An example of hybridising mosquito evolution in Africa

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January 2, 2020

1 Introduction

I describe a simple simulation using ODE dynamics defined by Beeton et al.[1], mainly as an example of the type of simulation that the code can perform.

2 Cell dynamics

The `CellDynamicsBeeton2_2` class is used. This solves the coupled equations

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \left[-\mu_X + \left(1 - \frac{X + \alpha_{XY}Y}{K_X}\right)\frac{X}{X + wY}\gamma_X\right]X \ , \tag{1}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \left[-\mu_Y + \left(1 - \frac{\alpha_{YX}X + Y}{K_Y}\right)\left(\gamma_Y + \frac{wX}{X + wY}\gamma_X\right)\right]Y \ , \tag{2}$$

for populations $X(t)$ and $Y(t)$. Beeton et al. describe various parameter scenarios, and here we explore the choice given in their Figure5(c). That is:

- $\mu_X = 0.7\,\mathrm{day}^{-1}$;
- $\mu_Y = 0.8\,\mathrm{day}^{-1}$;
- $\gamma_X = 1.0\,\mathrm{day}^{-1}$;
- $\gamma_Y = 1.0\,\mathrm{day}^{-1}$;
- $\alpha_{XY} = \alpha_{YX} = 0.4$;
- $w = 0.05$.

The carrying capacities, K_X and K_Y vary spatially and temporally, as described below.

In the spatially-dependent simulations, both X and Y are assumed to diffuse and advect. The ODE is solved for only occurs for 0.5 days out of each day.

[1] Beeton, Hosack, Wilkins, Forbes, Ickowicz and Hayes, Journal of Theoretical Biology 2019

3 Simulation of the ODE only

Beeton et al. describe a time-varying scnario where the carrying capacity $K_X = K_Y$ oscillates between 1 and 0.2 in a stepwise fashion, as in Figure 1

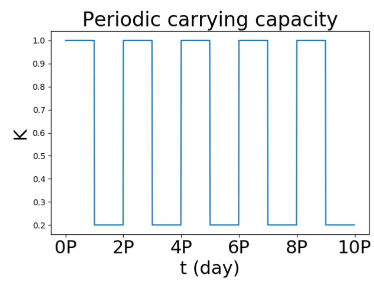

Figure 1: The periodic carrying capacity, depending on period P

They demonstrate that the evolution of X and Y can depend on the period of the oscillation. This may be simulated using the current code (see the "runner" script `ode_only.py`) and results in Figure 2 that replicates Beeton et al.'s result

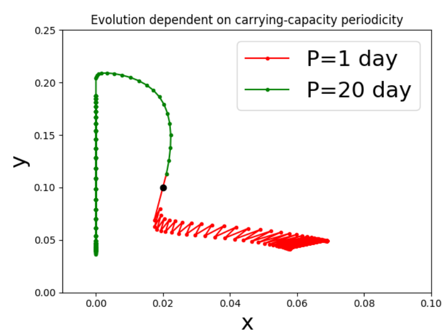

Figure 2: Replicating the scenario studied in Beeton et al., with oscillating carrying capacity. The black dot shows the initial condition. The dynamics depends on the periodicity, P (measured in days) of the oscillation. (Each dot in this diagram corresponds to the result after 1 day of simulation.)

4 The spatial domain

The spatial domain is defined by a $5\,\mathrm{km}\times 5\,\mathrm{km}$ grid with lower-left corner at $(x, y) = (-4614, -3967)$ (km) and $n_x = 1517$ and $n_y = 1667$, that is

```
Grid(-4614.0, -3967.0, 5.0, 1517, 1667, False)
```

The inactive cells are defined to be those in the ocean, while everything else is active. This results in 1.4×10^6 cells, as shown in Figure 3.

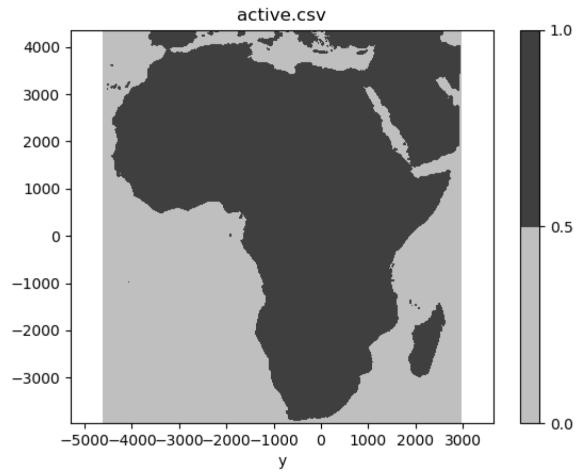

Figure 3: The spatial domain. Active cells are dark-coloured.

5 Carrying capacity

Carrying capacity is based on the spatially varying function

$$C(x, y) = \text{spatially-varying carrying capacity of Figure 4 .} \tag{3}$$

A time-dependent carrying capacity is used in some simulations, described below.

6 Diffusion

Diffusion is assumed to have a constant, uniform diffusion coefficient of $0.1\,\mathrm{km}^2.\mathrm{day}^{-1}$.

7 Advection by wind

Just one wind file is used, so wind is spatially-varying but temporally constant throughout the simulation. It is assumed that 1% of the population, P, of each cell enters the wind steam every day.

The probability distribution for mosquitoes to "drop out" of the wind stream is assumed to be exponential:

$$p(t) \propto \begin{cases} e^{-6t} & \text{for } t \leq 0.5 \\ 0 & \text{for } t > 0.5 \end{cases} \tag{4}$$

where t is measured in days. This gives $p(0) \approx 15\%$ and $p(0.5) \approx 0.7\%$. Hence, regardless of the time-step size, mosquitoes advect for up to 0.5 days every time step.

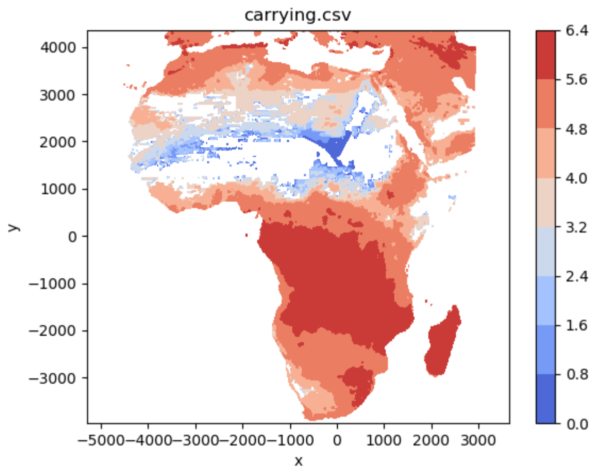

Figure 4: $\log_{10} C$ (mosquitoes per grid cell).

8 Initial conditions

It is assumed that initially

$$Y(x, y, t = 0) = C(x, y) \ , \tag{5}$$

and that $X(t = 0) = 0$, except for at cell $(x, y) = (-2000, 700)$, which is in the south of Nigera. At this cell 10,000 mosquitoes are introduced. This is shown in Figure 5.

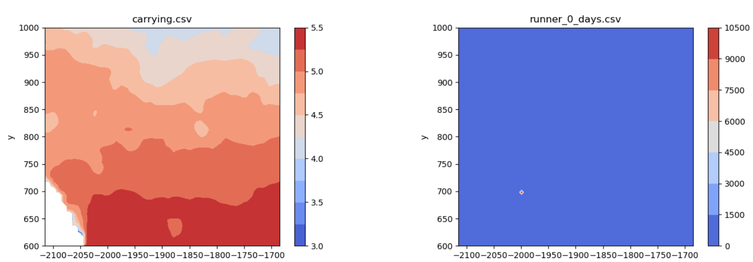

Figure 5: Zoomed views into the region of interest. Left: $\log_{10} C$. Right: the initial population of X.

9 Diffusion and advection only

(Using `runner.py`) When the ODE dynamics is not included in the simulation, the 10,000 “X” mosquitoes simply diffuse and advect. The result after 200 days is shown in Figure 6

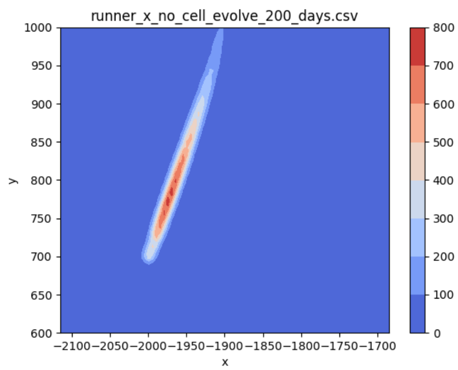

Figure 6: The population of “X” after 200 days of diffusion and advection without any ODE dynamics.

10 Invasion of X

(Using `runner.py`) When

$$K_X = C \quad \text{and} \quad K_Y = C/5 \ , \tag{6}$$

the ODE dynamics evolves towards an X-dominant steady state. Coupling this with advection and diffusion, the results after 200 days is shown in Figure 7.

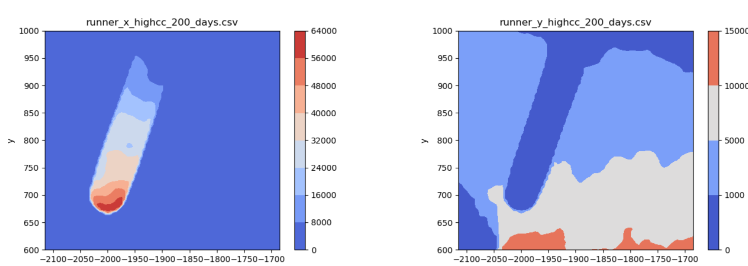

Figure 7: The population of X (left) and Y (right) after 200 days of evolution, diffusion and advection when the carrying capacity of X is five times that of Y.

11 Time-varying carrying capacities

(Using `runner.py`) Here I study a spatially-varying analogue of Section 3. That is

$$K_X = K_Y = \beta(t)C(x, y) \ . \tag{7}$$

where $\beta(t)$ oscillates between 1 and 0.2 in a stepwise fashion with periodicity P (days): see Figure 1. The results after 200 days is shown in Figure 8.

Although the ODE dynamics of shown in Figure 2 suggests there might be a difference between $P = 1\,$day and $P = 20\,$days, there is very little difference, and $X = 0$ in cells neighbouring the release site. What happens is that any X that spreads to neighbouring cells gets immediately out-competed by the Y that is present there.

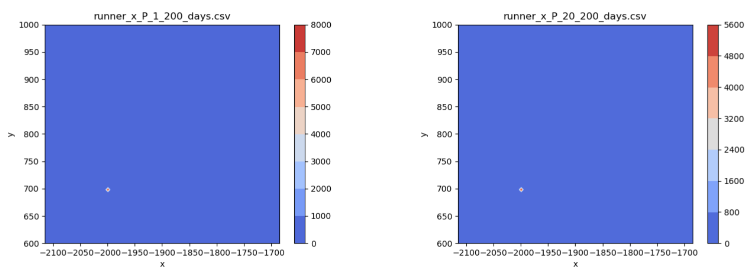

Figure 8: The population of X after 200 days of evolution, diffusion and advection, when the carrying capacity oscillates between C and $0.2C$ with periodicity P. Left: $P = 1\,$days. Right: $P = 20\,$days.

12 Time-varying K_X

(Using `runner.py`) Here I study a spatially-varying analogue of Section 3. That is

$$K_X = \beta(t)C(x, y) \quad \text{and} \quad K_Y = 0.2C(x, y) \ . \tag{8}$$

where $\beta(t)$ oscillates between 1 and 0.2 in a stepwise fashion with periodicity P (days). The results after 200 days is shown in Figure 9.

Evidently, there is a difference between $P = 1\,$day and $P = 20\,$days. Comparing with Figure 7 there is less lateral diffusion (perpendicular to the prevailing wind direction) as the X that spreads laterally to neighbouring cells gets out-competed by the Y that is present there.

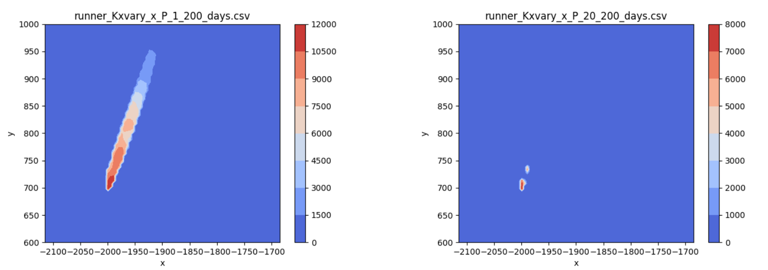

Figure 9: The population of X after 200 days of evolution, diffusion and advection, when the carrying capacity K_X oscillates between C and $0.2C$ with periodicity P, and $K_Y = 0.2C$. Left: $P = 1$ days. Right: $P = 20$ days.