Is margin preserved after random projection?

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Errata

In Theorem 6, "linearly separable by margin $\gamma - \frac{2\epsilon}{1-\epsilon}$ " should be "linearly separable by margin $(\frac{1+\epsilon}{1-\epsilon})\gamma - \frac{2\epsilon}{1-\epsilon}$ ", although the latter implies the former.

Fixing derivation of inequality (9) *i.e.*

$$\|\frac{\mathbf{R}\,\mathbf{x}}{\|\,\mathbf{x}\,\|} - \frac{\mathbf{R}\,\mathbf{w}}{\|\,\mathbf{w}\,\|}\|^2 \le \|\sqrt{(1+\epsilon)}(\frac{\mathbf{R}\,\mathbf{x}}{\|\,\mathbf{R}\,\mathbf{x}\,\|} - \frac{\mathbf{R}\,\mathbf{w}}{\|\,\mathbf{R}\,\mathbf{w}\,\|})\|^2 + (\sqrt{(1+\epsilon)} - \sqrt{(1-\epsilon)})^2. \tag{9}$$

Proof Let $\mathbf{a} = \frac{\mathbf{R}\,\mathbf{x}}{\|\,\mathbf{R}\,\mathbf{x}\,\|}$, $\mathbf{b} = \frac{\mathbf{R}\,\mathbf{w}}{\|\,\mathbf{R}\,\mathbf{w}\,\|}$, $\alpha = \frac{\|\,\mathbf{R}\,\mathbf{x}\,\|}{\|\,\mathbf{x}\,\|}$, $\beta = \frac{\|\,\mathbf{R}\,\mathbf{w}\,\|}{\|\,\mathbf{w}\,\|}$ and $\eta = (\sqrt{(1+\epsilon)} - \sqrt{(1-\epsilon)})^2$. We know $\alpha, \beta \in [\sqrt{1-\epsilon}, \sqrt{1+\epsilon}]$ via inequality (7), thus $\alpha\beta \le 1+\epsilon$. We also know $\mathbf{a}^2 = 1$, $\mathbf{b}^2 = 1$ and $\mathbf{a}^\intercal\,\mathbf{b} \le 1$ by definition. RHS of (9) $-$ LHS of (9) yields,

$$\begin{aligned}
&\eta + \|\sqrt{(1+\epsilon)}(\mathbf{a} - \mathbf{b})\|^2 - \|\,\mathbf{a}\,\alpha - \mathbf{b}\,\beta\|^2 \\
&= \eta + (1+\epsilon)(\mathbf{a}^2 + \mathbf{b}^2) - (1+\epsilon)(2\,\mathbf{a}^\intercal\,\mathbf{b}) - \alpha^2\,\mathbf{a}^2 - \beta^2\,\mathbf{b}^2 + 2\alpha\beta\,\mathbf{a}^\intercal\,\mathbf{b} \\
&= \eta + 2(1+\epsilon) - 2\,\mathbf{a}^\intercal\,\mathbf{b}[(1+\epsilon) - \alpha\beta] - \alpha^2 - \beta^2 \\
&\ge \eta + 2(1+\epsilon) - 2[(1+\epsilon) - \alpha\beta] - \alpha^2 - \beta^2 \qquad (\because [(1+\epsilon) - \alpha\beta] > 0, \mathbf{a}^\intercal\,\mathbf{b} \le 1) \\
&= \eta - (\alpha - \beta)^2 \ge 0.
\end{aligned}$$

Thus inequality (9) holds. This proof is suggested by Lijun Zhang (MSU).