Efficient Bayesian inference using approximate Riemannian geometry of stochastic population models

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We introduce an efficient and novel approach for Bayesian statistical inference of stochastic population models with complex dynamics, unobserved species and large numbers of parameters. Despite their increasing ubiquity, Bayesian analysis of such stochastic models still presents major challenges, as the exact likelihood is often computationally intractable for realistically sized models and full exploration of the parameter space may be inhibited by very strong correlation structure. Our methodology exploits the natural underlying approximate Riemannian geometry of the model parameters induced by estimating the likelihood using moment closure techniques. We demonstrate how this geometric approach allows us to perform inference in a statistically and computationally efficient manner, and in doing so we highlight the link between the Riemannian structure of the parameters and sensitivity analysis of the dynamical system. We illustrate the performance of our approach using synthetic examples, as well as a realistically-sized partially observed ecological model with 50 unknown parameters, with data describing the evolution of aphid populations. For the examples in this paper, efficient computer code written in Matlab and C is available from the authors upon request.

Introduction

The use of stochastic models is rapidly becoming ubiquitous in many areas of the natural sciences, where it is necessary to model the intrinsic stochasticity of a dynamical system in order to fully characterise its main modes of behaviour [REFs].

Markov Jump Process models - what are they and why are they useful?

Mathematical approaches are essential for making sense of these complex models; well developed for deterministic models, such as ordinary differential equations, but still underdeveloped for stochastic models.

Why? Likelihood is often computationally intractable for all but the simplest of Markov Jump Process models.

Sensitivity analysis plays an important role in elucidating the behaviour of mathematical models with complex dynamics - how do variations in parameters affect the output? Only fairly recently have methods been published for estimating the local sensitivity information for a stochastic chemical kinetic model via use of the linear noise approximation [REFs].

local versus global sensitivity analysis

Sensitivity information at the maximum likelihood is based on a linearisation around this point and is thus a local measure. Given the strong nonlinearities present in many current models, this provides limited information about the dynamics of the system.

FIM is often used to describe the local sensitivity at the maximum likelihood however this makes strong assumption of normality around that set of parameters, which may be inappropriate for strongly nonlinear systems.

Bayesian methodology in particular offers a consistent, probabilistic framework for reasoning under uncertainty, which may occur both in the data and the model structure itself.

Posterior distribution as a global measure of sensitivity of the system. However drawing samples from this is generally very challenging.

FIM provides a link between local and global sensitivity analysis.

FIM is usually used to measure the reliability of inferences around the maximum likelihood and is often understood geometrically as the curvature of the log-likelihood around this point. However, the expected Fisher Information in fact gives us much more geometric information than this; it defines a metric tensor at any point of parameter space and thus provides a natural representation of the parameter space as a Riemannian manifold, where distances between two sets of parameters are defined in terms of the relative changes they induce on the likelihood.

We use a moment closure technique to obtain local approximations to the likelihood at the current point - such an approach extends the LNR as we may now choose the pdf with which to approximate the local likelihood. We demonstrate how the expected Fisher Information may be obtained from this moment closure approximation and that this forms a Riemannian manifold that approximates the local sensitivity of the model at each possible set of parameters. Further, we demonstrate the utility of this using differential geometric MCMC algorithms that allow us to sample from the posterior distribution of the parameters of stochastic population models, by utilising the local sensitivity information at each point. Essentially a locally adaptive algorithm that exploits the local sensitivity at each point, allowing us to move from local to global sensitivity information in a computationally and statistically efficient manner.

- Inference for MJP is difficult: non-linear likelihood, computational expensive, proposals expensive, analytically intractable likelihood
- Data: Partially observed discrete, noisy time-course.
- Existing work for likelihood evaluations:
  - Likelihood calculations
    - Exact: Boys et al, slow, not noise
    - Approx SDE - Andy and Darren (with Noise)
    - Approx MC - Andy and Colin (no noise)

Reserved for Publication Footnotes