

1. Consider  $N$  independent Bernoulli trials in which every trial has probability  $p$  of success. Let  $X$  be the number of successes observed, and let  $Y = N - X$  be the number of failures.
  - (a) If  $N \sim \text{Pois}(\lambda)$ , the conditional PMF  $p_{X|N}(\cdot|n)$  is binomial with parameters  $n$  and  $p$  (representing the number of successes obtained in Bernoulli trials), and define  $Y = N - X$ , which represents the number of failures observed. Determine the pmf of  $X$  and  $Y$ .
  - (b) Are  $X$  and  $Y$  independent? Why?
2. Gubner Chapter 2: 25.
3. Gubner Chapter 3, Problem 7 (No need to find the generating function).
4. Let  $X$  be a random variable that takes integer values and is symmetric, that is,  $\mathbb{P}(X = k) = \mathbb{P}(X = -k)$  for all integers  $k$ . What is the expected value of  $Y_1 = \cos(\pi X)$  and  $Y_2 = \sin(\pi X)$ ?
5. Gubner Chapter 3, Problem 24.
6. Suppose that  $X_1, \dots, X_n$  are independent,  $\text{Geo}_0(p)$  random variables. Compute  $\mathbb{P}(\min(X_1, \dots, X_n) > k)$  and  $\mathbb{P}(\max(X_1, \dots, X_n) \leq k)$ .
7. Let  $G_1, G_2$ , and  $G_3$  be independent geometric random variables with the same pmf:  $p_{G_1}(k) = p_{G_2}(k) = p_{G_3}(k) = p(1 - p)^{k-1}$  where  $p$  is a scalar with  $0 < p < 1$ . What is  $\mathbb{P}(G_1 = k | G_1 + G_2 + G_3 = n)$ ? Hint: Try thinking in terms of coin tosses.
8. Gubner Chapter 3, Problem 26. Remember that Gubner uses probability of failure as the parameter of the Geometric random variable.
9. Gubner Chapter 3, Problem 30.
10. Gubner Chapter 2, Problem 37.
11. Simulate Problem 1a using R for  $p = 0.5$  and  $\lambda = 1$ .
12. (Extra Practice) Bertsekas and Tsitsiklis: 2.2, 2.3, 2.8, 2.9, 2.10, 2.11. Grimmet and Stirzaker: 3.3.7, 3.3.8, 3.6.2, 3.6.3, 3.6.6.
13. (Extra Practice) Bertsekas and Tsitsiklis: 2.33, 2.36, 2.37, 2.42, 2.46. Grimmet and Stirzaker: 3.1.4, 3.4.1, 3.4.3, 3.5.1, 3.5.4, 3.6.8