

Digital Logic

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Outline

- What is Computer Architecture
- What is a Central Processing Unit (CPU)
- Start with gates - AND / OR / NOT / XOR ...
- Adding numbers with gates – half and full adders
- Number representation – Base 2 and Base 10 (hexadecimal)
- Storing data with gates – Latches and Flip-Flops

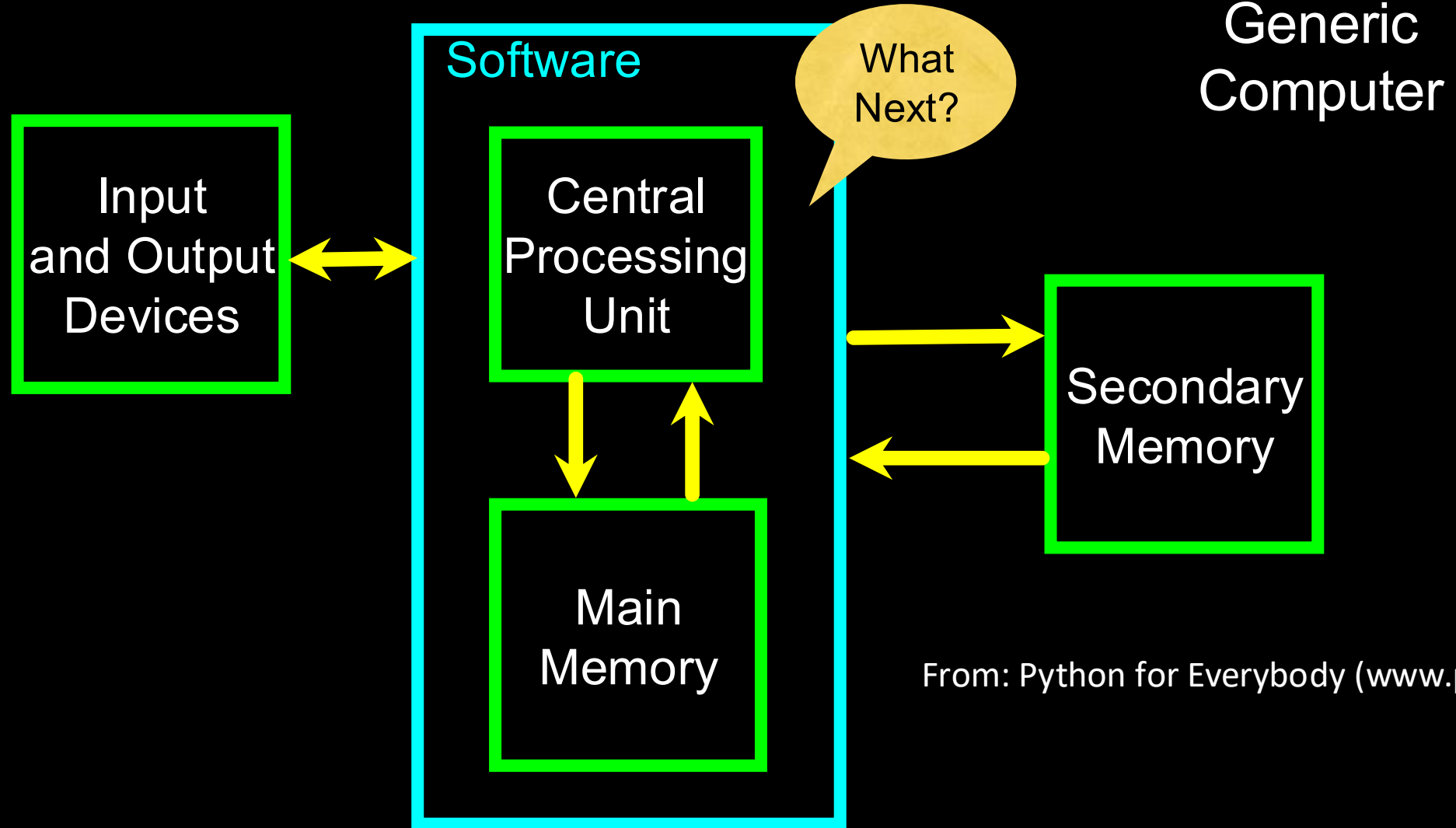
Computer Architecture

- Within the computer data is moved using electric wires
- Each wire generally has a voltage that is either a "0" or "1"
- Wires connect components like the CPU, Memory, or other devices

https://en.wikipedia.org/wiki/Printed_circuit_board

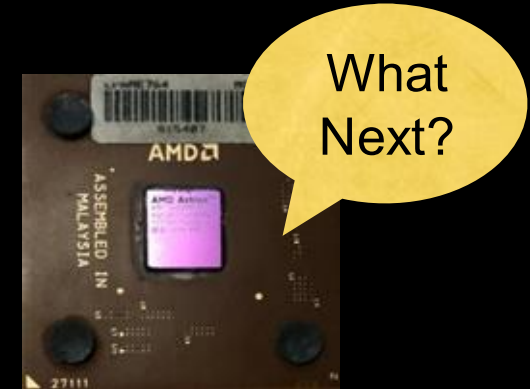


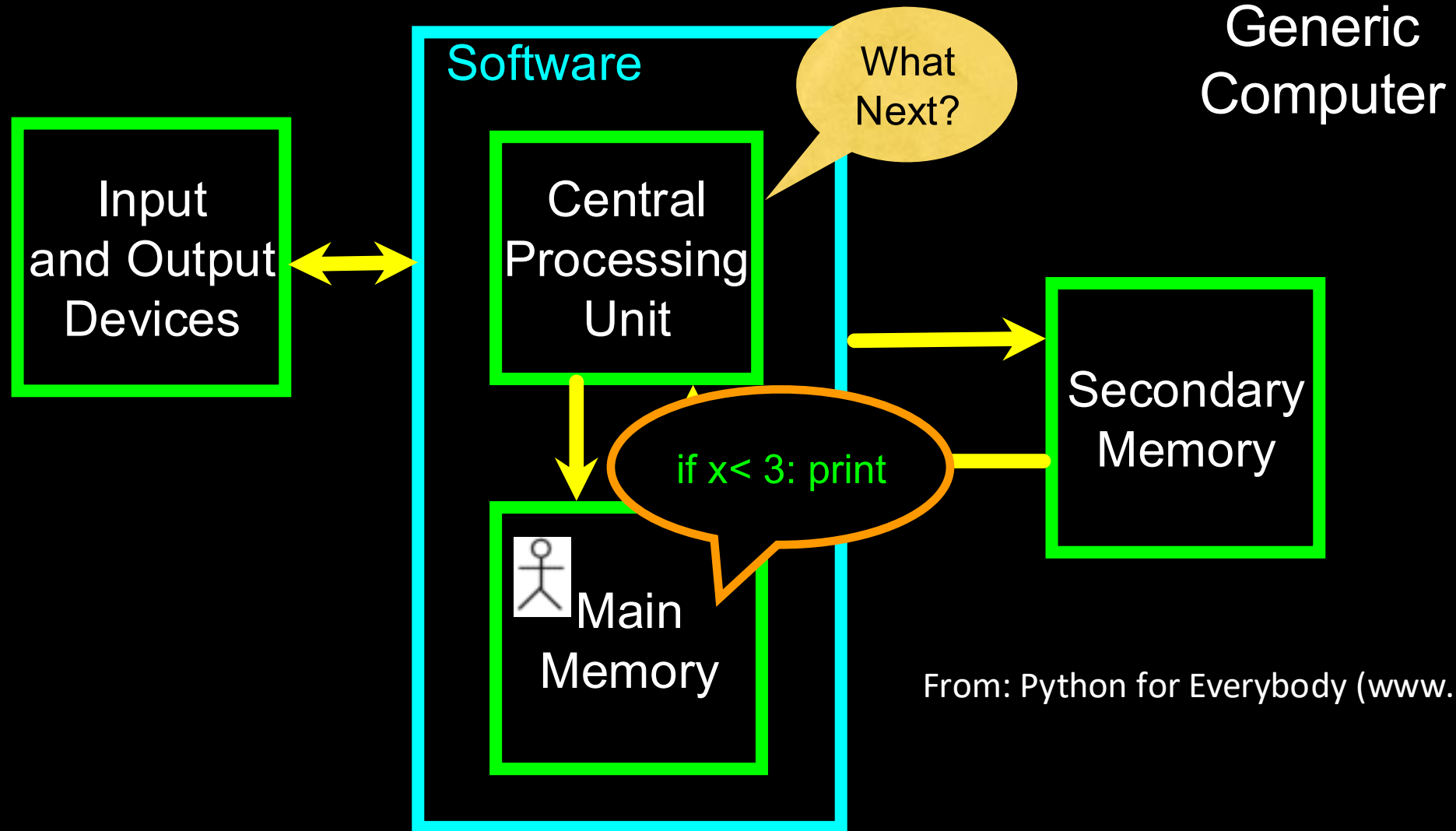
A sound chip from a 1980's Commodore 64 connected to the motherboard



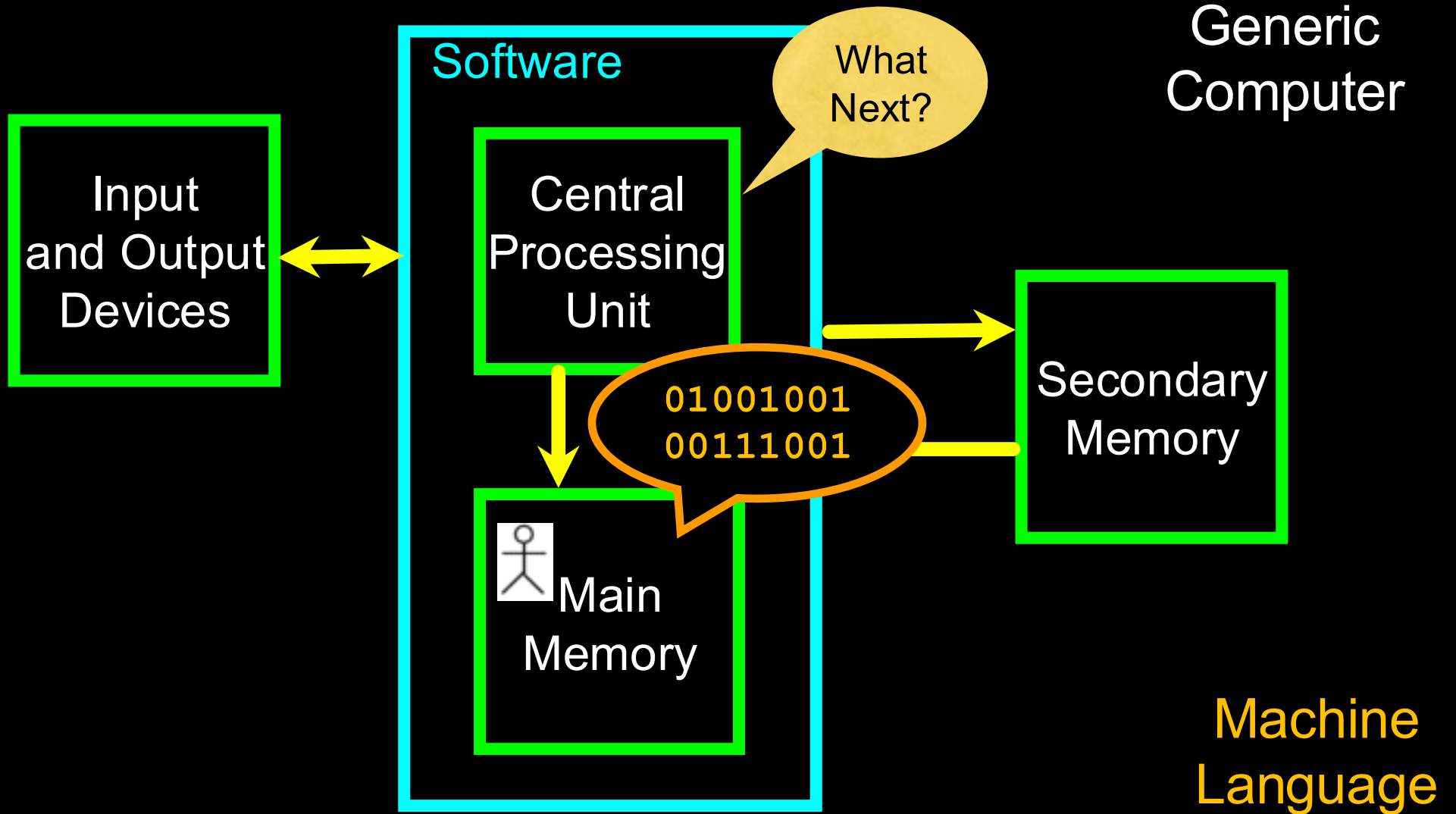
Definitions

- **Central Processing Unit:** Runs the Program - The CPU is always wondering “what to do next”. Not the brains exactly - very dumb but very very fast
- **Input Devices:** Keyboard, Mouse, Touch Screen
- **Output Devices:** Screen, Speakers, Printer, DVD Burner
- **Main Memory:** Fast small temporary storage - lost on reboot - aka RAM
- **Secondary Memory:** Slower large permanent storage - lasts until deleted - disk drive / memory stick





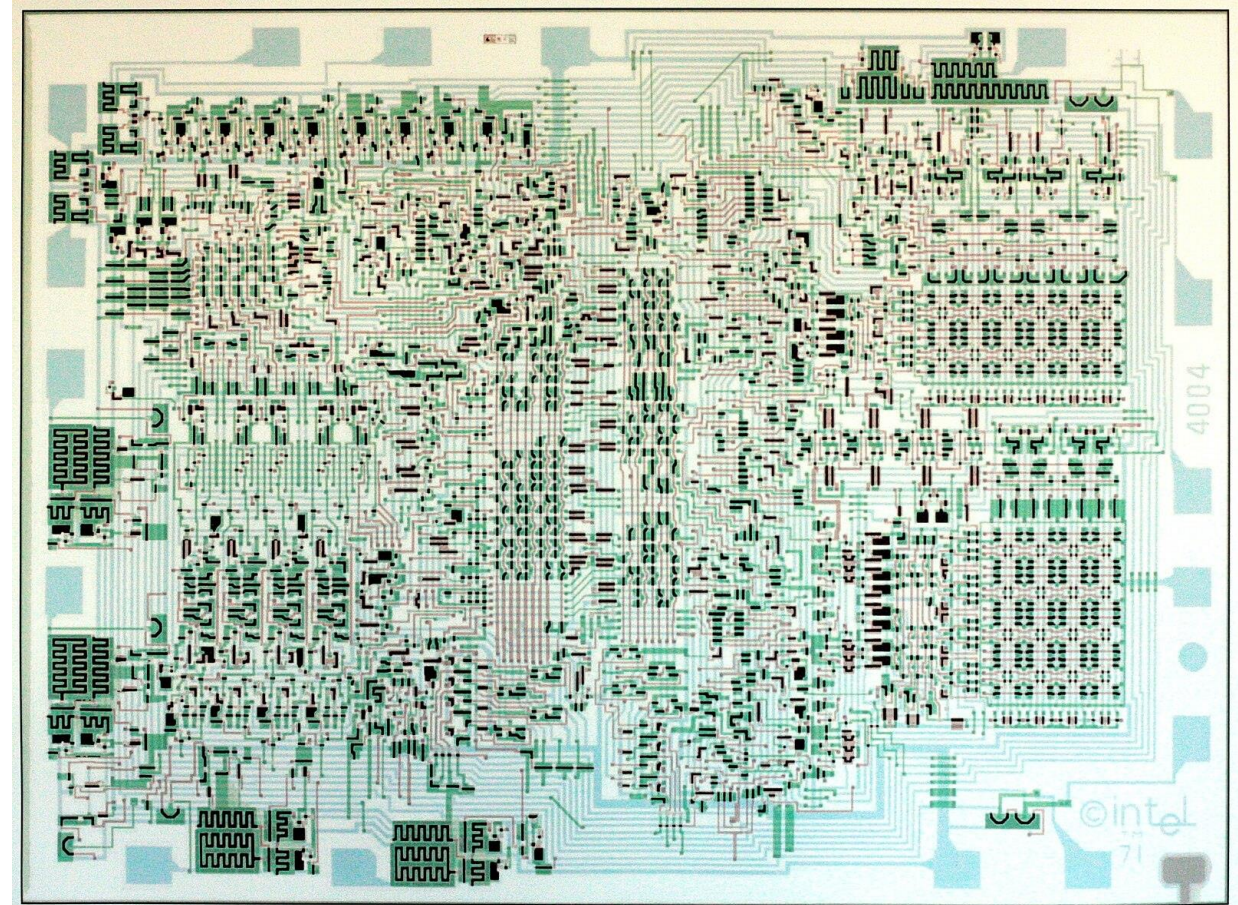
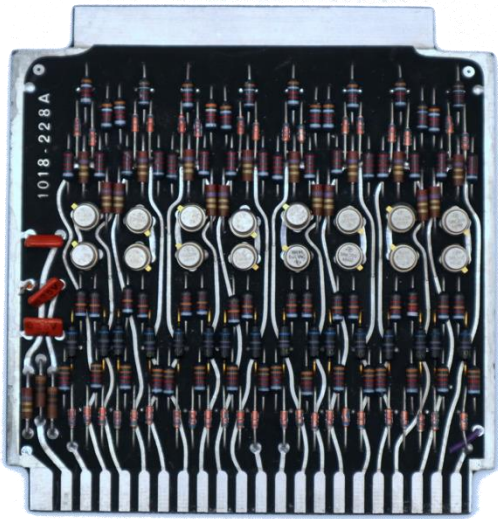
From: Python for Everybody (www.py4e.com)



Building Microprocessors

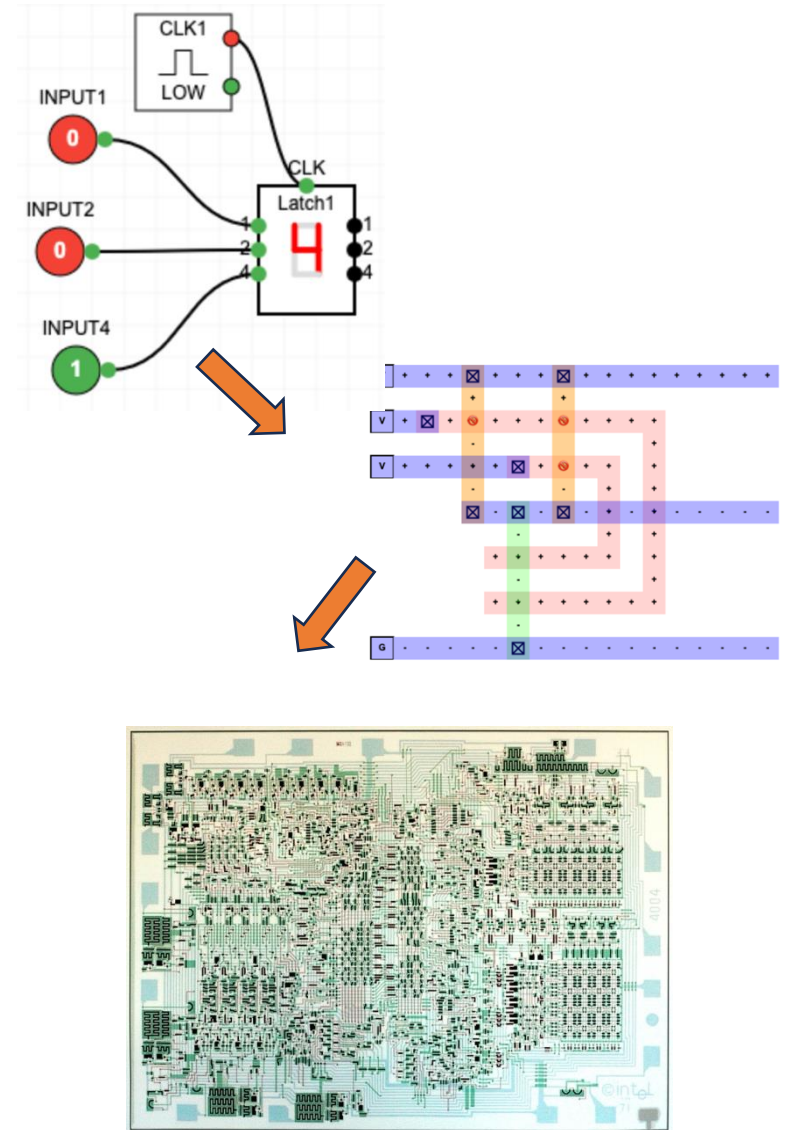
Very Large Scale Integration

- We went from 16 transistors on a board 4 inches on a side to billions of transistors in a chip 0.5 inches on a side



CPU Design Process

- We start with gates and build reusable components and then combine those components to implement programmable digital logic
- Software translates our abstract design into transistors and then creates an optimal layout for our chip
- Photo lithography is then used to build the chip



Adding numbers with gates

Gates are now our low-level building blocks – we can combine them to solve problems

Classic Mathematical Logic Gates

Not



A	Q
0	1
1	0

And



A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

Or



A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

Exclusive Or



A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

Representing Numbers with 0's and 1's

- If we are going to do significant computations, we will need a way to represent numbers as electrical values
- "Normal" numbers are Base-10
- We know the place values of Base-10 numbers
- To manipulate numbers with gates, we use Base-2 numbers

Base-10

$$\begin{aligned} 123_{10} &= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 \\ &= 1 \times 100 + 2 \times 10 + 3 \times 1 = 123_{10} \end{aligned}$$

Base-2

$$\begin{aligned} 110_2 &= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = \\ &= 4 + 2 + 0 = 6_{10} \end{aligned}$$

Keep it simple for now

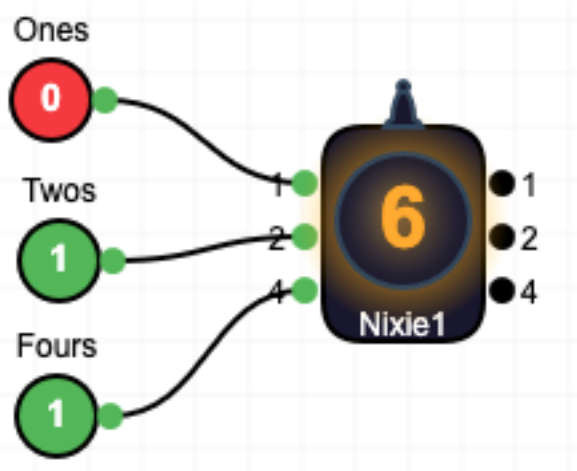
- Base-2 to Base-10 for numbers less than 8
- A.k.a. 3-bit numbers

4s 2s 1s

$$1 \ 1 \ 0 = 6$$

Base-10	Base-2
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

Nixie Tubes – Binary to Decimal Display

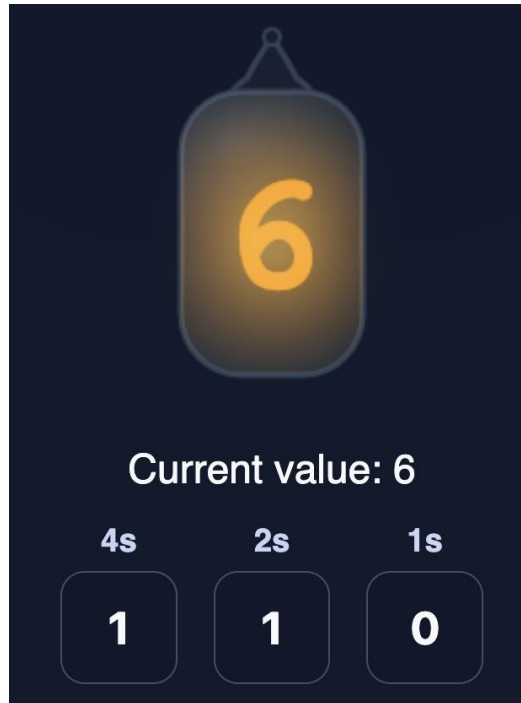


4 2 1

$$1 \ 1 \ 0 = 6$$



https://en.wikipedia.org/wiki/Nixie_tube



<https://www.ca4e.com/tools/nixie/>

NIXIE CHALLENGE

Add binary numbers one bit at a time and track the carry.

Binary Addition Score: 1/10

Add the two binary numbers. Enter each sum bit (include the carry on the left).

	8s	4s	2s	1s	
A		0	1	1	(3)
B		1	1	0	(6)
Sum	1	0	0	1	(9)

Base-2 Math

$$\begin{array}{r} \\ \\ \hline 1 \end{array}$$

$$\begin{array}{r} \\ \\ \hline 1 \end{array}$$

$$\begin{array}{r} \\ \\ \hline 1 \end{array}$$

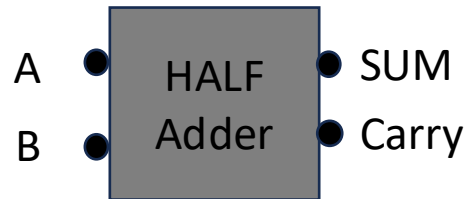
$$\begin{array}{r} \\ \\ \hline 1 \end{array}$$

$$\begin{array}{r} \\ \\ \hline 1 \end{array}$$

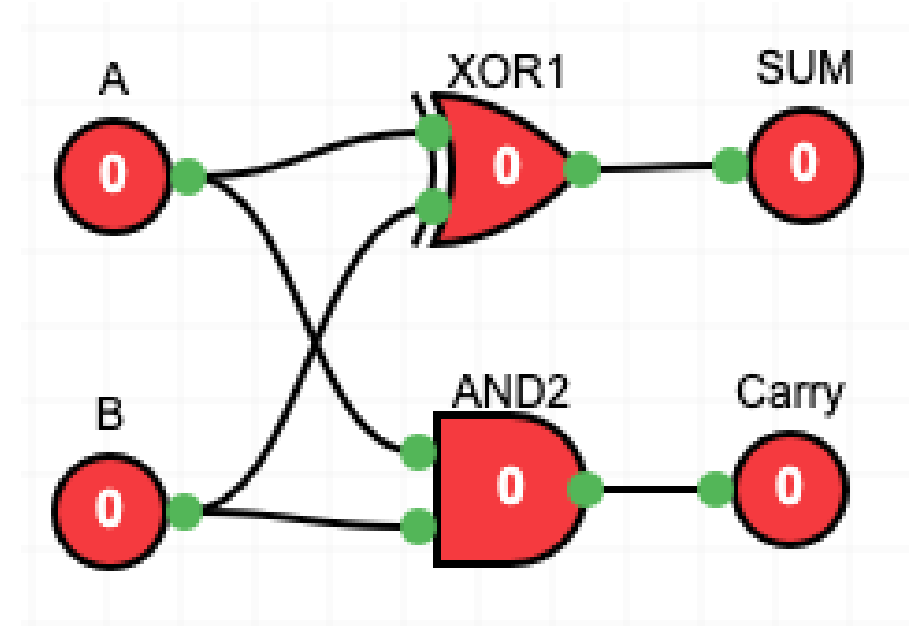
$$\begin{array}{r} \\ \\ \hline 1 \end{array}$$

Half Adder

- Compute the sum of two base-2 digits and produce a sum and carry.

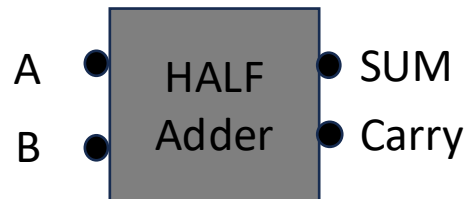


A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

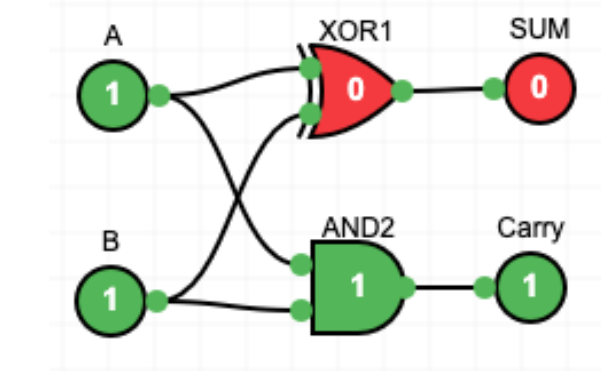
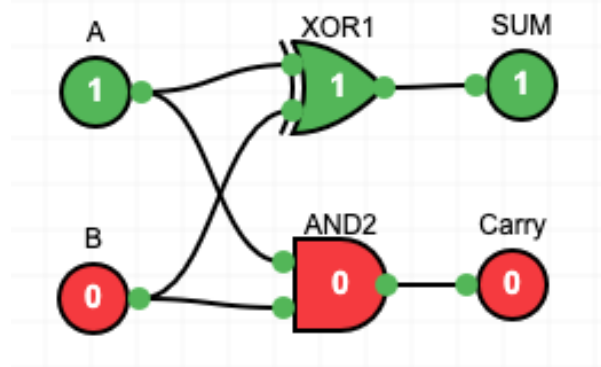
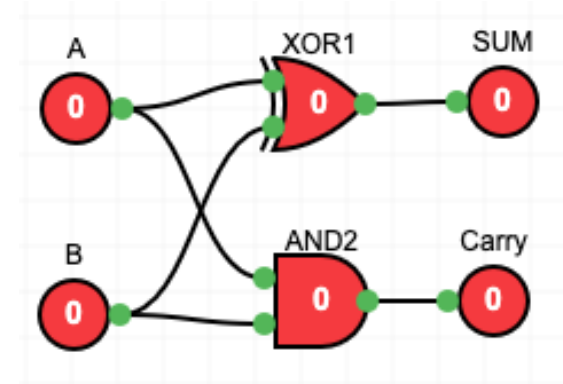


Half Adder

- Compute the sum of two base-2 digits and produce a sum and carry.

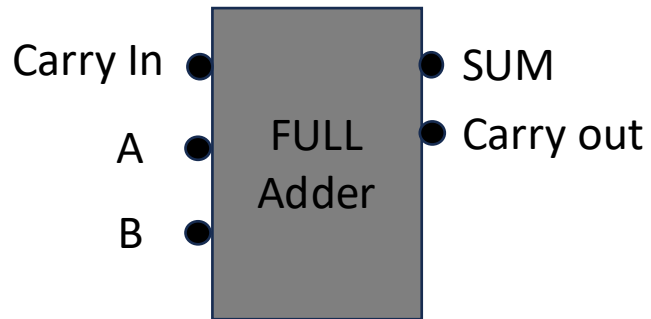


A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



Full Adder

- Adds three 0/1 numbers for a sum/carry between 0 and 3



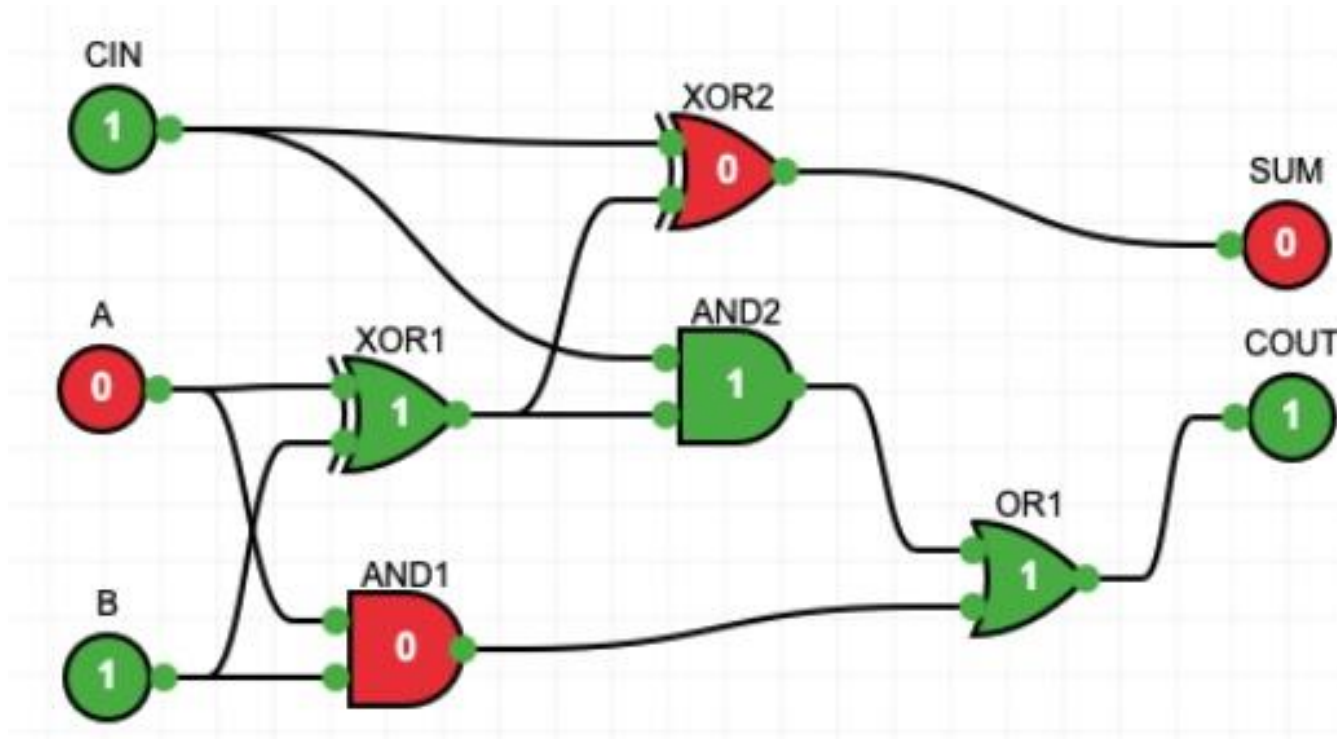
$$\begin{array}{r} 1 \text{ (CIN)} \\ 1 \text{ (A)} \\ + 1 \text{ (B)} \\ \hline 1 \ 1 \text{ (3)} \end{array}$$

Carry in	A	B	Sum	Carry Out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1



Full Adder

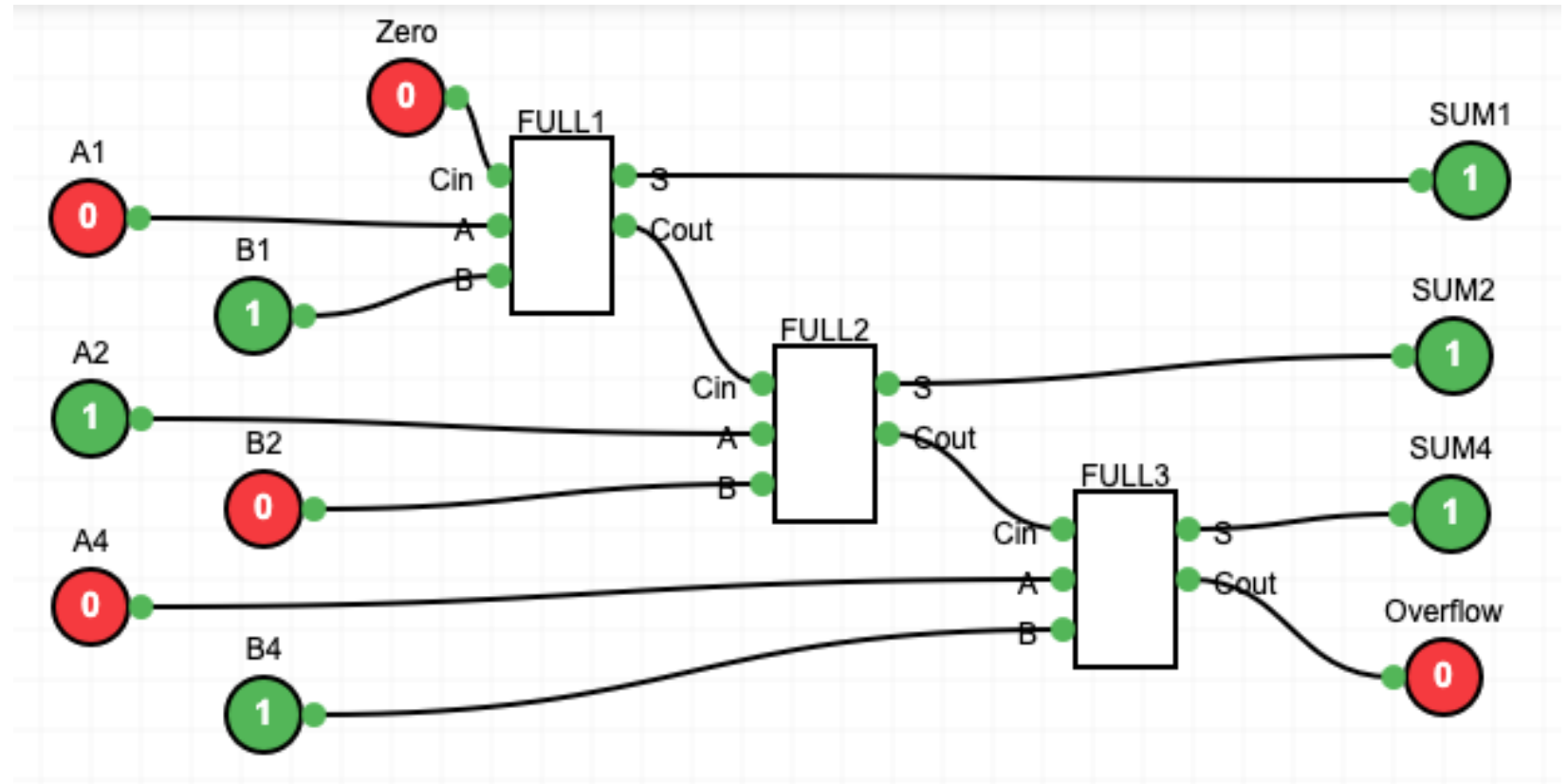
- Adds three 0/1 numbers for a sum/carry between 0 and 3



$$\begin{array}{r} 1 \text{ (CIN)} \\ 0 \text{ (A)} \\ + \quad 1 \text{ (B)} \\ \hline 1 \ 0 \text{ (2)} \end{array}$$

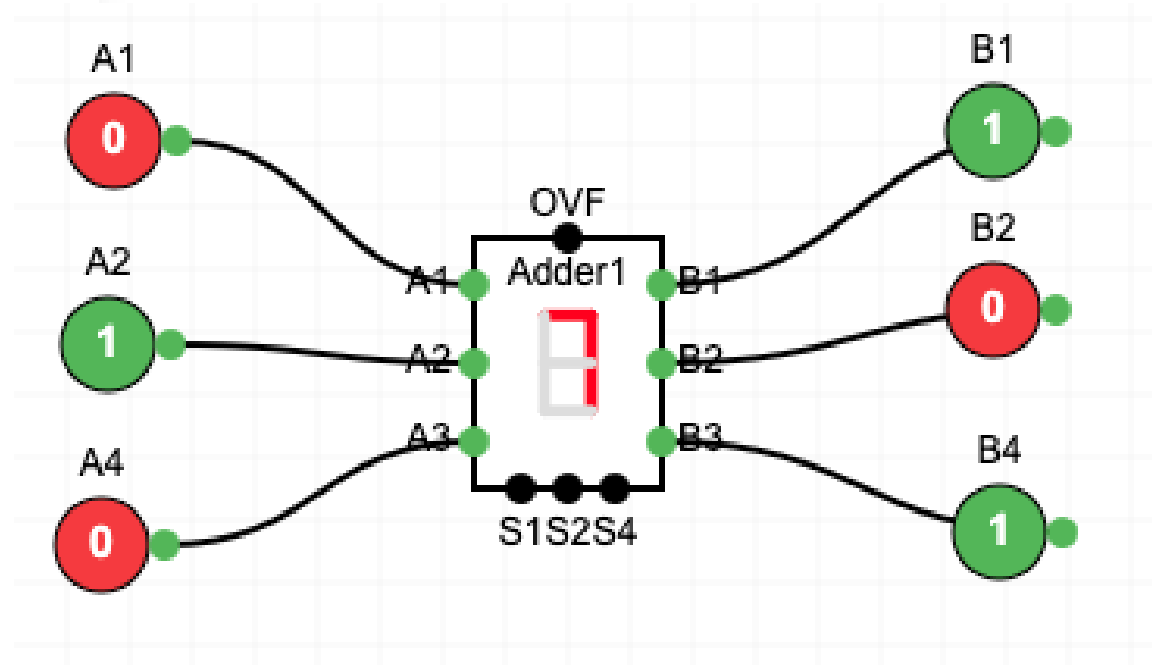
Chaining full adders

$$\begin{array}{r} 010 \text{ (2)} \\ + 101 \text{ (5)} \\ \hline 111 \text{ (7)} \end{array}$$



Three Bit Adder Component

$$\begin{array}{r} 0 \ 1 \ 0 \ (2) \\ + \ 1 \ 0 \ 1 \ (5) \\ \hline 1 \ 1 \ 1 \ (7) \end{array}$$



- Once we know how to make something, add a component to our design system

Storing data with gates

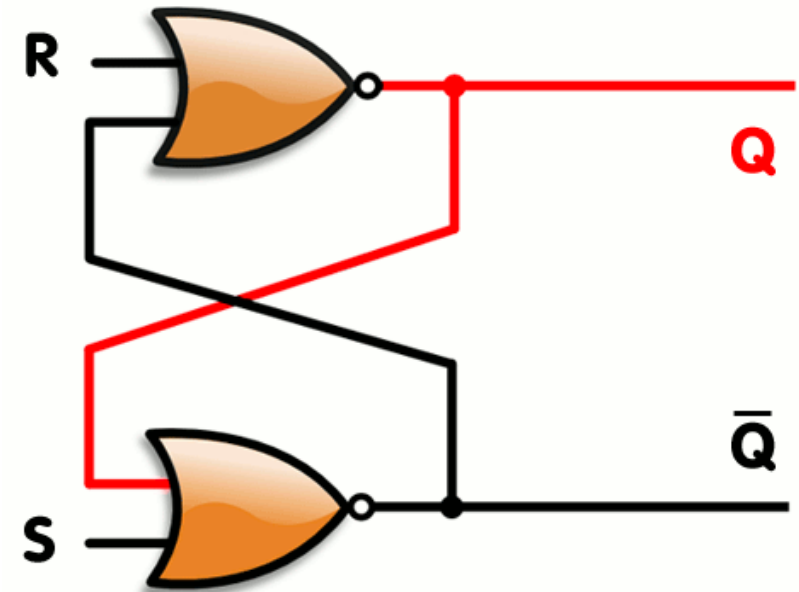
Latches and Flip-Flops

Storing Data Using Feedback Loops

- The simplest latch is a "Set-Reset" latch using two NOR gates
- The output of each NOR gate is connected to the one of the inputs of the other NOR gate

A	B	NOR
0	0	1
0	1	0
1	0	0
1	1	0

S	R	Q
0	0	Maintain
0	1	0
1	0	1
1	1	???

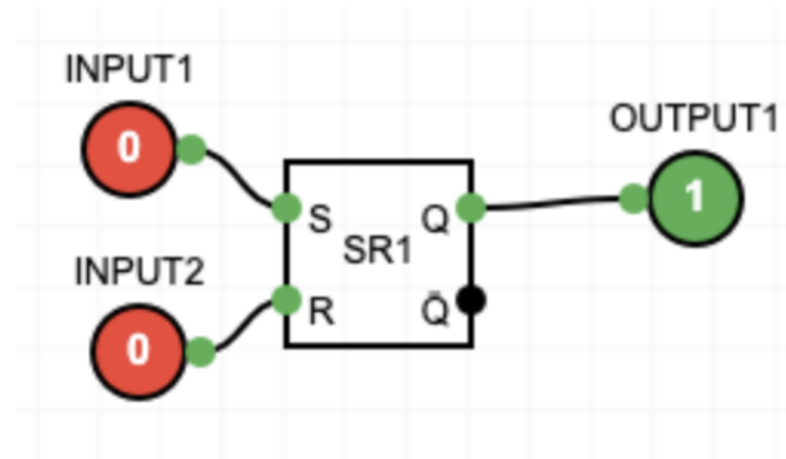


https://en.wikipedia.org/wiki/Flip-flop_%28electronics%29

SR Flip-Flop / Latch

- When S and R are zero, the latch maintains its value
- When S=0 and R=1 the internal value becomes 0
- When S=1 and R=0 the internal value becomes 1
- Setting both to 1 means output is undefined

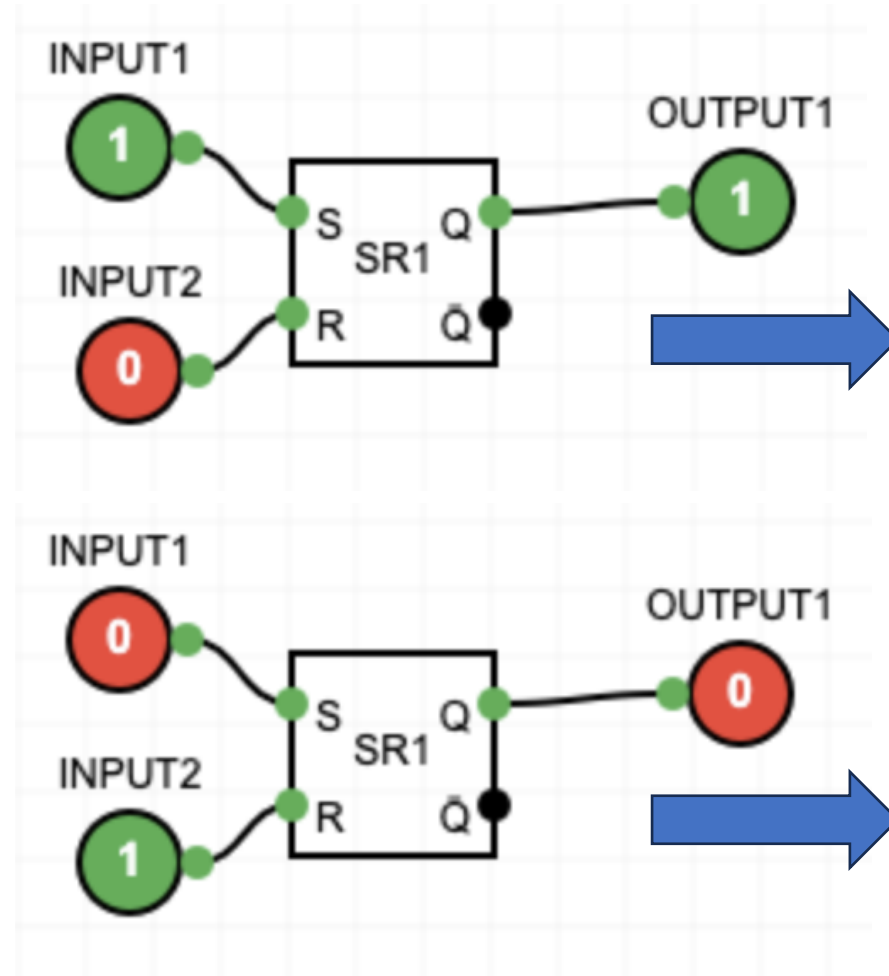
S	R	Q
0	0	Maintain
0	1	0
1	0	1
1	1	???



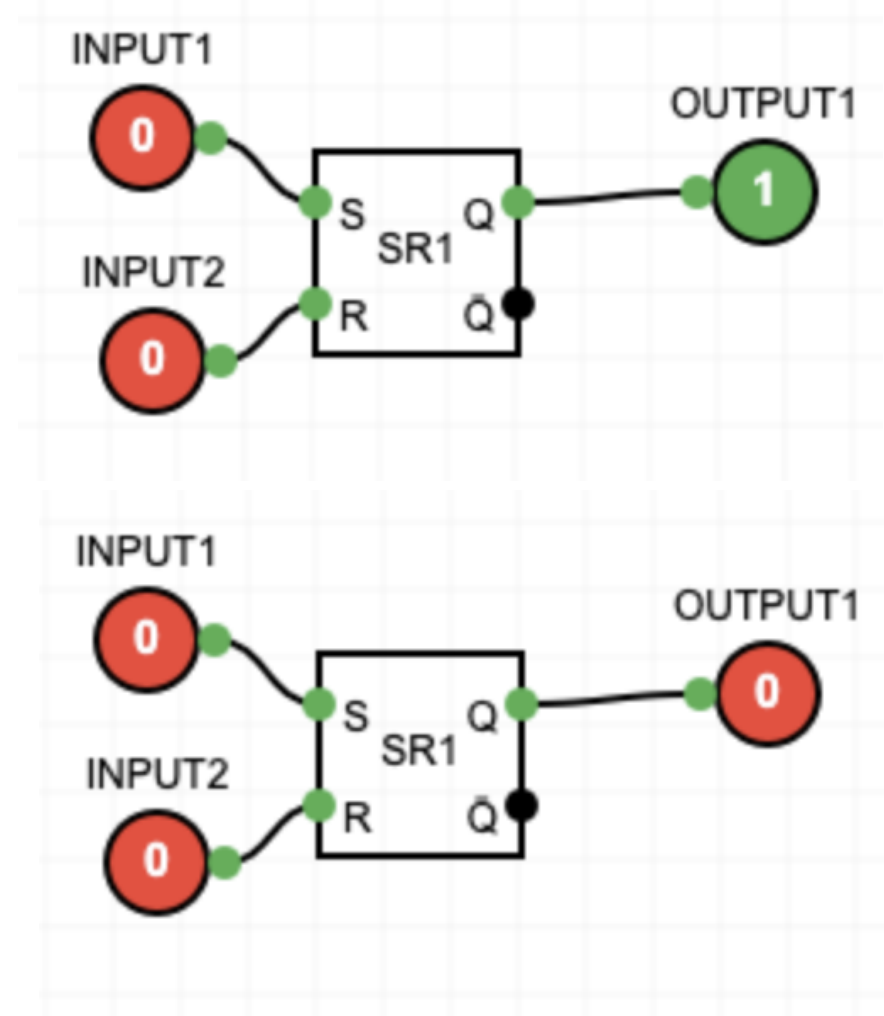
SR Flip-Flop / Latch

S	R	Q
0	0	Maintain
0	1	0
1	0	1
1	1	???

Set / Reset

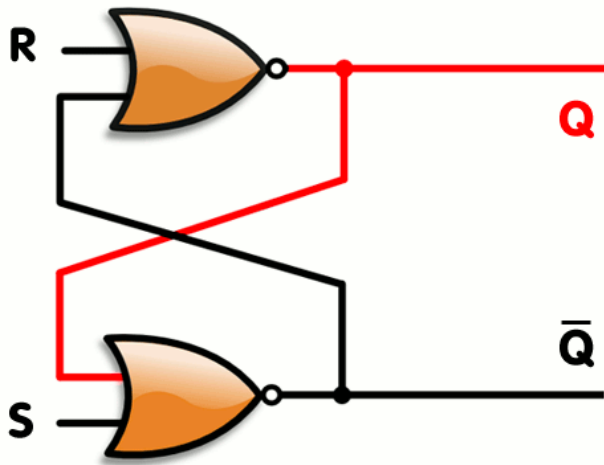


Stored Value

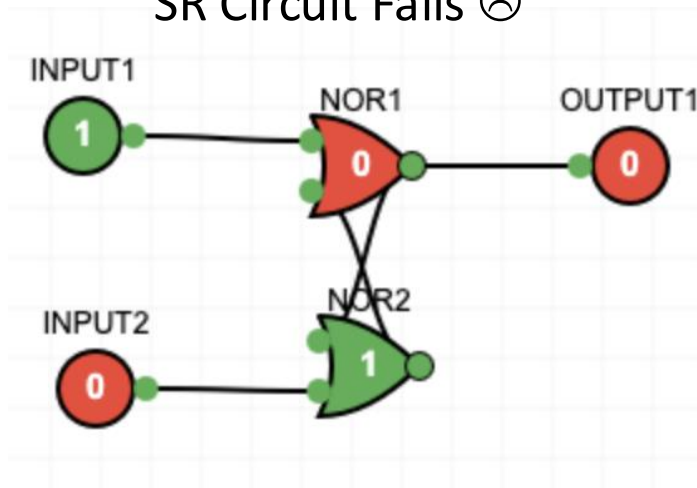


My Simulators Can't Build an SR Latch :(

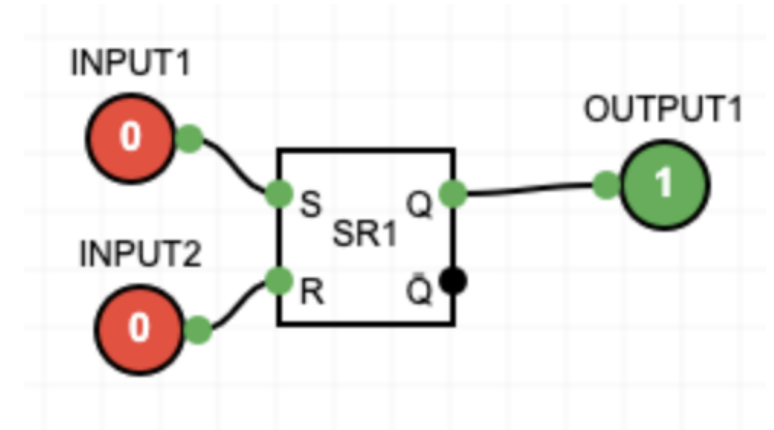
- My Digital Logic Builder does not handle feedback loops so you can't build a SR latch from two NOR gates (unless I fix it)



SR Circuit Fails ☹️



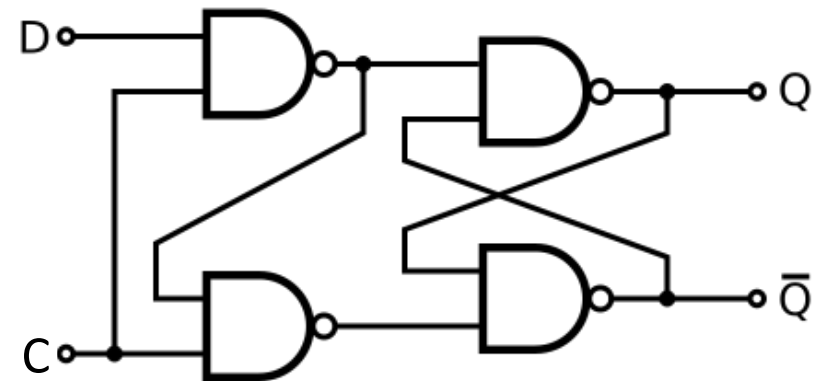
SR Component Works



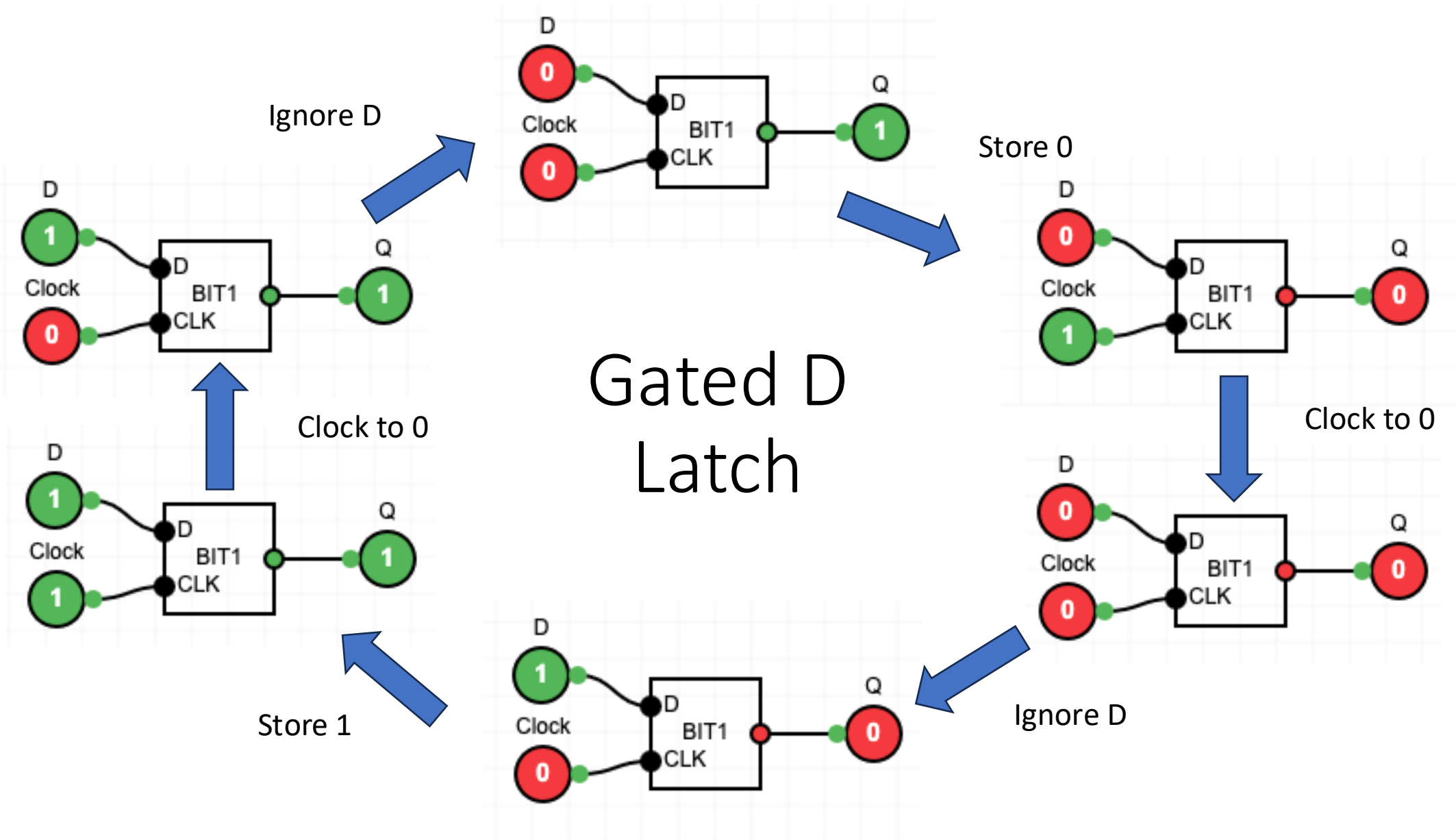
Gated D Latch

- The Gated D Latch has a D (data) and a C (clock) inputs.
- If C is zero, D is ignored, and the output is the most recent stored value
- If C is 1, D is copied into the internal state

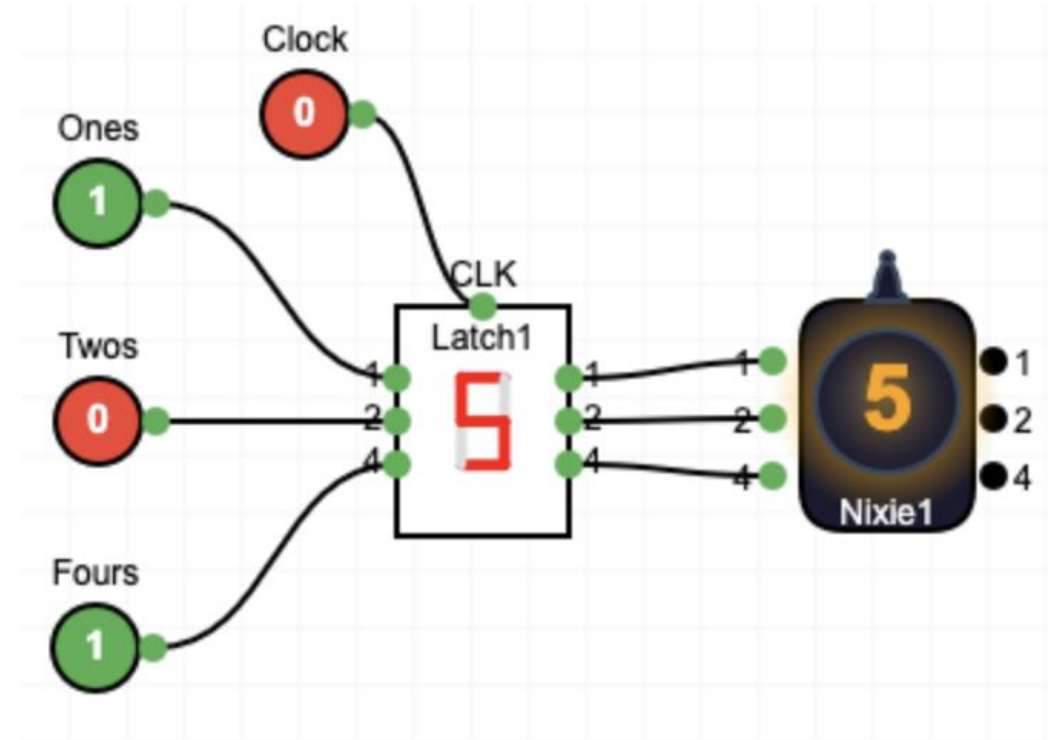
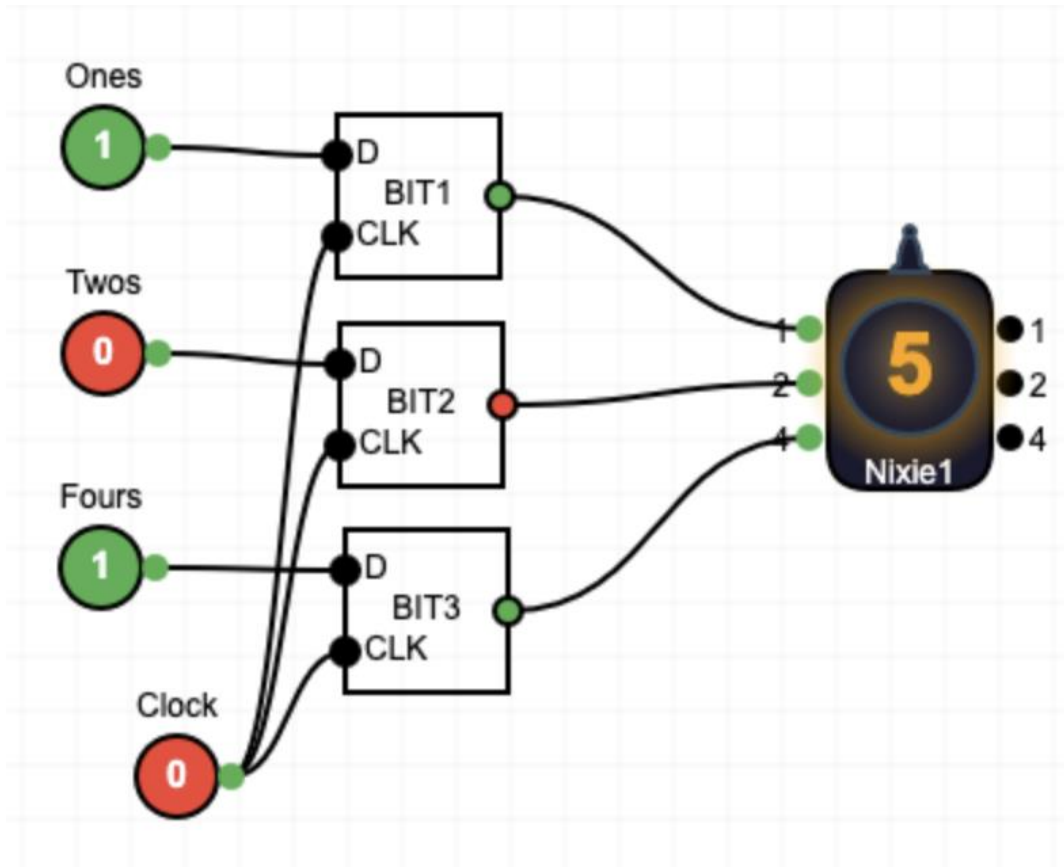
C	D	Q
0	0	Maintain
0	1	Maintain
1	0	0
1	1	1



Gated D Latch



Stack D Latches to Make a 3-bit Register



Summary

- In this lecture we started with simple gates like AND and OR
- We built latches / registers for storage

Acknowledgements / Contributions

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