

(100)

HW1

R10922129

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① 設有  $n$  個 sinusoids, 角頻率相同為  $\omega$

分別表示為  $A_i \cos(\omega t + \phi_i)$

$$\text{Let } A_i \cos(\phi_i) = A_i' \quad -A_i \sin(\phi_i) = A_i''$$

$$\sum_{i=1}^n A_i \cos(\omega t + \phi_i) = \sum_{i=1}^n A_i \cos(\omega t) \cos(\phi_i) - A_i \sin(\omega t) \sin(\phi_i)$$

$$= \sum_{i=1}^n A_i' \cos(\omega t) + A_i'' \sin(\omega t) = \underbrace{\left[ \sum_{i=1}^n A_i' \right]}_{B'} \cos(\omega t) + \underbrace{\left[ \sum_{i=1}^n A_i'' \right]}_{B''} \sin(\omega t)$$

$$\text{Let } C \cos(\phi_c) = B' \quad -C \sin(\phi_c) = B''$$

$$C = \sqrt{(B')^2 + (B'')^2} \quad \phi_c = \tan^{-1}\left(\frac{-B''}{B'}\right)$$

$$C \cos(\phi_c) \cos(\omega t) - C \sin(\omega t) \sin(\phi_c)$$

$$= C \cos(\omega t + \phi_c) \quad \text{still sinusoid}$$

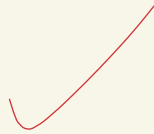
$$\textcircled{2} \quad \frac{1}{T_0} \int_0^{T_0} x(t) e^{j0t} dt = \frac{1}{T_0} \int_0^{T_0} x(t) dt$$

$$= \frac{1}{T_0} \left[ \int_0^{T_0/2} 50t dt + \int_{T_0/2}^{T_0} 2 - 50t dt \right]$$

$$= \frac{1}{0.04} \left[ \left( 25t^2 \right)_0^{0.02} + \left( 2t - 25t^2 \right)_{0.02}^{0.04} \right]$$

$$= \frac{1}{0.04} [0.01 + 0.01] = \frac{1}{2}$$

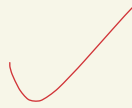
$$\text{DC component} = \frac{1}{2}$$



$$\textcircled{3} \quad \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt = -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty}$$

$$= -\frac{1}{a+j\omega} (0 - 1) = \frac{1}{a+j\omega}$$



$$\textcircled{4} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}} \cdot e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2} [t^2 + 2\sigma^2 j\omega t]} dt$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2} [t^2 + 2\sigma^2 j\omega t + (\sigma^2 j\omega)^2 - (\sigma^2 j\omega)^2]} dt$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2} (t^2 + 2\sigma^2 j\omega t)} \cdot e^{\frac{1}{2\sigma^2} (\sigma^2 j\omega)^2} dt$$

$$= \sqrt{2\pi\sigma^2} \cdot e^{\frac{\sigma^4 j^2 \omega^2}{2\sigma^2}} = \sqrt{2\pi\sigma^2} \cdot e^{-\frac{\sigma^2 \omega^2}{2}}$$

still gaussian

5)

$$x(t) = [5 + 4 \cos(40\pi t)] \cos(2\pi \cdot 700 \cdot t)$$

$$= 5 \cos(1400\pi t) + 4 [\cos(40\pi t) \cdot \cos(1400\pi t)]$$

$$= 5 \cos(1400\pi t) + 2 \cos(1360\pi t) + 2 \cos(1440\pi t)$$

$$= \frac{5}{2} e^{j1400\pi t} + \frac{5}{2} e^{-j1400\pi t} + e^{j1360\pi t} + e^{-j1360\pi t} + e^{j1440\pi t} + e^{-j1440\pi t}$$

