DSP HW3

r10922129 劉旭庭

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$$H_{hp}(w) = \begin{cases} 1 & ; \text{if} \quad w_c < |w| < \pi \\ 0 & ; \text{otherwise} \end{cases}$$

$$h_{hp}[n] = \frac{1}{2\pi}\int_{-\pi}^{-w_c} e^{jwn}\,dw + \frac{1}{2\pi}\int_{w_c}^{\pi} e^{jwn}\,w$$

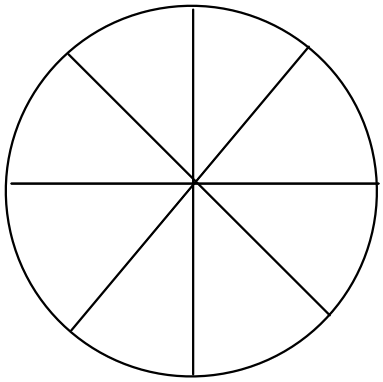
$$= \frac{1}{2\pi jn} e^{jwn}\Big]_{-\pi}^{-w_c} + \frac{1}{2\pi jn} e^{jwn}\Big]_{w_c}^{\pi}$$

$$= \frac{1}{2\pi jn}\left(2j \sin(-w_c n)\right)$$

$$= - \frac{\sin(w_c n)}{\pi n} \quad \#$$

② (a)

given $(e^{-j2\pi})^N = 1$, $e^{-j2\pi\frac{n}{N}}$ will be the roots



when we sum all roots together,

the diagonal roots will against each other results in 0

thus $\sum_{n=0}^{N-1} e^{-j2\pi\frac{n}{N}} = 0$ #

(b)

since $DFT(x[n]) = F_n \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

$$X_k = \begin{cases} N, & \text{if } k=0 \\ 0, & \text{otherwise} \end{cases}$$ #

(sum of first row $= N$)

(sum of the non-first row $= 0$ by (a) since it's the sum of the roots)

③ $x[n] = [2, 0, 1, 0]$, $y[n] = [1, -1, 0, 0]$

(a) $g[n] = x[n] \circledast_4 y[n] = \sum_{m=0}^{3} x[m]\, y[((n-m))_4]$

$g[0] = \sum_{m=0}^{3} x[m]\, y[((-m))_4] = 2+0+0+0 = 2$

$g[1] = \sum_{m=0}^{3} x[m]\, y[((1-m))_4] = -2+0+0+0 = -2$

$g[2] = \sum_{m=0}^{3} x[m]\, y[((2-m))_4] = 0+0+1+0 = 1$

$g[3] = \sum_{m=0}^{3} x[m]\, y[((3-m))_4] = 0+0-1+0 = -1$

$g[n] = [2, -2, 1, -1]$ #

(b)
$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-2\pi j \frac{1}{4}} & e^{-2\pi j \frac{2}{4}} & e^{-2\pi j \frac{3}{4}} \\ 1 & e^{-2\pi j \frac{2}{4}} & e^{-2\pi j \frac{4}{4}} & e^{-2\pi j \frac{6}{4}} \\ 1 & e^{-2\pi j \frac{3}{4}} & e^{-2\pi j \frac{6}{4}} & e^{-2\pi j \frac{9}{4}} \end{bmatrix}$$

$$F_4 \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix} \quad , \quad F_4 \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+j \\ 2 \\ 1-j \end{bmatrix}$$

$x[k] = [3, 1, 3, 1]$

$y[k] = [0, 1+j, 2, 1-j]$ #

(c)

$$Z[k] = [3\times 0,\ 1\times(1+j),\ 3\times 2,\ 1\times(1-j)]$$

$$= [0,\ 1+j,\ 6,\ 1-j] \ \#$$

(d)

$$F_4^{-1} = \frac{1}{4} F_4^{*} \qquad \frac{1}{4} F_4^{*} \cdot \begin{bmatrix} 0 \\ 1+j \\ 6 \\ 1-j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ -8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \\ -1 \end{bmatrix} \ \#$$

④

$$V[k] = \sum_{n=0}^{2N-1} v[n] e^{-j\frac{2\pi k n}{2N}} = \sum_{n=0}^{2N-1} v[n] e^{-j\frac{\pi k n}{N}}$$

$$= \sum_{n=0}^{N-1} \left(v[2n] e^{-j\frac{\pi k(2n)}{N}} + v[2n+1] e^{-j\frac{\pi k(2n+1)}{N}} \right)$$

$$= \sum_{n=0}^{N-1} \left(v[2n] e^{-j\frac{\pi k(2n)}{N}} + e^{-j\frac{\pi k}{N}} \cdot v[2n+1] e^{-j\frac{\pi k(2n)}{N}} \right)$$

$$= \sum_{n=0}^{N-1} v[2n] e^{-j\frac{\pi k(2n)}{N}} + e^{-j\frac{\pi k}{N}} \cdot \sum_{n=0}^{N-1} v[2n+1] e^{-j\frac{\pi k(2n)}{N}}$$

$$= G[k \bmod N] + e^{-j\frac{\pi k}{N}} H[k \bmod N]$$

$$= G[k \bmod N] + f[k] H[k \bmod N]$$

$$\Rightarrow \quad f[k] = e^{-j\frac{\pi k}{N}} \quad \#$$