There are lots of "d-type" and "x-type" variables in this document, sigh!

The equation

$$\frac{\mathrm{d}}{\mathrm{d}t}X(x,y,t) = -dX(x,y,t) + B(x,y,t-\delta) + \nabla\left[D\nabla X(x,y,t) - AV(x,y,t)X(x,y,t)\right] \ , \tag{1}$$

is solved numerically using an operator split. Specifically, the equation describing the lifecycle:

$$\frac{\mathrm{d}}{\mathrm{d}t}X(x,y,t) = -dX(x,y,t) + B(x,y,t-\delta) \ , \tag{2}$$

is solved using a timestep of Δt, and then the equation describing the spatial dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}X(x,y,t) = \nabla\left[D\nabla X(x,y,t) - V(x,y,t)X(x,y,t)\right] \ , \tag{3}$$

is solved using the same timestep Δt.

The lifecycle equation is solved using

$$X(x,y,t+\Delta t) = \frac{B(x,y,t-\delta)}{d} + \left(X(x,y,t) - \frac{B(x,y,t-\delta)}{d}\right)e^{-d\Delta t} \ . \tag{4}$$

For small Δt, $e^{-d\Delta t} \approx 1 - d\Delta t$, so this reduces to $X(t+\Delta t) = X(t) + \Delta t(-dX(t) + B(t-\delta))$, which is the forward Euler form. For large Δt, this reduces to $X(t+\Delta t) = B(t-\delta)/d$, which is the long-time equilibrium solution of the lifecycle equation. Equation (4) may be motivated by considering the variable $Y(t) = X(t) - B(t-\delta)/d$, which would satisfy the equation $\mathrm{d}Y/\mathrm{d}t = -dY$ if B were time independent. This equation has analytic solution $Y(t+\Delta t) = Y(t)e^{-d\Delta t}$, which is exactly Eqn (4). Of course, B is time dependent, so Eqn (4) is not an exact solution. Nevertheless, it has superior numerical properties compared with the forward Euler form.

If desired, Eqn (4) could be used with multiple small timesteps, Δt_i, such that $\sum_i \Delta t_i = \Delta t$. This would increase the accuracy of the final solution.

The spatial dynamics is solved using a finite-difference spatial discretisation with a 5-point stencil for the Laplacian, a fully-upwind advection approach, and a forward Euler temporal discretisation:

$$\begin{aligned} X(x,y,t+\Delta t) &= X(x,y,t) + \frac{\Delta t}{\Delta x \Delta y}\left[X(x-\Delta x,y,t) + X(x+\Delta x,y,t) + X(x,y+\Delta y,t) + X(x,y-\Delta y,t)\right. \\ &\quad \left.-4X(x,y,t)\right] - AX(x,y,t) + A\sum_{\tilde{x},\tilde{y}} X(\tilde{x},\tilde{y},t) \ . \end{aligned} \tag{5}$$

Here $\Delta x = \Delta y$ is the cell size, and $(\tilde{x},\tilde{y})$ are all the points from which advected mosquitoes originated from in this timestep:

$$\tilde{x} + \Delta t V_x(\tilde{x},\tilde{y},t) = x \quad \text{and} \quad \tilde{y} + \Delta t V_y(\tilde{x},\tilde{y},t) = y \ . \tag{6}$$

Given (x,y) there could be many $(\tilde{x},\tilde{y})$, for instance, if all wind vectors pointed towards one point, hence the $\sum_{\tilde{x},\tilde{y}}$ in Eqn (5). Since the spatial equations are solved on a grid, the equalities in Eqn (6) are rounded to the nearest grid point.