

# Capacitated lot sizing models: a literature review

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13th February 2008

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## Abstract

*In the present paper, we discuss one of the many processes arising in the context of supply chain management, namely production planning. We focus on one type of production planning models called capacitated lot sizing models. These models appear to be well suited for the usually rather inflexible production resources found in the process industries. We review the literature on single-level single-resource lot sizing models as well as their extensions to multi-level and/or multi-resource problems.*

## Introduction

Production planning is the process of determining a tentative plan for how much production will occur in the next time periods, during an interval of time called planning horizon. It is an important challenge for industrial companies because it has a strong impact on their performance in terms of customer service quality and operating costs. However, production planning often proves itself to be a very complex task, mainly for the following reasons:

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- Most often a production resource is not fully dedicated to the production of a single product but is rather used to produce different types of product. In the context of process industries, the production resources available are usually not very flexible and can produce only one type of product at a time with a given production rate. Thus a production planner is faced with a competition between products sharing the same production facility and has to decide which products should be produced, when and in which quantities, while taking into account all constraints arising from the production system. In some cases, these constraints can be so tight that even finding a feasible production plan can be very difficult.
- A production plan has to meet several conflicting objectives, namely guaranteeing an excellent customer service level and minimizing production and inventory costs. Thus basic policies like not satisfying the demand exceeding the production capacity or keeping high levels of inventory to be able to meet any demand are usually not commercially acceptable or much too expensive. A good production plan is therefore the result of a trade-off between conflicting objectives.
- A production plan is never fixed for ever. Its validity is restricted to a predefined planning horizon so that at the latest, when reaching the end of the planning horizon, a new plan has to be designed that reflects the current status of the production system. Moreover reality will nearly always deviate from the plan and if the discrepancy between the plan and the actual situation is too large, the plan has to be revised before the end of the planning horizon.

Production planning is thus a difficult and recurring problem for industrial companies and there is a strong need for decision support systems. The development of such decision support systems has been the focus of a large body of the operations research literature for the last fifty years and there is now a wide variety of models available for production planning and inventory management.

In the present paper, we focus on one type of production planning models that appear especially suitable for process industries: capacitated dynamic lot sizing models. Capacitated lot sizing models are based on the following assumptions:

- Production resources have a limited capacity, can produce only one type of product at a time and are not very flexible. This means that a significant amount of setup is required to change production from one type of products to another.
- Demand for all products is deterministic and time varying. It has to be

satisfied without backlogging, i.e. the production plan should be built so that a perfect customer service level is achieved.

- There are two type of costs to be taken into account:
  - setup costs. Setup costs are the costs incurred when changing the resource configuration from one type of products to another one. They account for the loss of potential production during the duration of the setup, the additional workforce needed, the additional raw material consumed during the setup...
  - inventory holding costs. Inventory holding costs account for the opportunity costs of capital as well as for the direct costs of storing goods (warehousing, handling...).

To minimize setups costs, production should be run with large batches but at the expense of high inventory costs. On the contrary, inventory levels can be kept low if production of a product is run in frequent and small batches, but at the expense of high setup costs. Thus capacitated lot sizing models aim at finding a production schedule achieving an optimal trade-off between setup and inventory holding costs, while complying with given capacity constraints and insuring that demand for all products is satisfied without backlogging. Recent overviews on the lot sizing literature can be found among others in [DK97], [Wol02], [JD07] and [JD08].

The practical relevance of capacitated lot sizing is supported by the numerous examples of their application in various industries: tile manufacturing ([dMG94]), tire industry ([JD04]), plastic injection molding ([DN05]), textile industry ([DV00], [SM06]), paper production ([GM05]), metallic alloy moulding ([dSMdSA02], packaging lines in process industries ([SDSD86], [MNS07])... Moreover, multi-level multi-resource lot sizing models are promising candidates to replace the traditional MRP-II logic which provides only suboptimal production schedules.

The purpose of this paper is to present a general survey on capacitated lot sizing models. We will review the main contributions to this long standing but active research field, focusing particularly on recent developments.

The complexity of lot sizing models depends on the features taken into account in the model. As a first step for classification, we use the following characteristics because they strongly impact the complexity of lot sizing decisions:

- *number of resources.* The products can be made on one single machine (single-resource models) or on multiple machines (multi-resource models). The use of parallel machines complicates the problem as we not only have to determine the timing and level of production, but we also have to assign production lots to machines.

- *number of levels.* Production systems may be single-level or multi-level. In single-level systems, the final products are obtained directly from raw materials after processing by a single operation with no intermediate subassembly. Demand on products is assessed directly from customer orders or market forecasts. In multi-level systems, there is a parent-component relationship between items. Raw materials after processing through several operations change to end products. The output of an operation (level) is an input for another operation. Therefore the demand at one level depends on the lot sizing decisions made at the parents' level. As a consequence, multi-level problems are more difficult to solve than single-level problems.
- *planning horizon discretization.* Lot sizing problems can be either big bucket or small bucket problems. Big bucket problems are those where the time period is long enough to produce multiple type of items while for small bucket problems the time period is so short that only one type of item can be produced in each time period.

The paper is organized as follows. Section 1 provides a general review on established single-level single-resource models. In section 2, we then discuss single-level multi-resource models. Finally, section 3 deals with multi-level extensions of lot sizing models.

## 1 Single-level single-resource models

In this section, we deal with single-level single-resource models: all products to be made are end items and make use of the same resource with a limited production capacity.

### 1.1 Big bucket models

#### 1.1.1 The capacitated lot sizing problem (CLSP)

The capacitated lot sizing problem (**CLSP**) is a typical example of a big bucket problem, where many different items can be produced on the same resource in one time period. The classical CLSP consists in determining the amount and timing of the production of products in the planning horizon: the outcome is a production plan giving for each planning period the quantity (lot size) of each item that should be produced. However detailed scheduling decisions are not integrated in the CLSP. The usual approach is therefore to solve the CLSP first and to solve a scheduling problem for each period separately afterwards.

In the CLSP, it is required that the resource is setup for a given item in each period where it is produced. The resulting setup costs and times may vary for

each item and each period but, as the exact sequence of production within each time period is not defined, they should be *sequence-independent*, i.e. they should not depend on the exact sequence followed to make the products on the resource.

Before going on with the literature review, we briefly present the mixed-integer programming (MIP) formulation for the basic CLSP with zero setup times. We wish to optimize the production schedule for a set of  $N$  items over an horizon featuring  $T$  planning periods. A period is indexed by  $t = 1, \dots, T$ , an item by  $i = 1, \dots, N$ .

We use the following notation for the parameters:

- $D_{it}$ : deterministic demand (in units) for item  $i$  in period  $t$ ,
- $P_t$ : available production capacity (in time units) on the resource in period  $t$ ,
- $v_{it}$ : capacity needed (in time units) to produce one unit of  $i$  in period  $t$ ,
- $h_i$ : holding costs per unit and period for item  $i$ ,
- $c_{it}$ : setup costs for item  $i$  in period  $t$ .

In the CLSP, the items to be produced can have different production rates on the resource. This is why the production capacity is not expressed as the number of items that can be produced in a planning period, but rather as an available amount of time ( $P_t$ ) that will be consumed by the produced items with an item-specific production rate ( $v_{it}$ ).

Decision variables are defined as follows:

- $I_{it}$ : inventory level corresponding to item  $i$  at the end of period  $t$ ,
- $x_{it}$ : production quantity for item  $i$  in period  $t$ ,
- $y_{it}$ : binary setup variables.  $y_{it} = 1$  if the resource is setup for item  $i$  in period  $t$ , and 0 otherwise.

Using this notation, the CLSP can be formulated as a MIP model:  
(CLSP)

$$\min \sum_{i=1}^N \sum_{t=1}^T (h_{it} I_{it} + c_{it} y_{it}) \quad (1)$$

$$\forall i, \forall t, I_{it} = I_{i,t-1} + x_{it} - D_{it} \quad (2)$$

$$\forall i, \forall t, v_{it} x_{it} \leq P_t y_{it} \quad (3)$$

$$\forall t, \sum_{i=1}^N v_{it} x_{it} \leq P_t \quad (4)$$

$$\forall i, \forall t, I_{it} \geq 0 \quad (5)$$

$$\forall i, \forall t, x_{it} \geq 0 \quad (6)$$

$$\forall i, \forall t, y_{it} \in \{0, 1\} \quad (7)$$

The objective, to minimize the sum of inventory holding costs and setup costs, is expressed by (1). Constraints (2) express the inventory balance. Due to restrictions (3), production of an item can only take place if the resource is setup for that particular item. Constraints (4) are the capacity constraints. The set of constraints (2) and (5) ensure that demand for each item is fulfilled without backlogging. Inequalities (6) are the non negativity conditions on the production quantities. The binary character of the setup variables is expressed by (7).

A recent review on the literature about the CLSP can be found in [KFGW03]. The authors classify solution methods into three main categories: exact methods, common-sense or specialized heuristics and mathematical programming-based heuristics.

The use of exact methods to solve the CLSP is described among others in [BVRW84], [EM87], [BW00], [BW01] and [Wol02]. The goal of this line of research is to improve the MIP formulation of the problem using reformulations and valid inequalities so that commercial solvers like CPLEX or XPRESS-MP are able to solve practical instances using a standard Branch & Bound type procedure.

Common-sense or specialized heuristics can be found for instance in [DS81] and [KK94]. In [DS81], a first production plan is built using a greedy period-by-period heuristic based on the single-item Silver-Meal approach ([SM73]). In a second step, this initial plan is modified so that feasibility is guaranteed and costs are reduced. [KK94] have developed a heuristic algorithm using an iterative item-by-item strategy for generating solutions to the problem. In each iteration, a subset of items from those not already scheduled is selected and production schedules over the planning horizon for this set of items are determined. To ensure feasibility of the overall problem, each item is scheduled by solving a bounded single item lot sizing problem where production capacity is restricted to take into account the production of already scheduled items.

A general drawback of common-sense heuristics is that they can be rather difficult to adapt for different variants or extensions of the problem because in most cases we have to alter the heuristic completely. On the contrary, mathematical programming-based heuristics which use an optimum seeking mathematical programming procedure to generate a solution are more general and allow for extensions to different problems. Another advantage is that many of these heuris-

tics provide lower bounds on the optimal solution cost, thus providing guidance for the assessment of the quality of the obtained solution. However they usually require much more computational effort for real-world problems and due to their technical concepts cannot be implemented easily by practitioners. Many mathematical programming-based procedures used to solve the CLSP rely upon a Lagrangian relaxation of the capacity constraints. By dualizing capacity constraints into the objective function, the problem decomposes into a series of single item uncapacitated problems, each of which can be solved using an efficient single-item algorithm. This approach is applied among others by [TvW85], [TTM89] and [DBKZ92]. Some other heuristic solution approaches based on different methods like column generation or metaheuristics can also be found in the literature. The reader is referred to [KFGW03] for more details.

### 1.1.2 Extensions of the CLSP

As mentioned above, in the CLSP, the decision variables are the production quantities of every item in every period, which can be considered as production orders to be released and submitted to the shop floor. This type of model does not involve the sequence of the lots within a period: this decision has to be determined by an additional scheduling step. However the need for simultaneous lot sizing and scheduling arises in the case of sequence-dependent setup costs which is frequently encountered in process industries. Therefore recent research has focused on extending the CLSP to incorporate scheduling decisions and deal with sequence-dependent setup costs. This problem is called General Lot Sizing Problem (GLSP) in some papers.

The integration of scheduling decisions in the CLSP formulation can be done in several ways. In [Haa96] and [GM05], the production sequence within a period is defined through the use of setup state variables giving the resource configuration at the beginning of each period and a series of setup transition variables linked by flow conservation constraints. In both papers, the resulting problem is solved thanks to a specialized heuristic. [FM97] and [Mey00] use a different approach where each period of the planning horizon is divided into a fixed number of micro-periods with variable length. The production sequence within each period is obtained by assigning an item to each micro-period. Their solution method is based on the use of a local search algorithm called threshold accepting. Finally, in [HK00], the authors build a predetermined sets of efficient production sequences. In this case, the production planning problem consists in selecting for each planning period a production sequence among those already identified as efficient and in determining the corresponding lot sizes. A tailored enumeration method of the branch-and-bound type is used to optimally solve medium-size instances of the problem.

## 1.2 Small bucket models

In small bucket models, the assumption is made that during each time period, at most one type of item can be produced on the resource. Thanks to this assumption, lot sizing and scheduling decisions can be made simultaneously: namely a unique item is assigned to each planning period and the resulting sequence of item-period assignments naturally defines the production schedule. Note that in small bucket models, the production of a lot may last several periods and setup costs should be incurred in a period only if the production of a new lot begins. To model this, new decision variables often called start-up variables or changeover variables are used. In the sequel, we use the binary variable  $z_{it}$  to indicate whether the production of a new lot of item  $i$  is beginning in period  $t$  ( $z_{it} = 1$ ) or not ( $z_{it} = 0$ ).

A first small bucket model is the so-called Continuous Setup Lot sizing Problem (CSLP). In the CSLP, only one item can be produced by period and the quantity produced can be any value between 0 and the resource capacity.

Using the same notation as in section 1.1.1, a MIP model of the CSLP can be stated as follows:

(CSLP)

$$\min \sum_{i=1}^N \sum_{t=1}^T (h_{it}I_{it} + c_{it}z_{it}) \quad (8)$$

$$\forall i, \forall t, I_{it} = I_{i,t-1} + x_{it} - D_{it} \quad (9)$$

$$\forall i, \forall t, v_{it}x_{it} \leq P_t y_{it} \quad (10)$$

$$\forall t, \sum_{i=1}^N y_{it} \leq 1 \quad (11)$$

$$\forall i, \forall t, z_{it} \geq y_{it} - y_{i,t-1} \quad (12)$$

$$\forall i, \forall t, I_{it} \geq 0 \quad (13)$$

$$\forall i, \forall t, x_{it} \geq 0 \quad (14)$$

$$\forall i, \forall t, y_{it} \in \{0, 1\} \quad (15)$$

$$\forall i, \forall t, z_{it} \in \{0, 1\} \quad (16)$$

The objective, to minimize the sum of inventory holding costs and startup costs, is expressed by (8). Constraints (9) express the inventory balance. (10) guarantee that production of an item can only take place if the resource is setup for that particular item and that capacity limits are respected. Constraints (11) ensure that only one item may be produced per period. The beginning of a new lot is defined by means of inequalities (12). The set of constraints (9) and (13) ensure that demand for each item is fulfilled without backlogging. Inequalities (6) are the non negativity conditions on the production quantities. The binary character of



the setup and startup variables is represented by (15) and (16).

[KS85] try to solve the CSLP using Lagrangian relaxation applied to the capacity constraints. More recently, [Con96] presents a cutting plane approach based on several families of valid inequalities derived for the single-item version of the problem. [Van98] develops an integer programming column generation algorithm to solve the same problem and uses the cutting-planes proposed by [Con96] to tighten the formulation of the master linear program at each node of the branch-and-bound tree.

The Discrete Lot sizing and Scheduling problem (**DLSP**) is another small bucket model. The difference with the CSLP is that a discrete production policy is assumed, implying that an item, if assigned to a planning period, must be produced at full capacity. This "all-or-nothing" assumption gives the problem some additional properties that make efficient implementation of mathematical programming approaches somewhat easier. It is enforced by replacing in the CSLP formulation the inequalities (10) by the equalities:

$$\forall i, \forall t, v_{it}x_{it} = P_t y_{it} \quad (17)$$

The first contributions on the DLSP used sequence-independent setup costs. [Fle90] solve medium-size instances using a branch-and-bound procedure where the lower bounds are determined by means of Lagrangian relaxation. [CSKvW93] describe a heuristic for the DLSP with positive setup times based on dual ascent and column generation techniques.

The DLSP with sequence-dependent setup costs is addressed in [Fle94] and [SSvW<sup>+</sup>97] who both reformulate the problem as as Travelling Salesman Problem with Time Windows. Studying the same variant, [JD98] show the equivalence between the DLSP with a single resource and a scheduling problem named Batch Sequencing Problem (BSP) and present a specific branch-and-bound type algorithm to solve the resulting BSP.

There is also a rather large amount of polyhedral results for the DLSP. Strong valid inequalities for the single-item variant can be found in [MV90], [vEvH97], [MS02] and [MW03]. These valid inequalities can be used to tighten the formulation of multi-item instances, thus improving the efficiency of the standard branch-and-bound procedure embedded in commercial solvers. A good overview on polyhedral results for the DLSP can be found in [Wol02] and [PW06].

## 2 Single-level multi-resource models

The lot sizing models presented in the previous section assume that the products are processed on a single machine. However in many cases a manufacturer has access to multiple machines or production lines, which can be used in parallel. In this section, we focus on the single level, parallel resources problem. A recent review on lot sizing problems involving parallel resources can be found in [Jan06]. As mentioned above, parallel resources further complicate the production planning problem. Namely, as an item can be produced on several machines, there is an additional decision to be made: the assignments of production lots to resources. As for the single-resource models, a distinction can be made between big bucket and small bucket models.

### 2.1 Big bucket models

We first consider extensions of the classical CLSP described in section 1.1.1 to the case of parallel resources. [OB98] consider a capacitated lot sizing problem with parallel machines. They assume that a lot can not be split among several machines so that in one specific period, an item can be produced on one machine at most. They develop hybrid heuristics combining local search techniques such as tabu search and a genetic algorithm to deal with the resulting problem. [TA06] address the same problem and propose a heuristic based on the Lagrangian relaxation of the capacity constraints and subgradient optimization: at each iteration, a series of single-item multi-resource problems are solved using a dynamic programming algorithm. Finally, [Jan06] focuses on the CLSP with parallel identical machines: all the resources have the same available capacity and the setup and production costs are identical on each of the resources. In this case, there exist a large number of equivalent solutions with the same total cost that differ only by the numbering of the machines. As this degeneracy will slow down the branch-and-bound algorithm, he proposes to add symmetry breaking constraints to the mixed integer programming formulation in order to obviate this problem.

There are also some papers extending the big bucket models with sequence-dependent setup costs presented in section 1.1.2 to the case of several parallel resources. Among them, [KMT98] propose an original model where the sequence of products produced on a machine in a period is modelled as a collection of subsequences. Each subsequence is made of at most 5 items and by enumeration, an optimal ordering for these items can be found. The lot sizing problem is then formulated as the problem of assigning a subsequence chosen among those predetermined to a position in the global production schedule on each resource. The authors propose a column-generation approach combined with a branch-and-bound procedure to solve the resulting problem. [CC00] use a model similar to the one

presented in [Haa96] to solve a variant of the CLSP with heterogeneous parallel resources and sequence-dependent setup times. The problem is solved heuristically using a rolling-horizon method. While planning production on a rolling horizon basis, only the lot sizing and sequencing decisions regarding the first periods of the horizon will be actually implemented in the production system. Namely after a few periods, the horizon is rolled forward and the model is applied once more with updated demand, inventory and capacity information. [CC00] propose to determine precisely the lot sizing and sequencing decisions only for the first planning periods. The other production decisions for the end of the planning horizon (which will not be actually implemented) are only approximately evaluated, without considering explicitly setup costs and times. This enable them to reduce the size of the mixed integer program to be solved and thus to save a significant amount of computing time while avoiding some drawbacks arising from a purely myopic approach. In a recent paper, [Mey02] extends his GLSP model to the case of parallel production lines and uses a solution procedure combining local search strategies with dual reoptimization to solve real problems gathered from the consumer goods industry.

## 2.2 Small bucket models

We now present extensions of the small bucket models to the case of multiple parallel resources.

In [SDSD86], a first extension of the CSLP is used to plan production on several packaging lines in a process industry. In their model, the authors consider that an item is a combination of a package size and a product to be filled into the packages. They assume that items can be grouped into families: a family can be either a package size or a product according to the industrial application. A major setup will occur if a transition between items belonging to different families has to be carried out whereas the transitions between two items belonging to the same family will lead only to a minor setup. In their model, they impose that only one family can be produced per planning period and focus on defining the exact sequence of family-period assignment. They use a standard branch-and-bound procedure to solve small instances involving 4 products, 5 periods and a single production line. Industrial applications of the CSLP with multiple parallel resources can be found in [DN05] for the planning of injection molding operations and [MNS07] for the planning of a yoghurt-packaging facility. In both papers, specialized heuristics are used to solve industrial instances of the problem.

[dMG94] consider production planning for the curing stage in a tile manufacturing facility. The problem is formulated as a DLSP with heterogeneous parallel resources (the curing kilns). They apply Lagrangian relaxation to the inventory balance constraints to decompose the problem into a series of single-resource independent subproblems. Combining this with a subgradient optimization method,

they are able to obtain strong lower bounds on the optimal cost. Feasible production schedules are generated from every Lagrangian solution using a so-called product-line assignment heuristic. Another industrial extension of the DLSP can be found in [JD04] who propose a production planning model for an international tire manufacturer. The problem involves multiple capacitated resources of different types: the molds and the heaters needed to build and cure the tires. It is solved by a column-generation-based algorithm combined with Lagrangian relaxation to reduce the degeneracy of the master problem. [SM06] solve a DLSP with multiple parallel machines arising in a company producing acrylic fibres using a problem-specific heuristic.

### 3 Multi-level multi-resource models

In a multi-level lot sizing problem, the production planning is not only considered for the final level (i.e. the end products), but also for the components and subassemblies that are needed to make the end products. Because of the parent-component relationship between items, production at one level leads to demand for components at a lower level (dependent demand). At the highest level, production is triggered by market demand (independent demand).

The parent-component relationship between items, also known as the bill of materials, is usually represented by an acyclic directed network where every node in the network is an item, an arc represents the assembly or distribution relation between items and the weight of an arc is the quantity relation (also called the "gozinto factor") between the two terminal nodes of the arc. Different kinds of product structures can be distinguished:

- serial product structure: each item has a single predecessor and a single successor in the network.
- assembly product structure: each item can be made from several predecessors (i.e. components) but has a single successor (i.e. parent).
- general product structure: each item can be made from several predecessors and can have several successors. Thus there may be several end products that have some components in common: this situation is sometimes referred to as component commonality.

Most contributions on multi-level lot sizing problem use big bucket models and a general product structure. They can thus be seen as extensions of the classical CLSP described in section 1.1.1 to the multi-level case. This is why we chose to classify the literature with respect to the type of solution approach used rather than with respect to the planning horizon discretization.

We classify solution methods into four main categories: exact methods, specialized heuristics, mathematical programming-based heuristics and metaheuristics.

### 3.1 Exact methods

Most single-level capacitated lot sizing problems are NP-hard. The multi-level extension makes them even harder because of the interdependency between levels created by the parent-component relationship between items. The demand at lower levels is namely the result of the lot sizing decisions made at highest levels. Most practical instances are too difficult to be optimally solved with a commercial integer optimization software. Therefore most existing solution approaches are based on heuristic techniques and the literature on exact solution methods to solve multi-level capacitated lot sizing problems is rather sparse.

Noticeable exceptions can be found in [PW91], [BW00], [BW01], [Wol02] and [PW06]. All these papers are based on the concept of echelon stock. The echelon stock of an item in a given period can be defined as the total stock of this item within the system, whether held directly as stock or as the stock of other items containing one or more units of this item. The problem can be reformulated using echelon stock variables and the obtained reformulation can be seen as a series of single-item lot sizing subproblems linked by capacity constraints. Thanks to this, valid inequalities available for single-item problem can be used. Some computational experiments based on a branch and cut procedure using these valid inequalities can be found in [BW00] and [BW01] but they are limited to instances involving a single resource.

### 3.2 Specialized heuristics

Several dedicated heuristics have been proposed for solving multi-level extensions of the CLSP. They mainly aim at building a good feasible solution for the problem, but without assessing the quality of the found solution with respect to some lower bounds on the optimal cost.

Most of them follow a level-by-level approach but modify the setup and inventory costs at lower levels to model the interdependencies. [BBM<sup>+</sup>94] study a multi-level CLSP with a serial product structure. They solve the problem by applying sequentially a multi-item single-level specialized heuristic to each level of the problem, beginning with the end products and proceeding through the raw materials. To compensate with this level-by-level myopic approach, before solving the lot sizing problem at a given level, they modify the setup and inventory costs following the procedure described in [BM82]. This cost-adjustment approach enables them to (approximately) model the impact of the lot sizing decisions made

at the given level on the lowest levels. A similar approach is used in [TH94] for general product structures.

Another type of special-purpose heuristic can be found in [CA95]. The authors study a multi-level CLSP with multiple resources and a general product structure. The starting point for their heuristic is a feasible production plan for the uncapacitated problem. It is obtained by applying sequentially the optimal Wagner-Whitin algorithm to solve single-item single-level problems, beginning with the end items and proceeding to items at the lowest levels. Afterwards, they try to achieve a feasible production plan for the capacitated problem by moving production backwards in time from overloaded periods to earlier underloaded ones while maintaining the feasibility of the plan with respect to demand satisfaction and component availability. With their heuristic, they were able to find good solutions for instances involving 40 items, 2 resources and 12 planning periods.

### 3.3 Mathematical programming-based heuristics

As already mentioned for the single-level single-resource CLSP, mathematical programming based heuristics make use of an optimum seeking mathematical programming methodology and adapt it to generate good feasible solutions for practical instances.

A first example of such an approach can be found in [TD96]. They propose to solve the multi-level multi-resource CLSP with a general product structure by Lagrangian relaxation applied to both the multi-level inventory balance constraints and the resource capacity constraints. Thanks to this relaxation, the overall problem is decomposed into single-level single-item lot sizing subproblems. They use subgradient optimization to update the Lagrangian multipliers and obtain good lower bounds on the cost of an optimal production plan. This procedure is combined with a sophisticated forward and backward scheduling heuristic to transform the obtained unfeasible solutions into good feasible solutions for the initial problem.

A second family of mathematical programming-based approaches involves various Linear Programming relaxations of a MIP formulation of the multi-level multi-resource CLSP. [MMvW91] solve a multi-level CLSP with an assembly product structure and several resources, each of them being dedicated to a specific product level. They reformulate the problem using extended production variables and solve the linear programming relaxation of the obtained tightened formulation. They try to build a feasible solution for the initial problem by applying a number of rounding heuristics on the linear programming solution. Their heuristic was tested on instances involving only serial product structures with up to 3 levels. [HL96] and [KLH98] describe a coefficient-modification heuristic where small LP restrictions of the original problem are repeatedly solved. At each iteration, capacity constraints

and objective function coefficients are modified in the linear program to account for the capacity consumed and the costs incurred by the setups on the resource.

[Sta03] proposes to reduce the complexity of the overall MIP model by using a time-oriented decomposition approach leading to the resolution of a series of reduced-size mixed integer programs. In this approach, lot sizing decisions are not made altogether for the entire planning horizon but sequentially, each time for a limited time interval called the lot sizing window. In each step, setup decisions are made only for the periods within the lot sizing windows while setup decisions already made for previous periods are taken into account and setup decisions for periods following the lot sizing window are only approximated through continuous variables. The resulting sub-model whose size is drastically reduced is solved by a commercial solver. Lot sizing windows are then deployed in internally rolling schedules up to the end of the planning horizon given by the initial decision problem so that a production schedule for the entire horizon is obtained. Their computational experiments show that the heuristic they propose provides a better solution quality than the heuristic found in [TD96].

### 3.4 Metaheuristics

In the past decade, meta-heuristics such as tabu search, simulated annealing and genetic algorithms have become more and more popular for solving complex combinatorial problems. One of the main reasons for their success is their flexibility and ability to handle large and complex problems. Thus these methods seem especially adapted for multi-level extensions of the standard lot sizing problems. But a major disadvantage is the fact that they do not provide a lower bound to assess the solution quality: it has to be calculated separately. Moreover, although their basic principle are easy to understand, this type of algorithms are in fact fairly complex because of all the special adaptations that are needed to make them work better.

Applications of metaheuristics to solve multi-level lot sizing problems can be found among others in [Kim99], [OB99], [OB00], [HC00], [XD02], [BR04] and [BFA05]. A detailed review on this subject can be found in [JD07].

## Conclusion

In the present paper, we reviewed the literature on single-level single-resource lot sizing models as well as their extensions to multi-level and/or multi-resource problems. Although research on capacitated lot sizing started some fifty years ago, lot sizing problems are still challenging because many extensions are very difficult to solve. This research field thus remains very active.

As mentioned by [JD08], the research on lot sizing is currently evolving towards two directions:

- Whereas the early models were usually more compact and captured only the main trade-off, there is now an increased attention to model into more detail specific characteristics of the production system such as sequence-dependent costs, multiple resources, backlogging... The objective is to better represent real life production planning problems and to provide more valuable decision support to managers. As a rider to this, [Poc01] indicates that modelling production planning problems in the process industries is a promising area for new research. Namely some distinguishing characteristics of process industries such as the use of flexible recipes, the existence of by-products, additional storage constraints... influence the planning and scheduling problem.
- Another new interesting research area deals with the integration of lot sizing models into more global models in order to better coordinate production and distribution decision. Examples of integrated production-distribution planning models can be found in [FV99], [OY99], [TK00], [CF01], [dMM04], [SY05], [Par05] and [EER07]. A literature review on these integrated models is currently underway.

Finally, solution approaches for such difficult extensions of the lot sizing problems should be based on previous research. Hybrid optimization procedures combining the strength of different methodologies like MIP formulation strengthening and meta-heuristics seem to be a promising research direction.



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