## Background

The mass concentration of a population of particle type *i* is

where *i* is individual particle density, *Ni* is the number concentration of particles of type *i* , and *Vp,i* is individual particle volume. It is possible to assume that the bulk optical response (e.g., backscattering or beam attenuation) of a sub-population *bx,i*, is the simple sum of the individual responses so that

 (1)

and that the bulk response to the combined sub-populations is

 (2)

where *αv,i*, is a volume-specific optical property*.* *αv,i* are typically calculated using Mie theory.

However, this neglects interactions among particles. I wanted to investigate the effect of occlusion (shadowing) of particles by other particles. When there is only one particle in the sample volume, the sampled area is identical to the area of that particle. If there is more than one particle in the sample volume, it is possible that they will overlap, and the area sampled for backscattering or transmissivity will be less than the sum of the particle areas and, as the number of particles increases, the likelihood that sampled area is significantly less than the particle area increases.

## Method

I approached this using a Monte Carlo simulation of randomly positioned spherical particles and compared the sampled area with the particle area for a range of concentrations and particle sizes. For a given volume concentration and uniform particle radius *r*, the number of particles *N* in a sample volume is

where is the volume of an individual particle. (The index denoting particle type has been dropped, as we are only considering homogenous particle populations for now). Assuming there is no occlusion, the projected area is the sum of all of the particle areas:

The sampled area was calculated by creating an array of pixels and mapping the projected area of *N* particles with centers at random locations in the sample area. An array *p* was used to track occluded pixels. The array was initialized with zeros, and set to one if the pixel it represented it overlapped with the area of any particle. The sample area was determined as , where *i* and *j* are indices into array and *a* is the area of a pixel. equals when there are no overlapping particles, but as the number of particles increase and they begin to occlude each other, .

There is a small error associated representing round shapes with a pattern of square pixels. This error decreases as the relative particle radius increases and is less than 1% for particles with pixels. There is also an edge effect. If particles are not fully contained in the sample area, is still correct, but as calculated above is overestimated because it includes areas of particles only partially within the sample area. This was resolved by using a second array *as* which was incremented (rather than set to one) each time a pixel fell within the radius of a particle. The sum over this array produces a value of that correctly represents the area of all particles, accounting for the fractional areas of particles that are only partially included in the sample area. It also compensates for the discrepancy between the circular area and the pixel representation by using the pixel areas for both and .

## Results

Sample particle distributions (Figs. 1 -3) illustrate the results of individual simulations of particle locations in a 1-mm2 sample area represented by a grid of 1000 x 1000 pixels, each 1 square *µ*m. The particles (colored circles) are randomly located in the sample area, and may overlap. Overlapping particles are indicated by hotter colors. The number of particles required to represent a given volume concentration decreases with particle concentration, so only a few of the large (50 *µ*m ) particles in Figure 1 overlap. When no particles overlap, the particle area *Ap* and the sampled particle area *As* are identical. But when portions of one or more particles are occluded, and thus obscured from view, the sampled particle area *As* decreases relative to *Ap*.

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| Figure 1. Four simulations with a sample area 106 *µ*m2 with *Vc* = 0.005 and *r* = 50 *µ*m. When no particles are occluded, the sample area *As* is the same as the summed area of particles *Ap*, but the ratio *As*/*Ap* drops when some particles are occluded. |

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| Figure 2. Four simulations with a sample area 106 *µ*m2 with *Vc* = 0.005 and *r* = 12 *µ*m. Some particles are occluded in all of the simulations and, because of the large number of particles, there is little variation in *As*/*Ap* among simulations. |

Increasing the number of particles increases the opportunity for occlusion, so the ratio of particle area to sampled particle area *As*/*Ap* increases with smaller particle size or higher volume concetrations (Figure 2). In addition, the amount of occlusion varies less between simulations, so fewer simulations are required to form a stable estimate of *As*. When particles are large, the suspension consists of only a few particles, even at relatively large volume concentrations (Fig. 3), and occlusion is relatively rare. More simulations are required to characterize the average *As*.

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| Figure 3. Four simulations with a sample area 106 *µ*m2 with *Vc* = 0.01 and *r* = 100 *µ*m. Because of their large volume, only two particles are required to fill the volume concentration. None of these four simulations produced occluding particles, so *As*/*Ap* was always 1. |

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| Figure 4. Plot showing the convergence of the mean and standard deviation of *As* as the number of iterations increases. Convergence occurs more quickly with more particles. |

Trial and error showed that estimates of *As* converged after 100 simulations. Fig. 4 shows the normalized mean (top panel) and normalized standard deviation (bottom panel), as a function of the number of iterations for a case with relatively large particles and low concentrations. This plot suggests that the mean and standard deviation of *A*s change by less than 1% beyond 100 simulations.

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| Figure 5. Simulated particle areas for a range of volume concentrations *Vc* (0.001 to 0.1) and four grain radii. Top panel: Particle areas *Ap* (dashed) and sampled particle areas *As* (solid) after accounting for occlusion. The total sample area was 1 mm2 (dashed line), so *As* saturates as it approaches this value. Bottom panel: ratio *As*/*Ap*, which can be interpreted as the decrease in the amplitude of the backscatter signal caused by occlusion. |

Results from a range of particle sizes (12 to 100 *µ*m) and range of volume concentrations (0.001 to 0.1; Fig. 5) show that rises rapidly for small particles as concentrations increase. increases linearly as with volume concentration, at rates inversely proportional to size. At very low concentrations, tracks *Ap*, but as concentrations rise, increasing occlusion causes to lag *Ap*. This occurs at lower concentrations for smaller particles. At yet higher concentrations, particle areas for the 12-*µ*m particles approach the total sample area. To the extent that represents the potential backscatter signal, this represents saturation. values for larger particles (50 and 100 *µ*m) do not approach saturation but, as the ratio *As*/*Ap* shows, the potential backscatter signal is decreased to about 75% at *Vc* =0.1 for 100-*µ*m particles, and to about half for 50-*µ*m particles (lower panel, Fig. 5).

The transmissivity can be estimated as the fraction of total measurement area not occluded by any particles, equal to where is the total measurement area and is the measurement volume. In the simulations, the measurement area = 1 mm2, and if we envision a sample volume of 1 mm3, the pathlength *l* is 1 mm. Transmissivity (Fig. 6, top panel) drops rapidly with concentration for the finest particles, and less rapidly for the coarser particles.

Finally, we can combine the potential backscatter signal () with the transmissivity to calculate the relative response *R* of a backscatter sensor sampling a water volume of 1 mm3 as , which has units of mm2. This response is the relative signal associated with the sampled particle area tempered by the loss of signal associated with transmissivity *T* over a pathlength *l*. The responses for fine particles are classic (Fig. 6, bottom panel). At low concentrations, the backscatter response to increasing concentrations is linear, but as concentrations increase, the response is attenuated by increasing turbidity. Eventually, the potential backscatter response is saturated at particle area approaches total measurement area . As concentrations continue to increase, the signal is increasingly attenuated as transmissivity declines. All of this happens later, at higher concentrations, as particle size increases. These plots look exactly like the OBS laboratory calibration curves for fluid muds (e.g., Kineke & Sternberg, 1992).

## Conclusion

A complete forward model would combine the optical parameter with *R*. The only inconsistency is that this has all been based on area, but the optical parameters are volume-specific. Thoughts?

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| Figure 6. Top panel: Transmissivity *T* (1/mm), calculated as the non-occluded sample area . Bottom panel: Simulated backscatter response, calculated as *TAs* (mm) |