1 Equations

The equations solved by `Mosquito23` are based on the following

$$
\begin{aligned}
\frac{\partial X(a,s,g,m)}{\partial t} &= -d_a X + b_{s,g,m}\left[\overbrace{X(a-1,s,g,m)}^{a>0} - \overbrace{X}^{a<N}\right] + \overbrace{\nabla\cdot\left(D_{s,g,m}\nabla X - \mathbf{V}_{s,g,m}X\right)}^{a=N} + \\
a = 0 \quad & \begin{cases} \left(\dfrac{K - \sum_{m^*}\alpha_{m,m^*}\sum_{s,g}\sum_{i<\max(N,1)} X(i,s,g,m^*)}{K}\right)\times \\ \displaystyle\sum_{\substack{g_M,g_F,\\ m_M,m_F}} h(m_M,m_F,m)\ \lambda(g_F,m_F)\ i(g_M,g_F,g)\ \frac{w(m_M,m_F)X(N,M,g_M,m_M)}{\sum_{g^*,m^*} w(m^*,m_F)X(N,M,g^*,m^*)}\times \\ p(g_M,g_F,m_M,m_F,s)X(N,F,g_F,m_F) \end{cases}
\end{aligned}
$$

where a = age (0 = newborn, N = adult, $[1, N-1]$ = intermediate stages),
s = sex (M or F),
g = genotype (ww wildtype, Gw heterozygous GM or GG homozygous GM),
m = mosquito species (C = *An. coluzzii*, G =*An. gambiae*)
and $X(a,s,g,m)$ is abbreviated to X.

2 Parameters

Parameter name	Explanation	Degrees of freedom
d_a	Death rate $\begin{cases} d_{\text{adult}} \text{ for } a = N \\ d_{\text{larvae}} \text{ for } a < N \end{cases}$	2
b	Larval emergence rate has distribution $\Gamma(k = N, \theta = b/N)$	1
D	Diffusion rate	1
$\mathbf{V}$	Advection vector field	Distribution of flight times
K	Carrying capacity for species (spatially explicit, dependent on rainfall)	1 (for now)
$h(m_M, m_F, m)$	Hybridisation rate (following Beeton et al. 2019) $= \begin{cases} 1 & \text{where } m = C \text{ and } \{m_M, m_F\} = \{C, C\} \\ 1 & \text{where } m = G \text{ and } \{m_M, m_F\} = \{G, G\} \\ 1 & \text{where } m = G \text{ and either } \{m_M, m_F\} = \{C, G\} \text{ or } \{G, C\} \\ 0 & \text{elsewhere} \end{cases}$	0
λ	Larvae per female	1
$i(g_M, g_F, g)$	Inheritance of genotype where $g = \{ww, Gw, GG\}$ $\begin{array}{c\|c\|c\|c} & ww & Gw & GG \\ \hline ww & \{1,0,0\} & \{\frac{1}{2},\frac{1}{2},0\} & \{0,1,0\} \\ Gw & \{\frac{1}{2},\frac{1}{2},0\} & \{\frac{1}{4},\frac{1}{2},\frac{1}{4}\} & \{0,\frac{1}{2},\frac{1}{2}\} \\ GG & \{0,1,0\} & \{0,\frac{1}{2},\frac{1}{2}\} & \{0,0,1\} \end{array}$	0
$p(g_M, g_F, m)$	Proportion of offspring of given sex given genotypes of parents $= \begin{cases} p(Gw, g_F, F) & = \frac{1}{a} - 1 \\ p(GG, g_F, F) & = \frac{1}{a} - 1 \\ 0.5 & \text{elsewhere} \end{cases}$	1 (accuracy a)
$w(m_M, m_F)$	Relative probability of female of species m_F mating with male of species m_M $= \begin{cases} w(C,C) = w(G,G) = 1 \\ w(C,G) = w(G,C) = w \end{cases}$	1 (w)

3 `Mosquito23`

`Mosquito23` simplifies the above equations, by assuming that the diffusion and advection is handled by other parts of the code, so that the above equations reduce to a system of ODEs. To interface with `Mosquito23`, you need to know the ordering of its vector X. For species m, genotype g, sex s and age a, this is

$$X\left(m + gN_{\text{species}} + sN_{\text{species}}N_{\text{genotypes}} + aN_{\text{species}}N_{\text{genotypes}}N_{\text{sexes}}\right)\ , \tag{1}$$

where

- N_{species} is the number of species,
- $N_{\text{genotypes}} = 3$ is the number of genotypes (with ordering $ww = 0$, $Gw = 1$, and $GG = 1$)
- $N_{\text{sexes}} = 2$ is the number of sexes (with order male$= 0$ and female$= 1$)
- N_{ages} (denoted by N in the above sections) is the number of ages (with ordering newborn$= 0$ and adult$=N_{\text{ages}} - 1$, with intermediate stages in between these).

3.1 Adults

All adult populations are goverend by

$$\frac{\mathrm{d}X(N-1, s, g, m)}{\mathrm{d}t} = -d_{\text{adult}}X(N-1, s, g, m) + bX(N-2, s, g, m) \ . \tag{2}$$

Here the $N - 1$ indicates the adult age bracket, and the $N - 2$ is the eldest juvenile age bracket. If $N = 1$ there are no adults, and all populations are governed by the "Newborn larvae" equations, below.

3.2 Intermediate juveniles

For $0 < a < N - 1$, the populations are governed by

$$\frac{\mathrm{d}X(a, s, g, m)}{\mathrm{d}t} = -d_{\text{larvae}}X(a, s, g, m) + b\left(X(a-1, s, g, m) - X(a, s, g, m)\right) \ . \tag{3}$$

For $N \leq 2$ there are no such intermediate juveniles.

3.3 Newborn larvae

For $a = 0$, the populations are govened by

$$\frac{\mathrm{d}X(0, s, g, m)}{\mathrm{d}t} = -d_{\text{larvae}}X(0, s, g, m) - bX(a, s, g, m) + B(s, g, m) \ , \tag{4}$$

where

$$B(s, g, m) = \left(1 - \frac{C(m)}{K}\right) \ldots \tag{5}$$

The following terms have been defined in this equation

- $C(m)$ is the competition that a newborn feels from the rest of the larval populations. It is

$$C(m) = \sum_{a=0}^{N-2} \sum_{s=0}^{N_{\text{sexes}}-1} \sum_{g=0}^{N_{\text{genotypes}}-1} \sum_{m'=0}^{N_{\text{species}}-1} \alpha_{m,m'} X(a, s, g, m') \ . \tag{6}$$

  Notice that this does not include adults $a = N - 1$. If $N = 1$, it is assumed that the carrying-capacity still applies, and the sum over a runs from 0 to 0. The Lotka-Voltera matrix α accounts for inter-specific competition. It defaults to $a = I$, that is, newborns only feel competition from their own species.