1. Assume that $(\Omega, \mathcal{B}, \mathbb{P})$ is a probability space, where $\Omega = [0, 1)$ and $\mathbb{P}(B) = \int_B 1 d\omega, \forall B \in \mathcal{B}$.[1] $\mathcal{B}$ is a σ-field that contains all open and closed sub-intervals of $[0, 1)$ and their countable unions and intersections.[2] Assume $A_1 = [0, 1/2), A_2 = [0, 1/4) \cup [1/2, 3/4), A_3 = [0, 1/8) \cup [1/4, 3/8) \cup [1/2, 5/8) \cup [3/4, 7/8)$, determine whether or not $\{A_1, A_2, A_3\}$ is an independent set. Moreover, determine whether or not it is pairwise independent.[3] (20 pts)

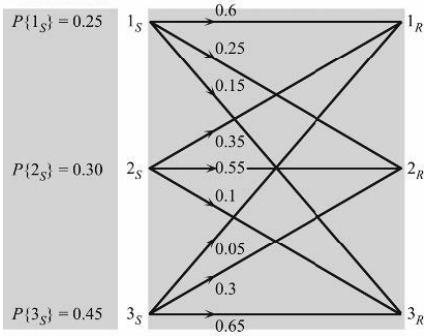
2. For a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, assume that A, B, C are $\mathcal{F}$-measurable, and $\mathbb{P}(C) > 0, \mathbb{P}(B \cap C) > 0$. Show that A, B are conditionally independent given C, if and only if $\mathbb{P}(A|B \cap C) = \mathbb{P}(A|C)$. (10 pts)

3. Anand and Ben turn in DSCI 564 homework weekly. What is the conditional probability that Anand does not solve all of the homework problems given that at least one of them does not solve all of the problems? Assume that they solve all of the problems independently and that each of them has a probability $0 < p < 1$ of solving all of the problems. (15 pts)

4. Three fair dice are rolled. Is the event A that their sum is 13 independent of the score shown by the second die? (20 pts)

5. Two cards are randomly selected from a deck of 52 standard cards without replacement. Use total probability theorem to find the probability that the second card is an Ace. Make sure that you rigorously define the probability space before using the total probability theorem. (20 pts)

6. A dysfunctional post office in Springfield delivers three types of parcels (1,2,3) to Simpsons. Because of some problems with the delivery process, the type of parcel that Simpsons receive may change. The diagram in below shows the probability that each type of parcel is sent to Simpsons, and the probability that they receive each type of parcel, given the type of parcel that was sent. For example, the probability that a type-2 parcel is received given that a type-1 parcel was sent is 0.25. If Simpsons receive a type-3 parcel, what is the probability that the parcel that was sent was type-2? (20 pts)



[1] You don't need to prove that $\mathbb{P}$ is a probability measure in this problem. Take it for granted.

[2] Later, we will see that $\mathcal{B}$ is called a *Borel* σ-field.

[3] Important Note: Posting the homework and its solutions to online forums or sharing it with other students is strictly prohibited. Instances will be reported to USC officials as academic dishonesty for disciplinary action.

7. Dobrow, Problem 2.31. (20 pts)

8. Reading Assignment: Dobrow, Examples 2.8 and 2.9, and Tree Diagrams on p. 42. (This will help you in Problem 7.)

9. Reading Assignment: Dobrow, Example 2.18. (Remember the blue and red cards example!)

10. (Extra practice. You do not need to turn the solutions in and they will not be graded) Bertsekas and Tsitsiklis: 1.22, 1.23, 1.33, 1.34, 1.51, 1.58. Grimmet and Stirzaker: 1.8.16, 1.8.19, 1.8.21,1.8.24, 1.8.37.