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① 設有 n 個 sinusoids，角頻率相同為 ω

分別表示為 $A_i \cos(\omega t + \phi_i)$

Let $A_i \cos(\phi_i) = A_i'$ $\quad -A_i \sin(\phi_i) = A_i''$

$$\sum_{i=1}^{n} A_i \cos(\omega t + \phi_i) = \sum_{i=1}^{n} A_i \cos(\omega t)\cos(\phi_i) - A_i \sin(\omega t)\sin(\phi_i)$$

$$= \sum_{i=1}^{n} A_i' \cos(\omega t) + A_i'' \sin(\omega t) = \underbrace{\left[\sum_{i=1}^{n} A_i'\right]}_{B'} \cos(\omega t) + \underbrace{\left[\sum_{i=1}^{n} A_i''\right]}_{B''} \sin(\omega t)$$

Let $C \cos(\phi_c) = B'$ $\quad -C \sin(\phi_c) = B''$

$$C = \sqrt{(B')^2 + (B'')^2} \qquad \phi_c = \tan^{-1}\left(\frac{-B''}{B'}\right)$$

$$C \cos(\phi_c) \cos(\omega t) - C \sin(\omega t) \sin(\phi_c)$$

$$= C \cos(\omega t + \phi_c)$$ still sinusoid

② $$\frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j0t}\, dt = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt$$

$$= \frac{1}{T_0}\left[\int_0^{T_0/2} 50t\, dt + \int_{T_0/2}^{T_0} 2-50t\, dt\right]$$

$$= \frac{1}{0.04}\left[\left(25t^2\right]_0^{0.02}\right) + \left(2t-25t^2\right]_{0.02}^{0.04}\left.\right)\right]$$

$$= \frac{1}{0.04}\left[0.01 + 0.01\right] = \frac{1}{2}$$

DC compnent $= \frac{1}{2}$

③ $\int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$

$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt = -\frac{1}{a+j\omega} e^{-(a+j\omega)t}\Big]_{0}^{\infty}$

$= -\frac{1}{a+j\omega}(0-1) = \frac{1}{a+j\omega}$

✓

(4) $$\int_{-\infty}^{\infty} e^{\left(-\frac{t^2}{2\sigma^2}\right)} \cdot e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}\left[t^2 + 2\sigma^2 j\omega t\right]}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}\left[t^2 + 2\sigma^2 j w t + (\sigma^2 j w)^2 - (\sigma^2 j w)^2\right]}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(t^2 + \sigma^2 j w)^2} \cdot e^{\frac{1}{2\sigma^2}(\sigma^2 j w)^2}\, dt$$

$$= \sqrt{2\pi\sigma^2} \cdot e^{\frac{\sigma^4 j^2 w^2}{2\sigma^2}} = \sqrt{2\pi\sigma^2} \cdot e^{-\frac{\sigma^2 w^2}{2}}$$

still gaussian

⑤

$$x(t) = [5 + 4\cos(40\pi t)]\cos(2\pi \cdot 700 \cdot t)$$

$$= 5\cos(1400\pi t) + 4[\cos(40\pi t) \cdot \cos(1400\pi t)]$$

$$= 5\cos(1400\pi t) + 2\cos(1360\pi t) + 2\cos(1440\pi t)$$

$$= \frac{5}{2}e^{j1400\pi t} + \frac{5}{2}e^{-j1400\pi t} + e^{j1360\pi t} + e^{-j1360\pi t} + e^{j1440\pi t} + e^{-j1440\pi t}$$