Two mathematical formulations of the Location Inventory Routing Problem (LIRP)

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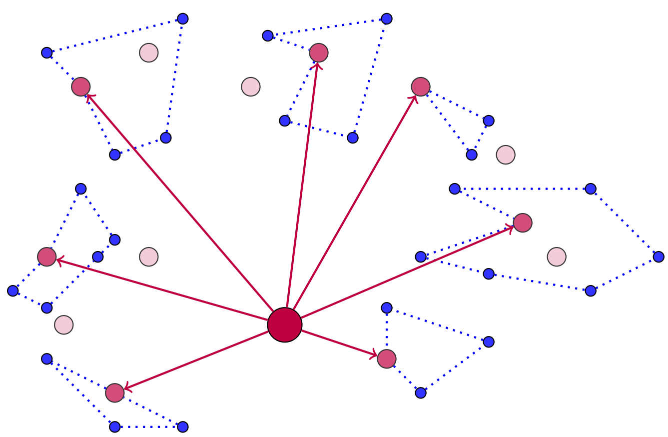
1 Model 1: a "direct+loop" LIRP model



1.1 Data Sets and parameters

Set	Definition
I	Set of customers
J	Set of distribution centers
P	Set of plants (1 plant here)
$T = \{0, \ldots, \lvert T\rvert\}$	set of time periods (days)
$T^* = T\backslash\{0\}$	
V	Set of all nodes $V = P \cup I \cup J$.
V^*	Set of depots and clients $V^* = I \cup J$.
R	Set of routes (in the second layer)

1.2 Data Sets and parameters

Data	Definition
f_j	Fixed cost of opening distribution center $j \in J$
Q	Capacity of vehicles (homogeneous fleet)
τ_{max}	Maximum shelf life.
d_i^t	Demand of customer $i \in I$ in period $t \in \{1, \dots, \lvert T\rvert + \tau_{max} - 1\}$.
h_i^t	Holding cost at facility $i \in V^*$ in time period $t \in T$
I_{i0}	Initial inventory at facility $i \in V^*$
c_j	Cost of delivering distribution center $j \in J$ (1^{st} layer)
c'_r	Cost of route $r \in R$ (2^{nd} layer)
α_{ir}	=1 if route $r \in R$ visits facility $i \in V^*$, 0 otherwise
$\bar{I}_i$	Capacity (max inventory) at facility $i \in V^*$

1.3 Variables

Binary Variables	
y_j	$=$ 1 if distribution center $j \in J$ is selected. 0 otherwise.
z_r^t	$=$ 1 if route $r \in R$ is selected in period $t \in T$, 0 otherwise
x_j^t	=1 if distribution center $j \in J$ is delivered in time period $t \in T$
Continuous Variables	
q_j^t	quantity delivered to distribution center $j \in J$ in time period $t \in T$.
u_{ir}^t	quantity delivered by route $r \in R$ to client $i \in I$ in time period $t \in T$.
I_i^t	inventory at facility $i \in I \cup J$ in time period $t \in T$

1.4 LIRP model 1 (1/3)

$$\min \sum_{j \in J} f_j y_j + \sum_{t \in T} \left(\sum_{j \in J} c_j x_j^t + \sum_{r \in R} c'_r z_r^t \right) + \sum_{t \in T} \sum_{i \in V^*} h_i^t I_i^t \tag{1}$$

$$\sum_{r\in R} \alpha_{ir} z_r^t \leq 1 \qquad \forall i \in I, \forall t \in T \tag{2}$$

$$q_{j,t} \leq Q\ x_j^t \qquad \forall j \in J, \forall t \in T \tag{3}$$

$$x_j^t \leq y_j \qquad \forall j \in J, \forall t \in T \tag{4}$$

$$\sum_{i\in I} u_{ir}^t \leq Q\ z_r^t \qquad \forall r \in R, \forall t \in T \tag{5}$$

$$z_r^t \leq \sum_{j\in J} \alpha_{jr} y_j \qquad \forall r \in R, \forall t \in T \tag{6}$$

$$\sum_{r\in R} z_r^t \leq |K| \qquad \forall t \in T \tag{7}$$

$$y_j = 1 \rightarrow I_j^t = I_j^{t-1} + q_j^t - \sum_{r\in R} \alpha_{jr} (\sum_{i\in I} \alpha_{ir}\ u_{ir}^t) \qquad \forall j \in J, \forall t \in T \tag{8}$$

$$I_i^t = I_i^{t-1} + \sum_{r\in R} \alpha_{ir}\ u_{ir}^t - d_i^t \qquad \forall i \in I, \forall t \in T \tag{9}$$

$$I_i^t \leq \sum_{t'\geq t}^{t'\leq t+\tau_{max}} d_i^{t'} \qquad \forall i \in I, \forall t \in T \tag{10}$$

$$u_{i,r}^t \leq M\alpha_{i,r} \qquad \forall i \in I, \forall r \in R, \forall t \in T \tag{11}$$

$$u_{i,r}^t \leq M z_{r,t} \qquad \forall i \in I, \forall r \in R, \forall t \in T \tag{12}$$

$$I_{i,t} \leq \bar{I}_i \qquad \forall i \in V^*, \forall t \in T \tag{13}$$

$$I_{j,t} \leq \bar{I}_j\ y_j \qquad \forall j \in J, \forall t \in T \tag{14}$$

$$y_j \in \{0,1\} \qquad \forall j \in J \tag{15}$$

$$z_r^t \in \{0,1\} \qquad \forall r \in R, \forall t \in T \tag{16}$$

$$x_j^t \in \{0,1\} \qquad \forall j \in J, \forall t \in T \tag{17}$$

$$I_i^t, I_j^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{18}$$

$$q_j^t, u_{ir}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall r \in R \tag{19}$$

1.5 Constraints of model 1

- Objective function (1)= facility fixed cost + routing cost + inventory cost
- (2) Each supplier is visited by at most one route at every period
- (3) Capacity constraint between the plant and the distribution centers.

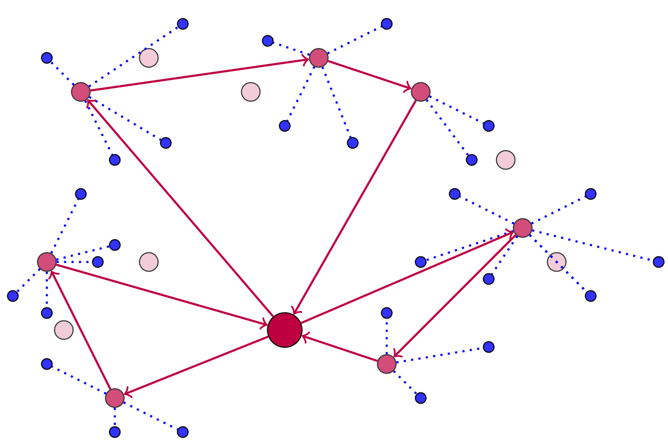
- (4) No routes to non-selected distribution centers
- (5) Capacity constraint on route r, if it is performed in time period t.
- (6) A route starts from a selected distribution centers
- (7) Fleet size limitation
- (8) Flow conservation at DC j
- (30) Flow conservation at customer i
- (10) Max inventory at customers (valid inequality)
- (11) If a customer $i \in I$ does not belong to a route $r \in R$, then it cannot be delivered by this route (this constraint is probably useless)
- (12) If a route is not performed on period $t \in T$, then it cannot deliver any customer (this constraint is probably useless)
- (13) capacity constraint at depots and clients
- (14) if a depot is not selected, then its inventory is zero

Constraints (8) can be linearised as follows

$$I_j^t - I_j^{t-1} - q_j^t + \sum_{r \in R} \alpha_{jr} (\sum_{i \in I} \alpha_{ir} \ u_{ir}^t) \leq (1 - y_j) \ \bar{I}_j \qquad \forall j \in J, \forall t \in T \tag{20}$$

$$-(1 - y_j) \ \bar{I}_j \leq I_j^t - I_j^{t-1} - q_j^t + \sum_{r \in R} \alpha_{jr} (\sum_{i \in I} \alpha_{ir} \ u_{ir}^t) \qquad \forall j \in J, \forall t \in T \tag{21}$$

2 Model 2: a "loop+direct" LIRP model



Data	Definition
R	Set of routes (in first layer)
c_r	Cost of route $r \in R$ (1^{st} layer)
c'_{ij}	Cost of delivering customer $i \in I$ from DC $j \in J$ (2^{nd} layer)
α_{jr}	=1 if route $r \in R$ visits DC $j \in J$, 0 otherwise

Binary Variables	
y_j	$=$ 1 if distribution center $j \in J$ is selected. 0 otherwise.
z_r^t	$=$ 1 if route $r \in R$ is selected in period $t \in T$, 0 otherwise
x_{ij}^t	=1 if customer i is delivered by DC $j \in J$ in time period $t \in T$
Continuous Variables	
v_{ij}^t	quantity delivered from DC $j \in J$ to customer $i \in I$ in period $t \in T$.

2.1 New notation

2.2 LIRP model 2

$$\min \sum_{j \in J} f_j y_j + \sum_{t \in T} \left(\sum_{j \in J} c_r z_r^t + \sum_{i \in I} \sum_{j \in J} c'_{ij} x_{ij}^t \right) + \sum_{t \in T} \sum_{i \in V^*} h_i^t I_i^t \tag{22}$$

$$\sum_{r \in R} \alpha_{jr} z_r^t \leq 1 \qquad \forall j \in J, \forall t \in T \tag{23}$$

$$q_j^t \leq Q \, z_r^t \qquad \forall j \in J, \forall r \in R, \forall t \in T \tag{24}$$

$$z_r^t \leq \alpha_{jr} \, y_j \qquad \forall j \in J, \forall t \in T \tag{25}$$

$$\sum_{r \in R} z_r^t \leq |K| \qquad \forall t \in T \tag{26}$$

$$v_{ij}^t \leq Q \, x_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{27}$$

$$v_{ij}^t \leq \left(\sum_{t \geq t}^{t' \leq t + \tau_{max}} d_i^{t'} \right) x_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{28}$$

$$x_{ij}^t \leq y_j \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{29}$$

$$I_j^t = I_j^{t-1} + q_j^t - \sum_{i \in I} v_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{30}$$

$$I_i^t = I_i^{t-1} + \sum_{j \in J} v_{ij}^t - d_i^t \qquad \forall i \in I, \forall t \in T \tag{31}$$

$$I_i^t \leq \sum_{t' \geq t}^{t' \leq t + \tau_{max}} d_i^{t'} \qquad \forall i \in I, \forall t \in T \tag{32}$$

$$y_j \in \{0,1\} \qquad \forall j \in J \tag{33}$$

$$z_r^t \in \{0,1\} \qquad \forall r \in R, \forall t \in T \tag{34}$$

$$x_{ij}^t \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{35}$$

$$I_i^t, I_j^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{36}$$

$$q_j^t, v_{ij}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall r \in R \tag{37}$$

2.3 Constraints of model 2 (vérifier numérso équations)

- Objective function = facility fixed cost + routing cost + inventory cost
- (21) Each DC is visited by at most one route in every time period
- (22) Capacity constraint on routes.
- (23) No routes to unselected distribution centers
- (24) Fleet size limitation
- (25) (22) Capacity constraints on route r, if it is performed in time period t.
- (26) A customer is served from a selected distribution center
- (27) Flow conservation at DC j
- (28) Flow conservation at customer i
- (29) Max inventory at customers (valid inequality)