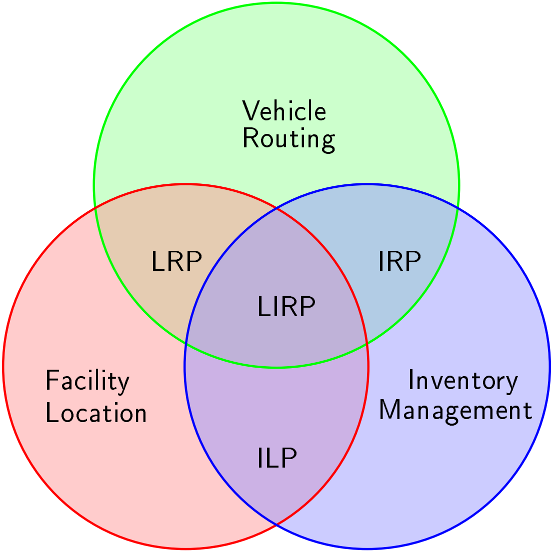
Location + Inventory + Routing = LIRP



State of the art

Reference	Year	Multi. prod.	Multi. period	Heterog. Fleet	RL	Routing	Layers	Flow
Ambrosino Scutella	2005		(✓)	✓		arc	4	direct+loops
Ago et al.	2007	✓	✓			no	3	direct+(loop)
Chanchan et al.	2008		✓		✓	arc	3	direct+loop
Zhang Bo et al.	2008		✓			arc	3	direct+loop
Ahmadi Javid, Azad	2010					arc	3	direct+loop
Hiassat Diabat	2011		✓			route	2	loop
Sajjadi, Cheraghi	2011	✓				arc	3	direct+loop
Ahmadi Javid, Seddighi	2012					arc	3	direct+loop
Li et al.	2013				✓	route	3	direct+loop
Ahmad et al.	2014		✓			arc	3	direct+loop
Nekooghadirli et al.	2014	✓	✓	✓		arc	3	direct+loop
Zhang et al.	2014		✓			arc	2	loop
Guerrero et al.	2015		✓			route	3	direct+loop
Liu et al.	2015				✓	route	3	direct+loop
Deng et al.	2016				✓	route	3	direct+loop
Yuchi et al.	2016					arc	3	direct+loop
Zhalechian et al.	2016	✓	✓	✓	✓	arc	3	direct+loop
Lerhlaly et al.	2016		✓	✓		route	3	direct+loop

(✓): model with multiple products, experiments with 1 product.

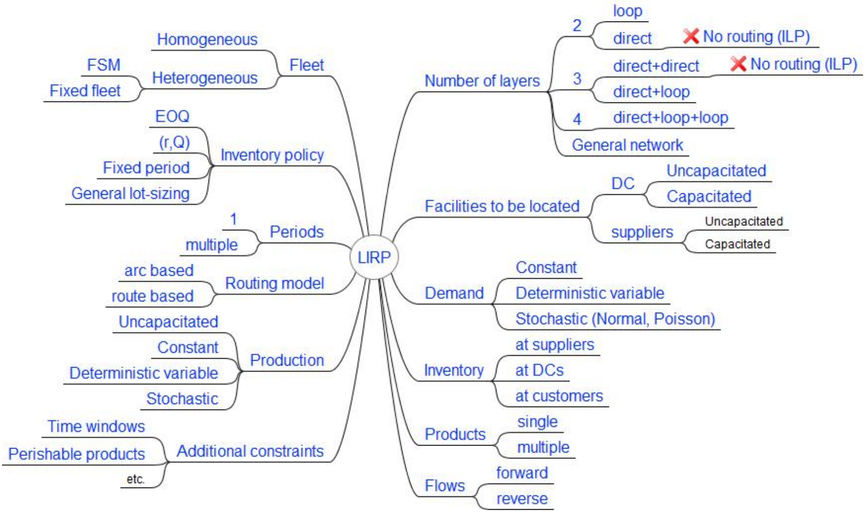
Inventory management in LIRP

Demand uncertainty	*Inventory policy* (r,Q)	fixed interval	lot-sizing
None	Ahmadi Javid, Seddighi (2012), Li et al. (2013), Deng et al. (2016)		Ambrosino Scutella (2005), Hiassat Diabat (2011), Ahmad et al. (2014), Zhang et al. (2014), Guerrero et al. (2015), Yuchi et al. (2016), Lerhlaly et al. (2016)
Poisson		Sajjadi, Cheraghi (2011)	Chanchan et al. (2008), Zhang Bo et al. (2008)
Normal	Ahmadi Javid, Azad (2010), Nekooghadirli et al. (2014), Liu et al. (2015), Zhalechian et al.(2016)		

Other interesting references

- 3 early papers with L+R + calculation of inventory costs: Liu and Lee (2003), Liu and Lin (2005), Ma and Davidrajuh (2005).
- 1 paper with I+L + a posteriori optimization of routing costs: Mete and Zabinsky (2010).
- 1 paper with I+L+ approximation of routing costs: Shen and Qi (2007).

A typology of LIRP models



Solution methods

References	Method	Facilities/T/P	Comments
Ambrosino & Scutella (2005)	General purpose code	20+130/1/	
Ago et al. (2007)	Lagrangean decomp.	3+21+3/3/	
Chanchan et al. (2008)	2 phase heuristic	20+5	
Zhang Bo et al. (2008)	GA, routing with savings	30+10	
Ahmadi Javid, Azad (2010)	Tabu/SA, nearest neighbor	50+400	nonlinear
Hiassat Diabat (2011)	GAMS/Cplex	4+2	
Sajjadi, Cheraghi (2011)	SA, savings	3+30+350//40	nonlinear
Ahmadi Javid, Seddighi (2012)	SA + ant colony	25+50+350	nonlinear
Li et al. (2013)	hybrid GA + SA	29+5	
Ahmad et al. (2014)	cost saving method	5+20	transshipment
Nekooghadirli et al. (2014)	MOICA, MOPSA, NSGA II, PAES	15+100/7/5	2 obj
Zhang et al. (2014)	NSGA-II	15+318	3 obj.
Guerrero et al. (2015)	CG,LR,local search	5+7/7/	
Liu et al. (2015)	Parallel GA	29+5	
Deng et al. (2016)	Ant Colony	10+100	
Yuchi et al. (2016)	tabu search	10+200	nonlinear
Zhalechian et al. (2016)	Hybrid GA, GAMS/Baron	2+5+6/2/2	3 obj., nl
Lerhlaly et al. (2016)	Cplex	5+20/5/	2 obj.

Main findings

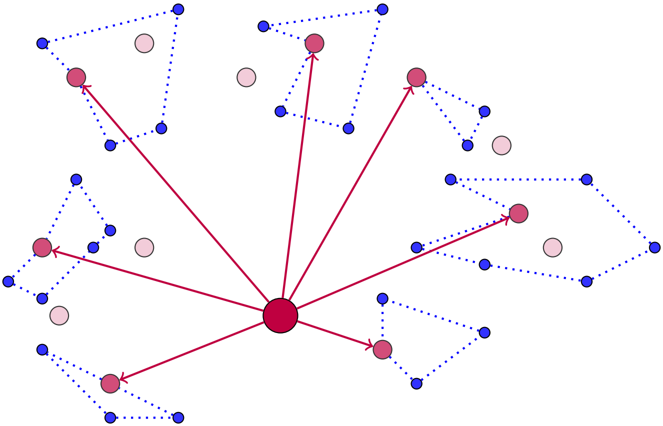
A "rich" problem by nature

- Large typology of problem variants
- Prevalence of the "direct+loop" flow
- Only 4 models with multiple products, 4 models with heterogeneous fleet

Main model features

- **Location:** even in multi-periodic models, the facility location decision is *static*
- **Routing:** more and more route based models.
- **Inventory:** simple policies.

Model 1: a "direct+loop" LIRP model



Data Sets and parameters

Set	Definition
I	Set of customers
J	Set of distribution centers
P	Set of plants (1 plant here)
$T = \{0, \dots, \lvert T \rvert\}$	set of time periods (days)
V	Set of all nodes $V = P \cup I \cup J$.
V^*	$= I \cup J$.
R	Set of routes (in the second layer)

Data Sets and parameters

Data	Definition
f_j	Fixed cost of opening distribution center $j \in J$
Q	Capacity of vehicles (homogeneous fleet)
τ_{max}	Maximum shelf life.
d_i^t	Demand of customer $i \in I$ in period $t \in \{1, \ldots, \lvert T \rvert + \tau_{max} - 1\}$.
h_i^t	Holding cost at facility $i \in V^*$ in time period $t \in T$
I_{i0}	Initial inventory at facility $i \in V^*$
c_j	Cost of delivering distribution center $j \in J$ (1^{st} layer)
c'_r	Cost of route $r \in R$ (2^{nd} layer)
α_{ir}	=1 if route $r \in R$ visits facility $i \in V^*$, 0 otherwise

Variables

Binary Variables	
y_j	$= 1$ if distribution center $j \in J$ is selected. 0 otherwise.
z_r^t	$= 1$ if route $r \in R$ is selected in period $t \in T$, 0 otherwise
x_j^t	$=1$ if distribution center $j \in J$ is delivered in time period $t \in T$
Continuous Variables	
q_j^t	quantity delivered to distribution center $j \in J$ in time period $t \in T$.
u_{ir}^t	quantity delivered by route $r \in R$ to client $i \in I$ in time period $t \in T$.
I_i^t	inventory at facility $i \in I \cup J$ in time period $t \in T$

LIRP model 1

$$\min \sum_{j \in J} f_j y_j + \sum_{t \in T} \left(\sum_{j \in J} c_j x_j^t + \sum_{r \in R} c'_r z_r^t \right) + \sum_{t \in T} \sum_{i \in V^*} h_i^t I_i^t \tag{1}$$

$$\sum_{r \in R} \alpha_{ir} z_r^t \leq 1 \qquad \forall i \in I, \forall t \in T \tag{2}$$

$$q_{j,t} \leq Q\ x_j^t \qquad \forall j \in J, \forall t \in T \tag{3}$$

$$x_j^t \leq y_j \qquad \forall j \in J, \forall t \in T \tag{4}$$

$$\sum_{i \in I} u_{ir}^t \leq Q\ z_r^t \qquad \forall r \in R, \forall t \in T \tag{5}$$

$$z_r^t \leq \sum_{j \in J} \alpha_{jr} y_j \qquad \forall r \in R, \forall t \in T \tag{6}$$

$$\sum_{r \in R} z_r^t \leq |K| \qquad \forall t \in T \tag{7}$$

LIRP model 1 (continued)

$$I_j^t = I_j^{t-1} + q_j^t - \sum_{r \in R} \alpha_{jr} \left(\sum_{i \in I} \alpha_{ir} \, u_{ir}^t\right) \qquad \forall j \in J, \forall t \in T \tag{8}$$

$$I_i^t = I_i^{t-1} + \sum_{r \in R} \alpha_{ir} \, u_{ir}^t - d_i^t \qquad \forall i \in I, \forall t \in T \tag{9}$$

$$I_i^t \leq \sum_{t' \geq t}^{t' \leq t + \tau_{max}} d_i^{t'} \qquad \forall i \in I, \forall t \in T \tag{10}$$

$$y_j \in \{0,1\} \qquad \forall j \in J \tag{11}$$

$$z_r^t \in \{0,1\} \qquad \forall r \in R, \forall t \in T \tag{12}$$

$$x_j^t \in \{0,1\} \qquad \forall j \in J, \forall t \in T \tag{13}$$

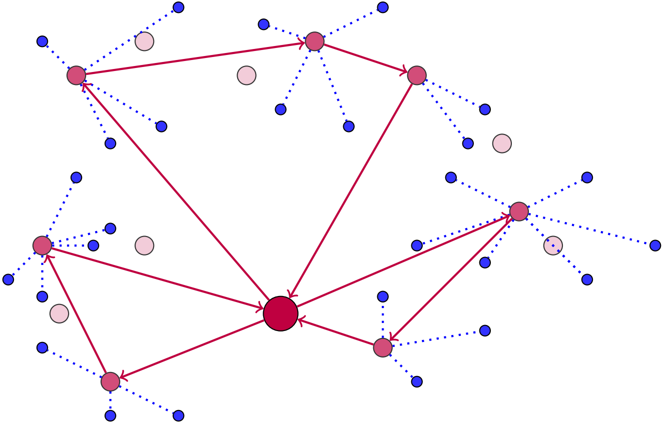
$$I_i^t, I_j^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{14}$$

$$q_j^t, u_{ir}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall r \in R \tag{15}$$

Constraints of model 1

- Objective function = facility fixed cost + routing cost + inventory cost
- (2) Each supplier is visited by at most one route at every period
- (3) Capacity constraint between the plant and the distribution centers.
- (4) No routes to non-selected distribution centers
- (5) Capacity constraint on route r, if it is performed in time period t.
- (6) A route starts from a selected distribution centers
- (7) Fleet size limitation
- (8) Flow conservation at DC j
- (9) Flow conservation at customer i
- (10) Max inventory at customers (valid inequality)

Model 2: a "loop+direct" LIRP model



New notation

Data	Definition
R	Set of routes (in first layer)
c_r	Cost of route $r \in R$ (1^{st} layer)
c'_{ij}	Cost of delivering customer $i \in I$ from DC $j \in J$ (2^{nd} layer)
α_{jr}	=1 if route $r \in R$ visits DC $j \in J$, 0 otherwise

Binary Variables	
y_j	$= 1$ if distribution center $j \in J$ is selected. 0 otherwise.
z_r^t	$= 1$ if route $r \in R$ is selected in period $t \in T$, 0 otherwise
x_{ij}^t	=1 if customer i is delivered by DC $j \in J$ in time period $t \in T$
Continuous Variables	
v_{ij}^t	quantity delivered from DC $j \in J$ to customer $i \in I$ in period $t \in T$.

LIRP model 2

$$\min \sum_{j \in J} f_j y_j + \sum_{t \in T} \left(\sum_{j \in J} c_r z_r^t + \sum_{i \in I} \sum_{j \in J} c'_{ij} x_{ij}^t \right) + \sum_{t \in T} \sum_{i \in V^*} h_i^t I_i^t \quad (16)$$

$$\sum_{r \in R} \alpha_{jr} z_r^t \leq 1 \qquad \forall j \in J, \forall t \in T \quad (17)$$

$$q_j^t \leq Q\, z_r^t \qquad \forall j \in J, \forall r \in R, \forall t \in T \quad (18)$$

$$z_r^t \leq \alpha_{jr}\, y_j \qquad \forall j \in J, \forall t \in T \quad (19)$$

$$\sum_{r \in R} z_r^t \leq |K| \qquad \forall t \in T \quad (20)$$

$$v_{ij}^t \leq Q\, x_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \quad (21)$$

$$v_{ij}^t \leq \left(\sum_{t \geq t}^{t' \leq t+\tau_{max}} d_i^{t'} \right) x_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \quad (22)$$

$$x_{ij}^t \leq y_j \qquad \forall i \in I, \forall j \in J, \forall t \in T \quad (23)$$

LIRP model 2 (continued)

$$I_j^t = I_j^{t-1} + q_j^t - \sum_{i \in I} v_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \qquad (24)$$

$$I_i^t = I_i^{t-1} + \sum_{j \in J} v_{ij}^t - d_i^t \qquad \forall i \in I, \forall t \in T \qquad (25)$$

$$I_i^t \leq \sum_{t' \geq t}^{t' \leq t + \tau_{max}} d_i^{t'} \qquad \forall i \in I, \forall t \in T \qquad (26)$$

$$y_j \in \{0, 1\} \qquad \forall j \in J \qquad (27)$$

$$z_r^t \in \{0, 1\} \qquad \forall r \in R, \forall t \in T \qquad (28)$$

$$x_{ij}^t \in \{0, 1\} \qquad \forall i \in I, \forall j \in J, \forall t \in T \qquad (29)$$

$$I_i^t, I_j^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \qquad (30)$$

$$q_j^t, v_{ij}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall r \in R \qquad (31)$$

Constraints of model 2

- Objective function = facility fixed cost + routing cost + inventory cost
- (17) Each DC is visited by at most one route in every time period
- (18) Capacity constraint on routes.
- (19) No routes to unselected distribution centers
- (20) Fleet size limitation
- (21) (22) Capacity constraints on route r, if it is performed in time period t.
- (23) A customer is served from a selected distribution center
- (24) Flow conservation at DC j
- (25) Flow conservation at customer i
- (25) Max inventory at customers (valid inequality)

Comparison of models

# variables	Model 1 direct+loop	Model 2 loop+direct
Binary	$\lvert J\rvert + \lvert R\rvert\lvert T\rvert + \lvert J\rvert\lvert T\rvert$	$\lvert J\rvert + \lvert R\rvert\lvert T\rvert + \lvert I\rvert\lvert J\rvert\lvert T\rvert$
Continuous	$\lvert J\rvert\lvert T\rvert + \lvert I\rvert\lvert R\rvert\lvert T\rvert + \lvert I\rvert\lvert T\rvert$	$\lvert I\rvert\lvert J\rvert\lvert T\rvert + \lvert J\rvert\lvert T\rvert$

- The number of routes is much larger in Model 1.
- Many variables x_{ij}^t can be set at 0.
- Stable allocation $x_{ij}^t = x_{ij}^{t'}, \forall t, t' \in T$
- Model 2 can be improved (variables x_{ij}^t can be replaced by several 2-index variables)

Applications

Features of potential applications

- many independent customers
- small frequent shipments
- possible consolidation at intermediate facilities
- agile networks, intermittent customers

Example of potential applications

- reverse logistics in construction and civil engineering
- reverse logistics: recovery of e-commerce goods
- collection of breast milk
- distribution of dairy products

Work proposal

Implementation

- Reading existing problem instances provided by Guerrero et al. (already available)
- Implementation and computation experiments with an MILP solver

Analysis

- Comparison of both models
- What if analysis: try to understand in which cases one model outperforms the other one.

Academic contribution

- State of the art
- Write a conference paper (8-10 pages)