I. EDGE-CLUSTERING COEFFICIENTS

Let $G = (V, E)$ be an undirected, unweighted graph. $N(u) = \{w \in V : (u, w) \in E\}$.

Definition 1 (Local clustering coefficient of an edge)**.** *Let $(u, v) \in E$. Then*

$$c(u, v) = \frac{|N(u) \cap N(v)|}{|N(u) \cup N(v) \backslash \{u, v\}|}.$$

This definition is similar to the one in [], which replaces the denominator above with $\min\{|N(u)| - 1, |N(v)| - 1\}$.

A. Longer cycles

II. EDGE-CLUSTERING IN MULTIPLEX NETWORKS

Let $\mathscr{G} = (G_\alpha = (V_\alpha, E_\alpha))_{\alpha \in \Lambda}$ be a multiplex. A vertex in the multiplex can be specified by a node-layer pair (u, α), where $u \in V_\alpha$ and $\alpha \in \Lambda$. Consider an edge in layer γ, specified by $((u, v), \gamma)$, where $u, v \in V_\gamma$. We consider triangles that include $((u, v), \gamma)$ and that have one edge in layer α and one in layer β, for any $\alpha, \beta \in \Lambda$. These triangles lead to a local clustering coefficient for $((u, v), \gamma)$ with respect to the pair of layers (α, β).

For a node $v \in V_\lambda$, let $N_\lambda(v)$ be the set of neighbors of (v, λ) in layer λ, for any $\lambda \in \Lambda$. That is,

$$N_\lambda(v) = \{w : (v, w) \in E_\lambda\}.$$

Definition 2 ((α, β)-local edge clustering coefficient)**.** *Let $\gamma \in \Lambda$, and let $e = (u, v) \in E_\gamma$. Then,*

$$c_{\alpha,\beta}(e, \gamma) = c_{\alpha,\beta}((u, v), \gamma) = \frac{|N_\alpha(u) \cap N_\beta(v)|}{|N_\alpha(u) \cup N_\beta(v) \backslash \{u, v\}|}.$$

Definition 2 is illustrated in Fig. 1. Notice that, in general, $c_{\alpha,\beta}(e) \neq c_{\beta,\alpha}(e)$. If $e = ((u, v), \gamma)$, and $\alpha = \beta = \gamma$, then $c_{\gamma,\gamma}(e)$ reduces to the monoplex edge-clustering coefficient of e in layer γ. To obtain an aggregate measure over all layers for local clustering of edge e, we propose the following method. Let weight $W = (w_{\alpha\beta\gamma} \in [0, 1])_{\alpha,\beta,\gamma \in \Lambda}$. W quantifies the impact of layers α, β on layer γ. Then, we define

$$c(e, \gamma) = \sum_{(\alpha,\beta) \in \Lambda^2} w_{\alpha\beta\gamma} c_{\alpha,\beta}(e),$$

and finally the local edge clustering coefficient of e:

$$c(e) = \frac{1}{|\{\gamma : e \in E_\gamma\}|} \sum_{\gamma : e \in E_\gamma} c(e, \gamma).$$

III. EDGE-CLUSTERING IN MULTILAYER NETWORKS

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