Magnetic Transport Along Translationally Invariant Obstacles

A Bachelor's Thesis Defense

Michal Grňo

September 10, 2021

Magnetic Transport Along Translationally Invariant Obstacles

Magnetic Transport Along Translationally Invariant Obstacles

A magnetic Hamiltonian

$$H_0 = \left(-\mathrm{i}\vec{\nabla} + \vec{A}\right)^2,$$

Magnetic Transport Along Translationally Invariant Obstacles

A magnetic Hamiltonian

$$H_0 = \left(-\mathrm{i}\vec{\nabla} + \vec{A}\right)^2 ,$$

$$\vec{\nabla} \times \vec{A} = \vec{B}_0 = \text{const.}$$

Magnetic Transport Along Translationally Invariant Obstacles

A magnetic Hamiltonian

$$H_0 = \left(-\mathrm{i}\vec{\nabla} + \vec{A}\right)^2 ,$$

$$\vec{\nabla} \times \vec{A} = \vec{B}_0 = \text{const.}$$

restricted to a 2D plane orthogonal to $\vec{B}_0$.

Magnetic Transport Along Translationally Invariant Obstacles

A magnetic Hamiltonian

$$H_0 = \left(-\mathrm{i}\vec{\nabla} + \vec{A}\right)^2 ,$$

$$\vec{\nabla} \times \vec{A} = \vec{B}_0 = \text{const.}$$

restricted to a 2D plane orthogonal to $\vec{B}_0$.

This is the *Landau Hamiltonian*.

Magnetic Transport Along Translationally Invariant Obstacles

Magnetic Transport Along Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x, y)$

Magnetic Transport Along Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x, y)$
- magnetic obstacle: $\vec{B} = \vec{B}_0 + \vec{b}(x, y)$

Magnetic Transport Along Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x, y)$
- magnetic obstacle: $\vec{B} = \vec{B}_0 + \vec{b}(x, y), \;\; \vec{b} \parallel \vec{B}_0$

Magnetic Transport Along Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x, y)$
- magnetic obstacle: $\vec{B} = \vec{B}_0 + \vec{b}(x, y), \;\; \vec{b} \parallel \vec{B}_0$
- geometric obstacle:
  - the system is not restricted to a plane, but to a thin layer

Magnetic Transport Along Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x, y)$
- magnetic obstacle: $\vec{B} = \vec{B}_0 + \vec{b}(x, y)$, $\vec{b} \parallel \vec{B}_0$
- geometric obstacle:
  - the system is not restricted to a plane, but to a thin layer
  - the layer is smoothly bent

Magnetic Transport Along Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x, y)$
- magnetic obstacle: $\vec{B} = \vec{B}_0 + \vec{b}(x, y), \;\; \vec{b} \parallel \vec{B}_0$
- geometric obstacle:
  - the system is not restricted to a plane, but to a thin layer
  - the layer is smoothly bent
  - Dirichlet boundary is assumed

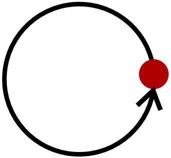
Magnetic Transport Along
Translationally Invariant Obstacles

- potential obstacle: $H = H_0 + V(x)$
- magnetic obstacle: $\vec{B} = \vec{B}_0 + \vec{b}(x)$, $\vec{b} \parallel \vec{B}_0$
- geometric obstacle:
    - the system is not restricted to a plane, but to a thin layer
    - the layer is smoothly bent and invariant under translation $y \mapsto y + c$
    - Dirichlet boundary is assumed

Magnetic Transport Along Translationally Invariant Obstacles

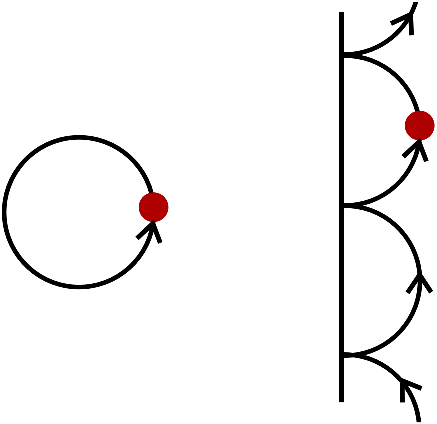
Magnetic Transport Along Translationally Invariant Obstacles

Classically:



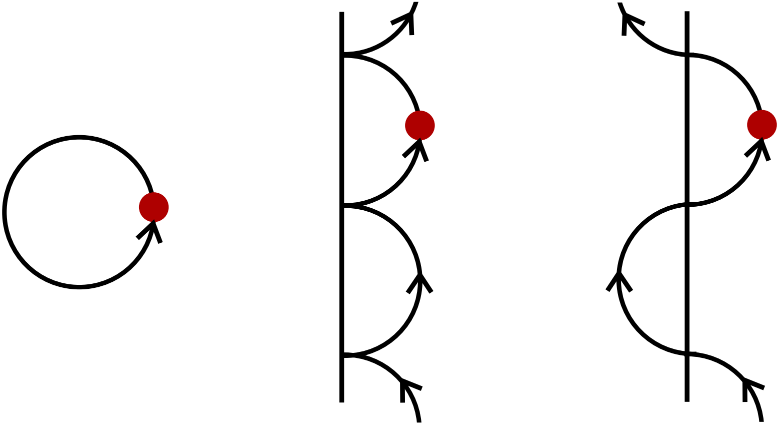
Magnetic Transport Along Translationally Invariant Obstacles

Classically:



Magnetic Transport Along Translationally Invariant Obstacles

Classically:



Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

- Spectrum of H is pure point

Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

- Spectrum of H is pure point
  $\leftrightsquigarrow$ there is a basis consisting of stationary states

Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

- Spectrum of H is pure point
  - $\leadsto$ there is a basis consisting of stationary states
  - $\leadsto$ time evolution is trivial

Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

- Spectrum of H is pure point
  - $\leftrightsquigarrow$ there is a basis consisting of stationary states
  - $\leftrightsquigarrow$ time evolution is trivial
- Spectrum of H is continuous

Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

- Spectrum of H is pure point
  - $\leftrightsquigarrow$ there is a basis consisting of stationary states
  - $\leftrightsquigarrow$ time evolution is trivial
- Spectrum of H is continuous
  - $\leftrightsquigarrow$ there are no stationary states

Magnetic Transport Along Translationally Invariant Obstacles

Quantum Mechanics:

- Spectrum of H is pure point
  - $\leftrightsquigarrow$ there is a basis consisting of stationary states
  - $\leftrightsquigarrow$ time evolution is trivial
- Spectrum of H is continuous
  - $\leftrightsquigarrow$ there are no stationary states
  - $\leftrightsquigarrow$ **Magnetic Transport!**

Magnetic Transport Along Translationally Invariant Obstacles

The Hamiltonian is either of these:

(a) $H = (-\mathrm{i}\vec{\nabla} + \vec{A})^2 + V(x)$ on $L^2(\Omega \subset \mathbb{R}^2)$
(b) $H = (-\mathrm{i}\vec{\nabla} + \vec{A}(x))^2$ on $L^2(\Omega \subset \mathbb{R}^2)$
(c) $H = (-\mathrm{i}\vec{\nabla} + \vec{A})$ on $L^2(\Omega)$, Ω being a thin layer in $\mathbb{R}^3$

And we are interested in its pure point / continuous spectrum.

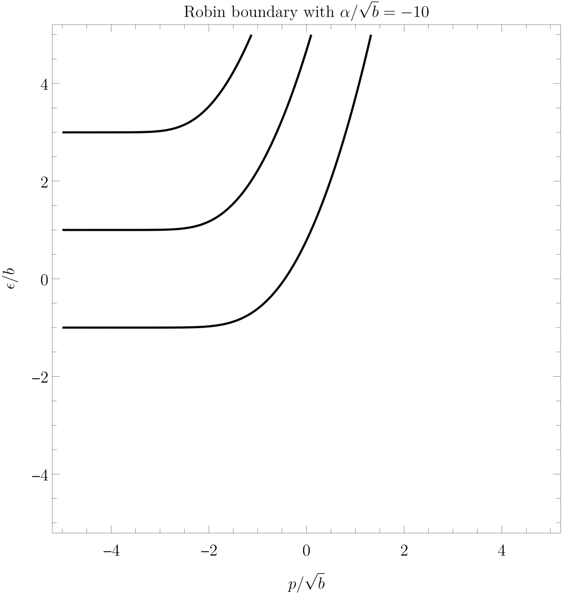
The two parts

1. Summary of known results
   - Steep potential wall (Macris et al., 1999) and (Fröhlich et al., 2000)
   - Half-plane with Dirichlet boundary (Fröhlich et al., 2000)
   - Bounded magnetic perturbation (Iwatsuka, 1983 and 1985)
   - Layer with one-sided fold, asymptotically flat layer, very thin layer (Exner et al., 2018)

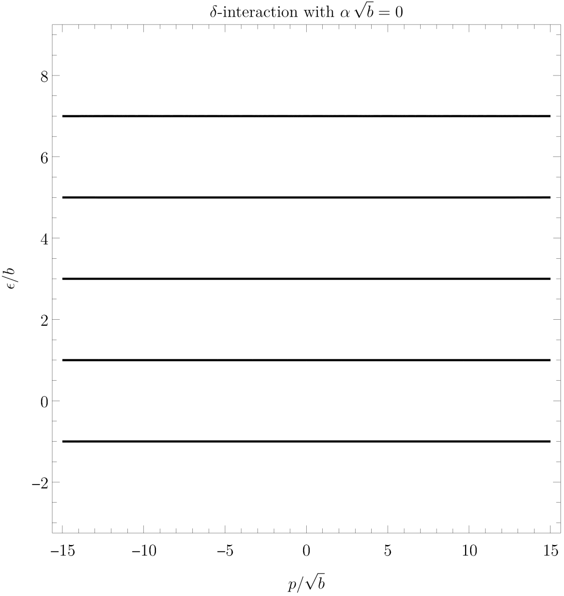
The two parts

1. Summary of known results
   - Steep potential wall (Macris et al., 1999) and (Fröhlich et al., 2000)
   - Half-plane with Dirichlet boundary (Fröhlich et al., 2000)
   - Bounded magnetic perturbation (Iwatsuka, 1983 and 1985)
   - Layer with one-sided fold, asymptotically flat layer, very thin layer (Exner et al., 2018)
2. Original work
   - Half-plane with Robin boundary
   - Dirac δ-interaction on a line

Frame Title


Robin boundary with $\alpha/\sqrt{b} = -10$
4
2
0
−2
−4
ϵ/b
−4
−2
0
2
4
$p/\sqrt{b}$

Frame Title


δ-interaction with $\alpha\sqrt{b} = 0$
8
6
4
2
0
−2
ϵ/b
−15
−10
−5
0
5
10
15
$p/\sqrt{b}$