How to Find a Critical Z Value

by Sophia

WHAT'S COVERED

This tutorial will cover how to find the critical z-value for the following tests:

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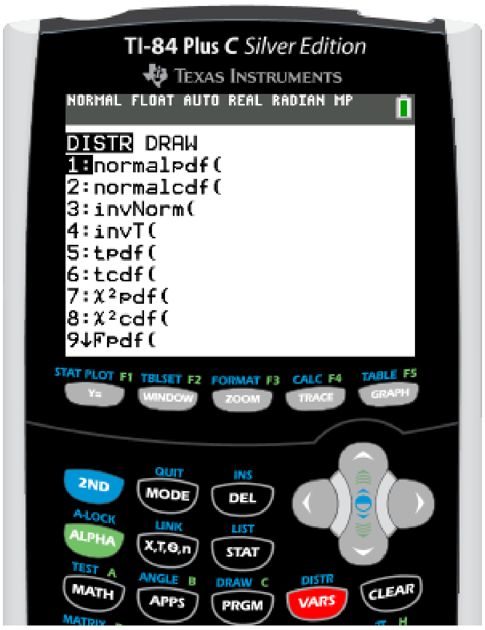
1. Left-Tailed Tests

For a left-tailed test, suppose we need to find the **critical z-value** for a hypothesis test that would reject the null hypothesis (H_0) at a 2.5% significance level. To do this, we want to find, on our normal distribution, the cutoff on the left tail that corresponds to the lower 2.5% of our distribution.

1a. Graphing Calculator

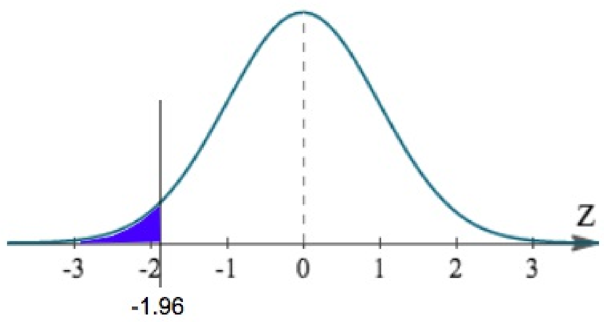
The first way is by using a graphing calculator. First, hit "2nd", then "DISTR" (This is above the button "VARS").



Then scroll down to the third function, which is inverse norm (invNorm). This is the inverse of the normal distribution. We're going to hit "Enter".

Next, we're going to input 0.025, because this is a left-tailed test, so we're looking at the lower 2.5% of our distribution. For this specific calculator (TI-84 Plus), we need to type 0.025 for the area, and 0 for mu and 1 for sigma because these are the values that correspond with a normal distribution. Hit "Enter", and we get a z-test statistic of negative 1.96.

At about -1.96, this is the cutoff for the lower 2.5% of our data.



Basically any z-score that is below a negative 1.96 means we're going to reject the null hypothesis. Any z-score that is above a negative 1.96 is going to fall in this unshaded region of our distribution. This means that we're willing to accept the variation in our sample from the center of our distribution due to chance, and we're going to fail to reject the null hypothesis.

1b. Z-Table

The second method is using a z-table. When using the z-table, we look for our significance level in the table. In this case, remember we were looking at a left-tailed test. This means we need to use the negative z-table with negative z-scores, not positive, because we're looking at the lower half of the distribution. Remember, the significance level is 0.025, or 2.5%, so we are going to look for that value or the closest thing to it. Here it is on a z-table:

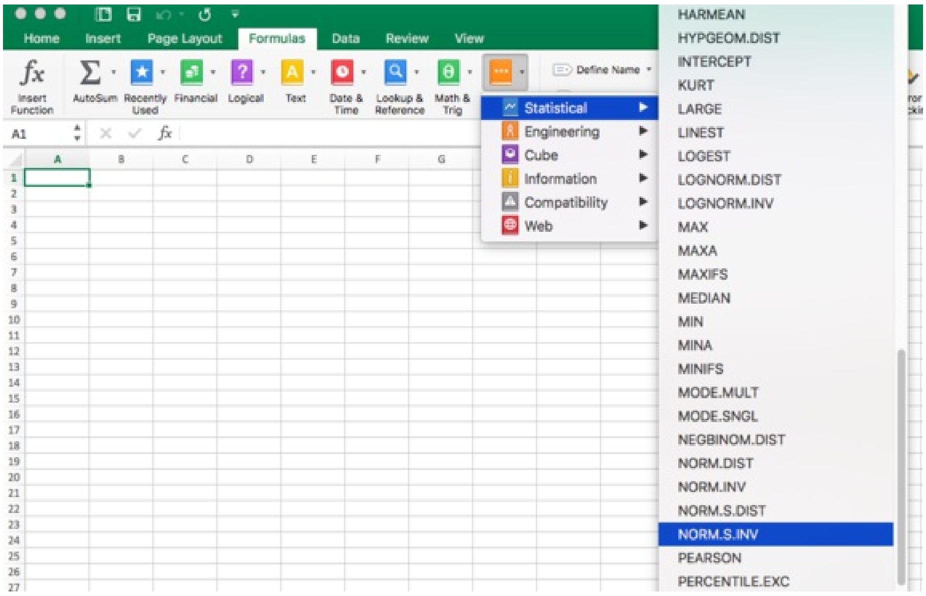
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455

-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559

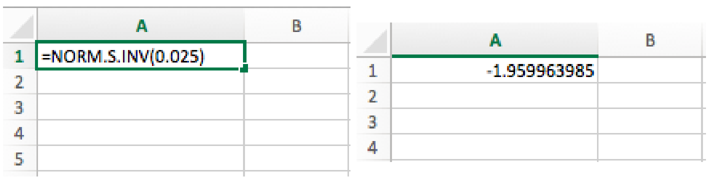
A significance level of 0.025 corresponds to a z-score of negative 1.96. Therefore, our z critical value is negative 1.96.

1c. Excel

A third way to find the critical z-value that corresponds to a 2.5% significance level for a left-tailed test is in Excel. All we have to do is go to our "Formulas" tab. We're going to insert under the "Statistical" column. We're looking for "NORM.S.INV", which is right here:



This is for the inverse of the normal distribution, and because it's a left-tailed test, we're looking at the lower half of our distribution. We're going to put in the 0.025 for the lower 2.5%. Hit "Enter", and notice how we get the same critical z-value that we did using the calculator and table:



TERM TO KNOW

Critical Value

A value that can be compared to the test statistic to decide the outcome of a hypothesis test

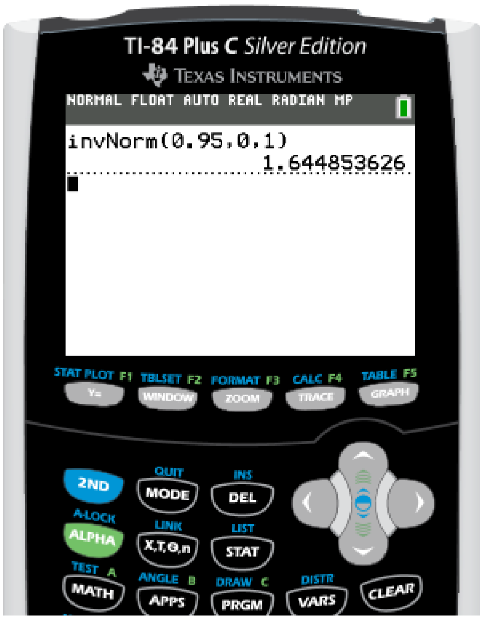
2. Right-Tailed Tests

For a right-tailed test, suppose we need to find the critical z-value for a hypothesis test that would reject the null (H_0) at a 5% significance level. To do this, we want to find, on our normal distribution, the cutoff on the upper part of the distribution where we are not going to attribute the difference in proportion due to chance.

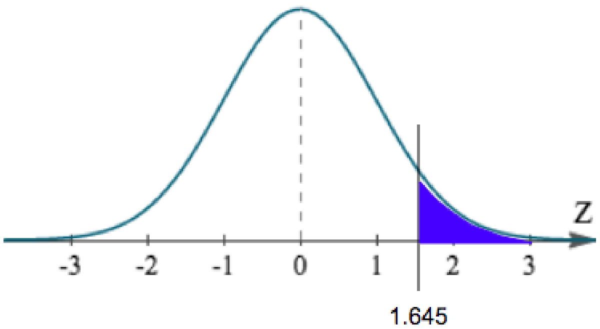
2a. Graphing Calculator

The first way is by using a graphing calculator. First, hit "2nd", then "DISTR" (This is above the button "VARS"). Then scroll down to the third function, which is inverse norm (invNorm). This is the inverse of the normal distribution. We're going to hit "Enter".

The significance level is 5%, but we're not going to put in 0.05 like we did with the left-tailed test, where the significance level was 2.5% and we entered 0.025. In the normal distribution, we always read left to right, and it always goes from 0 percent to 100 percent. We're looking at a right-tailed test, which is the upper portion of our distribution. That cutoff is the top 5% of our distribution. So, 100% minus 5% is going to be 95%. We are actually going to put in the inverse norm of 0.95, and that's going to get us a corresponding critical z-value of about 1.645.



Any z-test statistic that is greater than 1.645 falls in the upper 5% of our distribution, and therefore we would reject the null hypothesis.



2b. Z-Table

The second method uses the z-table. Because we're looking at a right-tailed test, we're going to have positive z-scores since we're looking at the upper half of the distribution. We'll use the positive z-table that corresponds with positive z-scores.

The significance level was 5%, but it was the upper 5%. Remember, this corresponds to the 95th percentile on our distribution. In the table, we need to look for the closest thing to 95%, or 0.95.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517

0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

This actually falls in between these two values, 0.9495 and 0.9505. This value corresponds to a z-score of 1.6 in the left column, and it falls between the 0.04 and the 0.05 in the top row. When we take the average of 1.64 and 1.65, we get a critical z-value of 1.645.

2c. Excel

A third way to find the critical z-value that corresponds to a 5% significance level for an upper tail test, or a right-tailed test, is by using Excel. Again, go to "Formulas" tab. We're going to insert under the "Statistical" column our "NORM.S.INV", but we're not going to put in 0.05 for the 5%. Because we're looking at the upper part of our distribution, this is going to correspond to the 95th percentile. We're going to enter 0.95, and notice how we get the same critical value we did from our table and our calculator, which is a positive 1.645.

	A	B
1	=NORM.S.INV(0.95)	
2		
3		
4		

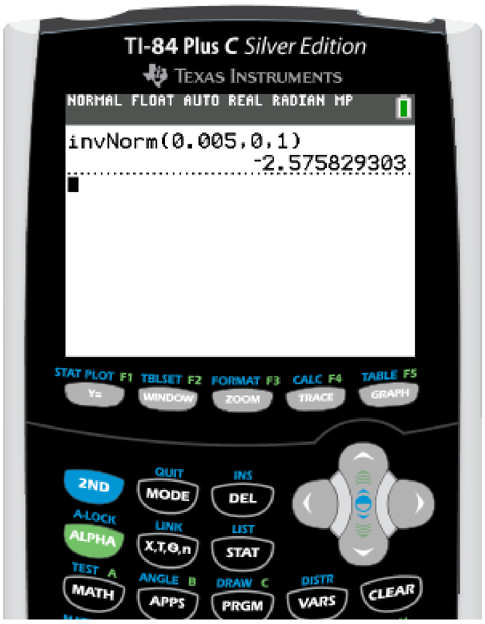
	A	B
1	1.644853627	
2		
3		
4		

3. Two-Sided Tests

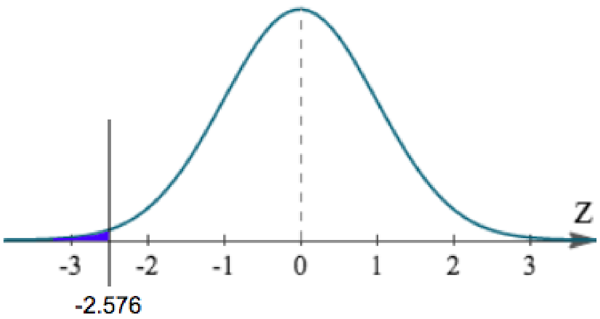
For a two-sided test, suppose we want to find the critical z-score for a hypothesis test that would reject the null at a 1% significance level. Because it's a two-sided test, we have to divide that 1% into each tail. Therefore, 1% divided by 2 means we're going to be looking for the cutoff at the lower 0.5% of the distribution, and the upper 0.5% of our distribution.

3a. Graphing Calculator

The first way to find this value is with a graphing calculator. Let's go ahead and first find the corresponding critical z-score for the lower part of our distribution. Hit "2nd", "DISTR", "invNorm". This tail is 0.5%, so we're going to put 0.005.

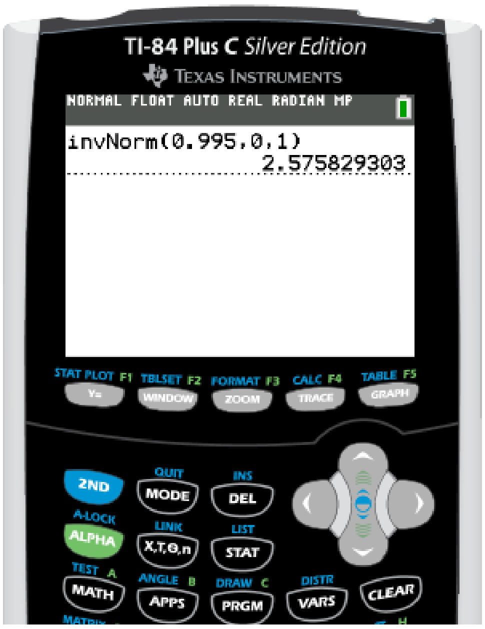


This gives us a corresponding z-score of negative 2.576. In a distribution, this falls right about here, negative 2.576.

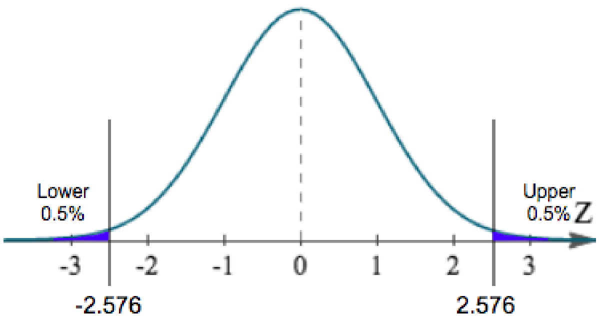


The shaded region corresponds to the lower 0.5% of the distribution. If we do this correctly, we should get the same z-score, but a positive value for the upper portion of our distribution for that 0.5% cut off.

Let's go ahead and do inverse norm again on our calculator. But we can't put in 0.005, because remember, our distribution reads from 0% to 100%. We actually have to do 100% minus 0.05%, or 99.5%. We are going to put in 0.995 and get a positive 2.576.



This positive 2.576 corresponds to the upper 0.5% of our distribution.



Any z-score that we would calculate that would be greater than a positive 2.576 or less than a negative 2.576 means we would reject the null hypothesis.

3b. Z-Table

Using our z-table, we first look for the corresponding critical value for the lower half of our distribution, since it's a two-sided test. Remember, we're not going to look for the closest thing to 1%, but we're going to look for the closest value to 0.5%, or 0.005.

Let's use the table to find the lower critical value for our two-sided test in a 1% significance level, we're going to find the closest thing to 0.5%.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019

-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183

The closest value to 0.005 is between these two values, 0.0051 and 0.0049. This corresponds to a negative 2.5 in the left column, and in between the 0.07 and the 0.08 in the top row. If we're using the table, we're going to get an average critical z-value of negative 2.575, which is quite close to what the calculator gave us. Remember, sometimes the table can just give us an estimate.

Let's use the table to find the upper critical value for our two-sided test in a 1% significance level, we're going to try to find the closest to 100% minus 0.5%, or 99.5%.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

The closest value to 99.5%, or 0.995, is in between these two values, 0.9949 and 0.9951. This corresponds to a positive 2.5 in the left column, and falling between the 0.07 and the 0.08 in the top row. If we're using the table, we would get a critical z-value of a positive 2.575, taking the average between those two values.

3c. Excel

In Excel, we're going to find the two critical z-values that correspond to the 1% significance level for our two-sided test. Again, go under your "Formulas" tab. We're going to insert the "NORM.S.INV" under the "Statistical" column. We'll first find the lower critical value that corresponds to the lower 0.5%, so enter 0.005.

	A	B
1	=NORM.S.INV(0.005)	
2		
3		
4		

	A	B
1	-2.575829304	
2		
3		
4		

You can see that we get our first critical z-value of negative 2.576. Now, if we do this correctly, we should get a positive 2.576. Again, we're going to insert to get the second critical value for the upper part of our distribution. The upper percentage that corresponds to the top 0.5% is going to be our 99.5%, so 0.995.

	A	B
1	=NORM.S.INV(0.995)	
2		
3		
4		

	A	B
1	2.575829304	
2		
3		
4		

We get the positive critical z-value of 2.576.

SUMMARY

We calculated a critical z-score for a left-tailed, right-tailed, and two-tailed test, utilizing three methods for each test: graphing calculator, z-table, and Excel.

Good luck!

Source: Adapted from Sophia tutorial by RACHEL ORR-DEPNER.

TERMS TO KNOW

Critical Value

A value that can be compared to the test statistic to decide the outcome of a hypothesis test

