1. Let $X_1, ..., X_n$ be iid $N(\theta, 1)$. A 95% confidence interval for $\theta$ is $\bar{x} \pm 1.96/\sqrt{n}$. Let $p$ denote the probability that an additional independent observation, $X_{n+1}$ will fall in this interval. Is $p$ greater than, less than, or equal to 0.95? Why?

2. The concentration of a certain air pollutant in Springfield has been known for several years to have mean $\mu = 34$ ppm (parts per million) and standard deviation $\sigma = 8$ ppm. Mayor Quimby is now claiming that they have lowered the average with improved filtration devices for factories. A group of environmentalists will test to see if this is true at the 4% level of significance. Assume that their sample of size 50 gives a mean of 32.5 ppm. Perform a (one-sided) hypothesis test at the 4% level of significance and state your decision. Repeat the problem assuming that a sample standard deviation $S = 8$ was obtained and the concentration obeys a normal distribution whose $\sigma$ is unknown. [1]

3. Derive the Maximumn Likelihood estimator of the parameter of a Poisson random variable based on iid observations $X_1, ..., X_n$. Is the estimator unbiased and consistent?

4. Let $X$ be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} cx^2 & 0 < x < 2, \\ 0 & \text{otherwise} \end{cases}$$

   Also, suppose that $Y|X = x \sim Geo(x/2)$. Find the MAP estimate of $X$ given $Y = 3$.

5. Gubner 6.3

6. Gubner 6.5

7. Gubner 6.6

8. Gubner 6.16

9. Dobrow 10.16

10. Dobrow 10.17

11. Draw an i.i.d sample from a Pois(5) with sample size 100. Assume that you know the variance of the population, but not its mean. Find a 95% confidence interval for the mean of the population. Repeat this process 1000 times and find in what fraction of the experiments the confidence interval contains the actual populatiuon mean.

12. Use Monte-Carlo simulation to estimate $\pi$ and build a 95% confidence interval for it

[1] For an online t-table, you can visit `http://stattrek.com/online-calculator/t-distribution.aspx`