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ABSTRACT: This paper addresses the joint carrier's selection and inventory control problem in two stage supply chains. To meet the random market demand and to deal with periods of unavailability of the internal transportation fleet, the shipper has two disparate decisions to make. At the first stage, the random lack of internal and external transportation capacities needs to be governed and compensated at the accurate moment by another carrier who is required from the spot market. At the second stage, the distribution center needs to respond to the market demand with minimal inventory and backlog costs. Our objective is to find joint optimal strategies for the carriers selection and the distribution center inventory control that minimizes the expected discounted cost of transportation, inventories and backlogs/late deliveries. It is shown that from a mathematical point of view, the considered problem is difficult to tackle and it calls upon dynamic programming and optimal control theory notions. A dynamic programming formulation of the problem is thus proposed. A numerical schema is then proposed to solve the obtained optimality conditions equations. A complete joint strategy is finally developed and analyzed. To illustrate the practical usefulness and the robust behavior of the developed strategy, several sensitivity analyses are carried out.

KEYWORDS: *carrier's selection, inventory control, optimal control, dynamic programming, numerical schema*

1 INTRODUCTION

Nowadays, outsourcing is becoming an important component of competitive strategy for companies. Outsourcing is defined as passing a part or all of the logistics functions to another organization (Parashkevova, *2007*). According to *Transport Intelligence (2006)*, around US$265 billion is outsourced of the total US$972 billion logistics market in the world to contract logistics providers and freight forwarders. There are many factors that may act as driving forces behind outsourcing. Globalization of business has been viewed by many authors as the most prominent one (Trunick, *1989;* Sheffi, *1990;* Pirannejad et al., *2010)*. The lack of internal capacity also forces sometimes companies to use an external logistics provider. Another major factor promoting outsourcing is the adaptation of the just-in-time (JIT) principles in many firms. With the shift to JIT delivery, inventory and logistics control have become more crucial to manufacturing and distribution operations *(Sheffi, 1990; MA Razzaque and Sheng, 1998)*. The main goal of outsourcing is the reduction of labor costs. Elliot (2006) states that, "in most cases the objective of outsourcing is a targeted 20% cost reduction, with actual savings coming from direct labor and variable costs."

The most frequently outsourced service is freight transportation. Compared to all outsourced logistics activities, the percentage of the outsourced domestic and international transportation service reached 91% and 87% in Europe, 85% and 89% in Asia-Pacific and 77% and 68% in North America, in 2007 as reported by *The State of Logistics Outsourcing*.
Freight transportation carriers present the physical connection between shippers and their customers. Shippers are the beneficial owners of freight; they can be for example manufacturers, distributors, and retailers. Carriers are transportation companies such as trucking, railroads, airlines, and ocean transport providers. When outsourcing their transportation activities, companies cope with an important decision problem: carriers' selection.
Several studies pointed out that shippers generally consider several criteria when selecting carriers. Premeaux Shane *(2002)*, for example, reported six criteria considered as important by shippers: information access, consistent performance, solid customer relations, flexible rates, service quality, and the availability of certain desired services such as effective responses in emergency or unexpected situations. Hong et al. (2004) emphasize on four criteria which are service quality, rate level, service reliability, and service speed. Voss *et* al. *(2006)* and Liao and Rittscher (2007) reported that the reliability of delivery and transfer prices are the first two criteria for selecting carriers. The reader is referred to Meixell *and* Norbis *(2008)* for an extended survey on carrier's selection criteria.

Building on these criteria, many researchers have tackled the problem of carriers' selection at a strategic level and proposed many approaches to solve it. Liberating and Miller (1995), for example, suggested the use of the Analytic Hierarchy Process (AHP) by incorporating transportation costs and service quality. Other authors used mathematical programming methods and heuristics for carrier's selection, in cases where demand is deterministic or static (Moore et al. (1991), Bolduc et al. (2007), Mohammaditbar and Teimoury (2008)). Moore et al. (1991) studied the case of Reynolds Metal Company for selecting and deploying truck carriers. They developed a MIP (Mixed Integer Programming) model to formulate the problem. The MIP has a single objective that minimizes the costs incurred in moving goods (freight costs) subject to individual carrier capacity constraints, equipment commitments, and other transportation-specific concerns. Bolduc et al. (2007) discussed the problem of simultaneously selecting customers to be served by external carriers and routing a heterogeneous internal fleet. They propose a SRI (selection, routing and improvement) heuristic which aims to minimize the sum of the external carrier cost and the variable and fixed costs of the internal fleet. Mohammaditbar and Teimoury (2008) addressed the problem of selecting carriers for serving specific company needs. They developed a linear programming model to determine the number of shipments allocated to selected carriers with the objective of maximizing profit and minimizing the inventory and transportation costs. Caplice and Sheffi (2003, 2006, and 2007) propose an auction-based trading mechanism for the procurement of transportation services. In such auctions, the shipper submits its transportation requests, also called lanes, to the participating carriers. Then carriers compete by submitting bids on package lanes, called combinatorial bids, to the shipper. The latter selects winning bids so as to minimize the total transportation procurement cost. This problem is known as the winner determination problem (WDP). The authors developed mathematical models for solving the WDP problem under several contexts.

Others papers proposed approaches to select carriers in case where demand is dynamic. Liao and Rittscher (2007), for example, treated the integration of three decisions: (1) supplier selection, (2) dynamic procurement lot sizing or replenishment and (3) carrier selection for a single purchasing item over multiple planning periods. A multi-objective programming model is developed and a genetic algorithm is proposed to obtain good solutions. The multiple objectives include the total logistics cost, the total quality rejected items and the total late deliveries. Lin and Yeh (2010) discussed the carrier selection with the network reliability including air routes and land routes. The objectives of this study are first to find the optimal choice of carriers that maximizes the network reliability and second to determine the routes assigned to each selected carrier. They developed a genetic algorithm to solve this problem, named OCSNR (Optimal Carrier Selection problem based on Network Reliability).

To the best of our knowledge, the issue of selecting carriers at the operational level in a dynamic stochastic context remains open. This paper discusses this issue in the case where a company should move a family of products from a warehouse to a distribution center. To ensure this shipment service, the decision maker has to decide whether using its internal fleet or an external fleet belonging to a for-hire contract carrier already selected at a strategic level. In addition to these two fleets, one can decide to go for carriers from the spot market, called "spot carriers" in the rest of the paper. Considering spot carriers becomes necessary in the case where the capacities of internal and external fleets are insufficient due to the uncertainty characterizing the carrier's availabilities and the market demand. Indeed, Caplice (2007) reported that three most common contractual arrangements are traditionally used by shippers for truckload services: private fleets, for-hire contract carriers, and spot. For-hire contracts assume a relatively long-time period (one year or longer) on which the shipper and the carrier engage on. The main goal of this paper is to determine the optimal policy for selecting the internal, for-hire contract and spot carriers subject to their availabilities, the variability of demand and the variability of inventory in the distribution center so as to minimize the transportation, storage and backlog costs. We propose a continuous dynamic programming formulation of the problem. A numerical approach is adopted to solve the associated optimality equations. Several sensitivity analyses are carried out and show the robust behavior of the developed policy.

The reminder of the paper is organized as follows. Section 2 presents the statement of the problem. Section 3 presents the numerical approach. Section 4 describes the resolution approach. The obtained results and the related carriers' selection strategy are presented and developed in Section 5. Section 6 contains discussions and concluding remarks.

2 PROBLEM STATEMENT

The problem under study consists of choosing the best carriers selection strategy to ship a family of products P from a warehouse to a distribution center belonging to the same company. In a dynamic stochastic context, the decision maker has to find the best compromise regarding which carrier is willing to insure the shipments given the real conditions under which the system dynamic evolves. These conditions are related to the available inventory in the distribution center, the availability of carriers and the demand status. The overall optimal decision policy is defined here as one that minimizes the expected long term total cost of inventory, backlog and transportation costs, in the presence of random availability of carriers and random demand. Figure 1 illustrates the system under study, its dynamic behavior, and the associated costs to be minimized.

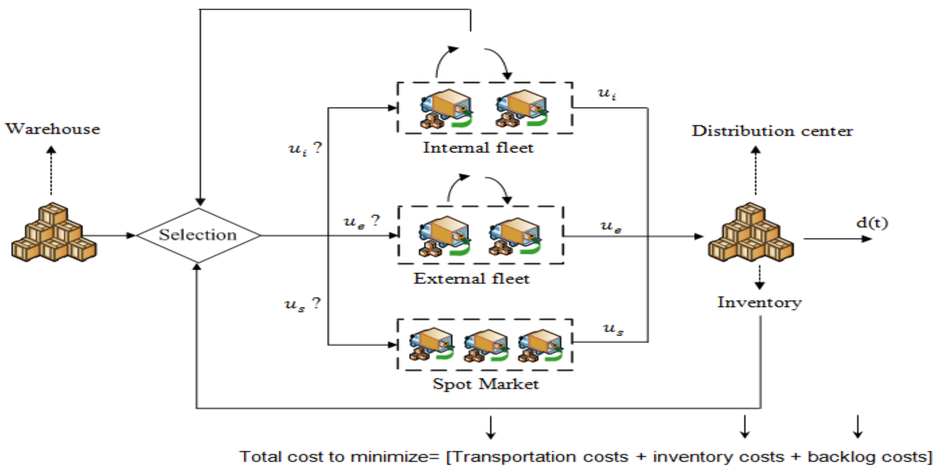

Figure 1: System under study

The following notations are used in the rest of this paper:
i : Internal fleet index;
e: External carrier index;
s: Spot carrier index;
$d(t)$: Demand rate at time t;

$x(t)$: Inventory/backlog stock levels of the distribution center at time t;
$u_i(t)$: The internal fleet transportation capacity available at time t;
$u_e(t)$: The external fleet transportation capacity available at time t;
$u_s(t)$: The spot carrier transportation capacity available at time t;
U_i^{max} : Maximal transportation capacity of the internal fleet;
U_e^{max} : Maximal transportation capacity of the external fleet;
U_s^{max} : Maximal transportation capacity of the spot carriers;

$\zeta_c^i(t)$: Continuous time and finite state Markov process of internal fleet i;

$\zeta_c^e(t)$: Continuous time and finite state Markov process of external fleet e;

$\zeta_c^d(t)$: Continuous time and finite state Markov process of demand;
H : Per unit time inventory cost of one unit product P;
B : Per unit time backlog cost of one unit product P;
C_i : Per unit product transportation cost of internal fleet;
C_e : Per unit product transportation cost of external fleet;
C_s : Per unit product transportation cost of spot carrier;
ρ : Discounted rate of the incurred cost;
J(.) : Expected and discounted total cost function;

The state of the system at time t has three components:

- a continuous part, measured by $x(t)$, which describes the cumulative surplus of product P at the distribution center,
- a discrete part which describes the internal and the external fleet availability state. It is assumed here that spot carriers are always available,
- a discrete part which describes the demand level at time t.

The state of the internal fleet is described by the random variable $\zeta_c^i(t)$ with values in $M_i = \{1, 0\}$. This state is assumed to evolve according to a continuous time discrete state Markov chain where:

$$\zeta_c^i(t) = \begin{cases} 1 \text{ if the internal fleet is available} \\ 0 \text{ if the internal fleet is unavailable} \end{cases} \quad (1)$$

Similarly, the state of the external fleet is represented by a continuous time discrete state Markov chain $\zeta_c^e(t)$ with values in $M_e = \{1, 0\}$ where:

$$\zeta_c^e(t) = \begin{cases} 1 \text{ If the external fleet is available} \\ 0 \text{ If the external fleet is unavailable} \end{cases} \quad (2)$$

The operational mode of the carriers can be described by the random vector $\boldsymbol{\zeta_c}$ (ζ_c^i, ζ_c^e) taking values in $M = M_i \times M_e$. $\zeta_c(t)$ can be expressed as follows:

$$\zeta_c(t) = \begin{cases} 1 = (1,1) & \text{internal \& external fleets are available} \\ 2 = (1,0) & \text{internal fleet available \& external unavailable} \\ 3 = (0,1) & \text{internal fleet unavailable \& external available} \\ 4 = (0,0) & \text{internal \& external fleets unavailable} \end{cases} \quad (3)$$

The internal and external fleet uptimes and downtimes are assumed to be exponentially distributed with their availability rate. The transportation system state evolves according to a continuous-time Markov process with states in M and with a generator matrix Q^c such that: $\mathbf{Q}^c = (q_{\alpha\beta}^c)$, where $q_{\alpha\beta}^c$ denotes the transition rate from state α to β, $q_{\alpha\beta}^c \geq 0$ and $q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}$, $\alpha, \beta \in M$. The transition rate matrix Q^c is expressed as follows:

$$\mathbf{Q}^c = \begin{vmatrix} q_{11}^c & q_{12}^c & q_{13}^c & 0 \\ q_{21}^c & q_{22}^c & 0 & q_{24}^c \\ q_{31}^c & 0 & q_{33}^c & q_{34}^c \\ 0 & q_{42}^c & q_{43}^c & q_{44}^c \end{vmatrix} \quad (4)$$

The demand of product P to be satisfied from the distribution center is random and its rate can take two values d_1 or d_2. The random process of demand is described by a continuous time discrete space stochastic process $\zeta_d(t)$ with values in $M_d = \{1, 2\}$. $\zeta_d(t)$ takes 1 if $d=d_1$ and 2 if $d=d_2$.
The generator matrix $\mathbf{Q}^d = (\boldsymbol{q^d_{\gamma\delta}})$, where $q^c_{\gamma\delta}$ denotes the transition rate from state $\boldsymbol{\gamma}$ to $\boldsymbol{\delta}$, $q^d_{\gamma\delta} \geq 0$ and $q^d{}_{\gamma\gamma} = -\sum_{\delta\neq\gamma} q^d{}_{\gamma\delta}$, $\gamma, \delta \in M_d$ is expressed as follows:

$$\mathbf{Q}^d = \begin{vmatrix} q^d_{11} & q^d_{12} \\ q^d_{21} & q^d_{22} \end{vmatrix} \tag{5}$$

The dynamic of the stock levels is given by the following differential equations:

$$\dot{x}(t) = u_i(t) + u_e(t) + u_s(t) - d(t) \tag{6}$$

At any given time, the available transportation capacity of the internal fleet, the external fleet and the spot carriers have to satisfy the overall capacity constraint. These constraints are given by equations (7), (8) and (9) where $U_i{}^{max}$, $U_e{}^{max}$ and $U_s{}^{max}$ denote the maximal transportation capacities of the internal fleet, external fleet and spot carriers, respectively.

$$0 \leq u_i(t) \leq U_i^{max} \tag{7}$$

$$0 \leq u_e(t) \leq U_e^{max} \tag{8}$$

$$0 \leq u_s(t) \leq U_s^{max} \tag{9}$$

Without loss of generality, due to the random availability of the internal and the external fleet, noted AVY_i and AVY_e respectively, we assume that the expected average demand rate cannot be met with internal and external fleets. This constraint is given by the following equation:

$$U_i^{max} \times AVY_i + U_e^{max} \times AVY_e < \tilde{d}(t) \tag{10}$$

Where $\tilde{d}(t)$ denote the expected average demand rate given the stochastic process $\zeta_d(t)$.
One should note that AVY_i and AVY_e can be easily found from the transitions rates expressed from the individual or the overall Markov chains given by equations (1) to (4).
As mentioned previously, the presence of spot carriers makes it possible to meet the demand, constraints (10) becomes in this case:

$$U_i^{max} \times AVY_i + U_e^{max} \times AVY_e + U_s^{max} \geq \tilde{d}(t) \tag{11}$$

The set of admissible capacity of all carriers is given by:

$$\mathrm{A} = \left\{ \begin{matrix} (u_i(t), u_e(t), u_s(t)); \\ 0 \leq u_k(t) \leq U_k^{max}; k = i, e, s;\ U_i^{max} \times AVY_i \\ + U_e^{max} \times AVY_e + U_s^{max} > d(t) \end{matrix} \right\} \tag{12}$$

Our decision variables are the per-period transportation capacities assigned to the internal fleet, external fleet and spot carrier given by $(u_i(t), u_e(t), u_s(t))$. Given the maximal available capacities of the fleets and the overall state of the system (i.e., random availability of internal and external fleets, demand rate and inventory level in the distribution center), admissible decisions cover the two boundaries of the decision space either internal fleet, external fleet and spot carrier are not selected (i.e, $u_i(t)$=0, $u_e(t)$=0, $u_s(t)$=0), or selected and they moved frets under their maximum transportation rate (i.e, $u_i(t)=U_i^{max}$, $u_e(t)=U_e^{max}$, $u_s(t)=U_s^{max}$).
The decision made by the shipper is also conditioned by the involved transportation costs. Let C_i, C_e and C_s denote the per unit product transportation cost of internal fleet, external fleet and spot carrier, respectively. Without loss of generality, it is reasonable to assume that moving goods by the spot carrier is much more costly than using the external and internal fleets. This assumption is given by:

$$C_i < C_e \ll C_s \tag{13}$$

The inventory, backlog costs applied in the distribution center and the transportation cost are expressed by the following instantaneous cost function g(.) :

$$\mathrm{g}(x, u_i, u_e, u_s) = Hx^+ + Bx^- + C_i u_i(t) + C_e u_e(t) + C_s u_s(t) \tag{14}$$

Where $x^+ = \max(0, x)$, $x^- = \max(-x, 0)$; H and B are positive constants representing the inventory /backlog costs due to surplus and late delivery, respectively.

The discounted total cost J(.) can be defined by the following equation:

$$\mathrm{J}(\mathrm{x}, \alpha, \gamma, u_i, u_e, u_s) = \mathrm{E}\left[\int_0^\infty e^{-\rho t} g(x, u_i, u_e, u_s).dt\right] \tag{15}$$

Where ρ denotes the discounted rate of the incurred cost and $E[.|x_0\alpha_0\gamma_0]$ is the expectation operator conditional to the initial overall system state conditions.

The considered problem consists in finding an admissible decision or control policy that minimizes the discounted total cost J(.) (15), subject to the system dynamic and constraints (3) to (14). Such a feedback control policy, as illustrated in Figure 1, determines the per period transportation volume to affect to the available carriers function of the surplus level $\boldsymbol{x}$, the state of the system $\boldsymbol{\alpha}$ and the state of demand $\boldsymbol{\gamma}$.

The value function of the transportation problem is described as follows:

$$v(x, \alpha, \gamma) = \min_{(u_i, u_e, u_s) \in \mathrm{A}} J(x, \alpha, \gamma, u_i, u_e, u_s) \quad \forall\, \alpha \in M, \gamma \in M_d \tag{16}$$

Hamilton Jacobi Bellman (HJB) equations associated with the value function (16) are a hyperbolic system of partial differential equations. These equations describe the optimal control strategy of the carrier's selection and inventory control problem and are given by:

$$\rho . v\,(x,\alpha,\gamma) = \min_{(u_i,u_e,u_s)\in A}\Big\{(u_i + u_e + u_s - d)\frac{\partial}{\partial x} v\,(x,\alpha,\gamma) + g(x,u_i,u_e,u_s) + \sum_\beta q_{\beta\alpha} v\,(x,\beta,\gamma) + \sum_\delta q_{\delta\gamma} v\,(x,\alpha,\delta)\Big\} \quad (17)$$

One can show that the value function (16) is locally Lipschitz and is the unique viscosity solution to the HJB equations (17). We refer the reader to Hajji et al. (2009), Sethi and Zhang (1994) and the references given therein for more details.

The carrier's selection policy that we are seeking for is obtained when the value function is known. While it's very complex if not impossible to analytically solve the HJB equations (17), the next section proposes a numerical method to obtain the approximation of the value function and the associated control policy.

3 NUMERICAL APPROACH

Numerical approaches are used to discretize and reduce the infinite space state associated to the problem. Thus, the unlimited domain, which is associated with broad control horizon, is replaced by a limited area including proper boundary conditions of the borders. The solution of this system must converge to the solution of the initial problem when the discretization step tends to zero.

In order to approximate the solution of the HJB equations (17) corresponding to the stochastic optimal control problem, and to solve the corresponding optimality conditions, a numerical method based on Kushner and Dupuis (1992) approach is adopted. The basic idea consists in using an approximation scheme for the gradient of the value function $v\,(x,\alpha,\gamma)$. Let h denotes the length of the finite difference interval of the variable *x*. Using the finite difference approximation, $v(x,\alpha,\gamma)$ is given by $v^h\,(x,\alpha,\gamma)$. The gradient of the value function $v_x(x,\alpha,\gamma)$ is approximated by:

$$(v)_x^h(x,\alpha,\gamma) = \begin{cases} \frac{1}{h}(v^h(x+h) - v^h(x)) \text{ if } u_i + u_e + u_s - d \geq 0 \\ \frac{1}{h}(v^h(x) - v^h(x-h)) \text{ if } u_i + u_e + u_s - d < 0 \end{cases} \quad (18)$$

Also, we could see that:

$$(u_i + u_e + u_s - d)\frac{\partial}{\partial x} v\,(x,\alpha,\gamma) = \frac{|u_i+u_e+u_s-d|}{h_1} .\ v^h(x+h_1)\,K_1^+ + \frac{|u_i+u_e+u_s-d|}{h_1} . v^h(x-h_1)K_1^- - \frac{|u_i+u_e+u_s-d|}{h_1} . v^h(x)$$

Where $K_1^+ = \begin{Bmatrix} 1 \ if \ u_i + u_e + u_s - d \geq 0 \\ 0 \ otherwise \end{Bmatrix}$ (19)

$K_1^- = \begin{Bmatrix} 1 \ if \ u_i + u_e + u_s - d < 0 \\ 0 \ \ otherwise \end{Bmatrix}$ (20)

Using this approximation, the HJB equations (17) can be expressed in terms of $v^h\,(x,\alpha,\gamma)$, as shown in equation (21).

$$v^h\,(x,\alpha,\gamma) = \min_{(u_i,u_e,u_s)\,\in A}\left\{\frac{Q_h^{\alpha\gamma}}{\rho+Q_h^{\alpha\gamma}} . \left(P_h^{\alpha\gamma}(v^h\,(x+h,\alpha,\gamma)K_1^+ + v^h\,(x-h,\alpha,\gamma)K_1^-) + \frac{g(x)}{Q_h^{\alpha\gamma}} + \sum_{\substack{\beta=1\\ \beta\neq\alpha}}^4 \widetilde{P_h^{\alpha}} v^h\,(x,\beta,\gamma) + \sum_{\substack{\delta=1\\ \delta\neq\gamma}}^2 \widetilde{P_h^{\gamma}}\, v^h\,(x,\alpha,\delta)\right)\right\} \quad (21)$$

Where

$$Q_h^{\alpha\gamma} = |\,q_{\alpha\alpha}| + |\,q_{\gamma\gamma}| + \frac{|\,u_i+u_e+u_s-d|}{h_1} \quad (22)$$

$$P_h^{\alpha\gamma} = \frac{|\,u_i+u_e+u_s-d|}{h_1\,Q_h^{\alpha\gamma}} \quad (23)$$

$$\widetilde{P_h^{\alpha}} = \frac{q_{\alpha\beta}}{Q_h^{\alpha\gamma}} \quad (24)$$

$$\widetilde{P_h^{\gamma}} = \frac{q_{\gamma\delta}}{Q_h^{\alpha\gamma}} \quad (25)$$

$P_h^{\alpha\gamma}$, $\widetilde{P_h^{\alpha}}$ and $\widetilde{P_h^{\gamma}}$ denote transition probability of Markov chain controlled in a discrete state space $M \times G_h$, where G_h represents a description of the numerical grid. For more details, we refer reader to see Boukas and Haurie (1990).

4 IMPLEMENTATION OF SUCCESSIVE APPROXIMATION TECHNIQUE

The implementation of the approximation technique needs the use of a finite grid denoted herein G_h. Thus, some boundary conditions are needed to describe the behaviour of the system at the border of G_h. These boundary conditions are realistic and their influence will be negligible since the value function is Lipschitz. In addition, if we consider that the optimal policy changes rarely when |x| is very large i.e. go over the boundary of our grid, then the optimal solution will never be at the boundaries of the domain. For the numerical implementation, the set of constraints presented in Yan and Zhang (1997) were used as boundary conditions and given by (26).

The area of the grid G_h is defined by the following set:

$$G_h = \{\,x;\ -b\ \leq x\ \leq b\}\,;\ b \text{ is a positive integer}$$

$$\begin{cases} v^h\,(-b-h,\alpha,\gamma) = v^h\,(-b,\alpha,\gamma) + \frac{B}{\rho}\,h \\ v^h\,(b+h,\alpha,\gamma) = v^h\,(b,\alpha,\gamma) + \frac{H}{\rho}\,h \end{cases} \quad (26)$$

Recall that *h* is the discretization step and let $\left(v^h\,(x,\alpha,\gamma)\right)^n$ the value function at the n^th^ iteration of the point x at state α and γ such as:

$$\left(v^h(x,\alpha,\gamma)\right)^n = \min_{(u_i,u_e,u_s)\in A}\left\{\frac{Q_h^{\alpha\gamma}}{\rho+Q_h^{\alpha\gamma}}\cdot\left(P_h^{\alpha\gamma}\left(\left(v^h(x+h,\alpha,\gamma)\right)^{n-1}K_1^+ + \left(v^h(x-h,\alpha,\gamma)\right)^{n-1}K_1^-\right)+\frac{g(x)}{Q_h^{\alpha\gamma}}+\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{4}\widetilde{P_h^{\alpha}}\left(v^h(x,\beta,\gamma)\right)^{n-1} + \sum_{\substack{\delta=1\\ \delta\neq\gamma}}^{2}\widetilde{P_h^{\gamma}}\left(v^h(x,\alpha,\delta)\right)^{n-1}\right)\right\} \quad (27)$$

where $\left(v^h(x+h,\alpha,\gamma)\right)^{n-1}$ denote the value function at (n-1)th iteration of the upstream of point $x(x+h)$ at state α and γ and $\left(v^h(x-h,\alpha,\gamma)\right)^{n-1}$denote the function at (n-1)th iteration of the downstream of point x $(x-h)$ at state α and γ.

The algorithm of successive approximation technique runs as follows:

Step1: Initialisation: take $\sigma \in R^+$ a great error, n=1 and $\left(v^h(x,\alpha,\gamma)\right)^n = 0 \;\; \forall x \in D, \forall \alpha \in M, \gamma \in M_d$

Step 2: $\left(v^h(x,\alpha)\right)^{n-1} = \left(v^h(x,\alpha)\right)^n$; $\forall x \in D$, $\alpha \in M, \gamma \in M_d$

Step 3: Calculate value function expressed in equation (27); $\forall x \in D, \alpha \in M, \gamma \in M_d$. And the corresponding u_i^*, u_e^*, u_s^*.

Step 4: Convergence test

$$\bar{c}_1 = \mathbf{max}_{x\in \boldsymbol{D}}\left[\left(v^h(x,\alpha,\gamma)\right)^n - \left(v^h(x,\alpha,\gamma)\right)^{n-1}\right], \quad \forall \alpha \in M,\ \gamma \in M_d.$$

$$\bar{c}_2 = \mathbf{min}_{x\in \boldsymbol{D}}\left[\left(v^h(x,\alpha,\gamma)\right)^n - \left(v^h(x,\alpha,\gamma)\right)^{n-1}\right], \quad \forall \alpha \in M, \gamma \in M_d.$$

$$C_{min} = \frac{\rho}{1-\rho}\ \min_{\alpha\in \boldsymbol{M}}(\bar{c}_1) \quad \text{et} \quad C_{max} = \frac{\rho}{1-\rho}\ \max_{\alpha\in \boldsymbol{M}}(\bar{c}_2)$$

$$If\ |C_{max} - C_{min}| \le \sigma\ then\ Stop,\ u_i = u_i^*,\ u_e = u_e^* \text{ And } u_s = u_s^*$$
$$Else\ n = n+1,\ Return\ to\ step\ 2$$

The successive approximation algorithm is used to solve equation (27) which defines the optimum conditions for the stochastic optimal control problem. The algorithm was developed on Matlab R2009b. The obtained results are analysed and discussed in the next section.

5 NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

The numerical results used to characterize the optimal selection carriers' policy are analyzed in this section in two steps. First a basic case study is considered to illustrate the overall structure and the dynamic behavior of the obtained control policy. Then, a sensitivity analysis is conducted to insure the robust behavior of the results.

5.1 Numerical results

As mentioned previously, the implementation of the successive approximation technique requires the definition of a computation domain which is given by:

$$G_h = \{x;\ -10 \le x \le 20, h = 0.1\}$$

Table 1 shows the operational and cost parameters of the considered case study.

Parameters	**H**	**B**	$\boldsymbol{C_i}$	$\boldsymbol{C_e}$	$\boldsymbol{C_s}$	$\boldsymbol{U_i^{max}}$	$\boldsymbol{U_e^{max}}$	$\boldsymbol{U_s^{max}}$
Values	4	20	5	8	14	12	5	3

Table 1: Data parameters

Demand levels d_1 and d_2 are equal to 10 and 18, respectively. The discounted rate of the incurred cost ρ is 0.8 and the transition matrixes Q^c and Q^d are expressed as follows:

$$Q^c = \begin{bmatrix} -0.5 & 0.4 & 0.1 & 0 \\ 0.3 & -0.4 & 0 & 0.1 \\ 0.25 & 0 & -0.55 & 0.3 \\ 0 & 0.45 & 0.25 & -0.7 \end{bmatrix}$$

$$Q^d = \begin{bmatrix} -0.4 & 0.4 \\ 0.6 & -0.6 \end{bmatrix}$$

From the Q^c transition matrix, one can calculate the availability rates of the internal and external fleets. They are equal in the considered case study to 71% and 42% respectively. It follows from Q^d that demand level d_1 and d_2 occur in average 60% and 40% of the time, respectively.

To ensure a clear characterization of the optimal control policy in the whole system space, we should solve numerically the optimality conditions in the computational domain and the other 8 discrete states defining the stochastic processes governing the availability of the internal and external fleets as well as the applied demand levels.

As explained in section 2, these states are defined by (α, $\boldsymbol{\gamma}$) where α is the couple (α1, α2) defining the availability state of the internal and external fleets and $\boldsymbol{\gamma}$ defines the demand sate. Thus, states 1 to 8 are a given combination of the triplet (α1, α2, $\boldsymbol{\gamma}$). As given by equation (1) and (2) α1 takes values in M_i = {1, 0}, α2 takes values in M_e = {1, 0} and $\boldsymbol{\gamma}$ takes values in M_d = {1, 2}. For example, the overall state 1 = (1, 1, 1) implies that internal and external fleets are available and demand rate is equal to d_1. The overall state is called ψ= {1,..,8} in the rest of the paper.

It follows from the numerical results that the optimal carriers' selection policy has a multiple base stock (MBS) structure, which we call the "State Dependent Multiple Base Stock Policy" SDMBSP. Within this policy, three thresholds of the available inventory in the distribution center are governing the carriers' selection strategy. Let $Z_1(\psi)$, $Z_2(\psi)$ and $Z_3(\psi)$ define these thresholds. Their values are function of the whole discrete state of the system ψ. The structure of the policy is illustrated in figure 2.

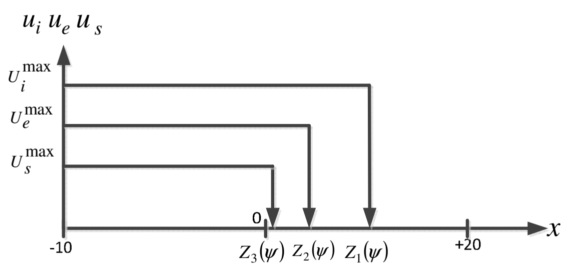

Figure 2: Carriers' selection policy

According to this carriers' selection strategy, the internal fleet must insure the transfer of the products at its maximum available capacity U_i^{max} as long as the inventory level in the distribution center is lower than $Z_1(\psi)$. This level defines the maximum security stock level to keep in the distribution center in the real state of the system ψ and one should stop the shipment of the product (i.e., u_i is fixed to zero) above $Z_1(\psi)$. The external fleet is also governed by a base stock policy but according to a different threshold $Z_2(\psi)$. Thus, the external fleet is requested to insure the shipment of products, in addition to internal fleet, up to the inventory level $Z_2(\psi)$. Finally, the spot carriers are requested in addition to the two other fleets up to the inventory level $Z_3(\psi)$. As mentioned previously, the values of the three stock levels governing the selection carriers' policy are dependent of the whole discrete state of the system ψ. Thus, if the system is in a state where the internal, respectively, the external carriers are unavailable, $Z_2(\psi)$, respectively, $Z_1(\psi)$ will disappear. This issue will be illustrated later.

Figure 3 shows the obtained policy in the system state 1 (i.e., internal and external fleet are available to insure the transportation with their maximum capacity and demand rate is equal to d_1). In this case, $Z_1(1)$, $Z_2(1)$ and $Z_3(1)$ are equals to -3, 0 and 1 respectively. The obtained results make sense since the transportation cost of the sport carriers and the external fleets are higher than that of the internal fleet. Therefore, they are used in support to the internal fleet. Moreover, one should note that in the negative area of the distribution center stock (i.e., shortage of stock), the contribution of the spot carriers and the external fleet is necessary but according to different degrees (up to -3 for the spot carriers and up to 0 for the external fleet). In fact, in the presence of shortages, using only the available capacity of the internal fleet makes it impossible to respond to the demand and build a safety stock to hedge against future periods of transportation service unavailability.

To insure an accurate analysis of the obtained results and to propose a robust general structure of the carriers' selection strategy to be applied in a stochastic dynamic context, one should observe carefully the obtained results in the other states of the system.

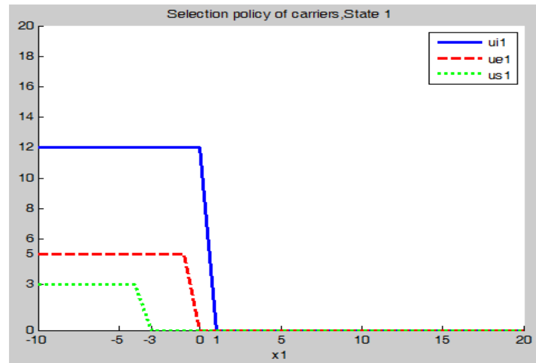

Figure 3: Selection policy of carriers at state 1

Figure 4 shows the obtained policy in the system state 5 (i.e., internal and external fleet are available and demand rate is equal to d_2). In this case, the demand is in its highest rate. Therefore, the spot carriers and the external fleet are requested more than it was the case in the previous situation (state 1) which leads to higher $Z_2(5)$ and $Z_3(5)$. Moreover, the three fleets must contribute to build a higher security stock in the distribution center to hedge against future unavailability (i.e., $Z_1(5)$=4 compared to 1 in the system state 1). This result also makes sense since with a higher demand rate the distribution center needs a higher security stock to hedge against possible future shortage during the period of unavailability.

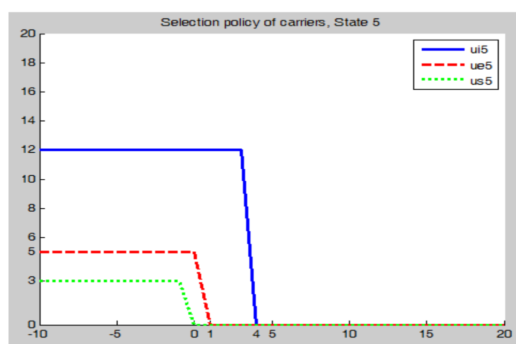

Figure 4: Selection policy of carriers at state 5

Let's look to the results when the external fleet is unavailable (state 6). Figure 5 shows the obtained policy and the corresponding thresholds. This result confirms our expectations and the previous analysis. In fact, the spot carriers are requested not only in the negative area of the distribution center stock but also to build with the internal fleet a security stock level.

The analysis of the other states was done and all the obtained results are in coherence with the analysis developed in this section. Table 2, presents the values of $Z_1(\psi)$, $Z_2(\psi)$ and $Z_3(\psi)$ for the 8 states. When the fleet is not available, N/A means that the corresponding threshold is not applicable.

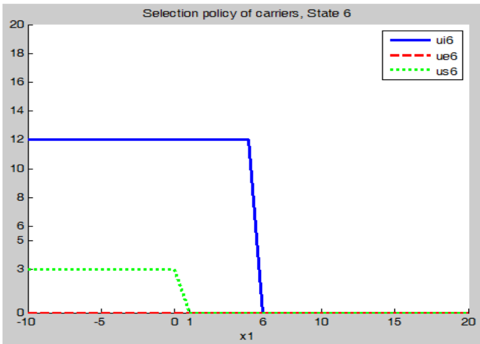

Figure 5: Selection policy of carriers at state 5

State	1	2	3	4	5	6	7	8
$Z_1(\psi)$	1	1	N/A	N/A	4	6	N/A	N/A
$Z_2(\psi)$	0	N/A	5	N/A	1	N/A	10	N/A
$Z_3(\psi)$	-3	-1	2	3	0	1	5	6

Table 2: selection policy thresholds

To interpret these results, figure 6 illustrates the dynamic behavior of the carriers' selection strategy to adopt over time. As explained previously four areas can be defined:

- Area 1: under the hedging level $Z_3(\psi)$, the three fleets must insure the transportation according to their maximum capacities ($U_i^{max} + U_e^{max} + U_s^{max}$).
- Area 2: between the hedging levels $Z_2(\psi)$ and $Z_3(\psi)$ the spot carriers are no longer needed and only the internal and external fleets insure the transportation according to their maximum capacities ($U_i^{max} + U_e^{max}$).
- Area 3: between the hedging levels $Z_2(\psi)$ and $Z_1(\psi)$ the external fleet is no longer needed and only the internal fleet insure the transportation according to its maximum capacity (U_i^{max}).
- Area 4: when the stock level reaches $Z_1(\psi)$, internal fleet must move freights according to the demand rate.

These areas are not static over time, they move according to the state of the system and the corresponding thresholds given in table 2 for the considered case study. This issue is illustrated in figure 6 by points A, B and C.

In fact, since unavailability periods occur randomly over time, figure 6 illustrates at point A the situation where the internal and external fleets become unavailable. In this case, the state of the system changes from 1 to 4 and the contribution of the spot carriers is needed since $Z_3(4)$ is equal to 3. At point B, the internal fleet becomes available and takes the place of the spot carriers.

Point C illustrates the situation where the internal fleet is still unavailable but the external fleet becomes available. In this case, the external fleet will take the place of the spot carriers or they will insure together the transportation according to the distribution stock level at that time.

In the next section, sensitivity analyses are conducted to ensure that the structure of the obtained SDMBSP policy is maintained and can be considered as a generalized policy for the general problem under study.

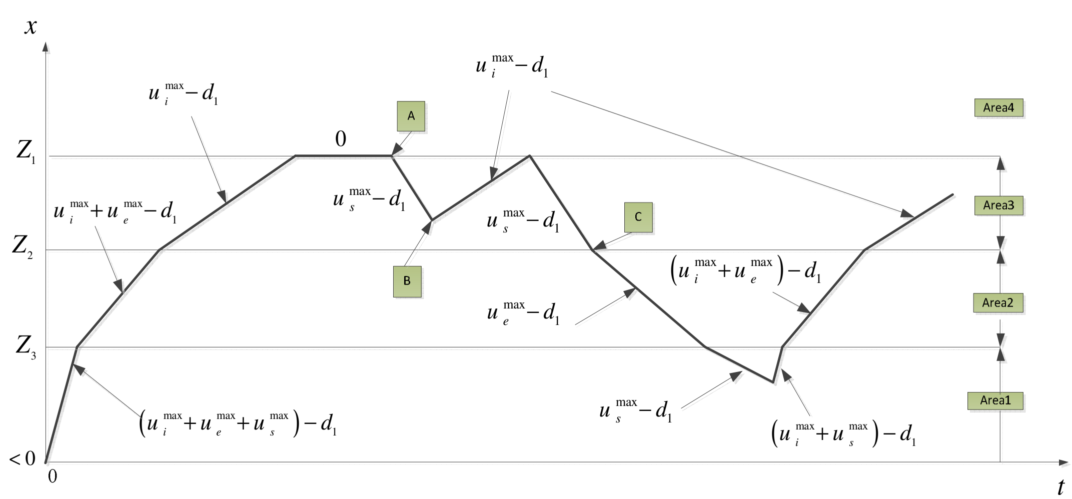

Figure 6: Dynamic behavior of the distribution center stock over time and the corresponding carriers' selection policy

5.2 Sensitivity analysis

To illustrate the effect of system parameters changing on the proposed SDMBSP policy, a sensitivity analysis has been performed (Table 3). Three sets of parameter variations have been conducted:

- Set I consists in 3 cases with backlog cost variation.
- Set II consists in 3 cases with external fleet transportation costs variation.
- Set III consists in 3 cases with spot carrier transportation costs variation.

Note that the given thresholds are those of the system state 1. For the other states the outlines of the analysis remain the same.

	Cases	B	C_e	C_s	(Z_3,Z_2,Z_1)
Set I	Basic	20	8	14	(-3; 0; 1)
	1	**50**	8	14	(0; 0; 1)
	2	**80**	8	14	(0; 0; 2)
Set II	Basic	20	8	14	(-3; 0; 1)
	1	20	**10**	14	(-2; 0; 1)
	2	20	**12**	14	(-2; -1; 1)
	3	20	**14**	14	(-1; -2; 1)
Set III	Basic	20	10	14	(-3; 0; 1)
	1	20	10	**18**	(-5; 0; 1)
	2	20	10	**30**	(-<<0; 5; 9)

Table 2 : Data parameters for the sensitivity analysis cases

Set I shows that when the backlog cost raises, the values of stock levels Z_1, Z_2 and Z_3 increase accordingly to ensure the availability of enough stocks to hedge against future backlogs. Moving from the basic case of Set I to the cases 1 and 2 of the same set, the three fleets are requested more often. Since backlog costs are higher than the transportation costs, it is more profitable for the shipper to call the spot carrier.

It is also interesting to observe the results of set II where the external fleet transportation costs increase. In this case, the value of stocks levels Z_2 decrease. This result makes sense since transportation cost of external fleet draw near the transportation cost of spot carrier, thus shipper has interest to call the spot carrier due to his constant availability.

Set III results show that when the transportation cost of the spot carrier increases, the stocks level Z_1 and Z_2 increase, but the stock level Z_3 decreases to reach values much lower than zero. This result makes sense since a very costly spot carrier should be called in support only for critical situations. Moreover, the security stock level in the distribution center is higher to hedge against future shortage and to avoid the situation where the shipper will be forced to call the sport carrier.

From the above analysis, it clearly appears that the results obtained make sense, and that the structure of the policy defined by the 3 parameters Z_1, Z_2 and Z_3 is always maintained. This allows the development of a parameterized selection policy of carriers defined by the following equation:

$$CSP = \begin{cases} [i,e,s] & if \quad x(t) \le Z_3(\psi) \\ [i,e] & if \quad x(t) \le Z_2(\psi) \,\&\, x(t) > Z_3(\psi) \\ [i] & if \quad x(t) \le Z_1(\psi) \,\&\, x(t) > Z_2(\psi) \\ \emptyset & if \quad x(t) > Z_1(\psi) \end{cases}$$

Where CSP denote the Carrier's Selection Policy; i, e and s designate internal fleet, external fleet and spot carrier respectively.

6 CONCLUSION

In this paper, we studied the carrier selection problem in the presence of three fleets, internal, external and spot carriers. Availability periods of internal and external fleets evolve according to a random process and this fact makes impossible to meet the random demand. In order to respond to the demand, shipper should call in support spot carriers. The carrier selection problem was formulated as a continuous time dynamic programming problem and HJB equations were derived. Numerical approach was also proposed to solve the HJB equations of the problem and to obtain near-optimal carriers selection policy that minimizes the expected discounted cost of transportation, inventories and backlogs/late deliveries.

The optimal control policy has been shown to be described by a State Dependent Multiple Base Stock Policy" SDMBSP. Within this policy and according to the carriers' availabilities, three thresholds of the available inventory in the distribution center are governing the carriers' selection strategy.

In conclusion, this paper makes an important contribution to the carriers' selection problem in the dynamic stochastic context of supply chains. As it may interest the reader to know, extensions to cover more complex situations including more actors and products are under study.

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