## A Novel Dual-Loop-Controlled Latent Factor Analysis Model for Highly-Efficient Representation Learning to High-Dimensional and Incomplete Matrices: Supplementary File

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## I. INTRODUCTION

THIS supplementary file supports the paper titled "A Novel Dual-Loop-Controlled Latent Factor Analysis Model for Highly-Efficient Representation Learning to High-Dimensional and Incomplete Matrices." It includes detailed experimental configurations.

## II. EXPERIMENTAL CONFIGURATIONS

**Other settings.** To ensure fair and consistent empirical studies, the following experimental settings and configurations are adopted:

- 1) According to previous studies [1], [2], the LF dimension f for each LFA model is set to 20, and the values of the LF matrices are initialized with a normal distribution having a mean of 0 and a variance of 0.05;
- 2) Training of each model is terminated when the number of iterations reaches 1000, or when the difference of evaluation error (RMSE or MAE) between the previous iteration and the current iteration smaller than 10<sup>-4</sup>;
- 3) The learning rate and regularization coefficient for each competitor on the HDI matrices are optimized using grid search within the range of [10<sup>-5</sup>, 10<sup>-1</sup>]. The optimal hyperparameters for each model are shown in Table S1;
- 4) According to previous studies [1], [2], the fuzzy subsets in Table S1 can set as:  $\Delta_1$  to  $\Delta_5$  are (2<sup>-12</sup>, 2<sup>-11</sup>, 2<sup>-10</sup>, 2<sup>-9</sup>, 2<sup>-8</sup>),  $P_1$  to  $P_5$  are (1.25e-5, 2.5e-5, 1e-4, 2e-4),  $I_1$  to  $I_5$  are (2e-4, 1e-4, 5e-5, 2.5e-5, 1.25e-5),  $D_1$  to  $D_5$  are (1.25e-5, 2.5e-5, 5e-5, 1e-4, 2e-4), and  $\lambda_1$  to  $\lambda_5$  are  $\lambda \times (0.6, 0.8, 1, 1.2, 1.4)$ ;
- 5) All experiments are performed on a machine equipped with 1 TB RAM, two 2.1GHz Montage Jintide(R) C6230R CPUs, each containing 26 cores;
- 6) All models are implemented in C++.

TABLE S1
HYPERPARAMETER SETTINGS FOR COMPETITORS

Abbr	: M2		M3		M4	M5		M6		M7		M8	M9	M10
D1	$\eta$ =5e-2,	$\lambda$ =8e-1,	$\eta$ =2e-2,	$\lambda$ =7e-1,	$\eta$ =9e-1,	$\eta$ =8e-3,	$\lambda$ =7e-1,	$\eta$ =3e-2,	$\lambda$ =9e-1,	$\eta$ =2e-2,	$\lambda$ =7e-1,	$\eta$ =2e-2,	η=9e-4,	$\eta$ =9e-4,
	$\beta_1 = 9e-1, \beta_1$	$\beta_2 = 9.99e-1$ ,	$K_P = 5e-1$ ,	$K_I = 5e-4$ ,	$\lambda$ =8e-1,	$\beta$ =9e-1,	$\epsilon$ =1e-8	$\beta_1 = 9e-1$ ,	$\beta_2$ =9.99e-1,	$K_P = 5e-5$ ,	$K_D = 5e - 5$	$\lambda$ =7e-1	$\lambda$ =9e-1,	$\lambda$ =9e-1,
	$\epsilon$ =1e-8		$K_D = 5e-4$		$\epsilon$ =1e-8			$\epsilon$ =1e-8					$\beta$ =9e-1	$\beta$ =9e-1
D2	η=6e-3,	$\lambda$ =4e-2,	$\eta$ =3e-2,	$\lambda$ =4e-2,	$\eta$ =8e-2,	$\eta$ =1e-3,	$\lambda$ =4e-2,	$\eta$ =9e-3,	$\lambda$ =7e-2,	$\eta$ =3e-2,	$\lambda$ =4e-2,	$\eta$ =3e-2,	$\eta$ =3e-3,	$\eta$ =3e-3,
	$\beta_1 = 9e-1, \beta_1$	$B_2 = 9.99e-1$ ,	$K_P = 5e-1$ ,	$K_I = 5e-4$ ,	$\lambda$ =4e-2,	$\beta$ =9e-1,	$\epsilon$ =1e-8	$\beta_1$ =9e-1,	$\beta_2 = 9.99e-1$ ,	$K_P = 5e-5$ ,	$K_D = 5e - 3$	$\lambda$ =4e-2	$\lambda$ =4e-2,	$\lambda$ =4e-2,
	$\epsilon$ =1e-8		$K_D = 5e - 2$		$\epsilon$ =1e-8			$\epsilon$ =1e-8					$\beta$ =9e-1	$\beta$ =9e-1
D3	η=9e-3,	$\lambda$ =3e-1,	$\eta$ =8e-3,	$\lambda$ =3e-1,	$\eta$ =1e-1,	$\eta$ =4e-3,	$\lambda$ =4e-2,	$\eta$ =7e-3,	$\lambda$ =3e-1,	$\eta$ =8e-3,	$\lambda$ =3e-1,	$\eta$ =8e-3,	η=9e-4,	$\eta = 9e-4$ ,
	$\beta_1 = 9e-1, \beta_1$	$B_2 = 9.99e-1$ ,	$K_P = 5e-1$ ,	$K_I = 5e-4$ ,	$\lambda$ =3e-1,	$\beta$ =9e-1,	$\epsilon$ =1e-8	$\beta_1$ =9e-1,	$\beta_2 = 9.99e-1$ ,	$K_P = 5e-4$ ,	$K_D = 5e-4$	$\lambda$ =3e-1	$\lambda$ =3e-1,	$\lambda$ =3e-1,
	$\epsilon$ =1e-8		$K_D = 5e-4$		$\epsilon$ =1e-8			$\epsilon$ =1e-8					$\beta$ =9e-1	$\beta$ =9e-1
D4	$\eta$ =8e-3,	$\lambda$ =3e-1,	$\eta$ =2e-2,	$\lambda$ =9e-2,	$\eta$ =5e-2,	$\eta$ =5e-3,	$\lambda$ =2e-1,	$\eta$ =8e-3,	$\lambda$ =3e-1,	$\eta$ =2e-2,	$\lambda$ =9e-2,	$\eta$ =2e-2,	$\eta$ =2e-3,	$\eta$ =2e-3,
	$\beta_1 = 9e-1, \beta_1$	$\beta_2 = 9.99e-1$ ,	$K_P = 5e-1$ ,	$K_I = 5e-4$ ,	$\lambda$ =3e-1,	$\beta$ =9e-1,	$\epsilon$ =1e-8	$\beta_1 = 9e-1$ ,	$\beta_2$ =9.99e-1,	$K_P = 5e-4$ ,	$K_D = 5e-4$	$\lambda$ =9e-2	$\lambda$ =9e-2,	$\lambda$ =9e-2,
	$\epsilon$ =1e-8		$K_D = 5e - 1$		$\epsilon$ =1e-8			$\epsilon$ =1e-8					$\beta$ =9e-1	$\beta$ =9e-1
D5	η=4e-2,	$\lambda$ =9e-1,	$\eta$ =2e-2,	$\lambda$ =8e-1,	$\eta$ =4e-1,	$\eta$ =9e-3,	$\lambda$ =8e-1,	$\eta$ =2e-2,	λ=9e-1,	$\eta$ =2e-2,	$\lambda$ =8e-1,	$\eta$ =2e-2,	η=9e-4,	$\eta = 9e-4$ ,
	$\beta_1 = 9e-1, \beta_1$	$\beta_2 = 9.99e-1$ ,	$K_P = 5e-1$ ,	$K_I = 5e-4$ ,	$\lambda$ =6e-1,	$\beta$ =9e-1,	$\epsilon$ =1e-8	$\beta_1$ =9e-1,	$\beta_2$ =9.99e-1,	$K_P = 5e-4$ ,	$K_D = 5e-4$	$\lambda$ =8e-1	$\lambda$ =9e-1,	$\lambda$ =9e-1,
	$\epsilon$ =1e-8		$K_D = 5e-1$		$\epsilon$ =1e-8			$\epsilon$ =1e-8					$\beta$ =9e-1	$\beta$ =9e-1
D6	η=9e-3,	$\lambda$ =6e-2,	$\eta$ =8e-3,	$\lambda$ =5e-2,	$\eta$ =3e-2,	$\eta$ =9e-4,	$\lambda$ =6e-2,	$\eta$ =8e-3,	$\lambda$ =5e-2,	$\eta$ =8e-3,	$\lambda$ =5e-2,	$\eta$ =8e-3,	$\eta$ =5e-4,	$\eta$ =9e-4,
	$\beta_1 = 9e-1, \ \beta_1 = 9e-1$	$\beta_2 = 9.99e-1$ ,	$K_P = 5e-1$ ,	$K_I$ =5e-3,	$\lambda$ =1e-2,	$\beta$ =9e-1,	$\epsilon$ =1e-8	$\beta_1$ =9e-1,	$\beta_2$ =9.99e-1,	$K_P = 5e-5$ ,	$K_D = 5e-4$	$\lambda$ =5e-2	$\lambda$ =9e-2,	$\lambda$ =6e-2,
	$\epsilon$ =1e-8		$K_D = 5e - 5$		$\epsilon$ =1e-8			$\epsilon$ =1e-8					$\beta$ =9e-1	$\beta$ =9e-1

## REFERENCES

<sup>[1]</sup> J. Li, Y. Yuan, T. Ruan, J. Chen, and X. Luo, "A proportional-integral-derivative-incorporated stochastic gradient descent-based latent factor analysis model," *Neurocomputing*, vol. 427, pp. 29–39, 2021.

<sup>[2]</sup> W. Qin and X. Luo, "Asynchronous parallel fuzzy stochastic gradient descent for high-dimensional incomplete data representation," *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 2, pp. 445–459, 2024.