1. Let $\{X_n\}$ be a sequence of random variables defined on the same probability space.
   (a) Suppose that $\lim_{n\to\infty} \mathbb{E}[|X_n|] = 0$. Show that $X_n$ converges to zero, in probability.
   (b) Suppose that each $X_n$ can only take the values 0 and 1 and, that $\mathbb{P}(X_n = 1) = 1/n$.
      i. Given an example in which we have almost sure convergence of $X_n$ to 0.
      ii. Given an example in which we do not have almost sure convergence of $X_n$ to 0.

2. Let $X_1, X_2, \ldots$ be a sequence of independent random variables that are uniformly distributed between 0 and 1. For every $n$, let $Y_n$ be the median of the values of $X_1, X_2, \ldots, X_{2n+1}$. [That is, order $X_1, \ldots, X_{2n+1}$ in increasing order and let $Y_n$ be the $(n+1)^{\text{st}}$ element in this ordered sequence.] Show that that the sequence $Y_n$ converges to 1/2, in probability.

3. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. normal random variables, with zero mean and unit variance. The corresponding characteristic function is $\phi_n(t) = e^{-t^2/2}$. Define:

$$Y_n = \sum_{i=1}^{n} \frac{X_i}{2^i}$$

   Use Characteristic Functions to prove that $Y_n$ converges in distribution, and to identify the nature of the limit distribution.

4. Homer Simpson has been having trouble keeping his weight constant. In fact, at the end of each week, he notices that his weight has changed by a random amount, uniformly distributed between -0.7 and 0.7 pounds. Assuming that the weight change during any given week is independent of the weight change of any other week, find the probability that Homer Simpson will gain or lose more than 3 pounds in the next 100 weeks.

5. Let $S_n$ be the number of successes in $n$ independent Bernoulli trials, where the probability of success in each trial is $p = 1/2$. Provide a numerical value for the limit as $n$ tends to infinity for each of the following three expressions.
   (a) $\mathbb{P}(n/2 - 10 \le S_n \le n/2 + 10)$
   (b) $\mathbb{P}(n/2 - n/10 \le S_n \le n/2 + n/10)$
   (c) $\mathbb{P}(n/2 - \sqrt{n}/2 \le S_n \le n/2 + \sqrt{n}/2)$

6. Gubner 3.3.17

7. Gubner 3.3.19

8. Gubner 5.6.51

9. Gubner 5.6.52

10. Dobrow 9.35

11. Sample 1000 points from $\sum_{i=1}^{n} U_i$ and plot their histogram, where $U_i$'s are i.i.d U$[-1, 1]$ and compare it with the pdf provided by the CLT. Repeat for $n = 1$, 2, 3, 4, 5, 10, 15, 20, 25, 50, 100, 1000.

12. (Extra Practice) Bertsekas and Tsitsiklis: 4.29-4.45. Grimmet and Stirzaker: 5.1.1-5.1.9, 5.2.3-5.2.9, 5.7.1-5.7.11, 5.8.3-5.8.11. Schaum's Outline of Probability and Statistics:[1] 3.14-3.22. Leon Garcia: 4.102-4.121. Gubner: Chapter 3: 1-7, Chapter 4: 38-51.

13. (Extra Practice) Bertsekas and Tsitsiklis: 5.1-5.7. Grimmet and Stirzaker: 7.1.1, 7.3.1, 7.3.7, 7.3.11. Schaum's Outline of Probability and Statistics:[2] 3.87-3.89. Leon Garcia: 7.40-7.50. Gubner: Chapter 2: 51, 52, Chapter 4: 66,

[1] https://www.amazon.com/Schaums-Outline-Probability-Statistics-4th/dp/007179557X

[2] https://www.amazon.com/Schaums-Outline-Probability-Statistics-4th/dp/007179557X