

## 1 Definitions

$P$  is the set of program variables used in rust program. Common names  $a, b, c$

$L$  is the set of logic variables used in refinement types. Common names:  $\alpha, \beta$

$\Gamma = (\mu, \sigma)$  is a tuple containing a function  $\mu : P \rightarrow L$  mapping all program variables to their (current) logic variable and a set of formulas  $\sigma$  over  $L$ . During execution of statements, the set increases monotonically

$\tau$  is a user defined type  $\{\alpha : b \mid \varphi\}$ . Where  $\alpha$  is a logic variable from  $L$ ,  $b$  is a base type from Rust (like `i32`) and  $\varphi$  is a formula over variables in  $L$ .

## 2 Typing Rules

**Abbreviations** We write:

- $\Gamma, c$  for  $(\mu, \sigma \wedge c)$
- $\Gamma[a \mapsto \alpha]$  for  $(\mu[a \mapsto \alpha], \sigma)$

### 2.1 Expression Typing: $\Gamma \vdash e : \tau$

$$\begin{array}{c}
\text{LIT} \frac{l \text{ fresh} \quad \text{base\_ty}(v) = b}{\Gamma \vdash v : \{l : b \mid l \doteq v\} \Rightarrow \Gamma} \\
\\
\text{VAR} \frac{\beta \text{ fresh} \quad \{\beta : b \mid \beta \doteq \alpha\}}{\Gamma \vdash x : \{\alpha : b \mid \varphi\} \Rightarrow \Gamma} \\
\\
\text{VAR-REF} \frac{\beta \text{ TODO} \quad \{\beta : b \mid \beta \doteq \alpha\}}{\Gamma \vdash x : \{\alpha : b \mid \varphi\} \Rightarrow \Gamma} \\
\\
\text{IF} \frac{\Gamma \vdash c : \text{bool} \Rightarrow \Gamma_c \quad \Gamma_c, c \vdash c_t : \tau \Rightarrow \Gamma_t \quad \Gamma_c, \neg c \vdash c_e : \tau \Rightarrow \Gamma_t}{\Gamma \vdash \text{if } c \text{ then } c_t \text{ else } c_e : \tau \Rightarrow \Gamma_t} \\
\\
\text{WHILE} \frac{\Gamma_I, c \vdash s \Rightarrow \Gamma'_I \quad \Gamma'_I \preceq \Gamma_I}{\Gamma_I \vdash \text{while}(c)s \Rightarrow \Gamma_I, \neg c} \\
\\
\text{SEQ} \frac{\Gamma \vdash s_1 : \tau_1 \Rightarrow \Gamma' \quad \Gamma' \vdash \bar{s} : \tau \Rightarrow \Gamma''}{\Gamma \vdash s_1; \bar{s} : \tau \Rightarrow \Gamma''} \\
\\
\text{ADD} \frac{\Gamma \vdash e_1 : \{v_1 : b \mid \varphi_1\} \Rightarrow \Gamma' \quad \Gamma' \vdash e_2 : \{v_2 : b \mid \varphi_2\} \Rightarrow \Gamma''}{\Gamma \vdash e_1 + e_2 : \{v : b \mid v \doteq [e_1] + [e_2]\} \Rightarrow \Gamma'', \varphi_1, \varphi_2}
\end{array}$$

$$\begin{array}{c}
\text{ASSIGN} \frac{\Gamma \vdash e : \{\beta \mid \varphi\} \Rightarrow \Gamma'}{\Gamma \vdash x = e : \tau \Rightarrow \Gamma'[x \mapsto \beta], \varphi} \\
\\
\text{ASSIGN-STRONG} \frac{\Gamma \vdash e : \{\beta \mid \varphi\} \Rightarrow \Gamma' \quad \Gamma \vdash x : \{\alpha : \&b \mid \alpha \doteq \&y\}}{\Gamma \vdash *x = e : \tau \Rightarrow \Gamma'[y \mapsto \beta], \varphi} \\
\\
\text{ASSIGN-WEAK} \frac{\Gamma \vdash e : \tau \Rightarrow \Gamma' \quad \Gamma \vdash x : \{\alpha : \&b \mid \alpha \doteq \&y \vee \alpha \doteq \&z \vee \dots\} \quad \Gamma \vdash \tau \preceq \tau_y \quad \Gamma \vdash y : \tau_y}{\Gamma \vdash *x = e : \tau \Rightarrow \Gamma'} \\
\\
\text{FN-CALL} \frac{(\{a \mapsto \tau_a, \dots\}, \{\alpha \doteq \mu(a), \dots, \varphi_\alpha, \dots\}) \preceq \Gamma \quad f : (\{\alpha \mid \varphi_\alpha\} \Rightarrow \{\alpha' \mid \varphi'_\alpha\}, \dots)}{\Gamma \vdash f(a, \dots) \Rightarrow (\mu[a \mapsto \alpha', \dots], \sigma \wedge \varphi'_\alpha \wedge \dots)} \\
\\
\text{INTRO-SUB} \frac{\Gamma \vdash e : \tau \quad \Gamma \vdash \tau \preceq \tau'}{\Gamma \vdash e \text{ as } \tau' : \tau'}
\end{array}$$

## 2.2 Sub-Typing Rules: $\Gamma \vdash \tau \preceq \tau'$

$$\preceq\text{-TY} \frac{\sigma \wedge \varphi'[\beta \triangleright \alpha] \models \varphi}{\Gamma = (\mu, \sigma) \vdash \{\alpha \mid \varphi\} \preceq \{\beta \mid \varphi'\}}$$

alternative (should be equivalent):

$$\preceq\text{-TY-ALT} \frac{\Gamma[f \mapsto \alpha], \varphi \preceq \Gamma[f \mapsto \beta], \varphi' \quad f \text{ fresh}}{\Gamma \vdash \{\alpha \mid \varphi\} \preceq \{\beta \mid \varphi'\}}$$

## 2.3 Sub-Context Rules: $\Gamma \preceq \Gamma'$

$$\preceq\text{-CTX} \frac{\sigma'[\mu(\alpha) \triangleright \mu'(\alpha) \mid \alpha \in \text{dom}(\mu)] \models \sigma \quad \text{dom}(\mu) \subseteq \text{dom}(\mu')}{(\sigma, \mu) \preceq (\sigma', \mu')}$$