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A location-routing problem with disruption risk

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ABSTRACT

This paper considers a location-routing problem in a supply-chain network with a set of producer–distributors that produce a single commodity and distribute it to a set of customers. The production capacity of each producer–distributor varies randomly due to a variety of possible disruptions, and the vehicles involved in the distribution system are disrupted randomly. The goal is to determine the location, allocation and routing decisions that minimize the annual cost of location, routing and disruption, under one of the moderate, cautious or pessimistic risk-measurement policies. Exact formulations and an efficient heuristic are presented for the problem.

1. Introduction

Risk management plays an important role in managing and reducing risks in supply chains, which may be classified into two major categories: operational and disruption. Operational risks do not affect the functionality of the elements of supply chains and only influence operational factors that are basically supposed to be uncertain. They are rooted in inherent uncertainties such as those involved in customer demand, procurement prices for raw materials or required resources, production costs, process performance, quality of a manufacturing system, production rate of a machine, lead times or transportation times. However, disruption risks can either completely or partly stop functioning of the elements of supply chains, typically for an uncertain amount of time. This kind of risk stems from major disruptions that may be caused by nature or disasters, such as earthquakes, floods, hurricanes, and terrorist attacks; by economic or financial crises, such as currency crisis; by social events, such as strikes; or, by machine breakdowns.

Disruptions lead to uncertainties in the supply, production or distribution capacities. These uncertainties are mostly discrete and major, while the uncertainties originating from operational sources are usually continuous and minor. Despite the traditional belief that considers supply-chain disruptions as rare events, which may plausibly be true for extraordinary forms of disruptions like natural disasters, it is highly probable that disruptions affect supply chains frequently, especially large-scale and worldwide ones. On the other hand, disruptions caused by rare events like hurricanes or earthquakes are naturally very ruinous and consequently their associated risks cannot be ignored. In most cases, the business impact associated with disruption risks is much greater than that of operational risks. This is why recently the study of approaches that mitigate disruption risks has become a very hot topic in supply chain management. The papers by Kleindorfer and Saad (2005), Tang (2006), and Oke and Gopalakrishnan (2009) reviewed and classified recent progress in this area. In what follows, the papers focusing on supply-chain design problems with disruption risks are reviewed in brief.

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Several papers have studied disruption within location problems. Snyder and Daskin (2005) looked at facility location problems in which some facilities fail with a given probability and where they assume that customers are served by the nearest non-disrupted facility. Drawing inspiration from these models, Berman et al. (2007), Berman et al. (2009) and Shen et al. (2011) developed and analyzed new facility-location models with disruption.

Tomlin (2006) presented a dual-sourcing model in which orders may be placed with either a cheap but unreliable supplier or an expensive but reliable supplier. He considered a very general supplier-recovery process. Snyder and Daskin (2007) compared models for reliable facility location under a variety of operating strategies. For other kinds of disruptions in supply chains, the reader is referred to Snyder et al. (2006). Lim et al. (2010) considered a facility location problem incorporating two types of facilities: one, unreliable; and the other, reliable and not subject to disruption but more expensive. Cui et al. (2010) proposed a compact mixed-integer programming formulation and an approximation model to study the reliable uncapacitated fixed-charge location problem. Li and Ouyang (2010) studied a similar problem, where facilities are subject to spatially correlated disruptions that occur with location-dependent probabilities.

Other studies considered disruption within routing problems. Wilson (2007) investigated the effect of a transportation disruption on supply chain performance, using system-dynamics simulation. Li et al. (2009) introduced real-time vehicle-rerouting problems with time windows in which disruptions are due to vehicle breakdowns and where therefore, one or more vehicles must to be rerouted to perform uninitiated services. Mu et al. (2010) introduced a disrupted vehicle-routing problem that deals with disruptions that occur at the execution stage of a vehicle-routing plan. They focused on the case where a vehicle breaks down during delivery and a new routing solution must be quickly generated to minimize the total cost. Wang et al. (2010) studied an emergency-vehicle routing problem with disruption. Wang et al. (2012) studied a vehicle-routing problem with time windows considering combinational disruption recovery.

Location-routing problems (LRPs) are binary-integration problems, which have been extensively examined by several authors in the literature. Min et al. (1998) and Nagy and Salhi (2007) surveyed papers dealing with LRPs and proposed classifications for them. For recent progress in LRPs, one may see Ahmadi-Javid and Azad (2010) and Ahmadi-Javid and Seddighi (2012), who present ternary-integration problems where LRPs are integrated with inventory decisions. No study has yet considered an LRP in a supply-chain network subjected to disruptions, although various types of LRPs have been investigated in the literature and there are papers incorporating disruption considerations into other binary-integration problems such as location and inventory problems (see, for example, Qi et al., 2010; Mak and Shen, 2012). This paper presents for the first time an LRP with production and distribution disruption risks.

In the proposed LRP (henceforth called LRP with production and distribution disruption risk or abbreviated by LR-PDDR-P) there is a two-echelon supply-chain network consisting of a set of producer–distributors (PDs) that produce a single commodity and distribute it to customers (or retailers). Various disruptions may affect the PDs' production capacities or the vehicles used in the distribution system. The goal is to determine the location and routing decisions that minimize the total cost of location, routing and disruption under three risk-measurement policies: moderate, cautious and pessimistic.

The LR-PDDR-P is NP-hard since its special case is the classical LRP belonging to the class of NP-hard problems (Perl and Daskin, 1985). Therefore, only small instances of the problem can be solved optimally within a reasonable time. To optimally solve small-sized instances, the problem is cast as mixed-integer linear programming models under each of the three risk-measurement policies. Next, an efficient heuristic is developed to find near-optimal solutions of large-sized problem instances. The heuristic is decomposed into two stages: constructive stage and improvement stage. In the constructive stage, an initial solution is randomly built and iteratively improved in the other stage, which then encompasses two phases: location phase and routing phase. In each phase of the second stage, a simulated annealing algorithm is used to improve the initial solution. A 2-OPT heuristic is used to improve routing decisions in the second phase. Using reformulated relaxations of the linear programming models, numerical error bounds are also provided to assess the quality of the heuristic solutions for medium-sized instances.

The remainder of the paper is organized as follows. Section 2 formally gives the problem statement and then presents a formulation under a general risk-measurement policy. Section 3 covers three possible risk-measurement policies and presents the linearizations of the formulation given in Section 2 under different risk-measurement policies. Section 4 compares analytically the improvements obtained by incorporating disruption risk under different risk-measurement policies. Section 5 presents the heuristic algorithm to approximately solve large-sized instances of the problem. The computational results are presented in Section 6. Finally, Section 7 concludes the paper.

2. Problem statement and formulation

2.1. Problem statement

The goal of the proposed location-routing problem with production and distribution disruption risks (LR-PDDR-P) is to choose, locate and allocate a set of potential producer–distributors (PDs) and to build vehicles' routes to meet supply-chain customers' demands, such that the total cost of location, routing and disruption is minimized. A customer may be an end customer, a retailer, a central depot or even another production unit. Each customer has a certain demand. In Fig. 1, an instance of the supply-chain network is illustrated.

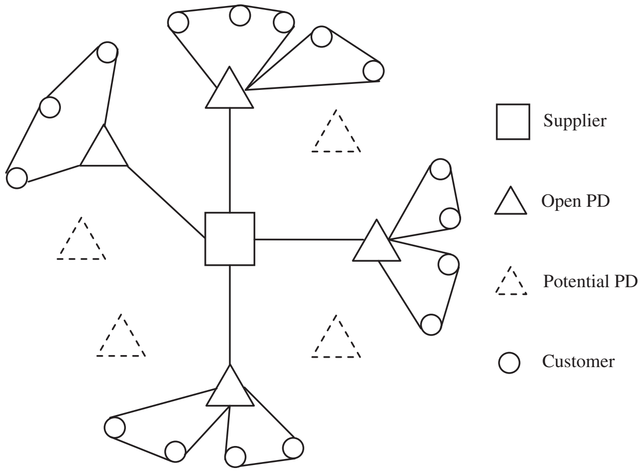

Fig. 1. An illustration of the supply-chain network under consideration.

The annual production capacity (APC) of a potential PD is stochastically uncertain due to different kinds of disruptions; in addition, a vehicle servicing customers may also be disrupted randomly. If a disruption causes a decrease in the APC of an open PD, then depending on the level of decrement, a certain cost (e.g., lost-sales cost, delay cost, outsourcing cost, etc.) is incurred in order to service the unfulfilled portion of the customers' demands. Disruptions may also decrease the annual number of visits (ANV) of each vehicle. When a vehicle experiences a disruption, it cannot perform its route, which incurs certain cost because another vehicle must do the route.

In the following subsections, the problem's assumptions and the decisions determined by the problem are given.

2.1.1. Assumptions

- Each customer has a certain demand and should be assigned to only one open PD.
- Each open PD produces and distributes to meet the demand of all allocated customers through a periodic routing system. The total of the customers' demands assigned to an open PD cannot exceed the maximum possible value of its annual production capacity (APC).
- Vehicle capacities are the same, and the fleet type is homogeneous. Each vehicle visits its allocated customers within a certain amount of time, which is the same for all vehicles.
- Each open PD may face frequent disruptions that decrease its APC. In this event, some penalty must be paid, or the unfulfilled customers' demands must be satisfied through some form of compensation depending on the mitigation strategies used. This however incurs an extra annual cost, which is assumed to linearly depend on the annual volume of the unsatisfied demands.
- For each PD, risk-mitigation strategies are already known and may differ when encountering different disruptions. These may include outsourcing the unfulfilled customers' demands, re-assigning them to other operating PDs, negotiating with customers to have them agree to backorder, accepting and considering them as lost sales, or procuring the required resources from alternate suppliers in the case where the PD faces a supply disruption.
- Each vehicle may experience frequent disruptions in a year, which decreases its annual number of visits (ANV), i.e., the number of times per year that the vehicle visits the customers allocated to it. If a vehicle is disrupted, then an unscheduled vehicle will be used to do the disrupted vehicle's route, for which an additional expenditure must be incurred. The additional cost is a specified percentage of the cost in a normal situation, and is assumed to be the same for all vehicles.
- The APCs of potential PDs and ANVs of vehicles are random variables that are represented by a joint discrete distribution with a finite support set.

2.1.2. Decisions

- *Location and allocation decisions*: which potential PDs to open and how to allocate customers to open PDs.
- *Routing decisions*: how to build the vehicles' routes starting from an open PD to serve its allocated customers.

2.2. Problem formulation

Under the assumptions in Section 2.1.1, a mathematical programming model is proposed to determine decisions considered in Section 2.1.2. Before presenting the model, the notation used throughout the paper is introduced.

2.2.1. Index sets

K	The index set of customers
J	The index set of potential PDs
V	The index set of available vehicles
S	The support set of the joint discrete distribution of APCs of potential PDs and ANVs of available vehicles
N	The index set associated with support set S, i.e., $N = \{1, \ldots, \lvert S\rvert\}$
M	The merged set of all nodes of customers and potential PDs, i.e., $K \cup J$

2.2.2. Parameters

d_k	The annual demand of customer k $(k \in K)$
f_j	The annual fixed cost for opening and operating PD j $(j \in J)$
t_{kl}	The fixed transportation cost between node k and node l $(k, l \in M)$
β	The additional percentage of the routing cost that must be paid when a vehicle is disrupted
vc	A given vehicle's delivery capacity
Ψ_j	The APC of potential PD j, which is a discrete random variable whose support set is finite $(j \in J)$
$\sup \Psi_j$	The maximum possible value of the APC of potential PD j, i.e., the right endpoint of random variable Ψ_j $(j \in J)$
$\mathbf{\Psi}$	The row vector of all APCs of potential PDs, i.e., $\mathbf{\Psi} = (\Psi_1, \ldots, \Psi_{\lvert J\rvert})$
Γ_v	The ANV of vehicle v, which is a discrete random variable with finite support set $(v \in V)$
$\sup \Gamma_v$	The maximum possible value of the ANV of vehicle v, i.e., the right endpoint of random variable Γ_v $(v \in V)$
$\mathbf{\Gamma}$	The row vector of all ANVs of vehicles, i.e., $\mathbf{\Gamma} = (\Gamma_1, \ldots, \Gamma_{\lvert V\rvert})$
$\mathbf{s}^n = (\mathbf{s}^n_{\mathbf{\Psi}}, \mathbf{s}^n_{\mathbf{\Gamma}})$	The nth member of the support set of random vector $(\mathbf{\Psi}, \mathbf{\Gamma})$, i.e., S, which can be represented in the extended form as $\mathbf{s}^n = \left(\mathbf{s}^n_{\mathbf{\Psi},1}, \ldots, \mathbf{s}^n_{\mathbf{\Psi},\lvert J\rvert}, \mathbf{s}^n_{\mathbf{\Gamma},1}, \ldots, \mathbf{s}^n_{\mathbf{\Gamma},\lvert V\rvert}\right)$ $(n \in N)$
p_n	The probability of occurrence of the nth member of the support set of random vector $(\mathbf{\Psi}, \mathbf{\Gamma})$, i.e., $p_n = \Pr\{(\mathbf{\Psi}, \mathbf{\Gamma}) = \mathbf{s}^n\}$ with $0 < p_n \leqslant 1$ and $\sum_{n=1}^{\lvert S\rvert} p_n = 1,\ (n \in N)$
dc_j	The cost per unit of demand that is not fulfilled because of the disruption of potential PD j $(j \in J)$
$\rho(\cdot)$	A risk measure considered to scalarize the stochastic cost function, which must naturally be chosen based on the decision maker's risk-measurement policy (see Section 3.2)

2.2.3. Decision variables

$$R_{klv} = \begin{cases} 1 & \text{if node } k \text{ precedes node } l \text{ in the route of vehicle } v \\ 0 & \text{otherwise} \end{cases} \quad (k, l \in M,\ v \in V)$$

$$Y_{jk} = \begin{cases} 1 & \text{if customer } k \text{ is assigned to open PD } j \\ 0 & \text{otherwise} \end{cases} \quad (j \in J,\ k \in K)$$

$$U_j = \begin{cases} 1 & \text{if potential PD } j \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \quad (j \in J)$$

M_{kv}: An auxiliary variable used to eliminate subtours for customer k and vehicle v $(k \in K, v \in V)$.

The row vectors that include all variables of each above-defined type are denoted by $\boldsymbol{R}$, $\boldsymbol{Y}$, $\boldsymbol{U}$ and $\boldsymbol{M}$, respectively. Also, random variable $TC(\boldsymbol{Y}, \boldsymbol{U}, \boldsymbol{R}, \boldsymbol{M}; \mathbf{\Psi}, \mathbf{\Gamma})$ represents the model's real-world total annual cost function that corresponds with decision vectors $\boldsymbol{R}$, $\boldsymbol{Y}$, $\boldsymbol{U}$ and $\boldsymbol{M}$, and random input vectors $\mathbf{\Psi}$ and $\mathbf{\Gamma}$.

2.2.4. Objective function

The objective function includes the following annual cost components.

1. The fixed annual cost for opening and operating PDs, given by $\sum_{j \in J} f_j U_j$.
2. The annual routing cost from open PDs to customers, given by $\sum_{v \in V} \Gamma_v \sum_{k \in M} \sum_{l \in M} t_{kl} R_{klv}$.

3. The annual distribution disruption cost, given by $\sum_{v\in V}(\sup \Gamma_v - \Gamma_v)(1+\beta)\sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv}$.
4. The annual production disruption cost, given by $\sum_{j\in J} dc_j \times \max\{\sum_{k\in K} d_k Y_{jk} - \Psi_j, 0\}$.

Summing up the above costs together, the real-world total annual cost becomes

$$\begin{aligned} TC(\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M};\boldsymbol{\Psi},\boldsymbol{\Gamma}) = &\sum_{j\in J} f_j U_j + \sum_{v\in V}\sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + \beta\sum_{v\in V}(\sup \Gamma_v - \Gamma_v)\sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + \sum_{j\in J} dc_j \\ &\times \max\left\{\sum_{k\in K} d_k Y_{jk} - \Psi_j, 0\right\} \end{aligned} \tag{1}$$

which is a random variable due to the last two terms, and thus cannot be optimized by existing numerical optimization methods. One way to overcome this issue is to use a risk measure ρ to scalarize the random cost (1). Risk measure ρ is chosen based on the decision maker's risk-measurement policy. Section 3 presents an introduction to risk measures and lists suitable risk measures ρ under different risk-measurement policies used by decision makers.

2.2.5. Formulation and description of constraints

Assuming that risk measure ρ is predetermined, the problem can be formulated as follows:

$$\min_{\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M}} \quad \rho(TC(\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M};\boldsymbol{\Psi},\boldsymbol{\Gamma})) \tag{2}$$

s.t.

$$\sum_{v\in V}\sum_{l\in M} R_{klv} = 1 \quad k\in K \tag{3}$$

$$\sum_{k\in K} d_k \sum_{l\in M} R_{lkv} \leqslant (\sup \Gamma_v)\times vc \quad v\in V \tag{4}$$

$$M_{kv} - M_{lv} + (|K|\times R_{klv}) \leqslant |K| - 1 \quad k,l\in K, \quad v\in V \tag{5}$$

$$\sum_{l\in M} R_{klv} - \sum_{l\in M} R_{lkv} = 0 \quad k\in M, \quad v\in V \tag{6}$$

$$\sum_{j\in J}\sum_{k\in K} R_{jkv} \leqslant 1 \quad v\in V \tag{7}$$

$$\sum_{l\in M} R_{klv} + \sum_{l\in M} R_{jlv} - Y_{jk} \leqslant 1 \qquad j\in J, \quad k\in K, \quad v\in V \tag{8}$$

$$\sum_{k\in K} d_k Y_{jk} \leqslant (\sup \Psi_j)\times U_j \quad j\in J \tag{9}$$

$$\begin{aligned} &Y_{jk}\in\{0,1\} \quad j\in J, \quad k\in K \\ &U_j\in\{0,1\} \quad j\in J \\ &R_{klv}\in\{0,1\} \quad k,l\in M, \quad v\in V \end{aligned} \tag{10}$$

$$M_{kv} \geqslant 0 \quad k\in K, \quad v\in V. \tag{11}$$

The model's objective function given in (2) minimizes a scalarization of real-world total annual cost (1), which consists of the location, routing and disruption costs. Constraints (3) make sure that each customer is placed on exactly one vehicle route. Constraints (4) are the vehicle-capacity constraints. Constraints (5) are the subtour-elimination constraints that guarantee that each tour must contain an open PD from which it originates, i.e., each tour must consist of an open PD and some customers. Constraints (6) are flow-conservation constraints which enforce that whenever a vehicle enters a node; either a customer or an open PD, it must leave the node, and also ensure that the routes remain circular. Constraints (7) imply that only one open PD is included in each route. Constraints (8) link the allocation and routing parts of the model; if vehicle v starts its trip from open PD j and also serves customer k, then customer k will be assigned to open PD j. Constraints (9) assure that the total demand of customers assigned to an open PD cannot exceed the maximum possible value of the PD's APC. Constraints (10) impose integrality restrictions on the binary variables, and constraints (11) enforce non-negativity restrictions on the auxiliary variables.

2.2.6. Extension

The proposed model can be easily extended in several directions. For example, it can be extended straightforwardly for a supply chain with more than two echelons. Indeed, two echelons of warehouses and distributors can be added in a simple manner. It is also adaptable for multi-commodity supply chains. Moreover, the model can practically handle the case where the APCs of potential PDs and ANVs of vehicles are arbitrary and independent random variables if risk measure ρ in (2) is considered as the entropic value-at-risk (see Section 3.2 for more details).

3. Risk-measurement policies

As seen in the previous section, real-world cost (1) is a random variable, and to incorporate it in a numerical optimization procedure, one needs a risk measure to scalarize it. Section 3.1 first briefly reviews the concept of a risk measure and its desirable properties in a mathematical framework, and then Section 3.2 introduces the risk measures that will be used in this paper under three risk-measurement polices. Section 3.3 shows how model (2)–(11) can be linearized for these selected risk measures.

3.1. Risk measures

A *risk measure* is a function ρ assigning to a random outcome or risk position X a real value $\rho(X)$, which makes it possible to select a suitable uncertain outcome from a set $\mathbf{X}$ of allowable uncertain outcomes. The associated optimization problem for this selection is

$$\min_{X\in\mathbf{X}} \rho(X). \tag{12}$$

To precisely define the concept of risk measure, let $(\mathbf{\Omega}, \mathbf{F}, P)$ be a probability space, where $\mathbf{\Omega}$ is a set of all simple events, $\mathbf{F}$ is a σ-algebra of subsets of $\mathbf{\Omega}$ and P is a probability measure on $\mathbf{F}$. Also, suppose that $\mathbf{X}$ is a linear space of Borel measurable functions (random variables) $X : \mathbf{\Omega} \to \Re$, including all constant functions. A risk measure ρ is then defined as $\rho : \mathbf{X} \to \overline{\Re}$ where $\overline{\Re} = \Re \cup \{-\infty, +\infty\}$ is the extended real line. Several desirable properties are introduced for a suitable risk measure in the literature. The following are the most important properties for risk measure $\rho : \mathbf{X} \to \overline{\Re}$.

(P1) (Translation Invariance) For any $X \in \mathbf{X}$ and $c \in \Re$, $\rho(X + c) = \rho(X) + c$.
(P2) (Positive Homogeneity) For any $X \in \mathbf{X}$ and $\lambda \geqslant 0$, $\rho(\lambda X) = \lambda\rho(X)$.
(P3) (Subadditivity) For any $X_1, X_2 \in \mathbf{X}$, $\rho\ (X_1 + X_2) \leqslant \rho(X_1) + \rho(X_2)$.
(P4) (Monotonicity) For any $X_1, X_2 \in \mathbf{X}$ with $X_1 \geqslant X_2$, $\rho(X_1) \geqslant \rho(X_2)$.

A risk measure satisfying P1–P4 is called *coherent*. Note that this definition of a coherent risk measure is taken from the operations research literature (Ruszczynski and Shapiro, 2006), which differs slightly from the original definition considered by Artzner et al. (1999) in the context of mathematical finance, where the risk measure ρ is called coherent if $\psi(X) = \rho(-X)$ satisfies the above four properties.

In practice, one usually uses composite random variables such as $X = G(\boldsymbol{Z}; \mathbf{\Lambda})$ where $\boldsymbol{Z}$ is a real decision vector in $\Im \subseteq \Re^w$, $\mathbf{\Lambda}$ is a η-dimensional random vector with a known probability distribution, and $G(\boldsymbol{Z}; \cdot) : \Re^\eta \to \Re$ is a Borel measurable function for all $\boldsymbol{Z} \in \Im$. Then, problem (12) becomes

$$\min_{\boldsymbol{Z}\in\Im} \rho(G(\boldsymbol{Z}; \mathbf{\Lambda})). \tag{13}$$

Note that model (2)–(11) is a special case of (13) where $\boldsymbol{Z}$ is the vector including $\boldsymbol{Y}$, $\boldsymbol{U}$, $\boldsymbol{R}$ and $\boldsymbol{M}$; $\mathbf{\Lambda}$ is the random vector including $\mathbf{\Psi}$ and $\mathbf{\Gamma}$; $\Im$ is the region defined by constraints (3)–(11); and $G(\boldsymbol{Z}; \mathbf{\Lambda}) = TC(\boldsymbol{Y}, \boldsymbol{U}, \boldsymbol{R}, \boldsymbol{M}; \mathbf{\Psi}, \mathbf{\Gamma})$.

3.2. Risk-measurement policies and corresponding risk measures

Let $\mathbf{L}$ be the set of all Borel measurable functions $X : \mathbf{\Omega} \to \Re$ on probability space $(\mathbf{\Omega}, \mathbf{F}, P)$ and $\mathbf{L}_p$ with $p \geqslant 1$ be the subset of functions X in $\mathbf{L}$ for which $\int_\Omega |X(\omega)|^p dP(\omega)$ is finite. Moreover, let $\mathbf{L}_M$ denote the set of functions X in $\mathbf{L}$ whose moment-generating functions $M_X(z) = \mathrm{E}(e^{zX})$ exist for every $z \in \Re$.

For a decision maker, three risk-measurement policies (*moderate*, *cautious* and *pessimistic*) can be considered. Under the moderate risk-measurement policy, the decision maker just considers the mean of the possible values of the risk, i.e., the total cost $TC(\boldsymbol{Y}, \boldsymbol{U}, \boldsymbol{R}, \boldsymbol{M}; \mathbf{\Psi}, \mathbf{\Gamma})$, while he or she considers the worst possible value of the risk under the pessimistic policy. Under the cautious risk-measurement policy, the decision maker selects a value between these two values, i.e., the mean and the worst value of the risk, preferably such that the selected value can be interpreted in a rational fashion.

Two other risk-measurement polices, which are seldom preferred by decision makers, are *optimistic* and *incautious* ones. Under the optimistic case, the decision maker considers the best possible value of the risk, but under the incautious case, he or she considers a value between the best possible value and the mean of the risk. Note that the definitions used here for being incautious, moderate and cautious differ from being risk-seeking, risk-neutral and risk-averse, which are defined in financial economics on the basis of utility functions. There are, however, some similarities between them.

In the following, the risk measures considered for quantifying the risk $TC(\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M};\boldsymbol{\Psi},\boldsymbol{\Gamma})$ under the aforementioned three risk-measurement policies are introduced.

Expectation risk measure for the moderate risk-measurement policy

$$\rho(X) = \mathrm{E}(X), \quad X \in \mathbf{L}_1. \tag{14}$$

Conditional Value-at-Risk risk measure for the cautious risk-measurement policy

$$\rho(X) = \mathrm{CVaR}_{1-\alpha}(X) = \inf_{t \in \Re}\left\{t + \frac{1}{\alpha}\mathrm{E}[X-t]_+\right\}, \quad X \in \mathbf{L}_1 \tag{15}$$

where $[s]_+ = \max\{0,s\}$ and $\alpha \in (0,1]$.

Worst-Case risk measure for the pessimistic risk-measurement policy

$$\rho(X) = \mathrm{ess\ sup}(X), \quad X \in \mathbf{L}. \tag{16}$$

The first and third risk measures are the most natural choices for measuring risk under the moderate and pessimistic risk-measurement policies, respectively. Both the expectation and the worst-case risk measures are coherent. The second risk measure, conditional value-at-risk (CVaR) at $1-\alpha$ confidence level (Rockafellar and Uryasev, 2002), is one of the best choices for quantifying risk $X \in \mathbf{L}_1$ under the cautious risk-measurement policy. The CVaR is coherent and can be interpreted by the value-at-risk (VaR), i.e., $\mathrm{VaR}_{1-\alpha}(X) = \inf\{t\colon \Pr(X \leqslant t) \geqslant 1-\alpha\}$, as follows:

$$\mathrm{CVaR}_{1-\alpha}(X) = \frac{1}{\alpha}\int_0^{\alpha} \mathrm{VaR}_{1-t}(X)dt \tag{17}$$

which shows that for small values of α (in practice, often $\alpha = 0.05$), the CVaR focuses on the worst losses of the random outcome and is more sensitive than the VaR to the shape of the distribution of X in the right tail. Also, it can be observed that

$$\begin{aligned}
&\mathrm{E}(X) \leqslant \mathrm{CVaR}_{1-\alpha}(X) \leqslant \mathrm{ess\ sup}(X)\\
&\mathrm{CVaR}_0(X) = \mathrm{E}(X)\\
&\lim_{\alpha\to 0} \mathrm{CVaR}_{1-\alpha}(X) = \mathrm{ess\ sup}(X).
\end{aligned}$$

This means that the extreme cases of the CVaR are the ones considered for the moderate and pessimistic risk-measurement polices.

An alternative to the CVaR is the entropic value-at-risk (EVaR), which was recently introduced by Ahmadi-Javid (2011, 2012a,b). The EVaR at $1-\alpha$ confidence level is defined as follows:

$$\mathrm{EVaR}_{1-\alpha}(X) = \inf_{z>0}\{z^{-1}\ln(M_X(z)/\alpha)\} \tag{18}$$

where moment-generating function $M_X(z)$ assumed to exist for all $z > 0$. The EVaR is in fact the tightest possible upper bound that can be obtained from the Chernoff inequality for the VaR, which is also an upper bound for the CVaR, i.e.,

$$\mathrm{VaR}_{1-\alpha}(X) \leqslant \mathrm{CVaR}_{1-\alpha}(X) \leqslant \mathrm{EVaR}_{1-\alpha}(X).$$

The EVaR can be represented by using the concept of relative entropy (see Lemma 2 in Section 4). Many properties of the EVaR are similar to those of the CVaR (see Ahmadi-Javid, 2012). For example, similarly to the CVaR, the following facts hold for the EVaR:

$$\begin{aligned}
&\mathrm{E}(X) \leqslant \mathrm{EVaR}_{1-\alpha}(X) \leqslant \mathrm{ess\ sup}(X)\\
&\mathrm{EVaR}_0(X) = \mathrm{E}(X)\\
&\lim_{\alpha\to 0} \mathrm{EVaR}_{1-\alpha}(X) = \mathrm{ess\ sup}(X).
\end{aligned}$$

An important feature of the EVaR is that it is computationally tractable in important cases where the CVaR is not. For example, if $\Psi_1 \cdots \Psi_{|J|}$ and $\Gamma_1, \ldots, \Gamma_{|V|}$ are independent arbitrary random variables whose moment-generating functions exist everywhere, e.g. bounded random variables, then the EVaR is the preferable choice from a computational perspective. The reader may consult Ahmadi-Javid (2012a) for further technical details. Since it is assumed here that the random variables involved in the problem have a discrete joint distribution, for which both the CVaR and the EVaR are efficiently computable, then, for the sake of brevity, in the mathematical formulations and the numerical study under the cautious risk-measurement policy only the CVaR is considered. However, the analytical results in Section 4 are given for both.

It should be noted that, for the cautious risk-measurement policy, it is quite difficult to present a suitable risk measure that is coherent, computable and intuitively interpretable. Most well-known risk measures which are frequently used in the literature are not coherent. For example, the *mean-standard deviation* risk measure M-SD$_\lambda$, for $\lambda > 0$:

$$\text{M-SD}_\lambda(X) = \mathrm{E}(X) + \lambda\sqrt{\mathrm{var}(X)}, \quad X \in \mathbf{L}_2,$$

does not have the property of monotonicity, or the VaR lacks the subadditivity property and is not efficiently computable in general.

3.3. Reformulation of model (2)–(11) for three risk-measurement policies

In this section, model (2)–(11) is linearized for the three risk measures presented in the previous subsection.

3.3.1. Moderate risk-measurement policy

By considering $\rho(X) = \mathrm{E}(X)$ in (2) and introducing some new variables and constraints, model (2)–(11) can be formulated as the following mixed integer linear program:

$$\min \quad \sum_{j\in J} f_j U_j + (1+\beta)\sum_{v\in V}\sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + \sum_{n=1}^{|S|} p_n\left[-\beta\sum_{v\in V}\mathbf{s}^n_{\Gamma,v}\sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + \sum_{j\in J} dc_j w^n_j\right] \tag{19}$$

$$\text{s.t.} \quad (3)-(11)$$

$$\sum_{k\in K} d_k Y_{jk} - \mathbf{s}^n_{\Psi,j} \leqslant w^n_j \quad j\in J,\ n\in N \tag{20}$$

$$w^n_j \geqslant 0 \quad j\in J,\ n\in N. \tag{21}$$

3.3.2. Cautious risk-measurement policy

By considering $\rho(X) = \mathrm{CVaR}_{1-\alpha}(X)$ in (2) and linearizing model (3)–(11), (20), (21), one obtains

$$\min \quad \sum_{j\in J} f_j U_j + (1+\beta)\sum_{v\in V}\sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + t + \frac{1}{\alpha}\sum_{n=1}^{|S|} p_n v^n_j \tag{22}$$

$$\text{s.t.} \quad (3)-(11),\ (20),\ (21)$$

$$-\beta\sum_{v\in V}\mathbf{s}^n_{\Gamma,v}\sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + \sum_{j\in J} dc_j w^n_j - t \leqslant v^n_j \quad j\in J,\ n\in N \tag{23}$$

$$v^n_j \geqslant 0 \quad j\in J,\ n\in N \tag{24}$$

$$t\in\mathfrak{R}. \tag{25}$$

3.3.3. Pessimistic risk-measurement policy

By substitution of $\rho(X) = \text{ess sup}(X)$ in (2), and introducing some new variables and constraints, model (3)–(11) becomes:

$$\min \quad \sum_{j\in J} f_j U_j + (1+\beta)\sum_{v\in V}\sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + z \tag{26}$$

$$\text{s.t.} \quad (3)-(11),\ (20),\ (21)$$

$$-\beta\sum_{v\in V}\mathbf{s}^n_{\Gamma,v}\sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv} + \sum_{j\in J} dc_j w^n_j \leqslant z \quad j\in J,\ n\in N \tag{27}$$

$$z\in\mathfrak{R}. \tag{28}$$

4. Impact of risk-measurement policy on improvement level

This section analytically investigates the impact of the risk-measurement policy on cost reduction, obtained by incorporating the disruption risk in the classic location-routing problem (CLRP). As a reminder, the CLRP and the LR-PDDR-P are respectively as follows:

$$\begin{aligned}\min_{\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M}} \quad & \sum_{j\in J} f_j U_j + \sum_{v\in V}\sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl}R_{klv}\\ \text{s.t.} \quad & (3)-(11)\end{aligned} \tag{CLRP}$$

and

$$\begin{aligned}\min_{\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M}} \quad & \rho(TC(\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M};\boldsymbol{\Psi},\boldsymbol{\Gamma}))\\ \text{s.t.} \quad & (3)-(11).\end{aligned} \tag{LR-PDDR-P}$$

It will be shown that, as the level of caution increases, one gets more improvement by considering the LR-PDDR-P over the CLRP where the disruption risk is not accounted for. The result is rooted in a very general analysis and holds for a wide range of settings which include the problem considered in this paper as a special case. To continue the discussion, the following two optimization problems must be defined:

$$\min_{x\in S} G(x) \tag{A}$$

$$\min_{x\in S} T(x) = G(x) + \rho(H(x,\xi)) \tag{B}$$

which respectively include the CLRP and the LR-PDDR-P as special cases, provided that risk measure ρ has the property of translation invariance. Functions $G(x)$ and $H(x,z)$ are non-negative bounded functions over bounded set S for any $z \in S_\xi$ where S_ξ is the support set of random vector ξ. For any $x \in S$, function $H(x,\cdot)$ is supposed to be Borel measurable.

Problem B is obtained from problem A by adding term $\rho(H(x,\xi))$, which is actually a scalarization of stochastic part $H(x,\xi)$. In fact, one may consider problem A as a naive approximation of problem B where stochastic part $H(x,\xi)$ is ignored. The absolute and relative improvements obtained by solving problem B instead of problem A for specific risk measure ρ with the translation-invariance property, denoted by AI_ρ and RI_ρ, are defined as follows:

$$AI_\rho := T(\bar{x}) - T(x^*)$$
$$RI_\rho := \frac{T(\bar{x}) - T(x^*)}{T(\bar{x})}$$

where $\bar{x} \in \arg\ \min_{x\in S} G(x)$ and $x^* \in \arg \min_{x\in S}\{G(x) + \rho(H(x,\xi))\}$.

In the sequel, for $\rho(X) = \mathrm{CVaR}_{1-\alpha}(X)$ or $\rho\ (X) = \mathrm{EVaR}_{1-\alpha}(X)$, the dependence between the quantities AI_ρ, RI_ρ and parameter α are studied. Recall that for $\alpha = 1$, $0 < \alpha < 1$ and $\alpha{\downarrow}0$, one retrieves, respectively, the moderate, cautious and pessimistic risk-measurement polices that are discussed in Section 3. As α decreases, the decision maker's level of caution increases. Corollaries 1 and 2 analyzes the impact of α on the tight upper bounds obtained for AI_ρ and RI_ρ in Theorems 1 and 2.

Theorem 1. *For any coherent risk measure ρ, e.g., the CVaR or the EVaR,*

$$0 \leqslant AI_\rho \leqslant \rho[AI(\xi)]$$

where $AI(\xi) := G(\bar{x}) + H(\bar{x},\xi) - \min_{x\in S}\{G(x) + H(x,\xi)\}$. The upper bound is tight, as S is singleton; $H(\cdot,z)$ is constant for any $z \in S_\xi$; or, ξ is a constant vector.

Proof. The coherency of ρ implies

$$\begin{aligned} AI_\rho &= T(\bar{x}) - T(x^*) = G(\bar{x}) + \rho(H(\bar{x},\xi)) - \min_{x\in S}\{G(x) + \rho(H(x,\xi))\} \\ &= \rho(G(\bar{x}) + H(\bar{x},\xi)) - \min_{x\in S}\rho(G(x) + H(x,\xi)) \\ &\leqslant \rho(G(\bar{x}) + H(\bar{x},\xi)) - \rho(\min_{x\in S}\{G(x) + H(x,\xi)\}) \\ &\leqslant \rho(G(\bar{x}) + H(\bar{x},\xi) - \min_{x\in S}\{G(x) + H(x,\xi)\}). \end{aligned}$$

The tightness of the bound under the given conditions is clear. □

By this theorem, for $\rho(X) = \mathrm{CVaR}_{1-\alpha}(X)$ or $\rho\ (X) = \mathrm{EVaR}_{1-\alpha}(X)$, one has

$$0 \leqslant AI_\rho \leqslant \mathrm{CVaR}_{1-\alpha}[AI(\xi)] \text{ or } 0 \leqslant AI_\rho \leqslant \mathrm{EVaR}_{1-\alpha}[AI(\xi)].$$

An important feature of these proposed bounds are that they only depend on parameter α since $AI(\xi)$ is a random variable whose distribution is independent of α. This clarifies the impact of parameter α on the range of possible absolute improvements achievable by considering problem B instead of problem A. Hence, the following observation follows.

Corollary 1. *For $\rho(X) = \mathrm{CVaR}_{1-\alpha}(X)$ or $\rho(X) = \mathrm{EVaR}_{1-\alpha}(X)$, the length of the range of possible values for the absolute improvement obtained by solving the LR-PDDR-P over the CLRP increases as the decision maker's caution increases, or equivalently, as parameter α decreases.*

Proof. The CLRP and the LR-PDDR-P are special cases of problems A and B, respectively, obtained by setting

$$G(x) = \sum_{j\in J} f_j U_j + \sum_{v\in V} \sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl} R_{klv}$$

$$H(x,\xi) = \beta \sum_{v\in V} (\sup \Gamma_v - \Gamma_v) \sum_{k\in M}\sum_{l\in M} t_{kl} R_{klv} + \sum_{j\in J} dc_j \times \max\left\{\sum_{k\in K} d_k Y_{jk} - \Psi_j, 0\right\}$$

where x is a vector including $\boldsymbol{Y}$, $\boldsymbol{U}$, $\boldsymbol{R}$, $\boldsymbol{M}$; S is the set defined by constraints (3)–(11); and ξ is a random vector containing $\boldsymbol{\Psi}$, $\boldsymbol{\Gamma}$. By Theorem 1, the length of the range of possible values for the absolute improvement equals $\mathrm{CVaR}_{1-\alpha}(AI(\xi))$, which is decreasing in α. □

The same analysis for the relative improvement RI_ρ can be presented, but four lemmas must first be stated. In these lemmas, an arbitrary probability measure on space $(\mathbf{\Omega}, \mathbf{F})$ is denoted by Q, with or without subscript, and symbol $Q \ll P$ means that probability measure Q is absolutely continuous with respect to P.

Lemma 1 (*Dual representation for CVaR*). *For any $X \in \mathbf{L}_1$,*

$$\mathrm{CVaR}_{1-\alpha}(X) = \sup_{Q\in RE_\alpha} (E_Q(X))$$

where $RE_\alpha = \{Q : 0 \leqslant \frac{dQ}{dP} \leqslant \frac{1}{\alpha}\}$.

Proof. See Delbaen (2002) and Ruszczynski and Shapiro (2006). □

Lemma 2 (*Dual representation for EVaR*). *For any* $X \in \mathbf{L}_M$,

$$\mathrm{EVaR}_{1-\alpha}(X) = \sup_{Q \in RE_\alpha} (E_Q(X))$$

where $RE_\alpha = \{Q \ll P : E_P(\frac{dQ}{dP} \ln \frac{dQ}{dP}) \leqslant -\ln \alpha\}$.

Proof. See Ahmadi-Javid (2012a,b). □

Lemma 3. *Let X, Y* $\in \mathbf{L}_1$ *be two independent positive random variables such that X/Y* $\in \mathbf{L}_1$. *Then,*

$$\frac{\mathrm{CVaR}_{1-\alpha}(X)}{\mathrm{CVaR}_{1-\alpha}(Y)} \leqslant \mathrm{CVaR}_{1-\alpha^2}\left(\frac{X}{Y}\right).$$

Proof. Without any loss of generality, suppose that $(\mathbf{\Omega}, \mathbf{F}, P)$ is a probability space where $\mathbf{\Omega} = (0,1]$, $\mathbf{F}$ is a σ-field of Borel subsets of $(0,1]$, and P is the Lebesgue measure restricted to $(0,1]$. This probability space is very flexible because not only all random variables but also any sequence of independent real-valued random variables or random vectors with arbitrary distribution functions can be defined on this space.

By Lemma 1 and Jensen's inequality one has

$$\begin{aligned}\frac{\mathrm{CVaR}_{1-\alpha}(X)}{\mathrm{CVaR}_{1-\alpha^2}(Y)} &= \frac{\sup_{Q\in RE_\alpha}(E_Q(X))}{\sup_{Q\in RE_\alpha}(E_Q(Y))} = \frac{E_{Q_1}(X)}{E_{Q_2}(Y)} \leqslant E_{Q_1}(X)E_{Q_2}\left(\frac{1}{Y}\right) = E_P(ZX)E_P\left(\frac{T}{Y}\right) = E_P(Z'X')E_P\left(\frac{T'}{Y'}\right) = E_P\left(Z'T'\frac{X'}{Y'}\right)\\ &\leqslant \sup_{0\leqslant W\leqslant\alpha^{-2},\ E_P(W)=1} E_P\left(W\frac{X'}{Y'}\right) = \sup_{Q\in RE_{\alpha^2}}\left(E_Q\left(\frac{X'}{Y'}\right)\right) = \mathrm{CVaR}_{1-\alpha}\left(\frac{X'}{Y'}\right) = \mathrm{CVaR}_{1-\alpha^2}\left(\frac{X}{Y}\right)\end{aligned}$$

where $Q_1 \in \arg\ \max_{Q\in RE_\alpha}(E_Q(X))$, $Q_2 \in \arg\ \max_{Q\in RE_\alpha}(E_Q(Y))$, $Z = \frac{dQ_1}{dP}$ and $T = \frac{dQ_2}{dP}$. Moreover, vectors (Z',X') and (T',Y') are independent and defined such that $(Z',X') \sim (Z,X)$ and $(T',Y') \sim (T,Y)$. Note that $E_P(Z'T') = 1$ and $0 \leqslant Z'T' \leqslant \alpha^{-2}$. The rest of the proof is straightforward. □

Lemma 4. *Let X, Y* $\in \mathbf{L}_M$ *be two independent positive random variables such that X/Y* $\in \mathbf{L}_M$. *Then,*

$$\frac{\mathrm{EVaR}_{1-\alpha}(X)}{\mathrm{EVaR}_{1-\alpha}(Y)} \leqslant \mathrm{EVaR}_{1-\alpha^2}\left(\frac{X}{Y}\right).$$

Proof. By applying the dual representation given in Lemma 2, the proof is quite similar to the one presented for Lemma 3. □

Theorem 2. *Let* $[G(\bar{x}) + H(\bar{x}, \xi)]^{-1}$ *be a bounded random variable. For* $\rho(X) = \mathrm{CVaR}_{1-\alpha}(X)$ *or* $\rho(X) = \mathrm{EVaR}_{1-\alpha}(X)$, *relative improvement* RI_ρ *can be bounded as follows, respectively:*

$$0 \leqslant RI_\rho \leqslant \mathrm{CVaR}_{1-\alpha^2}[RI(\xi, \xi^{IC})]$$

or

$$0 \leqslant RI_\rho \leqslant \mathrm{EVaR}_{1-\alpha^2}[RI(\xi, \xi^{IC})]$$

with

$$RI(\xi, \xi^{IC}) = \frac{G(\bar{x}) + H(\bar{x}, \xi) - \min_{x\in S}\{G(x) + H(x, \xi)\}}{G(\bar{x}) + H(\bar{x}, \xi^{IC})}$$

where ξ^{IC} *is an independent copy of* ξ. *The upper bound is tight as set S is singleton;* $H(\cdot, z)$ *is constant for any* $z \in S_\xi$; *or,* ξ *is a constant vector.*

Proof. The proof immediately follows from Lemmas 3 and 4. □

Note that $[G(\bar{x}) + H(\bar{x}, \xi)]^{-1}$ will be a bounded random variable if, for example, $G(\bar{x}) > 0$. This follows from the assumption that function $H(\bar{x}, z)$ is non-negative and bounded for any $z \in S_{\xi}$.

The tight upper bound provided in Theorem 2 for the relative improvement depends only monotonically on parameter α. This leads to the following observation, which is the same as the one given in Corollary 1.

Corollary 2. *For $\rho(X) = CVaR_{1-\alpha}(X)$ or $\rho(X) = EVaR_{1-\alpha}(X)$, the length of the range of possible values for the relative improvement obtained by solving the LR-PDDR-P over the CLRP increases as the decision maker's caution increases, or equivalently, as parameter α decreases.*

Proof. The proof immediately follows from Theorem 2 similarly as in the proof of Corollary 1. □

Corollaries 1 and 2 indicate that as the level of caution increases, more improvement (absolute or relative) is expected to be achieved by solving the LR-PDDR-P over the CLRP. The numerical study in Section 6.6 backs up these analytical observations.

5. Heuristic algorithm

As the LR-PDDR-P belongs to the class of NP-hard problems, this section develops a heuristic algorithm to find near-optimal solutions for large-sized instances of the LR-PDDR-P. The heuristic algorithm is decomposed into two stages: the constructive stage, where an initial solution is built at random; and the improvement stage, where the solution is iteratively improved by modifying the location and routing decisions in two phases: the location phase and the routing phase. The stopping criterion for the algorithm is *count* = *max-count* with initial settings *count* = 1 and *max-count* = 4. If this criterion is met, the algorithm terminates; otherwise, after setting *count* = *count* + 1, it continues to improve the current solution in the location and routing phases. Fig. 2 illustrates the proposed heuristic.

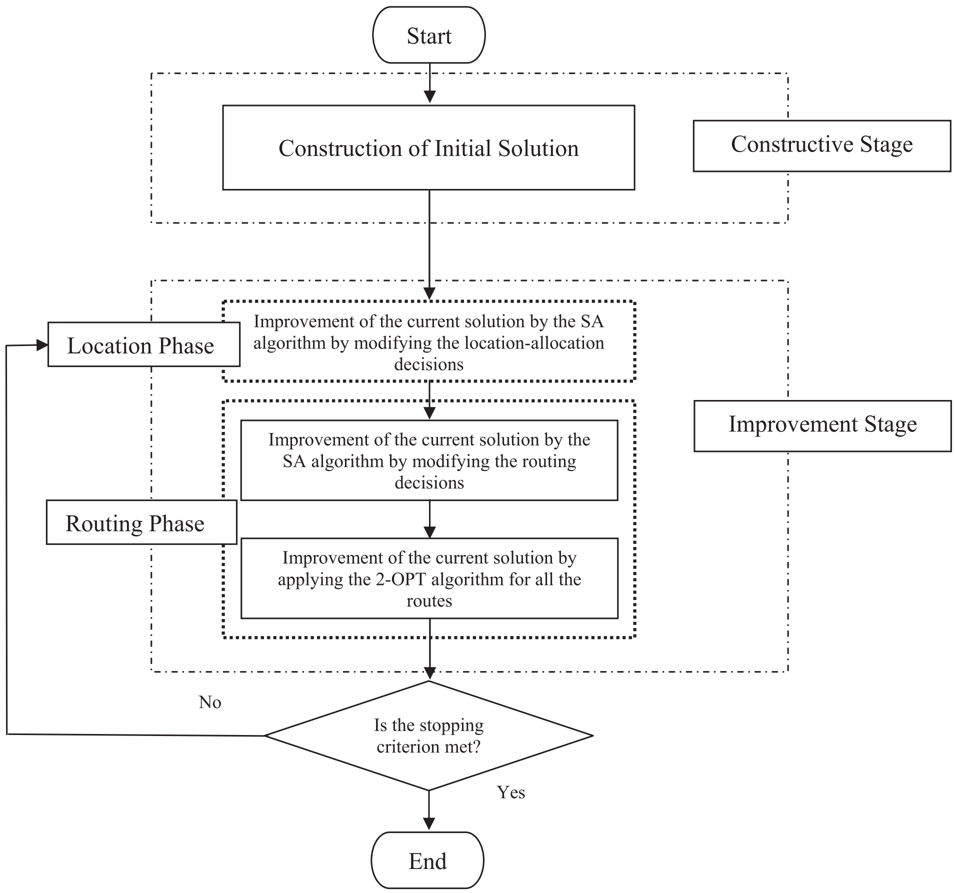

Fig. 2. The flowchart of the heuristic algorithm.

In both phases, a heuristic based on simulated annealing (SA) is used to improve the current solution. SA is an important heuristic optimization approach that is successful at approximately solving hard combinatorial problems. SA uses a stochastic approach to proceed with the search in a neighboring state even if the move causes the value of the objective function to worsen. This prevents search from falling into local optimum traps. The SA algorithm that is used in every phase of the improvement stage is described in the following.

The parameters of the algorithm are as follows:

IT	The initial temperature
CS	The decreasing rate of the current temperature
FT	The freezing temperature
MNT	The maximum number of accepted solutions at each temperature
nt	The counter for the number of accepted solutions at each temperature
x_0	The initial solution
x	The current solution in the algorithm
x_{nh}	A solution that is selected in the neighborhood of x at each iteration
x_{best}	The best solution obtained in the algorithm
$C(x)$	The objective function value for solution x

The steps of the SA algorithm, which is used in the improvement phase, are as follows:

Step 1: Take the initial temperature IT, the initial solution x_0, and set $T = IT$, $x_{best} = x_0$ and $x = x_0$.
Step 2: Generate solution x_{nh} in the neighborhood of x.
Step 3: Let $\Delta C = C(x_{nh}) - C(x)$.
  3.1. If $\Delta C \leqslant 0$, then set $x = x_{nh}$ and, if $C(x_{nh}) < C(x_{best})$, set $x_{best} = x_{nh}$.
  3.2. If $\Delta C > 0$, generate y from the uniform distribution over (0,1). If $y < \exp\{-T^{-1}\Delta C\}$, then set $x = x_{nh}$.
Step 4: If the number of iterations under temperature T is not greater than MNT, go to Step 2, else go to Step 5.
Step 5: Set $T = CS \times T$.
Step 6: If the SA stopping criterion ($T < FT$) is matched, stop; else go to Step 2.

This algorithm will be used in each phase. For both phases, all the settings are the same with the exception of how the algorithm generates a solution x_{nh} in the neighborhood of the current solution x. This means that different moves are used to generate neighboring solutions in the location and routing phases.

It is worth mentioning that under the cautious risk-measurement policy where the CVaR is used, objective value (2) is efficiently computed by formula (17) considering the fact that uncertain parameters of the problem are supposed to have a discrete joint distribution.

Section 5.1 describes the constructive stage, where it is shown how an initial solution is generated. Section 5.2 studies the moves used in the two phases of the improvement stage.

5.1. Constructive stage

In the constructive stage, an initial solution is built at random. First, customers are randomly assigned to the open PDs. For each open PD, routes are built through the customers by using the nearest-neighbor algorithm (see Section 3.1 in Ahmadi-Javid and Seddighi, 2012). This procedure is explained in the following steps.

Step 1: Put all the customers into set K_0.
Step 2:
  2.1. Randomly select a customer from K_0.
  2.2. Delete the selected customer from K_0.
Step 3: Select an open PD randomly. If the PD's remaining APC is greater than the demand of the customer selected in Step 2, then assign the customer to the PD and go to Step 5; otherwise, go to Step 4.
Step 4: Open an potential PD randomly. If the PD's APC is greater than the demand of the customer selected in Step 2, then assign the customer to the PD and go to Step 5; otherwise, open another potential PD randomly and continue in the same way.
Step 5: Is K_0 empty? If yes, go to step 6; if not, go to step 2.
Step 6: Build routes through the customers assigned to each open PD.

5.2. Improvement stage

The improvement stage has two phases: the location phase and the routing phase. In this stage, the current solution is improved by alternately modifying the location and routing decisions. First the initial solution is improved in the location phase, and then the resulting solution is improved in the routing phase. Next the solution obtained in the routing phase is improved in the location phase, and the procedure continues in this manner until the stopping criterion is matched. Sections 5.2.1 and 5.2.2 present the moves exploited by the SA algorithm in the location and routing phases, respectively, to generate a solution in the neighborhood of the current solution.

5.2.1. Location phase

The main goal of the location phase is to improve the current solution by modifying the number and locations of open PDs, and the allocation of the customers. The solution obtained in this phase is used as an input in the routing phase. This phase uses the SA algorithm, which randomly selects a move (Mov1, Mov2 and Mov3) to obtain a new solution x_{nh} in the neighborhood of the current solution x. In Mov1, one of the open PDs, for example j, is closed by the roulette-wheel method. Then, each of its customers is re-allocated among the other open PDs by the roulette-wheel method. If the remaining APCs of the other open PDs are not enough to satisfy the customers of PD j, then a new move is randomly selected. Then, by using the nearest-neighbor algorithm, the routes of all open PDs to which the customers of PD j have been assigned are rebuilt. In Mov2, two open PDs, for example i and j, are randomly selected, and then their customers are changed. The nearest-neighbor algorithm is used for PDs i and j to build the required routes. In Mov3, one of the open PDs is closed and a potential PD is opened by the roulette-wheel method. At this phase, the SA algorithm proceeds with the above three moves until $T < FT$. Then, the obtained solution will be an input to the routing phase described in the next subsection.

5.2.2. Routing phase

This phase tries to improve the current solution by modifying the routes. First, it uses the SA algorithm which randomly selects one of the moves (Mov4 and Mov5) to obtain a new solution x_{nh} in the neighborhood of the current solution x. Then, the 2-OPT algorithm is applied to each of the routes of the final solution obtained from the SA algorithm. In Mov4, one of the routes is selected randomly and one of its customers is chosen randomly. This customer is then inserted into another randomly selected route. Next, the best location in the route is selected for the insertion of the new customer without any change in the other customers' order. In Mov5, two routes are first selected randomly, for example v_i and w_j. Then two customers are randomly selected, for example k_i and k_j, in routes v_i and w_j, respectively, and then the routes of customers k_i and k_j are swapped. The best locations of customers k_j and k_i in their new routes are then determined without any change in the order of the other customers.

The SA algorithm is applied to the final solution obtained from the location phase, and it uses Mov4 and Mov5 to generate neighboring solutions. This process is repeated until $T < FT$ at this phase. Then, each route is improved by the 2-OPT algorithm. When this phase ends, the stopping criterion of the heuristic algorithm (i.e., $count = max - count$) is verified. If it is met, the heuristic algorithm stops; else, the counter $count$ is updated as $count = count + 1$ and the solution obtained from the routing phase is used as an input for the location phase and the heuristic algorithm continues in this fashion.

6. Computational results

This section presents an extensive computational experiment to evaluate the performance of the heuristic algorithm proposed in Section 5 on a set of randomly generated instances. The demands of the customers are drawn from a uniform distribution over interval [400, 1500]. Transportation costs t_{kl} are uniformly drawn from interval [0.1, 1], and disruption costs dc_j are uniformly drawn from interval [0.05, 0.075]. The fixed set-up cost of locating and operating PD j is determined by $f_j = [\pi_j]$ where π_j is uniformly generated between 130 and 200.

The vehicle capacity is computed as $vc = 2[D|V|^{-1}]$, where D represents the total of all customers' demands. It is also assumed that the number of working days per year is 300 and that a vehicle visits its customers every three days, i.e., $\sup\Gamma_v = 100$. The parameter β is set as 0.2.

For each $j \in J$, $\mathbf{s}^n_{\Psi,j}$ is drawn from the uniform distribution over interval $(0, H_j)$ with $H_j = [c_j D|J|^{-1}]$ where c_j is a random number between 1.5 and 1.8. Next, the right endpoint $\sup\Psi_j$ is computed on the basis of all generated scenarios. For each $v \in V$, $\mathbf{s}^n_{\Gamma,v}$ is a random number generated from the Binomial distribution $B(100, 0.1)$. For all scenarios, it is assumed that $p_n = |S|^{-1}$.

The program of the heuristic algorithm is coded in C++ and run on a Pentium 4 with 2.8 GHz processor. The linear models are solved by CPLEX 10. For each instance, the heuristic was run 10 times, and then the best objective value is reported in the tables. Tuning is done by random experiments for each instance. The CPU times reported in tables are given in seconds. In the following subsections, the parameter α of the CVaR is set to 0.05, and "E", "CVaR" and "W", respectively, stand for the moderate policy (expectation risk measure), cautious policy (CVaR risk measure) and pessimistic policy (worst-case risk measure). In Table 3, the coefficient of variation (CV) for each instance is reported, to measure the algorithm's stability. The CV for a set of data is computed as the ratio of the mean of the data and the standard deviation of the data.

6.1. Comparison of optimal and heuristic solutions

To evaluate the proposed heuristic, several small-sized instances were solved optimally using the MILPs presented in Section 3.3, and were also solved by the heuristic. For all instances, the heuristic optimally solved the instances very efficiently compared to CPLEX. Table 1 reports the CPU times.

6.2. Comparison of heuristic solutions and lower bounds

This section evaluates the heuristic algorithm by presenting lower bounds for the solutions obtained by the heuristic algorithm for medium-sized instances where there is no distribution disruption, or equivalently, $\beta = 0$. Here it is assumed that the distances between nodes satisfy the triangular inequality.

To find a lower bound, it is assumed that the vehicle capacity is unlimited. This relaxation implies that each PD can serve all its allocated costumers with only one route in the optimal solution. Hence constraints (4) can be omitted and then one can substitute decision variables R_{klv} in model (2)–(11) with the following:

$$R_{klj} = \begin{cases} 1 & \text{if } k \text{ precedes } l \text{ in the route starting at PD } j \\ 0 & \text{otherwise.} \end{cases} \quad k,l \in M, \quad j \in J$$

Constraints (5) and (8), which make solving the model very difficult, are also omitted and some valid constraints for tightening the lower bound are added. The whole model used to find a lower bound is given in (29)–(41) as follows:

$$\min \quad \rho\left(\sum_{j\in J} f_j U_j + \sum_{j\in J} \sup \Gamma_j \sum_{k\in M}\sum_{l\in M} t_{kl} R_{klj} + \sum_{j\in J} dc_j \times \max\left\{\sum_{k\in K} d_k Y_{jk} - \Psi_j, 0\right\}\right) \tag{29}$$

s.t.

$$\sum_{k\in K} d_k Y_{jk} \leqslant (\sup \Psi_j) \times U_j \quad j \in J \tag{30}$$

$$\sum_{j\in J}\sum_{l\in M} R_{klj} = 1 \quad k \in K \tag{31}$$

$$\sum_{l\in M} R_{klj} - \sum_{l\in M} R_{lkj} = 0 \quad k \in M, \quad j \in J \tag{32}$$

$$R_{kkj} = 0 \quad k \in M, \quad j \in J \tag{33}$$

$$\sum_{l\in J} R_{jlj} = 0 \quad j \in J \tag{34}$$

$$\sum_{l\in M} R_{klj} = 0 \quad k,j \in J, \quad k \neq j \tag{35}$$

$$\sum_{j\in J} Y_{jk} = 1 \quad k \in K \tag{36}$$

$$\sum_{k\in K} R_{jkj} = U_j \quad j \in J \tag{37}$$

$$\sum_{l\in M} R_{klj} \leqslant Y_{jk} \quad k \in K, \quad j \in J \tag{38}$$

$$\sum_{k\in M} R_{klj} \leqslant Y_{jl} \quad l \in K, \quad j \in J \tag{39}$$

$$R_{klj} + R_{lkj} \leqslant 1 \quad k,l \in K, \quad j \in J \tag{40}$$

$$\begin{aligned} & Y_{jk} \in \{0,1\} \quad k \in K, \quad j \in J \\ & U_j \in \{0,1\} \quad j \in J \\ & R_{klj} \in \{0,1\} \quad k,l \in M, \quad j \in J. \end{aligned} \tag{41}$$

Table 1
Comparison of the optimal and heuristic solutions under the three risk-measurement policies.

No.	#Customer	# Potential PD	# Vehicle	# Scenario	MILP CPU time			Heuristic CPU time		
					E	CVaR	W	E	CVaR	W
S1	4	2	2	40	1	1	1	2	6	4
S2	4	3	2	40	2	4	5	2	6	4
S3	6	3	2	40	5	5	6	3	8	6
S4	6	4	3	40	32	37	34	2	5	6
S5	7	3	5	40	161	263	110	2	5	5
S6	7	4	5	40	1647	3705	2633	4	6	7
S7	8	3	3	40	325	885	95	3	10	7
S8	8	4	5	40	1166	1415	3832	3	9	5
S9	9	3	2	40	150	130	43	3	10	6
S10	9	4	5	40	200	1422	687	4	12	8

Constraints (33) ensure that any node is not connected to itself in each route. Constraints (34) and (35) guarantee that two open PDs cannot be placed on the same route. Constraints (36) make sure that all customers are assigned to open PDs. Constraints (37) imply that routes can be constructed only for open PDs. Constraints (38) and (39) assure that each customer is assigned to the open PD from which the route that includes it starts. Constraints (40) eliminate subtours involving exactly two customers.

The error bounds are computed as follows:

$$\text{Error bound } (\%) = 100 \times (\text{Heuristic solution value} - \text{Lower bound value})/\text{Lower bound value}$$

The results are given in Table 2. It can be seen that the quality of the solutions obtained by the heuristic algorithm is promising and that the errors bounds are relatively small under any risk-measurement policy.

6.3. Comparison of heuristic algorithm with an adapted heuristic

This section compares the proposed heuristic with a heuristic designed on the basis of the heuristic proposed by Ahmadi-Javid and Azad (2010) for a different LRP. Improvements are computed as follows:

$$\begin{aligned}\text{Improvement}(\%) = 100 \times (&\text{Solution value obtained by adapted heuristic} \\ &- \text{Solution value obtained by proposed heuristic})/\text{Solution value obtained by proposed heuristic}\end{aligned}$$

The results, given in Table 3, indicate that the proposed heuristic considerably outperforms the one based on an old heuristic. The results also demonstrate that the CVs are under 0.02 and that the CPU times are less than 6 min for the very-large-sized instances L7–L10, under the three measurement policies. This means that the heuristic is stable and efficient enough to solve large-sized instances.

6.4. Improvements over initial solutions

Fig. 3 depicts the improvements made by the heuristic over the initial solutions for the instances given in Table 3. Improvements are computed as follows:

$$\text{Improvement}(\%) = 100 \times (\text{Initial solution value} - \text{Final solution value})/\text{Initial solution value}$$

From Fig. 3, it can be seen that the heuristic considerably improves the initial solutions. The heuristic also behaves similarly under the three risk-measurement policies. This shows that the heuristic is effective under each of the risk-measurement policies, thought the improvement level decreases as the decision maker's level of caution increases.

6.5. Contribution of each component of heuristic to overall improvement

This section examines the impact of each component of the heuristic on the overall improvements for the instances given in Table 3. This study tells us whether the components are properly selected or whether some are unnecessary and can be eliminated. In Fig. 4, the contribution percentages for each component are given. Contribution percentages are computed as follows:

$$\text{Contribution of a component}(\%) = 100 \times \text{Improvement obtained by the component}/\text{Overall improvement}$$

Fig. 4 shows that each component has a considerable portion of the overall improvement, so removing one of them would negatively impacts the algorithm's performance.

Table 2
The error bounds to assess the quality of the solutions obtained by the heuristic under the three risk-measurement policies.

No.	#Customer	# Potential PD	# Vehicle	# Scenario	CPU time – lower bound			CPU time – heuristic			Error bound (%)		
					E	CVaR	W	E	CVaR	W	E	CVaR	W
M1	20	8	8	50	195	722	647	35	13	26	1.73	0	2.38
M2	20	10	10	50	503	1220	901	42	15	24	3.89	0	2.6
M3	20	12	12	50	273	359	2337	43	16	23	0	0	0.87
M4	20	10	10	100	11,532	3730	4875	57	22	33	0.54	2.69	1.82
M5	40	12	12	50	5808	20,320	33,200	55	19	30	5.33	0.2	2.84
M6	40	15	15	50	117,970	103,763	100,902	70	24	39	4.9	0.91	5.31
M7	40	12	12	100	137,109	101,952	117,181	81	28	45	5.17	5.92	4.25
M8	40	15	15	100	34,882	69,172	44,278	130	34	56	6.5	5.14	5.28
M9	60	15	15	50	29,027	69,130	16,020	93	29	34	5.02	0.36	6.8
M10	60	17	17	50	64,400	88,570	91,905	106	99	165	5.39	0.25	5.45

Table 3
The improvements obtained by the proposed heuristic over the heuristic adapted from the literature under the three risk-measurement policies.

No.	#Customer	# Potential PD	# Vehicle	# Scenario	CPU time			CV			Improvement over adapted heuristic (%)		
					E	CVaR	W	E	CVaR	W	E	CVaR	W
L1	50	15	15	100	18	36	27	0.009	0.011	0.006	5.57	3.65	3.20
L2	50	15	15	200	27	61	36	0.009	0.007	0.007	5.50	5.36	2.87
L3	100	20	22	100	37	58	51	0.007	0.006	0.005	7.66	5.85	4.52
L4	100	20	22	200	46	84	62	0.005	0.013	0.009	6.90	6.15	6.79
L5	150	25	30	100	70	151	95	0.007	0.006	0.010	8.32	7.76	7.33
L6	150	25	30	300	55	76	74	0.005	0.006	0.005	8.70	8.25	8.26
L7	200	30	35	200	71	117	99	0.007	0.008	0.007	9.58	9.62	7.89
L8	200	30	35	400	102	191	139	0.008	0.010	0.011	9.62	8.60	8.07
L9	200	30	35	600	132	278	185	0.013	0.006	0.014	9.23	9.22	6.23
L10	200	30	35	800	165	342	221	0.005	0.007	0.009	10.10	8.08	8.58

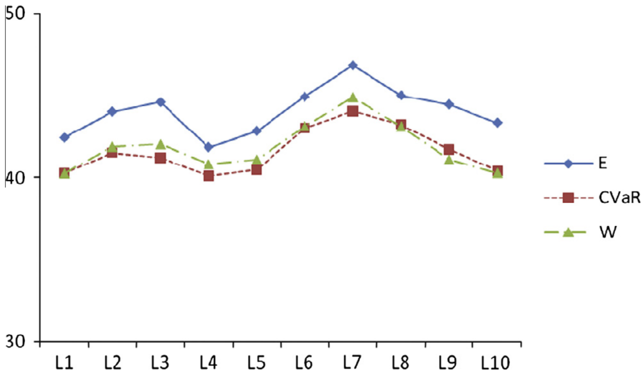

Fig. 3. The improvements obtained by the heuristic over the initial solutions under the three risk-measurement policies.

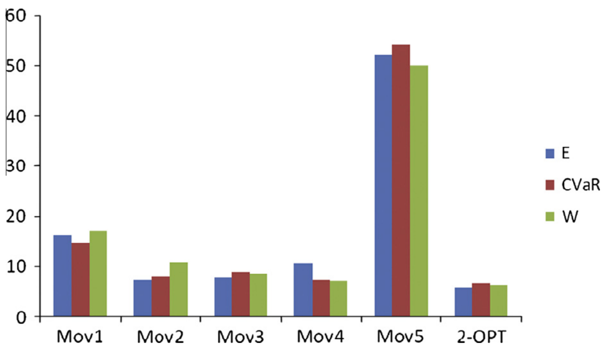

Fig. 4. The average contribution (in %) of the proposed heuristic's components in the overall improvements.

6.6. Advantage of incorporating disruption risk in CLRP

In Figs. 5–7, the classic location-routing problem (CLRP) is compared, respectively, to the location-routing problem with (production and distribution) disruption risk (LR-PDDR-P), the location-routing problem with production-disruption risk (LR-PDR-P) and the location-routing problem with distribution-disruption risk (LR-DDR-P). Note that the LR-PDR-P and LR-DDR-P are

$$\min_{\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M}} \quad \rho\left(\sum_{j\in J} f_j U_j + \sum_{v\in V} \sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl} R_{klv} + \sum_{j\in J} dc_j \times \max\{\sum_{k\in K} d_k Y_{jk} - \Psi_j, 0\}\right) \qquad \text{(LR-PDR-P)}$$
$$\text{s.t.} \quad (3)\text{–}(11)$$

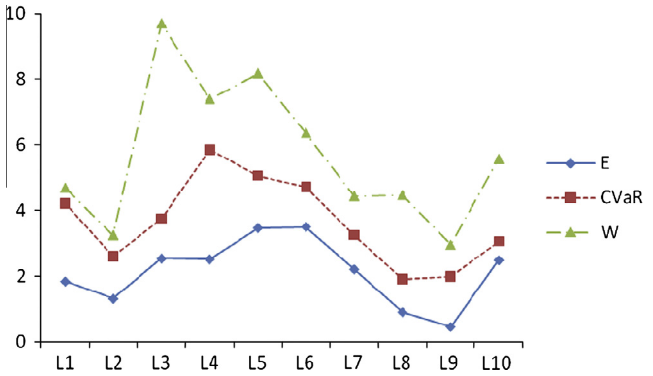

Fig. 5. Comparison of the CLRP and the LR-PDDR-P.

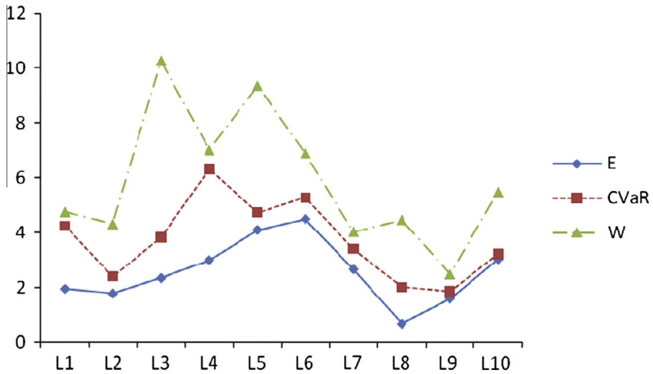

Fig. 6. Comparison of the CLRP and the LR-PDR-P.

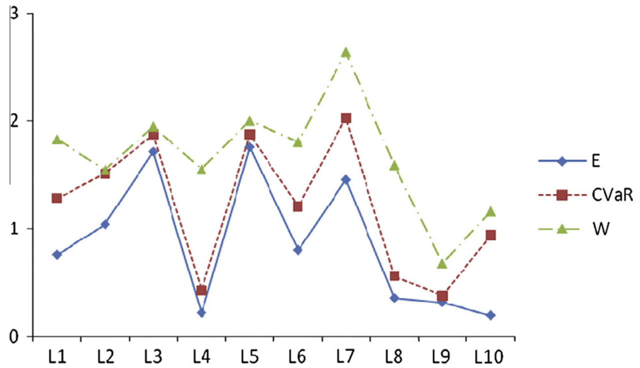

Fig. 7. Comparison of the CLRP and the LR-DDR-P.

$$\min_{\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M}} \quad \rho\left(\sum_{j\in J} f_j U_j + \sum_{v\in V} \sup \Gamma_v \sum_{k\in M}\sum_{l\in M} t_{kl} R_{klv} + \beta \sum_{v\in V} (\sup \Gamma_v - \Gamma_v) \sum_{k\in M}\sum_{l\in M} t_{kl} R_{klv}\right) \tag{LR-DDR-P}$$
$$\text{s.t.} \quad (3){-}(11).$$

The improvements in Figs. 5–7 are computed as follows:

$$\begin{aligned}\text{Improvement}(\%) = 100 \times (&\text{Objective function of LR-PDDR-P (LR-PDR-P or LR-DDR-P)}\\ &\text{for solution obtained from CLRP} - \text{Best known value of LR-PDDR-P (LR-PDR-P or LR-DDR-P))}/\\ &\text{Objective function of LR-PDDR-P (LR-PDR-P or LR-DDR-P) for solution obtained from CLRP}\end{aligned}$$

The results illustrate that the incorporation of any kind of disruption risk, production or disruption, in the CLRP significantly improves the total cost of location, routing and disruption under any risk-measurement policy. From Figs. 5–7, it can also be seen that, in all cases, the improvement increases as the decision maker's level of caution increases, which consistently verifies the analytical results proven in Corollaries 1 and 2. Figs. 6 and 7 also reveal that, at least for the instances solved here, incorporating production disruption yields more improvement than does incorporating distribution disruption.

Figs. 8 and 9 compare the LR-PDDR-P with the LR-PDR-P and the L-DDR-P, respectively. The improvements are computed as follows:

$$\begin{aligned}\text{Improvement}(\%) = {} & 100 \times \text{Objective function of LR-PDDR-P for solution obtained from LR-PDR-P} \\ & (\text{or LR-DDR-P}) - \text{Best known value of LR-PDDR-P})/\text{Objective function of LR-PDDR-}P \\ & \text{for solution obtained from LR-PDR-P (or LR-DDR-P)}\end{aligned}$$

For the instances solved here, the latter two figures indicate that ignoring the production–disruption risk in the procedure of designing a supply-chain network incurs more cost than does ignoring the distribution–disruption risk under any risk-measurement policy.

6.7. Comparison of risk-measurement policies from dispersion and magnitude perspectives

In this section, the risk-measurement policies are compared in terms of the magnitude and dispersion of the real-world cost function, which is random variable $TC(\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M};\boldsymbol{\Psi},\boldsymbol{\Gamma})$ given in (1). This cost function is quantified as $\rho(TC(\boldsymbol{Y},\boldsymbol{U},\boldsymbol{R},\boldsymbol{M};\boldsymbol{\Psi},\boldsymbol{\Gamma}))$ in (2) by the use of a risk measure ρ depending on the decision maker's risk-measurement policy. The mean and standard deviation of a random variable are well-known magnitude and dispersion measures, respectively. Traditionally, inspired from the famous Markowitz model (Markowitz, 1959), several authors have considered combinations of the mean and standard deviation, or variance; however, as discussed in Section 3.2, such risk measures unfortunately lack the monotonicity property, which is necessary for a desirable risk measure. This is the main reason that other risk measures, like the CVaR or the EVaR, have been used recently in various research studies.

Nonetheless, since decision makers still use the mean and the standard-deviation, they can be exploited here to numerically compare different risk-measurement policies from the points of view of magnitude and dispersion. To this end, the

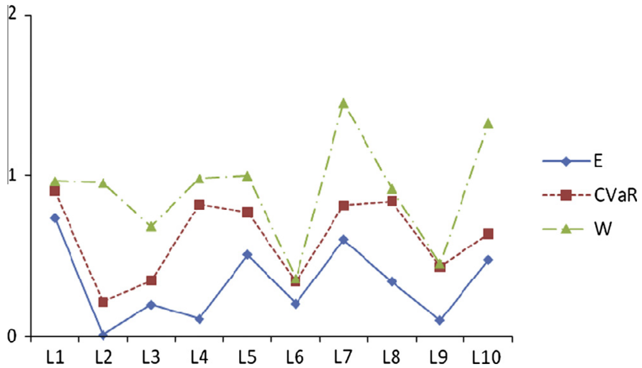

Fig. 8. Comparison of the LR-PDDR-P and the LR-PDR-P.

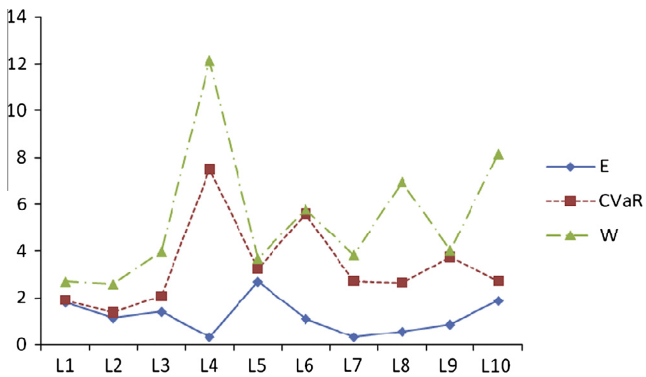

Fig. 9. Comparison of the LR-PDDR-P and the LR-DDR-P.

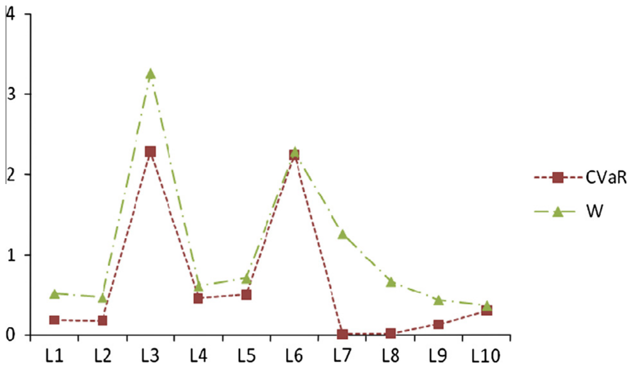

Fig. 10. The relative increment percentage of the means of the real-world objective values under the cautious and pessimistic risk-measurement polices, compared to the means under the moderate risk-measurement policy.

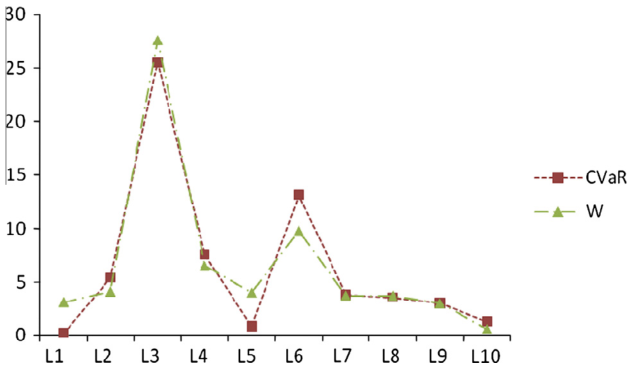

Fig. 11. The relative decrement percentage of the standard deviations of the real-world objective values under the cautious and pessimistic risk-measurement polices, compared to the standard deviations under the moderate risk-measurement policy.

mean and standard deviation of real-world cost (1) are compared by considering the solutions obtained under the three risk-measurement policies. Fig. 10 plots the relative percentage increments of the means under cautious and pessimistic risk-measurement policies, over the mean under the moderate risk-measurement policy. Fig. 11 similarly shows the relative percentage decrements. The following observations can be made from Figs. 10 and 11:

- For each instance, the means of the real-world objective value (1) obtained under cautious and pessimistic risk-measurement policies are larger than the mean under the moderate risk-measurement policy.
- For each instance, the standard deviations of the real-world objective value (1) obtained under cautious and pessimistic risk-measurement policies are less than the standard deviation under the moderate risk-measurement policy.
- For each instance, the mean of the real-world objective value (1) obtained under cautious risk-measurement policy is less than the one under the pessimistic risk-measurement policy.
- There is no conclusion to be made about comparing the standard deviations under cautious and pessimistic risk-measurement policies, but one can see that they are very close to each other.

From the above points, one can see first that the cautious and pessimistic risk-measurement polices successfully decrease dispersion in all instances. Secondly, the cautious policy mostly gives more suitable solutions in terms of the magnitude and dispersion of real-world cost (1). In other words, the cautious policy reduces the dispersion through a lower increment in the mean compared to the pessimistic policy. Moreover, in all the instances, the solutions obtained from the cautious policy are non-dominant, and they often dominate the solutions obtained from the pessimistic policy.

7. Conclusions

The aim of this paper is to study a location-routing problem with production and distribution disruption risks in a supply-chain network consisting of a set of PDs that produce and distribute a single commodity to a set of customers. Following some disruptions the annual production capacity of each PD may vary randomly, resulting in the PD either having to pay a delay cost, to lose the profit gained by supplying the unfulfilled demand, or meet the unfulfilled customers' demands through more expensive means, e.g., by outsourcing. Moreover, each vehicle may randomly be disrupted, which incurs an extra cost because another vehicle must do the route. The goal is to determine the location–routing decisions that minimize the total annual cost of location, routing and disruption. Three risk-measurement polices, moderate, cautious and pessimistic, are considered. The expectation, conditional value-at-risk and worst-case risk measures are respectively used to measure the risk under these policies.

The problem is formulated as mixed-integer linear programs (MILPs) under the three risk-measurement policies. Also, a powerful two-stage heuristic based on simulated annealing is designed to find near-optimal solutions for large-sized instances. Based on the MILPs, error bounds are also provided for medium-sized instances. The means of the error bounds are less than 5%. For large-sized instances, the average relative improvements obtained in reasonable amounts of times over initial solutions are greater than 40%. The numerical results show the effectiveness, stability and efficiency of the heuristic under each of the risk-measurement policies.

An important contribution of this paper to the literature is that the incorporation of the disruption risk significantly improves the total cost of establishing a supply-chain distribution network. The numerical study shows that the average reductions in total cost under the three risk-measurement policies are 2.1%, 3.6% and 5.7%, respectively, over the classical approach, where disruption risk is ignored. This additionally reveals that the cost reduction increases as the decision-maker's level of caution increases. This is also analytically shown to be an expected trend in general.

Also, a numerical study was done to compare risk-measurement polices in terms of the magnitude and dispersion of the real-world cost, using the mean and standard deviation of the real-world cost. It is observed that both cautious and pessimistic policies are suitable to control the real-world cost's dispersion. Furthermore, the cautious policy always gives non-dominant solutions, when compared to the pessimistic policy from magnitude and dispersion standpoints.

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