Calculating Sand Transport under Asymmetrical Waves

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12 September, 2017

The final objective is net wave-averaged sand transport. We choose to use the use van der A et al. (2013) formulae, which adds transport from onshore and offshore components. It does not include streaming, bottom slope, or variable roughness, and is probably not suitable when wave-orbital ripples exist, but it might be ok when ripple wavelengths are much less than the orbital diameter and a suitable roughness is used.

Required as input are vector components of the crest and trough velocities:



where



and  is steady current speed,  is angle from the *x* axis (direction of wave propagation, positive in the crest direction), and  , the representative orbital velocity. Also required are: T (wave period) Tc (duration of the crest part of the cycle), Tcu (duration of the acceleration to maximum crest velocity), Tt (duration of the trough part of the cycle), and Ttu (duration of acceleration to maximum trough velocity). How do we get these from wave statistics?

1) Need as input: *Hs*, *T*, *h* (not sure what the ideal period *T* is…probably *Tbot*)

2) Compute wavenumber *k* from the dispersion relationship



where angular frequency. Because *k* is implicit in , use function qkhfs to calculate.

1) Compute Ursell number (Ruessink et al., 2012, eqn. 6)



where wave amplitude .

2) Use Reussink et al. (2012) eqns. 9 and 10 to find fit to find non-dimensional non-linearity parameters *B* and





where *p*1 = 0, *p*2 = 0.857, *p*3 = -0.471, *p*4 = 0.297, *p*5 = 0.815, and *p*6 = 0.672.

3) Use Ruessink et al. (2013) eqns. 11 and 12 to convert *B* and to *b* and . Eqn. 11 is



and



Malarkey and Davies (2012; hereafter MD12) showed that eqn. can be rearranged to give non-linearity parameters *b* and *r*:



where



and



Abreu et al. (2010; their eqns. 7 and 8) introduced analytic forms for wave-orbital velocity and acceleration that incorporate velocity and acceleration asymmetry through parameters *r* and  . MD12 (eqns. 16a and 16b) express the Abreu et al. (2010) equations in terms of *b* and  (with the sign of reversed) as





4) Use the non-linearity parameters *b* and  to calculate time series of wave-orbital velocity parameters. The non-dimensional phase of the peak crest and peak trough velocities are (MD12, eqn. E3)



And the maximum and minimum non-dimensional velocities are (MD12, Appendix E)





where . The phase of zero upcrossing (MD12, p. 84, after eqn. 18) is



and the phase of zero downcrossing (MD12, p. 84, after eqn. 23) is



(Note that I had to add the *tzu* to make it work. There may be a typo in MD12 here).

5) Transport formula



where  is volumetric net transport per unit width,  ,  is median grain size,  is Shields parameter (non-dimensional bed stress,  are non-dimensional transport rates, and T is (portion of) wave period. Subscripts *c* ant *t* indicate crest and trough, respectively.

Calculating Roughness and Bed Shear Stress

The final results of these calculations are the x and y components of Shields parameter during the crest and trough intervals of the wave period. The x and y components are computed by multiplying the magnitude of the

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| --- | --- | --- | --- |
| Matlab variable name | Symbol in A13 | Description | Source / value |
| mag\_u\_d |  | steady current vector magnitude | A13, Eqn. 2, Fig. 2. |
| uhat |  | representative orbital amplitude | A13, Eqn. 8. |
| uhatc |  | peak crest orbital velocity | A13, Eqn. 6 & 7 |
| alpha |  | relative influence of mean current | A13, Eqn. 19 |
|  |  |  |  |
| uhatt |  | peak trough orbital velocity | A13, Eqn. 6 & 7 |
| utildecr |  | representative half-cycle crest orbital velocity | A13, Eqn. 10 |
| utildetr |  | representative half-cycle trough orbital velocity | A13, Eqn. 11 |
| ucrx |  | *x* component of combined wave-current crest velocity | A13, Eqn. 12 |
| ucry |  | *y* component of combined wave-current crest velocity | A13, Eqn. 12 |
| mag\_utc |  | magnitude of combined wave-current crest velocity |  |
| utrx |  | *x* component of combined wave-current trough velocity | A13, Eqn. 13 |
| utry |  | *y* component of combined wave-current trough velocity | A13, Eqn. 13 |
| mag\_utr |  | magnitude of combined wave-current trough velocity |  |
| s |  | ratio of sediment density to water density | This is defined in A13 as  but think this is wrong |
|  |  |  |  |
|  |  |  |  |
| Sc |  |  |  |
| Scx |  |  |  |
| Scy |  |  |  |
| St |  |  |  |
| Stx |  |  |  |
| Sty |  |  |  |
| tauwre |  | wave-averaged Reyolds stress from streaming | A13, Eqn. 22 |