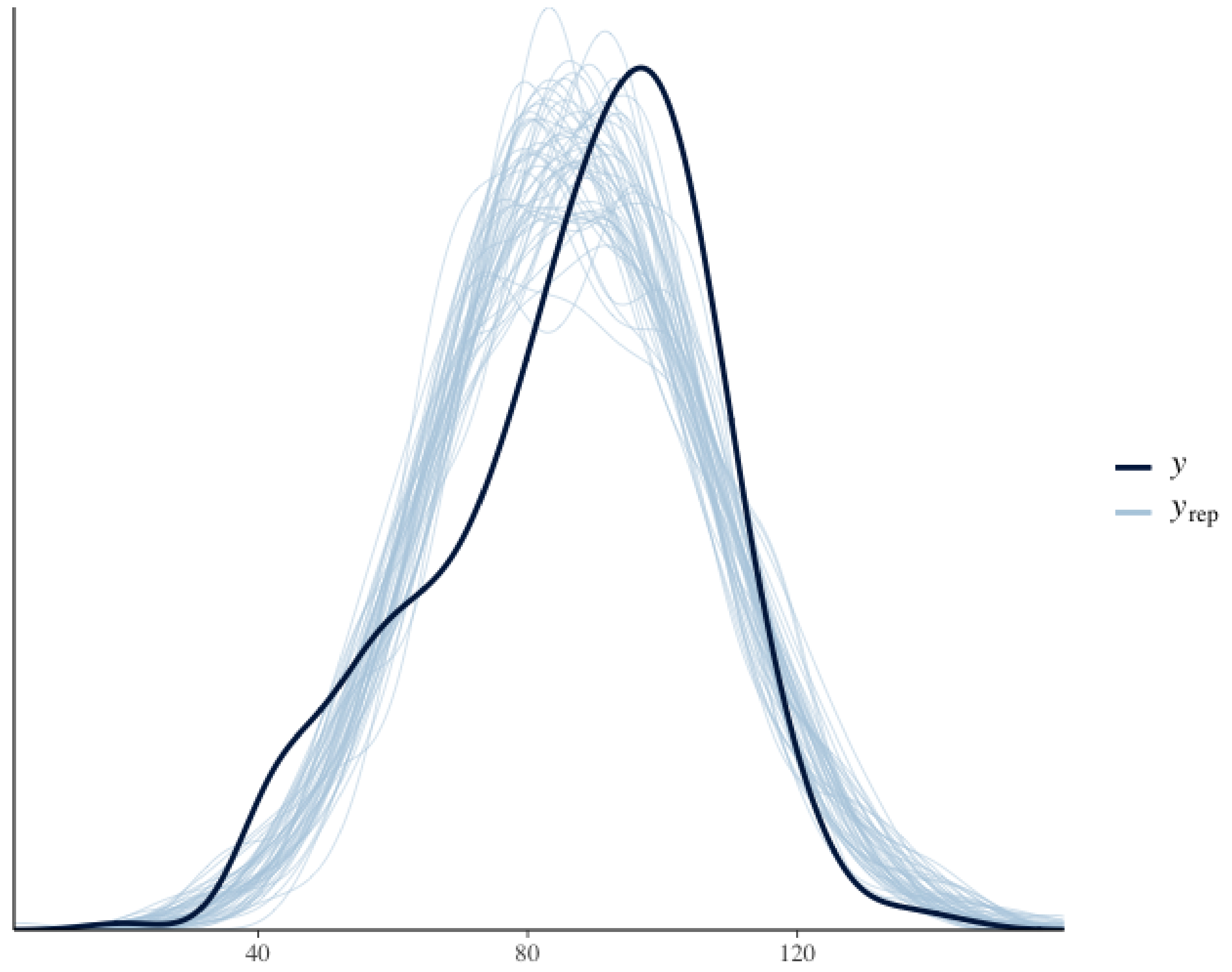
R^2 histogram

```
hist(r2_posterior)
```



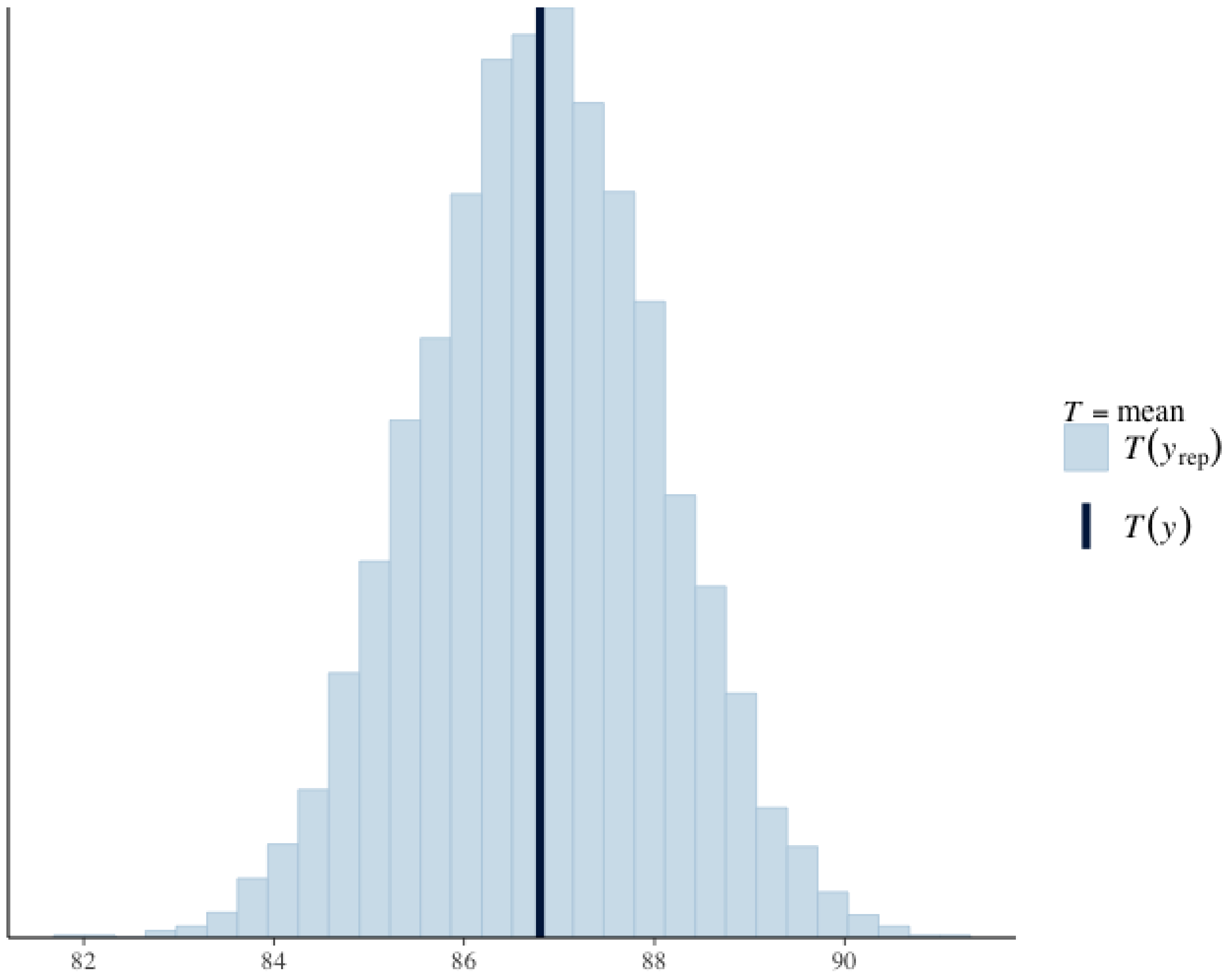
Density Overlay

```
pp_check(stan_model, "dens_overlay")
```



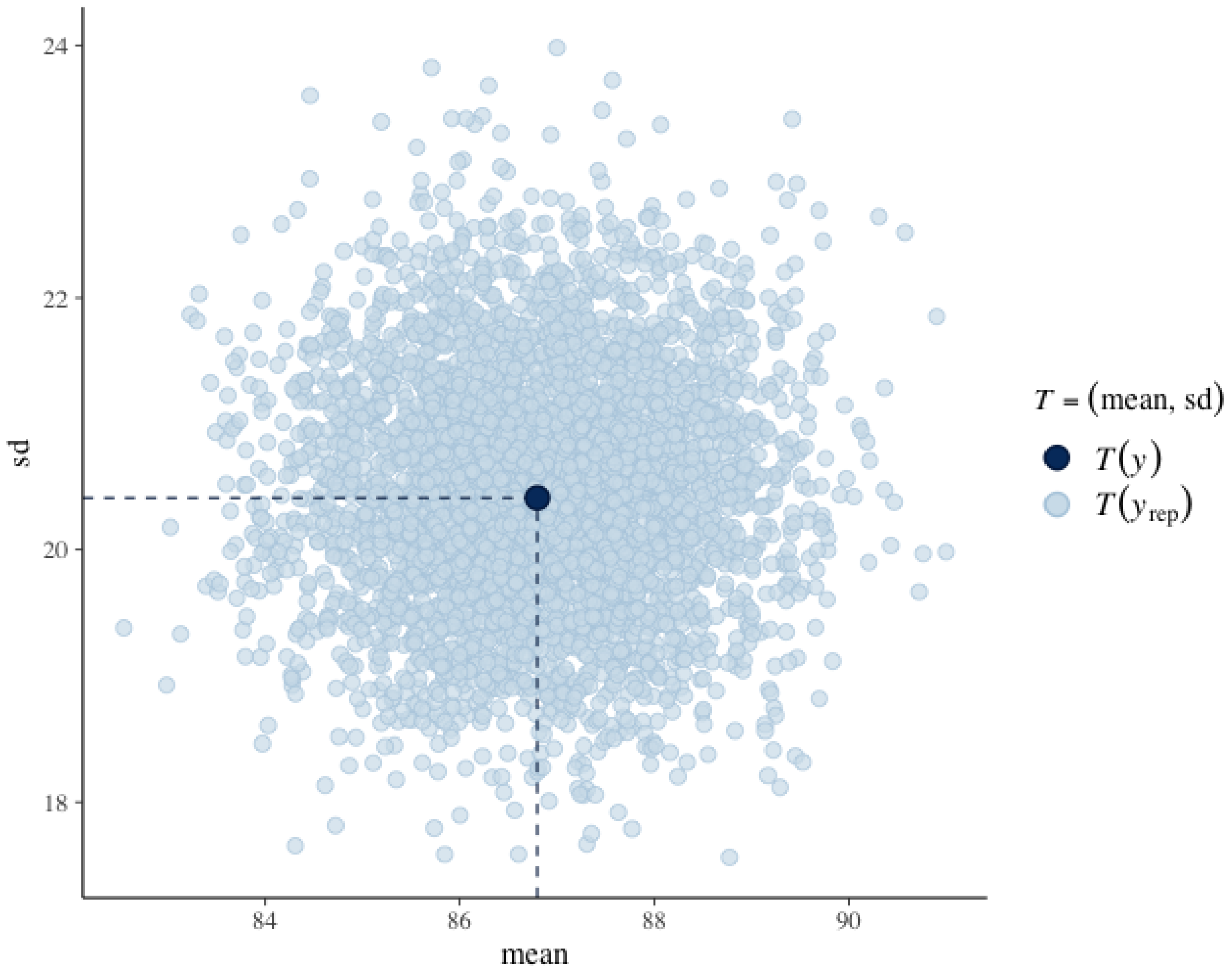
Posterior predictive tests

```
pp_check(stan_model, "stat")
```



Posterior predictive tests

```
pp_check(stan_model, "stat_2d")
```



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Bayesian Model Comparisons

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The loo package

- LOO = leave-one-out
  - Approximated cross validation
  - `?loo-package`
  - Using `loo` for model comparisons

Using loo on a single model

```
library(rstanarm)
library(loo)

stan_model <- stan_glm(kid_score ~ mom_iq, data = kidiq)

loo(stan_model)

#>
#> Computed from 4000 by 434 log-likelihood matrix
#>
#>          Estimate   SE
#> elpd_loo  -1878.5 14.5
#> p_loo         2.9  0.3
#> looic      3757.1 29.0
#> ------
#> Monte Carlo SE of elpd_loo is 0.0.
#>
#> All Pareto k estimates are good (k < 0.5).
#> See help('pareto-k-diagnostic') for details.
```

Model comparisons with loo

```
model_1pred <- stan_glm(kid_score ~ mom_iq, data = kidiq)
model_2pred <- stan_glm(kid_score ~ mom_iq * mom_hs, data = kidiq)

loo_1pred <- loo(model_1pred)
loo_2pred <- loo(model_2pred)

compare(loo_1pred, loo_2pred)
#> elpd_diff        se
#>       6.1       3.9
```

Model comparisons with loo

```
compare(loo_1pred, loo_2pred)
#> elpd_diff        se
#>       6.1       3.9
```

- Positive = prefer second model
- Negative = prefer first model
- Significant difference?
    - Absolute value of difference relative to standard error

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