$\textbf{Algorithm}\ PROCLUS\_SPMMP\ (No.of\ clusters: k, Average\ number\ of\ dimensions: l)$
$\{C_i\ is\ the\ i^{th}\ cluster\}$
$\{D_i\ is\ the\ set\ of\ dimension\ associated\ with\ cluster\ C_i\}$
$\{M_{best}\ is\ the\ best\ set\ of\ medoids\ found\ so\ far\}$
$\{N\ is\ the\ final\ set\ of\ medoids\ with\ associated\ dimension\}$
$\{A, B\ are\ constant\ integers\}$
$\{D\ is\ the\ set\ of\ all\ dimension\ (all\ the\ process\ stages)\}$
$\{P\ is\ the\ set\ of\ all\ points\ (all\ the\ production\ paths)\}$

$\textbf{begin}$
  $\{1.Initialization\ Phase\}$
$S = random\ sample\ of\ size\ A\ \cdot k$
$M = Greedy(S, B \cdot k)$

  $\{2.Iterative\ Phase\}$
$BestObjective = \infty$
$M_{current} = Random\ set\ of\ medoids\ \{m_1, m_2, \dots, m_k\} \subset M$
$\textbf{repeat}$
  $\{approximate\ the\ optimal\ set\ of\ dimensions\}$
  $(L_1, \dots, L_k) = FindLocality(S, M_{current})$
  $(D_1, D_2, \dots, D_k) = FindDimensions(k, l, L)$
  $\{form\ the\ clusters\}$
  $(C_1, \dots, C_k) = AssignPoints(D_1, \dots, D_k)$
  $ObjectiveFunction = EvaluateClusters(C_1, \dots, C_k, D_1, \dots, D_k)$
  $\textbf{if}\ ObjectiveFunction < BestObjective\ \textbf{then}$
  $\textbf{begin}$
    $BestObjective = ObjectiveFunction$
    $M_{best} = M_{current}$
  $\textbf{end}$
  $Compute\ M_{current}\ by\ replacing\ the\ bad\ medoids\ in\ M_{best}\ with\ random\ points\ from\ M$
$\textbf{until}\ (termination\_criterion)$

  $\{3.Refinement\ Phase\}$
$L = \{C_1, \dots, C_k\}$
$(D_1, \dots, D_k) = FindDimensions(k, l, L)$
$(C_1, \dots, C_k) = AssignPoints(D_1, \dots, D_k)$
$N = (M_{best}, D_1, \dots, D_k)$
$\textbf{return}(N)$
$\textbf{end}$

$\boldsymbol{Algorithm}$ $PROCLUS\_SPMMP$ $(No.of\ clusters{:}\,k, Average\ number\ of\ dimensions{:}\,l, number\ of\ agent\ T)$
$\{C_i\ is\ the\ i^{th}\ cluster\}$
$\{D_i\ is\ the\ set\ of\ dimension\ associated\ with\ cluster\ C_i\}$
$\{M_{best}\ is\ the\ best\ set\ of\ medoids\ found\ so\ far\}$
$\{N\ is\ the\ final\ set\ of\ medoids\ with\ associated\ dimension\}$
$\{A, B\ are\ constant\ integers\}$
$\{D\ is\ the\ set\ of\ all\ dimension\ (all\ the\ process\ stages)\}$
$\{P\ is\ the\ set\ of\ all\ points\ (all\ the\ production\ paths)\}$

$\boldsymbol{begin}$
    $\{1. Initialization\ Phase\}$
$S = random\ sample\ of\ size\ A\ \cdot k$
$M = Greedy(S, B \cdot k)$
$BestObjective = \infty$
$\boldsymbol{for}$ $agent\ t \in T$ $\boldsymbol{do}$
    $\boldsymbol{begin}$
    $t.M_{current} = Random\ set\ of\ medoids\ \{m_1, m_2, \dots, m_k\} \subset M$
    $t.L = FindLocality(S, t.M_{current})$
    $t.D = FindDimensions(k, l, t.L)$
    $t.C = AssignPoints(t.D)$
    $t.ObjectiveFunction = EvaluateClusters(t.C, t.D)$
    $\boldsymbol{if}$ $ObjectiveFunction < BestObjective$ $\boldsymbol{then}$
        $\boldsymbol{begin}$
            $BestObjective = t.ObjectiveFunction$
            $M_{best} = t.M_{current}$
            $C_{best} = t.C$
        $\boldsymbol{end}$
    $\boldsymbol{end}$

    $\{2. Iterative\ Phase\}$
$\boldsymbol{repeat}$
$\boldsymbol{for}$ $agent\ t \in T$ $\boldsymbol{do}$
    $\boldsymbol{begin}$
    $\{find\ better\ solution\ with\ slightly\ change\ medoid\}$
    $\boldsymbol{if}$ $rv \sim u(0,1) < 0.5$ $\boldsymbol{then}$
        $\boldsymbol{begin}$
            $\boldsymbol{if}$ $bad\ medoids\ in\ t.M_{current}$ $\boldsymbol{then}$
                $t.M_{current} = Change\ M_{best}'s\ bad\ medoid\ with\ random\ points\ from\ M$
            $\boldsymbol{else\ then}$
                $t.M_{current} =\ Change\ M_{best}'s\ one\ medoid\ with\ random\ points\ from\ M$
        $\boldsymbol{end}$
    $\{find\ better\ solution\ from\ totally\ random\ medoids\}$
    $\boldsymbol{else\ then}$ $t.M_{current} = Random\ set\ of\ medoids\ \{m_1, m_2, \dots, m_k\} \subset M$
    $t.M_{current} = Random\ set\ of\ medoids\ \{m_1, m_2, \dots, m_k\} \subset M$
    $t.L = FindLocality(S, t.M)$
    $t.D = FindDimensions(k, l, t.L)$
    $t.C = AssignPoints(t.D)$
    $t.ObjectiveFunction = EvaluateClusters(t.C, t.D)$
    $\boldsymbol{if}$ $ObjectiveFunction < BestObjective$ $\boldsymbol{then}$
        $\boldsymbol{begin}$
            $BestObjective = t.ObjectiveFunction$
            $M_{best} = t.M_{current}$
        $\boldsymbol{end}$
    $\boldsymbol{end}$
$\boldsymbol{until}$ $(termination\_criterion)$

    $\{3. Refinement\ Phase\}$
$L = C_{best}$
$(D_1, \dots, D_k) = FindDimensions(k, l, L)$
$(C_1, \dots, C_k) = AssignPoints(D_1, \dots, D_k)$
$N = (M_{best}, D_1, \dots, D_k)$
$\boldsymbol{return}(N)$
$\boldsymbol{end}$

$\boldsymbol{Algorithm}$ $FindLocality(Set\ of\ points\ S, Set\ of\ medoids\ M)$
$\boldsymbol{begin}$
$\boldsymbol{for\ each}$ $medoid\ m_i \in M$ $\boldsymbol{do}$
  $\boldsymbol{begin}$
  $\delta_i = \min\{Distance(m_i, m_j, D)\ i \neq j\}$
  $L_i = \{x | x \in S\ and\ Distance(x, m_i, D) < \delta_i\}$
  $\boldsymbol{end}$
$\boldsymbol{return}((L_1, \dots, L_k))$

$\boldsymbol{Algorithm}$ $Greedy(Set\ of\ points\ S, Number\ of\ medoids{:}\ k)$
$\boldsymbol{begin}$
$M = \{m_1\}, \{m_1\ is\ a\ random\ point\ of\ S\}$
$\boldsymbol{for\ each}$ $x \in S \backslash M$ $\boldsymbol{do}$ $dist(x) = Distance(x, m_1, D)$
$\boldsymbol{for}$ $i = 2\ to\ k$ $\boldsymbol{do}$
  $\boldsymbol{begin}$
    $\boldsymbol{let}$ $m_i \in S \backslash M\ be\ s.t.\ dist(m_i) = \max\{dist(x) | x \in S \backslash M\}$
    $M = M \cup \{m_i\}$
    $\boldsymbol{for\ each}$ $x \in S \backslash M$ $\boldsymbol{do}$ $dist(x) = \min\{dist(x), Distance(x, m_i)\}$
  $\boldsymbol{end}$
$\boldsymbol{return}$ M
$\boldsymbol{end}$

$\boldsymbol{Algorithm}$ $Distance(point\ x_1, point\ x_2, dimension\ set\ D_x)$
$\boldsymbol{begin}$
$CountNun = 0$
$\boldsymbol{for}$ $dimension\ d \in D_x$ $\boldsymbol{do}$
  $\boldsymbol{begin}$
  $\boldsymbol{if}$ $x_1(d) = x_2(d)$
    $CountNum = CountNum + 1$
  $\boldsymbol{end}$
$\boldsymbol{return}$ $(CountNum/|D_x|)$
$\boldsymbol{end}$

$\boldsymbol{Algorithm}$ $FindDimensions(k, l, L)$
$\boldsymbol{begin}$
$\{X_{i,j}\ is\ the\ average\ distance\ from\ the\ points\ in\ L_i\ to\ medoid\ m_i, along\ dimension\ j\}$
$\boldsymbol{for}$ each medoid i $\boldsymbol{do}$
  $begin$
  $Y_i = \frac{\sum_{j=1}^{|D|} X_{i,j}}{|D|}$
  $D_i = \emptyset$
  $\sigma_i = \sqrt{\frac{\sum_{j=1}^{|D|} (X_{i,j} - Y_i)^2}{d-1}}$
  $\boldsymbol{for}$ each dimension j $\boldsymbol{do}$ $Z_{i,j} = (X_{i,j} - Y_i)/\sigma_i$
  $\boldsymbol{end}$
Pick the k·l numbers with the least (most negative)values of $Z_{i,j}$
subject to the constraint that there are at least 2 dimensions for each clusters
$\boldsymbol{if}$ $Z_{i,j}$ *is picked* **then** *add dimension* j *to* D_i
$\boldsymbol{return}$ $(D_1, D_2, \dots, D_k)$
$\boldsymbol{end}$

$\boldsymbol{Algorithm}\ AssignPoints(D_1, D_2, .., D_k)$
$\boldsymbol{begin}$
$\boldsymbol{for\ each}\ i \in \{1, .., k\}\ \boldsymbol{do}\ C_i = \emptyset$
$\boldsymbol{for\ each}\ \ p \in P$
$\quad Find\ i\ with\ the\ lowest\ value\ of\ Distance(p, m_i, D_i)\ and\ add\ p\ to\ C_i$
$\boldsymbol{return}\ (C_1, \dots, C_k)$
$\boldsymbol{end}$

$\boldsymbol{Algorithm}\ EvaluateClusters(C_1, .., C_k, D_1, \dots, D_k)$
$\boldsymbol{begin}$
$\boldsymbol{for\ each}\ C_i\ \boldsymbol{do}$
$\quad \boldsymbol{begin}$
$\quad \boldsymbol{for\ each}\ dimension\ j \in D_i\ \boldsymbol{do}$
$\quad\quad \boldsymbol{begin}$
$\quad\quad Y_{i,j} = CentroidDistance(C_i, d_j)$
$\quad\quad \boldsymbol{end}$
$\quad w_i = \frac{\sum_j Y_{i,j}}{|D_i|}$
$\quad \boldsymbol{end}$
$\boldsymbol{return}\ \left(\frac{\sum_{i=1}^{k} |C_i| \cdot w_i}{|P|}\right)$

$\boldsymbol{Algorithm}\ CentroidDistance(C_i, d_j)$
$\{d_j\ is\ a\ dimension\ in\ D_i\}$
$Centroid(C_i, d_j)\ is\ an\ average\ vector\ of\ OneHotEncoding(p, d_j)\ \forall p \in C_i$
$\boldsymbol{for\ each}\ p \in C_i\ \ dist(p) = \left\| OneHotEncoding(p, d_j) - Centroid(C_i, d_j) \right\|_1 / 2$
$\boldsymbol{return}\ \left(\frac{\sum_{p \in C_i} dist(p)}{|C_i|}\right)$