1. Consider random variables X, Y with pdf $f(x, y)$ such that:

$$f_{X,Y}(x,y) = \begin{cases} c(x+y) & 0 < x < 1, 0 < y < 1, 0 < x+y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Determine the value of c. (5 pts)
   (b) Determine the marginal pdf of X and Y. (5 pts)
   (c) Determine the expected value and variance of X and Y. (5 pts)
   (d) Calculate $\mathbb{P}(X > Y)$. (5 pts)

2. A person whose level of exposure to Treponema Pallidum bacterium is $x \in [0, 1]$ contracts syphilis with probability $s(x) = x^\gamma$. Suppose that the exposure level of a randomly chosen person is modeled by a Beta random variable $X \sim Beta(a, b)$.

   (a) Find the conditional density of the exposure level of that person given that the person has syphilis. (10 pts)
   (b) Find the conditional expected value $\mathbb{E}[X|\text{the person has syphilis}]$. (5 pts)

3. Assume for simplicity that all people in the world are either completely introverted or completely extroverted. Draw a country at random, and let R and $1-R$ be respectively the ratios of people in that country that are extrovert or introvert. Suppose that R is uniformly distributed, $R \sim U(0, 1)$. This model is reasonable if we know nothing about the true distribution of extroverts. If some information is available however, then more realistic models can be used, such as the beta distribution, which is often used for modeling proportions[1]. Next, draw M people at random from that counrty, and let X be the number of those people who are extrovert.

   (a) Given $R = r$, what is the conditional pmf of X, i.e. $p_{X|R=r}$? (5 pts)
   (b) Find $\mathbb{E}[X]$ and σ_X^2. (10 pts)
   (c) Answer the above questions again, assuming that $R \sim Beta(a, b)$. (10 pts)

4. Gubner, Chapter 4, Problem 28. (10 pts)

5. Gubner, Chapter 4, Problem 35. (10 pts)

6. Gubner, Chapter 4, Problem 52. (10 pts)

7. Gubner, Chapter 5, Problem 6. (5 pts)

8. Gubner, Chapter 7, Problem 39. (20 pts)

9. Dobrow, 8.34. (10 pts)

10. Reading Assignment: Dobrow, Examples 8.6, 8.7, and p. 332.

[1] The uniform distribution on [0, 1] is a special case of the beta distribution $Beta(a, b)$ with $a = b = 1$.

11. (Extra Practice) Bertsekas and Tsitsiklis: 3.10, 3.11, 3.12, 3.15, 3.18, 3.19, 3.20, 3.22, 3.23, 3.29, 3.30. Grimmet and Stirzaker: 4.5.1, 4.5.4, 4.5.5, 4.5.6, 4.5.7, 4.5.8, 4.5.9, 4.6.1, 4.6.3, 4.6.4, 4.6.5, 4.6.6, 4.6.8, 4.6.9. Hwei Hsu-Schaum's Outline of Probability, Random Variables and Random Processes: Solved problems in chapter 3.