Homework 3

DSCI 564
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1. $A_1 = [0, 1/2)$ $A_2 = [0, 1/4) \cup [1/2, 3/4)$ $A_3 = [0, 1/8) \cup [1/4, 3/8) \cup [1/2, 5/8) \cup [3/4, 7/8)$

$P(A_1) = 1/2$ $P(A_2) = 1/2$ $P(A_3) = 1/2$

$P(A_1 \cap A_2) = P([0, 1/4)) = 1/4 = P(A_1)P(A_2)$

$P(A_1 \cap A_3) = P([0, 1/8) \cup [1/4, 3/8)) = 1/4 = P(A_1)P(A_3)$

$P(A_2 \cap A_3) = P([0, 1/8) \cup [1/2, 5/8)) = 1/4 = P(A_2)P(A_3)$

$P(A_1 \cap A_2 \cap A_3) = P([0, 1/8)) = 1/8 = P(A_1)P(A_2)P(A_3)$

$\therefore \{A_1, A_2, A_3\}$ is an independent set

independen set $\Rightarrow \{A_1, A_2, A_3\}$ pairwise independent

2. show $P(A|C)P(B|C) = P(A \cap B|C) \leftrightarrow P(A|B \cap C) = P(A|C)$

i) $P(A|C)P(B|C) = P(A \cap B|C) \Rightarrow P(A|C)\dfrac{P(B \cap C)}{P(C)} = \dfrac{P(A \cap B \cap C)}{P(C)}$

$\Rightarrow P(A|C) = \dfrac{P(A \cap B \cap C)}{P(C)} \cdot \dfrac{P(C)}{P(B \cap C)}$ $\because P(C) > 0$

$\Rightarrow P(A|C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} \Rightarrow P(A|C) = P(A|B \cap C)$

ii) $P(A|B \cap C) = P(A|C) \Rightarrow \dfrac{P(A \cap B \cap C)}{P(B \cap C)} = P(A|C) \Rightarrow \dfrac{P(A \cap B \cap C)}{P(C)} \cdot \dfrac{P(C)}{P(B \cap C)} = P(A|C)$

$\Rightarrow \dfrac{P(A \cap B \cap C)}{P(C)} = P(A|C) \cdot \dfrac{P(B \cap C)}{P(C)} \Rightarrow P(A \cap B|C) = P(A|C)P(B|C)$

Q.E.D.

3. define event A, B

A: Anand solve all of homework problems.
B: Ben solve all of homework problems

$P(A) = P(B) = p$ and $P(A \cap B) = P(A)P(B)$

$$P(A^c \mid (A \cap B)^c) = \frac{P(A^c \cap (A^c \cup B^c))}{1 - P(A \cap B)} = \frac{P(A^c)}{1 - P(A \cap B)} = \frac{1 - P(A)}{1 - P(A)P(B)}$$

$$= \frac{1-p}{1-p^2} = \frac{1}{1+p}$$

4. three fair dice

event A, B_i

A: sum of dice is 13
B_i: second dice is i

$$P(A) = \frac{21}{6 \times 6 \times 6}$$

1	6	6	1	
2	5, 6		2	21
3	4, 6		3	
4			4	
5			5	
6			6	

$P(B_i) = 1/6$ $\quad P(A \mid B_i) = \dfrac{P(A \cap B_i)}{P(B_i)} = \dfrac{i/(6 \times 6 \times 6)}{1/6} = \dfrac{i}{36}$

$\therefore P(A \mid B_i) \neq P(A)$ not an independent events.

S H C D

5. $\Omega = \{(x, y) \mid (x, y) \in \{S1, S2, \cdots SK, H1, \cdots HK, C1, \cdots CK, D1, \cdots DK\}, x \neq y\}$

define $A = \{(x,y) \mid (x,y) \in \Omega, x \in \{S1, H1, C1, D1\}\}$
$B = \{(x,y) \mid (x,y) \in \Omega, y \in \{S1, H1, C1, D1\}\}$

$$P(B) = P(B \mid A)P(A) + P(B \mid A^c)P(A^c) = \frac{P(A \cap B)}{P(A)} P(A) + \frac{P(A^c \cap B)}{P(A^c)} P(A^c)$$

$$= \frac{4/52 \times 3/51}{4/52} \, 4/52 + \frac{48/52 \times 4/51}{48/52} \, 48/52$$

$$= \frac{1}{13} \times \frac{1}{17} + \frac{12}{13} \cdot \frac{4}{51} = \frac{3+48}{13 \cdot 51} = \frac{1}{13}$$

6. $P(2_R | 1_S) = 0.25$

if Simpsons receive a type 3 parcel (3_R), what is the probability that the parcel that was sent was type 2 (S_2)?

$$P(2_S|3_R) = \frac{P(3_R|2_S)P(2_S)}{P(3_R|1_S)P(1_S) + P(3_R|2_S)P(2_S) + P(3_R|3_S)P(3_S)}$$

$$= \frac{0.1 \times 0.3}{0.15 \times 0.25 + 0.1 \times 0.3 + 0.65 \times 0.45} = \frac{0.03}{0.0375 + 0.03 + 0.2925} = \frac{0.03}{0.36} = 1/12$$

7 Dobrow. Problem 2.31

$$P(A) = \frac{13 \times 12}{52 \times 51} \times 4 = \frac{12}{51}$$

8. Example 2.8).

$$P(S_1 \cup S_2 \cup S_3) = 1 - P(S_1^c \cap S_2^c \cap S_3^c)$$

$$= 1 - P(S_3^c | S_1^c \cap S_2^c) \cdot P(S_2^c | S_1^c) P(S_1^c)$$

$$= 1 - [1 - P(S_3 | S_1^c \cap S_2^c)][1 - P(S_2 | S_1^c)][1 - P(S_1)]$$

$$= 1 - 0.7 \cdot 0.5 \cdot 0.35 = 0.8775$$

Example 2.9)

$$P(A_1T_2 \cup A_2T_1) = P(A_1T_2) + P(A_2T_1) \quad \because A_1T_2 \cap A_2T_1 = \phi$$

$$= P(T_2|A_1)P(A_1) + P(T_1|A_2)P(A_2)$$

$$= \frac{16}{51} \times \frac{4}{52} + \frac{4}{51} \times \frac{16}{52} = \frac{64 \times 2}{51 \times 52} = 0.048$$

9. Example 2.18 : Bertrand's box paradox

G_1, G_2 events

$$P(G_2 | G_1) = \frac{P(G_1 \cap G_2)}{P(G_1)} = \frac{1/3}{1/2} = 2/3$$