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A SEMI-VIRTUAL WATERSHED MODEL BY NEURAL NETWORKS

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Abstract: *A semi-virtual watershed model is presented in this study. This model places the design rainfall distribution on the input layer and the predicted runoff hydrograph on the output layer. The optimization scheme developed in this study can train the model to establish a set of weights under the guidance of the kinematic wave theory. The weights are time dependent variables by which rainfall signals can be converted to runoff distributions by weighting procedures only. With the consideration of time dependence, the computational efficiency of virtual watershed models is greatly enhanced by eliminating unnecessary visitations between layers. The weighting procedure used in the semi-virtual watershed model expands the Rational method from peak runoff predictions to complete hydrograph predictions under continuous and non-uniform rainfall events.*

INTRODUCTION

Continuous processes in a hydrologic cycle can be simulated by a chain of systems. A system consists of input, output, and throughput elements[2]. For instance, a watershed is a system with rainfall as input, runoff as output, and drainage network as throughput. A detention basin located at the outfall point of a watershed will receive the runoff hydrograph as an input element. The operation of the detention basin is the throughput process, and the outflow hydrograph released from the detention basin becomes another input to the downstream outfall channel system. Hydrologic models reproduce the responses between input-output pairs through the imitations of hydrologic processes. Mathematically, a set of physical laws and numerical algorithms are employed to describe the natural process in a slow motion fashion, i.e. one step at a time or a finite distance. System identification is a calibration process using the observed input-output pairs to determine the values of system constants and to confirm the applicability of the assumed functional relations. To model a hydrologic process, the physical layout is converted into a node-link illustration in which nodes represent design points to receive inputs to the system, and links represent flow elements to convey signals through the system. Nodes and links are connected by following physical laws described by mathematical algorithms. Reliability of a hydrologic model depends on the consistent reproducibility between input and output pairs. Similarly, a neural network system consists of an input layer, output layer, and hidden layers[8, 1]. Neurons on the input layer receives input information and pass them to the neurons on the hidden layers by means of weighting procedures, but not by laws or equations in physics [10]. The learning process of a neural network uses back-propagation training procedures which require historical or desired outputs to adjust the weights among neurons. The performance of a neural computing network is examined by the data processing using the test sets. Architecture of a neural network is similar to a hydrologic system, but the operation is different. It applies random weighting procedures using a virtual watershed other than calculations by governing equations of water movements through an actual watershed.

This study presents a semi-virtual watershed model which applies a combined approach to model the rainfall/runoff processes. The virtuosity of this watershed model is due to the runoff prediction algorithm that only applies weights to rainfall blocks as inputs without using any flow movement equation, and the actuality of this watershed model is due to the derivation of its weights which are determined under the guidance of the kinematic wave approach. This model places a non-uniform rainfall distribution on the input layer, and the outflow hydrograph on output layer. The weights can be identified during a training process. Although the time of concentration and runoff coefficient are employed as system identifiers, both of them are derived from the weighting process, not from any hydrologic empirical formula. Once the system is identified by the training process, the runoff predictions can be achieved by weighting procedures only. The weighting procedure derived for the semi-virtual watershed model involves an optimization scheme. In this study, the conventional Rational method has been expanded by the neuro-weighting procedures from peak runoff predictions to complete hydrograph predictions for continuous non-uniform rainfall events.

VIRTUAL WATERSHED MODEL BY NEURAL NETWORK

A neuron serves as a node to receive input signals, evaluates the strength of the signals, and determines how to respond to the input signal. To simulate a rainfall/runoff event, the multiple input signals include the incremental rainfall amounts and hydrologic losses as time-depend variables. The input signal is expressed as

$$I = (P(\Delta t), P(2\Delta t), P(3dt), \ldots\ldots\ldots\ldots, P(n\Delta t)) \tag{1}$$

in which I = input vectors in time including incremental rainfall depth P(Δt), P(2Δt) etc., Δt = incremental time interval, and n = the maximum number of input. The outputs are represented by

$$O = (Q(\Delta t), Q(2\Delta t), Q(3\Delta t), ..., Q(m\Delta t)) \tag{2}$$

In which O = output vectors including runoff rates Q(Δt), Q(2Δt) etc., and m = the maximum number of output. Neuron computing approach is to mimic the process using a multiple dimensional interconnection network among nodes which can be established during the training. Between inputs and outputs, the computing procedure used in this study is defined by a matrix of weights, W(t, T):

$$Q(T) = \sum_{i=1}^{i=n} P(t)W(t,T) \qquad 0 \le T \le T_B \tag{3}$$

and

$$t = i\Delta t \tag{4}$$

in which Q(T) = runoff rate at the specified time, T, and T_B = base time on the hydrograph. Eq 3 is a matrix operation with rainfall and runoff vectors. Each neuron produces one and the only output if the input exceeds a threshold value. For instance, a positive impulse implies that a rainfall input has an intensity exceeding the infiltration losses, and a negative impulse implies that the weak rainfall input does not produce any runoff during the computing time interval. Unlike hydrologic models, the neural network approach applied weighting procedures which does not require complete and accurate knowledge about the relations among parameters involved in the process. However, as Carriere et al.[1] indicated, the regular back-propagation neural network can not be used for the rainfall/runoff processes because they are highly time dependent. Back propagation neural networks are static neural models and will not perform well for a process in which time dependency is a major factor. Many times a visitation to a layer resulted from random process in a neural network ignores the order of time sequence. This means that the later rainfall blocks are still considered in the neural computing process to have possible contribution to earlier runoff rates. In fact, the time sequence is a factor of great importance in the rainfall/runoff process because it defines the tributary watershed area and contributing rainfall blocks on a hyetograph to the runoff rate at a specified time. Haykin[5] suggested that in a spatio-temporal network, a feedback process is essential to capture the nature of the process. In this study, the nature of the rainfall/runoff process is derived by a compatible hydrologic model using the kinematic wave approach. Comparison between the neural network and hydrologic model, provides a guidance to derive the weights. With this guidance, a matrix of weights can be established using the training sets and then applied to test sets of data.

DETERMINATION OF WEIGHTS BY KINEMATIC WAVE

Complexities of this process result in a wide diversity of modeling techniques. Rainfall/runoff models can be as complicated as the dynamic wave approach which considers the spatial and temporal variations in the rainfall distribution, or as simple as the kinematic wave approach. The kinematic wave approach assumes that the gravitational force in an overland flow is balanced by the friction force[13]. Solutions for kinematic wave from a unit-width catchment take a form similar to the rating curve relationship[12], namely:

$$q = \alpha y^m \tag{5}$$

in which q = flow rate per unit width, y = runoff depth α = conveyance factor that is determined by the overland slope and surface roughness, and m = empirical exponent. The total derivative for the unit-width runoff rate, q, is expressed as[6] :

$$\frac{dq}{dt} = V_w \bar{I}_e \tag{6}$$

in which t = time variable, $V_w =$ celerity of kinematic wave, $\bar{I}_e$= net average rainfall intensity. The celerity of the kinematic wave in Eq 6 is

$$\frac{dx}{dt} = V_w = \alpha m y^{m-1} \tag{7}$$

in which x = distance along the flow path. The initial condition and boundary condition at time, T, for Eq's 6 and 7 are[7]

$$q(T) = 0 \quad for \quad 0 \le x \le L \quad for \quad T = 0 \tag{8}$$

$$q(T) = 0 \quad for \quad x = 0 \qquad for \quad T \geq 0 \tag{9}$$

As shown in Figure 1, the rainfall amount contributing to the runoff rate, q(t), is confined by the drainage area in terms of its waterway length, i.e. from x = 0 to x = L.

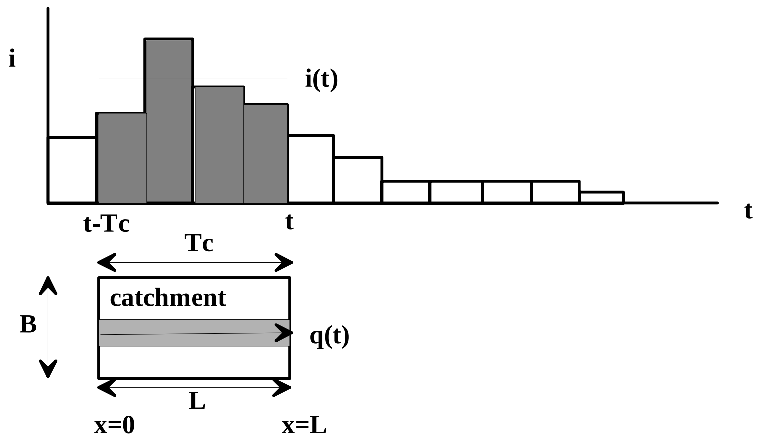

Figure 1 Illustration of Contributing Rainfall Blocks

This waterway length is then converted to its equivalent flow time, i.e. the time of concentration of the catchment, T_c. At time, T, and $T>T_c$, the runoff rate, q(T), is determined by the rainfall excess between time, T, and $(T-T_c)$ [9, 4]. As a result, Eq's 6 and 7 may be integrated as:

$$L = \int_{t=(T-T_c)}^{t=T} amy^{m-1}dt \tag{10}$$

$$q(T) = \int_{t=(T-T_c)}^{t=T} V_m \bar{I}_e(T)dt = \bar{I}_e(T) \int_{t=(T-T_c)}^{t=T} amy^{m-1}dt = L\,\bar{I}_e(T) \tag{11}$$

As suggested by EPA SWMM [3], an urban catchment can be converted into an equivalent rectangular watershed with a width, B and length, L. To expand the runoff per unit width to the entire watershed yields

$$Q(T) = Bq(T) = BL\,\bar{I}_e(T) = A\,\bar{I}_e(T) \tag{12}$$

in which O(T) = runoff output, B= catchment width, and A = catchment area. The net rainfall intensity serves as an indicator for determining the strength of input. Numerically, there are many methods available to perform rainfall reductions. For simplicity, the runoff coefficient is used as an example for deriving the net precipitation depths as follows:

$$P_e(t) = C\,P(t) \tag{13}$$

and

$$\bar{I}_e(T) = \frac{1}{T_c} \sum_{t=(T-T_c)}^{t=T} CP(t) \tag{14}$$

in which $P_e(t)$ = net rainfall depth at time t, C = runoff coefficient. Substituting Eq's 13 and 14 into Eq 12 yields

$$Q(T) = \frac{1}{T_c} \sum_{t=(T-T_c)}^{t=T} ACP(t) \qquad for \quad T_d \geq T \geq T_c \tag{15}$$

Comparing Eq 15 with Eq 3, the weights are:

$$W(t,T) = \frac{CA}{T_c} \qquad for \quad (T - T_c) \le t \le T \le T_d \tag{16}$$

$$W(t,T) = 0 \qquad for \quad t \le (T - T_c) \quad or \quad t \ge T \tag{17}$$

On the rising hydrograph, as shown in Eq's 10 and 11, the contributing area to the runoff rate at time, T, is proportional to the ratio of time, T, to the time of concentration. Thus, we have

$$Q(T) = \frac{1}{T} \sum_{t=0}^{t=T} C(\frac{T}{T_c}A)P(t) \qquad for \quad T_c \ge T \ge 0 \tag{18}$$

Or, the weighs are:

$$W(t,T) = \frac{CA}{T_c} \qquad for \quad T_c \ge T \ge t \ge 0 \tag{19}$$

The receding kinematic wave is slower than the rising one. Runoff rates on the recession hydrograph are often described by an exponential decay. In this study a linear approximation is adopted as an example to model the recession through a small watershed as:

$$Q(T) = Q(T_d)[1 - \frac{T - T_d}{T_c}] = \frac{1}{T_c} \sum_{t=(T_d - T_c)}^{t=T_d} CAP(t)\left[1 - \frac{T - T_d}{T_c}\right] \quad for \ T_d \le T \le (T_d + T_c) \tag{20}$$

in which $Q(T_d)$ = runoff rate when the rainfall ceases. The weights are

$$W(t,T) = \frac{CA}{T_c}\left[1 - \frac{T - T_d}{T_c}\right] \qquad for \quad (T_d - T_c) \le t \le T_d \quad and \quad T_d \le T \le (T_d + T_c) \tag{21}$$

The matrix of weights derived in this study is time dependent. When time T is assigned, a set of weights can be determined by Eq's 16, 17, 19, and 21.

MODEL TRAINING PROCESS

From USGS Open File 82-873[11], Kennedy Drive Watershed in the Denver metropolitan areas, Colorado was selected for model tests. The watershed has an area of 83 acres, a waterway length of 2500 ft, and average watershed slope of 0.0231 ft/ft. In this study, the weighting factors suggested by Eq's 16, 17, 19, and 21 are adopted as initial values with an estimated T_c and C. Consider the case with Δt=5.0 minutes, T_c =15 minutes, and C= 0.45. The matrix of weights for the pair (T_c , C)=(15, 0.45) is

$$\begin{bmatrix} P(\Delta t) \\ P(2\Delta t) \\ P(3\Delta t) \\ p(4\Delta t) \\ P(5\Delta t) \\ . \\ . \\ P(n\Delta t) \end{bmatrix} \begin{bmatrix} 0.03A & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.03A & 0.03A & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.03A & 0.03A & 0.03A & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.03A & 0.03A & 0.03A & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.03A & 0.03A & 0.03A & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0.00. & 0.00. & 0.00. & 0.00. & 0.00. & 0.00. & 0.00. & 0.03A & 0.03A & 0.03A \end{bmatrix} = \begin{bmatrix} Q(\Delta t) \\ Q(2\Delta t) \\ Q(3\Delta t) \\ Q(4\Delta t) \\ Q(5\Delta t) \\ . \\ . \\ .Q(n\Delta t) \end{bmatrix}$$

For a training data set, all possible combinations between T_c's and C's shall be developed and placed on different hidden layers. In general, a total of 10 to 15 combinations needs to be investigated for a training data set. During the training, each hidden layer is identified by the pair of (T_c , C). Input vectors are applied to the matrix of weight to produce output vectors. This process continues until the pair of (T_c, C) which produces the least square error is obtained by

$$E = \sum_{T=0}^{T=T_B} (Q_o(T) - Q(T))^2 \tag{22}$$

in which $Q_o(T)$ = observed runoff at time t, and E = accumulated squared error. To minimize the squared error between the predicted and observed runoff rates yields the following derivatives:

$$\frac{\partial E}{\partial C} = 0 \tag{23}$$

and

$$\frac{\partial E}{\partial T_c} = 0 \tag{24}$$

For instance, the event observed in August 1979 at Kennedy Drive Watershed was analyzed for runoff coefficients of 0.40, 0.50, 0.52, and 0.60 with times of concentration from 10- to 100 minutes. The distributions of squared discharge error for the assumed T_c's under different runoff coefficients are plotted in Figure 2. It can be seen that the minimum error was achieved with T_c=15 minutes and C= 0.52.

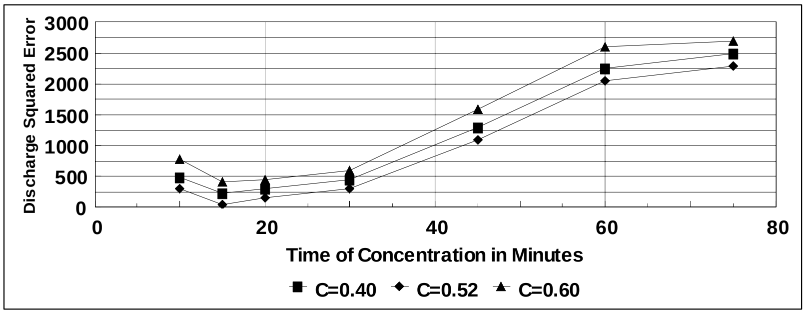

FIgure 2 Distribution of Model Errors

MODEL VERIFICATION AND COMPARISONS

The virtual watershed model developed by Carriere et. al.[1] includes 270 connections between seven input neurons, 35-hidden neurons, and one output neuron. The semi-virtual watershed model used n-neurons for rainfall inputs, one neuron for one output at a time for a total of m time steps, and 10 to 15 hidden layers to identify the optimal values for T_c and C. The operation of the semi-virtual watershed model follows the time sequence between rainfall and runoff amounts. At each time step, the weights are initialized using Eq's 16, 17, 19, and 21 with estimated T_c and C when time, T, is assigned. These weights are updated after each sweep of computation until the predicted values converged to the observed. With the consideration of time dependence, Eq 17 uses zero weights to eliminate all visitations that violate the time sequence in the rainfall/runoff process. Obviously, this screening process using the time of concentration as a parameter greatly enhances the model's computational efficiency.

The storm event recorded on August 18, 1979 at Kennedy Drive Watershed near Denver, Colorado is used as an example to illustrate the application of the semi-virtual watershed model. The records show that the total precipitation for this event was 0.55 inch with a duration of approximately two hours and the peak runoff rate was 23.0 cfs. For comparison, the time of concentration of this catchment was predicted by four widely used empirical formulas. Table 1 shows that the predicted time of concentration varies between 44.6 minutes and 13.8 minutes. The optimization process discussed above chose 15.0 minutes for T_c. The corresponding storm hydrographs predicted by Eq's 15, 18, and 20 under the observed nonuniform hyetograph are summarized in Table 2 and plotted in Figure 3. It can be seen that the hydrograph predicted by the semi-virtual model reflects the temporal variations of the continuous rainfall distribution and results in the least accumulative squared error for this case.

Empirical Method	Estimation of the Time of Concentration in Minutes
FAA Airport Formula	$0.3938\frac{(1.1-C)\sqrt{L}}{S^{0.33}} = 0.3938\frac{(1.1-0.52)*\sqrt{2500}}{0.0231^{0.33}} = 39.60$
Kirpitch Formula	$0.0078(\frac{L}{\sqrt{S}})^{0.77} = 0.0078(\frac{2500}{\sqrt{0.00231}})^{0.77} = 13.76$
Kinematic Wave Formula	$\frac{0.94}{i^{0.4}}(\frac{nL}{\sqrt{S}})^{0.6} = \frac{0.94}{0.275^{0.4}}(\frac{0.016*2500}{\sqrt{0.0231}})^{0.6} = 44.62$
SCS-Upland Method for Gutter Flow	$\frac{L}{60*20\sqrt{S}} = \frac{2500}{60*20\sqrt{0.0231}} = 18.28$

Table 1 Predicted Times of Concentration for Kennedy Drive Watershed near Denver, Colorado

				Predicted Hydrographs		
Time	Incremental	Observed	Kinematic	Airport	Semi-Virtual	SCS Upland
	Precipitation	Hydrograph	Wave	Formula	Model	Formula
			Time of Concentration in Minutes			
			45	40	15	20
minutes	inch	cfs	cfs	cfs	cfs	cfs
0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.00	0.00	0.00	0.00	0.00	0.00	0.00
10.00	0.01	1.00	0.58	0.65	1.73	0.65
15.00	0.02	3.30	1.73	1.94	5.18	3.88
20.00	0.02	6.00	2.88	3.24	8.63	6.47
25.00	0.02	11.00	4.03	4.53	10.36	9.06
30.00	0.05	17.00	6.91	7.77	15.54	15.54
35.00	0.05	21.00	9.78	11.01	20.72	20.72
40.00	0.04	23.00	12.08	13.60	24.17	23.31
45.00	0.04	22.00	14.39	16.19	22.44	25.90
50.00	0.03	15.00	16.11	18.13	18.99	27.19
55.00	0.01	11.00	16.69	18.13	13.81	22.01
60.00	0.01	7.70	16.69	17.48	8.63	16.83
65.00	0.01	6.90	16.11	16.83	5.18	12.95
70.00	0.02	6.90	16.11	16.83	6.91	10.36
75.00	0.01	8.30	15.54	14.24	6.91	7.77
80.00	0.03	11.00	14.39	12.95	10.36	10.36
85.00	0.03	13.00	13.24	12.30	12.08	12.95
90.00	0.02	14.00	12.08	11.01	13.81	14.24
95.00	0.02	13.00	10.93	10.36	12.08	14.24
100.00	0.03	13.00	10.93	11.65	12.08	16.83
105.00	0.01	12.00	10.93	11.65	10.36	14.24
110.00	0.02	8.80	11.51	12.30	10.36	12.95
115.00	0.01	7.20	11.51	11.65	6.91	11.65
120.00	0.01	5.70	10.93	11.65	6.91	10.36
125.00	0.01	4.50	10.93	10.36	5.18	7.77
Sum of Discharge Error Square			922	872	55.00	515

Table 2 Simulation of August 18, 1979 Event at Kennedy Drive Watershed near Denver, Colorado

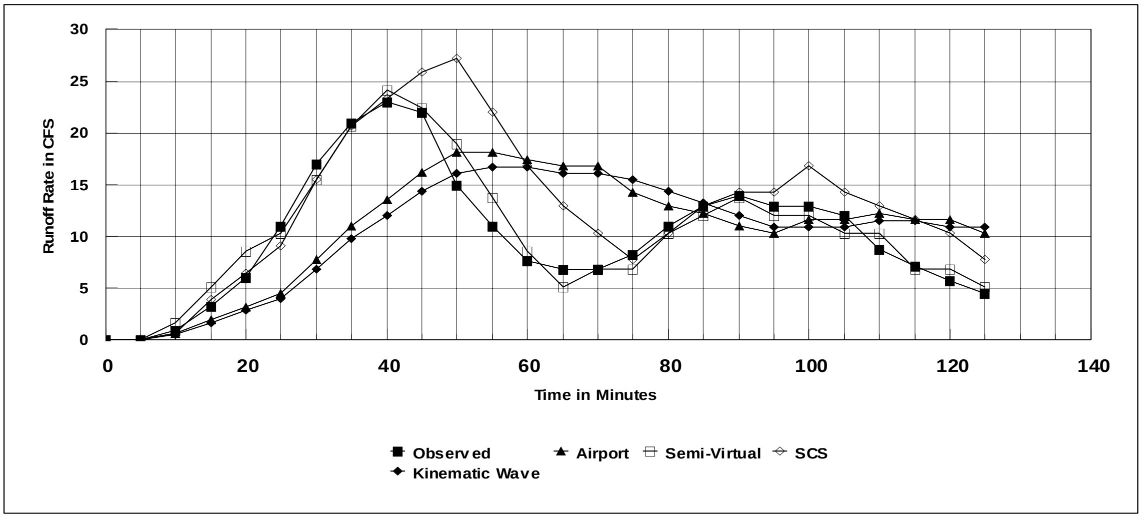

Figure 3 Simulation of August 18, 1979 Event at Kennedy Drive Watershed near Denver, Colorado

CONCLUSIONS

A semi-virtual watershed model is presented in this study. Similar to a virtual watershed model using the neural computing approach, this model applies a matrix of time-dependent weights to rainfall and runoff vectors. However, the determination of weights used in this model is guided by the kinematic wave theory. With the consideration of time dependence, the computational efficiency is greatly improved by the pre-determined interconnections between input and output layers. This approach eliminates all random visitations which violate the time sequence during the rainfall/runoff process. Optimization procedures used in the semi-virtual watershed model identify the time of concentration and runoff coefficient as the system parameters for determining the weights between input and output vectors. Review of the current practice indicates that most of synthetic hydrograph methods were developed for larger watersheds. Predictions of hydrographs from a small urban watershed rely on the kinematic wave method applied to an equivalent rectangular watershed. The semi-vitual watershed model expands the Rational method from peak flow predictions to complete hydrograph predictions. This model is applicable to small urban watersheds with a drainage area less than 150 acres.

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