1. Let X and Y be random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ and $A \in \mathcal{F}$ be an event. Show that $Z : \Omega \to \mathbb{R}$, defined as $Z(\omega) = X(\omega)I_A(\omega) + Y(\omega)I_{A^c}(\omega)$ is a random variable, where I_A is the indicator function of A.

2. Let X have the following distribution function

$$F_X(x) = \begin{cases} 0 & x < -3/4 \\ 1/3x + 1/4 & -3/4 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Also, let $Y = 2X^2$. Find the following probabilities:

   (a) $\mathbb{P}(1/2 < X < 3/2)$
   (b) $\mathbb{P}(X + Y \le 2)$
   (c) $\mathbb{P}(Y \le X/2)$
   (d) $\mathbb{P}(Y > 1/2)$
   (e) $\mathbb{P}(Y = 8)$

3. Gubner Chapter 5, 49.

4. Gubner Chapter 2: 5. [1]

5. Assume $N(\omega)$ and $K(\omega)$ have the joint pmf $p_{N,K}(n,k) = c/(n^2 k!),\ n,k \in \mathbb{N}$. Note: if finding a closed form for a series seems hopeless, use a computer program to approximate it numeically).

   (a) Determine c. (Hint: $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$.)
   (b) Find the marginal pmfs $p_N(n)$ and $p_K(k)$.
   (c) Find the probability of $\{\omega \| N(\omega) - K(\omega)| = 1\}$ if it is measurable.

6. Assume that $p_X(x) = c/(x^2 + 0.5),\ x \in \mathbb{Z}$, where c is a constant that makes p_X be a valid pmf (you do not need to determine c).

   (a) Find the pmfs of $Y = -X$ and $Z = X$.
   (b) Find the joint pmfs p_{XY} and p_{XZ}.
   (c) Which one of the concepts marginal pmf and joint pmf better captures the relationship between X and Y or X and Z?

7. A card deck that has 52 cards is distributed to four persons: 13 cards to each person. All partitions are equally likely.

[1] Important Note: Posting the homework and its solutions to online forums or sharing it with other students is strictly prohibited. Instances will be reported to USC officials as academic dishonesty for disciplinary action.

(a) We say a royal hand occurs if the first person receives all four Kings, four Queens, and four Aces. Find the probability that a royal hand occurs.

(b) If this experiment is repeated for 40 years (30 normal years and 10 leap years) three times a day, what is the probability that a royal hand occurs at least once?

(c) Use an approximate distribution to solve (7b).

8. Gubner Chapter 2: 14.

9. Gubner Chapter 2: 21.[2]

10. Dobrow 2.28.

11. Verify your answer to Problem 7 using R. You can use the method that Dobrow 3.46 suggests, or material on pages 91 and 100.

12. (Extra practice. You do not need to turn the solutions in and they will not be graded) Grimmet and Stirzaker: 2.1.1, 2.1.2, 2.1.3, 2.1.4, 2.1.5, 2.3.2, 2.3.4, 2.3.5, 2.4.1, 2.4.2

[2]Note that Gubner uses the probability of failure as the parameter of Geometric.