1. Suppose that $\mathbb{E}[X|Y] = \mathbb{E}[X]$, with probability 1.
   (a) Show that X and Y are uncorrelated.
   (b) Give an example to show that X and Y need not be independent.

2. Let X, Y be independent geometrically distributed random variables with parameter p. Let $Z = \mathbb{E}[X|X+Y]$. Find the expected value and the variance of Z. Hint: $\mathbb{E}[X+Y|X+Y] = X+Y$.

3. Every day in the afternoon, Hannan visits the kitchen to pick up one, two, or three fruits from the refrigerator with equal probability 1/3. If she picks up three fruits, she does not return to the kitchen again that day. If she picks up one or two fruits, she will make one additional trip to the kitchen, where she again will pick up one, two, or three fruits with equal probability 1/3. (The number of fruits taken in one trip will not affect the number of fruits taken in any other trip.) Calculate the following:
   (a) The probability that Hannan gets a total of three fruits on any particular day.
   (b) The conditional probability that she visited the kitchen twice on a given day, given that it is a day in which she got a total of three fruits.
   (c) $\mathbb{E}[K]$ and $\mathbb{E}[K|A]$, where $\mathbb{E}[K]$ is the unconditional expectation of K, the total number of fruits Hannan gets on any given day, and $\mathbb{E}[K|A]$ is the conditional expectation of K given the event $A = \{K > 3\}$.
   (d) $\sigma_{K|A} = \sqrt{\mathbb{E}[K^2|A] - (\mathbb{E}[K|A])^2}$, the conditional standard deviation of the total number of fruits Hannan gets on a particular day, where K and A are as in part 3c.
   (e) The probability that she gets more than three fruits on each of the next 16 days.
   (f) The conditional standard deviation of the total number of fruits he gets in the the next 16 days given that she gets more than three fruits on each of those days.

4. Using the properties of conditional expectation, show that $\text{Var}(X) = \mathbb{E}[\text{Var}(X|Y)] + \text{Var}(\mathbb{E}[X|Y])$. Let N be a random variable that takes nonnegative integer values and has finite variance. Let $X_1, X_2, ...$ be a sequence of i.i.d. discrete random variables that have finite expectation and variance and are independent from N . Using the previous result, show that $\text{Var}(\sum_{i=1}^{N} X_i) = \mathbb{E}[N]\text{Var}(X_1) + (\mathbb{E}[X_1])^2\text{Var}(N)$.

5. Dobrow 5.22

6. (Extra Practice) Bertsekas and Tsitsiklis: 2.25, 2.27, 2.28, 2.34. Grimmet and Stirzaker: 3.7.3, 3.7.4, 3.7.5, 3.7.10.