1. Let $\{A_i | i \in I\}$ be a collection of sets. Show De Morgan's laws: $\left(\bigcup_{i\in I} A_i\right)^c = \bigcap_{i\in I} A_i^c$ and $\left(\bigcap_{i\in I} A_i\right)^c = \bigcup_{i\in I} A_i^c$. (10 pts)
2. Let A and B be two sets. Under what conditions is the set $A \cup (A \cup B)^c$ empty? (10 pts)
3. Show $A \subseteq B$ if and only if $A \cup B = B$. (10 pts)
4. Show $A \times (B \cup C) = (A \times B) \cup (A \times C)$. (10 pts)
5. Show that if $A \subset B$ and $C \subset D$, then $(A \times C) \subset (B \times D)$.
6. Let $A_1, A_2, \ldots, A_m$ and $B_1, B_2, \ldots, B_n$ be partitions of a set Ω. Show that the collection of sets $A_i \cap B_j; i = 1, \ldots, m, j = 1, \ldots, n$ is also a partition (called the cross partition) of Ω. (20 pts)
7. Gubner[1] Chapter 1, Problem 8. (10 pts)
8. Gubner Chapter 1, Problem 13. (10 pts)
9. Gubner Chapter 1, Problem 14. (10 pts)
10. Assume that we have three decks of standard cards: the first deck has cards whose both sides are blue, the second deck has cards whose both sides are red, and the third deck has cards that have a blue side and a red side. We pick a card and see one of its sides, which is blue. We wish to calculate the probability of the other side being blue using simple computer simulations. To do this, perform the following steps using R language of choice (there are other ways of simulating this experiment as well). Carry out the following steps n times, where n is a large number (e.g., 100,000). (20 pts)
    (a) Construct a 156×2 matrices A to simulate three decks of 52 cards. Assign $[0, 0]$ to the first 52 rows of A, to simulate card that have red on both sides. Assign $[0, 1]$ to next 52 rows of the matrix, to simulate the cards that have blue on one side, and red on the other side. Assign $[1, 1]$ to the next 52 rows to simulate card whose both sides are blue. 1 represents blue and 0 represents red.
    (b) Select an entry of the matrix at random (you can do this by generating a random integer i between 1 and 156 and a random integer j to be either 1 or 2 and selecting the (i, j) element of the matrix.)
    (c) If the entry that you selected is 0, do nothing and repeat selecting another entry until you see a 1 (because we assume we pick a card and see a blue side). After seeing a 1, go to the other column of the matrix: if you selected $(m, 1)$, see $(m, 2)$ and if you selected $(m, 2)$, go to $(m, 1)$. This simulates looking at the other side of the card. If you get another 1, it means the other side of the card is blue too.

[1]Gubner uses $\subset$ instead of $\subseteq$. *Only in Gubner problems* consider $\subset$ as $\subseteq$

(d) Count the number of double blues (1 and then 1) you obtained, and the total number of the cases in which you, at first, got a blue side (no matter what the second side's color was). Their ratio is a relative frequency estimate of the probability that the other side is blue in the above problem.

(e) Submit your program and the answer that you obtained.