

1. A non-negative interger-valued random variable X has one of the following characteristic functions:

$$\phi(t) = e^{2(e^{jt}-1)} \quad (1)$$

$$\phi(t) = e^{2(e^{jt}-1)} \quad (2)$$

- (a) Explain why one of them cannot possibly be a characteristic function.
 - (b) Use the true characteristic function to find $\mathbb{P}(X = 0)$.
2. Assume that X has a normal distribution with fixed standard deviation σ and mean Y , and $Y \sim \text{Exp}(\lambda)$. Find $M_{X+Y}(s)$.
 3. The MGF and the mean of a discrete random variable X are given by $M_X(s) = ae^s + be^{4(e^s-1)}$, $\mathbb{E}[X] = 3$. Find:
 - (a) The parameters a and b .
 - (b) $p_X(1), \mathbb{E}[X^2], \mathbb{E}[3^X]$.
 - (c) $\mathbb{P}(X + Y = 2)$ where Y is a random variable that is independent from X and is identically distributed with X .
 4. The MGF of a random variable X is

$$M_X(s) = \frac{6 - 3s}{2(1 - s)(3 - s)}, \quad s < 1$$

Find its cdf.

5. Show that if X_1, X_2 are iid $\text{Exp}(1)$ distributions, then the symmetrized variable $Y = X_1 - X_2$ has a bilateral exponential (Laplace) density $f_Y(z) = \frac{1}{2}e^{-|z|}$. Show that the Characteristic Function of Y is $1/(1 + t^2)$.
6. A biased coin, which lands heads with probability $1/10$ each time it is flipped, is flipped 200 times consecutively. Give an upper bound on the probability that it lands heads at least 120 times.
7. Gubner Chapter 3, 3
8. Gubner Chapter 3, 6
9. Gubner Chapter 4, 40
10. Gubner, Chapter 4, 66
11. (Extra Practice) Bertsekas and Tsitsiklis: 4.29-4.45. Grimmet and Stirzaker: 5.1.1-5.1.9, 5.2.3-5.2.9, 5.7.1-5.7.11, 5.8.3-5.8.11. Schaum's Outline of Probability and Statistics:¹ 3.14-3.22. Leon Garcia: 4.102-4.121. Gubner: Chapter 3: 1-7, Chapter 4: 38-51.

¹<https://www.amazon.com/Schaums-Outline-Probability-Statistics-4th/dp/007179557X>

12. (Extra Practice) Bertsekas and Tsitsiklis: 5.1-5.7. Grimmet and Stirzaker: 7.1.1, 7.3.1, 7.3.7, 7.3.11. Schaum's Outline of Probability and Statistics:² 3.87-3.89. Leon Garcia: 7.40-7.50. Gubner: Chapter 2: 51, 52, Chapter 4: 66,

²<https://www.amazon.com/Schaums-Outline-Probability-Statistics-4th/dp/007179557X>