Tradeoffs and Comparison Complexity*

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Abstract

This paper develops a theory of how tradeoffs govern the difficulty of comparisons. In our model, options are easier to compare when they involve less pronounced tradeoffs – when they are 1) more similar feature-by-feature and 2) closer to dominance. These postulates yield tractable measures of comparison complexity in three domains: multiattribute, lottery, and intertemporal choice. Our model rationalizes multiple behavioral regularities, such as context effects, preference reversals, and apparent probability weighting and hyperbolic discounting. In choice data spanning all three domains, our model predicts errors, inconsistency, and cognitive uncertainty. Manipulating tradeoffs reverses classic behavioral regularities, in line with model predictions.

Keywords: Complexity, multi-attribute choice, choice under risk, intertemporal choice, experiments

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1 Introduction

We often face difficult tradeoffs when comparing choice options, and standard economic models abstract from this difficulty. In choosing utility services, households trade off fixed fees and variable usage fees. In shopping for mortgages, homebuyers weigh brokerage fees and closing costs against interest rates. In selecting health insurance plans, individuals balance premium and deductible costs as well as other financial and non-financial features of coverage. In each of these settings, choosing optimally requires the decision-maker to engage in a cognitively challenging process in the presence of tradeoffs: they must both assess the relative values of different option features, and aggregate value across features.

Tradeoffs are a feature of many economic decisions. Yet despite evidence from psychology that individuals struggle to navigate tradeoffs (e.g. Tversky, 1969; Tversky and Shafir, 1992; Shugan, 1980), economists lack a formal understanding of how tradeoffs make options hard to compare, and how this difficulty distorts choice. At the same time, a growing body of evidence suggests that well-known behavioral regularities from different choice domains may be partially driven by complexity. Taken together, this raises the question of whether a formal model of tradeoff difficulty can make sense of existing findings and generate new implications for economic behavior.

This paper makes three contributions. First, we develop a theory of comparison complexity in the domains of multiattribute, lottery, and intertemporal choice, which formalizes the principle that comparisons are difficult when they involve pronounced tradeoffs across option features. Second, we show how our theory can rationalize an array of documented biases and instabilities ranging across multiple choice domains, including context effects, preference reversals, and apparent probability weighting and hyperbolic discounting. Third, we use rich experimental data in all three domains to test the behavioral implications of our theory, including novel predictions on how these choice patterns can be eliminated or even reversed by manipulating the nature of tradeoffs.

Theory of comparison complexity. We model a decision-maker (DM) who is uncertain about the values of two options, x and y, and chooses based on a noisy signal on how these values compare. The precision of this signal, τ_{xy}, captures the ease of comparison between x and y, and governs the decision-maker's likelihood of choosing the higher-valued option. We develop a theory of how the ease of comparison τ_{xy} depends on the features of choice options in multiattribute, lottery, and intertemporal choice.

Our theory is motivated by the idea that decision-makers struggle to aggregate tradeoffs

(a) Multiattribute		(b) Lottery		(c) Intertemporal	
x :	\$11/month, \$3.45/GB	x :	\$27 w.p. 25%, \$3 w.p. 75%	x :	\$60 in 61 days
x' :	\$32/month, \$1.6/GB	x' :	\$9 for sure	x' :	\$100 in 3 years
y :	\$10.95/month, \$4.45/GB	y :	\$20 w.p. 20%, \$3.2 w.p. 80%	y :	\$40 in 60 days

Figure 1: Choice Domains. Comparisons between a) phone plans characterized by a monthly and data use fee, b) monetary lotteries, and c) payoff flows.

across option features, and that not all comparisons require the same degree of aggregation. Tradeoffs complicate comparisons because decision-makers may be uncertain over the relative importance of utility-relevant features, and even absent preference uncertainty, may find it difficult to aggregate advantages and disadvantages across features. To illustrate, consider the three choice environments in Figure 1. Across these domains, the comparison between x and y is simple — there is little need to make tradeoffs to see that x is better than y. On the other hand, the comparison between x' and y is less obvious, as the DM must now engage with non-trivial tradeoffs: 1a) involves a tradeoff between the phone plan monthly fee vs. usage fee, 1b) involves trading off a higher maximum payout against a lower payout probability, and 1c) involves a tradeoff between payout amounts and delays.

Our theory is built on two formal principles that capture this notion of tradeoff complexity: similarity and dominance. First, we posit that holding fixed their value difference, options are easier to compare if they are more *similar* — that is, that the ease of comparison is an increasing transformation H of the *value-dissimilarity* ratio:

$$\tau_{xy} = H\left(\frac{|v_x - v_y|}{d(x,y)}\right),$$

where the numerator contains the value difference between the two options and the denominator is a distance metric measuring their dissimilarity. Intuitively, similar options require less aggregation of tradeoffs to compare, as the DM can divert attention from features that are similar across options and so more easily assess differences. This intuition echoes work in psychology and economics (Tversky and Russo, 1969; Rubinstein, 1988), and we follow recent work in stochastic choice (He and Natenzon, 2024) in our formalism.

To pin down the specific dissimilarity measure, we appeal to our second principle: that options are maximally easy to compare in the presence of *dominance* – that is, when there are no tradeoffs. The relevant dominance notions in each of our domains — attribute-wise dominance in multiattribute choice, first-order stochastic dominance in lottery choice, and temporal dominance in intertemporal choice — give rise to the appropriate dissimilarity

Domain	Representation for τ_{xy}	Distance Metric
Multiattribute $U(x) = \sum_k \beta_k x_k$	$\tau_{xy} = H\left(\frac{\lvert U(x) - U(y)\rvert}{d_{L1}(x,y)}\right)$	$d_{L1}(x,y) = \sum_k \lvert\beta_k(x_k - y_k)\rvert$
Lottery $EU(x) = \sum_w u(w) f_x(w)$	$\tau_{xy} = H\left(\frac{\lvert EU(x) - EU(y)\rvert}{d_{CDF}(x,y)}\right)$	$d_{CDF}(x,y) = \int_0^1 \lvert u(F_x^{-1}(q)) - u(F_y^{-1}(q))\rvert \, dq$
Intertemporal $PV(x) = \sum_t \delta^t x_t$	$\tau_{xy} = H\left(\frac{\lvert PV(x) - PV(y)\rvert}{d_{CPF}(x,y)}\right)$	$d_{CPF}(x,y) = \ln\left(\frac{1}{\delta}\right) \int_0^\infty \delta^t \lvert M_x(t) - M_y(t)\rvert \, dt$

Table 1: Complexity Measures. $F_x^{-1}(q) = \inf\{w \in \mathbb{R} : q \leq F_x(w)\}$ denotes the quantile function of a lottery x. $M_x(t) = \sum_{t'<t} x_{t'}$ denotes the cumulative payoff function of a payoff flow x.

measures in each domain, summarized in Table 1; for each of these measures, options are maximally easy to compare when they have a dominance relationship.

The postulates of similarity and dominance are not only satisfied by our representations, but also are key in characterizing them; we show that axioms on binary choice behavior corresponding to the postulates of similarity and dominance, in tandem with other easily understood axioms, characterize our representations for τ_{xy} in each domain.

Behavioral implications. To study the behavioral implications of tradeoff-based comparison complexity beyond binary choice, we embed our theory of complexity in a multinomial choice model. In the model, the DM faces a menu of options and chooses based on noisy signals of the ordinal value comparison between each pair of options, where the precision of each signal is governed by the ease of comparison.

This model generates two key implications. First, in binary choice, comparison complexity leads to noisy, but unbiased, choice. Second, in larger menus, comparison complexity generates systematic distortions: a) context effects in multinomial choice, which occur when competing alternatives are hard to compare to each other but differ in their comparability to other options in the menu, and b) "pull-to-center" effects in the valuation of choice options – that is, when the option being valued is hard to compare to prices in a multiple price list, valuations are compressed towards the center of the price list.

To build intuition, consider the options $x \succ z^1 \succ z^2$, where $\{z^1, z^2\}$ are themselves perfectly comparable, but hard to compare to x. First, consider the binary choice x vs. z^1.

As the DM receives only an imprecise signal on how x and z^1 compare, she may err in her choice, but is more likely to choose the better option x. Next, consider the addition of z^2 to the menu: x vs. z^1 vs z^2. The DM is uncertain how x compares to either z^1 or z^2, but is certain that z^1 is superior to z^2: this additional information inflates the DM's assessment of z^1 relative to x, and so distorts choice toward z^1, producing a context effect. Note that this information also worsens the DM's assessment of z^2 relative to x, and so can pull her valuation of x between that of the unambiguously ranked options z^1 and z^2. This logic generalizes to valuation tasks, where the DM values x against $\{z^1, z^2, .., z^n\}$ in a multiple price list: the difficulty of comparing x to the unambiguously ranked items in the price list pulls her valuation of x toward the center of that ranking.

These forces rationalize a range of documented behavioral regularities. In multiattribute choice, the context effects predicted by our model straightforwardly generate familiar decoy/asymmetric dominance effects. Given our lottery and intertemporal complexity measures, which predict which risky and intertemporal prospects are hard to compare to prices, our model can rationalize documented regularities in valuation: preference reversals and apparent probability weighting and hyperbolic discounting, specifically because these patterns can be generated or exaggerated by a pull-to-center bias in valuation.

To illustrate, consider a canonical paradigm used to estimate risk preferences: eliciting the certainty equivalents of a simple lottery l that pays $\$\overline{w}$ with probability $p \in [0, 1]$. For p sufficiently close to 0 or 1, l has a near-dominance relationship with the certain payments in the price list, so valuations are close to accurate. For interior values of p, however, tradeoffs between payoff amounts and probabilities make l harder to compare to the price list, producing pull-to-center distortions: small probabilities are overvalued and large probabilities are undervalued, generating inverse-S probability weighting. The same logic can generate apparent hyperbolic discounting in the valuation of delayed payments.

Importantly, our model predicts that these patterns are not generic, but instead arise from the difficulty of comparing options to prices. As such, it predicts that one can *reverse* these distortions by manipulating tradeoffs in the valuation task. Suppose that instead of valuing lotteries against certain payments, our decision-maker is tasked with valuing certain payments against lotteries: assessing the payoff probability p that makes l indifferent to $\$w \in [0, \overline{w}]$ for sure. The pull-to-center distortions that produce classic probability weighting in certainty equivalents now generate the *opposite* pattern in this paradigm: low certain payments are overvalued and high payments are undervalued, causing the agent to appear relatively risk-averse (risk-seeking) over low (high) probabilities, consistent with experimental work comparing the two valuation methods (Sprenger, 2015; Feldman and

Ferraro, 2024). Our model also makes the novel prediction that hyperbolic discounting can be similarly reversed by inverting the role of the price list and the option being valued.

Since these pull-to-center distortions affect valuations but not direct choice, our model predicts apparent inconsistencies between these preference elicitation modes. Our model straightforwardly rationalizes documented preference reversals in lottery choice: cases where subjects favor lower-risk options when choosing between lotteries, yet exhibit the opposite preference in their lottery valuations. The model also predicts that these reversals can be *eliminated* by manipulating the relative ease of comparing each option to prices – specifically by changing the numeraire against which lotteries are valued, consistent with findings from experimental manipulations (Butler and Loomes, 2007; Slovic et al., 2002). Again, the model makes analogous novel predictions for intertemporal preference elicitations.

Experimental tests. We conduct two sets of experiments. First, we test the validity of our proposed complexity measures using three large-scale binary choice datasets in our domains of interest: choice between induced-values multiattribute goods, lotteries, and time-dated payoff streams. In line with the theory, our complexity measures are strongly predictive of choice inconsistency, choice errors, and subjective uncertainty over choices in all three datasets. When a choice problem involves a lower value-dissimilarity ratio, subjects are more likely to make inconsistent choices across repeat instances of that problem, and are more likely to make choice "errors", where we define an error as a) choosing the lower-value option in multiattribute choice and b) choosing the less-preferred option according to a best-fit preference model in lottery and intertemporal choice. These relationships are quantitatively meaningful: multiattribute choice inconsistency and error rates are more than four times as large for choice problems with the lowest versus highest value-dissimilarity ratio, with similarly pronounced relationships in our other domains. Furthermore, the value-dissimilarity ratio is strongly predictive of subjects' reported uncertainty over their choices.

In all three domains, our model explains a large share of variation in choice rates not captured by existing models, and substantially outperforms leading models in lottery and intertemporal choice by 10-22%. We also conduct *completeness* and *restrictiveness* exercises (Fudenberg et al., 2022, 2023). In terms of completeness, our model explains 70% of the predictable variation in multiattribute choice, and over 90% in lottery and intertemporal choice. In terms of restrictiveness, our model is less restrictive than the leading model in multiattribute choice, but *more* restrictive than leading models in lottery and intertemporal choice, suggesting that our gains in predictive power do not come at substantial costs to parsimony relative to benchmark models.

Second, we run a series of experiments to demonstrate that both preference reversals and classic distortions in lottery and intertemporal valuation are in part driven by comparison complexity — specifically by showing that these patterns can be eliminated or reversed by manipulating tradeoffs as our model predicts. In our preference reversal experiments, we 1) reproduce classic reversals in lottery choice and document similar reversals in intertemporal choice, and 2) show that these reversals can be eliminated by manipulating the relative ease of comparing options to prices. In line with the model, the documented inconsistency between valuations and direct choice disappears when subjects instead value lotteries (delayed payments) using probability equivalents (time equivalents).

Finally, we document that two classic valuation patterns, probability-weighting and hyperbolic discounting, reverse when we manipulate tradeoffs in the valuation task. In line with model predictions and existing findings, we find that using probability equivalents rather than certainty equivalents to estimate risk preferences results in apparent *underweighting* of small probabilities and *overweighting* of large probabilities. Similarly, relative to the discounting revealed by their present value equivalents, subjects' time equivalents exhibit *overvaluation* for delays close to the present and *undervaluation* for longer delays.

Contribution and relation to prior work. This paper adds to a theoretical literature on complexity by formalizing a distinct source of choice difficulty and establishing its empirical relevance. Existing theories tend to emphasize what makes individual options difficult to value (Puri, 2025; Hu, 2023; Salant and Spenkuch, 2022; Gabaix and Laibson, 2017), or study how menu size affects complexity (de Lara and Dean, 2024; Gerasimou, 2018). These accounts do not speak to the difficulty of assessing tradeoffs, which plays an important role in choice complexity. For instance, two lotteries can be complex in the sense of having many states, and yet easy to compare if one dominates the other; and even in binary menus, choice can be difficult in the presence of tradeoffs. By formalizing a notion of tradeoff difficulty, our theory provides a complementary account that can capture these features of complexity.

Our paper also contributes to a growing literature studying the cognitive foundations of economic decision-making, which aims to explain multiple empirical regularities from different choice domains using a set of common cognitive principles. We show how a single mechanism – the difficulty of making tradeoffs – can rationalize phenomena as distinct as probability weighting, hyperbolic discounting over money, preference reversals, context effects, and the large variation of noise in binary choice. Our paper thus relates to work on the cognitive foundations of prospect theory-like behavior (Khaw et al., 2021; Frydman and Jin, 2021; Enke and Graeber, 2023; Oprea, 2024) and hyperbolic discounting (Gabaix and

Laibson, 2017; Enke et al., 2025). In contrast to the majority of the literature, we develop a model and experimental tests that tie together regularities across different choice domains.

Our theory not only rationalizes these behavioral phenomena, but also makes progress in organizing a body of evidence documenting their instability and context-dependence. Whereas canonical distortions such as probability weighting and hyperbolic discounting are typically modeled as a fixed part of the agent's value function, experiments have shown that they are unstable across choice contexts: for instance, probability weighting is far less pronounced in direct choice between lotteries and certain payments relative to valuation tasks (Harbaugh et al., 2010; Bouchouicha et al., 2023). Relatedly, a long-standing literature has documented inconsistencies between preference elicitation methods (see Seidl, 2002, for a review). These findings pose basic issues for economists hoping to apply insights from behavioral research to a given setting, or to draw inferences from choice data. We propose a theory that organizes these findings, which we validate by testing model predictions on how manipulating tradeoffs can eliminate or even reverse these choice patterns.

Our model builds on a recent stochastic choice literature (Natenzon, 2019; He and Natenzon, 2024, 2023) studying the *moderate utility* class – models wherein binary choice probabilities are a function of the value difference between two options normalized by a distance metric – as well as research on more general forms of heteroskedastic noise in choice under risk (e.g. Hey, 1995; Buschena and Zilberman, 2008; Loomes, 2005). We make two contributions to these literatures. First, we propose specific moderate utility representations in three choice domains, and show how these representations are characterized by a common set of properties reflecting tradeoff complexity. Second, we formalize how our form of noise can rationalize systematic biases in richer choice settings beyond binary choice, and bring experimental evidence to bear on these predictions.

Section 2 develops the measure of comparison complexity. Section 3 develops the multinomial choice model and studies its implications. Section 4 describes the experimental tests of the model. Section 5 discusses relationships between our theory and existing models, and Section 6 concludes. Appendix C contains proofs of all results stated in the main text.

2 Theory of Comparison Complexity

Let X denote the set of options, and let v_x denote the value of each $x \in X$. We consider a decision-maker (DM) who is uncertain about the value of each option, and when faced with a binary choice $\{x, y\}$, chooses based on a noisy signal on how v_x and v_y compare. In particular, the DM has continuous, i.i.d. priors over each $(v_x)_{x\in X}$ given by the symmetric distribution Q, and receives a signal s_{xy} on the ordinal value comparison between x and y:

$$s_{xy} = \text{sgn}(v_x - v_y) + \frac{1}{\sqrt{\tau_{xy}}}\epsilon_{xy},$$
$$\epsilon_{xy} \sim N(0,1)$$

and chooses the option with the highest posterior expected value.[1] Here, the precision τ_{xy} governs the *ease of comparison* between x and y. Letting $\rho(x,y)$ denote the likelihood of choosing x over y conditional on the true values v_x and v_y, we have $\rho(x,y) = \Phi(\text{sgn}(v_x - v_y)\sqrt{\tau_{xy}})$, for Φ the standard normal CDF.[2] That is, the DM's likelihood of choosing the higher-valued option is increasing in the ease of comparison τ_{xy}. Let $\tau : \{(x,y) \in X^2 : x \neq y\} \to \mathbb{R}^+$ denote the associated mapping from each pair of options to their ease of comparison. In what follows, we propose a theory of how τ depends on the structure of choice options in the domains of multiattribute, lottery, and intertemporal choice.

2.1 Comparison Complexity: General Principles

Our theory of τ formalizes the intuition that the difficulty of a comparison is governed by the degree to which it requires the DM to aggregate tradeoffs. The theory is grounded in two principles that capture this intuition: similarity and dominance.

First, we posit that holding fixed the value difference, options are easier to compare when they are more *similar*. Echoing work in psychology and economics (Tversky and Russo, 1969; Rubinstein, 1988; He and Natenzon, 2024), this property reflects the intuition that if options are more similar, the DM can divert attention from the features that are similar across the options and so more easily assess the differences between them, thereby reducing the need to aggregate tradeoffs. Formally, we posit that the ease of comparison τ_{xy} is an increasing transformation H of the *value-dissimilarity* ratio:

$$\tau_{xy} = H\left(\frac{|v_x - v_y|}{d(x,y)}\right),$$

where the numerator contains the value difference between the two options and the denominator is a distance metric measuring their dissimilarity.

Second, we posit that options are maximally easy to compare when there are no trade-

[1] Normality of ϵ_{xy} can be relaxed. Appendix C.2 shows that our results hold for any ϵ_{xy} with a continuous density that is symmetric around 0 and satisfies a monotone likelihood ratio property.

[2] In particular, $\rho(x,y) \equiv \mathbb{P}(\mathbb{E}[v_x|s_{xy}] > \mathbb{E}[v_y|s_{xy}] \,|\, v)$, where the DM randomizes if $\mathbb{E}[v_x|s_{xy}] = \mathbb{E}[v_y|s_{xy}]$.

offs – that is, when they have a *dominance* relationship. As formalized below, this principle gives rise to specific distance metrics given the domain-relevant dominance notion.

2.2 Multiattribute Choice

Consider the domain of multiattribute choice, where each option $x \in X \equiv X_1 \times ... \times X_n$ is defined on n real-valued attributes, i.e. $X_i = \mathbb{R}$.[3] Utility is linear in attributes, where the value of each option x is given by $v_x = U(x) = \sum_k \beta_k x_k$ for attribute weights $\beta \in \mathbb{R}^n$. We propose that the ease of comparison is governed by the following representation:

Definition 1. *Say that τ has an L_1-complexity representation, denoted τ^{L1}, if there exists $\beta \in \mathbb{R}^n$, $\beta_k \neq 0$ for all k, such that*

$$\tau_{xy} = H\left(\frac{|U(x) - U(y)|}{d_{L1}(x, y)}\right)$$

for H continuous, strictly increasing with $H(0) = 0$, where $d_{L1}(x, y) = \sum_k |\beta_k(x_k - y_k)|$ is the L_1 distance between x and y in value-transformed attribute space.

In words, under the L_1-complexity representation, the ease of comparison between two options is governed by their *value-dissimilarity ratio*: their *aggregated* value difference, normalized by a distance metric equal to the summed *feature-by-feature* value differences.

Note that this complexity representation satisfies the properties of *similarity* and *dominance*: holding fixed the value difference between two options, the ease of comparison is increasing in the similarity between x and y as measured by a distance metric, where this metric is chosen so that τ_{xy} achieves its maximal value in the presence of a dominance relationship. Specifically, if there is an attribute-wise dominance relationship between x and y: i.e. $\beta_k x_k \geq \beta_k y_k$ for all k, the ease of comparison τ_{xy} takes on its maximal value of $H(1)$.

L_1-complexity also satisfies a *simplification* property, wherein reducing the number of attributes along which there is a value difference increases the ease of comparison. To illustrate, suppose $n = 3$ and $\beta = (1, 1, 1)$, and consider the following comparisons:

$$\begin{array}{llcll} & (x, y) & & & (x', y) \\ x & = (10, 7, 9) & & x' & = (3, 14, 9) \\ y & = (3, 15, 5) & & y & = (3, 15, 5) \end{array}$$

[3] In Appendix B.1, we provide guidance on how the analyst should specify the attributes in a given setting: specifically, how attributes can be identified from choices.

Note that (x', y) is formed by eliminating the value difference along the first attribute and redistributing that value to the second attribute. Our complexity representation predicts that the DM finds (x', y) easier to compare than (x, y), i.e. $\tau_{x'y} > \tau_{xy}$. More generally, our model predicts that eliminating a value difference along some attribute i and redistributing it to another attribute j makes options easier to compare.[4] This property again reflects tradeoff complexity: if an individual finds it difficult to aggregate tradeoffs across features, we might expect that a simplification operation of the kind above, where some of that aggregation is done for the individual, makes the comparison easier.

2.2.1 Axiomatic Foundations

We now show that the above properties are not only satisfied by our complexity representation, but are also key properties in its characterization: specifically, L_1-complexity is characterized by axioms on binary choice behavior corresponding to the properties of similarity, dominance, and simplification, along with two other easily understood axioms.

Let $\mathcal{D} = \{(x, y) \in \mathbb{R}^n \times \mathbb{R}^n : x \neq y\}$ denote the set of all pairs of distinct options. Consider a *binary choice rule* $\rho : \mathcal{D} \to [0, 1]$ satisfying $\rho(x, y) = 1 - \rho(y, x)$ for all x, y; here, $\rho(x, y)$ denotes the likelihood of choosing x over y. In our binary choice framework, τ has an L_1-complexity representation if and only if binary choice probabilities take the form below, wherein $\rho(x, y)$ is an increasing function of the signed value difference between the two options, normalized by their L_1 distance.

Definition 2. *Say that a binary choice rule ρ has an L_1-complexity representation if there exists $\beta \in \mathbb{R}^n$, $\beta_k \neq 0$ for all k, such that*

$$\rho(x, y) = G\left(\frac{U(x) - U(y)}{d_{L1}(x, y)}\right)$$

for some continuous, strictly increasing G satisfying $G(r) = 1 - G(-r)$.[5]

Let $x_{\{k\}}y$ denote the option obtained by replacing the kth attribute of y with x_k[6]. Say that x *dominates* y, written $x >_D y$, if $\rho(x_{\{k\}}y, y) \geq 1/2$ for all k with a strict inequality for at least one k – that is, if x is revealed better than y along each attribute when considering attributes in isolation. Say that attribute k is *null* if $\rho(x_{\{k\}}z, y_{\{k\}}z) = 1/2$ for all $x, y, z \in X$. Consider the following conditions on ρ:

[4]That is, given any x, y, for x' satisfying $x'_i = y_i$, $x'_k = x_k$ for all $k \neq i, j$, with $v_{x'} = v_x$, we have $\tau_{x'y} \geq \tau_{xy}$.

[5]There is a one-to-one correspondence between G and H, which maps the value-dissimilarity ratio into signal precisions τ_{xy}. In particular, for $r \in [0, 1]$, $H(r) = (\Phi^{-1}(G(r))^2$, where Φ is the standard normal CDF.

[6]That is $(x_{\{k\}}y)_k = x_k$ and $(x_{\{k\}}y)_j = y_j$ for all $j \neq k$.

M1. **Continuity:** $\rho(x, y)$ is continuous on its domain.

M2. **Linearity:** $\rho(x, y) = \rho(\alpha x + (1-\alpha)z, \alpha y + (1-\alpha)z)$.

M3. **Moderate Transitivity:** If $\rho(x, y) \geq 1/2$ and $\rho(y,z) \geq 1/2$, then either $\rho(x,z) > \min\{\rho(x, y), \rho(y,z)\}$ or $\rho(x,z) = \rho(x, y) = \rho(y,z)$.

M4. **Dominance**: If $x >_D y$, then $\rho(x, y) \geq \rho(w,z)$ for any $w,z \in X$, where the inequality is strict if $w \not>_D z$.

M5. **Simplification**: If $\rho(x, y) \geq 1/2$: for any $x' \in X$ satisfying

(1) $x'_i = y_i$ for some i,

(2) $x'_j \neq x_j$ for at most one $j \neq i$,

such that $\rho(x', x) = 1/2$, we have $\rho(x', y) \geq \rho(x, y)$.

Continuity and Linearity are standard axioms, and the latter reflects the fact that both utility and the L_1 distance are linear in attributes. Moderate Transitivity is a stochastic transitivity condition that allows choice probabilities to depend on a notion of the similarity between the options being compared, in addition to just their value difference.[7]

Dominance and Simplification are the exact counterparts of the properties of τ^{L1} discussed earlier, stated in terms of choice probabilities. Dominance says that if x is revealed better than y on every attribute, then the likelihood of correctly choosing x takes on its maximal value. Simplification says that eliminating the value difference between x and y along some attribute i and redistributing that value to another attribute j increases the DM's likelihood of correctly choosing x.[8]

When there are three or more attributes, M1–M5 characterize the behavioral implications of our representation for binary choice data. Moreover, the parameters (G, β) of our representation can be identified from binary choice data.

Theorem 1. *Suppose that all attributes are non-null and $n > 2$. $\rho(x, y)$ has an L_1-complexity representation (G, β) if and only if it satisfies M1–M5. Moreover, if $\rho(x, y)$ also has an L_1-complexity representation (G', β'), then $G' = G$ and $\beta' = C\beta$ for some $C > 0$.*

[7]He and Natenzon (2024) show that in a finite domain, ρ satisfies Moderate Transitivity if and only if $\rho(x, y)$ is increasing in the value difference between x and y, normalized by *some* distance metric $d(x, y)$. While this result does not apply to our domain as it is not finite, our other axioms can be interpreted as adding structure to this distance metric.

[8]Hammond et al. (1998) argue on normative grounds that such an operation can be used to rationally simplify tradeoffs between multi-dimensional options.

In Appendix B.1, we extend Theorem 1 to the two attribute case, and also characterize a generalization of the L_1-complexity representation that allows for non-linear preferences.

2.3 Risky and Intertemporal Choice

2.3.1 Lotteries

Consider the lottery domain, where each option x is a finite-support lottery over $\mathbb{R}$; that is each x is described by the mass function $f_x : \mathbb{R} \to [0,1]$ where $f_x(w) > 0$ for finitely many w. Let F_x and F_x^{-1} denote the CDF and quantile function of x. Tastes are given by expected utility: $v_x = EU(x) = \sum_w u(w) f_x(w)$ for u strictly increasing.

Definition 3. *τ has a CDF-complexity representation, denoted τ^{CDF}, if for u strictly increasing,*

$$\tau_{xy} = H\left(\frac{|EU(x) - EU(y)|}{d_{CDF}(x,y)}\right)$$

for H continuous, strictly increasing with $H(0) = 0$, where the CDF distance d_{CDF} is given by

$$d_{CDF}(x,y) = \int_0^1 |u(F_x^{-1}(q)) - u(F_y^{-1}(q))| \, dq$$

As with the L_1-complexity representation, the ease of comparison between two options under the CDF-complexity representation is governed by their value-dissimilarity ratio — that is, the value difference between the two options normalized by a measure of their dissimilarity. The specific dissimilarity measure in our representation, $d_{CDF}(x,y)$, is a metric equal to the area between the utility-valued CDFs of x and y, and so captures how similarly the payoffs in x and y are distributed.[9]

This measure provides a formal foundation for empirical work documenting a tight connection between the CDF distance and choice rates (Enke and Shubatt, 2023; Erev et al., 2008; Buschena and Zilberman, 2008). In particular, this work shows that the performance of lottery choice models markedly improves when choice noise is allowed to vary with a special case of d_{CDF} in which u is linear. Fishburn (1978) proposes and axiomatizes a closely related model of binary choice over lotteries in which choice probabilities are increasing in the CDF ratio.[10] As discussed below and in Appendix B.1, we provide an alternative axiomatic foundation for this choice model using easily understood conditions analogous to our multiattribute axioms.

[9] d_{CDF} is a special case of the Wasserstein metric, a distance notion defined over probability distributions.
[10] We thank Paulo Natenzon for bringing our attention to this paper.

2.3.2 Intertemporal Payoff Flows

Consider the intertemporal domain, where each option x is a finite payoff stream described by the *payoff function* $m_x : [0,\infty) \to \mathbb{R}$, where $m_x(t) \neq 0$ for finitely many t. Here, $m_x(t)$ describes how much x pays off at time t. Let $M_x(t) = \sum_{t' \leq t} m_x(t')$ denote the *cumulative payoff function* of x, which describes how much money x pays in total up to time t. Utility is given by exponential discounting, with $v_x = PV(x) = \sum_t \delta^t m_x(t)$, for $\delta < 1$.

Definition 4. *τ has a CPF-complexity representation, denoted τ^{CPF}, if for $\delta < 1$ we have*

$$\tau_{xy} = H\left(\frac{|PV(x) - PV(y)|}{d_{CPF}(x,y)}\right)$$

for H continuous, strictly increasing with $H(0) = 0$, where the CPF distance d_{CPF} is given by

$$d_{CPF}(x,y) = \ln(1/\delta)\int_0^\infty |M_x(t) - M_y(t)| \cdot \delta^t \, dt.$$

As with our previous complexity measures, the ease of comparison between two options under the CPF-complexity representation is governed by their value-dissimilarity ratio. Here, the dissimilarity measure $d_{CPF}(x,y)$ is a metric that is proportional to the present value of the difference between the cumulative payoff functions of x and y, and captures how similarly x and y distribute their payoffs across time.

2.3.3 Shared Properties and Axiomatic Foundations

As with our multiattribute complexity measure, τ^{CDF} and τ^{CPF} satisfy our core properties of similarity and dominance. Holding fixed the value difference, τ^{CDF} and τ^{CPF} are increasing in the similarity between options as measured by a distance metric — where the metric is chosen so that τ takes on its maximal value when the options have a dominance relationship. In particular, say that a lottery x *first-order stochastically dominates* y when $F_x(w) \leq F_y(w)$ for all w, and say that a payoff flow x *temporally dominates* y if $M_x(t) \geq M_y(t)$ for all t — that is, if x will have paid out more in total than y at any point in time. τ^{CDF}_{xy} takes on its maximal value of $H(1)$ when there is a first-order stochastic dominance relationship between lotteries x and y, and τ^{CPF}_{xy} takes on its maximal value of $H(1)$ when there is a temporal dominance relationship between payoff flows x and y.

τ^{CDF} and τ^{CPF} also satisfy analogs of the simplification property for τ^{L1}. As formally stated in Appendix B.1, concentrating value differences from different regions in the distribution of two lotteries into the same region increases the ease of comparison under τ^{CDF}.

Likewise, concentrating value differences from different time periods of two payoff flows into the same period increases the ease of comparison under τ^{CPF}.

The binary choice behavior implied by τ^{CDF} and τ^{CPF} can be characterized using axioms analogous to M1—M5. In Appendix B.1, we show how the binary choice rules corresponding to τ^{CDF} and τ^{CPF} are characterized by five axioms: direct translations of Continuity, Linearity, Moderate Transitivity, and Dominance, as well as an analog of Simplification.

Our complexity measures for risk and time not only share the same properties of L_1 complexity, but can also be seen as an extensions of the complexity notion. In Appendix B.2, we show that the CDF (CPF) complexity between two lotteries (payoff flows) is equivalent to the L_1 complexity computed over a common attribute representation of those options — specifically, the attribute representation that maximizes their ease of comparison. This suggests the following two-stage cognitive interpretation of our CDF and CPF complexity measures: in a "representation stage", the DM first represents the options using a common attribute structure to make them easier to compare, and then compares the options along these attributes in an "evaluation stage".

2.4 Parameterizing the Model

In each domain, choice probabilities under our model take the form $\rho(x,y) = G\left(\frac{U(x)-U(y)}{d(x,y)}\right)$, where the signed value-dissimilarity ratio $\left(\frac{U(x)-U(y)}{d(x,y)}\right)$ is specified according to Definitions 1, 3, and 4, and G is an increasing transformation satisfying $G(r) = 1 - G(-r)$. To obtain quantitative predictions, the analyst needs to specify the preference parameters that enter the value-dissimilarity ratio — the attribute weights β in multiattribute choice, the Bernoulli utility u in lottery choice, and the discount factor δ in intertemporal choice — as well as the transformation G. Each of these objects can be identified from binary choice data, as stated in Theorem 1 for multiattribute choice, and in Appendix B.1 for our other two domains.

Our preferred specification of G is given by the two-parameter functional form

$$G(r) = \begin{cases} (1-\kappa) - (0.5-\kappa)(1-r)^{\gamma} & r \geq 0 \\ \kappa + (0.5-\kappa)(1+r)^{\gamma} & r < 0 \end{cases} \tag{1}$$

Here κ is a tremble parameter that governs the error rates at dominance, and γ governs the curvature in the relationship between choice rates and the value-dissimilarity ratio. [11]

[11] In Appendix E we also consider a three-parameter functional form that nests (1), wherein the additional parameter governs the sensitivity of choice rates to r around indifference.

3 Multinomial Choice

So far, we have focused on comparison complexity in binary choice. We now extend our model to multinomial choice and show how comparison complexity can rationalize a range of documented behavioral regularities, including preference reversals and apparent probability weighting and hyperbolic discounting in valuation tasks. We develop novel predictions on how manipulating tradeoffs can cause these patterns to disappear or even reverse.

3.1 Multinomial Choice Extension

Consider the same setting as in our binary choice framework. There is a set of options X, and the DM has continuous, i.i.d. priors over v_z for all $z \in X$, distributed according to a symmetric distribution Q. Let $\mathcal{A}$ denote the collection of non-empty finite subsets of X, and let $\mathcal{C} = \mathcal{A} \cup \{\varnothing\}$. The DM faces a *choice problem* $(A, C) \in \mathcal{A} \times \mathcal{C}$, comprised of a *menu* of options A and a *choice context* C – a set of options the DM observes but cannot choose, i.e. *phantom options*. The DM chooses from A based on signals on how each pair of options in $A \cup C$ compares.

In particular, for each pair of distinct options $x, y \in A \cup C$, the DM observes the signal

$$s_{xy} = \text{sgn}(v_x - v_y) + \frac{1}{\sqrt{\tau_{xy}}}\epsilon_{xy},$$
$$\epsilon_{xy} \sim N(0,1).$$

Let s denote the collection of these signals. The DM chooses the option $x \in A$ with the maximal posterior expected value $\mathbb{E}[v_x|s]$. We are interested in the resulting choice probabilities, which are given by[12]

$$\rho(x, A|C) = \mathbb{P}(\{s : \mathbb{E}[v_x|s] > \mathbb{E}[v_y|s] \; \forall \, y \in A/\{x\}\} \,|\, v)$$

Note that on the set of binary choice problems, i.e. (A, C) such that $|A| = 2$ and $C = \varnothing$, this model is exactly the binary choice model studied in Section 2. With some abuse of notation, we let $\rho(x, y) = \rho(x, \{x, y\}|\varnothing)$ denote binary choice probabilities, and let $\rho(x, y|C) = \rho(x, \{x, y\}|C)$ denote binary choice probabilities given a choice context C.

To apply the choice model, the analyst needs to specify the ranking of the choice options according to v_z and the precision parameters τ_{xy}. While these parameters can be identified

[12] This formulation for $\rho(x, A|C)$ holds when ties in posterior expectations occur with probability 0. In the case of ties, we assume a symmetric tiebreaking rule. See Appendix B.3 for details.

using binary choice data (see Appendix B.4), we will discipline the model using our theory of comparison complexity, which pins down v_x and τ_{xy} in our domains of interest.

3.2 Model Properties

This model has two key implications. First, it predicts that comparison complexity leads to noisy but unbiased choice in binary menus — the DM may err, but is more likely to pick the higher-value option. Second, it predicts systematic biases in choice from larger menus: when facing hard-to-compare alternatives, the DM relies on information from comparisons to other options, which can distort choice. This generates a) context effects when options are hard to compare to each other but differ in their ease of comparison to other options in the choice context, and b) systematic pull-to-center distortions in valuations as a function of the difficulty of comparing the option being valued to the numeraire.

3.2.1 Context Effects

To illustrate how the model generates systematic context effects, consider an example in which $X = \{x, y, z\}$, with $v_x > v_y > v_z$ and $\tau_{xy} = \tau_{xz} = 0$, $\tau_{yz} = \infty$. That is, the DM has no idea how x compares to y and z, but knows y is better than z. Here, the model predicts that $\rho(y, x|\{z\}) = 1$: the presence of z in the choice context provides additional information that rules out posterior beliefs over (v_x, v_y, v_z) in which $v_y < v_z$, thus distorting the DM's choice in favor of the inferior option y. This is generalized in the following proposition, which says that if x and y are sufficiently hard to compare, the presence of an inferior option z that is easier to compare to y distorts choice in favor of that option.[13]

Proposition 1. *Let $v_x, v_y > v_z$. If $\tau_{yz} > \tau_{xz}$, then there exists $\epsilon > 0$ such that if $\tau_{xy} < \epsilon$, $\rho(y, x|\{z\}) > 1/2$.*

When combined with our theory of complexity in multiattribute choice, this result rationalizes documented decoy and asymmetric dominance effects (e.g. Huber et al., 1982).

Corollary 1.1. *Consider options from $X = \mathbb{R}^n$, with $v_x = \sum \beta_k x_k$, and let τ have an L_1-complexity representation. Let $v_x, v_y > v_z$.*

(i) If $v_x = v_y$, then $d_{L1}(x, z) > d_{L1}(y, z)$ implies $\rho(y, x|\{z\}) > 1/2$.

[13] In Appendix C, we also consider the analogous result that the addition of a superior option z to the choice context can bias choice in favor of x if $\tau_{yz} > \tau_{xz}$.

(ii) For any value difference $\Delta = |v_x - v_y|$, there exists $\underline{d} \in \mathbb{R}^+$ such that if $d_{L1}(x,y) > \underline{d}$, there exists $z \in X$ with $d_{L1}(x,z) > d_{L1}(y,z)$ such that $\rho(y,x|\{z\}) > 1/2$.

Part (i) says that if x and y are indifferent, then introducing an inferior phantom option z that is more similar to y than x distorts choice in favor of y. Part (ii) says that if x and y are sufficiently dissimilar relative to their value difference, there exists a decoy that distorts choice in favor of y. In Appendix B.5, we discuss how our model can rationalize a range of documented decoy effects that other explanations cannot capture.

This model belongs to class of menu-dependent learning models in which the DM chooses based on a signal that depends on the menu of options (e.g. Safonov, 2022; Natenzon, 2019); as in our model, these menu-dependent signals in these models can generate context effects. In Section 5, we discuss how this model relates to others in this class.

3.2.2 Compression Effects in Valuation

We model the valuation of a choice option as a sequence of choices structured as a multiple price list, a workhorse experimental procedure for eliciting valuations. There is an option $x \in X$ to be valued against a *price list* $Z = \{z^1, z^2, .., z^n\} \subseteq X$: a set of options for which the ranking $v_{z^1} > v_{z^2} > ... > v_{z^n}$ is unambiguous, i.e. $\tau_{z^i z^j} = \infty$ for all $z^i, z^j \in Z$. For each price $z^k \in Z$, the DM chooses between z^k and x, revealing her valuation of x in terms of Z.

To capture this setting, we extend our multinomial choice framework. The DM now faces a finite *menu sequence* $A^1, A^2, ..., A^n \in \mathcal{A}$ in a choice context C, generates a set of signals s for each pairwise comparison in $A^1 \cup A^2 \cup ..A^n \cup C$, and chooses the option from each menu with the highest posterior expected value, yielding joint choice frequencies[14]

$$\rho((x^1, ...x^n), (A^1, ..., A^n)|C) = \mathbb{P}\left(\bigcap_{i=1}^{n} \{s : \mathbb{E}[v_{x^i}|s] > \mathbb{E}[v_y|s] \;\forall\, y \in A^i/\{x^i\}\} \,\Big|\, v\right).$$

Here, $\rho((x^1, ...x^n), (A^1, ..., A^n)|C)$ records the frequency of choosing $x^i \in A^i$ for $i = 1, ..., n$. Given an option x and a price list Z, a *valuation task* (x, Z) is simply the binary menu sequence $A^1, ..., A^n = \{x, z^1\}, ..., \{x, z^n\}$.

Since the DM learns the ranking of prices in Z, this procedure yields a single switching point: for any signal realization, there is an index $R \in \{1, ..., n, n+1\}$ for which the DM chooses the option $x \in A^k$ for all $k \geq R$, and the price $z^k \in A^k$ for all $k < R$. R reveals where the DM believes the object x falls within the ranking of prices, i.e. the sub-

[14]As before, this formulation for choice probabilities holds when ties in posterior expectations occur with probability 0. In the case of ties, we assume a symmetric tiebreaking rule; See Appendix B.3 for details.

ject's valuation in terms of Z. We will be interested in the distribution over R induced by $\rho((x^1, ...x^n), (A^1, ..., A^n)|Z)$, which we denote by $R(x, Z)$.[15] With a slight abuse of notation, we denote $v_k \equiv v_{z^k}$ and $\tau_{xk} \equiv \tau_{xz^k}$.

Our model predicts that when x is hard to compare to prices, valuations will be systematically compressed towards the center of the price list. Let $R^*(x, Z) \in \{1, ..., n, n+1\}$ denote the true ranking of x relative to Z.[16]

Proposition 2. *Given a valuation task (x, Z), where $\tau_{xk} = \tau$ and $v_x \neq v_k$ for all $k = 1, ..., n$, we have the following:*

(i) If $\tau = 0$, $\mathbb{E}[R(x, Z)] = (n+2)/2$.

(ii) As $\tau \to \infty$, $R(x, Z)$ converges in distribution to $\delta_{R^*(x,Z)}$.[17]

Proposition 2 says i) when x is incomparable to prices, valuations exhibit "pull-to-center" effects, and ii) as x becomes more comparable to prices, valuations converge to the truth. Intuitively, if $\tau = 0$, the DM receives no information on where x falls within the ranking of prices – her posterior puts equal probability on each possible ranking, so she values x in the middle of the price list. As τ increases, valuation of x becomes increasingly accurate. Crucially, this "pull-to-center" force depends on how difficult x is to compare to prices. When combined with our theory of comparison complexity, this force rationalizes documented preference reversals and biases in valuation.

3.3 Preference Reversals

Consider the classic preference reversal phenomenon in risky choice. Lottery x pays a high amount with a low probability, while y pays a modest sum with a high probability, e.g.

$$x: \quad \$23.50 \text{ with } 19\%$$
$$y: \quad \$4.75 \text{ with } 94\%$$

Most subjects choose y over x in direct choice, yet state a higher certainty equivalent for x. A number of explanations for these reversals have been proposed, such as intransitive

[15]Given a signal s, the DM's posterior switching point R is computed by calculating $\mathbb{E}[v_x|s]$ and $\mathbb{E}[v_j|s]$ for all $j \in \{1, 2, ..., n\}$, and finding the unique index R such that $\mathbb{E}[v_x|s] < \mathbb{E}[v_{R-1}|s]$ and $\mathbb{E}[v_x|s] > \mathbb{E}[v_R|s]$ (in the case of ties, we assume the DM randomizes as described in Appendix B.3).

[16]That is, $v_x > v_k$ if $k \geq R^*(x, Z)$ and $v_x < v_k$ otherwise. For ease of exposition, we assume x is not indifferent to any price in Z.

[17]$\delta_{R^*(x,Z)}$ denotes the probability distribution which places a unit mass on $R^*(x, Z)$.

preferences or independence violations (see Seidl (2002) for a review). Our model can rationalize these reversals between choice and valuation as expressions of the same underlying preferences, taking into account errors produced by the differential ease of comparing options to money. Under our complexity notion, y is easier to compare to money than x, which results in differential pull-to-center distortions. As this force distorts valuations but not direct choice, reversals can occur. As we show below, this logic not only generates reversals in lottery choice: it predicts similar reversals in intertemporal choice, and that these reversals can be eliminated by manipulating the ease of comparing options to prices.

3.3.1 Lottery Reversals

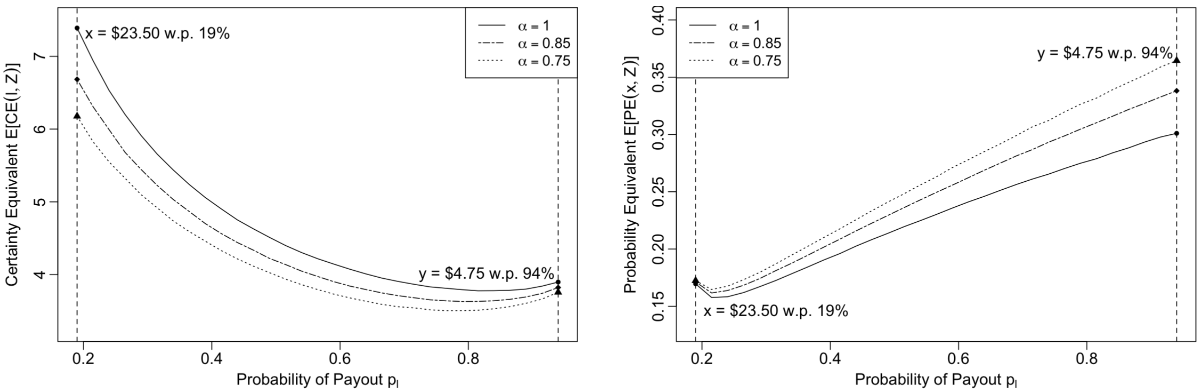
Consider the lottery domain, where $v_x = \sum_w u(w) f_x(w)$ for u strictly increasing, and where τ has a CDF-complexity representation $\tau_{xy}^{CDF} = H\left(\frac{|EU(x)-EU(y)|}{d_{CDF}(x,y)}\right)$ for which $H(1) = \infty$; that is, the DM perfectly learns the ranking between lotteries with a dominance relationship. Call $l = (w_l, p_l)$ a simple lottery if it pays out w_l with probability p_l, and nothing otherwise.

Example 1. (Classic preference reversals). Suppose the DM is weakly risk-averse (u is concave), and again consider the simple lotteries $x = (\$23.5, 0.19)$, $y = (\$4.75, 0.94)$. First, consider binary choice between the (equal expected value) lotteries. Since any risk-averse DM weakly prefers y to x, our model predicts that the DM is more likely to choose y.

Now consider a DM tasked with assessing certainty equivalents of the lotteries. Formally, the DM faces a valuation task (l, Z) in which the simple lottery $l = (w_l, p_l)$ is valued against a price list $Z = \{z^1, ..., z^n\}$, where each $z^k = (w_k, 1)$ is a sure payment. We work with "adapted" price lists, where Z is *adapted* to l if Z contains equal-sized steps and contains the minimal and maximal support points of l (i.e. $w_k - w_{k+1}$ is constant in k, and $w_n = 0$, $w_1 = w_l$). Recall that each valuation task (l, Z) produces a distribution of switching points $R(l, Z)$, and denote $CE(l, Z) = 1/2\left[w_{R(l,Z)-1} + w_{R(l,Z)}\right]$ the resulting distribution over certainty equivalents.

Figure 2a plots the expected certainty equivalents $\mathbb{E}[CE(l, Z)]$ simulated from our model for simple lotteries l with the same expected value as x and y. Notice that the high-risk lottery x is valued higher than the low-risk lottery y on average, despite the fact that y is weakly preferred to x for our risk-averse DM. Intuitively, x is dissimilar to and therefore difficult to compare to money, so its valuation is inflated toward the midpoint of the undominated range of prices $[0, w_x]$. On the other hand, y is easier to compare to money, so its valuation is less distorted, and if anything is pulled *downward* toward the midpoint of undominated prices $[0, w_y]$. We have a reversal: $\rho(y, x) \geq 1/2$ and yet $\mathbb{E}[CE(x, Z)] > \mathbb{E}[CE(y, Z)]$.

The rest of the figure traces our model's predictions for preference reversals in general:



(a) Certainty Equivalents $\mathbb{E}[CE(l,Z)]$

(b) Probability Equivalents $\mathbb{E}[PE(l,Z)]$

Figure 2: Simulated average certainty equivalents and probability equivalents for simple lotteries $l = (w_l, p_l)$ with expected value equal to that of $x = (23.50, 0.19)$ as a function of p_l. Z is adapted to l and we set $|Z| = 15$. τ has a CDF-complexity representation parameterized by $u(w) = w^\alpha$ and $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0,1]$.

for a high-risk lottery x' and a low-risk lottery y', we have $\mathbb{E}[CE(x',Z)] > \mathbb{E}[CE(y',Z)]$ — even though y' is preferred to x', and so $\rho(y',x') \geq 1/2$.

In our model, reversals result from the differential ease of comparing lotteries to money. This echoes past work suggesting that the difficulty of valuing lotteries against an incongruent response scale may generate preference reversals (Tversky et al., 1990; Slovic et al., 2002; Butler and Loomes, 2007). Unlike previous work, however, we provide a formal account of both what makes lotteries hard to value, and how this difficulty distorts valuation. As such, our model generates novel predictions: in particular, that one can eliminate these reversals by manipulating the ease of comparing each lottery to prices — specifically, by changing the units against which the lotteries are valued.

Example 2. (Reversals with probability equivalents). Consider the lotteries $x = (\$23.50, 19\%)$ and $y = (\$4.75, 94\%)$ from Example 1. Instead of valuing x and y against money, suppose the DM assesses their *probability-equivalents*: the probability p that makes the lottery $z = (\$24, p)$ indifferent to each. Whereas y was easier to compare to money, x is easier to compare to this new numeraire, which is more similar to x than to y. Our model predicts that this change in numeraire *reverses* the distortions in the valuation of x and y.[18]

[18]Slovic et al. (2002) experimentally study a related procedure of "probability matching", in which subjects indicate the probability p that makes $x' = (\$23.5, p)$ indifferent to y, from which a preference between x and y can be inferred. This can be seen as a special case of our manipulation in which the price list is made perfectly comparable to the high risk lottery x, i.e. $z = (\$23.50, p)$. They document that the inconsistency with direct choice disappears when high-risk/low-risk lottery pairs are assessed using probability matching, consistent with the predictions of our model.

Formally, the DM now values $l = (w_l, p_l)$ against a *probability list*: a price list of lotteries $Z = \{z^1, ..., z^n\}$, where each $z^k = (24, p_k)$. Analogous to Example 1, we work with *adapted* probability lists, for which those containing equal-sized steps with $p_1 = p_l$, $p_n = 0$. Denote by $PE(l, Z) = 1/2[p_{R(l,Z)-1} + p_{R(l,Z)}]$ the distribution over the DM's probability equivalents.

Figure 2b plots the simulated probability equivalents $\mathbb{E}[PE(l, Z)]$ for the same set of simple lotteries l as in Figure $2a$. Notice that the valuation distortions in certainty equivalents *reverse* when the lotteries are valued using probability equivalents. Intuitively, y is harder to compare to prices, so its valuation is compressed upward toward the middle of the range of undominated probabilities, whereas x is easier to compare, so its valuation is closer to the truth. Thus, $\mathbb{E}[PE(y, Z)] > \mathbb{E}[PE(x, Z)]$, eliminating the reversal.

While probability equivalents are consistent with direct choice in this example, note that our model does *not* predict that probability equivalents are more accurate than certainty equivalents. To see this, focus on the case of risk neutral preferences. Here, *both* methods of valuation are subject to bias: x and y are indifferent in truth, and yet we have $\mathbb{E}[CE(x, Z)] > \mathbb{E}[CE(y, Z)]$ and $\mathbb{E}[PE(x, Z)] < \mathbb{E}[PE(y, Z)]$.

3.3.2 Intertemporal Reversals

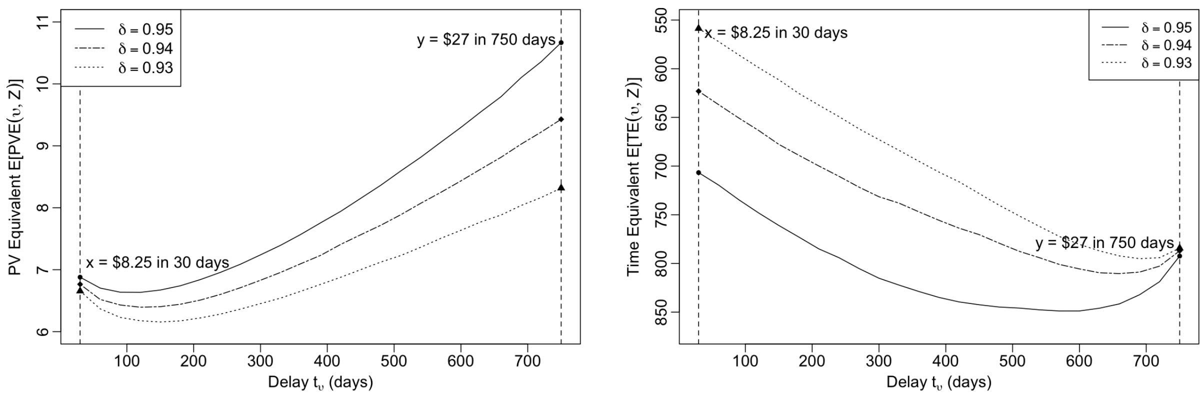
As preference reversals result from the difficulty of comparing options to prices in our model, it predicts that this phenomenon is not limited to lottery choice. Consider the intertemporal domain, where $v_x = \sum_t \delta^t m_x(t)$ and $\tau_{xy} = \tau_{xy}^{CPF} = H\left(\frac{|PV(x)-PV(y)|}{d_{CPF}(x,y)}\right)$, with $H(1) = \infty$.

Example 3. (Intertemporal reversals). We have the following delayed payments:

$$x: \quad \$27 \text{ in } 750 \text{ days}$$
$$y: \quad \$8.25 \text{ in } 30 \text{ days}$$

Consider a DM with a monthly discount factor $\delta \leq 0.95$ choosing between these payments: since $v_y \geq v_x$ our model says the DM is more likely to choose y. Now suppose the DM must value each option in terms of dollars today. Under τ^{CPF}, the two payments differ in their ease of comparison to money today; following the same logic as in Example 1, the resulting distortions cause x to be valued higher than y.

Formally, the DM faces a valuation task (υ, Z), where $\upsilon = (m_\upsilon, t_\upsilon)$ is a delayed payment that pays out $m_\upsilon > 0$ at time $t_\upsilon > 0$, valued against a price list $Z = \{z^1, ..., z^n\}$, where each $z^k = (m_k, 0)$ is an immediate payment. This setting is similar to those described in Examples 1 and 2, so we reserve further details for Appendix B.6. Figure 3a plots the present value



(a) Present Value Equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ (b) Time Equivalents $\mathbb{E}[T(\upsilon, Z)]$

Figure 3: Simulated average present value equivalents and time equivalents for delayed payments $\upsilon = (m_\upsilon, t_\upsilon)$ with present value equal to that of $x = (8.25, 30)$ under $\delta = 0.95$. For PVEs, Z is adapted to υ with $|Z| = 15$. For TEs, $Z = \{z^1, ..., z^n\}$, where $z^k = (27.5, t_\upsilon + t_k)$, for $(t_1, ..., t_n) = (0, 7, 30, 60, 120, 180, 240, 360, 480, 600, 720, 900, 1080, 1260, 1440)$ days. τ has a CPF-complexity representation with $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0, 1]$.

equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ simulated from our model for delayed payments υ with the same present value as x and y (assuming a monthly discount factor $\delta = 0.95$). Here, the high-delay option x has a higher valuation than y, despite the fact that $\rho(y, x) \geq 1/2$.[19]

As we saw in lotteries, the direction of these distortions may be reversed by changing the units of valuation. Suppose the DM values options in terms of *time-equivalents*: the time t that makes the delayed payment ($27.50, t) indifferent to x or y. Formally, the DM values υ against a *time list* $Z = \{z^1, ..., z^n\}$, where each $z^k = (\$27.50, t_k)$. As x is more similar to the numeraire under this valuation mode, the model predicts the distortions in time equivalents will be flipped relative to present value equivalents, as Figure 3b illustrates. Specifically, we have $\mathbb{E}[T(y, Z)] < \mathbb{E}[T(x, Z)]$ and so the "reversal" relative to direct choice disappears.

3.4 Biases in Valuation of Risk and Time

The same pull-to-center effects that produce preference reversals in our model can also generate classical patterns in the valuation of risk and time: probability-weighting and hyperbolic discounting. The same logic underlies both domains: consider a simple lottery with payout probability p, or a payoff flow that pays out with delay t. The presence of tradeoffs can make both options hard to compare to money, generating pull-to-center distortions: valuations of small (large) payoff probabilities are compressed upward (downward), generating apparent probability weighting; likewise, valuations of small (large) delays are com-

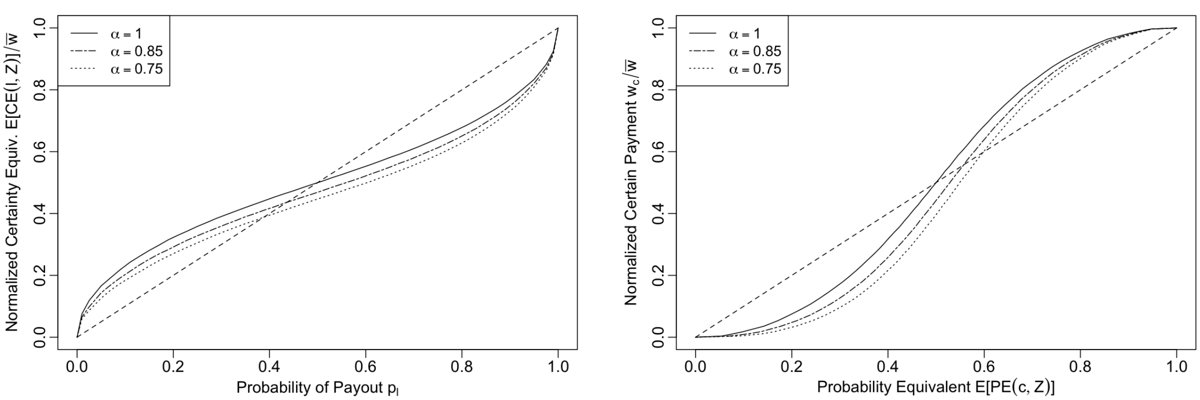
[19]Using choice vignettes, Tversky et al. (1990) document similar intertemporal preference reversals.

pressed downward (upward), generating apparent hyperbolic discounting. Furthermore, our model predicts that these biases can be *reversed* by inverting the role of the numeraire.

3.4.1 Apparent Probability Weighting in Lottery Valuation

Consider the standard paradigm used to estimate risk preferences, in which the DM provides certainty equivalents of simple lotteries $l = (\overline{w}, p_l)$. Note that for p_l sufficiently close to 0 or 1, l is easy to compare to prices and so valuations are close to accurate, whereas for interior values of p_l, tradeoffs between payoff amounts and probabilities make l harder to compare to prices, producing pull-to-center distortions: small probabilities are overvalued and large probabilities are undervalued. These two forces generate apparent inverse-S probability weighting in the DM's valuations, as seen in Figure 4a, which plots our model's predicted normalized certainty equivalents $\mathbb{E}[CE(l,Z)]/\overline{w}$ as a function of p_l.

Importantly, our model does not predict that probability weighting occurs generically in choice; instead, it results from the difficulty of comparing a lottery to a price list of certain payments. This is important for two reasons. First, our model can generate probability weighting in valuation even absent such distortions in binary choice, which is consistent with evidence that the fourfold pattern of probability weighting is far more prominent in valuation tasks than in direct choice (Harbaugh et al., 2010; Bouchouicha et al., 2023). Second, we predict that these patterns of probability weighting will be sensitive to the specific valuation paradigm. In particular, we predict that it is possible to *reverse* the pattern



(a) Simulated normalized average certainty equivalents $\mathbb{E}[CE(l,Z)]/\overline{w}$ for simple lotteries $l = (\overline{w}, p_l)$ as a function of p_l.

(b) Simulated average probability equivalents $\mathbb{E}[PE(c,Z)]$ for certain payments $c = (w_c, p_l)$ and normalized payment amounts $w_c/\overline{w}$.

Figure 4: Probability weighting functions implied by valuations in certainty equivalents (left) and probability equivalents (right). Z is adapted to l with $|Z| = 15$. τ has a CDF-complexity representation with $u(w) = w^\alpha$ and $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0,1]$.

of apparent probability-weighting with an appropriate choice of price list currency.

Consider an alternative paradigm for estimating risk preferences, in which the DM provides probability equivalents of a certain payment: the probability p that makes the lottery $z = (\overline{w}, p)$ indifferent to $c = (w_c, 1)$. Tracing out the probability equivalents p as a function of the normalized certain payments $w_c/\overline{w}$ should — absent complexity-driven distortions — reveal the same preferences as the certainty equivalents discussed above. Figure 4b plots the predicted relationship between the certain payment amount $w_c/\overline{w}$ (y-axis) and the associated probability equivalent $\mathbb{E}[PE(c, Z)]$ (x-axis), in which the certain payment $c = (w_c, 1)$ is valued against a probability list $Z = \{z^1, ..., z^n\}$, for $z^k = (\overline{w}, p_k)$. Here we see a reversal of the inverse S-shaped pattern: the difficulty of comparing sure payments against the numeraire good causes probability equivalents to be compressed towards the middle of the price list, generating apparent *underweighting* of small probabilities and *overweighting* of large probabilities.[20]

3.4.2 Apparent Hyperbolic Discounting in Intertemporal Valuation

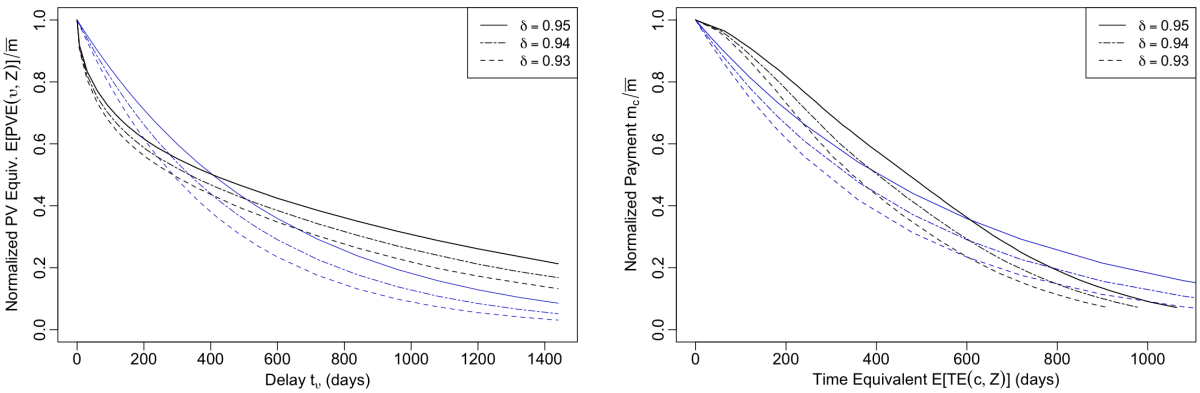
Consider a standard paradigm used to estimate discounting over money: the DM values delayed payments $\upsilon = (\overline{m}, t_\upsilon)$ in terms of money today. Following identical logic as in lottery valuation, pull-to-center effects result in payments closer to the present to be *undervalued* and payments further in the future to be *overvalued*, with valuations becoming accurate as $t_\upsilon \to 0$. This produces complexity-driven hyperbolic discounting (Enke et al., 2025).

To illustrate, Figure 5a plots the normalized valuations $\mathbb{E}[PVE(\upsilon, Z)]/\overline{m}$ as a function of the delay t_υ, where υ is valued against a price list $Z = \{z^1, ..., z^n\}$ of immediate payments adapted to υ. The DM's valuations exhibit apparent hyperbolicity: she undervalues payments close to the present and overvalues payments with longer delays. As this pattern is driven by tradeoff complexity rather than a preference for the present, our model also predicts that hyperbolicity persists when front-end delays are incorporated, and so reconciles a puzzle in the literature wherein valuations of delayed payments exhibit marked hyperbolicity and yet are stationary with respect to front-end delays (Cohen et al., 2020).[21]

Now consider an alternative valuation paradigm in which the DM assesses the *time equivalents* of an immediate payment: the delay t that makes the delayed payment $(\overline{m}, t)$

[20]While Sprenger (2015) shows that a model of stochastic reference points can predict a larger degree of risk aversion in probability vs. certainty equivalents, this account cannot explain the overweighting of large probabilities that our model predicts in probability vs. certainty equivalents. Feldman and Ferraro (2024) shows that by allowing utility curvature to differ across valuation procedures, a model of stochastic reference points can generate a difference between probability vs. certainty equivalents similar to our model predictions.

[21]See Supplemental Appendix J for details.



(a) Simulated average present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ (in black) for delayed payments $\upsilon = (\overline{m}, t_\upsilon)$ as a function of t_υ.

(b) Relationship between simulated average time equivalents $\mathbb{E}[TE(c, Z)]$ (in black) for immediate payment $c = (m_c, 0)$ and normalized amount $m_c/\overline{m}$.

Figure 5: Discount functions implied by present value equivalents (left) and time equivalents (right). For PVEs, Z is adapted to υ with $|Z| = 15$. For TEs, $Z = \{z^1, \dots, z^n\}$, where $z^k = (\overline{m}, t_k)$, for $(t_1, \dots, t_n) = (0, 7, 30, 60, 120, 180, 240, 360, 480, 600, 720, 900, 1080, 1260, 1440)$ days. Blue curves plot distortion-free discount functions given the true discount rate δ. τ has a CPF-complexity representation parameterized by $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0, 1]$.

indifferent to the immediate payment $c = (m_c, 0)$. Figure 5b plots the predicted relationship between the immediate payment amount $m_c/\overline{m}$ (y-axis) and the associated time equivalents $\mathbb{E}[TE(c, Z)]$ (x-axis). Here, the model predicts an apparent reversal of hyperbolic discounting: the difficulty of comparing immediate payments to the numeraire good causes time equivalents to be compressed toward the middle of the price list, generating overvaluation of payments close to the present and undervaluation of payments with longer delays.

We do not claim that observed hyperbolic discounting is purely a complexity-driven distortion; rather, we provide a mechanism explaining why canonical valuation tasks may overstate the degree of hyperbolic discounting present in direct choice. In Appendix B.7, we repeat the simulation exercise in Figure 5 for a DM with a hyperbolic discount function, and show how comparison complexity magnifies the extent of true hyperbolic discounting in present value equivalents, and reduces it in time equivalents.

The pull-to-center distortions in our model echo the predictions of Bayesian cognitive noise models, in which noise compresses the DM's action toward an intermediate default. Unlike our model, these accounts cannot explain why these compression effects disappear near boundary parameters (i.e. $p_l = 0, 1$ in lottery valuation, and $t_\upsilon = 0$ in intertemporal valuation) without the auxiliary assumption that cognitive noise vanishes at these boundaries (Enke and Graeber, 2023; Enke et al., 2025). As our model explains *why* options are easier to value at these boundaries, it provides a theoretical basis for such an assumption.

4 Experimental Tests

We turn to testing the key predictions of our model. We assess whether 1) in binary choice, our complexity measures predict choice noise and errors; 2) classic preference reversals can be eliminated by manipulating the ease of comparing options to the price list; and 3) probability weighting and hyperbolic discounting can be reversed by eliciting valuations of money in terms of probability and time equivalents. We then quantify the predictive power of our binary choice model relative to benchmark models.

4.1 Tests of Complexity Measures

We test the validity of our proposed measures of comparison complexity against data from three binary choice experiments in multiattribute, intertemporal, and lottery choice. Below, we provide an overview of the goals and design features shared across the three experiments. We then present domain-specific details and results.

The purpose of these experiments is two-fold. First, we show that our proposed complexity measures indeed capture the difficulty of comparisons. We consider three natural indicators of choice complexity: choice errors, choice inconsistency, and subjective uncertainty, and show that they are decreasing in the value-dissimilarity ratio, as our theory predicts. Second, we show that our model captures quantitatively important features of choice not explained by standard models. In each domain, we structurally estimate our model and compare its performance against leading behavioral models.

We carry out these analyses in three parallel binary choice datasets. We run new experiments in multiattribute and intertemporal choice and compile existing data from Enke and Shubatt (2023) and Peterson et al. (2021) to study lottery choice. In our experiments, we recruit participants through an online survey platform to make 50 incentivized binary choices. For each problem, we elicit participants' subjective certainty in the optimality of their decision. In order to measure choice consistency, 10 of these problems are randomly repeated throughout the survey. We collect an average of 37 choices for each of 662 multiattribute choice problems and 1,097 intertemporal problems – a total of more than 66,000 individual decisions. The compiled risk dataset includes nearly 10,000 binary choice problems (over 1 million decisions) and includes similar measures of cognitive uncertainty and choice consistency.[22]

[22]Cognitive uncertainty is elicited only for the 500 problems from Enke and Shubatt (2023). Problems are only repeated in the Peterson et al. (2021) experiment. Unlike our experiments, subjects see these repeated problems immediately after giving an initial response.

Multiattribute Choice. Participants make binary choices between hypothetical phone plans characterized by either two, three, or four attributes, which include a device cost, a monthly flat fee, a data usage fee, and a quarterly wi-fi fee. Each choice problem has an objective payoff-maximizing answer, which allows us to observe choice errors: subjects choose on behalf of a hypothetical consumer with a known budget and data usage, and are incentivized to choose the plan that will save the consumer the most money. If a participant is selected to earn a bonus (1 in 2 chance), we select one of their choices at random and pay them based on the money saved. Across all choice problems, choosing the more expensive plan results in an average bonus of $4.20, and choosing the cheaper plan results in an average bonus of $8.98. For more detail on the design and pre-registration, see Appendix D.

Figures 6a, 6b, and 6c show binned scatterplots relating the L_1 ratio (x-axis) to choice errors, cognitive uncertainty, and inconsistency. All three complexity indicators are strongly decreasing in the ratio. The average error rate, around 5% for problems near-dominance, increases five-fold for problems with the lowest values of the L_1 ratio ($R^2 = 0.32$). We take this as strong evidence that (1) the ratio indeed captures choice complexity, and (2) decision-makers respond to this complexity by making more random choices. Importantly, these relationships are not merely driven by variation in value differences: they are unchanged when controlling for value differences, as the regression analysis in Appendix Table 3 shows.[23]

Intertemporal Choice. Participants make binary choices between time-dated payoff streams. Each option has up to two payoffs ranging between $1 and $40, to be received at delays ranging between the present and 2 years in the future. If a participant is selected to win a bonus (1 in 5 chance), we select a decision at random and pay out the chosen payoff stream on the specified dates. For more details on the design and pre-registration, see Appendix D.

Unlike in multiattribute choice, we cannot directly observe choice errors: the definition of an "error" depends on the decision-maker's discount function. The CPF ratio also depends on this discount function, so we proceed by estimating a representative-agent exponential discount factor δ from the choice data (monthly $\hat{\delta} = 0.94$). For details on the specification, see Appendix E.2. Importantly, the discounting estimated from our choice data, which involves choices over money, should *not* be interpreted as subjects' "pure" time preferences (Cohen et al., 2020): it could reflect, for instance, access to outside credit or beliefs about repayment risk. Rather than identifying time preferences, we are interested in studying which

[23]We pre-registered analyses restricting to subjects who do not report using a calculator in the experiment (82.5% of the sample), reported in Appendix Table 4. Quantitative relationships are virtually unchanged.

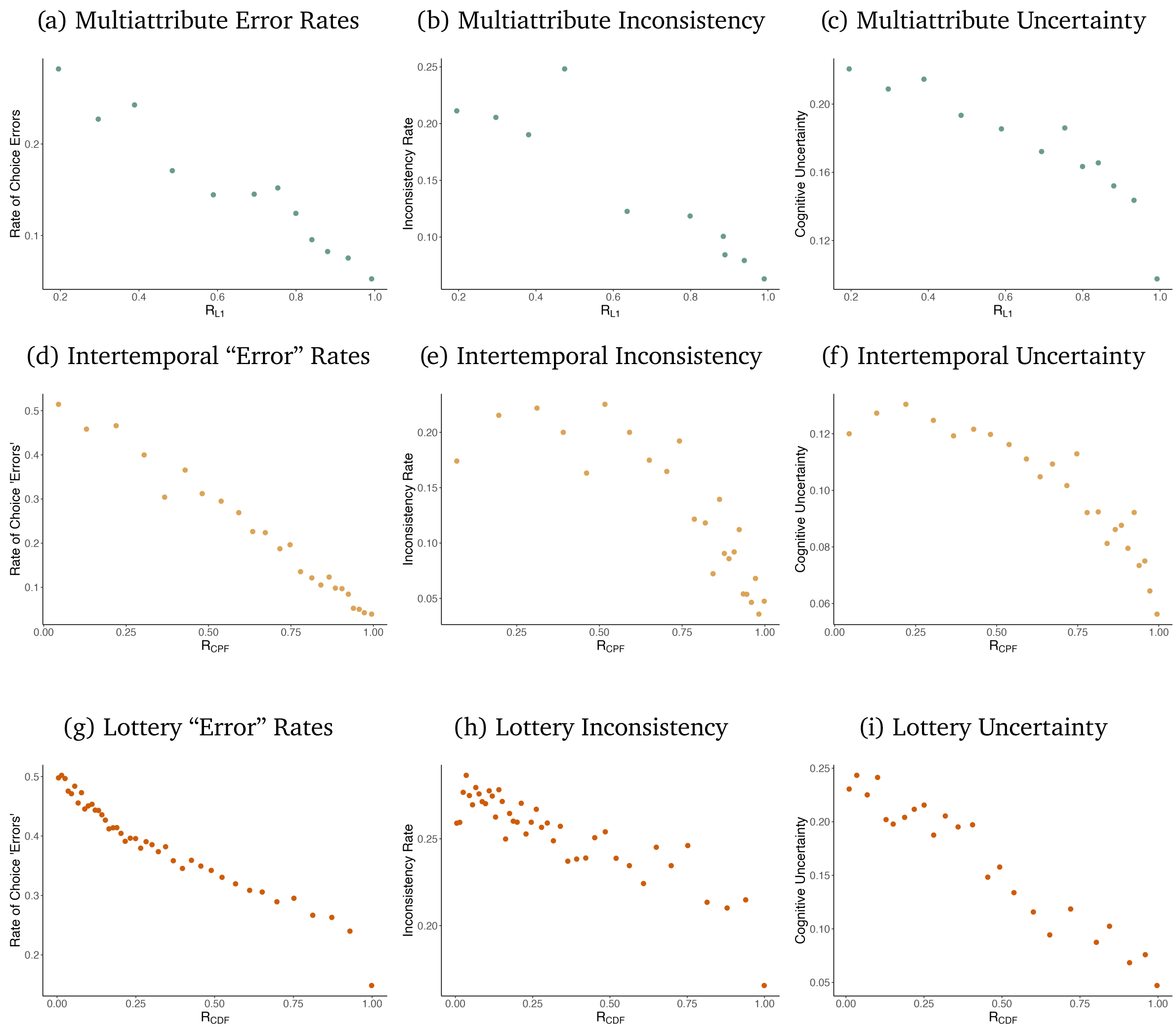


Figure 6: Binscatter of problem-level error rates, choice inconsistency, and cognitive uncertainty versus the value-dissimilarity ratio. Problem-level error rates in panels (a), (d), and (g) are constructed as the rate of choosing (a) the lower-value option, (d) the less-preferred option according to the best-fit exponential discounted utility model, and (g) the less-preferred option according to the best fit expected utility model (see Appendix E for estimation details). Problem-level inconsistency rates in panels (b), (e), and (h) are constructed as the percent chance subjects choose differently in any repeated instance of the choice problem. Problem-level cognitive uncertainty in panels (c), (f), (i) is constructed as the average subjective likelihood that subjects assign to making an error in that choice problem, i.e. choosing the less-preferred option. The value-dissimilarity ratios R_{L1}, R_{CPF}, and R_{CDF} follow Definitions 1, 4, and 3, respectively, where the preference parameters β, δ, and u are given by the known attribute weights in multiattribute choice, the best-fit exponential discount rate in intertemporal choice, and the best-fit expected utility preferences in lottery choice, respectively. See Appendix Tables 3, 6, and 9 for corresponding regression analyses.

problems are hard given subjects' required rate of return in our experimental setting.

Figures 6d, 6e, and 6f relate the CPF ratio (x-axis) to our indicators of choice complexity, coding a choice as an "error" if the subject chooses the option with lower value according to $\hat{\delta}$. Again, we find strong relationships between the ratio and choice "errors," cognitive uncertainty, and inconsistency. The average "error" rate ranges from around 5% for problems with the highest value of the CPF ratio to 50% for problems with the lowest value of the CPF ratio ($R^2 = 0.6$). We also run analyses in which errors are classified using individually estimated discount factors $\hat{\delta}_i$ with similar results (see Appendix Figure 12 and Table 7). As in multiattribute choice, these relationships are unchanged when controlling for the value difference (see Appendix Table 6). This further indicates that the value-dissimilarity *ratio*, rather than the value difference alone, drives these relationships.

Lottery Choice. In both lottery choice experiments, participants are asked to choose between two lotteries which pay off different amounts with known probabilities. If selected to receive a bonus, they receive the outcome of a lottery they chose in a randomly selected decision. As in intertemporal choice, both the CDF ratio and our notion of choice "errors" depend on an unknown preference object – the Bernoulli utility function. We proceed by estimating a representative-agent CRRA utility function with additive logit noise (see Appendix E.3 for details), and code "errors" as departures from this estimated model. Figures 6g, 6h, and 6i show the results. Once again, all three outcomes are strongly decreasing in the ratio; in particular, the CDF ratio achieves an R^2 of 0.45 in variance explained over error rates. We additionally run analyses using individually estimated risk aversion parameters to classify choices as errors and find similar results (see Appendix Figure 13 and Table 10).[24] As in our other domains, these relationships are driven by the value-dissimilarity ratio, as opposed to the value difference alone (see Appendix Table 9).

4.2 Tests of Preference Reversal Predictions

We run experiments mirroring the simulation exercises in Section 3.3 to test our model predictions regarding preference reversals. Specifically, we establish the presence of reversals in lottery and intertemporal choice, and test the novel model prediction that these reversals can be eliminated by manipulating the ease of comparing options to the price list.

In each domain, we construct two sets of "base" options: 3 high-risk and 3 low-risk

[24]We restrict this analysis to the Enke and Shubatt (2023) data since the Peterson et al. (2021) data contain a small number of unique choice problems per subject (75% of subjects face ≤ 14 unique problems).

lotteries, and 3 high-delay and 3 low-delay payoff flows (see Table 2). We elicit binary choices between these sets of options, as well as multiple price list valuations of each option. In these valuation tasks, we experimentally vary the ease of comparing each option to the price list: for lotteries, we elicit certainty equivalents as well as probability equivalents using a yardstick lottery that pays \$24 with p% chance; for intertemporal choice, we elicit present value equivalents as well as time equivalents using a yardstick option that pays \$27.5 in t days. We elicit choices and valuations for both the base options as well as a "scaled-up" version of each option, where we multiply payouts by a scale factor of 1.6.

Recall the predictions developed in Section 3.3. First, binary choice probabilities will favor low-risk (low-delay) options over high-risk (high-delay) options.[25] Second, valuations will be higher for high-risk (high-delay) options compared to low-risk (low-delay) options when they are valued in terms of money. Third, these relative valuations will flip when options are instead valued using probability equivalents (time equivalents).

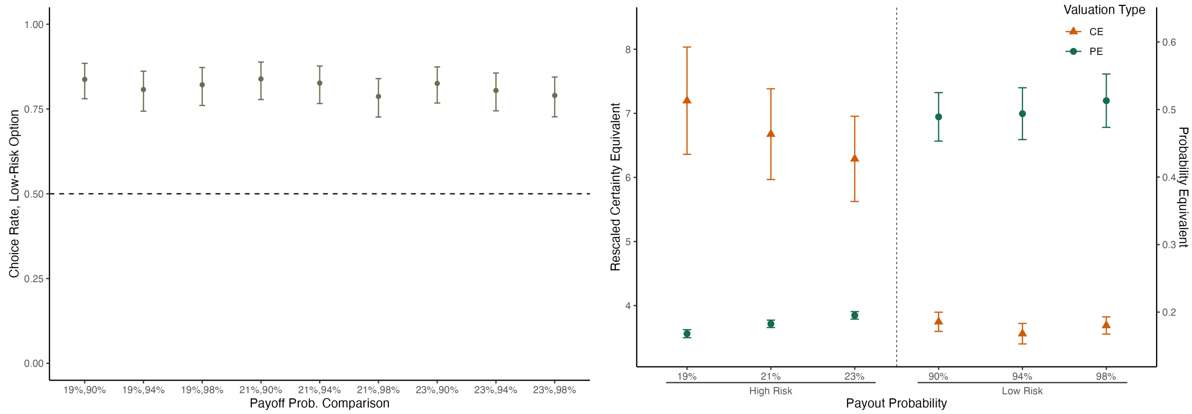
We run separate incentivized lottery and intertemporal choice experiments on an online survey platform with 151 and 152 subjects, respectively. All subjects complete two parts, in random order: *Binary Choice* and *Valuation*. In *Binary choice*, subjects make 16 direct choices between option pairs. In *Valuation*, participants value all 12 options (6 base options, at 2 scale factors) using multiple price lists corresponding to one of two randomly assigned valuation modes: certainty equivalents or probability equivalents in the lottery experiment, and present value equivalents or time equivalents in the intertemporal experiment. See Appendix D.3 for instructions and additional design details.

Figures 7 and 8 present results from our lottery and intertemporal choice experiments, aggregating across scale factors. Figures 7a and 8a show that binary choice rates favor the low-risk and low-delay options, with choice rates for those options well above 50%. In

	Lottery	*Intertemporal*
	\$4.50 w.p. 98%	\$8.25 in 30 days
Low-risk/delay	\$4.75 w.p. 94%	\$9.50 in 90 days
	\$5.00 w.p. 90%	\$11.00 in 150 days
	\$19.50 w.p. 23%	\$24.00 in 630 days
High-risk/delay	\$21.25 w.p. 21%	\$25.50 in 690 days
	\$23.50 w.p. 19%	\$27.00 in 750 days

Table 2: Base options used in reversal experiments. We consider every possible pairing of a low-risk, high-risk option for lotteries; and every possible pairing of a low-delay, high-delay option for delayed payments.

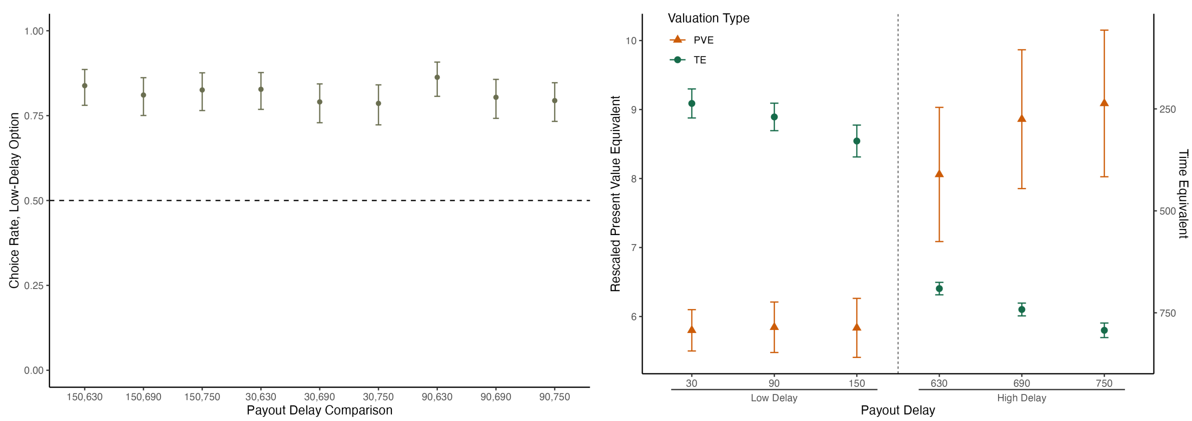
[25]These options were calibrated so that under the preferences estimated from our binary choice data, the low-risk/low-delay options are preferred.



(a) Binary Choice Probabilities

(b) Certainty and Probability Equivalents

Figure 7: Preference reversal experiments for simple lotteries, aggregated across scale factor. Panel (a) presents binary choice rates for each high/low risk lottery comparison. Panel (b) presents average rescaled certainty equivalents, computed by dividing each certainty equivalent by the scale factor (scale on left axis), and average probability equivalents (scale on right axis). Error bars reflect 95% confidence intervals.



(a) Binary Choice Probabilities

(b) Present Value and Time Equivalents

Figure 8: Preference reversal experiments for delayed payments, aggregated across scale factor. Panel (a) presents binary choice rates for each high/low delay comparison. Panel (b) presents average rescaled present value equivalents, computed by deviding each present value equivalent by the scale factor (scale on left axis) and average time equivalents (scale on right axis). The direction of the time equivalent axis is inverted, so that valuations are increasing in the vertical axis. Error bars reflect 95% confidence intervals.

contrast, Figures 7b and 8b show that there is an apparent reversal in preference when the options are valued via certainty equivalents and present value equivalents: subjects assign higher valuations to high-risk and high-delay options on average. In these same figures, however, we see the predicted *flipping* of these valuation patterns when we manipulate the ease of comparing each option to the price list. In particular, when options are instead valued via probability equivalents and time equivalents, subjects instead assign higher valuations to the low-risk and low-delay options, thus eliminating the apparent reversal.[26]

4.3 Tests of Valuation Predictions

We run a multiple price list experiment to 1) replicate patterns of inverse-S shaped probability weighting in certainty equivalents and hyperbolic discounting in present value equivalents, and 2) test the prediction, developed in Section 3.4, that these biases can be reversed by manipulating the units of valuation. Specifically, our model predicts that relative to the certainty equivalents of binary lotteries, probability equivalents of certain payments should exhibit underweighting of low probabilities and overweighting of high probabilities. Likewise, relative to the discounting revealed by the present value equivalents of delayed payments, time equivalents of immediate payments should exhibit overvaluation of delays close to the present and undervaluation of delays far in the future.

To test these predictions, we recruit 300 subjects through an online survey platform who each complete 24 incentivized multiple price lists: 12 for lotteries, and 12 for intertemporal payments. Each subject is randomly assigned to a valuation mode for each domain: certainty or probability equivalents for lotteries, and present value or time equivalents for intertemporal payments, and the order of the domains is randomized. See Appendix D.3 for additional design details.

Lottery Valuations. We elicit certainty equivalents $CE(l)$ for a simple lottery $l = (\overline{w}, p_l)$, and relate the normalized valuations $CE(l)/\overline{w}$ to payout probabilities p_l. We also elicit probability equivalents $PE(c)$ of a certain payment $c = (w_c, 1)$ against a yardstick lottery $(\overline{w}, p)$, and relate the normalized payouts $w_c/\overline{w}$ to probability equivalents $PE(c)$. In our experiment, we draw $\overline{w}$ from $\{\$9, \$18, \$27\}$. For certainty equivalents, we draw p_l from $\{0.03, 0.05, 0.10, 0.25, 0.5, 0.75, 0.90, 0.95, 0.97\}$. For probability equivalents, we draw w_c so that $w_c/\overline{w} \in \{0.033, 0.056, 0.11, 0.25, 0.5, 0.75, 0.89, 0.944, 0.967\}$.

Results for lotteries are presented in Figure 9. Certainty equivalent valuations follow the standard pattern: risk-averse over low probabilities and risk-seeking over high probabilities. Consistent with the model's predictions, the pattern *reverses* for probability equivalents. Relative to certainty equivalents, probability equivalents are more risk-averse over

[26]Butler and Loomes (2007) document a related result in lottery choice: examining inconsistency between direct choice and probability equivalents, they find that subjects are more likely to state higher PEs for the low-risk lottery yet choose the high-risk lottery in direct choice than to exhibit the opposite inconsistency. We find similar patterns in our data.

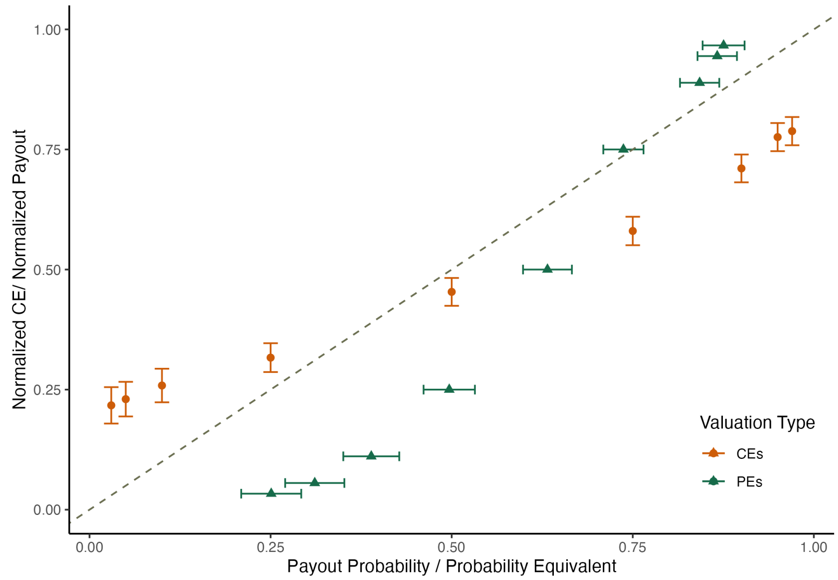


Figure 9: Probability weighting functions implied by lottery valuation experimental tasks. Orange dots plot the payout probability p_l against the average normalized certainty equivalent $CE(l)/\overline{w}$; turquoise triangles plot average probability equivalents $PE(c)$ against the normalized payout $w_c/\overline{w}$. The dashed black line represents linear probability weighting. Error bars reflect 95% confidence intervals.

low probabilities, and more risk-seeking over high probabilities.[27]

Intertemporal Valuations. We elicit present value equivalents $PVE(v)$ for a delayed payment $v = (\overline{w}, t_v)$, and relate the normalized valuations $PVE(v)/\overline{m}$ to payout delays t_v. We also elicit time equivalents $TE(c)$ of an immediate payment $c = (m_c, 0)$ against a yardstick delayed payment $(\overline{m}, t)$, and relate normalized payouts $m_c/\overline{m}$ to time equivalents $TE(c)$. In our experiment, we draw $\overline{m}$ from $\{25, 30, 35\}$. For present value equivalents, we draw t_v from $\{7, 30, 60, 120, 240, 360, 480, 720, 1080\}$ (in days). For time equivalents, we draw m_c so that $m_c/\overline{m} \in \{0.20, 0.35, 0.50, 0.65, 0.75, 0.85, 0.90, 0.95, 0.97\}$.

Results for are presented in Figure 10. Focusing first on the discount function implied by present value equivalents, we document the familiar pattern of short-run impatience and long-run patience (i.e. hyperbolicity). We also see that this hyperbolicity is *more extreme* than the hyperbolic discount function estimated from our binary choice experiment (traced in the dotted line; see Appendix E.2 for estimation details). This is consistent with the interpretation that the true hyperbolicity in temporal preferences, as revealed by subjects' binary choices, is exaggerated when measured through present value equivalents. However, when

[27]These results are consistent with Feldman and Ferraro (2024), who find similar patterns in an expert sample of commercial agricultural producers.

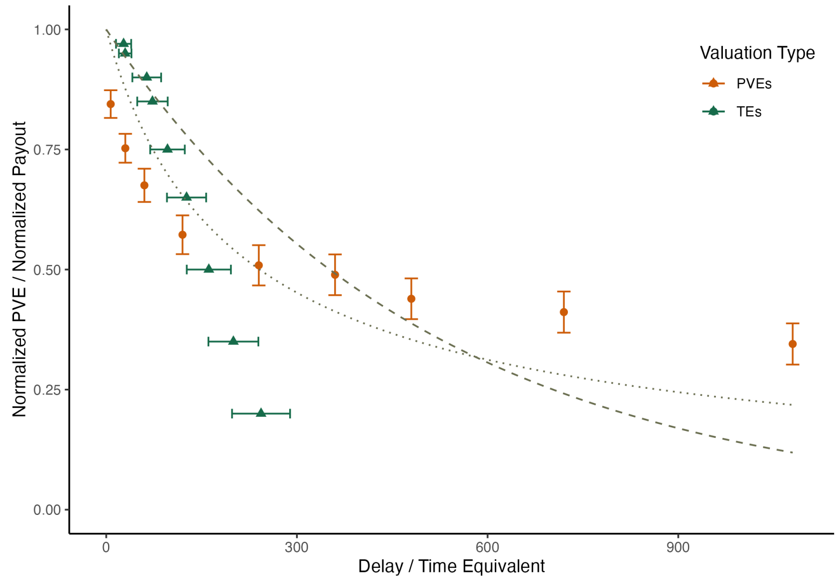


Figure 10: Discount functions implied by intertemporal valuation experimental tasks. Orange dots plot the payout delay t_v against the average normalized present value equivalent $PVE(l)/\overline{m}$; turquoise triangles plot average time equivalents $TE(c)$ against the normalized payout $m_c/\overline{m}$. The dashed line traces the exponential discount function estimated from binary choice data; the dotted line, the estimated hyperbolic discount function. Error bars reflect 95% confidence intervals.

instead we elicit valuations using time equivalents, the pattern of hyperbolicity *reverses*: relative to both the hyperbolic discounting estimated from our binary choice data, as well as the discounting implied by their present value equivalents, subjects exhibit short-run patience and long-run impatience.

4.4 Benchmarking Model Performance

Whereas the predominant approach in behavioral economics is to model non-standard elements in the DM's value function, this paper seeks to model an orthogonal feature of choice: how difficult the DM finds a choice, given her objectives. To study whether our theory indeed explains variation in choice that is not captured by existing models, we compare the explanatory power of these modeling approaches in our binary choice data.

First, we structurally estimate our choice model, which is parameterized by a transformation G that maps the value-dissimilarity ratio into choice probabilities and (in the case of lottery and intertemporal choice) domain-specific preference parameters. Across all domains, we use the same specification of G developed in Section 2.4. We then compare performance against a set of models specifying the DM's value function, which we fit to our data using logit errors. In particular, we estimate a "standard" benchmark model: a distortion-free logit model in multiattribute choice, exponential discounted utility in intertemporal choice, and expected utility in lottery choice. We also estimate a set of behavioral models: salience-weighting (Bordalo et al., 2013), focusing (Kőszegi and Szeidl, 2013), and relative thinking (Bushong et al., 2021) in multiattribute choice; quasi-hyperbolic and hyperbolic discounting in intertemporal choice; and simplicity theory (Puri, 2025) and cumulative prospect theory in lottery choice. Estimates are obtained using maximum likelihood; see Appendix E for details on each specification.

Figure 11 reports the variance explained over problem-level choice rates of 1) the standard benchmark model, 2) the leading behavioral model, and 3) our comparison complexity model. To characterize the extent to which our model captures orthogonal variation in choice relative to existing models, the final column reports the variance explained of an ensemble model that combines the predictions of the leading behavioral model and our comparison complexity model.[28] Appendix Tables 5, 8, and 11 report full estimation results.

In multiattribute choice, we estimate versions of our L_1-complexity model corresponding to the two- and three-parameter specifications of G developed in Section 2.4. Both the two- and three-parameter models (R^2 of 0.32 and 0.36, respectively) are comparable to the

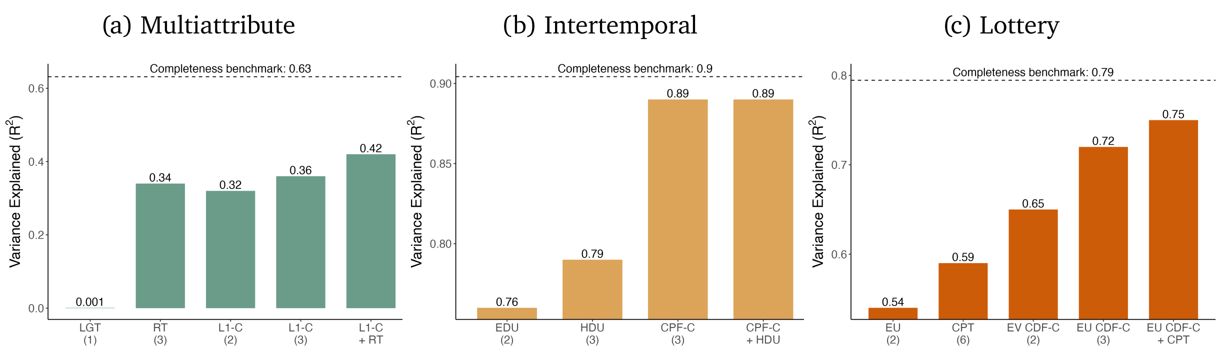


Figure 11: Variance explained of estimated models and completeness benchmarks. R^2 values are observation-weighted. Number of free model parameters in parentheses. “LGT”, “RT”, “L1-C” refer to the Distortion-Free, Relative Thinking, and L_1-complexity models described in Appendix E.1. “EDU”, “HDU”, and “CPF-C” refer to the Exponential Discounting, Hyperbolic Discounting, and CPF-complexity models desdribed in Appendix E.2. ‘EU", “CPT", “EV CDF-C", and “EU CDF-C" refer to the Expected Utility, Cumulative Prospect Theory, risk-neutral CDF-complexity, and expected utility CDF-complexity models described in Appendix E.3. Completeness benchmarks are obtained using an ensemble of fitted models and a neural network; see Supplemental Appendix G for details.

[28]Specifically, we report the variance explained of an observation-weighted OLS regression of choice rates against the predicted choices rates of the two models.

leading behavioral model of relative thinking ($R^2 = 0.34$).[29] Importantly, L_1-complexity explains a substantial amount of variation in choices not captured by the relative thinking model. Combining the three-parameter L_1-complexity model and relative thinking results in an R^2 of 0.42, a 24% increase in variance explained compared to relative thinking alone.

In both intertemporal and lottery choice, our model captures significantly more variation in choice rates than leading models. In intertemporal choice, our CPF-complexity model ($R^2 = 0.89$) delivers a 13% improvement in variance explained over hyperbolic discounting ($R^2 = 0.79$), using the same number of parameters. In lottery choice, we estimate two versions of the CDF-complexity model — one that assumes risk-neutral preferences, and one that allows for utility curvature. Despite having far fewer free parameters, the risk-neutral CDF-complexity model ($R^2 = 0.65$) delivers a 10% improvement over cumulative prospect theory ($R^2 = 0.59$). This is in line with Enke and Shubatt (2023), which finds that allowing complexity to enter the noise term of a logit choice model substantially improves performance over standard models.[30] Adding a parameter to capture utility curvature in our model yields an R^2 of 0.72 – a 22% improvement over cumulative prospect theory.

Completeness and Restrictiveness. Following Fudenberg et al. (2022), we establish "completeness benchmarks" in each of our three domains by training flexible predictive models to predict choice rates based on problem features; these benchmarks represent the predictable variation in choice rates that any model could hope to capture. We use an ensemble approach to form these benchmarks, described in detail in Supplemental Appendix G, where we combine parametric model predictions with those of a neural network trained on the data. The dashed lines in Figure 11 report the R^2 of these benchmarks, and Appendix Tables 5, 8, and 11 report the completeness index of each model following Fudenberg et al. (2022). Our model captures 70% of the predictable variation in our multiattribute choice data, and over 90% in lottery and intertemporal choice. Note that this high level of completeness in lottery and intertemporal choice suggests that there is limited scope for systematic heterogeneity *across problems* in how well the model captures behavior. If our model fit substantially worse on a class of problems in the dataset relative to other models, this would manifest in large performance gains of the neural network. However, this exercise cannot speak to heterogeneity *across subjects*; we characterize completeness only with reference to the best representative agent model.

[29]The performance of the distortion-free logit model is due to the low variation in value differences in our multiattribute data, which by design, could only take on one of two values.

[30]The "complexity index" developed by Enke and Shubatt (2023) loads heavily on "excess dissimilarity," which is equal to the denominator of the CDF ratio minus the numerator, assuming risk-neutral preferences.

One concern is that the predictive power of our model may come at the cost of parsimony — i.e. our model is flexible enough to explain any dataset. To address this, we estimate the restrictiveness of each model: a measure of how well it fits synthetic data. Following Fudenberg et al. (2023), we simulate synthetic data in each domain according to a set of common restrictions imposed by the models we evaluate, and assess how well each model fits the synthetic data (see Appendix F for details). We obtain a restrictiveness measure $r \in [0,1]$, where $r = 1$ means the model fits the synthetic data no better than a baseline model on average, and $r = 0$ means the model perfectly fits the synthetic data. Appendix Tables 5, 8, and 11 report this measure in each domain. While our model is less restrictive than relative thinking in multiattribute choice ($r = 0.45$ vs. 0.46), our model is *more* restrictive than cumulative prospect theory ($r = 0.60$ vs. 0.53) and hyperbolic discounting ($r = 0.59$ vs. 0.58) in lottery and intertemporal choice, respectively. These results suggest that the performance gains of our model do not come at substantial costs to model parsimony.

5 Relationship to Existing Models

Relationship to Linear Differentiation. He and Natenzon (2023) propose a *Linear Differentiation Model* (LDM) in multiattribute choice, where choice probabilities are given by

$$\rho(x,y) = G\left(\frac{U(x)-U(y)}{\sqrt{(x-y)'\Sigma(x-y)}}\right)$$

for linear utility $U(x) = \sum_k \beta_k x_k$, an $n \times n$ positive definite matrix Σ, and a continuous, strictly increasing G. Note that this model is identical to the L_1-complexity model save for the choice of distance metric in the denominator: in the LDM, dissimilarity is measured by the generalized Euclidean distance, in contrast to the L_1 distance. Below, we outline two key formal dimensions along which the models differ.

Dominance. As Theorem 1 implies, the L_1-complexity model respects dominance: the likelihood the DM chooses the superior option x over the inferior option y is maximized when x attribute-wise dominates y (written $x >_D y$). The LDM violates this property. For example, consider a parameterization of the LDM with $n = 3$, $\beta = (1,1,1)$, and $\Sigma = I$, i.e. the distance in the ratio is Euclidean: $d_{L2}(x,y) = \sqrt{\sum_k (x_k - y_k)^2}$. Consider the choice options $x = (4,0,0), x' = (-1,2,3)$, and $y = (0,0,0)$. In this case, the LDM predicts a dominance violation $\rho(x',y) > \rho(x,y)$: while both comparisons involve the same value difference, we have $d_{L2}(x,y) > d_{L2}(x',y)$.[31] In Appendix B.9, we show that such dominance violations arise for any parameterization of the LDM when there are 3 or more attributes.

Monotonicity. The L_1-complexity model also satisfies a monotonicity property, wherein $x' >_D x$ implies $\rho(x',y) \geq \rho(x,y)$: that is, improving a choice option along each attribute cannot lead to a decrease in its probability of being chosen.[32] Just as the LDM generates dominance violations, the LDM also violates monotonicity. For example, take the the parameterization of LDM discussed above and consider the options $x = (5,5,0)$, $x' = (10,5,0)$, and $y = (0,0,0)$. The LDM predicts the monotonicity violation $\rho(x',y) < \rho(x,y)$. In Appendix B.9, we show that such monotonicity violations arise generically in the LDM.

This discussion highlights a key difference in how the analyst should interpret the attributes within each model, and the types of applications appropriate for each model. As the L_1-complexity model satisfies dominance and monotonicity, it describes settings where the ranking within each attribute is unambiguous to the DM, and where choice rates principally reflect the difficulty of making tradeoffs across attributes. On the other hand, since the LDM violates dominance and monotonicity, it is better suited to applications where choice rates also reflect difficulty in processing within-attribute differences, or alternatively, reflect disagreement in a population over the valence of attributes.

Relationship to Bayesian Probit. Our multinomial choice model shares similarities with the Bayesian Probit model developed in Natenzon (2019). In this model, the DM has i.i.d. Gaussian priors over v_x, and chooses based on signals $s_x = v_x + \frac{1}{p}\epsilon_x$ received for each option in the menu, where $\epsilon_x \sim N(0,1)$ are jointly normal across options. The pairwise correlations of (ϵ_x, ϵ_y) allow the model to capture a notion of the ease of comparison between options, where x, y are more comparable if (ϵ_x, ϵ_y) are more highly correlated.

Recall that in our model, the DM only receives information on *ordinal* value comparisons. In Bayesian Probit, the DM learns about the *cardinal* value differences between choice options, which rules out certain intuitive choice patterns. For instance, consider the the following choice options in the multiattribute domain: $x = (2,0), y = (0,1), z = (0,0)$.

Since z is dominated by both x and y, we might expect that $\rho(x,z) = \rho(y,z) = 1$ and also $\rho(x,y) < 1$; that is, the DM does not err in the presence of dominance but

[31] Intuitively, the Euclidean metric penalizes the concentrated attribute difference in the comparison (x,y) more heavily than the less concentrated attribute differences in (x',y).

[32] See Proposition 9 in the Appendix. More generally, each of L_1, CDF, and CPF representations satisfy monotonicity with respect to the appropriate dominance notions; see Lemma 1 in the Appendix.

finds tradeoffs across attributes difficult.[33] Bayesian Probit cannot rationalize this choice data; $\rho(x,z) = \rho(y,z) = 1$ implies that $\text{Cor}(\epsilon_x, \epsilon_z) = \text{Cor}(\epsilon_y, \epsilon_z) = 1$, which implies $\text{Cor}(\epsilon_x, \epsilon_y) = 1$; as a result, we have $\rho(x, y) = 1$.[34]

Intuitively, in Bayesian Probit the DM receives information on the *cardinal* value difference between choice options. As such, when the Bayesian Probit DM perfectly learns the cardinal value differences $v_x - v_z$ and $v_y - v_z$, they also learn $v_x - v_y$. In our model, on the other hand, the DM only receives information on the ordinal value comparison between choice options, which allows for situations in which the DM perfectly learns that $v_x > v_z$ and $v_y > v_z$, yet remains uncertain regarding the ranking between v_x and v_y.

Importantly, this feature of Bayesian Probit means it cannot capture key choice patterns in several of the applications considered in this paper. For instance, in the application to lottery valuation, we model settings where the DM finds a given lottery trivial to compare against certain options in the price list and difficult to compare to others — for instance, the DM is certain that the lottery that pays \$10 with 80% chance is worse than \$10 for sure and finds the same lottery difficult to compare to \$5 for sure — yet perfectly understands the ranking within the price list, i.e. that \$10 is better than \$5. Bayesian Probit cannot accommodate these choice patterns, whereas our choice model can.

6 Conclusion

This paper presents a theory of tradeoff-based comparison complexity in multiattribute, lottery, and intertemporal choice. We show how our theory can be deployed to organize a range of documented biases and choice instabilities across different domains of behavioral economics, and generates novel predictions on how these patterns can be eliminated or reversed by manipulating the nature of tradeoffs. We demonstrate the predictive power of our complexity measures using large-scale binary choice data, and experimentally test the novel predictions of our theory. Here, we close by discussing limitations of the paper and potential avenues for future work.

Incorporating systematic biases. This paper shows that a parsimonious theory of noise in binary comparisons can rationalize systematic distortions that resemble commonly modeled biases. Throughout, we abstract away from potential biases in the DM's objective function

[33] The L_1-complexity model generates the choice probabilities for $\beta_1, \beta_2 > 0$, and $G(-1) = G(1) = 1$.

[34] More generally, when $\rho(x,z), \rho(y,z)$ are close but not necessary equal to 1, the Bayesian Probit model places lower bounds on the choice probability $\rho(x, y)$. As Appendix B.10 discusses, these bounds imply that there are binary choice rules rationalizable by L_1, CDF, and CPF complexity but not by Bayesian Probit.

v_x. However, even biased decision-makers presumably face tradeoffs, and so there may be value in integrating models of systematic biases — that is, models that augment v_x — with our model of tradeoff-driven complexity, and understanding their behavioral implications. Some models can be easily integrated with our approach: for instance, many non-standard multiattribute choice models simply reflect distorted attribute weights, which can directly be incorporated into the L_1 complexity measure. However, incorporating other distortions may require more involved adaptations of our measure.

Developing implications for measurement. This paper develops a theory that can help explain apparent inconsistencies across preference elicitation methods, such as the gap between certainty equivalents and binary choice found in lottery preference reversals. However, we do not view our theory nor experimental results as an indication that any one elicitation method should be *a priori* favored over others — the suitability of a given measure ultimately depends on the goals of the researcher and how response data is interpreted in service of those goals.[35] Instead, we view a theory like ours, which models how noise may manifest differently across elicitation methods, as a tool to help facilitate the interpretation of existing elicitation methods and motivate the development of new measures. We view both aims as promising directions for future work.

Extending to additional domains. Although we formulated our theory in the domains of multiattribute, lottery, and intertemporal choice, the difficulty of tradeoffs is likely a more general feature of decision-making. One straightforward extension is to adapt our L_1 complexity measure to choice under uncertainty by reinterpreting the attributes as states of the world. Another potential extension is to apply the model to derive solution concepts in normal-form games that account for tradeoff-induced noise.

Heterogeneity in tradeoff difficulty. Our theory implies that agents with identical preferences will agree on the *relative* difficulty of comparisons, since the value dissimilarity ratio is pinned down by the DM's preference parameters. However, these agents could differ in the *absolute* difficulty they face in a given comparison — there could be heterogeneity in the G transformation that maps the value-dissimilarity ratio into binary choice rates. In-

[35]For instance, even if valuations are subject to pull-to-center effects as in our model, comparing the valuations of two options could still be informative about option the DM prefers, so long as pull-to-center effects impact valuations in the same way for both options. Likewise, inference of preference intensities on the basis of binary choice rates may be counfounded by the presence of heteroskedastic noise; see McGranaghan et al. (2024) for an illustration of this idea.

tuitively, decision-makers may differ not only along their preferences, but also in how they respond to complexity, for instance due to differences in cognitive ability or attentional costs. Measuring this latter form of heterogeneity could allow researchers to better understand correlations (or lack thereof) across behavioral regularities and preference measures.

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ONLINE APPENDIX

A Appendix: Tables and Figures

Table 3: Complexity Responses vs. L_1 Ratio

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
L_1 Ratio	-0.26^{***}	-0.26^{***}	-0.19^{***}	-0.19^{***}	-0.12^{***}	-0.12^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
Global Value Difference		0.00		-0.01		-0.00
		(0.00)		(0.01)		(0.00)
(Intercept)	0.32^{***}	0.32^{***}	0.26^{***}	0.28^{***}	0.25^{***}	0.27^{***}
	(0.01)	(0.02)	(0.01)	(0.03)	(0.00)	(0.01)
R^2	0.32	0.32	0.03	0.03	0.30	0.31
Adj. R^2	0.32	0.32	0.03	0.03	0.30	0.31
Num. obs.	662	662	4880	4880	662	662

OLS Estimates. Standard errors (in parentheses) are robust. “L_1 Ratio" and “Global Value Difference" are the L_1 ratio and the monetary value difference for each choice problem.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 4: Complexity Responses vs. L_1 Ratio, No Calculator Users

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
L_1 Ratio	-0.30^{***}	-0.29^{***}	-0.20^{***}	-0.20^{***}	-0.12^{***}	-0.12^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
Global Value Difference		-0.00		-0.01		-0.00^{*}
		(0.00)		(0.01)		(0.00)
(Intercept)	0.36^{***}	0.36^{***}	0.28^{***}	0.32^{***}	0.26^{***}	0.28^{***}
	(0.01)	(0.03)	(0.01)	(0.03)	(0.01)	(0.01)
R^2	0.32	0.32	0.03	0.03	0.30	0.30
Adj. R^2	0.32	0.32	0.03	0.03	0.30	0.30
Num. obs.	662	662	4020	4020	662	662

OLS Estimates. Standard errors (in parentheses) are robust. “L_1 Ratio" and “Global Value Difference" are the L_1 ratio and the monetary value difference for each choice problem.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 5: Structural Estimates: Multiattribute Choice

	DF	BGS	Focus	RT	L_1-C 2P	L_1-C, 3P
Parameter Estimates						
δ		1				
θ			0			
ω				0.84		
ξ				1.62		
κ					0.09	0.04
γ					2.36	0.86
ψ						0.55
η	0.37	0.37	0.37	1.54		
R^2	0.001	0.001	0.001	0.339	0.323	0.357
Completeness	0	0	0	0.56	0.62	0.7
Restrictiveness	0.483	0.483	0.483	0.461	0.454	0.452
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)

"DF", "BGS", "Focus", and "RT" refer to the Distortion-Free, Salience, Focusing, and Relative Thinking models described in Appendix E.1. "L_1-C, 2P" and "L_1-C, 3P" refer to the 2 and 3 parameter L_1-Complexity models described in Appendix E.1. Completeness and Restrictiveness measures are defined in Appendix F. Standard errors for Restrictiveness estimates in parentheses.

Table 6: Complexity Responses vs. CPF Ratio

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
Global CPF Ratio	-0.51^{***}	-0.51^{***}	-0.20^{***}	-0.17^{***}	-0.07^{***}	-0.07^{***}
	(0.01)	(0.01)	(0.01)	(0.02)	(0.00)	(0.00)
Global Value Difference		-0.00		-0.01^{***}		-0.00
		(0.00)		(0.00)		(0.00)
(Intercept)	0.55^{***}	0.55^{***}	0.27^{***}	0.28^{***}	0.15^{***}	0.15^{***}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
R^2	0.59	0.59	0.02	0.03	0.22	0.22
Adj. R^2	0.59	0.59	0.02	0.03	0.22	0.22
Num. obs.	1097	1097	8290	8290	1097	1097

OLS estimates. Standard errors (in parentheses) are robust. "Global CPF" Ratio" and "Global Value Difference" are the representative-agent CPF ratio and value difference for each choice problem, computed using the value of δ estimated in the Exponential Discounting model described in Appendix E.2.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 7: Individual-Level Error Rates vs. CPF Ratio

	Dependent Variable: Binary Error (Indiv. $\hat{\delta}$)			
	(1)	(2)	(3)	(4)
Global CPF Ratio	-0.18^{***}	-0.17^{***}		
	(0.01)	(0.01)		
Indiv. CPF Ratio			-0.37^{***}	-0.32^{***}
			(0.01)	(0.01)
Indiv. Value Difference		-0.02^{***}		-0.01^{***}
		(0.00)		(0.00)
(Intercept)	0.26^{***}	0.32^{***}	0.40^{***}	0.40^{***}
	(0.00)	(0.01)	(0.01)	(0.01)
R^2	0.02	0.06	0.10	0.10
Adj. R^2	0.02	0.06	0.10	0.10
Num. obs.	41450	41450	41450	41450

OLS estimates. Standard errors (in parentheses) are robust. “Global CPF" Ratio" is the representative-agent CPF ratio for each subject-choice problem, computed using the value of δ estimated in the Exponential Discounting model described in Appendix E.2. “Indiv. CPF" Ratio" and “Indiv. Value Difference" are the individual-level CPF ratio and value difference for each subject-choice problem, computed using the individual-level δ_i estimates under the same model.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 8: Structural Estimates: Intertemporal Choice

	EDU	QDU	HDU	CPF-C
Parameter Estimates				
δ	0.95	0.96		0.96
β		0.84		
ι			0.16	
ζ			0.12	
κ				0.03
γ				0.85
η	0.35	0.41	0.43	
R^2	0.76	0.76	0.79	0.89
Completeness	0.76	0.77	0.82	0.99
Restrictiveness	0.592	0.584	0.584	0.59
	(0.001)	(0.001)	(0.001)	(0.001)

“EDU", “QDU", “HDU", and “CPF-C" refer to the Exponential Discounting, Quasi-Hyperbolic Discounting, Hyperbolic Discounting, and CPF Complexity models described in Appendix E.2. For these estimates, each time period is 24 days. Completeness and Restrictiveness measures are defined in Appendix F. Standard errors for Restrictiveness estimates in parentheses.

Table 9: Complexity Responses vs. CDF Ratio

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
Global CDF Ratio	-0.32^{***}	-0.28^{***}	-0.10^{***}	-0.11^{***}	-0.19^{***}	-0.21^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Global Value Difference		-0.01^{***}		0.00^{***}		0.01^{***}
		(0.00)		(0.00)		(0.00)
(Intercept)	0.48^{***}	0.51^{***}	0.28^{***}	0.28^{***}	0.25^{***}	0.22^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
R^2	0.45	0.47	0.10	0.11	0.31	0.35
Adj. R^2	0.45	0.47	0.10	0.11	0.31	0.35
Num. obs.	10920	10920	10420	10420	500	500

OLS estimates. Standard errors (in parentheses) are robust. "Global CDF" Ratio" and "Global Value Difference" are the representative-agent CDF ratio and value difference for each choice problem, computed using the value of α estimated in the Expected Utility model described in Appendix E.3.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 10: Individual-Level Error Rates vs. CDF Ratio

	Dependent Variable: Binary Error (Indiv. $\hat{\alpha}$)			
	(1)	(2)	(3)	(4)
Global CDF Ratio	-0.27^{***}	-0.27^{***}		
	(0.01)	(0.01)		
Indiv. CDF Ratio			-0.35^{***}	-0.35^{***}
			(0.01)	(0.01)
Indiv. Value Difference		-0.00		0.00
		(0.00)		(0.00)
(Intercept)	0.36^{***}	0.36^{***}	0.41^{***}	0.41^{***}
	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.04	0.04	0.07	0.07
Adj. R^2	0.04	0.04	0.07	0.07
Num. obs.	12500	12500	12500	12500

OLS estimates. Standard errors (in parentheses) are robust. "Global CDF" Ratio" is the representative-agent CDF ratio for each subject-choice problem, computed using the value of α estimated in the Expected Utility model described in Appendix E.3. "Indiv. CDF" Ratio" and "Indiv. Value Difference" are the individual-level CPF ratio and value difference for each subject-choice problem, computed using the individual-level α_i estimates under the same model.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 11: Structural Estimates: Lottery Choice

	EU	ST	RDEU	CPT	EV CDF-C	EU CDF-C
Parameter Estimates						
α	0.85	0.83	0.83	0.75		0.6
β			0.77	0.78		
λ			1.07	0.79		
χ				1.06		
ν				0.83		
ϕ		-2.6				
υ		0.6				
μ		0.81				
κ					0.15	0.15
γ					0.77	0.71
η	0.22	0.24	0.24	0.33		
R^2	0.54	0.56	0.55	0.59	0.65	0.72
Completeness	0.64	0.67	0.66	0.71	0.81	0.9
Restrictiveness	0.563	0.559	0.563	0.533	0.6	0.596
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

"EU", "ST", "RDEU", and "CPT" refer to the Expected Utility, Simplicity Theory, Reference-Dependent Expected Utility, and Cumulative Prospect Theory models described in Appendix E.3. "EV CDF-C" and "EU CDF-C" refer to the risk-neutral and expected utility CDF complexity models described in Appendix E.3. Completeness and Restrictiveness measures are defined in Appendix F. Standard errors for Restrictiveness estimates in parentheses.

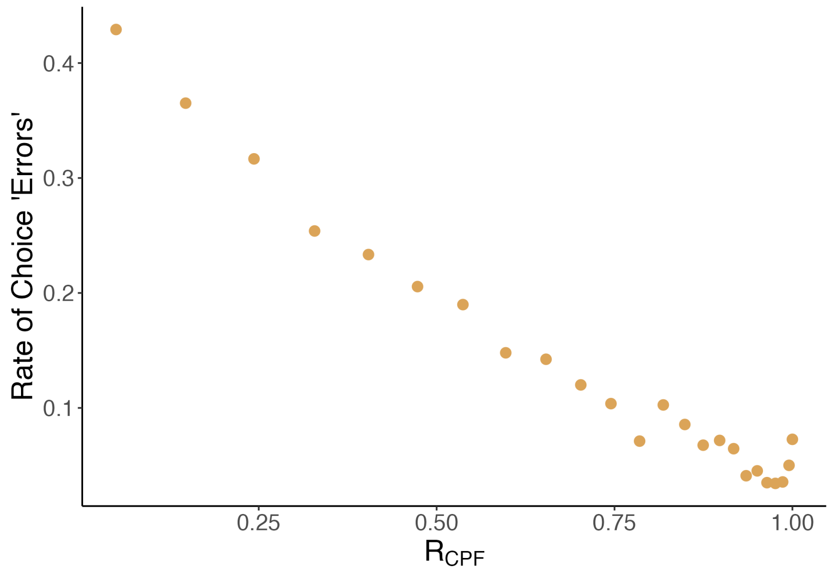


Figure 12: Binscatter of individual-level error dummies against individual-level CPF ratios. A choice is coded as an "error" if the individual chooses the lower-value option, according to the best-fit discount function estimated on their 50 experimental choices. We use the same 2-parameter structural model as in estimating global temporal preferences, which features exponential discounting and logit noise. The estimating equation is provided in Appendix E.2

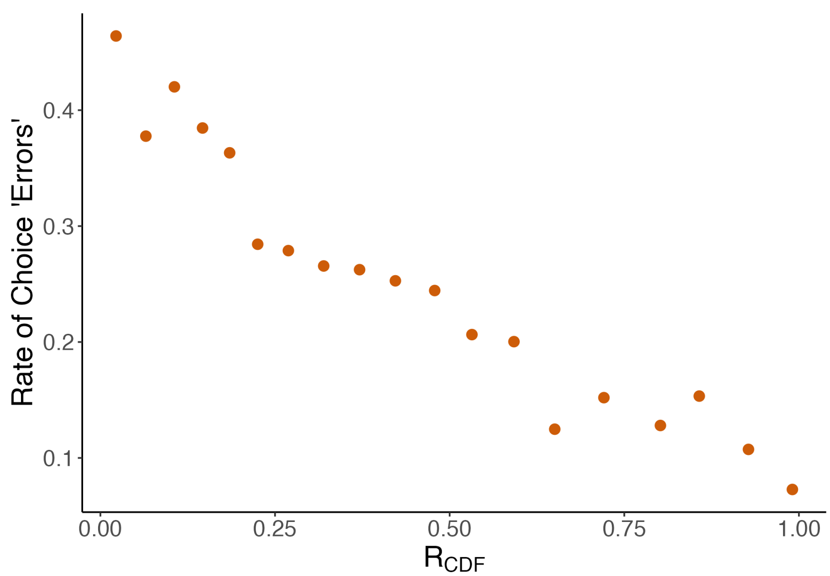


Figure 13: Binscatter of individual-level error dummies against individual-level CDF ratios. Analogously to time, a choice is coded as an "error" if the individual chooses the lower-value option, according to the utility function estimated on their 50 experimental choices. We use a 2-parameter structural model of risk preferences, which features symmetric CRRA utility and logit noise. The estimating equation is provided in Appendix E.3; we employ the parameter restriction $\alpha_i \geq 0.35$ in our estimations.

B Appendix: Additional Theoretical Results

Proofs of all results stated in this Appendix are compiled in Supplemental Appendix F.

B.1 Characterization Results

B.1.1 Multiattribute Choice: $n = 2$ Case

We state our characterization theorem in the case where $n \geq 2$. The $n = 2$ case requires an additional axiom. Let $x_{\{k\}} = x_{\{k\}}\vec{0}$.

M6. **Exchangeability**: If $\rho(x_{\{i\}}, x'_{\{j\}}) = 1/2$ and $\rho(x_{\{j\}}, x'_{\{i\}}) = 1/2$, with $x_k = x'_k = 0$ for all $k \neq i, j$, then $\rho(x, 0) = \rho(x', 0)$.

Exchangeability states that swapping attribute labels (adjusting for attribute weights) will not affect choice, and arises from the fact in our theory, the similarity in the denominator is defined over the same value-transformed attributes that govern preferences.

Theorem 2. *Suppose that all attributes are non-null. A binary choice rule ρ satisfies M1–M6 iff it has an L_1-complexity representation (G, β). If $n > 2$, ρ satisfies M1–M5 iff it has an L_1 complexity representation (G, β). Furthermore, if ρ also has an L_1-complexity representation (G', β') then $G' = G$ and there exists $C > 0$ such that $\beta' = C\beta$.*

B.1.2 Multiattribute Choice: Nonlinear Preferences

We consider a more general multiattribute domain. Each option in $X \equiv X_1 \times X_2 \times \ldots \times X_n$ is defined on n attributes, where each X_i is a connected and separable topological space. Preferences are additively separable in each attribute, where the value of each $x \in X$ is given by $U(x) = \sum_k u_k(x_k)$. Say that u_k is *non-trivial* if there exist $x_k, x'_k \in X_k$ such that $u_k(x_k) \neq u_k(x'_k)$. We propose that the ease of comparison in this domain is governed by the following representation:

Definition 5. *τ has an additively separable L_1-complexity representation if there exist continuous, non-trivial $u_i : X_i \to \mathbb{R}$ such that for $U(x) = \sum_k u_k(x_k)$ and $d_{L1}(x, y) = \sum_k |u_k(x_k) - u_k(y_k)|$, whenever $d_{L1}(x, y) \neq 0$,*

$$\tau_{xy} = H\left(\frac{|U(x) - U(y)|}{d_{L1}(x, y)}\right)$$

for H continuous, increasing with $H(0) = 0$, and $\tau_{xy} = 0$ otherwise. Similarly, a binary choice rule ρ has an additively separable L_1-complexity representation if there exist continuous, non-trivial $u_i : X_i \to \mathbb{R}$ such that whenever $d_{L1}(x, y) \neq 0$,

$$\rho(x, y) = G\left(\frac{U(x) - U(y)}{d_{L1}(x, y)}\right).$$

for G continuous, strictly increasing, and $\rho(x, y) = 1/2$ otherwise.

Note that this representation mirrors the linear L_1-complexity representation discussed in the main text, except each utility-weighted attribute value $\beta_k x_k$ is replaced by its potentially non-linear counterpart $u_k(x_k)$. We provide an axiomatic characterization for this more general representation, in which Linearity is relaxed and replaced with two axioms.

First some definitions. For $E \subseteq I$, let $x_E y$ denote the option that replaces the value of option y along attributes $k \in E$ with x_k. Say that comparisons $(x, y), (w, z) \in \mathcal{D}$ are *congruent* if for all $i \in I$, either $\rho(x_{\{i\}} y, y) \geq 1/2$ and $\rho(w_{\{i\}} z, z) \geq 1/2$, or $\rho(x_{\{i\}} y, y) \leq 1/2$ and $\rho(w_{\{i\}} z, z) \leq 1/2$. That is, if (x, y) and (w, z) are congruent, the advantages and disadvantages in the two comparisons are located in the same attributes.

M7. **Separability:** $\rho(x_E z, y_E z) = \rho(x_E z', y_E z')$ for all $x, y, z, z' \in X$, $E \subseteq I$.

M8. **Tradeoff Congruence**. Suppose that (x, y) is congruent to (y, z), and $\rho(x, y), \rho(y, z) \geq 1/2$. Then $\rho(x, z) \leq \max\{\rho(x, y), \rho(y, z)\}$.

Separability is the stochastic analog of the familiar coordinate independence axiom in deterministic choice, which says that $x_E z \succeq y_E z \implies x_E z' \succeq y_E z'$ for all $E \subseteq I, x, y, z, z' \in X$. The interpretation of Tradeoff Congruence is as follows: consider the attribute-wise tradeoffs involved in comparing z to y and x to y, where x is in fact better than y, and y is better than z. The condition says that if replacing y with x in the first comparison and replacing y with z in the second only increases the magnitude of these tradeoffs — i.e. if (x, y) and (y, z) are congruent — then (x, z) cannot be an easier comparison than both of the intermediate comparisons (x, y) and (y, z). Intuitively, both of these replacements only increase the size of the tradeoffs the DM must contend with, and so as revealed by choice probabilities, the DM cannot find the comparison (x, z) easier than both (x, y) and (y, z).

The following result states that Continuity, Moderate Transitivity, Dominance, Simplification, Separability, and Tradeoff Congruence characterize the additively separable representation, and that its primitives are identified from choice data.

Theorem 3. *Suppose that $n > 2$ and that all attributes are non-null. Then a binary choice rule ρ satisfies M1,M3–M5, M7–M8 if and only if it has an additively separable L_1-complexity representation. Morever, suppose that at least two attributes are non-null. If ρ has additively separable L_1 complexity representations $((u_i)_{i=1}^n, G)$ and $((u_i')_{i=1}^n, G')$, then there exists $C > 0$, $b_i \in \mathbb{R}$ such that $u_i' = Cu_i + b_i$ for all i, and $G' = G$.*

B.1.3 Mulitrattribute Choice: Specifying Attributes

The predictions of our theory depend on how the attributes are specified. Here, the Dominance axiom lends a simple interpretation to the attributes in the model: they are dimensions within which the ranking of values is unambiguous to the DM. That is, the DM has minimal difficulty resolving tradeoffs *within* an attribute X_i, which may itself consist of multiple features (e.g. if X_i encodes the base price of a good and its shipping fee), but struggles with tradeoffs *across* attributes X_i and X_j (e.g. if X_j is an aspect of the good's quality).

This suggests two approaches to specifying the attributes in a given application. First, the analyst can specify the attributes based on a-priori knowledge of which features are simple to trade off. Second, by observing how the DM responds to tradeoffs between features, the analyst can use choice data to recover the attributes of the model.

Formally, consider a generalized version of our additively separable representation. Options are described by a set of *features*: $X = X_1 \times X_2 \times ... \times X_n$, where each feature X_i is a connected and separable topological space. For a subset of indices $E \subseteq \{1, ..., n\}$, let $X_E = \times_{i\in E} X_i$, and for all $x \in X$, let $x_E \in X_E$ denote the restriction of x to indices E, where $(x_E)_i = x_i$. Say that the mapping $u : X_E \to \mathbb{R}$ is *trivial* if u is constant in any feature $i \in E$.

The DM forms a partition of these features into *attributes*, over which she has an L_1-complexity representation. The interpretation is that the DM has no issue with tradeoffs across features that are aggregated into a single attribute class, but struggles with tradeoffs across features belonging to different attribute classes.

Definition 6. *Say that a binary choice rule ρ has a generalized L_1-complexity representation if there exists a partition P over features $\{1, ..., n\}$ containing at least two elements, and for each $E \in P$, there exists $u_E : X_E \to \mathbb{R}$ continuous, non-trivial such that for $U(x) = \sum_{E\in P} u_E(x_E)$ and $d_{L1}(x, y) = \sum_{E\in P} |u_E(x_E) - u_E(y_E)|$, we have*

$$\rho(x, y) = G\left(\frac{U(x) - U(y)}{d_{L1}(x, y)}\right)$$

for G continuous, strictly increasing, whenever $d_{L1}(x, y) \neq 0$ and $\rho(x, y) = 1/2$ otherwise.

Example 4. (Price and Quantity). $X = \mathbb{R}^3$, where X_1 and X_2 are the base price and shipping fee, respectively, and X_3 is the quantity. Consider the following two representations for ρ:

1. ρ is represented by $(G, P, (u_E)_{E\in P})$ where $P = \{\{1\},\{2\},\{3\}\}$ and $u_{\{1\}}(x_1) = -x_1$, $u_{\{2\}}(x_2) = -x_2$, and $u_{\{3\}}(x_3) = v(x_3)$ for some concave v.

2. ρ' is represented by $(G, P', (u'_E)_{E\in P'})$ where $P = \{\{1,2\},\{3\}\}$ and $u'_{\{1,2\}}(x_1, x_2) = -(x_1 + x_2)$ and $u'_{\{3\}}(x_3) = v(x_3)$.

This corresponds to a setting where the DM's true preferences are quasilinear in total price, $V(x) = v(x_3) - x_1 - x_2$, but in the first case the DM treats the base price and shipping fee as two different attributes, and in the second, the DM aggregates the two features into a single “total price" attribute.

By observing how the DM responds to tradeoffs between X_1 and X_2, the analyst can distinguish between the two situations in the example above. More generally, the attributes in our model can be revealed using choice data, as Proposition 3 below states.

Proposition 3. *Suppose ρ is represented by $(G, P, (u_E)_{E\in P})$ and $(\tilde{G}, \tilde{P}, (\tilde{u}_E)_{E\in\tilde{P}})$. Then $G = \tilde{G}$ and $P = \tilde{P}$, and there exists $C > 0$, and $b_E \in \mathbb{R}$ such that for each $E \in P$, $\tilde{u}_E = Cu_E + b_E$.*

Supplemental Appendix J provides axiomatic foundations for the generalized L_1-complexity representation.

B.1.4 Lottery Choice

Consider the lottery choice domain, where X is the set of finite state lotteries over $\mathbb{R}$. The CDF-complexity representation for τ implies the following binary choice representation:

Definition 7. *A binary choice rule ρ has a CDF-Complexity representation if there exists $u : \mathbb{R} \to \mathbb{R}$ strictly increasing such that*

$$\rho(x, y) = G\left(\frac{EU(x) - EU(y)}{d_{CDF}(x, y)}\right)$$

for G continuous, strictly increasing.

Let $\geq$ denote the partial order on X corresponding to first-order stochastic dominance. Let $S_x = \{w \in \mathbb{R} : f_x(w) > 0\}$ denote the support of x. Consider the following axioms:

L1. **Continuity:** $\rho(x, y)$ is continuous on its domain.

L2. **Independence:** $\rho(x,y)=\rho(\lambda x+(1-\lambda)z,\lambda y+(1-\lambda)z)$ for $\lambda\in(0,1)$.

L3. **Moderate Transitivity:** If $\rho(x,y)\geq 1/2$ and $\rho(y,z)\geq 1/2$, then either $\rho(x,z)>\min\{\rho(x,y),\rho(y,z)\}$ or $\rho(x,z)=\rho(x,y)=\rho(y,z)$.

L4. **Dominance:** $x\geq y$, then $\rho(x,y)\geq\rho(w,z)$ for any $w,z\in X$, where the inequality is strict if $w\not\geq z$.

L5. **Simplification**. If $\rho(x,y)\geq 1/2$: for any $x'\in X$ with support in $S_x\cup S_y$ satisfying

(1) $F_{x'}(w^*)=F_y(w^*)$ for some $w^*\in S_x\cup S_y$,

(2) $F_{x'}(w)\neq F_x(w)$ for at most one $w\in S_x\cup S_y/\{w^*\}$,

such that $\rho(x',x)=1/2$, we have $\rho(x',y)\geq\rho(x,y)$.

Axioms L1–L4 are direct analogs of M1–M4 in the characterization of L_1 complexity. Axiom L5 says that concentrating value differences the same region of the distribution of two lotteries makes them easier to compare, and is an analog of the Simplification property (Axiom M5) for L_1 complexity. L1–L5 exhaust the behavioral content of CDF complexity.

Theorem 4. *A binary choice rule ρ satisfies L1-L5 if and only if it has a CDF-Complexity representation (G,u). Moreover, if (G',u') also represents ρ, then $G'=G$ and there exists $C>0, b\in\mathbb{R}$ such that $u'=Cu+b$.*

B.1.5 Intertemporal Choice

Consider the intertemporal choice domain, where X is the set of finite payoff streams. For a payoff flow $x\in X$, let $T_x=\{t:m_x(t)\neq 0\}$ denote the *support* of x, and for $x,y\in X$ let $T_{xy}=T_x\cup T_y\cup\{0,\infty\}$ denote the *joint support* of x and y. We consider the following extension of Definition 4 to general time discounting. Call $d:\mathbb{R}^+\cup\{+\infty\}\to\mathbb{R}^+$ a *discount function* if d is strictly decreasing and $d(\infty)=0$. We will consider discounted utility preferences of the form $DU(x)=\sum_t d(t)m_x(t)$. Note that d need not be continuous, and so can capture discontinuous time preferences such as quasi-hyperbolic discounting.

Definition 8. *τ has a generalized CPF complexity representation if there exists a discount function d such that*

$$\rho(x,y)=G\left(\frac{DU(x)-DU(y)}{d_{CPF}(x,y)}\right)$$

for H continuous, strictly increasing with $H(0) = 0$, where $d_{CPF}(x,y) = \sum_{k=0}^{n-1} |M_x(t_k) - M_y(t_k)| \cdot (d(t_k) - d(t_{k+1}))$, for $t_0 < t_1 < \ldots < t_n$ enumerating T_{xy}, is the generalized CPF distance. Similarly, a binary choice rule ρ has a generalized CPF complexity representation if

$$\rho(x,y) = G\left(\frac{DU(x) - DU(y)}{d_{CPF}(x,y)}\right)$$

for some continuous, strictly increasing G.

Note that if d is differentiable, d_{CPF} can be rewritten as $d_{CPF}(x,y) = \int_0^\infty |M_x(t) - M_y(t)| \cdot (-d'(t))\, dt$. In the case where $d(t) = \delta^t$, generalized CPF complexity reduces to Definition 4. Let $\geq$ denote the partial order X corresponding to temporal dominance (i.e., $x \geq y$ iff at every time $t \in \mathbb{R}^+ \cup \{+\infty\}$, $M_x(t) \geq M_Y(t)$). Consider the following axioms:

T1. **Continuity:** $\rho(x,y)$ is continuous on its domain.

T2. **Linearity:** $\rho(x,y) = \rho(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)z)$ for $\lambda \in (0,1)$.

T3. **Moderate Transitivity:** If $\rho(x,y) \geq 1/2$ and $\rho(y,z) \geq 1/2$, then either $\rho(x,z) > \min\{\rho(x,y), \rho(y,z)\}$ or $\rho(x,z) = \rho(x,y) = \rho(y,z)$.

T4. **Dominance:** $x \geq y$, then $\rho(x,y) \geq \rho(w,z)$ for any $w,z \in X$, where the inequality is strict if $w \not\geq z$.

T5. **Simplification**. If $\rho(x,y) \geq 1/2$, for any $x' \in X$ with support in $T_x \cup T_y$ satisfying

(1) $M_{x'}(t^*) = M_y(t^*)$ for some $t^* \in T_x \cup T_y$,

(2) $M_{x'}(t) \neq M_x(t)$ for at most one $t \in T_x \cup T_y / \{t^*\}$,

such that $\rho(x',x) = 1/2$, we have $\rho(x',y) \geq \rho(x,y)$.

Axioms T1–T4 are direct analogs of M1–M4 in the characterization of L_1 complexity. Axiom T5 says that concentrating value differences between payoff flows in the same region of time makes them easier to compare, and is an analog of the Simplification property (Axiom M5) for L_1 complexity. T1–T5 exhaust the behavioral content of CPF complexity.

Theorem 5. *A binary choice rule ρ satisfies T1 – T5 iff it has a generalized CPF-Complexity Representation (G,d). Moreover, if ρ is also represented by (G',d'), then $G' = G$, and there exists $C > 0$ such that $d' = Cd$.*

To characterize CPF complexity with exponential discounting preferences, an additional standard stationarity axiom is needed.

T6. **Stationarity**. If $\rho(x, y) > 1/2$, then for $x', y', k > 0$ such that $m_{x'}(t) = m_x(t-k)$, $m_{y'}(t) = m_y(t-k)$ for all $t \geq k$ and $m_{x'}(t) = m_{y'}(t) = 0$ for all $t < k$, $\rho(x', y') \geq 1/2$.

B.2 Relationship between CDF/CPF and L_1 Complexity

We formalize a connection between our complexity measures for lottery and intertemporal choice and our multiattribute complexity measure. In particular, we show that the CDF (CPF) complexity between two lotteries (payoff flows) is equivalent to the L_1 complexity computed over a common attribute representation of those choice options — specifically, the common attribute representation that maximizes their ease of comparison.

CPF Complexity and $\mathbf{L_1}$ Complexity. In the lottery domain, consider the set of couplings of lotteries x, y — that is, the set $\Gamma(x, y)$ of joint distributions $g(w_x, w_y)$ over payoffs such that $\sum_{w_y} g(w, w_y) = f_x(w)$ and $\sum_{w_x} g(w_x, w) = f_y(w)$ for all w. Note that each coupling g induces an attribute representation of x and y, in which the attributes are given by the set of joint utility-transformed payoff realizations in the support of g, weighted by the likelihoods of those payoff realizations.[36]

To take an example, consider a lottery x which pays \$18 w.p. 20%, and y which pays \$12 w.p. 25%, and consider the attribute structures induced by two different couplings:

	60%	20%	5%	15%
x	$u(0)$	$u(0)$	$u(18)$	$u(18)$
y	$u(0)$	$u(12)$	$u(12)$	$u(0)$

	75%	5%	20%
x	$u(0)$	$u(0)$	$u(18)$
y	$u(0)$	$u(12)$	$u(12)$

The attribute structure on the left corresponds to a coupling in which the lotteries are uncorrelated, and the attribute structure on the right corresponds to a coupling that imposes positive correlation between the lotteries. For each attribute structure induced by g, we can

[36]Such attribute representations of lotteries have been used in previous work, such as Bordalo, et al. (2012).

compute the ease of comparison under L_1 complexity, given by[37]

$$\tau_{xy}^{L1}(g) \equiv H\left(\frac{|\sum_{w_x,w_y} g(w_x,w_y)(u(w_x)-u(w_y))|}{\sum_{w_x,w_y} |g(w_x,w_y)(u(w_x)-u(w_y))|}\right).$$

Proposition 4 says that the attribute structure g that maximizes the ease of comparison according to $\tau_{xy}^{L1}(g)$ gives rise to exactly the CDF complexity representation.

Proposition 4. $\max_{g\in\Gamma(x,y)} \tau_{xy}^{L1}(g) = H\left(\frac{EU(x)-EU(y)}{d_{CDF}(x,y)}\right).$

Proposition 4 points to the following two-stage cognitive interpretation of our CDF complexity measure: in a “representation stage", the DM first represents the lotteries using a common set of attributes — specifically, the attribute structure that makes the lotteries maximally easy to compare — and then compares the lotteries along these attributes in an “evaluation stage".

CDF Complexity and $\mathbf{L_1}$ Complexity The CPF complexity measure can similarly be interpreted through this two-stage procedure. In what follows, we restrict attention to positively-valued payoff flows; i.e. $x \in X$ such that $m_x \geq 0$.

Consider a common attribute representation of payoff flows (x,y) in which the attributes are the discounted-delays of payoffs in (x,y), weighted by the payoff amount at each delay. Formally, let $B(x,y)$ denote the set of *joint payoff functions* $b : \mathbb{R}_+ \cup \{+\infty\} \times \mathbb{R}_+ \cup \{+\infty\} \to \mathbb{R}$ that map joint delays of x and y into payoff amounts, where $b(\infty,\infty) = 0$ and where $m_x(t) = \sum_{t_y} b(t,t_y)$ and $m_y(t) = \sum_{t_x} b(t_x,t)$ for all $t < \infty$; that is, the marginal payoff functions induced by b agree with those of x and y. For each attribute representation given by $b \in B(x,y)$, the ease of comparison under L_1 complexity is given by

$$\tau_{xy}^{L1}(b) \equiv H\left(\frac{|\sum_{t_x,t_y} b(t_x,t_y)(d(t_x)-d(t_y)|}{\sum_{t_x,t_y} |b(t_x,t_y)(d(t_x)-d(t_y))|}\right).$$

The following proposition states that the attribute structure g that maximizes the ease of comparison according to $\tau_{xy}^{L1}(b)$ gives rise to the CPF complexity representation.

Proposition 5. *For positively-valued payoff flows x, y, we have $\max_{b\in B(x,y)} \tau_{xy}^{L1}(b) = H\left(\frac{DU(x)-DU(y)}{d_{CPF}(x,y)}\right)$, where d_{CPF} is the generalized CPF distance.*

[37]For example, in the risk neutral case where $u(w) = w$, the ease of comparison is given by $H\left(\frac{0.05\cdot(6)+0.15\cdot(18)-0.2\cdot(12)}{0.05\cdot(6)+0.15\cdot(18)+0.2\cdot(12)}\right) = H(0.11)$ for the leftmost attribute representation and $H\left(\frac{0.20\cdot(6)-0.05\cdot(12)}{0.2\cdot(14)+0.6\cdot(4)}\right) = H(0.33)$ for the rightmost attribute representation

B.3 Tiebreaking in Multinomial Choice

Fix any choice problem (A, C). In the event that a signal s induces a tie among options that maximize posterior expected value, we assume a symmetric tiebreaking rule in which the DM randomizes between the maximal options. In particular, for any option $x \in A$ and signal realization s, let $\mathcal{N}(x,s) \equiv |\{y \in A : \mathbb{E}[v_y|s] = \mathbb{E}[v_x|s]\}|$ denote the number of options in A with the same posterior expected value as x, and define the random variable

$$c(x,s) \equiv \begin{cases} 1/\mathcal{N}(x,s) & \mathbb{E}[v_x|s] \geq \mathbb{E}[v_y|s]\,\forall\, y \in A \\ 0 & \text{otherwise} \end{cases}$$

Choice probabilities are given by

$$\rho(x,A|C) = \mathbb{E}[c(x,s)\,|\,v].$$

We assume the same tiebreaking rule in our extension to menu sequences wherein the DM independently randomizes between the maximal options in each menu. In particular, fix a choice problem $((A^1,...,A^n), C)$. For any option $x \in A^i$ and signal realization s, let $\mathcal{N}^i(x,s) \equiv |\{y \in A^i : \mathbb{E}[v_y|s] = \mathbb{E}[v_x|s]\}|$ denote the number of options in A^i with the same posterior expected value as x, and define

$$c^i(x,s) \equiv \begin{cases} 1/\mathcal{N}^i(x,s) & \mathbb{E}[v_x|s] \geq \mathbb{E}[v_y|s]\,\forall\, y \in A^i \\ 0 & \text{otherwise} \end{cases}$$

Choice probabilities are given by

$$\rho((x^1,...,x^n),(A^1,...,A^n)|C) = \mathbb{E}\left[\prod_{i=1}^{n} c^i(x^i,s)\,\middle|\, v\right].$$

B.4 Identification in Multinomial Choice

Call $\rho : X \times \mathcal{A} \times \mathcal{C} \to [0,1]$ a multinomial choice rule if $\sum_{x \in A} \rho(x,A|C) = 1$ for all $(A,C) \in \mathcal{A} \times \mathcal{C}$. Our multinomial choice model is parameterized by the prior distribution Q, the value function $v : X \to \mathbb{R}$, and the signal precisisons $\tau : \mathcal{D} \to \mathbb{R}^+$, where we make the additional assumption that $\tau(x,y) = 0$ if $v(x) = v(y)$. The following result states that v is ordinally identified and τ is exactly identified.

Proposition 6. *Suppose that a multinomial choice rule ρ is represented by (Q, v, τ) and (Q', v', τ'). Then $\tau' = \tau$ and there exists $\phi : \mathbb{R} \to \mathbb{R}$ strictly increasing such that $v' = \phi \circ v$.*

This identification result relies only on binary choice data, from which the prior distribution Q cannot be identified. We conjecture that Q can be identified using choice data from larger menus.

B.5 Decoy Effects

Example 5. (Classic decoy effects). Consider a setting where options have two attributes, where $\beta = (1, 1)$, and where τ has an L_1 complexity representation. Consider two indifferent choice options $x = (1, 2)$, $y = (2, 1)$, and consider the effect of including a phantom option on choice shares between x and y.

Case 1: $z = (1.8, 0.8)$. Since $d_{L1}(x, z) < d_{L1}(y, z)$, we have $\rho(y, x|\{z\}) > 0.5$. We recover the classic asymmetric dominance effect: the addition of an option that is dominated by the target option y but not by the competitor x distorts choice in favor of y.

Case 2: $z' = (1.5, 1.1)$. We again have $d_{L1}(x, z') < d_{L1}(y, z')$, we have $\rho(y, x|\{z'\}) > 0.5$. Here, the model predicts a "good deal" effect – z' is not dominated by either x or y, but its proximity to y makes the target option seem like a "good deal" relative to z, whereas its distance to x prevents the DM from drawing the same inference about the competing option.

Case 3: $z'' = (0.8, 0.5)$. Here, $d_{L1}(x, z'') = d_{L1}(y, z'')$, and so Corollary 1.1 implies that $\rho(y, x|\{z''\}) = 0.5$. That is, the model predicts that the addition of a mutually dominated option does not affect choice shares.

Comparison to other context-dependent models. Though each of the choice patterns above can be explained by familiar models, our model is distinct in simultaneously explaining all three. The salience (Bordalo et al., 2013) and focusing models (Kőszegi and Szeidl, 2013) cannot rationalize the decoy effects in Cases 1 and 2. The relative thinking model (Bushong et al., 2021), in which the DM weighs a given change along an attribute by less when there is a larger range of values along that attribute, can rationalize the decoy effect in Case 1 as a result of option z extending the range of attribute 2 more than attribute 1, but not Case 2, where z' has no effect on attribute ranges. The pairwise normalization model (Landry and Webb, 2021) predicts that z increases the relative of y relative to x whenever z_1/z_2 is closer to y_1/y_2 than it is to x_1/x_2, and so can rationalize the decoy effects in both Cases

1 and 2, but also delivers the prediction that the addition of a mutually dominated option z'' will also distort choice in favor of x. Furthermore, all of these models are formulated in multi-attribute choice, which means they cannot easily explain documented asymmetric dominance/decoy effects in lottery (Soltani et al., 2012) or intertemporal (Marini and Paglieri, 2019) choice. Our choice framework straightforwardly applies to lottery and intertemporal choice.

We also make a conceptual distinction from these existing models. In our model, biased choice does *not* arise from a mechanical bias, but instead as a rational response to imperfect comparability. We only expect decoy options to distort choice between x and y when their binary comparison is challenging. This is consistent with the fact that the attraction effect is muted when consumers face familiar choice contexts or have clear prior preferences (Huber et al., 2014) – a phenomenon that is not well-explained by existing models.

B.6 Intertemporal Preference Reversals: Details

Present Value Equivalents. The DM faces a valuation task (v, Z), where $v = (m_v, t_v)$ is a delayed payment that pays out $m_v > 0$ at time $t_v > 0$, to be valued against a price list $Z = \{z^1, ..., z^n\}$, where each $z^k = (m_k, 0)$ is an immediate payment. Call Z *adapted* to a delayed payment v if $m_k - m_{k+1}$ is constant in k and $m_1 = m_v$, $m_n = 0$; we restrict attention to adapted price lists. Let $PVE(v, Z) = 1/2[m_{R(v,Z)-1} + m_{R(v,Z)}]$ denote the distribution over the DM's present value equivalents obtained from assigning each switching point to a valuation at the midpoint of the adjacent prices.

Time Equivalents. The DM faces a valuation task (v, Z), where $v = (m_v, t_v)$ is a delayed payment to be valued against a price list $Z = \{z^1, ..., z^n\}$, where now each $z^k = (27.5, t_v + t_k)$ is an delayed payment; we restrict attention to time lists with $t_1 = 0$. We let

$$TE(c, Z) = \begin{cases} 1/2[t_{R(c,Z)-1} + t_{R(c,Z)}] & R(c, Z) < n + 1 \\ t_n + 1/2(t_n - t_{n-1}) & R(x, Z) = n + 1 \end{cases}$$

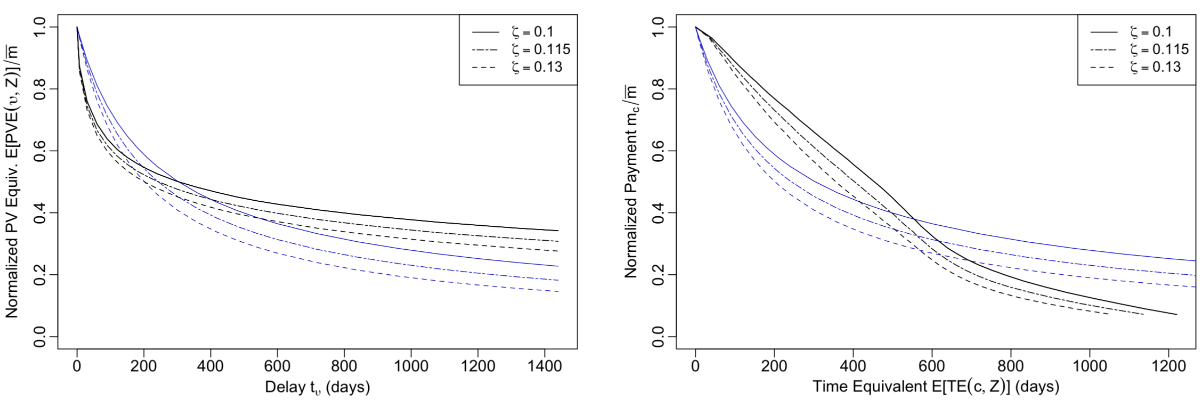
denote the distribution over the DM's time equivalents obtained from assigning each switching point to a valuation at the midpoint of the adjacent delays.

B.7 Intertemporal Valuations with Hyperbolic Preferences

We consider the intertemporal domain with hyperbolic discounting, where $v_x = DU(x) \equiv \sum_t d(t)m_x(t)$ for $d(t) = (1 + \iota t/24)^{-\zeta/\iota}$ and $\tau_{xy} = \tau_{xy}^{CPF} = H\left(\frac{DU(x)-DU(y)}{d_{CPF}(x,y)}\right)$, where d_{CPF} is the generalized CPF distance in Definition 8. Again, $H(1) = \infty$, meaning the DM perfectly learns the ranking between prospects with a temporal dominance relationship.

Figure 14 conducts the same simulation exercise as Figure 5 in the main text, except now the DM's true time preferences are given by hyperbolic discounting instead of exponential discounting. In particular, Figure 14a plots the normalized plots the normalized present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]/\overline{m}$ of a delayed payment $\upsilon = (\overline{m}, t_\upsilon)$ as a function of the delay t_υ, where the delayed payment $\upsilon = (\overline{m}, t_\upsilon)$ is valued against a price list $Z = \{z^1, ..., z^n\}$ of immediate payments adapted to υ. As the figure shows, the DM's valuations exaggerates the hyperbolicity inherent to her preferences: relative to her true discount rate (blue lines), she undervalues payments close to the present and overvalues payments with longer delays.

On the other hand, Figure 14b plots the time equivalents of an immediate payment $c = (m_c, 0)$: the predicted relationship between the immediate payment amount $m_c/\overline{m}$ (y-axis) and the associated time equivalents $\mathbb{E}[TE(c, Z)]$ (x-axis). Here, the model predicts overvaluation of payments close to the present and undervaluation of payments with longer delays relative to the DM's hyperbolic discount function.



(a) Simulated average present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ (in black) for delayed payments $\upsilon = (\overline{m}, t_\upsilon)$ as a function of t_υ.

(b) Relationship between simulated average time equivalents $\mathbb{E}[TE(c, Z)]$ (in black) for immediate payment $c = (m_c, 0)$ and normalized amount $m_c/\overline{m}$.

Figure 14: Discount functions implied by present value equivalents (left) and time equivalents (right). For PVEs, Z is adapted to υ with $|Z| = 15$. For TEs, $Z = \{z^1, ..., z^n\}$, where $z^k = (\overline{m}, t_k)$, for $(t_1, ..., t_n) = (0, 7, 30, 60, 120, 180, 240, 360, 480, 600, 720, 900, 1080, 1260, 1440)$ days. Blue curves plot distortion-free discount functions given the true hyperbolic parameters (ι, ζ), for $\iota = 0.159$, ζ varying. τ has a generalized CPF-complexity representation parameterized by $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0, 1]$.

B.8 Relationship to Random Utility

The choice probabilities generated by our L_1-complexity model shares the following intersection with random utility models:

Proposition 7. *Suppose ρ has an L_1-complexity representation (β, G), where $G(1) = G(-1) = 1$. Then, there exists an n-dimensional random vector $\tilde{\beta}$ such that for all $(x, y) \in \mathcal{D}$, $\rho(x, y) = \mathbb{P}\left\{\sum_k \tilde{\beta}_k x_k \geq \sum_k \tilde{\beta}_k y_k\right\}$, where for all k, $\mathbb{P}\left\{sgn(\tilde{\beta}_k) = sgn(\beta_k)\right\} = 1$.*

This result suggests two additional interpretations to the binary choice probabilities produced by the L_1-complexity model. First, we can interpret L_1-complexity as describing the binary choices of an agent who correctly understands the valence of each attribute – that is, the sign of each attribute weight – but has only a noisy perception over the exact attribute weights – in other words, the DM understands dominance but errs when making tradeoffs across attributes. Second, if ρ describes population-level data, we can interpret L_1-complexity as describing the choices of a population of agents who agree on the valence of each attribute, but disagree on the relative weights of attributes.

B.9 Relationship to Linear Differentiation Model

In this section, we contrast the L_1-Complexity model (Definition 2) against the Linear Differentiation Model (LDM) proposed in He and Natenzon (2023).

Definition 9. *(He and Natenzon, 2023). A binary choice rule ρ has a linear differentiation representation if there exists $\beta \in \mathbb{R}^n$, an $n \times n$ symmetric positive-definite matrix Σ, and a continuous, strictly increasing function G, such that*

$$\rho(x, y) = G\left(\frac{\beta'(x-y)}{\sqrt{(x-y)'\Sigma(x-y)}}\right).$$

From Definitions 2 and 9, it is immediate that the intersection between L_1-Complexity and the LDM is trivial. Below, we discuss two axes along which the models differ.

Dominance. As Theorem 1 implies, the L_1-Complexity model satisfies a dominance property (M4), wherein if x attribute-wise dominates y (written $x >_D y$), the choice probability $\rho(x, y)$ is maximal. Consider a weaker dominance notion, which requires only that if $x >_D y$ and $w \not>_D z$, then $\rho(x, y) \geq \rho(w, z)$. When there are three or more attributes, the

LDM fails this dominance notion[38].

Proposition 8. *Suppose ρ has a linear differentiation representation and at least 3 attributes are non-null. There exists $x, y, w, z \in \mathbb{R}$, such that $x >_D y$ and $w \not>_D z$, such that $\rho(x, y) < \rho(w, z)$.*

Monotonicity. Say that a binary choice rule ρ is *weakly monotonic* if $x' >_D x$ implies $\rho(x', y) \geq \rho(x, y)$ for any $y \in \mathbb{R}^n$: that is improving a choice option along each attribute cannot lead to a decrease in its probability of being chosen over some other choice option. The L_1-complexity model satisfies such a monotonicity property, whereas in general, the LDM violates monotonicity.

Proposition 9. *If ρ has an L_1-complexity representation, ρ is weakly monotonic. If instead ρ has a linear differentiation representation, and at least 2 attributes are non-null, then ρ is not weakly monotonic: there exists $x, x', y \in \mathbb{R}^n$ with $x' >_D x$ such that $\rho(x, y) > \rho(x', y)$.*

The intuition for the monotonicity violations produced by the LDM stem stems from a property of the model formalized in Proposition 1 from He and Natenzon (2023), which states that any x that solves $\max_x \rho(x, y)$ lies along along the same one-dimensional subset of $\mathbb{R}^n$ containing y. An immediate implication of this property is that for any x' that dominates x, we will have $\rho(x', y) < \rho(x, y)$ so long as x' does not also lie in that subset.

B.10 Relationship to Bayesian Probit

Consider the same example as in Section 6: we have $x = (2, 0), y = (0, 1), z = (0, 0)$, where $v_x = 2$, $v_y = 1$, $v_z = 0$, and $q \equiv \rho(x, z) = \rho(y, z)$. We are interested in the lower bound that the Bayesian Probit model places on $\rho(x, y)$.

Let $\sigma_{ij} = \mathrm{Cor}(\epsilon_i, \epsilon_j)$ and let Σ denote the correlation matrix between $(\epsilon_x, \epsilon_y, \epsilon_z)$. In Bayesian Probit, binary choice probabilities are given by $\rho(i, j) = \Phi\left(\frac{\sqrt{p}}{\sqrt{2}} \cdot (v_i - v_j) \cdot \frac{1}{\sqrt{1-\sigma_{ij}}}\right)$. The conditions on binary choice $\rho(x, z) = \rho(y, z) = q$, as well as the restriction that $\det(\Sigma)$

[38] When there are two attributes, there are certain parameterizations of the LDM that satisfy the dominance notion.

is positive, yields the set of restrictions

$$\sigma_{xz} = 1 - \frac{(v_x - v_z)^2 p}{2\Phi^{-1}(q)}$$
$$\sigma_{yz} = 1 - \frac{(v_y - v_z)^2 p}{2\Phi^{-1}(q)}$$
$$0 \leq 1 + 2\sigma_{xy}\sigma_{xz}\sigma_{yz} - \sigma_{xy}^2\sigma_{yz}^2\sigma_{xz}^2.$$

For any given q, p, we can numerically solve for the minimum level σ_{xy}^* satisfying the above restrictions, which in turn yields a lower bound on the choice probability $\rho(x,y)$ given by $\rho^*(x,y) = \Phi\left(\frac{\sqrt{p}}{\sqrt{2}} \cdot (v_x - v_y) \cdot \frac{1}{\sqrt{1-\sigma_{xy}^*}}\right)$, since $\rho(x,y)$ is increasing in σ_{xy}. Table 12 lists these bounds as a function of q, for a range of global precision parameters p; we see that the model cannot accomodate $\rho(x,y)$ close to $1/2$. Note that in contrast, the L_1-complexity model (with $\beta = (1,1)$) can accomodate any $\rho(x,y) \in (1/2, q)$. As such, there are binary choice rules rationalizable by L_1-complexity, and therefore our multinomial extension, that cannot be rationalized by Bayesian Probit. More generally, by considering similar examples where x and y dominate z but do not themselves have a dominance relationship in the domains of lottery and intertemporal choice, one can show that there are binary choice rules rationalizable by CDF and CPF complexity but not by Bayesian Probit.

Table 12: Bayesian Probit Bounds

	$q = 0.9$	$q = 0.92$	$q = 0.94$	$q = 0.96$	$q = 0.98$	$q = 0.99$
$p = 0.01$	0.666	0.680	0.698	0.720	0.753	0.781
$p = 0.1$	0.668	0.682	0.700	0.722	0.754	0.782
$p = 0.2$	0.670	0.685	0.702	0.724	0.756	0.783
$p = 0.4$	0.676	0.690	0.706	0.727	0.759	0.785
$p = 0.7$	0.686	0.698	0.713	0.733	0.763	0.789
$p = 1$	0.698	0.708	0.721	0.739	0.767	0.792

Minimal $\rho(x,y)$ under Bayesian Probit for $v_x = 2, v_y = 1, v_z = 0$ given $q \equiv \rho(x,z) = \rho(y,z)$ and p.

C Appendix: Proofs of Main Text Results

C.1 Characterization of L_1-Complexity

Begin with some basic definitions and observations. Let X be a space of options, and let $\mathcal{D} = \{(x,y) \in X \times X : x \neq y\}$. Say $\rho : \mathcal{D} \to [0,1]$ is a *binary choice rule* on X if $\rho(x,y) = 1 - \rho(y,x)$.

Call a (complete) binary relation $\succeq$ on X the *stochastic order* induced by a binary choice rule ρ if for all $x \neq y$, $x \succeq y$ if $\rho(x,y) \geq 1/2$, and for all $x \in X$, $x \succeq x$. Say that a binary choice rule ρ satisfies *moderate transitivity* if for $\rho(x,y), \rho(y,z) \geq 1/2$, then $\rho(x,z) > \min\{\rho(x,y), \rho(y,z)\}$ or $\rho(x,z) = \rho(x,y) = \rho(y,z)$. Say that a binary choice rule ρ satisfies *weak transitivity* if for $\rho(x,y), \rho(y,z) \geq 1/2$, $\rho(x,y) \geq 1/2$. Consider a partial order $\geq_X$ on X. Say that a binary choice rule ρ satisfies *monotonicity* with respect to $\geq_X$ if $x' \geq_X x$ implies $\rho(x',y) \geq \rho(x,y)$, where the inequality is strict whenever $x \not\geq_X x'$, $x \not\geq_X y$ and $y \not\geq_X x$. Say that ρ satisfies *dominance* with respect $\geq_X$ if whenever $x \geq_X y$, we have $\rho(x,y) \geq \rho(w,z)$ for all $w,z \in X$, where the inequality is strict if $w \not\geq_X z$.

Lemma 1. *If ρ defined on X satisfies moderate transitivity and dominance with respect to a partial order $\geq_X$, then it satisfies monotonicity with respect to $\geq_X$.*

Proof. Take any options x,y, and suppose $x' \geq_X x$. If $x \geq_X x'$, then $x' = x$ since $\geq_X$ is a partial order and is therefore antisymmetric, and we are done. Now consider the case where $x \not\geq_X x'$. Note that if $x \geq_X y$, since $\geq_X$ is transitive we also have $x' \geq_X y$, and so Dominance implies that $\rho(x',y) \geq \rho(x,y)$ and we are done.

Now consider the case where $x \not\geq_X y$. Let $\succeq$ denote the stochastic order induced by ρ; since ρ satisfies MST, $\succeq$ is complete and transitive. By dominance, we have $\rho(x',x) > \rho(x,x') \implies \rho(x',x) > 1/2$ and so $x' \succ x$. There are three cases to consider:

Case 1: $x' \succeq x \succeq y$. By moderate transitivity, $\rho(x',y) > \min\{\rho(x',x), \rho(x,y)\}$ or $\rho(x',y) = \rho(x',x) = \rho(x,y)$. But since $\rho(x',x) > \rho(x,y)$ by dominance, it must be the case that $\rho(x',y) > \rho(x,y)$.

Case 2: $x' \succeq y \succeq x$. By definition of $\succeq$, $\rho(x',y) \geq 1/2 \geq \rho(x,y)$. Also, since $x' \succ x$, we must have one of $x' \succ y$ or $y \succ x$, and so by definition of $\succeq$ we must have one of $\rho(x',y) > 1/2$ or $1/2 > \rho(x,y)$, which implies $\rho(x',y) > \rho(x,y)$.

Case 3: $y \succeq x' \succeq x$. Toward a contradiction, suppose that $\rho(y,x') > \rho(y,x)$. By moderate transitivity, we have $\rho(y,x) > \min\{\rho(y,x'), \rho(x',x)\}$ which implies $\rho(y,x) > \rho(x',x)$, which contradicts dominance and so $\rho(y,x') \leq \rho(y,x) \implies \rho(x',y) \geq \rho(x,y)$.

All that remains is to show that $\rho(x',y) > \rho(x,y)$ when $x \not\geq_X y$ and $y \not\geq_X x$. We have already shown that the inequality is strict in Cases 1 and 2; all that remains is to show that the inequality is strict in Case 3. Suppose $y \succeq x' \succeq x$. Toward a contradiction, suppose that $\rho(y,x') \geq \rho(y,x)$. Moderate transitivity then implies that either (i) $\rho(y,x) > \min\{\rho(y,x'), \rho(x',x)\}$ or (ii) $\rho(y,x) = \rho(y,x') = \rho(x',x)$. As we saw above, it cannot be the case that (i) holds. If (ii) holds, then dominance implies that $y \geq x$, a contradiction. □

For the following result, we consider the case where X is a convex set. Say ρ is *linear* if $\rho(x,y) = \rho(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)z)$ for all $x,y,z \in X$, $\lambda \in (0,1)$. Say ρ is *superadditive* if for any x,y,x',y' with $\rho(x,y), \rho(x',y') \geq 1/2$, for any $\lambda \in [0,1]$ we have $\rho(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y') \geq \min\{\rho(x,y), \rho(x',y')\}$.

Lemma 2. *Let X be a vector space. If ρ defined on X satisfies moderate transitivity and linearity, then ρ is superadditive.*

Proof. Since X is a vector space and so contains additive inverses, linearity implies $\rho(x,y) = \rho(Cx, Cy)$ and $\rho(x,y) = \rho(x-z, y-z)$ for any for any $C > 0, x,y,z \in X$. Now consider x,y,x',y' with $\rho(x,y), \rho(x',y') \geq 1/2$, and $\lambda \in [0,1]$. The above implies that

$$
\begin{aligned}
\rho(\lambda(x-y), 0) &= \rho(x,y) \geq 1/2 \\
\rho(0, -(1-\lambda)(x'-y')) &= \rho(x',y') \geq 1/2 \\
\rho(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y') &= \rho(\lambda(x-y), -(1-\lambda)(x'-y'))
\end{aligned}
$$

This, in conjunction with moderate transitivity, implies that

$$
\begin{aligned}
\rho(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y') &= \rho(\lambda(x-y), -(1-\lambda)(x'-y')) \\
&\geq \min\{\rho(\lambda(x-y), 0), \rho(0, -(1-\lambda)(x'-y'))\} \\
&= \min\{\rho(x,y), \rho(x',y')\}
\end{aligned}
$$

□

Proof of Theorem 1.

Necessity of the axioms is immediate from the definition. We now show sufficiency.

Assume that M1–M5 holds. Let $\succeq$ denote the stochastic order on $\mathbb{R}^n$ induced by ρ. By weak transitivity, $\succeq$ is transitive. Since ρ satisfies Continuity and Linearity, $\succeq$ satisfies axioms D1–D3 of Theorem 9.1 of Gilboa (2009). Invoking an intermediate step in the proof of this theorem, we conclude that there exists weights $\beta \in \mathbb{R}^n$ such that $U(x) = \sum_k \beta_k x_k$ represents $\succeq$. Since all attributes are non-null, we have that $\beta_k \neq 0$ for all k. For the remainder of the proof, we henceforth identify each option x with its weighted attribute values, so that $U(x) = \sum_k x_k$. Since ρ satisfies Dominance and MST, Lemma 1 implies that ρ satisfies monotonicity with respect to the component-wise dominance relation on $\mathbb{R}^n$.

For $z \in \mathbb{R}^n$, Let $d^+(z) = \sum_{k:z_k \geq 0} z_k$ and $d^-(x) = \sum_{k:z_k<0} |z_k|$ denote the summed advantages and disadvantages in the comparison between z and 0. Say that z has *no dominance relationship* if $d^+(z), d^-(z) > 0$.

Claim 1. For any $z \in \mathbb{R}^n$ satisfying $\sum_k z_k \geq 0$, $\rho(z,0) = \rho(d^+(z)e_1 - d^-(z)e_2, 0)$.

Proof. For $i, j \in \{1, ..., n\}$, $i \neq j$, define $z^{ij} \in \mathbb{R}^n$ satisfying

$$z_k^{ij} = \begin{cases} d^+(z) & k = i \\ -d^-(z) & k = j \\ 0 & \text{otherwise} \end{cases}$$

Note that because we have normalized utility weights 1, for all $i \neq j$, $l \neq m$, we have $U(z^{ij}) = d^+(z) - d^-(z) = U(z^{lm})$, and so $z^{ij} \sim z^{lm}$. We will first show that $\rho(z^{ij}, 0) = \rho(z^{lm}, 0)$ for all $i \neq j, l \neq m$. It is sufficient to show that for all i, j $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$. There are two cases to consider:

Case 1: $j > i$. Since $z^{1j} \sim z^{ij}$, and since $z_i^{1j} = 0$, $z_k^{1j} = z_k^{ij}$ for all $k \neq i, 1$, Simplification implies that $\rho(z^{1j}, 0) \geq \rho(z^{ij}, 0)$. Also, since $z_1^{ij} = 0$, and $z_k^{ij} = z_k^{1j}$ for all $k \neq 1, i$, Simplification implies $\rho(z^{1j}, 0) \leq \rho(z^{ij}, 0)$, and so $\rho(z^{1j}, 0) = \rho(z^{ij}, 0)$. A analogous argument yields $\rho(z^{12}, 0) = \rho(z^{1j}, 0)$, and so $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$.

Case 2: $j < i$. By analogous arguments as above, we have $\rho(z^{ij}, 0) = \rho(z^{nj}, 0)$, $\rho(z^{nj}, 0) = \rho(z^{n2}, 0)$ and $\rho(z^{n2}, 0) = \rho(z^{12}, 0)$, and so $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$ as desired.

Let $K^+ = \{i \in \{1, 2, ..., n\} : z_i \geq 0\}$ and $K^- = \{i \in \{1, 2, ..., n\} : z_i < 0\}$. Defining $\lambda_i = \frac{z_i}{\sum_{k \in K^+} z_k}$ for $i \in K^+$, and $\gamma_j = \frac{z_j}{\sum_{k \in K^-} z_k}$ for $j \in K^-$, note that $z = \sum_{i \in K^+} \sum_{j \in K^-} \lambda_i \gamma_j z^{ij}$, and so z can be expressed as a mixture of z^{ij}'s. Since ρ satisfies superadditivity by Lemma 2, by inductive application of superadditivity, we have $\rho(z, 0) \geq \rho(z^{ij}, 0)$ for all $i \neq j$, which in turn implies $\rho(z, 0) \geq \rho(z^{12}, 0)$.

Note that by repeated application of Simplification, we have $\rho(z, 0) \leq \rho(z^{ij}, 0)$, for some i where $z_i \geq 0$, and some j where $z_j \leq 0$. Since $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$, we have $\rho(z, 0) \leq \rho(z^{12}, 0)$, and so $\rho(z, 0) = \rho(z^{12}, 0)$ as desired. $\square$

Claim 2. For z with $\sum_k z_k \geq 0$, $\rho(z,0) = \tilde{G}\left(\frac{d^+(z)-d^-(z)}{d^+(z)+d^-(z)}\right)$ for some strictly increasing, continuous $\tilde{G} : [0,1] \to \mathbb{R}$.

Proof. Fix any z with $\sum_k z_k \geq 0$, and consider the case where z has no dominance relationship, that is $d^+(z) > 0$, $d^-(z) > 0$. By Claim 1, we have $\rho(z,0) = \rho(d^+(z)e_1 - d^- e_2, 0)$. Define $F : [1,\infty) \to [1/2,1)$ by $F(t) = \rho(te_1 - e_2, 0)$; by monotonicity, of ρ, F is strictly increasing. By Linearity, we have $\rho(d^+(z)e_1 - d^- e_2, 0) = \rho((d^+(z)/d^-(z))e_1 - e_2, 0) = F(d^+(z)/d^-(z))$.

Let $\varphi(z) = \frac{z-1}{z+1}$; and define $\tilde{G} : [0,1) \to \mathbb{R}$ where $\tilde{G}(z) = F(\varphi^{-1}(z))$; since φ and F are strictly increasing, $\tilde{G}$ is strictly increasing. By construction, we have $F(z) = \tilde{G}\left(\frac{z-1}{z+1}\right)$, and so $\rho(z,0) = \rho(d^+(z)e_1 - d^- e_2, 0) = \tilde{G}\left(\frac{d^+(z)-d^-(z)}{d^+(z)+d^-(z)}\right)$. Since ρ is continuous, $\tilde{G}$ is continuous on its domain $[0,1)$, and in particular is uniformly continuous since it is increasing and bounded. Take the continuous extension of $\tilde{G}$ to $[0,1]$.

Now consider the case where z has a dominance relationship; that is $d^+(z) > 0$, $d^-(z) = 0$. By Dominance, $\rho(z,0) = \rho(d^+(z)e_1 - d^- e_2, 0)$ takes on some constant value q such that $q > \rho(z', 0)$ for all z' without a dominance relationship, which implies that $q > \tilde{G}(t)$ for all $t \in [0,1)$. Since ρ is continuous, it must be the case that $q = \tilde{G}(1)$. $\square$

Now, let $G : [-1,1] \to \mathbb{R}$ be the symmetric extension of $\tilde{G}$ satisfying

$$G(z) = \begin{cases} \tilde{G}(z) & z \geq 0 \\ 1 - \tilde{G}(-z) & z < 0 \end{cases}$$

Claim 3. For any z, $\rho(z,0) = G\left(\frac{d^+(z)-d^-(z)}{d^+(z)+d^-(z)}\right)$.

Proof. Claim 1 implies that $\rho(z,0) = G\left(\frac{d^+(z)-d^-(z)}{d^+(z)+d^-(z)}\right)$ whenever $\sum_k z_k \geq 0$. Now consider the case where $\sum_k z_k < 0$. Note that

$$\begin{aligned} \rho(z,0) &= 1 - \rho(-z, 0) \\ &= 1 - \tilde{G}\left(\frac{d^-(z) - d^+(z)}{d^+(z) + d^-(z)}\right) \\ &= G\left(\frac{d^+(z) - d^-(z)}{d^+(z) + d^-(z)}\right) \end{aligned}$$

as desired, where the first equality uses symmetry and Linearity of ρ and the second equality uses Claim 2. $\square$

Take any x, y, and let $z = x - y$. Due to linearity, we have, as desired

$$\begin{aligned}
\rho(x,y) &= \rho(z,0) \\
&= G\left(\frac{d^+(z) - d^-(z)}{d^+(z) + d^-(z)}\right) \\
&= G\left(\frac{\sum_k z_k}{\sum_k |z_k|}\right) \\
&= G\left(\frac{U(x) - U(y)}{d_{L1}(x,y)}\right).
\end{aligned}$$

Finally, to show uniqueness, suppose (G, β) and (G', β') both represent ρ. Define the stochastic preference relation $\succeq$ as before. Since G and G' are both strictly increasing and symmetric around 0, $U(x) = \sum_k \beta_k x_k$ and $U'(x) = \sum_k \beta'_k x_k$ both represent $\succeq$, and so there exists $C > 0$ such that $\beta'_k = C\beta_k$. This in turn implies that for all $z \in \mathbb{R}^n$, we have

$$\begin{aligned}
G\left(\frac{\sum_k \beta_k z_k}{\sum_k |\beta_k z_k|}\right) &= G'\left(\frac{\sum_k \beta'_k z_k}{\sum_k |\beta'_k z_k|}\right) \\
&= G'\left(\frac{\sum_k \beta_k z_k}{\sum_k |\beta_k z_k|}\right)
\end{aligned}$$

Let $z = \alpha/\beta_1 e_1 + \gamma/\beta_2 e_2$. Note that for any $r \in [-1, 1]$, there exists α, γ such that $\frac{\sum_k \beta_k z_k}{\sum_k |\beta_k z_k|} = \frac{\alpha - \gamma}{|\alpha + \gamma|} = r$, and so $G'(r) = G(r)$ for all $r \in [-1, 1]$.

□

C.2 Multinomial Choice Results

We will prove our results for a more general signal structure, where

$$s_{xy} = \operatorname{sgn}(v_x - v_y) + \frac{1}{\sqrt{\tau_{xy}}} e_{xy}$$

where the e_{xy} are distributed according to a continuous distribution with density g that is symmetric around 0 and satisfies the monotone likelihood ratio property: that is $\frac{\partial}{\partial x} \frac{g(x-t)}{g(x)} > 0$ for all $t > 0$. We begin with the following basic observation:

Lemma 3. *Consider a continuous distribution with density g that is symmetric around 0 and satisfies the monotone likelihood ratio property. The function g then has the following properties:*

1. *$g'(x-t)g(x)-g(x-t)g'(x)>0$ for all $t>0$, x*

2. *g is unimodal; that is $g'(x)=-g'(-x)\leq 0$ for all $x>0$*

3. *$g(t-x)>g(-t-x)$ for any $t,x>0$*

Proof. 1) follows directly from the definition of MLRP. To see 2), towards a contradiction suppose $g'(x)>0$ for some $x>0$. Then for any $t>0$, 1) implies

$$g'(x-t)\geq \frac{g(x-t)g'(x)}{g(x)}\geq 0$$

So for any $y>x$, $g'(y)>0$. Symmetry implies that for any $y<-x$, $g'(y)<0$, and so g is not integrable, a contradiction. To see 3), note that by symmetry, $\frac{g(t-x)}{g(-t-x)}=1$ for $x=0$. MLRP of g implies that $\frac{g(t-x)}{g(-t-x)}>1$ for all $x>0$ as desired. □

The following observations pertain to finite set of options A. Enumerate A by $1,2,...,N$ and let $s=(s_{ij})_{i<j}$ collect all pairwise signals in A. Let $X^N_{(k)}$ denote the kth order statistic among N draws from the prior distribution q. Let $V^N_{(k)}=\mathbb{E}[X_{(N+1-k)}]$, that is, $V^N_{(k)}$ gives the expected value of an option if it is ranked kth. Let $\pi:A\to A$ denote a permutation function; let Π denote the set of permuation functions on A. With some abuse of notation, associate each π with the event that the v_i's are ordered according to π: that is $\pi(i)=n$ means that option i is ranked nth in the ordering. The posterior expected value of an option i given signal s is then given by

$$\mathbb{E}[v_i|s]=\sum_{n=1}^{N}V^N_{(n)}\cdot Pr(\pi(i)=n|s)$$

where

$$Pr(\pi(i)=n|s)\propto\sum_{\pi\in\Pi:\pi(i)=n}\prod_{k=1}^{N}\prod_{j<k}g\left(\sqrt{\tau_{jk}}(s_{jk}-\operatorname{sgn}(\pi(k)-\pi(j))\right)$$

Lemma 4. *Take any permutation π satisfying $\pi(i)<\pi(j)$. Then $\frac{\partial}{\partial e_{ij}}Pr(\pi|s)>0$.*

Proof. We have

$$\begin{aligned} Pr(\pi|s) &= \frac{\prod_{k<l} g\left(\sqrt{\tau_{kl}}(s_{kl}-\operatorname{sgn}(\pi(l)-\pi(k)))\right)}{\sum_{\pi'\in\Pi}\prod_{k<l} g\left(\sqrt{\tau_{kl}}(s_{kl}-\operatorname{sgn}(\pi(l)-\pi(k)))\right)} \\ &= \frac{\lambda g\left(\sqrt{\tau_{ij}}s_{ij}-\sqrt{\tau_{ij}}\right)}{\alpha g\left(\sqrt{\tau_{ij}}s_{ij}-\sqrt{\tau_{ij}}\right)+\beta g\left(\sqrt{\tau_{ij}}s_{ij}+\sqrt{\tau_{ij}}\right)} \\ &= \frac{\lambda g\left(e_{ij}+\eta-\sqrt{\tau_{ij}}\right)}{\alpha g\left(e_{ij}+\eta-\sqrt{\tau_{ij}}\right)+\beta g\left(e_{ij}+\eta+\sqrt{\tau_{ij}}\right)} \end{aligned}$$

where the $\lambda, \alpha, \beta, \eta$ are non-negative and do not depend on e_{ij}. This implies that

$$\frac{\partial}{\partial e_{ij}} Pr(\pi|s) = \frac{\partial}{\partial e_{ij}}\left(\frac{\lambda}{\alpha+\beta\frac{g\left(e_{ij}+\eta+\sqrt{\tau_{ij}}\right)}{g\left(e_{ij}+\eta-\sqrt{\tau_{ij}}\right)}}\right) > 0$$

by MLRP of g. □

Lemma 5. $1\{\mathbb{E}[v_i|s] > \mathbb{E}[v_j|s]\}$ *is increasing in* e_{ij}.

Proof. We show the stronger result that $\mathbb{E}[v_i|s]-\mathbb{E}[v_j|s]$ is increasing in s_{ij}. Note that

$$\begin{aligned} \mathbb{E}[v_i|s]-\mathbb{E}[v_j|s] &= \sum_{\pi\in\Pi}\left(V^N_{(\pi(i))}-V^N_{(\pi(j))}\right)Pr(\pi|s) \\ &= \sum_{\pi\in\Pi:\pi(i)<pi(j)}\left(V^N_{(\pi(i))}-V^N_{(\pi(j))}\right)Pr(\pi|s)+\sum_{\pi\in\Pi:\pi(i)>pi(j)}\left(V^N_{(\pi(i))}-V^N_{(\pi(j))}\right)Pr(\pi|s) \end{aligned}$$

Since $V^N_{(\pi(i))}-V^N_{(\pi(j))} > 0$ if $\pi(i) < \pi(k)$ and $V^N_{(\pi(i))}-V^N_{(\pi(j))} < 0$ otherwise, Lemma 4 implies that $\frac{\partial}{\partial e_{ij}}\left[\mathbb{E}[v_i|s]-\mathbb{E}[v_j|s]\right] > 0$. □

We now observe a result that will be useful for the proof of Proposition 1.

Lemma 6. *Consider any two options* x, y *with* $\tau_{xy} = 0$. *Then, for any* z *with* $v_z > \max\{v_y, v_x\}$ $\rho(x, y|\{z\})$ *is decreasing in* τ_{yz} *and increasing in* τ_{xz}. *Likewise, if* $v_z < \min\{v_y, v_x\}$ $\rho(x, y|\{z\})$ *is increasing in* τ_{yz} *and decreasing in* τ_{xz}.

Proof. Suppose that $v_z > \max\{v_y, v_x\}$; the proof for the case where $v_z < \max\{v_y, v_x\}$ is identical. For options i, j, k let π_{ijk} denote the the permutation that ranks i first, j second, and k last. We have

$$\begin{aligned}
Pr(\pi_{xyz}|s) &= g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}-2\sqrt{\tau_{yz}})/Pr(s)\\
Pr(\pi_{xzy}|s) &= g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz})/Pr(s)\\
Pr(\pi_{zxy}|s) &= g(e_{xz})g(e_{yz})/Pr(s)\\
Pr(\pi_{yxz}|s) &= g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}-2\sqrt{\tau_{yz}})/Pr(s)\\
Pr(\pi_{yzx}|s) &= g(e_{xz})g(e_{yz}-2\sqrt{\tau_{yz}})/Pr(s)\\
Pr(\pi_{zyx}|s) &= g(e_{xz})g(e_{yz})/Pr(s)
\end{aligned}$$

and so we have

$$\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]=\left(V^3_{(1)}-V^3_{(3)}\right)(Pr(\pi_{yzx}|s)-Pr(\pi_{xzy}|s))$$

Lemma 4 then implies that $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]$ is strictly increasing in e_{yz} and decreasing in e_{xz}. This implies that for $e^*_{yz}(\tau_{xz},e_{xz})$ defined implicitly by

$$g(e_{xz}-2\sqrt{\tau_{xz}})g(e^*_{yz}(\tau_{xz},e_{xz}))=g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})$$

for any realization of e_{xz}, we have $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]=0$ when $e_{yz}=e^*_{yz}(\tau_{xz},e_{xz})$, and so $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]>0$ whenever $e_{yz}>e^*_{yz}(\tau_{xz},e_{xz})$, and $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]\leq 0$ otherwise. Here we note three properties of $e^*_{yz}(\tau_{xz},e_{xz})$:

1. $e^*_{yz}(\tau_{xz},e_{xz})$ is strictly increasing in e_{xz}.
2. $e^*_{yz}(\tau_{xz},e_{xz})$ is decreasing in τ_{xz} whenever $e_{xz}\leq\sqrt{\tau_{xz}}$.
3. $e^*_{yz}\left(\tau_{xz},\sqrt{\tau_{xz}}\right)=\sqrt{\tau_{yz}}$, and $e^*_{yz}(\tau_{xz},e_{xz})\leq\sqrt{\tau_{yz}}$ whenever $e_{xz}\leq\sqrt{\tau_{xz}}$.

Property follows 1 by implicitly differentiating the equality $\frac{g(e_{xz}-2\sqrt{\tau_{xz}})}{g(e_{xz})}=\frac{g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})}{g(e^*_{yz}(\tau_{xz},e_{xz}))}$ and MLRP. Property 2 follows from differentiating the same equality, MLRP, and part 2 of Lemma 3. Property 3 follows from symmetry of g and Property 1. We have

$$\begin{aligned}
\rho(y;x|z)=&\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}}\int_{e_{yz}=-\infty}^{\infty}1\left\{\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]\geq 0\right\}g(e_{xz})g(e_{yz})\,de_{yz}de_{xz}\\
&+\int_{e_{xz}=\sqrt{\tau_{xz}}}^{\infty}\int_{e_{yz}=-\infty}^{\infty}1\left\{\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]\geq 0\right\}g(e_{xz})g(e_{yz})\,de_{yz}de_{xz}
\end{aligned}$$

Note that

$$
\begin{aligned}
&\int_{e_{xz}=\sqrt{\tau_{xz}}}^{\infty}\int_{e_{yz}=-\infty}^{\infty} 1\left\{\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]\geq 0\right\} g(e_{xz})g(e_{yz})\,de_{yz}de_{xz}\\
&=\int_{e_{xz}=\sqrt{\tau_{xz}}}^{\infty}\int_{e_{yz}=-\infty}^{\infty} 1\left\{g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz})-g(e_{xz})g(e_{yz}-2\sqrt{\tau_{yz}})\geq 0\right\} g(e_{xz})g(e_{yz})\,de_{yz}de_{xz}\\
&=\int_{e'_{xz}=-\infty}^{\sqrt{\tau_{xz}}}\int_{e'_{yz}=-\infty}^{\infty} 1\left\{g(e'_{xz})g(e'_{yz}-2\sqrt{\tau_{yz}})-g(e'_{xz}-2\sqrt{\tau_{xz}})g(e'_{yz})\geq 0\right\} g(e'_{xz}-2\sqrt{\tau_{xz}})g(e'_{yz}-2\sqrt{\tau_{yz}})de'_{yz}de'_{xz}\\
&=\int_{e'_{xz}=-\infty}^{\sqrt{\tau_{xz}}}\int_{e'_{yz}=-\infty}^{\infty} 1\left\{\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]\leq 0\right\} g(e'_{xz}-2\sqrt{\tau_{xz}})g(e'_{yz}-2\sqrt{\tau_{yz}})de'_{yz}de'_{xz}
\end{aligned}
$$

where the third line uses the change of variables $e'_{xz}=2\sqrt{\tau_{xz}}-e_{xz}$, $e'_{yz}=2\sqrt{\tau_{yz}}-e_{yz}$. This implies that

$$
\begin{aligned}
\rho(y;x|z)=&\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}}\int_{e_{yz}=e^*_{yz}(\tau_{xz},e_{xz})}^{\infty} g(e_{xz})g(e_{yz})\,de_{yz}de_{xz}\\
&+\int_{e'_{xz}=-\infty}^{\sqrt{\tau_{xz}}}\int_{e_{yz}=-\infty}^{e^*_{yz}(\tau_{xz},e_{xz})} g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}-2\sqrt{\tau_{yz}})\,de_{yz}de_{xz}\\
=&\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}}\int_{e_{yz}=e^*_{yz}(\tau_{xz},e_{xz})}^{\infty} g(e_{xz})g(e_{yz})\,de_{yz}de_{xz}\\
&+\int_{e_{xz}=-\infty}^{-\sqrt{\tau_{xz}}}\int_{e_{yz}=-\infty}^{e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})} g(e_{xz})g(e_{yz}-2\sqrt{\tau_{yz}})\,de_{yz}de_{xz}
\end{aligned}
$$

and so

$$
\begin{aligned}
&\frac{\partial}{\partial\tau_{xz}}\rho(y;x|z)\\
&=\frac{1}{2\sqrt{\tau_{xz}}}\left[\int_{e_{yz}=e^*_{yz}(\tau_{xz},\sqrt{\tau_{xz}})}^{\infty} g\left(\sqrt{\tau_{xz}}\right)g(e_{yz})\,de_{yz}-\int_{e_{yz}=-\infty}^{e^*_{yz}(\tau_{xz},\sqrt{\tau_{xz}})} g\left(-\sqrt{\tau_{xz}}\right)g(e_{yz}-2\sqrt{\tau_{yz}})\,de_{yz}\right]\\
&\quad+\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}}-\frac{\partial}{\partial\tau_{xz}}e^*_{yz}(\tau_{xz},e_{xz})g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}))\,de_{xz}\\
&\quad+\int_{e_{xz}=-\infty}^{-\sqrt{\tau_{xz}}}\left[\frac{\partial}{\partial\tau_{xz}}e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})+\frac{1}{\sqrt{\tau_{xz}}}\frac{\partial}{\partial e_{xz}}e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})\right]g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})-2\sqrt{\tau_{yz}})\,de_{xz}\\
&=\frac{g(\sqrt{\tau_{xz}})}{2\sqrt{\tau_{xz}}}\left[G\left(-\sqrt{\tau_{yz}}\right)-G\left(e^*_{yz}(\tau_{xz},\sqrt{\tau_{xz}}\right)-2\sqrt{\tau_{yz}})\right]\\
&\quad+\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}}-\frac{\partial}{\partial\tau_{xz}}e^*_{yz}(\tau_{xz},e_{xz})\left[g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}))-g(e_{xz}-2\sqrt{\tau_{xz}})g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})\right]de_{xz}\\
&\quad+\int_{e_{xz}=-\infty}^{-\sqrt{\tau_{xz}}}\frac{1}{\sqrt{\tau_{xz}}}\frac{\partial}{\partial e_{xz}}e^*_{yz}(\tau_{xz},e_{xz})g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})-2\sqrt{\tau_{yz}})\,de_{xz}
\end{aligned}
$$

The first term is equal to 0 since $e^*(\tau_{xz}, \sqrt{\tau_{xz}}) = \sqrt{\tau_{yx}}$. To see that the second term is non-negative, note that on the domain of integration, $\frac{\partial}{\partial \tau_{xz}} e^*_{yz}(\tau_{xz}, e_{xz}) \leq 0$ (Property 2), and $e^*_{yz}(\tau_{xz}, e_{xz}) \leq \sqrt{\tau_{yz}}$ (Property 3) and so applying part 3) of Lemma 3, $g(e^*_{yz}(\tau_{xz}, e_{xz})) \geq g(e^*_{yz}(\tau_{xz}, e_{xz}) - 2\sqrt{\tau_{yz}})$ and $g(e_{xz}) > g(e_{xz} - 2\sqrt{\tau_{xz}})$. To see that the third term is strictly positive, note that $\frac{\partial}{\partial e_{xz}} e^*_{yz}(\tau_{xz}, e_{xz}) > 0$ (Property 1). We therefore have $\frac{\partial}{\partial \tau_{xz}} \rho(y, x|\{z\}) > 0$. A symmetric argument shows that $\frac{\partial}{\partial \tau_{yz}} \rho(y, x|\{z\}) < 0$. □

Proof of Proposition 1

Suppose $v_x, v_y > v_z$, and $\tau_{yz} > \tau_{xz}$. Lemma 6 implies that if $\tau_{xy} = 0$, $\rho(y, z|\{z\}) > 1/2$. The desired result then follows from the fact that $\rho(y, z|\{z\})$ is continuous in τ_{xy}.

□

Proof of Proposition 2

Let π_k denote the ordering over $x, z^1, ..., z^n$ in which x is ranked kth and the z^j are ordered correctly, and let $p_k(s)$ denote the DM's posterior belief over π_k given signal s, where $p(s) = (p_1(s), ..., p_{n+1}(s))$. Note that

$$\mathbb{E}[v_x|s] = \sum_{k=1}^{n+1} V^N_{(k)} p_k(s)$$

$$\mathbb{E}[v_j|s] = \left(\sum_{k=1}^{j} p_k(s)\right) V^N_{(j+1)} + \left(\sum_{k=j+1}^{n+1} p_k(s)\right) V^N_{(j)} \quad \forall j = 1, ..., n$$

where $V^N_{(k)}$ is the expectation of the kth order statistic from $N = n+1$ draws over the DM's prior of Q.

To show (i), consider the case where $\tau = 0$. We have that with probability 1, $p_k(s) = 1/n+1$ for all $k \in \{1, ..., n+1\}$. Let μ denote the expectation of Q. By symmetry of Q, we have $V^N_{(k)} = 2\mu - V^N_{(N+1-k)}$ for all $k = 1, ..., N$, and so $\mathbb{E}[v_x|s] = \mu$ with probability 1.

First consider the case where n is odd, and let $j^* = \frac{n+1}{2}$. We have $\mathbb{E}[v_{j^*}|s] = \frac{1}{2} V^N_{j^*} + \frac{1}{2} V^N_{j^*+1} = \mu$ with probability 1, and so $\mathbb{E}[v_x|s] = \mathbb{E}[v_{j^*}|s]$ and $\mathbb{E}[v_x|s] \neq \mathbb{E}[v_k|s]$ for any $k \neq j^*$ with probability 1. This implies that $\mathbb{P}(R(x, Z) = j^*) = \mathbb{P}(R(x, Z) = j^* + 1) = 1/2$, and so $\mathbb{E}[R(x, Z)] = \frac{n+2}{2}$ as desired.

Now consider the case where n is even. Let $j^* = n/2$, $k^* = n/2 + 1$. Since $V^N_{(k)} = 2\mu - V^N_{(N+1-k)}$, we have $V^N_{(j^*)} > V^N_{(j^*+1)} = \mu = V^N_{(k^*)} > V^N_{(k^*+1)}$. This implies that with probability 1, $\mathbb{E}[v_{j^*}|s] = \frac{n/2}{n+1} V^N_{(j^*)+1} + \frac{n/2+1}{n+1} V^n_{(j^*)} > \mu$, and $\mathbb{E}[v_{k^*}|s] = \frac{n/2+1}{n+1} V^N_{(k^*)+1} + \frac{n/2}{n+1} V^n_{(k^*)} < \mu$, which in turn implies that $\mathbb{E}[v_{k^*}|s] < \mathbb{E}[v_x|s] < \mathbb{E}[v_{j^*}|s]$, and so we have $R(x,Z) = k^* = \frac{n+2}{2}$ with probability 1. This implies that $\mathbb{E}[R(x,Z)] = \frac{n+2}{2}$.

To show (2), Let $R(s)$ denote the DM's switching point given the signal s: that is, $R(s) = R$ if $\mathbb{E}[v_x|s] > \mathbb{E}[v_k|s]$ for all $k \geq R$ and $\mathbb{E}[v_x|s] < \mathbb{E}[v_k|s]$ for all $k < R$. Note that $R(s)$ is well defined for any $\tau > 0$ since ties in posterior expected values occur with probability 0 if $\tau > 0$. Note that there exists $\epsilon > 0$ such that whenever $p_k(s) > 1 - \epsilon$, $R(s) = k$. Since $p_{R^*(x,Z)}(s) \to_p 1$ as $\tau \to \infty$, $R(s) \to_p R^*(x,Z)$ as $\tau \to \infty$.

$\square$

D Appendix: Experiments

Here we provide more details on the design and pre-registration of our choice and valuation experiments experiments. Screenshots of experimental instructions, comprehension checks, and sample choice interfaces for these experiments are compiled in Supplemental Appendix I.

D.1 Multi-Attribute Binary Choice

Problem Selection. In our multiattribute choice experiments, we collected data on 662 choice problems in total: 582 problems in the *main* problem sample, and 80 problems in a *robustness* problem sample.

The main sample consists of 80 two-attribute problems, 432 three-attribute problems, and 104 four-attribute problems. The three-attribute choice options consist of a monthly fee, a per-GB usage rate (where the fictional consumer has a monthly usage of 6 GB), and an annual device cost; the two-attribute choice options consist of a monthly fee and usage rate, and the four-attribute choice options additionally contain a quarterly wi-fi charge. The two-attribute problems are generated by drawing a value difference (in bonus payment terms) from one of two values in $\{\$3.84, \$5.76\}$ and an L_1-ratio from one of 12 values in $\{1.00, 0.94, 0.89, 0.84, 0.80, 0.76, 0.70, 0.59, 0.48, 0.39, 0.30, 0.20\}$.[39] The three- and four-attribute problems are generated by similarly drawing a value difference and L_1 ratio value, which determines the summed attribute-wise advantages and disadvantages in the comparison, and randomizing how the advantages and disadvantages are split across the attributes.

[39]Due to rounding in the attribute values, the actual L_1 ratios deviate slightly from these values.

The robustness sample consists of 10 two-attribute problems, 60 three-attribute problems, and 10 four-attribute problems that are identical in structure to those main sample except for the attribute weights: in the robustnesss sample, the fictional consumer has a monthly usage of 12 GB. Each problem in the robustness sample is constructed to match the utility-weighted attribute values of a corresponding problem in the main sample.

Sample Collection and Screening. We collect choice data from the two problem samples in separate experiments. In the main experiment, each subject completes 50 choice problems in total: 30 randomly drawn unique three-attribute problems, 10 repeat problems drawn from these 30 unique problems, and 10 randomly drawn unique two- or four-attribute problems. Participants first complete the 40 three-attribute problems; for their last 10 problems, they will see either two- or four-attribute problems, with 30% of participants randomly assigned to the two-attribute problems and the remaining participants assigned to the four-attribute problems. The robustness experiment follows an identical structure, except that 50% of participants are randomly assigned to the two-attribute problems with the remaining participants assigned to the four-attribute problems.

Participants for both the main and robustness experiments were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. As pre-registered, data for both the main and robustness experiment were collected in waves to reach a pre-specified number of participants who did not report using a calculator in the experiment: 350 for the main experiment and 48 in the robustness experiment. In total, 428 subjects were recruited for the main experiment (357 non-calculator users) and 65 subjects were recruited for the robustness experiment (50 non-calculator users). The pre-registration for these expeirments can be accessed at `https://aspredicted.org/TNQ_XBQ`.

D.2 Intertemporal Binary Choice

Problem Selection. In our intertemporal choice experiments, we collected data on 1097 choice problems in total: 900 problems in the *broad* problem sample, and 197 problems in a *targeted* problem sample.

In the broad problem sample, choice options contain either one or two payouts; in total, there are 300 1-payout vs. 1-payout choice problems, 300 1-payout vs. 2-payout choice problems, and 300 2-payout vs. 2-payout choice problems. For each choice problem, the options are generated by sampling payout amounts and payout delays. The delays of each payout (in days) are drawn from {0, 12, 24, 48, 72, 108, 144, 180, 216, 264, 312, 360, 420, 480, 540, 600, 660, 720}, and the monetary amount of each payout is drawn from $\{\$0, \$0.50, ..., \$20\}$ for two-payout options and $\{\$0, \$0.50, ..., \$40\}$ for one-payout options. Rather than uniformly sampling from these ranges, we employ a sampling procedure that 1) undersamples dominance problems, 2) excludes problems involving very large value differences and problems near indifference, and 3) stratifies by CPF ratio and value difference (computed using a benchmark discount factor).

In the selected problem sample, problems are generated from sampling the same payout amounts and delays as for the broad problem sample, but are generated using a sampling procedure that holds fixed the threshold discount rate that makes the two options in the choice problem indifferent for a DM with exponential time preferences. In particular, 100 problems in the selected sample involve a threshold monthly discount rate of 1 (meaning that *any* individual with exponential time preferences should prefer the option that pays off earlier), and 97 problems involve a threshold monthly discount rate of 0.747. Within each of these subsamples of problems, approximately half involve 1-payout vs 2-payout options, and the other half involves 2-payout vs. 2-payout options. The sampling procedure for the selected problem sample was additionally designed to stratify by CPF ratio and to reduce variation in the value difference.

Sample Collection and Screening. In the main experiment, each subject completes 50 choice problems in total: 40 unique problems randomly drawn from the combined sample of 1100 problems, and 10 repeat problems randomly drawn from these 40 unique problems. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 829 subjects in total were recruited for the study. The pre-registration for this experiment can be accessed at `https://aspredicted.org/QCJ_S81`.

D.3 Preference Reversal and Valuation Experiments

D.3.1 Lottery Preference Reversals: Experimental Details

Price List and Choice Construction. The experiment concerns 12 lotteries: 6 "base" options consisting of 3 high-risk and 3 low-risk options as described in Table 2, and 6 "scaled-up" options constructed by multiplying the base option payments by a factor of 1.6. The experiment contains two parts, the order of which is randomized: *Binary choice* and *Valuation*.

In *Binary Choice*, subjects make 16 binary choices between lotteries, 12 of which are drawn from the 18 possible high/low risk lottery comparisons within the base and scaled-up options, and 6 of which are filler problems that are included to limit repetition. The order of problems are randomized, subject to the constraints that 1) no single choice option appears in consecutive choice problems, 2) no consecutive choice problems contain the same set of payoff probabilities, and 3) no consecutive choice problems are both filler problems.

In *Valuation*, subjects value all 12 options using one of two randomly assigned valuation modes: certainty equivalents and probability equivalents. In describing the price lists, we will denote each lottery being valued as $(\$\lambda w, p)$, where w is the base payoff amount, p is the payoff probability, and $\lambda \in \{1, 1.6\}$ is the scale factor. For certainty equivalent price lists, each lottery $(\$\lambda w, p)$ is valued against a price list $Z = (z^1, .., z^n)$ of certain payments with evenly-spaced payoff differences, i.e. $z^k = (\$\lambda[w-(k-1)d), 1)$ and $n = \lfloor w/d \rfloor$. We use $d = 0.25$ for the low-risk lotteries and $d = 1$ for the lottery $(\$\lambda \cdot 19.5, 0.23)$, and $d = 1.25$ for the remaining high-risk lotteries. For probability equivalents, each lottery $(\$\lambda w, p)$ is valued against a price list $Z = (z^1, .., z^n)$ of lotteries that pay off $\$\lambda \cdot 24$ with evenly-spaced payoff probability differences, i.e. $z^k = (\$\lambda{\cdot}24, p-(k-1)d)$ and $n = \lfloor p/d \rfloor$. We use $d = 0.05$ for the low-risk lotteries and $d = 0.01$ for the high-risk lotteries. The order of the valuation tasks is randomized subject to the constraint that no consecutive valuation tasks involve lotteries with the same payoff probability or scale factor.

If a participant is selected to win a bonus (1 in 5 chance), one part of the study (*Valuation* or *Binary Choice*) is selected at random. If *Binary Choice* is selected, subjects receive the option they chose in a randomly selected decision. If *Valuation* is selected, subjects receive the option they chose in a randomly selected decision within a randomly selected price list.

Sample Collection and Screening. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 151 subjects in total were recruited for the study. The median study completion time was 20 minutes. The pre-registration for this experiment can be accessed at `https://aspredicted.org/C62_GRK`.

D.3.2 Intertemporal Preference Reversals: Experimental Details

Price List and Choice Construction. The experiment concerns 12 delayed payments: 6 "base" options consisting of 3 high-delay and 3 low-delay options as described in Table 2, and 6 "scaled-up" options constructed by multiplying the base options payments by a scale factor of 1.6. The experiment consists of two parts, the order of which is randomized: *Binary choice* and *Valuation*.

In *Binary Choice*, subjects make 16 binary choices between delayed payments, 12 of which are drawn from the 18 possible high/low delay comparisons within the base and scaled-up options, and 6 of which are filler problems that are included to limit repetition. The order of problems are randomized, subject to the constraints that 1) no single choice option appears in consecutive choice problems, 2) no consecutive choice problems contain the same set of payoff delays, and 3) no consecutive choice problems are both filler problems.

In *Valuation*, subjects value all 12 options using one of two randomly assigned valuation modes: present value equivalents and time equivalents. In describing the price lists, we will denote each delayed payment being valued as $(\lambda m, t)$, where m is the base payoff amount, t is the payoff delay (in days), and $\lambda \in \{1, 1.6\}$ is the scale factor. For present value equivalent price lists, each option $(\$\lambda m, t)$ is valued against a price list $Z = (z^1, .., z^n)$ of immediate payments with evenly-spaced payoff differences, i.e. $z^k = (\$\lambda[w-(k-1)d), 0)$ and $n = \lfloor w/d \rfloor$. We use $d = 0.5$ for the low-delay options and $d = 1.25$ for the high-delay options. For time equivalents, each option $(\$\lambda m, t)$ is valued against a price list $Z = (z^1, .., z^n)$ of delayed payments $z^k = (\$\lambda \cdot 27.5, \tau_k)$ and where $\tau_k = t + d_k$, for $(d_1, ..., d_n) = (0, 7, 15, 30, 45, 60, 90, 120, 180, 240, 300, 360, 420, 480, 540, 600, 660, 720, 840, 960, 1080)$. The order of the valuation tasks is randomized subject to the constraint that no consecutive valuation tasks involve lotteries with the same payoff delay or scale factor.

If a participant is selected to win a bonus (1 in 10 chance), one part of the study (*Valuation* or *Binary Choice*) is selected at random. If *Binary Choice* is selected, subjects receive the option they chose in a randomly selected decision. If *Valuation* is selected, subjects receive the option they chose in a randomly selected decision within a randomly selected price list.

Sample Collection and Screening. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 152 subjects in total were recruited for the study. The median study completion time was 19 minutes. The pre-registration for this experiment can be accessed at `https://aspredicted.org/C62_GRK`.

D.3.3 Valuation Experiments: Experimental Details

Price List and Choice Construction. Subjects complete two parts of the experiment, in random order: *Risk* and *Time*. In *Risk* (*Time*), subjects complete 12 multiple price list valuation tasks using one of two randomly selected valuation modes: certainty equivalents and probability equivalents (present value equivalents and time equivalents).

For certainty equivalents, subjects value a simple lottery $l = (\overline{w}, p_l)$, against a price list $Z = \{z^1, ..., z^n\}$ of certain payments adapted to l, with $n = 19$. For probability equivalents, subjects value a certain payment $c = (w_c, 1)$ against a probability list $Z = \{z^1, ..., z^n\}$ of yardstick lotteries $z^k = (\overline{w}, p_k)$ adapted to c, with $n = 21$. We draw $\overline{w}$ from $\{\$9, \$18, \$27\}$. For certainty equivalents, we draw p_l from $\{0.03, 0.05, 0.10, 0.25, 0.5, 0.75, 0.90, 0.95, 0.97\}$. For probability equivalents, we draw w_c so that $w_c/\overline{w} \in \{0.033, 0.056, 0.11, 0.25, 0.5, 0.75, 0.89, 0.944, 0.967\}$. Subjects complete 12 price lists randomly selected from the 27 possible price lists; the order of the price lists are randomized, subject to the constraint that no consecutive price list contains the same payoff probability p_l (normalized payment $w_c/\overline{w}$) for certainty equivalents (probability equivalents).

For present value equivalents, subjects value a delayed payment $v = (\$\overline{m}, t_v)$, against a price list $Z = \{z^1, ..., z^n\}$ of immediate payments adapted to l, with $n = 21$. For time equivalents, subjects value a certain payment $c = (w_c, 1)$ against a probability list $Z = \{z^1, ..., z^n\}$ of yardstick delayed payments $z^k = (\overline{m}, t_k)$, with $(t_1, ..., t_n) = (0, 7, 15, 30, 45, 60, 90, 120, 150, 180, 240, 300, 360, 420, 480, 540, 600, 720, 840, 960, 1080)$. We draw $\overline{m}$ from $\{25, 30, 35\}$. For present value equivalents, we draw t_v from $\{7, 30, 60, 120, 240, 360, 480, 720, 1080\}$ (in days). For time equivalents, we draw m_c so that $m_c/\overline{m} \in \{0.20, 0.35, 0.50, 0.65, 0.75, 0.85, 0.90, 0.95, 0.97\}$. Subjects complete 12 price lists randomly selected from the 27 possible price lists; the order of the price lists are randomized, subject to the constraint that no consecutive price list contains the same payoff delay t_v (normalized payment $m_c/\overline{m}$) for present value equivalents (time equivalents).

If a participant is selected to win a bonus (1 in 8 chance), one part of the study (*Risk* or *Time*) is selected at random, and a price list within that part is randomly selected; Subjects receive the option they chose in a randomly selected decision within that price list.

Sample Collection and Screening. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 302 subjects in total were recruited for the study. The median completion time is 24 minutes. The pre-registration for this experiment can be accessed at `https://aspredicted.org/D8R_552`.

E Appendix: Structural Specifications

We estimate several standard models of value in multi-attribute objects, intertemporal payoffs, and lotteries, assuming logit choice probabilities:

$$\rho(x,y) = \text{sgm}_\eta(V(x) - V(y))$$

where $\text{sgm}_\eta(t) = 1/(1+\exp(-\eta t))$ is the sigmoid function for $\eta \geq 0$. For each of these models, we jointly estimate a parameterized V function and the logit noise parameter η. We also estimate our parameterized model of complexity from Section 2.4,

$$\rho(x,y) = G\left(\frac{U(x) - U(y)}{d(x,y)}\right),$$

$$G(r) = \begin{cases} (1-\kappa) - (0.5-\kappa)\dfrac{(1-r)^\gamma}{(r^\psi + (1-r)^\psi)^{1/\psi}} & r \geq 0 \\ \kappa + (0.5-\kappa)\dfrac{(1+r)^\gamma}{(r^\psi + (1-r)^\psi)^{1/\psi}} & r < 0 \end{cases}$$

for $\kappa \in [0, 0.5], \gamma, \psi > 0$. Unless stated otherwise, we will use the 2-parameter functional form of G in which we fix $\psi = 1$. In each domain, we jointly estimate the parameterized value-dissimilarity ratio and the G-function parameters κ and γ (and ψ, if applicable). Below we give the equations for each model estimated in the paper.

E.1 Multi-attribute Choice

In the following structural equations, we normalize utility the weights β_k to be equal to 1: that is, the true value of option x is given by $U(x) = \sum_k x_k$.

Distortion-Free Logit. Choice rates are given by logit noise applied to the value difference:

$$\rho(x,y) = \text{sgm}_\eta(U(x) - U(y)).$$

This model is parameterized by η.

Salience. We use the continuous salience-weighting model in Appendix C of Bordalo et al. (2013), where

$$\rho(x,y) = \mathrm{sgm}_\eta(V_{BGS}(x|\{x,y\}) - V_{BGS}(y|\{x,y\}))$$
$$V_{BGS}(x|\{x,y\}) \equiv \sum_k x_k \left(1 + \frac{|x_k - (x_k+y_k)/2|}{|x_k| + |(x_k+y_k)/2|}\right)^{1-\delta}$$

where $\delta \leq 1$. This model is parameterized by (η, δ).

Focusing. We use the parameterization proposed in Kőszegi and Szeidl (2013), where

$$\rho(x,y) = \mathrm{sgm}_\eta(V_{KS}(x|\{x,y\}) - V_{KS}(y|\{x,y\}))$$
$$V_{KS}(x|\{x,y\}) = \sum_k x_k |x_k - y_k|^\theta$$

where $\theta \geq 0$. This model is parameterized by (η, θ).

Relative Thinking. We use the parameterization proposed in Bushong et al. (2021), where

$$\rho(x,y) = \mathrm{sgm}_\eta(V_{BRS}(x|\{x,y\}) - V_{BRS}(y|\{x,y\}))$$
$$V_{BRS}(x|\{x,y\}) \equiv \sum_k x_k \left[(1-\omega) + \omega \frac{1}{|x_k - y_k| + \xi}\right]$$

where $\omega \in [0,1]$, $\xi > 0$. This model is parameterized by (η, ω, ξ).

$\mathbf{L_1}$ Complexity. Choice probabilities in our model are given by

$$\rho(x,y) = G\left(\frac{U(x) - U(y)}{d_{L1}(x,y)}\right)$$

where d_{L1} is defined as in Definition 1. We estimate both the 2 and 3 parameter versions of G; our model is parameterized by (κ, γ) for the former and (κ, γ, ψ) for the latter.

E.2 Intertemporal Choice

Exponential Discounting. Choice probabilities are given by

$$\rho(x,y) = \text{sgm}_\eta(PV(x) - PV(y))$$
$$PV \equiv \sum_t \delta^t m_x(t)$$

The parameters of the model are given by (η, δ). This model is also used for the estimation of individual-level discount factors used in Figure 12 and Table 7.

Quasi-Hyperbolic Discounting. Choice probabilities are given by:

$$\rho(x,y) = \text{sgm}_\eta(V_{qd}(x) - V_{qd}(Y))$$
$$V_{qd} \equiv \sum_{t>0} \beta\delta^t m_x(t) + m_x(0)$$

The parameters of this model are (η, δ, β).

Hyperbolic Discounting. We use the Loewenstein and Prelec (1992) discount function:

$$\rho(x,y) = \text{sgm}_\eta(V_{hd}(x) - V_{hd}(Y))$$
$$V_{hd} \equiv \sum_t (1 + \iota t)^{-\zeta/\iota} m_x(t)$$

for $\iota, \zeta > 0$. The parameters of this model are (η, ι, ζ).

CPF Complexity. Choice probabilities in our model are given by

$$\rho(x,y) = G\left(\frac{PV(x) - PV(y)}{d_{CPF}(x,y)}\right)$$

where $d_{CPF}(x,y)$ is defined as in Definition 4. Our model is parameterized by (δ, κ, γ).

E.3 Choice Under Risk

Expected Utility. To estimate the global preference parameters used in Figure 6 and Table 9, we assume agents have a Bernoulli utility function that exhibits constant relative risk aversion for both pure-gain and pure-loss lotteries:

$$\rho(x,y) = \text{sgm}_\eta(EU_{sym}(x) - EU_{sym}(y))$$
$$EU_{sym}(x) \equiv \sum_w f_x(w) u_{sym}(w)$$
$$u_{sym}(w) \equiv \begin{cases} w^\alpha & w \geq 0 \\ -(-w)^\alpha & w < 0 \end{cases}$$

for $\alpha > 0$. This model is parameterized by (η, α). This model is also used for the estimation of individual-level preferences used in Figure 13 and Table 10.

Simplicity Theory. The DM has EU preferences, but pay penalize (or favor) lotteries with larger support. We follow Puri (2025) in paramterizing the penalization term:

$$\rho(x,y) = \text{sgm}_\eta(V_{st}(x) - V_{st}(y))$$
$$V_{st}(x) = EU_{sym}(x) + CA(|S_x|)$$
$$CA(s) = \frac{\phi}{1 + \exp(\upsilon(s - \mu))} - \frac{\phi}{1 + \exp(\upsilon(1 - \mu))}$$

for $\alpha > 0$. This model is parameterized by $(\eta, \alpha, \phi, \upsilon, \mu)$.

Reference-Dependence. The DM has expected utility preferences, where the (two parameter) Bernoulli utility function allows for separate curvature parameters for positive and negative payouts, with a loss-aversion parameter λ.

$$\rho(x,y) = \text{sgm}_\eta(EU_{rd}(x) - EU_{rd}(y))$$
$$EU_{rd}(x) \equiv \sum_w f_x(w) u_{rd}(w)$$
$$u_{rd}(w) \equiv \begin{cases} w^\alpha & w \geq 0 \\ -\lambda(-w)^\beta & w < 0 \end{cases}$$

for $\alpha, \beta > 0$. This model is paramaterized by $(\eta, \alpha, \beta, \lambda)$.

Cumulative Prospect Theory. We also estimate a model where the agent exhibits probability weighting and loss aversion, following Tversky and Kahneman (1992). We use the probability weighting function given by Gonzalez and Wu (1999). Let the distinct payoffs in a lottery x be ordered by $w_{-m},...,w_{-1},w_0,w_1,...,w_n$, where $w_{-m},...,w_0$ indicate negative payoffs and $w_0,...,w_n$ indicate positive payoffs, with $p_{-m},...,p_n$ denoting the associated probabilities. The value of x is given by

$$
\begin{aligned}
V_{cpt}(x) &= \sum_{k=-m}^{0} u_{pt}(w_k)\pi_k + \sum_{k=0}^{n} u_{pt}(w_k)\pi_k, \\
&\pi_n = q(p_n),\ \pi_{-m} = q(p_{-m}) \\
&\pi_k = q(p_k + ... + p_n) - q(p_{k+1} + ... + p_n), \quad 0 \le k < n \\
&\pi_k = q(p_{-m} + ... + p_k) - q(p_{-m} + ... + p_{k-1}), \quad -m < k < 0 \\
q(p) &= \frac{\chi p^{\nu}}{\chi p^{\nu} + (1-p)^{\nu}} \\
u_{pt}(w) &\equiv \begin{cases} w^{\alpha} & w \ge 0 \\ -\lambda(-w)^{\beta} & w < 0 \end{cases}
\end{aligned}
$$

for $\alpha, \beta, \chi, \nu, \lambda > 0$. Choice probabilities are given by

$$\rho(x,y) = \mathrm{sgm}_{\eta}(V_{cpt}(x) - V_{cpt}(y))$$

This model is parameterized by $(\eta, \alpha, \beta, \chi, \nu, \lambda)$.

CDF Complexity. We estimate two versions of our model: one that assumes risk neutrality, and one that allows for utility curvature. In the risk neutral model, we have

$$\rho(x,y) = G\left(\frac{EV(x) - EV(y)}{d_{CDF}(x,y)}\right)$$

$EV(x) = \sum_w w f_x(w)$ and d_{CDF} are defined as in Definition 3 with the Bernoulli utility function $u(x) = x$. This model is parameterized by (κ, γ).

In the model that allows for utility curvature, we have

$$\rho(x,y) = G\left(\frac{EU(x) - EU(y)}{d_{CDF}(x,y)}\right)$$

$EU(x) = \sum_w u_{sym}(w) f_x(w)$ and d_{CDF} are defined as in Definition 3 with the Bernoulli utility function $u = u_{sym}$. This model is parameterized by (κ, γ, α).

F Appendix: Model Completeness and Restrictiveness

We adapt the completeness and restrictiveness measures in Fudenberg et al. (2022, 2023) to our setting. We have a set of binary choice problems $\mathcal{C}$, where each $c \in \mathcal{C}$ is a vector of objective problem features. For each $c \in \mathcal{C}$ there is an associated outcome $q \in \Delta(\{a, b\})$ – a distribution over the options in the choice problem; let $\mathcal{Q}$ denote the set of such distributions. Abusing notation, we will also let $q \in [0, 1]$ identify the rate of choosing option a.

In our actual dataset, there is a joint distribution over $\mathcal{C} \times \mathcal{Q}$ given by μ. Letting $\mu_{\mathcal{C}}$ denote the marginal distribution over the choice problems in our dataset, we have $\mu_{\mathcal{C}}(c) = n_c / \sum_{c' \in \mathcal{C}} n'_c$, where n_c denotes the number of observations for choice problem c, and letting $\mu_{\mathcal{Q}|\mathcal{C}}$ denote the conditional distribution over $\{\delta_a, \delta_b\}$ in our choice set, we have

$$\mu_{\mathcal{Q}|\mathcal{C}}(q|c) = \begin{cases} r(c) & q = \delta_a \\ 1 - r(c) & q = \delta_b \\ 0 & \text{otherwise} \end{cases}$$

where $r(c)$ denotes the empirical choice rate for option a for choice problem c.

We consider prediction rules of the form $p : \mathcal{C} \to \mathcal{Q}$ and denote the set of such functions by $\overline{\mathcal{P}}$; $p(c)$ denotes the rate of choosing option a in choice problem c under prediction rule p. We are interested in evaluating the completeness and restrictiveness of a parametric model $\mathcal{P}_\Theta = \{p_\theta\}_{\theta \in \Theta}$. Completeness and restrictiveness are evaluated with respect to a base model p^b: we take p^b to be the constant prediction rule $p^b(c) = \sum_{c \in \mathcal{C}} \frac{n_c}{\sum_{c' \in \mathcal{C}} n_{c'}} r(c)$ outputting the average choice rate in lottery and intertemporal choice, and the fitted Distortion-Free Logit model in multiattribute choice.[40]

F.1 Completeness

Definition. Let $l(q, q') = -[q' \log(q) + (1 - q') \log(1 - q)]$ denote the negative log-likelihood loss function. Analogous to the maximum-likelihood estimates discussed in Section 4.4, we

[40]In multiattribute choice, we use the Distortion-Free Logit model instead of the constant predictor as the base model since the constant predictor has *lower* prediction loss than Distortion-Free Logit; in the framework of Fudenberg et al. (2022, 2023), the base model should have higher loss than the model under evaluation.

measure the prediction loss of a model p by the expected negative log-likelihood:

$$\begin{aligned} e(p) &= \mathbb{E}_{\mu}[l(p(c), r)] \\ &= -\frac{1}{\sum_{c\in\mathcal{C}} n_c} \sum_{c\in\mathcal{C}} [n_c r(c) \log(p(c)) + n_c(1-r(c)) \log(1-p(c))] \end{aligned}$$

Let p^* denote the prediction rule that minimizes expected loss in the data, i.e. $p^* \in \arg\min_{p\in\overline{\mathcal{P}}} e(p)$. The completeness of a model $\mathcal{P}_\Theta$ is defined by

$$\kappa(\mathcal{P}_\Theta) = \frac{e(p_{base}) - \min_{p\in\mathcal{P}_\Theta} e(p_\theta)}{e(p_{base}) - e(p^*)}$$

Implementation. To form p^*, we use an ensemble approach to build a predictor that maps problem features into choice probabilities, where we combine parametric model predictions with those of a neural network trained on the data. This ensemble predictor is described in detail in Supplemental Appendix G.

F.2 Restrictiveness

Definition. Restrictiveness captures the degree to which a model is able to fit to pre-defined synthetic data – the better the fit, the less restrictive the model. This synthetic data is defined by the *admissable set* of prediction rules $\mathcal{P} \subseteq \overline{\mathcal{P}}$, characterized by restrictions so that $\mathcal{P}$ constitutes “reasonable" choice data, as absent restrictions on the admissable set, any model satisfying basic restrictions could have high restrictiveness. Following Fudenberg et al. (2023), we impose restrictions that are shared by the class of models we estimate:

1. *Weak Dominance*: If $x > y$, then $\rho(x, y) \geq 1/2$
2. *Monotonicity*: If $x > x'$, then $\rho(x', y) > \rho(x, y)$

Where $>$ denotes the domain-specific dominance notion.[41] Every model in Section E satisfies Weak Dominance except for Simplicity Theory, and every model satisfies Monotonicity except for Simplicity Theory and the Salience model.

[41]That is, $>$ denotes attribute-wise dominance, first-order-stochastic dominance, and temporal dominance in multiattribute, lottery, and intertemporal choice, respectively.

Let $d : \overline{\mathcal{P}} \times \overline{\mathcal{P}} \to \mathbb{R}_+$ denote the expected Kullback-Leibler divergence

$$\begin{aligned} d(p, p') &= \mathbb{E}_{\mu_C}[D(p'(c)||p(c))] \\ &= -\frac{1}{\sum_{c \in C} n_c} \sum_{x \in C} \left[n_c p'(c) \log\left(\frac{p(c)}{p'(c)}\right) + n_c (1 - p'(c)) \log\left(\frac{1 - p(c)}{1 - p'(c)}\right) \right]. \end{aligned}$$

Letting $\lambda_{\mathcal{P}}$ denote the uniform distribution on $\mathcal{P}$ and denoting $d(\mathcal{P}_\Theta, p) = \inf_{\theta \in \Theta} d(p_\theta, p)$, the restrictiveness of model $\mathcal{P}_\Theta$ is defined as

$$r(\mathcal{P}_\Theta) \equiv \frac{\mathbb{E}_{\lambda_{\mathcal{P}}}[d(\mathcal{P}_\Theta, p)]}{\mathbb{E}_{\lambda_{\mathcal{P}}}[d(p_{base}, p)]}$$

Implementation. Following (Fudenberg et al., 2023), we generate $K = 1000$ synthetic datasets by taking K i.i.d. samples from $\lambda_{\mathcal{P}}$, denoted $\{p^k\}_{k=1}^K$, and form the estimator

$$\hat{r}(\mathcal{P}_\Theta) = \frac{\frac{1}{K} \sum_{k=1}^K d(\mathcal{P}_\Theta, p^k)}{\frac{1}{K} \sum_{k=1}^K d(p_{base}, p^k)}$$

Standard errors for $\hat{r}(\mathcal{P}_\Theta)$ are computed following Fudenberg et al. (2023).

Due to the high-dimensional nature of restrictions characterizing $\mathcal{P}$ – the Monotonicity restriction generates 648, 6616, and 93529 independent pairwise inequality constraints between choice rates in our multiattribute, intertemporal, and lottery choice data, respectively – standard methods of sampling uniformly from $\mathcal{P}$ such as rejection sampling are computationally infeasible. Instead, we approximate uniform draws from $\mathcal{P}$ by implementing a hit-and-run sampler (Smith, 1984), a Markov chain Monte-Carlo algorithm designed to efficiently sample from a uniform distribution over a convex body. Supplemental Appendix H describes this sampling procedure in full.