Hidden Markov Model

By

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Contents

PROJECT INSTRUCTIONS

1. The dataset `hmm_pb1.csv` represents a sequence of dice rolls x from the Dishonest casino model. The states of Y are 1='Fair' and 2='Loaded'.
   a) Implement the Viterbi algorithm and find the most likely sequence y that generated the observed x. Use the log probabilities, as shown in the HMM slides. Report the obtained sequence y of 1's and 2's for verification.
   b) Implement the forward and backward algorithms and run them on the observed x. You should memorize a common factor u_t for the α_t^k to avoid floating point underflow, since α_t^k quickly become very small. The same holds for β_k^t. Report $\frac{\alpha_{133}^1}{\alpha_{133}^2}$ and $\frac{\beta_{133}^1}{\beta_{133}^2}$, where the counting starts from $t = 0$.

2. The dataset `hmm_pb2.csv` represents a sequence of 10000 dice rolls x from the Dishonest casino model but with other values for the a and b parameters. Having so many observations, you are going to learn the model parameters. Implement and run the Baum-Welch algorithm using the forward and backward algorithms that you already implemented. You can initialize the π, a, b with your guess, or with some random probabilities (make sure that π sums to 1 and that a_i^j , b_i^k sum to 1 for each i). The algorithm converges quite slowly, so you might need to run it for up 1000 iterations or more for the parameters to converge. Report the values of π, a, b that you have obtained.

PART 1.A

y = [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2
,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,
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,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,
2,2]

PART 1.B

$$\frac{\alpha_{133}^{1}}{\alpha_{133}^{2}} = \frac{0.84}{0.16} = 5.26$$

$$\frac{\beta_{133}^{1}}{\beta_{133}^{2}} = \frac{0.223}{0.777} = 0.287$$

PART 2

$$\pi = [0.001 \quad .999]$$

$$a = \begin{bmatrix} 0.625 & 0.375 \\ 0.012 & 0.988 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.079 & 0.11 & 0.062 & 0.039 & 0.625 & 0.084 \\ 0.201 & 0.205 & 0.193 & 0.201 & 0.107 & 0.094 \end{bmatrix}$$

REFERENCES

1. https://github.com/adeveloperdiary/HiddenMarkovModel/tree/master/part4
2. https://www.youtube.com/watch?v=6JVqutwtzmo
3. https://numpy.org/doc/stable/reference/generated/numpy.dot.html
4. https://stackoverflow.com/questions/8437964/python-printing-horizontally-rather-than-current-default-printing