Pergamon

Transpn Res.-E (Logistics and Transpn Rev.), Vol. 34, No. 3, pp. 173–186, 1998

Printed in Great Britain
1366-5545/98 $—see front matter

PII: S1366-5545(98)00010-6

INTEGRATING INVENTORY IMPACTS INTO A FIXED-CHARGE MODEL FOR LOCATING DISTRIBUTION CENTERS

LINDA K. NOZICK* and MARK A. TURNQUIST
School of Civil and Environmental Engineering, Cornell University, Ithaca, NY 14853, U.S.A

(Received 20 June 1997; in revised form 8 April 1998; accepted 13 April 1998)

Abstract—An important question in the design of efficient logistics systems is the identficaton of locations for distribution centers. Optimal designs should be based on consideration of inventory, transportation, construction and operating costs. This paper describes a method for including inventory costs within a fixed-charge facility location model, allowing such a model to be used more effectively for the development of optimal system designs. A realistic application of the model is discussed.

Keywords: location, inventory, distribution centers.

1. INTRODUCTION

Total logistics costs (inventory plus transportation) in the U.S. declined from about 13 per cent of GDP in 1985, to about 10.5 per cent in 1993, largely as a result of increasing emphasis on controlling inventories throughout the supply chain (Thomas, 1997). However, since 1993, logistics costs (as a percentage of GDP) have been essentially constant. The 'easy' cost reductions have been made, and further improvements will require more effective tools for an integrated analysis of inventory and transportation costs.

One key question in designing a production-distribution system is locating distribution centers (DCs). There is a rich history of analytic work on facility location problems (see, for example, the reviews in Dresner, 1995), and one of the prime motivations for such analyses is locating distribution centers. However, it is generally assumed that demands are known with certainty, and thus inventory costs are either neglected or assumed to be unrelated to the DC location decisions.

A basic premise of this paper is that inventory costs (in particular, the safety stock required to provide a desired level of service when demand is uncertain) should be considered together with other facility operation and transportation costs in determining the optimal number of DCs and their locations. Our analysis demonstrates a way of approximating the costs of safety stock for a set of products as a linear function of the number of DCs. This allows the costs of safety stock to be included directly in a fixed-charge form of the facility location model. Existing methods for solving the fixed-charge location problem can then be used to find the optimal number and locations for DC's with a more complete measure of total cost. The contribution of this paper is the integration of safety stock inventory costs with a facility location model. We illustrate the effectiveness of the procedure using a realistic case study.

The distribution system of interest in this paper includes one or more production plants, a set of distribution centers, retail outlets and customers. In this analysis, the number and location of the production plants is assumed fixed, as is the number and location of retail outlets which serve the customers. A wide range of individual products, with uncertain demands, move through this distribution system. The retail outlets order replenishment stock from the DCs on a one-for-one basis as products are sold, and the DCs in turn order replenishments from the production plant(s). Safety stocks of each product are held at the DCs to buffer against the uncertain demands at the retail outlets. The decision on which we focus is, given the product demand characteristics at the

*Author for correspondence. Fax: 001 607 255 9004; e-mail: lkn3@cornell.edu

retail outlets, the retail locations, and replenishment time characteristics from the production plant(s), how many DCs should there be, and where should they be located?

Section 2 of this paper provides a summary of prior related research. Section 3 describes the method for modelling the safety stock requirements across a set of DCs. The fourth section discusses how the safety stock costs can be integrated into the fixed charge facility location model. Section 5 discusses a realistic case study and the last section offers some conclusions and ideas for future research.

2. PRIOR RELATED WORK

Distribution center location is a subproblem of production-distribution system design. A recent review by Vidal and Goetschalckx (1997) of strategic production/distribution models highlights the fact that uncertainties are not generally considered in the models. Most of the formulations focus on a mixed integer programming (MIP) representation, in which the demands at a set of specified locations are assumed fixed and known with certainty. The models generally include integer variables for locating plants and/or DCs in given zones and allocating customers to DCs, and continuous variables for determining flows of products through the system. Examples of such models are described by Geoffrion and Graves (1974), Robinson (1989), Gao and Robinson (1992), and Arntzen *et al.* (1995).

A notable attempt to include demand uncertainty and safety stocks is the work by Cole (1995). He presents a formulation that includes a Normal distribution of demand, and focuses on the safety stocks required for maintaining a specified level of customer service, along with decisions on DC location and customer allocation. Cole's model is represented as a capacitated fixed-charge multi-commodity network flow model with side constraints. The side constraints are the nonlinear inventory service level constraints resulting from the assumption of Normally distributed demands. He suggests two solution procedures, and examines three example problems. The largest problem has four products, nine customers, three potential plant locations, and six potential warehouse locations. The model has about 2300 constraints and 2900 variables (of which about 900 are integers). He illustrates that as the customer service level increases, the effort required to solve the model increases sharply. Unfortunately his solution procedure is impractical for most problems of realistic size.

Masters (1993) has illustrated a very effective technique for determining safety stocks for various products in a multi-echelon distribution system. He uses a model for inventory based on Palin's Theorem (see Feeney and Sherbrooke, 1966), but his analysis does not consider location decisions for the DCs. We have adopted a very similar approach to modelling inventories at the DCs, but linked this to a facility location formulation.

3. INVENTORY ANALYSIS

Figure 1 illustrates of the flow of products from the plants to the retail outlets through the DCs. Retail outlets demand products from the DC to which they are assigned as products are demanded by their customers. DC's backorder excess demand. As products are supplied to retail outlets, orders are placed for the products at the plants. That is, the inventory policy adopted is continuous review with a one-for-one replacement policy at the DCs.

For a given number of outlets (and demand at those outlets), we can find the inventory which must be held so as to provide a given level of service to the outlets. Consider a single product or group of products which for inventory planning purposes can be analyzed together. We will also assume that the demand for each product is independent. That is, a temporary shortage of one product does not increase the demand for related products.

Conceptually, the analysis of inventory costs in the DC system for a fixed number of DCs (and retail outlet assignments to the DCs) is similar to the analysis of an inventory of repairable items using an $(s-1, s)$ continuous review policy where s is the order-up-to level. A repairable items supply system is composed of a repair depot and some number of bases, denoted by n. Equipment failures occur at each of these n bases according to a Poisson distribution with rate λ_i, $1 \leqslant i \leqslant n$. For each base i, a fixed percentage of the time, τ_i, the failure can be fixed at the base and the time required to fix the failure is governed by a repair time distribution. If the failure cannot be fixed

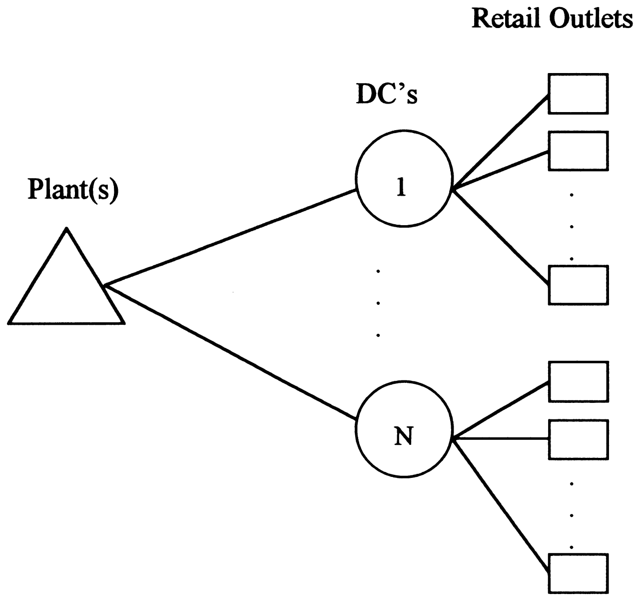

Fig. 1. Distribution system analyzed.

locally, it is sent to the depot for repair where the repair time is governed by a probability distribution. The number of repairs tasked to the depot at any given time is equivalent to the number of busy servers in an M/G/∞ queue where the arrival process is Poisson with mean rate:

$$\Lambda = \sum_{i=1}^{n} \lambda_i (l - \tau_i). \tag{1}$$

The service time distribution is governed by the in-transit time from the bases to the depot (which is assumed to be equal for each base) and the repair time at the depot. For a more complete discussion of the inventory analysis of repairable items, see Sherbrooke (1968), Simon (1971), and Graves (1985).

In the DC analysis, the items need not be repairable, but the idea of the service time distribution at the repair depot is used to represent the time for replenishment (from the plant to the DC) of an item which is out of stock at the DC. Assume that there are n retail outlets assigned to the DC, each with a Poisson demand process whose mean rate is λ_i where $1 \leqslant i \leqslant n$. Each time an item is demanded at a retail outlet, a replacement is ordered immediately from the DC. Therefore, the demand at the DC is Poisson with a mean rate:

$$\Lambda = \sum_{i=1}^{n} \lambda_i \tag{2}$$

Notice that this is equivalent to the mean of the Poisson arrival rate in the repairable item inventory when $\tau_i = 0$ for $1 \leqslant i \leqslant n$.

Assume that the DC has a total of s items either in inventory or on order, and it orders from the plant each time an item is sent to a retail outlet. We will define μ and σ^2 to be the mean and variance of the delivery time of a product from the plant to the DC once an order is placed. Then the performance measures for the single DC and assigned retail outlets are the same as those for an M/G/Is queue with an arrival rate of Λ, a mean and variance for the service time distribution of μ and σ^2, respectively, and s servers. This basic concept is also discussed by Diks *et al.* (1996), and van Houtum *et al.* (1996).

The inventory at the DC will include both cycle stock and safety stock. In terms of our queuing system representation, the cycle stock at the DC is given by $s\rho$, where ρ represents the average utilization rate of the s servers ($\rho = \lambda\mu/s$). The safety stock is $s(l-p)$, the amount of stock (number of servers, in the queuing analogy) in excess of the expected demand. The service level requirements will dictate how large this extra stock needs to be.

An important and commonly used level-of-service measure in inventory system m which customer demands are backordered when insufficient on-hand inventory is available is called the stockout rate (Masters, 1993). That is, the stockout rate is the percentage of demand that can not be satisfied from on-hand inventory. In the queuing system representation, this is analogous to the probability that when a customer enters the system, a server is not available (i.e. wait time, denoted by W, is greater than zero). The probability of waiting (stockout), $P(W>0)$, in an M/G/s queue can be approximated quite accurately by $P(W>0)$ in an M/M/s queue (Whitt, 1993), so we can use the following approximation for $P(W>0)$ from Iglehart (1965):

$$r = P(W > 0) \approx 1 - \Phi((s - s\rho - 0.5)/\sqrt{s\rho}) \tag{3}$$

where Φ is the standard normal CDF.

This approximation is designed for use in light traffic (in queuing terminology), where $P(W>0)$ is small. In most inventory systems, we want a small probability of stockout, r, so this will be an appropriate approximation. However, in the event that a large value of r is tolerable for the customer service level, an alternate approximation due to Halifin, and Whitt (1981) can be used:

$$r = P(W > 0) \approx [1 + \sqrt{2\pi s}(1-\rho)\Phi((1-\rho)s^{.5})exp(((1-\rho s^{.5})^2/2)]^{-1} \tag{4}$$

Equations 3 or (4) can be used to find the minimum inventory necessary for a given stockout rate. For instance, if r is the desired stockout rate, then we can find S_r, the smallest value of s such that either eqns (3) or (4), as appropriate, is less than or equal to r.

We can then examine several questions which are critical to understanding the impacts of a given inventory management strategy, and to our ability to integrate the inventory analysis into a broader analysis of distribution center location. For a given stockout rate, how does inventory vary with demand at a single distribution center? For a given demand at a distribution center, how does inventory stock vary with the required stockout rate? For a given fixed demand across the retail outlets, how does the number of distribution centers affect the total safety stock that must be held throughout the system?

Figures 2–4 contain illustrative numerical results that show the general pattern of inventory requirements for different sets of parameter values. Figure 2 shows the cycle and safety stock required for different values of yearly demand, with an allowable stockout rate of 5 per cent, assuming that the mean of the distribution of time for the delivery of an item from the plant to the DC is 21 days. Notice that cycle stock is a linear function of demand and that safety stock increases at a decreasing rate with expected demand, consistent with the "square root" rule-of-thumb first described by Eppen (1978), and expanded by Stulman (1987) and Chang and Lin (1991). Since inventory savings from consolidation result from reductions in the necessary safety stock, the remainder of the discussion will focus on safety stock only.

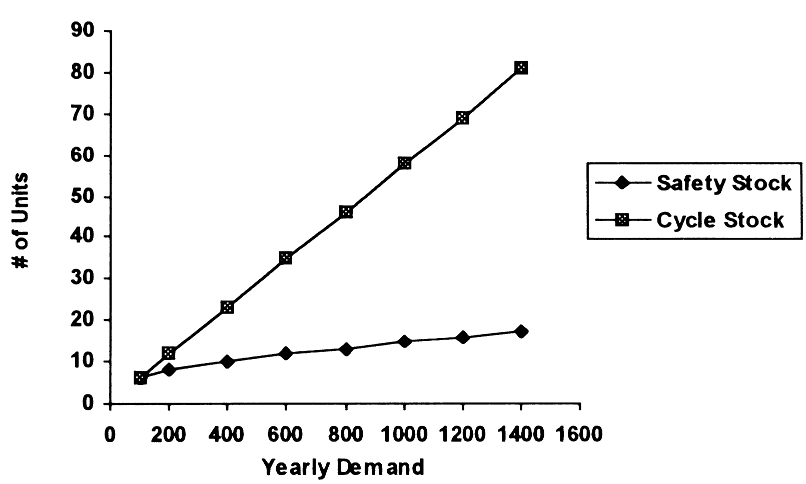

Fig. 2. Yearly demand vs cycle and safety stock when $P(W>0) \cong 0.05$.

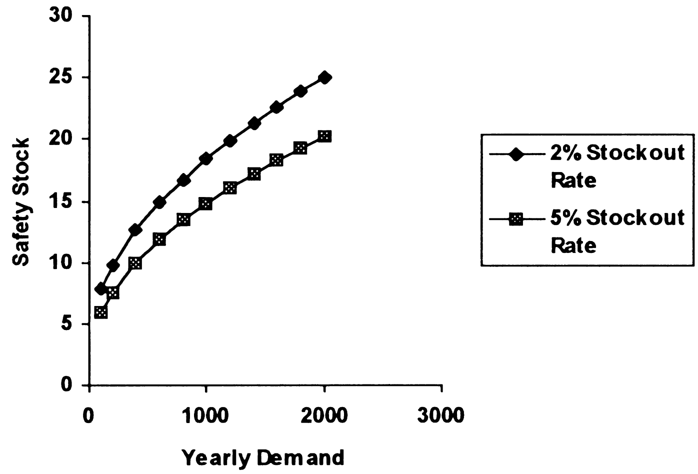

Fig. 3. Yearly demand vs safety stock for different levels of service.

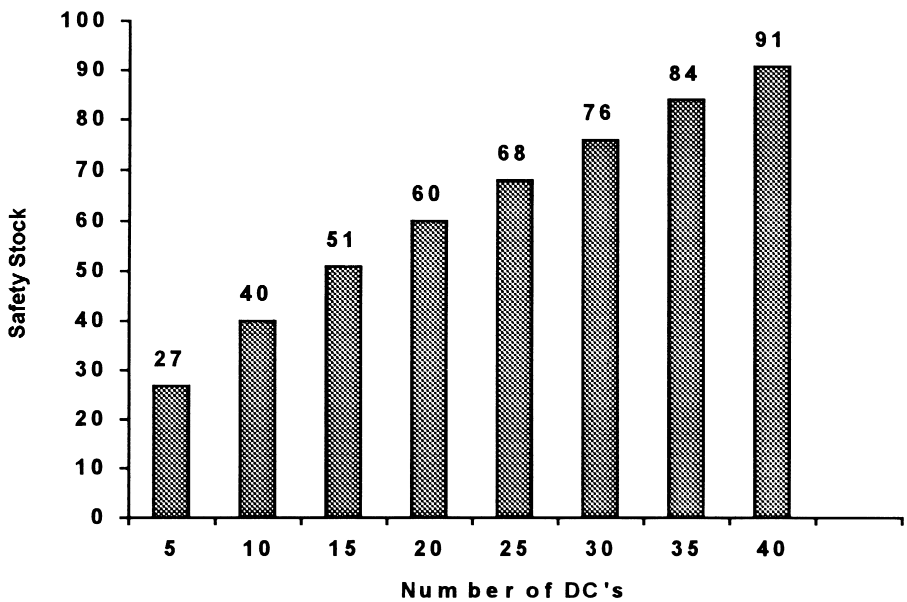

Fig. 4. Total safety stock requirements for various numbers of DCs (stockout rate = 5 per cent and total expected demand = 800 units).

Figure 3 presents the safety stock for different values of yearly demand for both a 5 per cent and 2 per cent stockout rate. Notice that a 2 per cent stockout rate requires from two to five extra units of safety stock beyond that required for a 5 per cent stockout rate as yearly demand increases from 100 to 2000 units. The safety stock as a percentage of demand declines rapidly as demand increases.

If there is a constant total expected demand across all retail outlets, the total safety stock required is related to the number of DCs used. Consider a product with an expected demand of 800 units per year. Further, assume that this expected demand is divided equally among N DCs. Based on these assumptions, Fig. 4 shows the safety stock necessary for various numbers of DCs, given a stockout rate of 5 per cent using eqn (3). Although eqns (3) and (4) imply that the required safety stock is nonlinear in the number of DCs, Fig. 4 illustrates that the degree of nonlinearity is quite modest over a substantial range in the number of DCs. If we have a product that is demanded in several hundred or more retail locations, and we consider consolidating inventory of the product in 10–40 DCs, the total safety stock requirements for different numbers of DCs can be approximated well by a linear function of the form:

$$\text{Safety stock} = a + bN. \tag{5}$$

The unit cost of holding safety stock of a particular product k can be specified as βV_k, where β is the cost of money (per cent per time period) and V_k is the value of a unit of product k. Then the quantity βV_k is the incremental cost associated with each DC. This cost can be incorporated as part of the fixed charge for opening a DC in a fixed-charge location model. This makes it possible to integrate the inventory analysis directly into a facility location model.

Figure 4 indicates an apparent suitability of a linear approximation for safety stock requirements as a function of the number of DCs, but the values shown pertain to specific assumed values for total expected demand and stockout rate. It is also assumed that the total expected demand is allocated equally across the DCs. It is necessary to analyze the effects of these assumptions more carefully before a general recommendation can be made for constructing linear approximations of safety stock to be included in a location analysis for a wide range of potential products. The next two subsections present an error analysis for the linear approximation, and an analysis of the effects of assuming equal allocation of demand across DCs.

3.1. Error analysis of the linear approximation

Previous work (Eppen, 1978; Stulman, 1987; Chang and Lin, 1991) has indicated that we should expect a theoretical square-root relationship between total safety stock and the number of DCs. Suppose that the actual expression for the safety stock requirements were of the form:

$$g(N) = c\sqrt{N} \tag{6}$$

where g(N) is the safety stock, N is the number of DCs, and c is a positive constant. We can construct a linear approximation to $f(N)$ using a Taylor series expansion about some value $N = N_0$. If the actual number of DCs sited is $N \neq N_0$, then the percent mean absolute error (%MAE) created by the approximation is:

$$\%MAE = 100\left|1 - \frac{N + N_0}{2\sqrt{N_0\sqrt{N}}}\right| \tag{7}$$

Figure 5 illustrates the %MAE for different choices of No and N. For example, if the Taylor series expansion is created about $N_0 = 15$ and the actual number of DCs located (N) is in the range of 10 to 20, the error created is negligible. For small values of N, the error in the approximation can be significant (even if No is also small). Thus, this linear approximation is most useful in situations where a relatively large number (approximately 10 or more) of DCs are to be located.

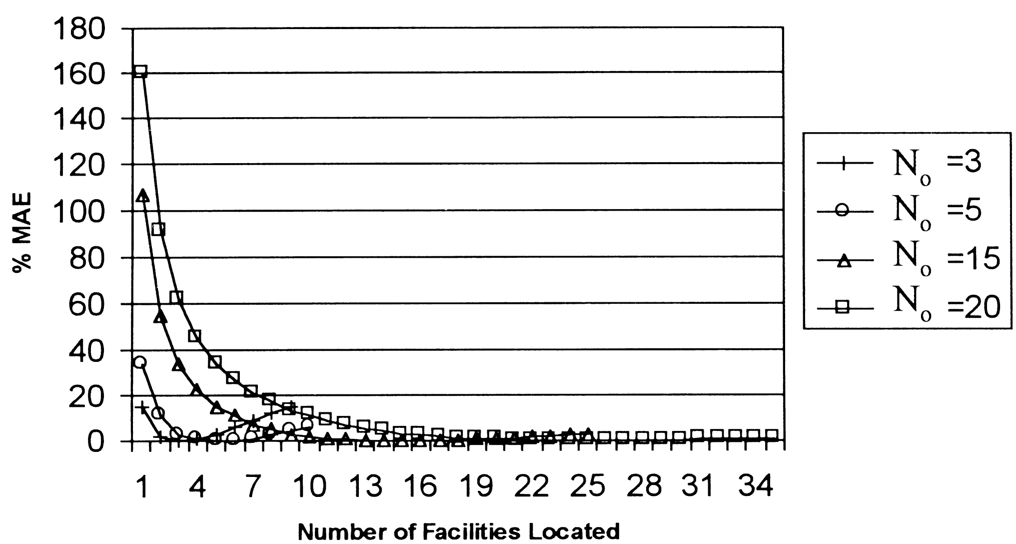

Fig. 5. Percentage mean absolute error from linear approximation to safety stock calculation.

3.2. Effects of the equal allocation assumption

The safety stock values shown in Fig. 4 are based on an equal allocation of expected demand to each of the DCs. It is also important to examine the effect of a departure from an equal allocation on the total safety stock requirements. In fact, the assumption that an equal amount of demand is allocated to each DC provides an upper bound on the actual expected safety stock required for any other assignment of demand. This can be demonstrated by an argument based on Taylor's theorem. Suppose N DCs are created to serve a total expected demand of D units. Define x, to be the demand allocated to be served by DC i, $1 \leqslant i \leqslant N$, and $f_r(x_i)$ as the safety stock at DC i when the stockout rate is r. Also define $\mu = D/N$ as the average demand at the N DC's.

The expressions in eqns (3) and (4) are twice differentiable, so Taylor's theorem allows us to express $f_r(x_i)$ as follows:

$$f_r(x_i) = f_r(\mu) + f_r'(\mu)(x_i - \mu) + \frac{1}{2}(x_i - \mu)^2 f_r''(y_i) \tag{8}$$

for some $y_i = \lambda x_i + (1 - \lambda)\mu$,where $0 \leqslant \lambda \leqslant 1$.

The sum of all the safety stock in the N DCs is then:

$$\sum_{i=1}^{N} f_r(x_i) = Nf_r(\mu)\sum_{i=1}^{N}(x_i - \mu) + \frac{1}{2}\sum_{i=1}^{N}(x_i - \mu)^2 f_r''(y_i) = Nf_r(\mu) + \frac{1}{2}\sum_{i=1}^{N}(x_i - \mu)^2 f_r''(y_i) \tag{9}$$

The second derivative term, $f_r''(y_i)$ is always negative because we have a "square roof" relationship of yearly demand to safety stock. Thus, the total safety stock required cannot be more than $Nf_r(\mu)$, the total safety stock required for the assignment based on equal allocation of demand to each DC. Therefore, an estimate of safety stock based on an equal allocation is conservative.

3.3. A linearization procedure

The linear approximation to the total safety stock requirements (as a fraction of the number of DCs used) allows incorporation of the safety stock requirements into the fixed-charge coefficient for candidate sites in a location model. To illustrate this, consider a product with a yearly expected demand of 800 units and a maximum stockout rate of 5 per cent (the same characteristics used to generate the safety stock calculations in Fig. 4). Figure 6 illustrates both the required safety stock for different numbers of distribution centers (previously shown in Fig. 4, and labeled in Fig. 6 as the "estimated values"), and the predicted safety stock based on a linear regression model:

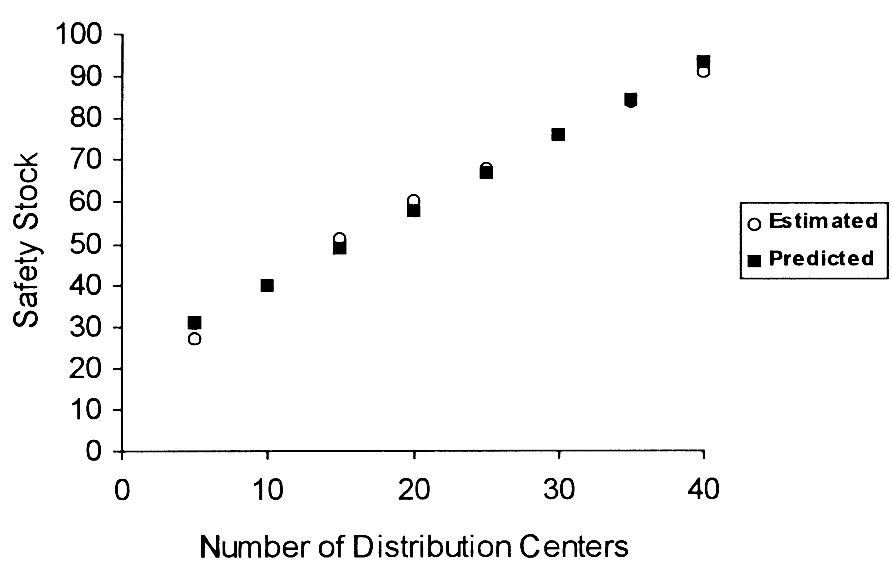

Fig. 6. Safety stock estimated from eqn (3) compared to predictions from eqn (9), for various numbers of DCs.

$$\text{Safety stock} = 22 + 1.8N \tag{10}$$

where N = number of DCs.

For the fixed-charge location model formulation, this implies that the inventory implications for the product considered (denoted as product k) of opening a distribution center at candidate site j, can be reflected in the fixed-charge coefficient by including the value $1.8\,\beta V_k$ in that coefficient along with other facility-related fixed charges. This value (for a given product) is the same for all candidate DC sites. The inventory implications of all products can be included in the fixed-charge formulation by developing constants like this for each product. The value of the multiplier (1.8, for example) will depend on the annual sales of each product and the stockout rate requirement.

For each product, a series of computations using eqns (3) or (4) for different numbers of DCs will produce estimated values like the set shown in Fig. 6. Then a simple regression analysis allows estimation of the slope coefficient for use in evaluating the costs of the stock at each DC. These values are aggregated over all products to create the total value to be included in the fixed-charge coefficient in the location model.

4. THE FIXED-CHARGE FACILITY LOCATION MODEL

The fixed-charge facility location model is an effective tool for analysis of how many distribution centers should be built and where they Should be located. The fixed charge facility location model is as follows (see, for example, Daskin, 1995):

$$\text{Min} \sum_j f_j X_j + \alpha \sum_i \sum_j h_i d_{ij} Y_{ij}$$

such that

$$\begin{aligned} &\sum_j Y_{ij} = 1 && \forall i \\ &Y_{ij} \leqslant X_j && \forall i, j \\ &X_j = 0, 1 && \forall j \\ &Y_{ij} \geqslant 0 && \forall i, j \end{aligned} \tag{11}$$

where:

f_j = fixed cost of locating a facility at candidate site j
h_i = demand at location i
d_{ij} = distance from demand location i to candidate site j
α = cost per unit distance per unit demand
X_j = 1 if a facility is to be located at candidate site j, and 0 otherwise
Y_{ij} = fraction of demand at location i which is served by a facility at j.

This model identifies the subset of potential facility locations that minimize the cost of serving a set of demand locations. Costs include a fixed cost which can be used to represent facility and operating expenses and a transportation cost which represents the expense of moving products from the located facilities to the demand locations. In the fixed charge location analysis literature, inventory costs have not been explicitly incorporated into the fixed charge coefficient (Daskin, 1995; Dresner, 1995), but the analysis in the previous section illustrates how they can be included.

Many solution procedures have been developed for this model, among them a variety of greedy heuristics including the add, drop, and exchange heuristics, Lagrangian relaxation, and branch and bound (Mirchandani and Francis, 1990; Daskin, 1995; Dresner, 1995). For our analysis we have used a hybrid heuristic (a combination of a greedy add and an improvement algorithm) as discussed by Daskin (1995).

As the DC locations are changed in this problem, the allocation of demand to DCs (the set of Y_{ij} values) changes also, but the analysis from the previous section indicates that we can develop a conservative estimate of total safety stock by assuming that each DC serves an equal proportion of the total demand. This allows us to uncouple the calculation of total safety stock for a given number of distribution centers from the assignment of retail outlets to those distribution centers in the location model.

5. CASE STUDY

As an example of this modelling approach consider a distribution system for finished automobiles. The current distribution system for finished automobiles relies on large networks of independent dealers to provide the inventory of vehicles from which customers choose. Dealers order new vehicles from the manufacturer in the models and colors (and with the accessories and trim packages) that they believe they can sell. These orders typically require 6–12 weeks to fill and make up the dealer stock. Occasionally dealers will order a specific vehicle for a customer, but most customers are unwilling to wait the 6–12 weeks for a vehicle, so most vehicles are sold out of the finished vehicle inventory held on dealer lots. If a customer wants a specific vehicle that the dealer does not have in stock, the dealer often attempts to locate such a vehicle at another dealership, and arrange an inter-dealer trade or purchase. The system of relying on dealers to maintain separate pools of inventory is costly because dealers are forced to maintain a large inventory of cars (generally 60–90 days of sales) in order to have access to an adequate supply of vehicles in a sufficiently wide variety of colors and options to satisfy customer requests. As an illustration of the magnitude of inventory costs in the auto industry. consider that in a typical year about 15 million new cars and light trucks are sold in the U.S., and the average value of those vehicles, is about \$18, 500 (Anonymous, 1994; Taylor, 1994). If the average dealer holds 60 days' inventory, total on-hand inventory is approximately 2.5 million vehicles, representing an investment of over \$46 billion. If inventory carrying costs are about 22 per cent of inventory value (Foster, 1994), then dealer stock generates annual inventory holding costs on the order of \$10 billion.

One alternative method of managing this inventory is through a series of regional distribution centers (RD(s) which would hold most of the inventory, with only small numbers of vehicles (primarily for display purposes) held on the dealer lots. All dealers served by a given RDC would have access to its entire inventory. In order for the RDC system to be successful, delivery of the selected vehicle from the RDC to the dealership must be prompt and reliable. If fast, reliable, deliveries from RDCs to dealers can be achieved, this alternative would provide an increased level of responsiveness to customer demands at a lower cost than is offered today through substantial dealer inventories and inter-dealer vehicle swaps. In a rational or international-scale implementation of the distribution center concept, the questions of how many RDC's are necessary, and where should they be located, are critical to the success of the venture. This example will focus on identifying how many RDCs are necessary and where they should be located in order to minimize transportation, fixed facility, and inventory costs for new vehicles.

Suppose an auto manufacturer sells 700 products or new vehicle configurations (a configuration is equivalent to a product in the previous discussion). Of those 700 configurations, 200 have an expected yearly demand of 8000 units in the continental U.S., 225 have expected yearly demand of 6000 units, and 275 have expected yearly demand of 4000 units. The total annual expected demand for all products is then 4,050,000 new vehicles. Figure 7 shows the required safety stock for different numbers of RDCs for each of the yearly volumes at a 5 per cent stockout rate. Notice that safety stock is relatively linear in the range of 15 to 50 RDCs. For this case study, that range is likely to contain the optimal number of RDCs.

Three regression equations can be estimated to predict the safety stock requirements for each of the three demand volumes in Fig. 7. We have used the data for the range from 15 to 50 RDC s in these regressions, because the optimal number of RDCs is very likely to be in that range and limiting the range of values improves the linear approximation from the regression model. Of course, the range of 15–50 is an initial estimate, and if the optimal solution suggested by the optimization were to he outside this range it would be necessary to recalibrate our safety stock equations based on a different range. We then multiplied the slope coefficients from each regression relationship by the number of configurations that have that demand volume, and summed the results, yielding the following regression relationship between safety stock and number of RDCs:

$$\text{Safety stock} = 58,836 + 2,140 \times N \tag{12}$$

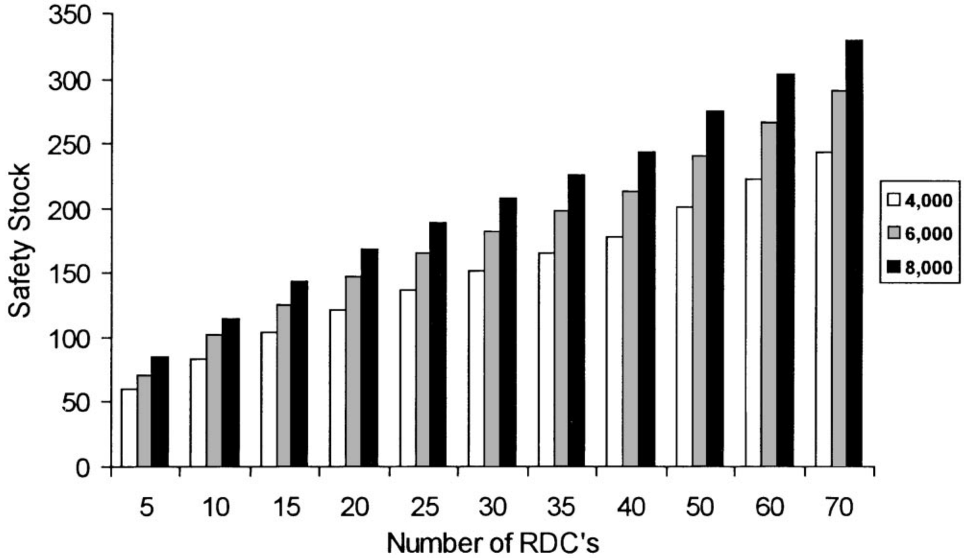

Fig. 7. Safety stock for different numbers of RDCs and annual demands of 4000, 6000 and 8000 vehicles.

If the average value of a vehicle is assumed to be \$15,000, and the yearly holding cost is 22 per cent, the inventory cost equation is as follows.

$$\$194,158,800 + \$7,063,523 \times N \tag{13}$$

This implies that the safety stock inventory cost for each additional RDC is slightly over \$7 million annually.

For the purposes of this case study, we have assumed that each of these RDCs will cost approximately \$10 million to construct, will have an expected life of 30 years, and that the cost of capital to the manufacturer is 15 per cent per year. Then the amortized annual facility cost is about \$1.5 million. Combining these facility costs with inventory costs given in (13), the coefficient f_j to be included in the fixed-charge formulation is \$8.5 million. In reality, some of the facility development costs will be site-specific, so the $f_j$values would likely vary across sites. However, we have used the simple assumption here that the f_j values are the same for all potential sites. Notice that most of the fixed costs associated with the establishment of an RDC are related to the inventory costs. In this case it is clear that the real trade-off in the RDC location analysis will be between and inventory costs, with the construction costs playing a more secondary role.

In this case study the manufacturer serves demands across the continental U.S., organized into 698 demand areas. For the purposes of this analysis, the demand within each of the demand areas is assumed to occur at the centroid of the demand area. The centroids scaled by demand are illustrated in Fig. 8. These 698 demand centroids will also serve as potential RDC locations.

We have assumed a transportation cost of 60 cents per unit-mile for delivering vehicles from the RDCs to demand area centroids, and we have used a widely-used representation of the U.S. highway network (Bureau of Transportation Statistics, 1997) to calculate the centroid-to-centroid distances.

The fixed-charge model yields a recommendation of 23 RDCs, with the locations shown in Fig. 9. Figure 9 also illustrates the assignment of demand areas to RDCs, and gives an indication (using line thickness) of the volume of demand allocated to each RDC.

Figure 10 illustrates the sensitivity of this solution to changes in the per-RDC fixed-charge coefficient. It is important to notice that within a range of plus or minus 10 per cent, the number of RDC's recommended does not change. In fact the coefficient can rise at least 30 per cent with little change in the solution. However, a 30 per cent reduction produces a significant change in the solution. From a managerial perspective, the stability of the locations suggested against small changes in the fixed-charge coefficients demonstrated in this application is very desirable. However, that stability is a function of the input data and hence may not occur in other applications.

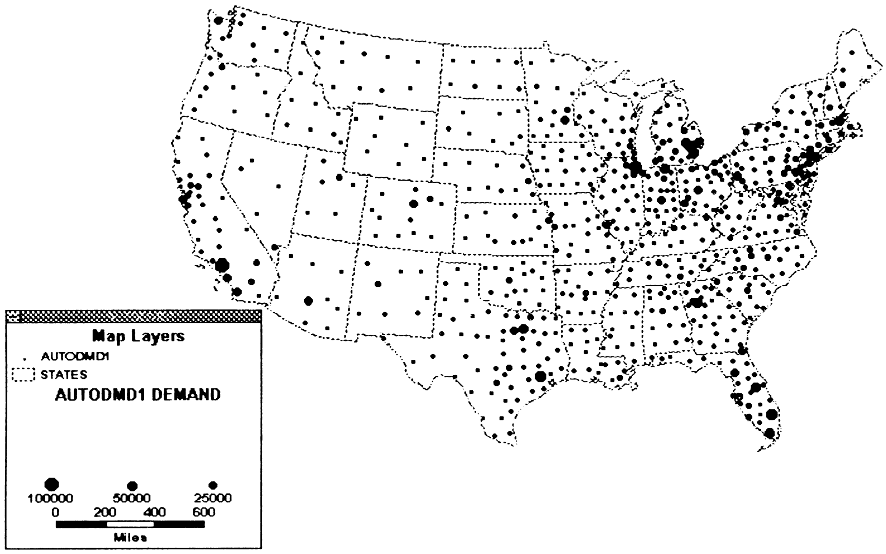

Fig. 8. Demand centroids scaled by volume.

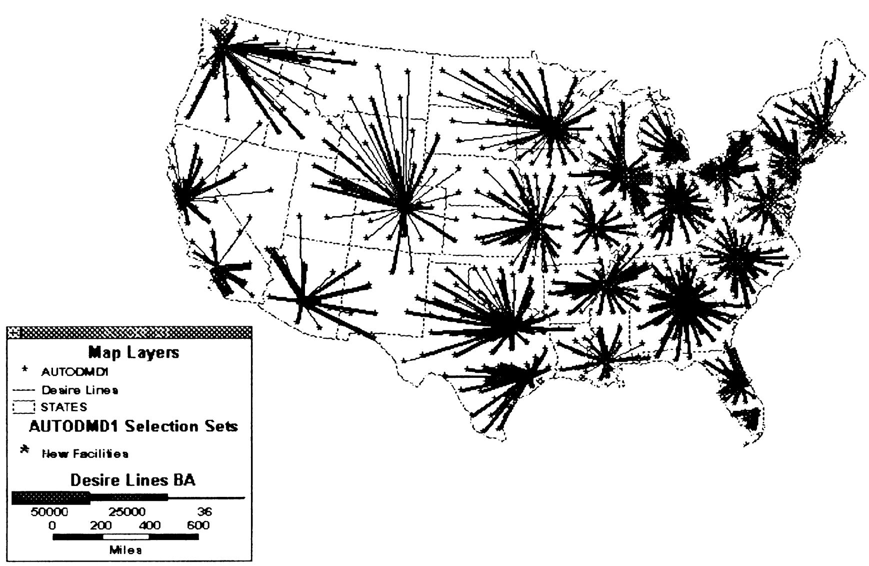

Fig. 9. Solution for a per-RDC coefficient of $8.5 million and a per-mile cost of 60 cents.

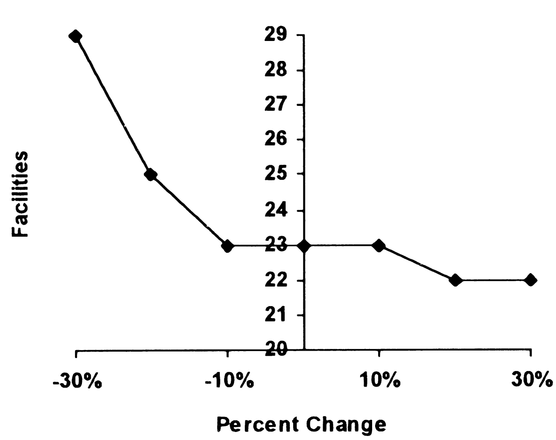

Fig. 10. Sensitivity of optimal number of RDCs to percentage changes in the fixed-charge coefficient.

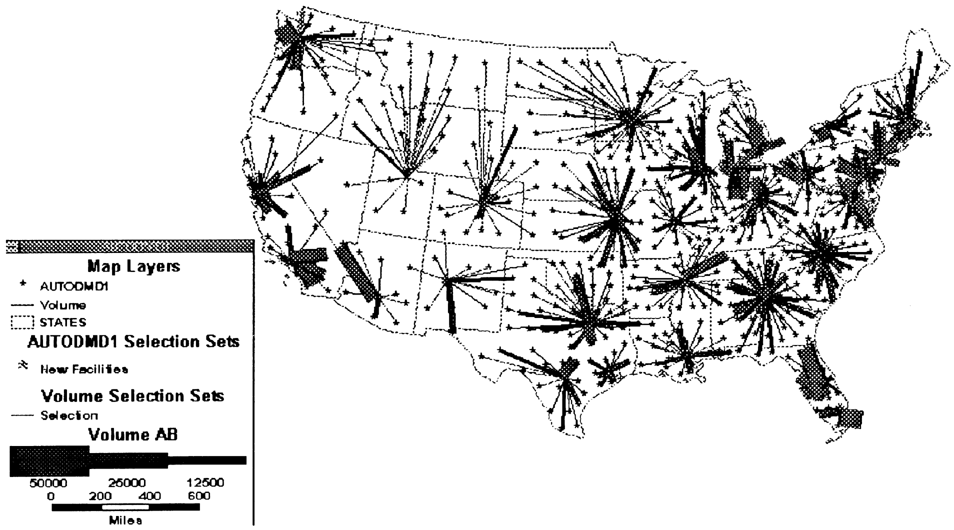

Fig. 11. Suggested locations based on a 30 per cent reduction in the fixed-charge coefficient.

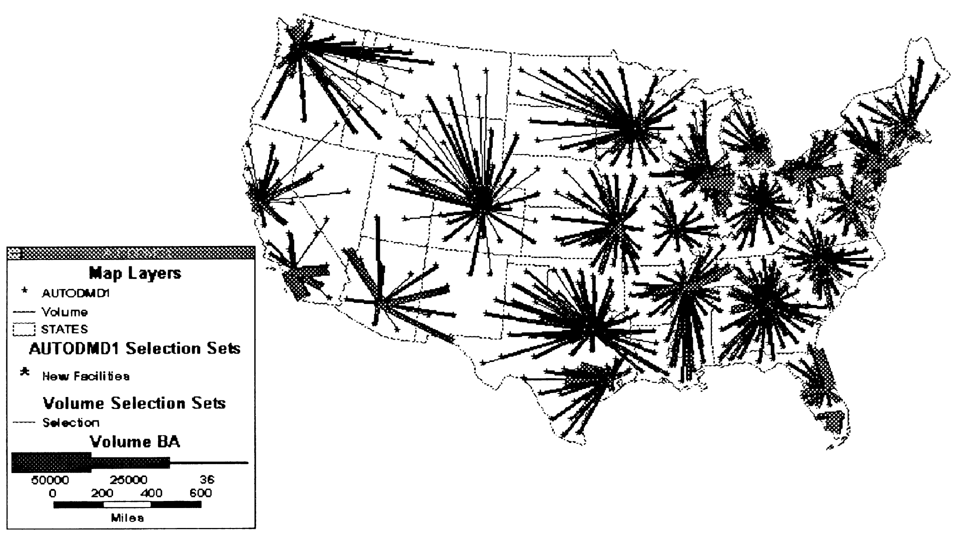

Fig. 12. Suggested locations based on a 30 per cent increase in the fixed-charge coefficient.

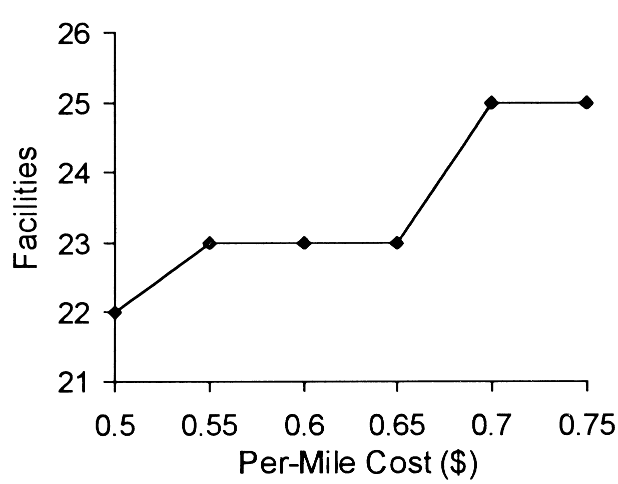

Fig. 13. Number of facilities vs per-mile cost (assuming a fixed-charge coefficient per RDC of $8.5 million).

It is useful to examine the solutions generated by the fixed-charge formulation at each of the extremes for the per-RDC coefficient. Figure 11 presents the suggested locations based on a decrease of 30 per cent and Fig. 12 presents the locations based on an increase of 30 per cent Both of these solutions are quite consistent with the base solution. That is, as the per-RDC coefficient decreases, the locations selected as RDC sites are still included in the solution but additional locations are added.

From an implementation perspective, this is a very attractive characteristic of the solution because the actual development of the RDCs would likely be staged over time. The results shown here suggest that solutions with fewer locations can be implemented and then augmented, without sacrificing effectiveness of the final solution.

Figure 13 illustrates the sensitivity of the number of RDCs to the per-mile transportation costs. For shifts of up to 10 per cent no change in the solution occurs, and for shifts of up to 30 per cent the solutions are quite similar.

6. CONCLUSIONS

The fixed-charge facility location model locates distribution centers so that all demand at each retail outlet is allocated to a DC while minimizing the sum of transportation and fixed costs. This paper has illustrated a method through which inventory costs can be integrated into the estimated fixed-charge coefficient in this location model, allowing a more complete exploration of the trade-offs between transportation, construction, operating and inventory costs when determining how many centres to create and where they should be located.

This paper has established a necessary link between fixed-charge facility location analysis and inventory planning. That link now needs to be strengthened in two important ways. First, this analysis was based on a one-to-one inventory replacement policy. It would be useful to extend the analysis beyond this single policy. Second, the analysis was based on a single level of inventory held at the distribution centre. It would be useful to extend the analysis to distribution systems for which inventory is held in other levels as well (i.e. to optimize the location of inventory in a multi-echelon system). Given the flexibility and the generally good performance of the greedy heuristics for the fixed-charge facility location problem, it is likely that excellent solutions in these areas are possible.

Acknowledgements—This research was sponsored in part by a grant from the National Science Foundation. This support is gratefully acknowledged but implies no endorsement of the findings. The authors also want to thank the Editor and two anonymous referees for suggestions that improved the presentation of the paper substantially.

REFERENCES

Anon (1994) U.S. car dealerships. *Automotive News,* May.

Arntzen, B. C., Brown, G., Ga., Harrison, P. and Trafton, L. L. (1995) Global Supply Chain Management at Digital Equipment Corporation. *Interfaces* **25**(1), 69–93.

Bureau of Transportation Statistics (1997) *National High way Planning Network Model, Version 2.1*. U.S. Department of Transportation, Washington, DC.

Chang, P. and Lin, C. (1991) On the effects of centralization on expected costs in a multi-location newsboy problem. *Journal of the Operational Research Society* **42**(11), 1025–1030.

Cole, M. I. L. (1995) Service considerations and the design of strategic distribution systems. Ph.D. thesis, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA.

Daskin, M. (1995) *Network and Discrete Location: Models, Algorithms, and Applications*. John Wiley and Sons, New York.

Diks, E. B. B., de Kok, A.-G. and Lagodimos, A.-G. (1996) Multi-echelon systems: a service measure perspective. *European Journal of Operations Research* **95**, 241–263.

Dresner, Z. (ed.) (1995) *Facility Location: A Survey of Applications and Methods*. Springer Series in Operations Research. Springer–verlag, New York.

Eppen, G. (1978) Effects of centralization on expected costs in a multi-location newsboy problem. *Management Science* **25**(5), 498–501.

Feeney, G. L. and Sherbrooke, C. C. (1966) The (s−1, s) Inventory Policy Under Compound Poisson Demand. *Management Science* **12**(5), 391–411.

Foster, T. A (1994) It's about time and inventory, distribution, July.

Gao, L. L. and Robinson, Jr., E. P. (1992) A Dual Based Optimization Procedure for the Uncapacitated Facility Location Problem. *Naval Research Logistics* **39**, 191–212.

Geoffrion, A. M. and Graves, G. W. (1974) Multicommodity Distribution System Design by Bender's Decomposition. *Mwogement Science* **20**(5), 822–844.

Graves, G. W. (1985) A Multi-Echelon Inventory Model for a Repairable Item with One-For-One Replacement. *Management Science* **12**, 1247–1256.

Halifin, S. and Whitt, W. (1981) Heavy Traffic Lirnits for Queues with Many Exponential Servers. *Operations Research* **29**, 567–598.

Iglehart, D. L. (1965) Limit Diffusion Approximations for the Many Server Queue and the Repairman Problern. *Journal of Applied Probability* **2**, 429–441.

Masters, J. M. (1993) Determination of Near Optimal Stock Levels for Multi-Echelon Distribution Inventories. *Journal of Business Logistics* **14**(2), 165–195.

Mirchandani P. B. and Francis, R. X. (1990) *Discrete Location Theory*. John Wiley & Sons, New York.

Robinson, E. P., Jr. (1989) Multi-Activity Uncapacitated Facility Location Problem: A New Tool for Logistics Planning. *Journal of Bussiness Logistics* **10**(2), 159–179.

Sherbrooke, C. C. (1968) METRIC: A Multi-Echelon Technique for Recoverable Inventory Control. *Operations Research* **16**, 122–141.

Simon, R. M. (1971) Stationary Properties of a Two-Echelon Inventory Model for Low Demand Items. *Operations Research* **19**, 791–773.

Stulman, A (1987) Benefits of Centralized Stocking for the Multi-Centre Newsboy Problem with First Come, First Serve Allocation. *Journal Of the Operational Research Society* **38**(9), 827–832.

Taylor, R. A. (1994), "The Golden Age Of Autos," Fortune, April 4, pp. 51–65.

Thomas, J. (1997) Where the money is. *Distribution*, July, 14–20.

Van Houtum, G. LInderfurth, K. and Zijm, W. H. M. (1996) Materials Coordination in Stochastic Multi-Echelon Systems. *European Journal of Operations Research* **95**, 1–23.

Vidal, C. J. and Coetscalckx, Y. (1997) Strategic Production–Distribution Models: A Critical Review with Emphasis on Global Supply Chain Models. *European Journal of Operation Research* **98**, 1–18.

Whitt, W. (1994) Approximations for the GI/G/m Queue. *Production and Inventory Management* **2**(4), 114–161.