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Event — an event is modeled by a set

⇒ require Set theory & Logic

Aristotelian Logic

- Proposition (Statement) : either true or false

Logical connectives — proposition p, q , negation $\neg p$, conjunction $p \wedge q$, disjunction $p \vee q$, implication (conditional statement) $p \Rightarrow q$, double implication $p \Leftrightarrow q$

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
0	0	1	0	0	1	1
0	1	1	0	1	1	0
1	0	0	0	1	0	0
1	1	0	1	1	1	1

$(p \Rightarrow q) \equiv (\neg p \vee q)$

p	q	$\neg p$	q	$(\neg p \vee q)$
0	0	1	0	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

Q.E.D.

Exclusive OR (XOR) / Exclusive disjunction — $(p \veebar q) \equiv (p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
0	0	0	1	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

De Morgan Laws — $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$p \vee q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

contrapositive $(p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$

p	q	$p \Rightarrow q$	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

Tautology logical expression that is always true for all of the values of the propositional variables

i.e. $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
0	1	1
1	0	1

Contradiction logical expression that is always false for all of the values of the propositional variables

i.e. $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
0	1	0
1	0	0

$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ is tautology

$\therefore (p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$

$p \Leftrightarrow p$ is always true Q.E.D.

Tautologic examples

Modus Ponens

$((p \Rightarrow q) \wedge p) \equiv q$

Modus Tollens

$((p \Rightarrow q) \wedge \neg q) \equiv \neg p$

Disjunction Syllogism

$(p \vee q) \wedge \neg p \equiv q$

Hypothetical Syllogism

$(p \Rightarrow q) \wedge (q \Rightarrow r) \equiv p \Rightarrow r$

Predicate Logic	proposition with a variable : boolean value function $P: X \to \{0,1\}$