Université de Nantes - UFR Sciences and Techniques

Academic Year 2018-2019 - M2 ORO

MultiObjective Meta Heuristics
Topic: Handling non-dominated points

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Planning:

- September 27: Lecture (2h)
- September 27: Exercices (2h, individual work)

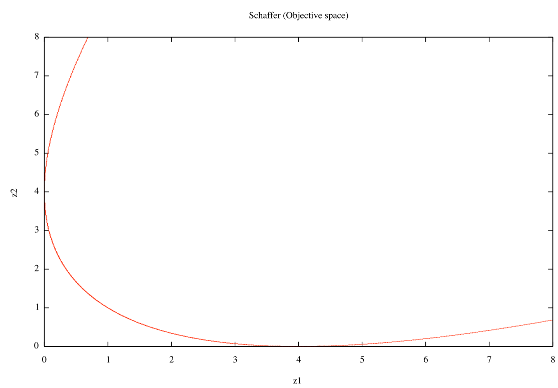
Context

A piece of software aiming to bootstrap your work is provided. It is written in language C, and available on Madoc.

1. it computes the outcome set $Y = z(X)$ for the Schaffer problem:

$$\begin{array}{lcl} \min\ z^1(x) & = & x^2 \\ \min\ z^2(x) & = & (x-2)^2 \\ \text{subject to} & & -10 \leq x \leq 10 \\ & & x \in \mathbb{R} \end{array}$$

2. it plots the outcome set with gnuplot:



Develop the software components issued from the exercices 1–3 for $z \in \mathbb{R}$ or $z \in \mathbb{N}$ or again mixed, and implement them in Julia (or C) language. They will be reused later as a routine of MOMH.

Exercice 1: filtering procedure which extract Y_N from Y

For the Schaffer's NLP problem[1] ($p \in \{2\}$), write a piece of code:

- for computing the outcome set for some values of $x \in [-10, 10]$
- for computing the non-dominated points Y_N

Exercice 2: procedure splitting Y_N into Y_{SN} and Y_{NN}

For the Kim's NLP problem[2] ($p \in \{2\}$):

A problem with 2 objectives and 2 variables with bound constraints

$$\left[\begin{array}{lll} \min & f_1(x) = & -(3(1-x_1)^2 \exp(-x_1^2-(x_2+1)^2) \\ & & -10(x_1/5.0 - x_1^3 - x_2^5)\exp(-x_1^2-x_2^2) \\ & & -3\exp(-(x_1+2)^2 - x_2^2) + 0.5(2x_1+x_2)) \\ \min & f_2(x) = & -(3(1+x_2)^2 \exp(-x_2^2-(1-x_1)^2) \\ & & -10(-x_2/5.0 + x_2^3 + x_1^5)\exp(-x_1^2-x_2^2) \\ & & -3\exp(-(2-x_2)^2 - x_1^2)) \end{array}\right],$$

with $-3 \leq \{x_1, x_2\} \leq 3$.

write a piece of code:

- for computing the outcome set for some random values (x_1, x_2)
- for computing the non-dominated points Y_N
- for splitting Y_N in supported and non-supported points (resp. Y_{SN} and Y_{NN})

Note: The Jarvis algorithm is designed to compute the convex envelop of a set of discrete points of $\mathbb{R}^2$. It can be used for the identification of Y_{SN} and Y_{NN}.

[1] David Schaffer. Multiple Objective Optimization with Vector Evaluated Genetic Algorithms. In *Proceedings of the 1st International Conference on Genetic Algorithms*, L. Erlbaum Associates Inc. pp. 93–100, 1985. `http://dl.acm.org/citation.cfm?id=645511.657079`,

[2] I.Y. Kim and O.L. de Weck. Adaptive weighted-sum method for bi-objective optimization: Pareto front generation. *Structural and Multidisciplinary Optimization.* Vol. 29, Num. 2, pp. 149–158, 2005. `https://link.springer.com/article/10.1007/s00158-004-0465-1`

Additional Exercise 1: filtering procedure which extract Y_N from Y

For the Przybylski's instance of linear assigment problem[3] [4] with $p \in \{2,3\}$:

$$C^1 = \begin{pmatrix} 2 & 5 & 4 & 7 \\ 3 & 3 & 5 & 7 \\ 3 & 8 & 4 & 2 \\ 6 & 5 & 2 & 5 \end{pmatrix}, \quad C^2 = \begin{pmatrix} 3 & 3 & 6 & 2 \\ 5 & 3 & 7 & 3 \\ 5 & 2 & 7 & 4 \\ 4 & 6 & 3 & 5 \end{pmatrix} \quad \text{and} \quad C^3 = \begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

write a piece of code:

- for computing the outcome set for some values
- for computing the non-dominated points Y_N

Additional Exercise 2

For the Mavrotas's MILP problem[5] with $p \in \{2,3\}$:

$$\begin{aligned}
&\max\{17x_1 - 12x_2 - 12x_3 - 19x_4 - 6x_5 - 73\delta_1 \\
&\qquad\qquad - 99\delta_2 - 81\delta_3\}, \\
&\max\{2x_1 - 6x_2 - 12x_4 + 13x_5 - 61\delta_1 - 79\delta_2 \\
&\qquad\qquad - 53\delta_3\}, \\
&\max\{-20x_1 + 7x_2 - 16x_3 - x_5 - 72\delta_1 \\
&\qquad\qquad - 54\delta_2 - 79\delta_3\} \\
&\text{subject to:} \\
&\delta_1 + \delta_2 + \delta_3 \leq 1, \\
&-x_2 + 6x_5 + 25\delta_1 \leq 52, \\
&-x_1 + 18x_4 + 18x_5 + 8\delta_2 \leq 77, \\
&7x_4 + 9x_5 + 19\delta_3 \leq 66, \\
&16x_1 + 20x_5 \leq 79, \\
&13x_2 + 7x_4 \leq 86, \\
&x_j \geq 0 \text{ and } \delta_j = 0 \text{ or } 1.
\end{aligned}$$

write a piece of code:

- for computing the outcome set for some values
- for computing the non-dominated points Y_N

[3] Anthony Przybylski, Xavier Gandibleux, Matthias Ehrgott. Two phase algorithms for the bi-objective assignment problem. *European Journal of Operational Research*, Vol. 185, Iss. 2, pp. 509–533, 2008. `http://www.sciencedirect.com/science/article/pii/S0377221707000781`

[4] Anthony Przybylski, Xavier Gandibleux, Matthias Ehrgott. A two phase method for multi-objective integer programming and its application to the assignment problem with three objectives. *Discrete Optimization*, Vol. 7, Iss. 3, pp. 149–165, 2010. `http://www.sciencedirect.com/science/article/pii/S1572528610000125`

[5] G. Mavrotas, D. Diakoulaki. A branch and bound algorithm for mixed zero-one multiple objective linear programming. *European Journal of Operational Research*, 107, pp. 530–541, 1998. `http://www.sciencedirect.com/science/article/pii/S0377221797000775`