



SUPERVISED LEARNING IN R: REGRESSION

Logistic regression to predict probabilities

Nina Zumel and John Mount

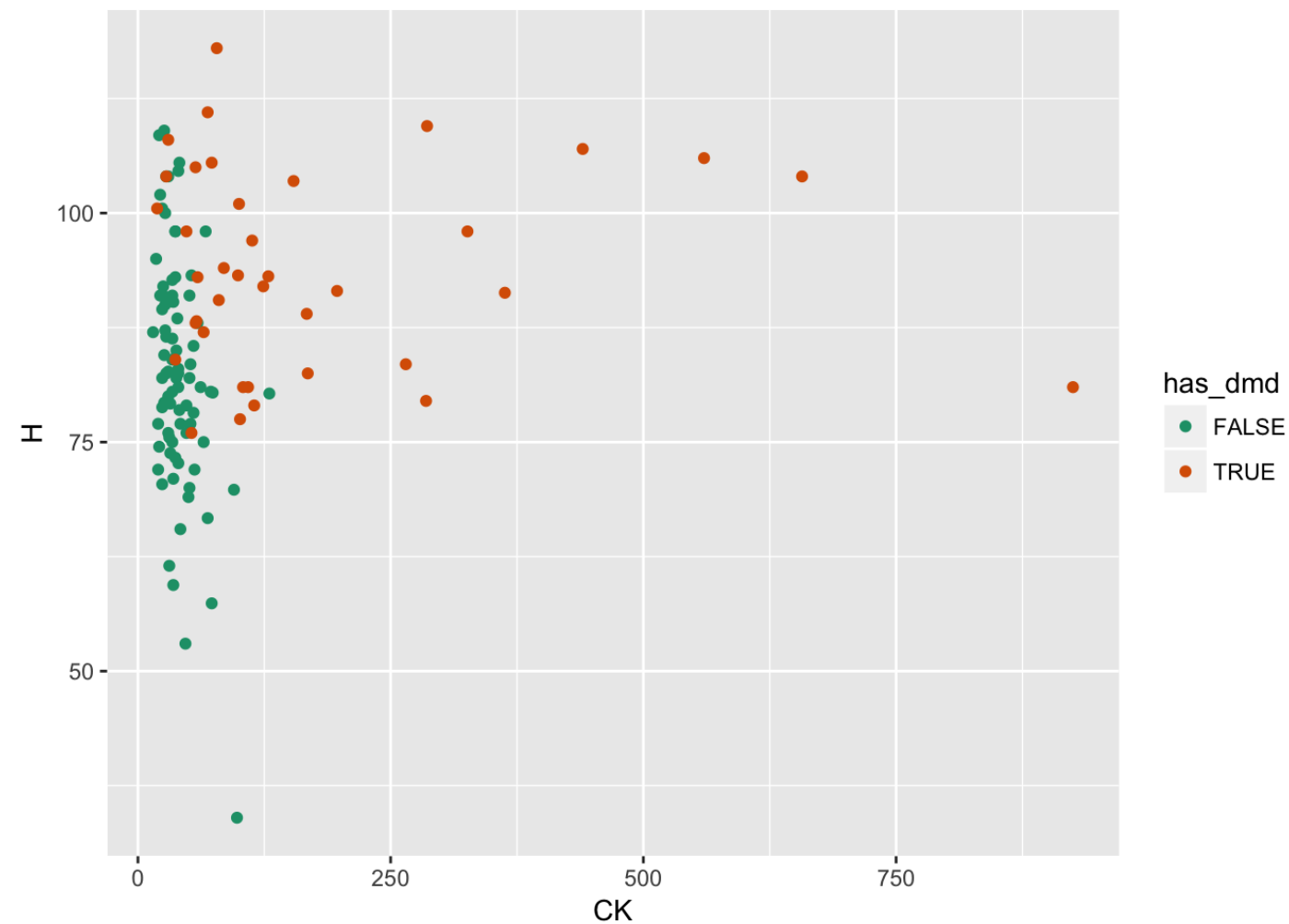
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Predicting Probabilities

- Predicting *whether* an event occurs (yes/no): **classification**
- Predicting *the probability* that an event occurs: **regression**
- Linear regression: predicts values in $[-\infty, \infty]$
- Probabilities: limited to $[0, 1]$ interval
 - So we'll call it non-linear

Example: Predicting Duchenne Muscular Dystrophy (DMD)



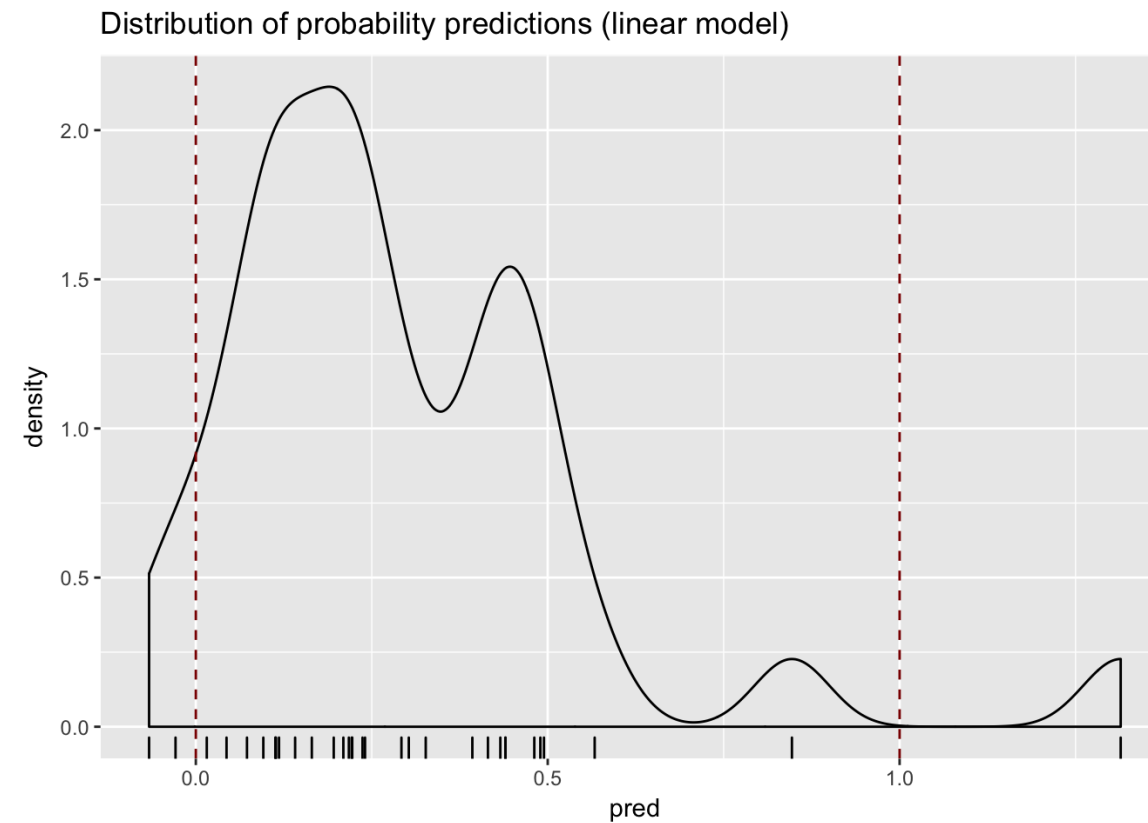
- **outcome:** `has_dmd`
- **inputs:** `CK`, `H`

A Linear Regression Model

```
> model <- lm(has_dmd ~ CK + H,  
+             data = train)  
  
> test$pred <- predict(  
+   model,  
+   newdata = test  
+ )
```

- outcome: $\text{has_dmd} \in \{0,1\}$
 - 0: FALSE
 - 1: TRUE

Model predicts values outside the range [0:1]



Logistic Regression

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots$$

```
glm(formula, data, family = binomial)
```

- Generalized linear model
- Assumes inputs additive, linear in *log-odds*: $\log(p/(1-p))$
- family: describes error distribution of the model
 - logistic regression: *family = binomial*



DMD model

```
> model <- glm(has_dmd ~ CK + H, data = train, family = binomial)
```

- outcome: two classes, e.g. a and b
- model returns $Prob(b)$
 - Recommend: 0/1 or FALSE/TRUE



Interpreting Logistic Regression Models

```
> model

## Call:  glm(formula = has_dmd ~ CK + H, family = binomial, data = train)
##
## Coefficients:
## (Intercept)          CK              H
##  -16.22046      0.07128      0.12552
##
## Degrees of Freedom: 86 Total (i.e. Null);  84 Residual
## Null Deviance:      110.8
## Residual Deviance: 45.16      AIC: 51.16
```

Predicting with a glm() model

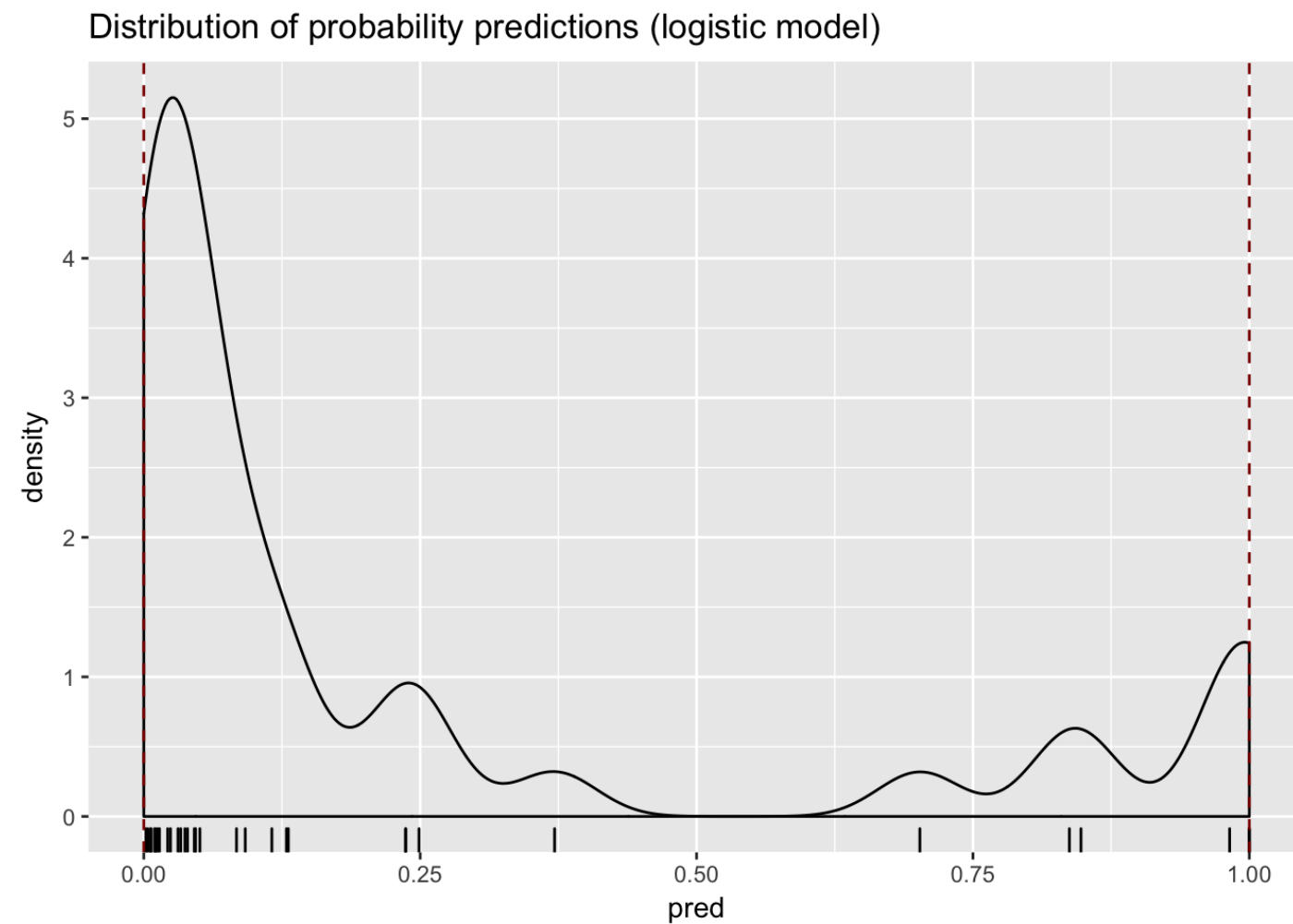
```
predict(model, newdata, type = "response")
```

- `newdata`: by default, training data
- To get probabilities: use `type = "response"`
 - By default: returns log-odds



DMD Model

```
> model <- glm(has_dmd ~ CK + H, data = train, family = binomial)
> test$pred <- predict(model, newdata = test, type = "response")
```



Evaluating a logistic regression model: pseudo- R^2

$$R^2 = 1 - \frac{RSS}{SS_{Tot}}$$

$$pseudoR^2 = 1 - \frac{deviance}{null.deviance}$$

- Deviance: analogous to variance (RSS)
- Null deviance: Similar to SS_{Tot}
- pseudo R^2 : Deviance explained



Pseudo- R^2 on Training data

Using `broom::glance()`

```
> glance(model) %>%  
+   summarize(pR2 = 1 - deviance/null.deviance)  
  
##      pseudoR2  
## 1 0.5922402
```

Using `sigr::wrapChiSqTest()`

```
> wrapChiSqTest(model)  
  
## "... pseudo-R2=0.59 ..."
```



Pseudo- R^2 on Test data

```
# Test data
> test %>%
+   mutate(pred = predict(model, newdata = test, type = "response")) %>%
+   wrapChiSqTest("pred", "has_dmd", TRUE)
```

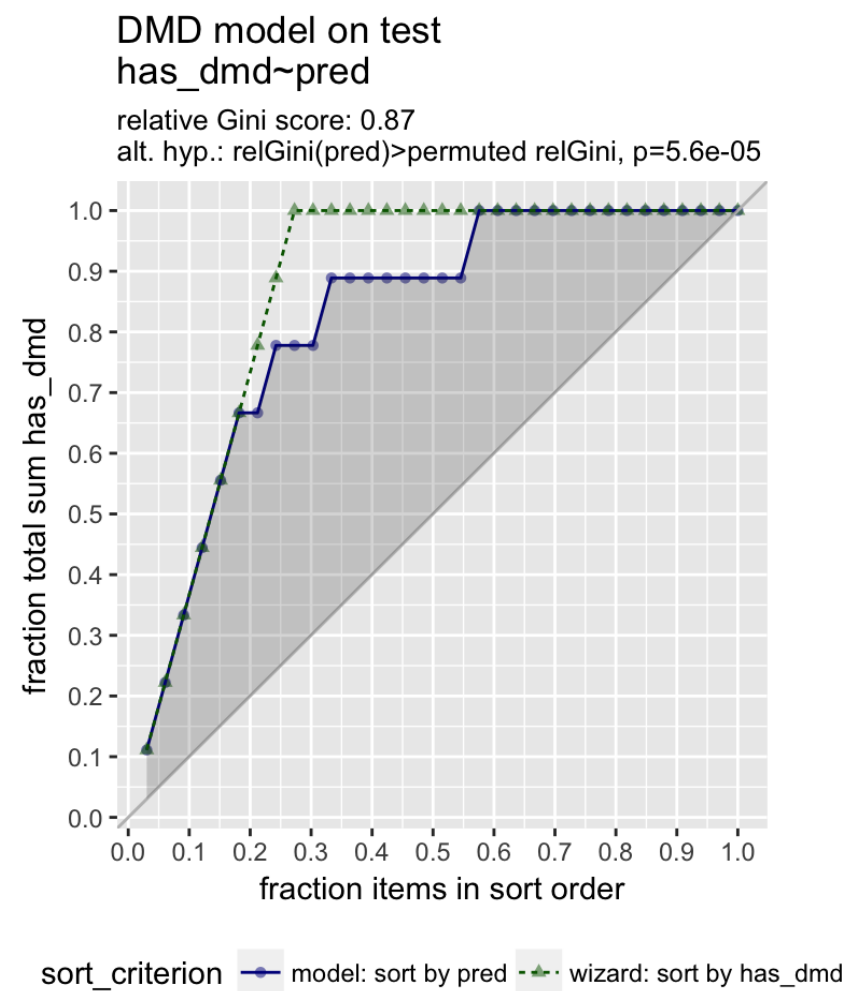
Arguments:

- data frame
- prediction column name
- outcome column name
- target value (target event)



The Gain Curve Plot

```
> GainCurvePlot(test, "pred", "has_dmd", "DMD model on test")
```





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Let's practice!



SUPERVISED LEARNING IN R: REGRESSION

Poisson and quasipoisson regression to predict counts

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Predicting Counts

- Linear regression: predicts values in $[-\infty, \infty]$
- Counts: integers in range $[0, \infty]$



Poisson/Quasipoisson Regression

```
glm(formula, data, family)
```

- **family:** either `poisson` or `quasipoisson`
- inputs additive and linear in $\log(\text{count})$

Poisson/Quasipoisson Regression

```
glm(formula, data, family)
```

- family: either `poisson` or `quasipoisson`
- inputs additive and linear in $\log(\text{count})$
- outcome: *integer*
 - counts: e.g. number of traffic tickets a driver gets
 - rates: e.g. number of website hits/day
- prediction: expected *rate* or *intensity* (not integral)
 - expected # traffic tickets; expected hits/day

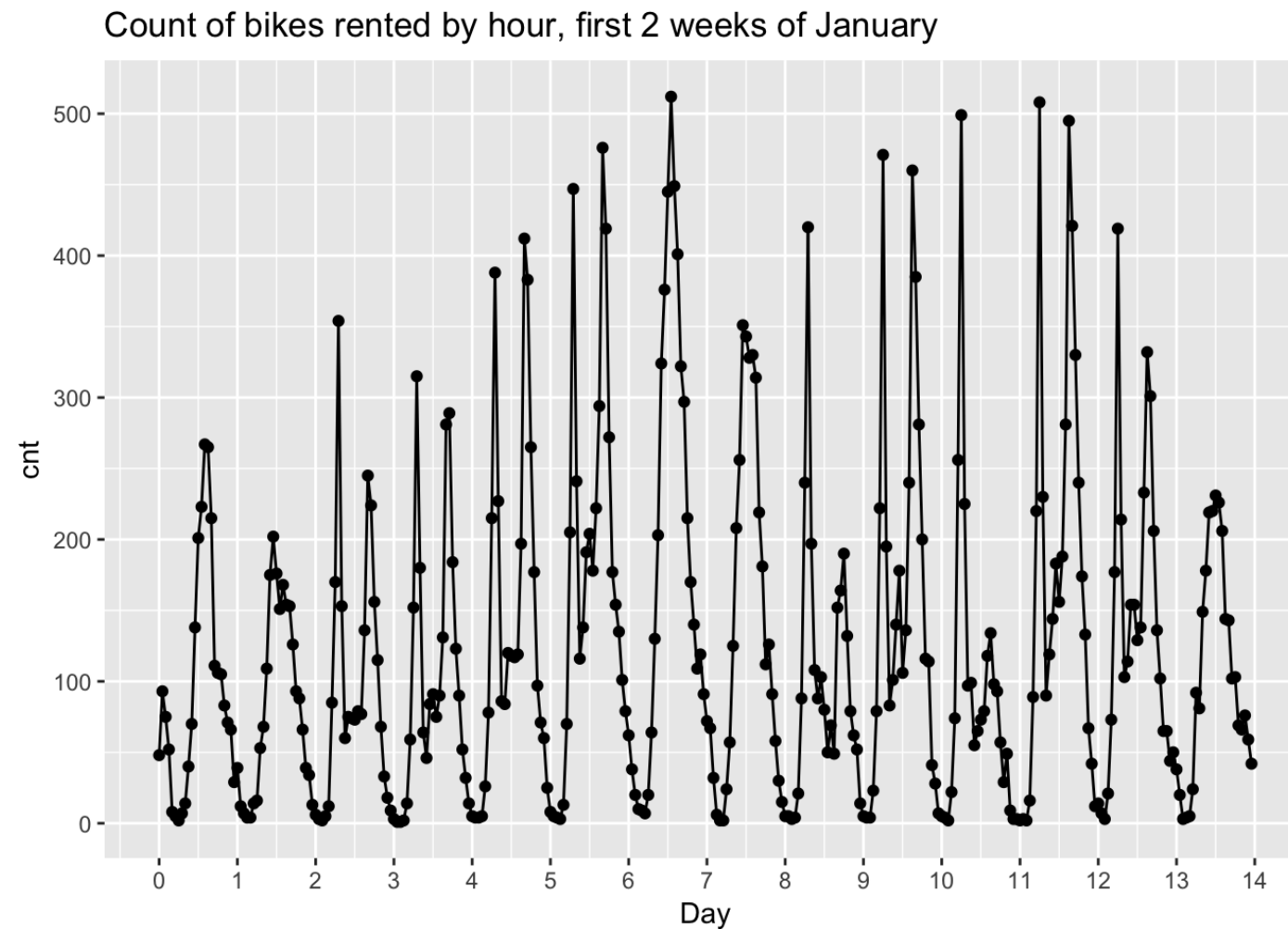


Poisson vs. Quasipoisson

- Poisson assumes that $\text{mean}(y) = \text{var}(y)$
- If $\text{var}(y)$ much different from $\text{mean}(y)$ - quasipoisson
- Generally requires a large sample size
- If rates/counts $\gg 0$ - regular regression is fine



Example: Predicting Bike Rentals





Fit the model

```
> bikesJan %>%  
+   summarize(mean = mean(cnt), var = var(cnt))  
  
##           mean          var  
## 1 130.5587 14351.25
```

Since $\text{var}(\text{cnt}) \gg \text{mean}(\text{cnt}) \rightarrow$ *use quasipoisson*

```
> fmla <- cnt ~ hr + holiday + workingday +  
+   weathersit + temp + atemp + hum + windspeed  
  
> model <- glm(fmla, data = bikesJan, family = quasipoisson)
```



Check model fit

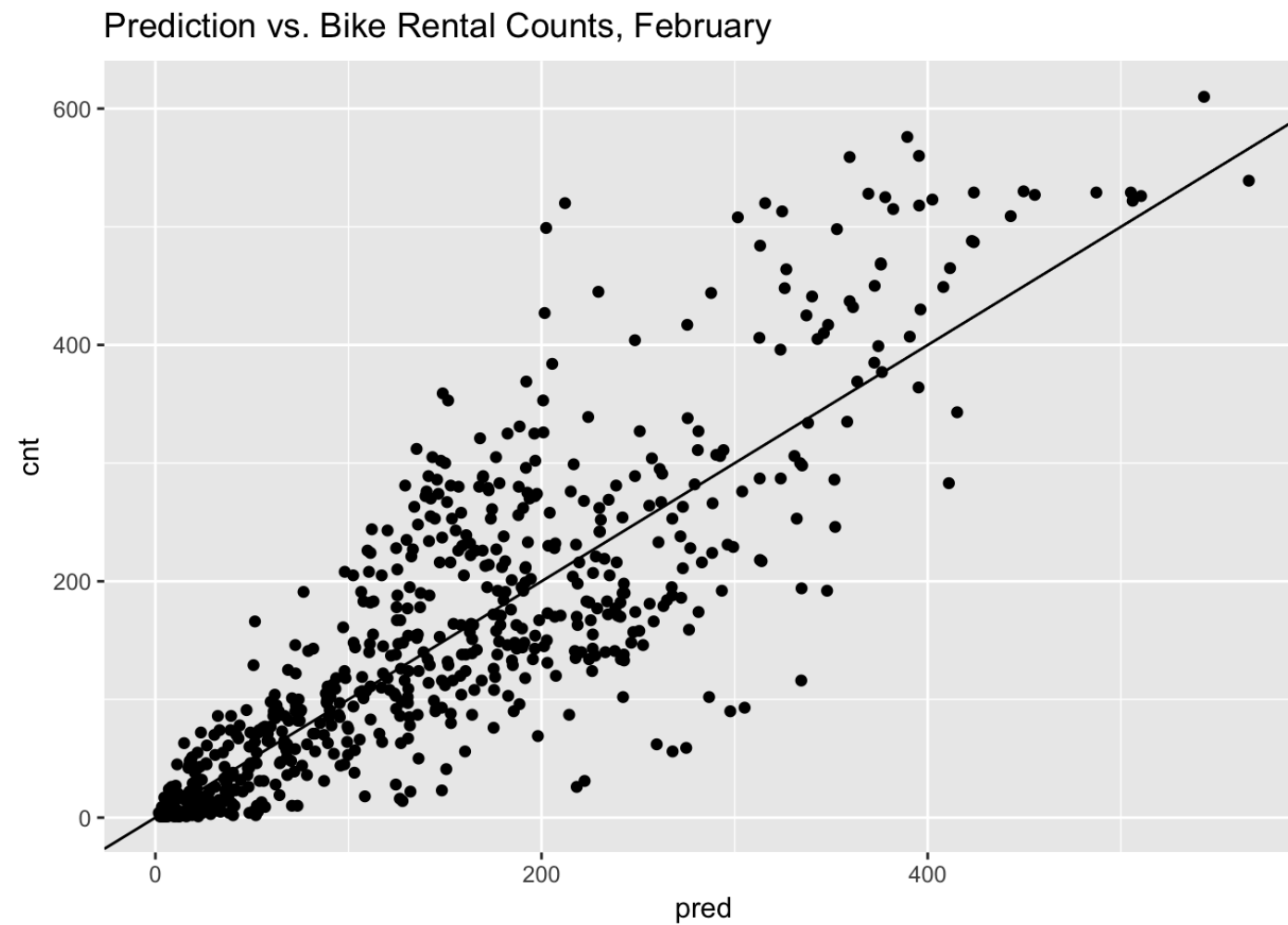
$$pseudoR^2 = 1 - \frac{deviance}{null.deviance}$$

```
> glance(model) %>%  
+   summarize(pseudoR2 = 1 - deviance/null.deviance)  
  
##      pseudoR2  
## 1 0.7654358
```



Predicting from the model

```
> predict(model, newdata = bikesFeb, type = "response")
```





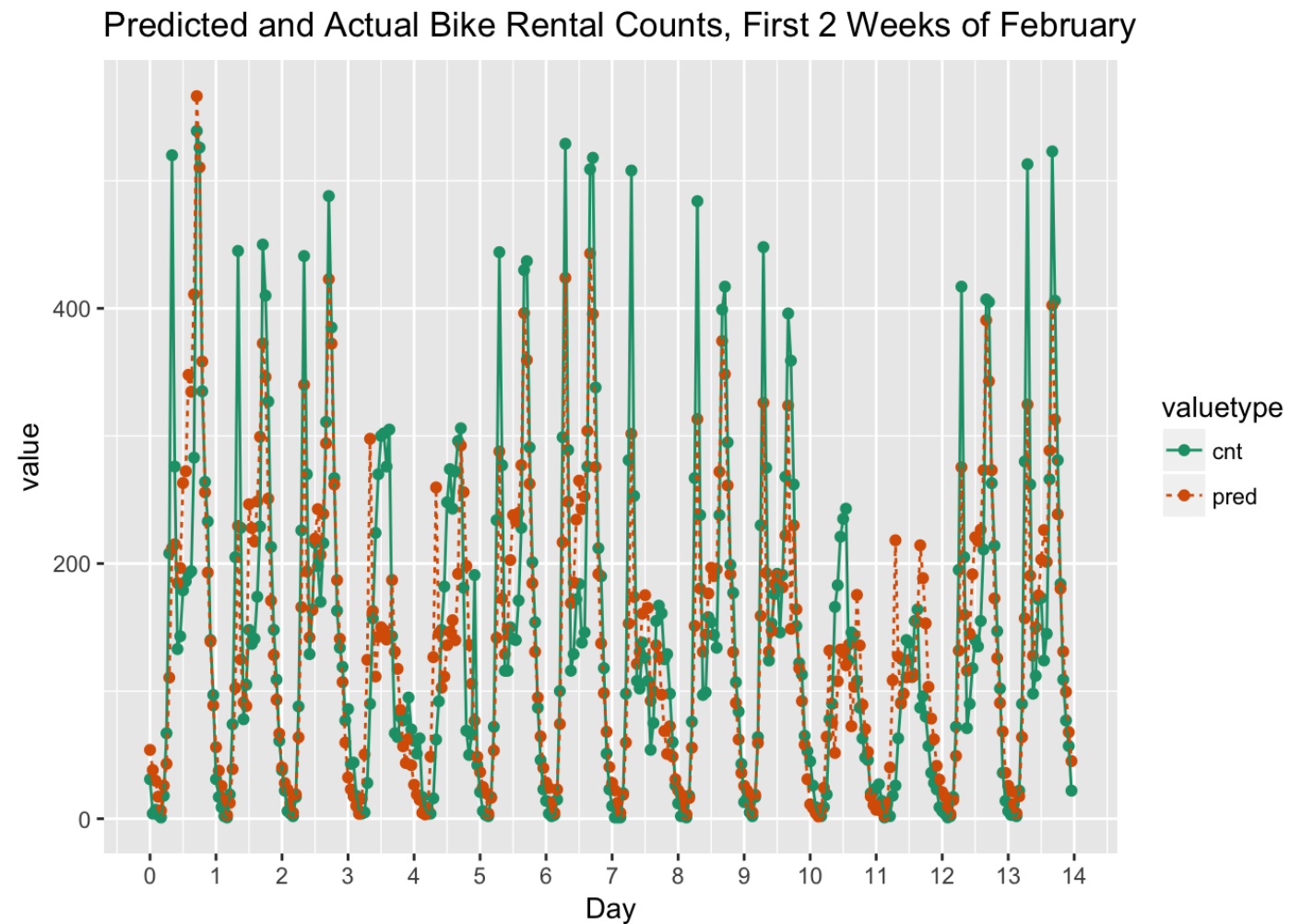
Evaluate the model

You can evaluate count models by RMSE

```
> bikesFeb %>%  
+   mutate(residual = pred - cnt) %>%  
+   summarize(rmse = sqrt(mean(residual^2)))  
  
##           rmse  
## 1 69.32869  
  
> sd(bikesFeb$cnt)  
[1] 134.2865
```




Compare Predictions and Actual Outcomes





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Let's practice!



SUPERVISED LEARNING IN R: REGRESSION

GAM to learn non-linear transformations

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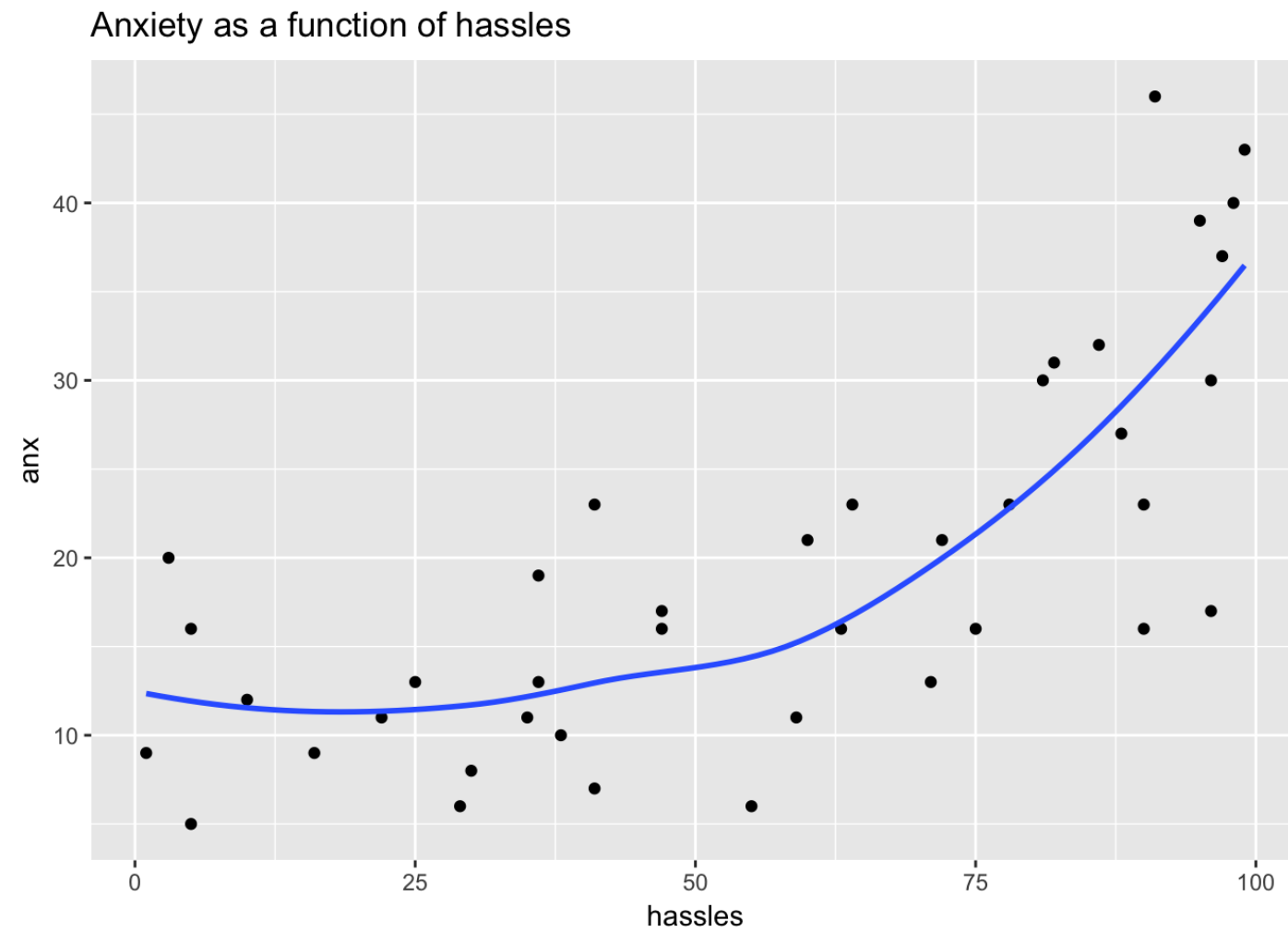


Generalized Additive Models (GAMs)

$$y \sim b_0 + s_1(x_1) + s_2(x_2) + \dots$$



Learning Non-linear Relationships





gam() in the mgcv package

```
gam(formula, family, data)
```

family:

- gaussian (default): "regular" regression
- binomial: probabilities
- poisson/quasipoisson: counts

Best for larger data sets



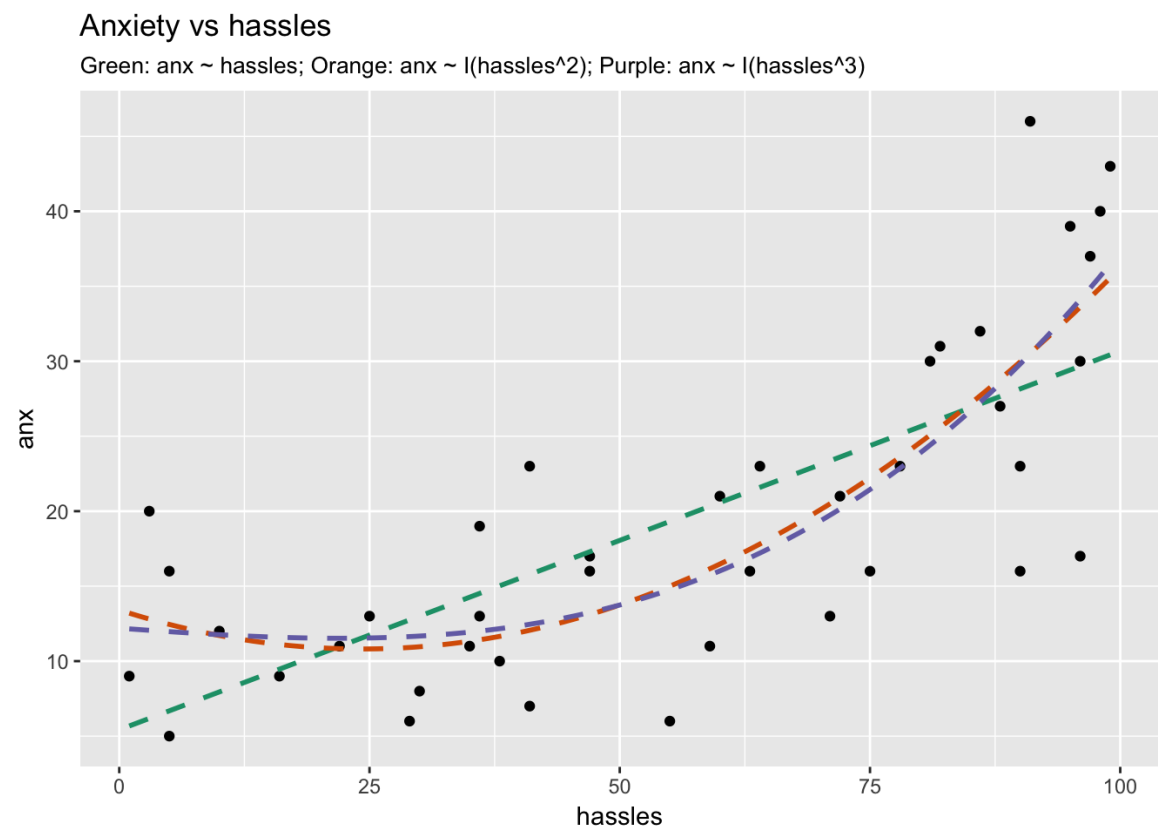
The `s()` function

```
> anx ~ s(hassles)
```

- `s()` designates that variable should be non-linear
- Use `s()` with continuous variables
 - More than about 10 unique values



Revisit the hassles data



Model	RMSE (cross-val)	R^2 (training)
Linear (<i>hassles</i>)	7.69	0.53
Quadratic (<i>hassles</i> ²)	6.89	0.63
Cubic (<i>hassles</i>³)	6.70	0.65



GAM of the hassles data

```
> model <- gam(anx ~ s(hassles), data = hassleframe, family = gaussian)

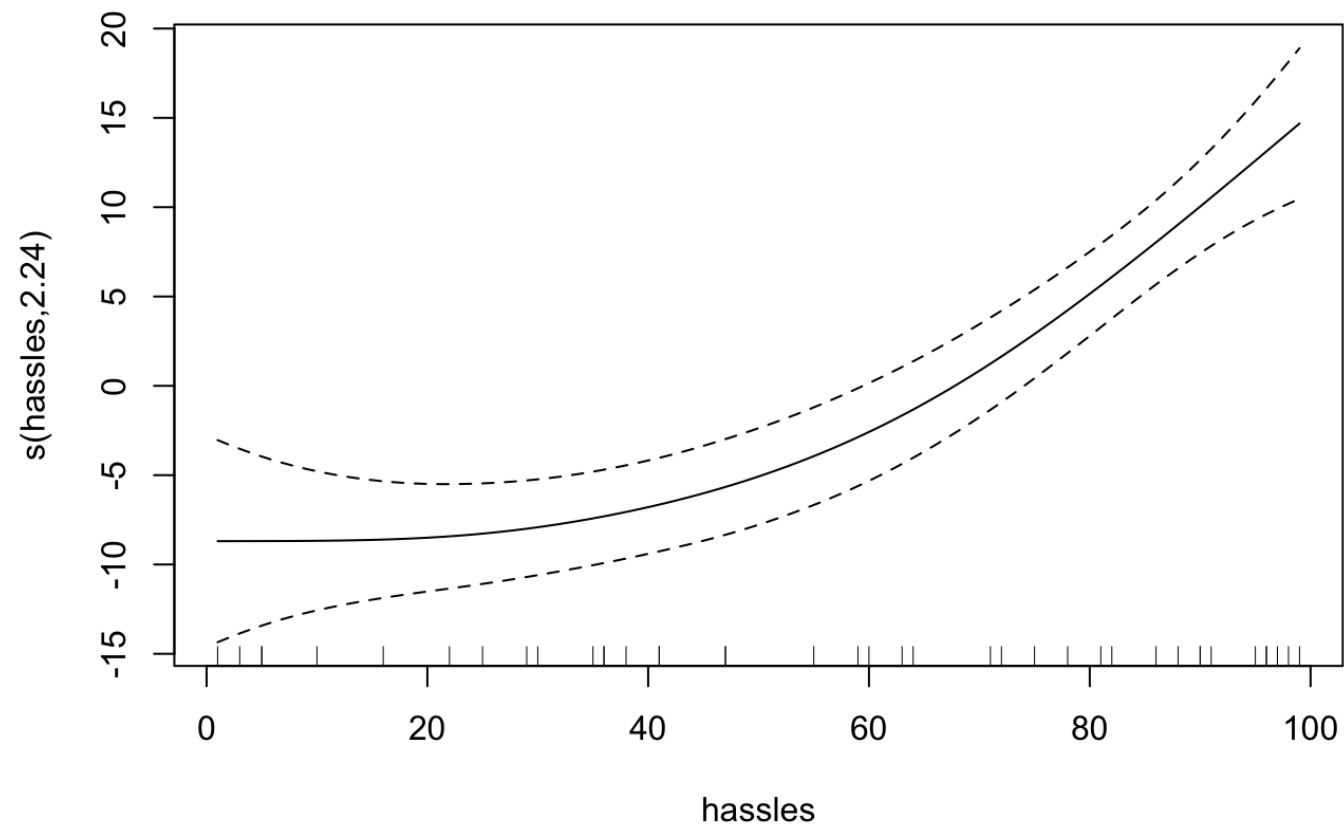
> summary(model)

## ...
##
## R-sq.(adj) = 0.619    Deviance explained = 64.1%
## GCV = 49.132    Scale est. = 45.153    n = 40
```



Examining the Transformations

```
> plot(model)
```

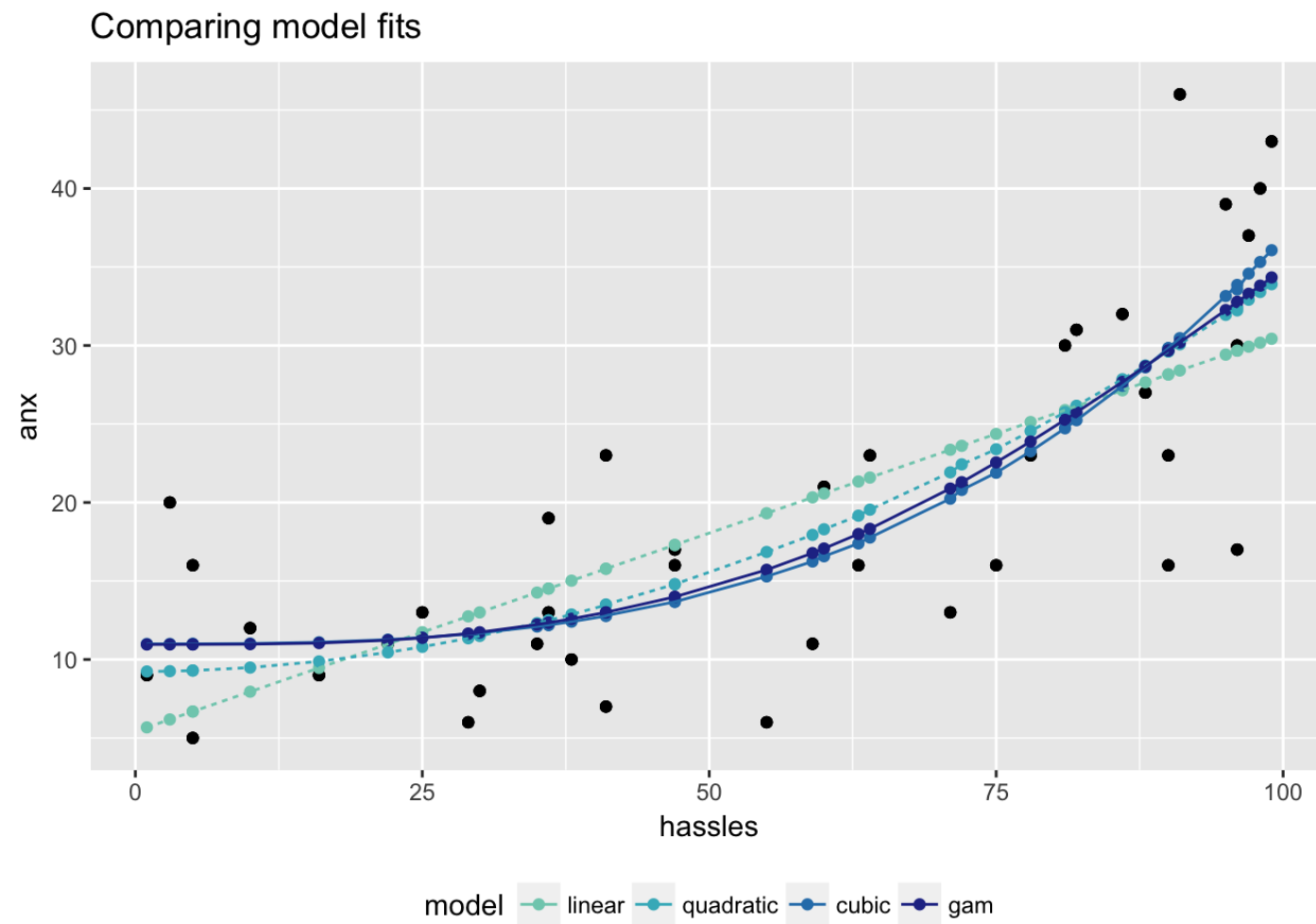


y values: `predict(model, type = "terms")`



Predicting with the Model

```
> predict(model, newdata = hassleframe, type = "response")
```



Comparing out-of-sample performance

Knowing the correct transformation is best, but GAM is useful when transformation isn't known

Model	RMSE (cross-val)	R^2 (training)
Linear (<i>hassles</i>)	7.69	0.53
Quadratic (<i>hassles</i> ²)	6.89	0.63
Cubic (<i>hassles</i> ³)	6.70	0.65
GAM	7.06	0.64

- Small data set → noisier GAM



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